Targeting Nominal GDP or Prices: Expectation Dynamics and the Interest Rate Lower Bound*

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Abstract

We examine global dynamics under infinite-horizon learning in New Keynesian models where monetary policy practices either price-level or nominal GDP targeting. These regimes are compared to inflation targeting. All of the interest-rate rules are subject to the zero lower bound. The targeted steady state is locally but not globally stable under adaptive learning. Robustness properties of the three rules in learning adjustment are compared using criteria for volatility of inflation and output, sensitivity to the speed of learning parameter and, most importantly, domain of attraction of the targeted steady state. Nominal income targeting with transparency is on the whole the best policy regime, and inflation targeting is fairly close second. Price-level targeting is the least robust regime according to these criteria.

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1 Introduction

The practical significance of the zero lower bound (ZLB) for policy interest rates has become evident in the US and Europe since the 2007-9 financial crisis and earlier in Japan since the mid 1990s. In monetary economics the Japanese experience initiated renewed interest in ways of getting out or avoiding the ZLB constraint.[1] The long-standing Japanese experience of persistently very low inflation and occasional deflation took place nearly concurrently with the rise of inflation targeting as the most common, "best practice" monetary policy since the early 1990's. The ongoing economic crisis in most advanced market economies and the already lengthy period of very low interest rates has recently led to suggestions that price-level or nominal GDP targeting can potentially be more appropriate frameworks for the conduct of monetary policy.

History-dependence is a key feature of nominal income and price-level targeting as it can provide more guidance to the economy than inflation targeting.[2] This guidance can be helpful and is arguably also good policy in a liquidity trap where the ZLB is a constraint on monetary policy. See Eggertsson and Woodford (2003) for a discussion of price-level targeting. Carney (2012) and Evans (2012) provide broad discussions of the need for additional guidance for the price level and possibly other variables including nominal income or GDP. Carney (2012) suggests that with policy rates at ZLB "there could be a more favorable case for nominal GDP targeting".

Theoretical analysis of the ZLB as a constraint on interest rate policy has recently been pursued in an inflation targeting framework that employs a Taylor-type interest rate rule. It has been shown that there necessarily exit multiple equilibria because the rule must respect the ZLB. Reifschneider and Williams (2000) pointed out the existence of two steady states and discussed other implications of ZLB using a backward-looking model with a piecewise linear Taylor rule. Benhabib, Schmitt-Grohe, and Uribe (2001) showed that in a New Keynesian (NK) model with a continuous Taylor rule there are two steady states, the targeted steady state π^* as well as a low-inflation (or even deflationary) steady state π_L. In addition, there can be a continuum

[1] For prominent early analyzes, see for example Krugman (1998), Eggertsson and Woodford (2003), and Svensson (2003).

[2] Price level targeting has received a fair amount of attention, see for example Svensson (1999), Vestin (2006). Nominal income targeting has occasionally been considered, for example see Hall and Mankiw (1994) and Jensen (2002) and the cited references.

of perfect foresight paths, starting from an initial $\pi < \pi^*$, which converge asymptotically to π_L. Using perfect foresight analysis, the multiple equilibria issue was studied further in Benhabib, Schmitt-Grohe, and Uribe (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002).

It is important to observe that the possibility of multiple steady states has broader applicability than inflation targeting and Taylor rules. In a standard NK model the Fisher equation $R = \beta^{-1}\pi$, where R is the gross interest rate, β is the subjective discount factor and π is the gross inflation rate, is a key equation for a nonstochastic steady state. Usually the interest rate rule has a specified target inflation rate $\pi^* \geq 1$ (and an associated output level) as a steady state. It is also possible that at a sufficiently low (or even less than unity) inflation rate $\pi \approx \beta$, together with the interest rate is very close to or at one, and then the Fisher equation again holds and a second steady state may thus exist. For example, if the rule specifies $R = 1$ for π in the interval containing the value $\pi_L = \beta$, then π_L, $R = 1 = \beta^{-1}\beta$ is a second steady state. This argument does not require that a Taylor rule describes the policy regime.[3] As will be seen below, the existence of multiple steady states also applies to standard versions of nominal GDP and price-level targeting.

The arguments in favor of nominal GDP or price-level targeting and the cited studies about multiple equilibrium under a Taylor rule are both firmly based on the rational expectations (RE) hypothesis. This standard approach makes strong assumptions about the agents' knowledge of the economy. The agents with RE are able to predict the future path of the economy, except for the consequences of unforecastable random shocks. The RE approach can be questioned especially in global nonlinear settings that include an unfamiliar region of outcomes that are caused by the ZLB. An alternative viewpoint to RE, which assumes that agents' economic knowledge have some form of imperfection, has received rising support in recent years. Imperfect knowledge has been specified in different ways in the recent literature. Probably the most popular formulation has been the adaptive learning approach in which agents maximize anticipated utility or profit subject to expectations that derived from an econometric model. The model is updated over time as new information becomes available.[4]

[3]Eggertsson and Woodford (2003), pp.193-194 note that such a deflationary trap can exist but they do not analyze it in detail.

[4]For discussion and analytical results concerning adaptive learning in a wide range of macroeconomic models, see for example Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009).

The adaptive learning approach has been applied to the multiple equilibria issue caused by ZLB with a Taylor rule in some papers, see Evans and Honkapohja (2005), Evans, Guse, and Honkapohja (2008), Evans and Honkapohja (2010), and Benhabib, Evans, and Honkapohja (2012). In this paper we use a standard nonlinear NK model to discuss nominal GDP targeting and price-level targeting from the viewpoint provided by global nonlinear analysis. We argue that the good properties of nominal GDP or price-level targeting advocated in the studies relying on the RE assumption no longer hold under when imperfect knowledge prevails and agents behave as suggested by adaptive learning.[5] These targeting regimes can have good properties near the normal steady state π^* but the regimes do not escape the problems caused by the ZLB.

We also compare key properties of nominal GDP targeting to price-level and also to inflation targeting. One of the main results is that nominal GDP and inflation targeting are clearly more robust than price-level targeting to deal with imperfect knowledge and learning. The second main result is that commitment and transparency to the policy regime are helpful as they add to robustness of the policy regimes in ways described below.

2 A New Keynesian Model

We employ a standard New Keynesian model as the analytical framework.[6] There is a continuum of household-firms, which produce a differentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses monetary policy, buys a fixed amount of output, and finances spending by taxes and issues of public debt as described below.

The objective for agent s is to maximize expected, discounted utility

[5] Williams (2010) makes a similar argument about price-level targeting under imperfect knowledge and learning. His work relies on simulations of a linearized model.

[6] The same economic framework is developed in Evans, Guse, and Honkapohja (2008). It is also employed in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2012).

subject to a standard flow budget constraint:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right) \tag{1}$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s}, \tag{2}$$

where $c_{t,s}$ is the consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t, $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,s}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,s}}{P_{t-1,s}} - 1 \right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982). We use the Rotemberg formulation rather than the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system.[7] The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha},$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve given by

$$P_{t,s} = \left(\frac{y_{t,s}}{Y_t} \right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. Y_t is aggregate output, which is exogenous to the firm.

[7] The linearizations at the targeted steady state are identical for the two approaches.

The government's flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1}, \tag{5}$$

where we will assume that $\beta^{-1} - 1 < \kappa < 1$. Thus fiscal policy is "passive" in the terminology of Leeper (1991) and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

We assume that g is constant and given by

$$g_t = \bar{g}. \tag{6}$$

From market clearing we have

$$c_t + g_t = y_t. \tag{7}$$

2.1 Optimal decisions for private sector

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$\begin{aligned} 0 \;=\; & -h_{t,s}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,s} - 1)\pi_{t,s}\frac{1}{h_{t,s}} \\ & +\alpha\left(1 - \frac{1}{\nu}\right)Y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}}c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}}E_{t,s}(\pi_{t+1,s} - 1)\pi_{t+1,s}. \end{aligned} \tag{8}$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s}\left(\pi_{t+1}^{-1}c_{t+1,s}^{-\sigma_1}\right)$$

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_{t,s}\pi_{t+1}^{\sigma_2 - 1}}\right)^{-1/\sigma_2},$$

where $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$. We now make use of the representative agent assumption. In the representative-agent economy all agents s have the same utility functions, initial money and debt holdings, and prices. We assume also that they make the same forecasts $E_{t,s}c_{t+1,s}$ $E_{t,s}\pi_{t+1,s}$, $E_{t,s}\pi_{t+1}$, as well as forecasts of other variables that will become relevant below. Under these assumptions all agents make the same decisions at each point in time, so that $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and all agents make the same forecasts. Imposing the equilibrium condition $Y_t = y_t = h_t^\alpha$, one obtains the equations

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^\varepsilon - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-\sigma_1}\right) + \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right],$$

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1}c_{t+1}^{-\sigma_1}\right),$$

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}.$$

For convenience we assume that the utility of consumption and of money is logarithmic ($\sigma_1 = \sigma_2 = 1$). It is also assumed that agents have point expectations, so that their decisions depend only on the mean of their subjective forecasts. This allows us to write the system as

$$m_t = \chi\beta(1 - R_t^{-1})^{-1}c_t, \tag{9}$$

$$c_t^{-1} = \beta r_{t+1}^e(c_{t+1}^e)^{-1}, \text{ where } r_{t+1}^e = R_t/\pi_{t+1}^e, \text{ and} \tag{10}$$

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^\varepsilon - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-1}\right) + \beta\frac{\alpha\gamma}{\nu}\left[\left(\pi_{t+1}^e - 1\right)\pi_{t+1}^e\right]. \tag{11}$$

Equation (11) is the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term $(\pi_t - 1)\pi_t$ arises from the quadratic form of the adjustment costs, and this expression is increasing in π_t over the allowable range $\pi_t \geq 1/2$. To interpret this equation, note that the bracketed expression in the first term on the right-hand side is the difference between the marginal disutility of labor and the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs.

Equation (10) is the standard Euler equation giving the intertemporal first-order condition for the consumption path. Equation (9) is the money demand function resulting from the presence of real balances in the utility function. Note that for our parameterization, the demand for real balances becomes infinite as $R_t \to 1$.

We proceed to rewrite the decision rules for c_t and π_t so that they depend on forecasts of key variables over the infinite horizon (IH). The IH learning approach in New Keynesian models was emphasized by Preston (2005) and Preston (2006), and was used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2012) to study the properties of a liquidity trap.

2.2 The infinite-horizon Phillips curve

Starting with (11), let

$$Q_t = (\pi_t - 1)\,\pi_t. \tag{12}$$

The appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. Making use of the aggregate relationships $h_t = y_t^{1/\alpha}$ and $c_t = y_t - g_t$ we can rewrite (11) as

$$Q_t = \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t (y_t - g_t)^{-1} + \beta Q_{t+1}^e.$$

Solving this forward with $g_t = \bar{g}$, we obtain

$$\begin{aligned} Q_t &= \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t (y_t - \bar{g})^{-1} + \\ &\quad \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j \left(y_{t+j}^e\right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left(\frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}} \right) \\ &\equiv \tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e \ldots), \end{aligned} \tag{13}$$

where government spending is assumed to be constant over time. The expectations are formed at time t and variables at time t are assumed to be in the information set of the agents. We will treat (13), together with (12), as the temporary equilibrium equations that determine π_t, given expectations $\{y_{t+j}^e\}_{j=1}^{\infty}$.

One might wonder why inflation does not also depend directly on the expected future aggregate inflation rate in the Phillip's curve relationship (13).[8] Equation (8) is obtained from the first-order conditions using (3) to eliminate relative prices. Because of the representative agent assumption, each firm's output equals average output in every period. Since firms can be assumed to have learned this to be the case, we obtain (13).[9]

[8] There is an indirect effect of expected inflation on current inflation via current output.

[9] An alternative procedure would be to start from (8), iterate it forward and use the

2.3 The consumption function

To derive the consumption function from (10) we use the flow budget constraint and the NPG condition to obtain an intertemporal budget constraint. First, we define the asset wealth

$$a_t = b_t + m_t$$

as the sum of holdings of real bonds and real money balances and write the flow budget constraint as

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1})m_{t-1}, \tag{14}$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t)y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (14) forward and imposing

$$\lim_{j\to\infty}(D^e_{t,t+j})^{-1}a_{t+j} = 0, \tag{15}$$

where

$$D^e_{t,t+j} = \frac{R_t}{\pi^e_{t+1}}\prod_{i=2}^{j}\frac{R^e_{t+i-1}}{\pi^e_{t+i}}$$

with $r^e_{t+i} = R_{t+i-1}/\pi^e_{t+i}$, we obtain the life-time budget constraint of the household

$$\begin{aligned} 0 &= r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}\Phi^e_{t+j} && (16) \\ &= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\phi^e_{t+j} - c^e_{t+j}), && (17) \end{aligned}$$

where

$$\begin{aligned} \Phi^e_{t+j} &= y^e_{t+j} - \Upsilon^e_{t+j} - c^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}, && (18) \\ \phi^e_{t+j} &= \Phi^e_{t+j} + c^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}. \end{aligned}$$

demand function to write the third term on the right-hand side of (8) in terms of the relative price. This would lead to a modification of (13) in which future relative prices also appear, but using the representative agent assumption and assuming that firms have learned that all firms set the same price each period, the relative price term would drop out.

Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

Invoking the relations

$$c^e_{t+j} = c_t \beta^j D^e_{t,t+j}, \tag{19}$$

which is an implication of the consumption Euler equation (10), we obtain

$$c_t(1-\beta)^{-1} = r_t a_{t-1} + y_t - \Upsilon_t + {\pi_t}^{-1}(1-R_{t-1})m_{t-1} + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}\phi^e_{t+j}. \tag{20}$$

As we have $\phi^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1-R^e_{t+j-1})m^e_{t+j-1}$, the final term in (20) is

$$\sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}) + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1-R^e_{t+j-1})m^e_{t+j-1}$$

and using (9) we have

$$\begin{aligned} & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1-R^e_{t+j-1})m^e_{t+j-1} \\ = & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(-\chi\beta R^e_{t+j-1}c^e_{t+j-1}) = -\frac{\chi\beta}{1-\beta}c_t. \end{aligned}$$

We obtain the consumption function

$$c_t\frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}).$$

The preceding derivation of the consumption function assumes households that do not necessarily act in a Ricardian way, i.e. they do not impose the intertemporal budget constraint (IBC) of the government. To simplify the analysis, we assume that consumers are Ricardian, which allows us to modify the consumption function as in Evans and Honkapohja (2010).[10] From (4)

[10] Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

one has

$$\begin{aligned} b_t + m_t + \Upsilon_t &= \bar{g} + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= \bar{g} - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

By forward substitution, and assuming

$$\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0, \tag{21}$$

we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j}. \tag{22}$$

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \Delta_{t+j}^e, \text{ where} \\ \Delta_{t+j}^e &= \bar{g} - \Upsilon_{t+j}^e - m_{t+j}^e + m_{t+j-1}^e (\pi_{t+j}^e)^{-1} \text{ for } j = 1, 2, 3, \ldots. \end{aligned}$$

A Ricardian consumer assumes that (21) holds. His flow budget constraint (14) can be written as:

$$\begin{aligned} b_t &= r_t b_{t-1} + \psi_t, \text{ where} \\ \psi_t &= y_t - \Upsilon_t - m_t - c_t + \pi_t^{-1} m_{t-1}. \end{aligned}$$

The relevant transversality condition is now (21). Iterating forward and using (19) together with (21) yields the consumption function

$$c_t = (1-\beta)\left(y_t - \bar{g} + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \bar{g}) \right). \tag{23}$$

For further details see Evans and Honkapohja (2010).

3 Temporary Equilibrium and Learning

We assume that agents form expectations using steady-state learning, which is formulated as follows. Steady-state learning with point expectations is formalized as

$$s^e_{t+j} = s^e_t \text{ for all } j \geq 1, \text{ and } s^e_t = s^e_{t-1} + \omega_t(s_{t-1} - s^e_{t-1}) \tag{24}$$

for $s = y, \pi, R$. Here ω_t is called the "gain sequence," and measures the extent of adjustment of estimates to the most recent forecast error. In stochastic systems one often sets $\omega_t = t^{-1}$ and this "decreasing gain" learning corresponds to least-squares updating. Also widely used is the case $\omega_t = \omega$, for $0 < \omega \leq 1$, called "constant gain" learning. In this case it is usually assumed that ω is small.

It is worth emphasizing here that the notation s^e_t for expectations refers to future periods (and not the current one). This is important when computing the stability conditions for learning (E-stability). The temporary equilibrium equations with steady-state learning are:

1. The aggregate demand

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y^e_t - \bar{g}) \left(\frac{\pi^e_t}{R_t}\right) \left(\frac{R^e_t}{R^e_t - \pi^e_t}\right) \\ &\equiv Y(y^e_t, \pi^e_t, R_t, R^e_t). \end{aligned} \tag{25}$$

   Here it is assumed that consumers make forecasts of future nominal interest rates, which are equal for all future period, given that we are assuming steady-state learning.

2. The nonlinear Phillips curve

$$\pi_t = Q^{-1}[\tilde{K}(y_t, y^e_{t+1}, y^e_{t+2}...)] \equiv Q^{-1}[K(y_t, y^e_t)], \tag{26}$$

   where $\tilde{K}(.)$ is defined in (13) and

$$Q(\pi_t) \equiv (\pi_t - 1)\,\pi_t \tag{27}$$

$$\begin{aligned} K(y_t, y^e_t) \equiv\ & \frac{\nu}{\gamma}\left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y_t}{(y_t - \bar{g})}\right) \\ & + \frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y^e_t)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right)\frac{y^e_t}{y^e_t - \bar{g}}\right)\right). \end{aligned} \tag{28}$$

3. Bond dynamics

$$b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}. \tag{29}$$

4. Money demand

$$m_t = \chi\beta \frac{R_t}{R_t - 1} c_t. \tag{30}$$

5. Different interest rate rules are considered below and are specified in the next section.

The state variables are b_{t-1}, m_{t-1}, and R_{t-1}. With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of the endogenous variables, though clearly the evolution of b_t and m_t is influenced by the dynamics of inflation and output. The system in general has three expectational variables: output y_t^e, inflation π_t^e, and the interest rate R_t^e. The evolution of expectations is given by

$$\begin{aligned} y_t^e &= y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e), && (31)\\ \pi_t^e &= \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e), && (32)\\ R_t^e &= R_{t-1}^e + \omega(R_{t-1} - R_{t-1}^e). && (33) \end{aligned}$$

Analysis of E-stability is a convenient way to analyze conditions for convergence of these kinds of learning models.[11] Below we derive E-stability and instability results for the steady states in this model. The E-stability conditions of RE equilibrium give conditions for local convergence of real-time learning rules such (31)-(33). The E-stability conditions apply directly for rules that employ a decreasing gain but the same conditions also apply to constant-gain rules in the limit, where the gain parameter is made arbitrarily small.

4 Monetary Policy Frameworks

4.1 Price-level targeting

Starting with price-level targeting (PLT), we first note that a number of different formulations exist in the literature. We consider a simple formulation,

[11] See e.g. Evans and Honkapohja (2001) and Evans and Honkapohja (2009) for details on E-stability analysis and its connections to real-time learning.

where the policy maker sets the policy instrument so that the actual price level moves gradually toward a targeted price-level path, which is specified exogenously. We assume that this target price-level path $\bar{p}_t$ involves constant inflation, so that

$$\bar{p}_t/\bar{p}_{t-1} = \pi^* \geq 1. \tag{34}$$

In the "gradualist" formulation the interest rate, which is the policy instrument, is set above (below, respectively) the targeted steady-state value of the instrument when the actual price level is above (below, respectively) the targeted price-level path $\bar{p}_t$, as measured in percentage deviations. The interest rate also responds to the percentage gap between targeted and actual levels of output. This idea leads to a Wicksellian interest rate rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p[(p_t - \bar{p}_t)/\bar{p}_t] + \psi_y[(y_t - y^*)/y^*], 0], \tag{35}$$

where the max operation takes account of the ZLB on the interest rate and $\bar{R} = \beta^{-1}\pi^*$ is the gross interest rate at the targeted steady state.[12] For analytical ease, we adopt a piecewise linear formulation of the interest rate rule.

4.2 Nominal GDP targeting

A standard formulation of nominal GDP targeting obtains from postulating that the monetary authority sets the interest rate in each period to meet a targeted nominal GDP path $\bar{z}_t$ in each period. In line with a standard NK model, assume also that there is an associated inflation objective calling for a non-negative (net) inflation rate and that the economy does not have any sources for trend real growth. Then formally, $\bar{z}_t = \bar{p}_t\bar{y}$ and $\bar{p}_t/\bar{p}_{t-1} = \pi^*$, and we obtain the commonly used concept of nominal GDP growth targeting, see e.g. Mitra (2003) and the references therein. Taking time differences,

$$\bar{z}_t - \bar{z}_{t-1} = \Delta\bar{z}_t = \pi^*. \tag{36}$$

Requiring that actual growth of nominal GDP equals the target, i.e.,

$$\frac{p_t y_t}{p_{t-1} y_{t-1}} = \Delta\bar{z},$$

[12] See pp.260-61 of Woodford (2003) and Giannoni (2012) for discussion of Wicksellian rules.

yields the equation

$$\Delta \bar{z} y_{t-1} = \pi_t y_t,$$

which can be coupled to the temporary equilibrium equations (25) and (27)-(28). We obtain the equation

$$\frac{\Delta \bar{z} y_{t-1}}{\pi_t} = Y(y_t^e, \pi_t^e, R_t, R_t^e). \tag{37}$$

Clearly, the right-hand side of (37) is decreasing in R_t, so that a unique interest rate in can be computed. When the right-hand side takes the specific form (25), it is possible to compute an explicit form for the interest rate R_t. It is given by

$$R_t = (\beta^{-1} - 1) \frac{\pi_t^e (y_t^e - \bar{g})}{\Delta \bar{z} y_{t-1}/\pi_t - \bar{g}} \left(\frac{R_t^e}{R_t^e - \pi_t^e} \right) \equiv R(\pi_t, y_t^e, \pi_t^e, R_t^e, y_{t-1}), \tag{38}$$

so that the current interest rate depends on expected inflation, expected future interest rate and also on past gross and expected net output.

The formulation (38) is, however, incomplete as it does not take into account the ZLB. The preceding solution is meaningful only when it does not violate the ZLB. If the ZLB constraint is binding in temporary equilibrium, then the monetary authority cannot meet the nominal GDP target. The constrained temporary equilibrium is then determined by $R_t = 1$ and equations (25) and (27)-(28). Incorporating the constraint, the interest rate rule takes the final form

$$R_t = \max[1, R(\pi_t, y_t^e, \pi_t^e, R_t^e, y_{t-1})]. \tag{39}$$

5 Steady States

A non-stochastic steady state (y, π, R) under PLT must satisfy the Fisher equation $R = \beta^{-1}\pi$, the interest rate rule (35), and steady-state form of the equations for output and inflation (25) and (26). One steady state clearly obtains when the actual inflation rate equals the inflation rate of the price-level target path, see equation (34). Then $R = R^*$, $\pi = \pi^*$ and $y = y^*$, where y^* is the unique solution to the equation

$$\pi^* = Q^{-1}[K(Y(y^*, \pi^*, R^*, R^*), y^*)].$$

Moreover, for this steady state $p_t = \bar{p}_t$ for all t.

Then consider the possible steady states under nominal GDP targeting. One of the steady states obtains when the economy follows the targeted nominal GDP path, so that $R = R^*$, $\pi = \pi^*$ and $y = y^*$ as under PLT and $\pi^* = \Delta\bar{z}$.

The targeted steady state under either PLT or nominal GDP targeting is, however, not unique.[13] It can be verified that there is a second steady state in which the ZLB condition is binding:

Lemma 1 *Assume that $\beta^{-1}\pi^* - 1 < \psi_p$. There exists a ZLB-constrained steady state under PLT in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and $\hat{y}$ solves the equation*

$$\hat{\pi} = Q^{-1}[K(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y})]. \tag{40}$$

Correspondingly, a ZLB-constrained steady state exists nominal GDP targeting. In this steady state the price-level target $\bar{p}_t$ or nominal GDP target $\Delta\bar{z}$, respectively, is not met.

Proof. Consider the interest rate rule (35). Imposing $\hat{\pi} = \beta < 1$ implies that $p_t \to 0$ while $\bar{p}_t \to \infty$ (or $\bar{p}$ if $\pi^* = 1$) as $t \to \infty$. It follows that $\bar{R} - 1 + \psi_p[(p_t - \bar{p}_t)/\bar{p}_t] + \psi_y[(y_t - y^*)/y^*] < 0$ for t sufficiently large when $y_t \to \hat{y} < y^*$, so that $R_t = 1$ in the interest rate rule. A unique steady state satisfying (40) is obtained. Thus, $\hat{y}$, $\hat{\pi}$ and $\hat{R}$ constitute a ZLB-constrained steady state.

Now consider the economy under nominal GDP targeting and impose $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$ where $\hat{y}$ solves (40) with $\hat{\pi} = \beta$. As just noted, these requirements yield a steady state state for the economy. Moreover, setting expectations and actual values of (y, π, R) equal to $(\hat{y}, \beta, 1)$ the equation (38) does not hold and

$$R(\beta, \hat{y}, \beta, 1, \hat{y}) < 1,$$

so that the growth of nominal GDP is below the target. Yet, $(y, \pi, R) = (\hat{y}, \beta, 1)$ is a ZLB-constrained steady state as the complete interest rate rule (39) holds. ■

We remark that the sufficient condition $\beta^{-1}\pi^* - 1 < \psi_p$ is not restrictive e.g. when $\beta = 0.99$ and $\pi^* = 1.02$.[14]

[13] Existence of the two steady states under PLT was pointed out in Evans and Honkapohja (2013), section 2.5.3. This paper also considers E-stability of the steady states under short-horizon (called Euler equation) learning.

[14] A weaker sufficient condition is $\beta^{-1}\pi^* - 1 - \psi_p + \psi_y(\hat{y}/y^* - 1) < 0$, in which the term $\hat{y}/y^*$is complicated function of all model parameters.

The lemma states that the commonly used formulations of price-level and nominal GDP targeting both suffer from global indeterminacy as the economy has two steady states under either monetary policy regime. We next consider global dynamics of the economy in these regimes under the hypothesis that agents operate under imperfect knowledge and their expectations are formed in accordance with steady-state learning as described above.

6 Expectations Dynamics: Theoretical Results

6.1 Price-level targeting

We begin to analyze the dynamics of learning under price-level targeting formulated in Section (4.1) above. Considering the Wicksellian rule, we have the system (25), (26), (35) and (34), together with the adjustment of expectations given by (31), (32) and (33). Since the model is nonstochastic, we impose steady-state learning as specified by the adjustment equations for expectations.[15]

The first step is to consider local dynamics near the two possible steady states. Here some theoretical results can be obtained by exploiting the E-stability method that is commonly used in the study of adaptive least-squares learning. However, for most part it is necessary to employ simulations of this system to obtain aspects of global properties of the economy under price-level targeting and with expectations formed by steady-state learning. In the numerical analyses we treat a general case and allow for trend inflation, i.e. $\pi^* > 1$.

In the special case $\pi^* = 1$ there is no explosive exogenous time-dependence and it is possible to partial analytical results of the local stability properties of the two possible steady states under adaptive learning. For this we write the temporary equilibrium system (25), (26), (35) and (34) in the abstract form

$$F(x_t, x_t^e, x_{t-1}) = 0,$$

[15] Preston (2008) discusses local learnability of the targeted steady state with IH learning when the central bank employs PLT. In the earlier literature Evans and Honkapohja (2006) and Evans and Honkapohja (2013) consider E-stability of the targeted steady state under Eular equation learning for versions of PLT.

where the vector $x_t = (y_t, \pi_t, R_t, p_t)^T$. Linearizing around a steady state we obtain

$$x_t = (-DF_x)^{-1}(DF_{x^e}x_t^e + DF_{x_{-1}}x_{t-1}) \equiv Mx_t^e + Nx_{t-1}, \tag{41}$$

where for brevity we use the same notation for the deviations for the steady state. Recall that x_t^e refers to the expected future values of x_t and not the current one. Below it is assumed that information about the current value of endogenous variables is not available when expectations in period t are formed. This system is in a standard form for the analysis of E-stability of RE equilibrium.

Consider the possible RE solutions of the form

$$x_t = \bar{b}x_{t-1} \tag{42}$$

for the system (41).[16] It can be shown that $\bar{b}$ solves the matrix equation

$$b = (I - Mb)^{-1}N. \tag{43}$$

The E-stability condition is that all eigenvalues of the matrices

$$M(I + \bar{b}) \text{ and } \bar{b}' \otimes M + I \otimes M\bar{b}$$

have real parts less than one.

In general, the analytical details for E-stability of PLT appear to be intractable, but results are available in the limiting case $\gamma \to 0$, i.e. price adjustment costs are sufficiently small. Moreover, we make the normalization $\bar{p} = 1$ for simplicity. It is possible to obtain the following result:[17]

Proposition 2 *Assume $\pi^* = 1$ and $\gamma \to 0$. Then under PLT the targeted steady state with $\pi = 1$ and $R = \beta^{-1}$ is E-stable if $\psi_p > 0$.*

[16] We note here that the formulation of E-stability outlined below allows for learning of both intercepts and AR-coefficients of agents' perceived law of motion. Complete E-stability conditions are then obtained. In contrast, analysis of real-time learning specified in (31)-(33) is restricted to learning of the steady state values because, for simplicity, the model does not include random shocks. See Evans and Honkapohja (1998) for methodological issues about modeling learning in deterministic and stochastic frameworks.

[17] Mathematica routines containing technical derivations in the proof are available upon request from the authors.

Proof. First consider the possible RE solutions of the form (42). Since only the lagged price level influences the RE dynamics, the first three columns of matrix $\bar{b}$ are zero vectors. Moreover, in the limit $\gamma \to 0$ it can be shown that the equation for y_t becomes

$$y_t = \frac{\beta}{\beta - 1} y_t^e,$$

so that the movement of y_t under learning influences other variables but not *vice versa*. The dynamics of y_t and y_{t+1}^e converge to the steady state as $\beta < 1$. From this it follows that the $(4,1)$ element of $\bar{b}$ is equal to zero, i.e. in what follows we can impose that $b_{14} = 0$.

In the limit $\gamma \to 0$ we have

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\beta^2\psi_y(\bar{g}-y^*)-y^*}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{-1}{(\beta-1)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \\ \frac{1}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{-1}{(\beta-1)\beta} & \frac{\beta}{\beta-1} & 0 \\ \frac{y^*+\beta^2\psi_y(y^*-\bar{g})}{(\bar{g}-y^*)(\beta-1)\beta\psi_p} & \frac{-1}{(\beta-1)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \end{pmatrix}, \ N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The MSV solutions for the system are given by equation (43), where

$$b = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{24} \\ 0 & 0 & 0 & b_{34} \\ 0 & 0 & 0 & b_{44} \end{pmatrix}.$$

It can be shown that the unknown coefficients b_{24}, b_{34} and b_{44} can be solved from the system

$$b_{24} = -1, b_{34} = 0 \text{ and } b_{44} = 0.$$

Next, calculate the eigenvalues of matrices $M(I + \bar{b}) - I$ and $\bar{b}' \otimes M + I \otimes M\bar{b} - I$, where $\bar{b}$ is a steady state. The eigenvalues of $M(I + \bar{b})$ are

$$-1, -1, \frac{1}{\beta - 1}, \frac{1}{\beta - 1},$$

and the eigenvalues of $\bar{b}' \otimes M + I \otimes M\bar{b}$ consist of 12 roots equal to -1 and four roots equal to

$$-1 - \frac{1}{\beta\psi_p(1-\beta)},$$

so that the MSV solution is E-stable whenever $\psi_p > 0$. ■

We emphasize that by continuity of eigenvalues, E-stability of the targeted steady state also obtains when γ is sufficiently small. Below we carry out numerical simulations for other parameter configurations and with strictly increasing price-level target. The learning dynamics do converge for many cases with non-zero value of γ.

As regards the other steady state in which ZLB binds, we have the result:

Proposition 3 *Assume $\pi^* = 1$. The steady state with binding ZLB $(\hat{y}, \beta, 1, \bar{p})$ is not E-stable under PLT.*

Proof. When the ZLB binds, the interest rate R_t is constant and the price level evolves exogenously in a neighborhood of the constrained steady state. The temporary equilibrium system and learning dynamics then reduce to two variables y_t and π_t together with their expectations. Moreover, no lags of these variables are present, so that the abstract system (41) is two dimensional with $x_t = (y_t, \pi_t)^T$ and $N = 0$.

It can be shown that

$$Det(M - I) = \frac{\hat{y}^{(1+\varepsilon)/\alpha}(1+\varepsilon)\nu(\bar{g} - \hat{y})^2 + \bar{g}\hat{y}\alpha^2(\nu - 1)}{(\bar{g} - \hat{y})\hat{y}\alpha^2(\beta - 1)^2\beta(2\beta - 1)\gamma}.$$

The numerator is positive whereas the denominator is negative. Thus, $Det(M - I) < 0$, which implies E-instability. ■

The preceding results show that the targeted steady state is locally but not globally stable under learning. In Section 7 we present a variety of numerical results for PLT. These results do not employ the restrictive assumption of a small γ that was invoked in Propositions 2 and 3.

6.2 Nominal GDP targeting

In this section, we consider the consequences of nominal GDP targeting under learning. We have the system (25), (26), (36) and (38), together with the adjustment of expectations given by (31), (32) and (33).

We first consider local dynamics near the two possible steady states by using the E-stability technique on the linearization (41). The state variable is now $x_t = (y_t, \pi_t, R_t)^T$. The following result holds for the targeted steady state:[18]

[18] Mitra (2003) discusses E-stability of the targeted steady state under Euler equation learning.

Proposition 4 *Assume $\gamma \to 0$. Then there exists $g_0 > 0$ such that for $\bar{g} \in [0, g_0)$ targeted steady state with $\pi^* = 1$ and $R = \beta^{-1}$ is E-stable under nominal GDP targeting.*

Proof. We consider the RE solution of the form (42) and the E-stability conditions that all eigenvalues of the matrices $M(I+\bar{b})$ and $\bar{b}' \otimes M + I \otimes M\bar{b}$ have real parts less than one. In the case $\gamma \to 0$ the coefficient matrices are given by

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 \\ \frac{-(\bar{g}-y^*)^2(\bar{g}(-1+y^*+\beta)-y^*(-2+2y^*+\beta))}{(\bar{g}-y^*)^2 y^{*2}(\beta-1)} & \frac{\bar{g}(1-y^*\beta)+y^{*2}(-2+y^*+\beta)}{(\bar{g}-y^*)y^*(\beta-1)} & \frac{(y^*-1)(\bar{g}-y^{*2})\beta^2}{(\bar{g}-y^*)y^*(\beta-1)} \\ \frac{-\bar{g}+2y^*}{(\bar{g}-y^*)y^*(\beta-1)\beta} & \frac{-1}{(\beta-1)\beta} & \frac{1}{\beta-1} \end{pmatrix},$$

$$N = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{y^*} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

From these matrices it is evident that the matrix b has second and third columns equal to zero in the RE equilibrium, i.e. it can be assumed that

$$b = \begin{pmatrix} b_{11} & 0 & 0 \\ b_{12} & 0 & 0 \\ b_{13} & 0 & 0 \end{pmatrix}$$

when solving for $\bar{b}$ using (43). Given M and N, we get $\bar{b}_{11} = 0$, $\bar{b}_{12} = \frac{1}{y^*}$, and $\bar{b}_{13} = 0$.

To assess E-stability, the eigenvalues of $M(I+\bar{b})$ are

$$-1, \frac{\bar{g} + (y^* - 2)y^{*2}}{(\bar{g} - y^*)y^*(\beta - 1)}, \frac{1}{\beta - 1}$$

while all eigenvalues of $\bar{b}' \otimes M + I \otimes M\bar{b} - I$ are equal to -1. All eigenvalues except $\frac{\bar{g}+(y^*-2)y^{*2}}{(\bar{g}-y^*)y^*(\beta-1)}$ are clearly negative. We need to examine this remaining eigenvalue in more detail.

From (27)-(28) one can obtain the steady-state relation between y^* and $\bar{g}$

$$\alpha^{-1}(y^*)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y^*}{(y^* - \bar{g})} = 0. \tag{44}$$

We note that $y^* < 1$ when $\bar{g} = 0$ and then the remaining eigenvalue is negative. From equation (44) one can solve $\bar{g}$ in terms of y^*. We obtain

$$\bar{g} = \nu^{-1}\alpha(1-\nu)(y^*)^{1-(1+\varepsilon)/\alpha} + y^*,$$

which is an increasing function in $(y^*, \bar{g})$-space and at point $y^* = 1$ it has a value $1 + \nu^{-1}\alpha(\nu - 1) < 1$. Requiring that the value of the numerator of the eigenvalue be negative yields the restriction

$$\bar{g} < (2 - y^*)y^{*2}.$$

The right-hand side of the last formula is increasing for $y^* < 4/3$ and decreasing for $y^* > 4/3$. At $y^* = 1$ the right-hand side equals 1. These Considerations imply the existence of the asserted $g_0 > 0$. ■

Recall that the ZLB-constrained steady state also exists under nominal GDP targeting. One then has $R_t = 1$ in the implied interest rate rule (39). The analysis of E-stability of this steady state is formally the same as in Proposition 3, and one has:

Proposition 5 *Assume* $\pi^* = 1$. *The steady state with binding ZLB* $(\hat{y}, \beta, 1)$ *is not E-stable under nominal GDP targeting.*

7 Numerical Analysis: Non-transparent Policy

7.1 Dynamics under PLT

We adopt the following calibration: $A = 2.5$, $\pi^* = 1.02$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 350$, $\upsilon = 21$, $\varepsilon = 1$, and $g = 0.2$. We choose $A = 2.5$ to clearly separate the intended and unintended steady states in the numerical analysis. Our results are robust to using $A = 1.5$, which is the usual value for the interest rate rule. The calibrations of π^*, β, α, and g are standard. We set the labor supply elasticity $\varepsilon = 1$. The value of $\upsilon = 21$ was chosen so that the implied markup of prices over marginal cost at the steady state is 5 percent, which is consistent with the evidence presented by Basu and Fernald (1997). Following Sbordone (2002), we set $\gamma = -17.5(1+\nu) = 350$. It is also assumed that interest rate expectations $r^e_{t+j} = R_{t+j-1}/\pi^e_{t+j}$ revert to the steady state value β^{-1} for $j \geq T$. We use $T = 28$, which under a quarterly calibration corresponds to 7 years. To facilitate the numerical analysis the lower bound on the interest rate R is set slightly above 1 at value 1.001. The gain parameter is set at $\omega = 0.002$, which is a low value. Sensitivity of this choice is discussed below.

The targeted steady state is $y^* = 0.944025$, $\pi^* = 1.02$ and the low steady state is $y_L = 0.942765$, $\pi_L = 0.99$. The initial conditions in the next simulation are set as follows: $y^e(0) = y(0) = 0.945$, $\pi^e(0) = \pi(0) = 1.025$, $R^e(0) = R(0) = R^*$ and $(p(0) - \bar{p}(0))/\bar{p}(0) = 1.03$. The initial value for $(y(0) - y^*)/y^*$ is determined by the value $y(0)$. For policy parameters we adopt the values $\psi_p = 2.00$ and $\psi_y = 0.18$ which approximate the values by Giannoni (2012), Table 1 (with *iid* shocks).

Figures 1-3 show the dynamics inflation and the interest rate for PLT from these initial conditions. It is seen that there is oscillatory convergence to the targeted steady state. There are significant initial oscillations and R hits the lower bound almost 20% of the time periods despite assuming initial conditions for inflation and output above the targeted steady state.[19]

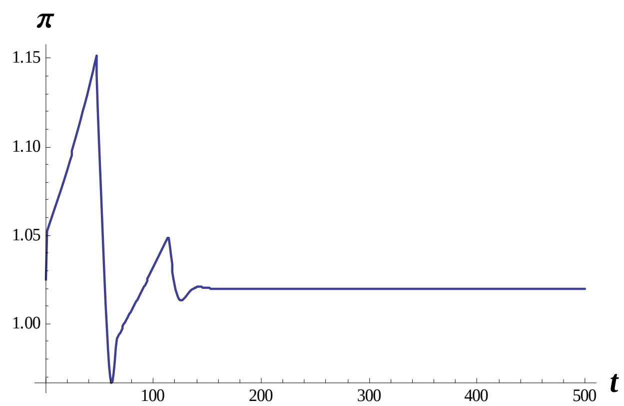

Figure 1: Inflation dynamics under PLT

[19] For numerical purposes, we define the zero lower bound to be the interval when R is between 1 and 1.005.

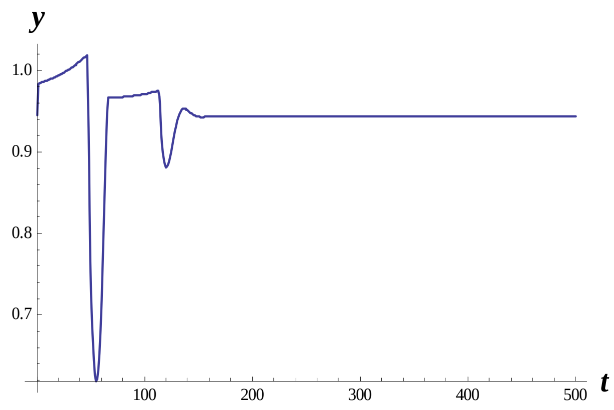

Figure 2: Output Dynamics under PLT

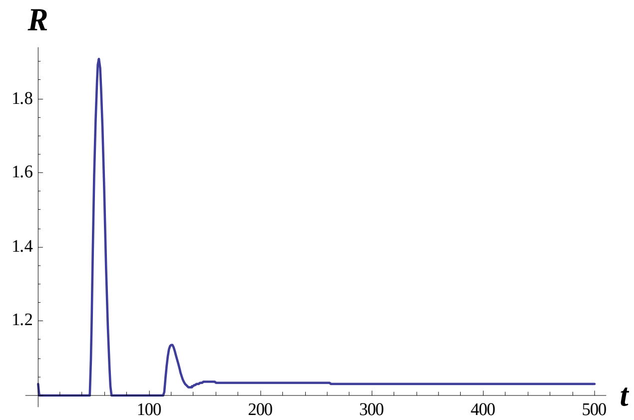

Figure 3: Interest Rate Dynamics under PLT

We then carried out a few sensitivity analyzes. First, the effect of the policy parameters ψ_p and ψ_y do not have a very big impact on the dynamics, except that very low values for ψ_p can lead to instability. Second, convergence to the targeted steady state is surprisingly sensitive to the magnitude of gain parameter. Raising the gain from the base value of $\omega = 0.002$ to $\omega = 0.004$ led to instability. If the target is unstable, then the dynamics will eventually enter the deflationary region for which $\pi < \pi_L$, $y < y_L$ and get trapped there.

This last observation raises questions of the robustness of PLT. It is worth examining the robustness of PLT in terms of sensitivity to (i) initial conditions, (ii) size of the gain parameter and (iii) magnitude of the fluctuations.

As a point of comparison we take the policy framework of inflation targeting (IT) by means of a Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi[(\pi_t - \pi^*)/\pi^*] + \psi_y[(y_t - y^*)/y^*], 0],$$

with the usual values $\psi_p = 1.5$ and $\psi_y = 0.5$, same initial conditions and the same calibrated model.

Looking first at convergence from initial conditions, the top part of Table 1 looks at implications of variations in initial output and output expectations (these two are kept equal to each other) when inflation and the interest rate and their expectations are kept at the steady state. "conv" and "div" indicate that there is convergence to (divergence from, respectively) to the targeted steady state for PLT and IT from the indicated initial conditions. The bottom half considers implications of variation in initial inflation and inflation expectations (the two are kept equal to each other) when initial output and output expectations are set at the steady-state value.

$\pi(0) = \pi^*, y(0) =$	0.94	0.941	0.942	0.946	0.948
PLT	div	div	conv	conv	div
IT	div	conv	conv	conv	conv
$y(0) = y^*, \pi(0) =$	1.008	1.01	1.03	1.04	1.05
PLT	div	conv	conv	div	div
IT	div	conv	conv	conv	conv

Table 1: Partial domains of attraction of learning under PLT and IT

Table 1 shows that IT is more robust than PLT in this sense. The set of initial conditions from which there is convergence to the targeted steady state seems to be slightly larger for IT than for PLT.

Second, we consider robustness of IT and PLT with respect to the magnitude of the gain parameter ω. The initial conditions were set at values that were used to generate Figures 1-3. The range of ω for which there is convergence to the targeted steady state under PLT is approximately equal to $(0, 0.003)$. The corresponding range for IT is approximately $(0, 0.125)$. Clearly, IT is much more robust than PLT in this sense.

The third comparison of robustness was done in terms of the range of variation in inflation and output during the learning adjustment for the basic simulation of Figures 1-3. The iterations were done for 2000 periods (with the gain $\omega = 0.002$). For PLT the ranges are $\pi \in [0.951, 1.175]$ and $y \in [0.561, 1.030]$. For IT we obtained $\pi \in [1.018, 1.041]$ and $y \in [0.940, 0.945]$. These results suggest that IT is more robust than PLT as it induces smaller fluctuations in inflation and output than PLT.

7.2 Dynamics under nominal GDP targeting

We now consider aspects of learning dynamics under nominal GDP targeting. For the basic simulation, we use the same calibration and initial conditions as in the preceding section. The gain parameter is also set at the same value $\omega = 0.002$ and below consequences of higher values for ω are considered. Figures 4-6 show the evolution of inflation, output and the interest rate under nominal GDP targeting.

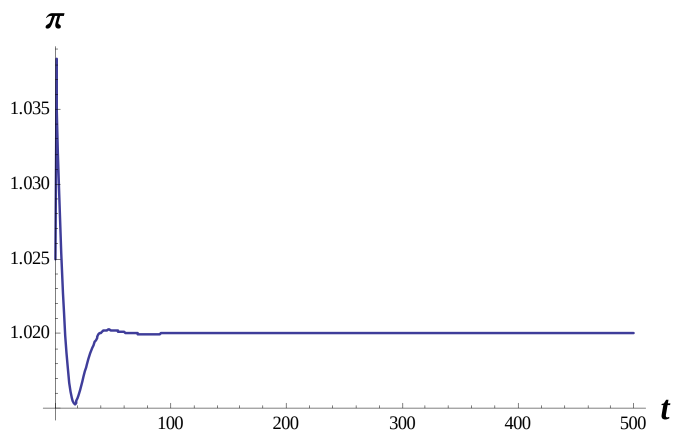

Figure 4: Inflation dynamics under NGDP targeting

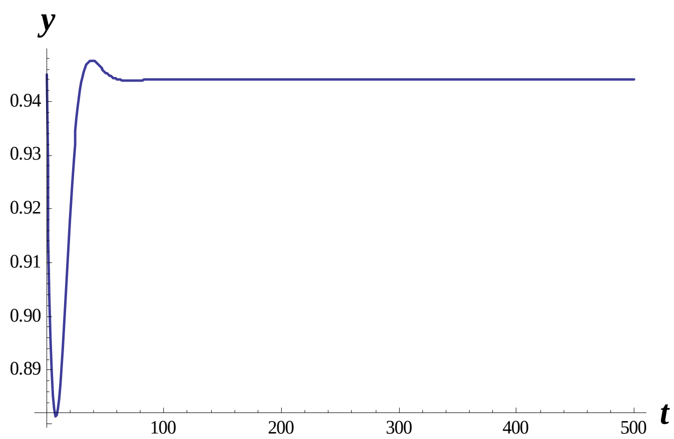

Figure 5: Output dynamics under NGDP targeting

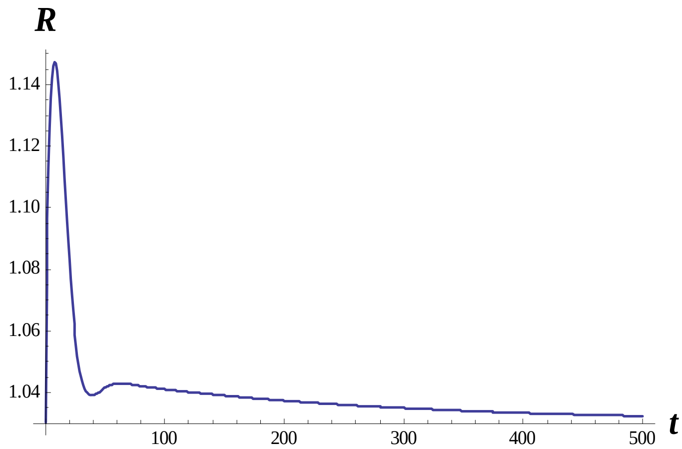

Figure 6: Interest rake under NGDP targeting

Comparing these figures to Figures 1-3, it is seen that under nominal GDP targeting the movements of inflation, output and interest rate in the early periods are oscillatory, which resembles dynamics under PLT. A major difference between the two policy regimes is that the oscillations are more persistent under PLT than under nominal GDP targeting. For the latter the fluctuations die out fairly quickly. Both PLT and nominal GDP targeting differ from IT in that the latter does not show any oscillations from the same initial conditions (figures are not shown but are available).

Next, we examine the robustness of nominal GDP targeting with respect to (i) the size of domain of attraction of the targeted steady state, (ii) gain parameter and (iii) magnitude of the fluctuations. The comparison is made to the IT regime, since IT performed better in the preceding contest between IT and PLT.

Looking first at convergence from initial conditions, the top part of Table 2 looks at implications of varying in initial output and output expectations (these two are kept equal to each other) when inflation and the interest rate and their expectations are kept at the steady state. As before, "conv" and "no conv" indicates that there is convergence (non-convergence, respectively) to the targeted steady state from the indicated initial conditions. The bottom half of the table considers implications of variations in initial inflation and inflation expectations (the two are kept equal to each other) when initial output and output expectations are set at the steady-state value.

$\pi(0) = \pi^*, y(0) =$	0.94	0.941	0.92	0.946	0.948	0.950
$NGDP$	div	conv	conv	conv	conv	div
IT	div	conv	conv	conv	conv	conv
$y(0) = y^*, \pi(0) =$	1.01	1.011	1.03	1.04	1.05	1.06
$NGDP$	conv	conv	conv	conv	conv	conv
IT	div	conv	conv	conv	conv	conv

Table 2: Partial domains of attraction of learning under nominal GDP targeting and IT

Table 2 shows that nominal GDP targeting and IT are nearly equally robust in this sense. There are some differences, but they occur at some distance away from the targeted steady state.

Second, we consider robustness of IT and nominal GDP targeting with respect to the magnitude of the gain parameter ω. The initial conditions were set at values that were used to generate Figures 4-6. The range of ω for which there is convergence to the targeted steady state under nominal GDP targeting is approximately equal to $(0, 0.18)$. The corresponding range for IT is approximately $(0, 0.105)$. The conclusion here is that nominal GDP targeting is somewhat more robust than IT, though it should noted that the ranges for ω giving convergence in both regimes are large and well cover the estimated or calibrated values for the gain parameter that are used in the applied literature.

The third comparison of robustness was done in terms of the range of variation in inflation and output during the learning adjustment for the basic simulation of Figures 4-6. The iterations were done for 2000 periods (given that the gain was kept small). For nominal GDP targeting the ranges were $\pi \in [1.015, 1.038]$ and $y \in [0.881, 0.948]$. Comparing these ranges to those obtained above for IT, which are $\pi \in [1.018, 1.041]$ and $y \in [0.940, 0.945]$, it is seen that for inflation fluctuations the ranges are close to each other. In terms of output fluctuation, IT does clearly better. We can conclude that IT is more robust than nominal GDP targeting in that it appears to induce smaller fluctuations in output.

8 Transparency of Policy

A key underlying assumption in the preceding analysis is that the behavior of the policy maker is not transparent, so that private agents must make forecasts of future interest rates. This is a restrictive assumption and major part of the recent discussions of the desirable properties of nominal GDP and price-level targeting has relied on credibility and transparency of the policy regime. However, it must be stressed that these earlier analyses additionally rely on the RE hypothesis. As was already pointed out in the introduction, it is important to consider the alternative assumption that expectations of private agents are not rational.

Even if private agents' knowledge is imperfect, credibility and transparency of policy regime can be built into the model by postulating that agents know the policy and use this knowledge in their forecasting of future interest rates. In this kind of setting agents do not directly forecast the interest rate but instead use the policy rule, so that agents use forecasts for the future values of the variables in the policy rule.

In this section we first develop the details of the formulation of transparent nominal GDP targeting. Then we compare its robustness properties to the non-transparent formulation. Later, we compare robustness of transparent nominal GDP targeting to the transparent forms of price-level and IT targeting. For brevity, we omit formal details of the latter, as they are straightforward: one simply substitutes the interest rate rule into the aggregate demand equation (25).[20] It should also be pointed out that for reasons of brevity we also omit the theoretical analysis of E-stability of the steady states under the different transparent policy regimes. Since the simulations converge, E-stability should not be a concern.

Under nominal GDP targeting the policy rules is given (39). There are two cases to consider for temporary equilibrium and learning. If ZLB is not expected to bind, interest rate expectations are computed from

$$R_t^e = (\beta^{-1} - 1)\frac{\pi_t^e(y_t^e - \bar{g})}{\Delta\bar{z}y_{t-1}/\pi_t^e - \bar{g}}\left(\frac{R_t^e}{R_t^e - \pi_t^e}\right)$$

as current data is not used in expectations formation. Substituting this expression and the rule for the actual interest rate (39) in the aggregate demand function yields

$$y_t = \Delta\bar{z}y_{t-1}/\pi_t.$$

Applying (31) and (32), the learning dynamics are then

$$\begin{aligned} y_{t+1}^e &= y_t^e + \omega(y_t - y_t^e), && (45)\\ \pi_{t+1}^e &= \pi_t^e + \omega(Q^{-1}[K(y_t, y_t^e)] - \pi_t^e, && (46) \end{aligned}$$

where $y_t = \Delta\bar{z}y_{t-1}/\pi_t$ for period $t+1$. If instead ZLB is expected to bind, then $R_t^e = 1$ and

$$\begin{aligned} y_t &= Y(y_t^e, \pi_t^e, R_t, 1), \text{ where}\\ R_t &= \max\left[1, (\beta^{-1} - 1)\frac{\pi_t^e(y_t^e - \bar{g})}{\Delta\bar{z}y_{t-1}/\pi_t - \bar{g}}\left(\frac{1}{1 - \pi_t^e}\right)\right]. \end{aligned}$$

These equations are then coupled with temporary equilibrium for inflation (27)-(28) and the learning system (45)-(46).

It turns out that transparency of the policy rule has a major impact on the domain of attraction of the targeted steady state, so we consider it first. This general result holds for the three interest rate rules considered, but the improvement is biggest from nominal GDP and inflation targeting. Transparency about the policy rule yields some improvement also for PLT and lack of robustness of PLT relative to nominal GDP and inflation targeting is accentuated. Table 3 presents the results.[21]

[20] For example, see Eusepi and Preston (2010) and Preston (2006).

[21] The initial conditions and other parameter values were the same as those for Figures 1-3. In PLT simulation, the initial price and output deviations from their targets were set at 3 percent.

$\pi(0) = \pi^*, y(0) =$	0.6	0.935	0.938	0.94	0.942	0.94	0.946	0.948
$NGDP$	conv	conv	conv	conv	conv	conv	conv	conv
IT	div	div	conv	conv	conv	conv	conv	conv
PLT	div	div	conv	conv	conv	conv	conv	conv
$y(0) = y^*, \pi(0) =$	0.985	0.99	0.991	1.002	1.003	1.02	1.04	1.05
$NGDP$	div	div	conv	conv	conv	conv	conv	conv
IT	div	conv	conv	conv	conv	conv	conv	conv
PLT	div	div	div	div	conv	conv	conv	conv

Table 3: Partial domains of attraction for nominal GDP, inflation and price-level targeting with transparent policy

The results show that in terms of domain of attraction nominal GDP targeting is the best on the whole. IT is nearly as robust as nominal GDP targeting, whereas PLT is clearly less robust. The most remarkable feature for both nominal GDP and inflation targeting is that the lowest value for the initial condition on $\pi(0)$ (and $\pi^e(0) = \pi(0)$) can be pushed to arbitrarily close to the low steady state value $\pi_L = 0.99$ in the partial domains shown.

Comparing to the results in Tables 1 and 2, it is seen that there is major improvement to the corresponding regime without transparency of policy. Considering at the partial domain of attraction in terms of $y(0)$, the improvement is huge for nominal GDP targeting but less dramatic for both IT and PLT. The general conclusion here is that nominal GDP targeting is the most robust policy regime. IT is somewhat less robust and PLT is clearly least robust of the three rules considered according to this criterion.

Next, we look at the impact of policy transparency for the three rules with respect to the other measures of robustness, namely speed of learning (the gain parameter) and the range of variation. The results make use of the same initial conditions as in the corresponding analysis in Section 7.[22]

The results for the maximal value of the gain parameter ω for the three policy rules under transparency are:

$NGDP$	$0 < \omega < 0.190$
IT	$0 < \omega < 0.126$
PLT	$0 < \omega < 0.024$

[22]Note that there is no need for an assumption about $R^e(0)$ in this section.

Table 4: Robustness with respect to the gain parameter with transparent policy

It is seen that policy transparency improves robustness with respect to speed of learning parameter for nominal GDP, PLT and inflation targeting. Comparing to the corresponding results with non-transparent policy, the changes are rather small, except for PLT.

The results about volatility in terms of ranges for inflation and output are as follows for robustness of the three policy rules under transparency:

$NGDP$	$\pi \in [1.014, 1.038]$	$y \in [0.892, 0.953]$
IT	$\pi \in [1.019, 1.039]$	$y \in [0.929, 0.945]$
PLT	$\pi \in [0.967, 1.148]$	$y \in [0.667, 1.090]$

Table 5: Robustness with respect to volatility with transparent policy

It is seen that IT performs the best in terms of inflation and output volatility. Nominal GDP targeting has more variability for both variables then IT, while performance of PLT is clearly the worst in terms of both inflation and output volatility.

9 Conclusion

Our objective has been to provide an assessment of price-level and nominal GDP targeting that have been recently suggested as improvements over inflation targeting policy. There are two important starting points for our analysis.

First, it is assumed that agents have imperfect knowledge and therefore their expectations need not be rational. Instead, agents make their forecasts using an econometric model that is updated over time. It should be emphasized that our analysis covers both non-transparent and transparent policy regimes within this approach.

Second, we have carefully introduced the nonlinear global aspects of a standard framework, so that the implications of the interest rate lower bound can be studied. It is well-known that inflation targeting with a Taylor rule suffers from global indeterminacy and it was shown here that the same problem exists for both standard versions of price-level and nominal GDP targeting. Theoretical results for local (but not global) stability under learning of the targeted steady state were derived for the latter two policy rules[23], whereas the low inflation steady state is locally unstable under learning with each policy rule.

We then examined further properties of the three policy rules under cases of non-transparency or transparency of policy. The goal was to consider different dimensions of robustness of the policies. The three criteria for robustness were as follows. First, sensitivity with respect to the speed of private agents' learning was studied, i.e. for what values of learning speed is there convergence of learning dynamics to the targeted steady state. Second, we looked at the range of fluctuation in inflation and output in a typical path of learning adjustment for each of the three policy rules under non-transparency and transparency. Third and most importantly, what is the domain of attraction of the desired steady state under the different rules, i.e., how big is the set of initial conditions from which there is convergence to the desired steady state.

The current numerical results are preliminary. A more systematic study of especially the third criterion of robustness is complex in a multidimensional dynamic system that as multiple steady states. Also the other criteria can be done with more systematic methods and we will do this. The preliminary results are, however, suggestive and surprising.

The results are surprising in our opinion. Overall, nominal GDP targeting with transparency of the policy rule appears to have the best robustness properties. Inflation targeting with transparency reaches the second place in the comparison and its robustness is not that far below that of nominal GDP targeting. For volatility of inflation and output, inflation targeting has the smallest range of variation. Transparency about the policy rules is helpful under both nominal GDP and inflation targeting. Price-level targeting is clearly the least robust policy rule of the alternatives considered.

[23] The corresponding result for IT is known. See e.g. Evans and Honkapohja (2010) for the forward-looking version of the Taylor rule.

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