The Making Of A Great Contraction With A Liquidity Trap and A Jobless Recovery*

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Abstract

The great contraction of 2008 pushed the U.S. economy into a protracted liquidity trap (i.e., a long period with zero nominal interest rates and inflationary expectations below target). In addition, the recovery was jobless (i.e., output growth recovered but unemployment lingered). This paper presents a model that captures these three facts. The key elements of the model are downward nominal wage rigidity, a Taylor-type interest-rate feedback rule, the zero bound on nominal rates, and a confidence shock. Lack-of-confidence shocks play a central role in generating jobless recoveries, for fundamental shocks, such as disturbances to the natural rate, are shown to generate recessions featuring recoveries with job growth. The paper considers a monetary policy that can lift the economy out of the slump. Specifically, it shows that raising the nominal interest rate to its intended target for an extended period of time, rather than exacerbating the recession as conventional wisdom would have it, can boost inflationary expectations and thereby foster employment. *Keywords*: Downward Nominal Wage Rigidity, Liquidity Trap, Taylor Rule, Jobless Recovery

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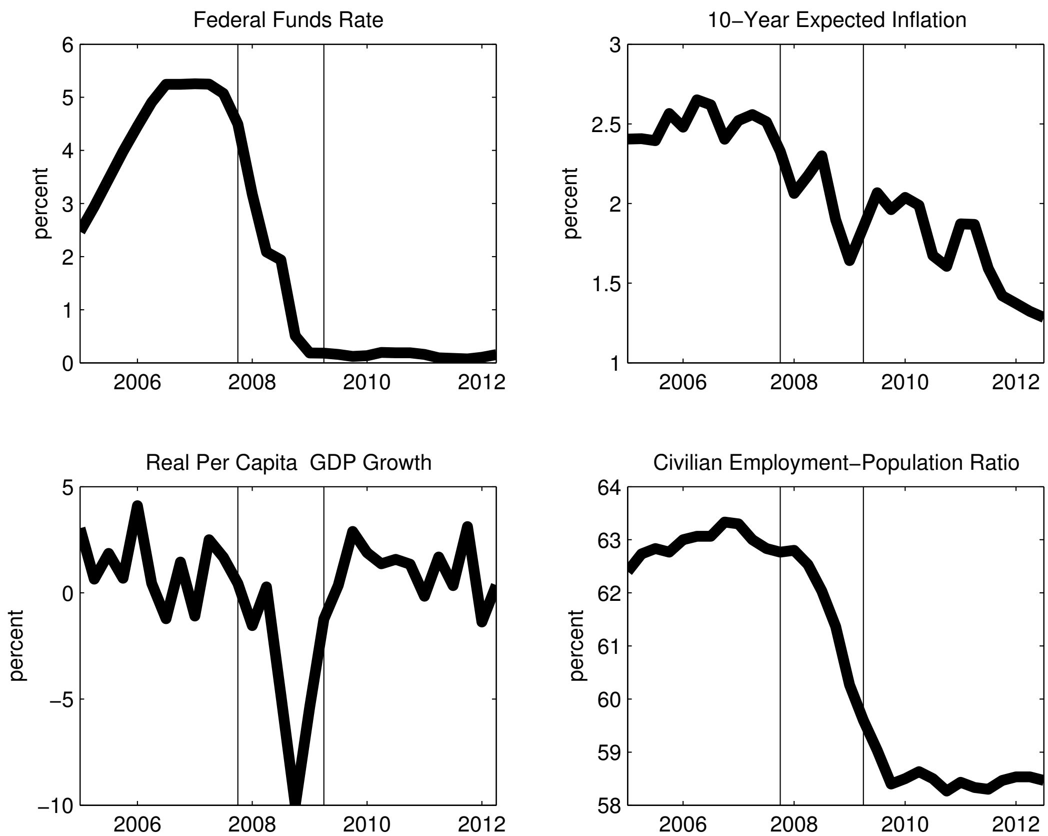
1 Introduction

The great contraction of 2008 pushed the U.S. economy into a protracted liquidity trap with zero nominal rates and declining inflationary expectations. A second characteristic of the great contraction is that the recovery in output growth was swift but jobless, in the sense that unemployment lingered long after output growth had returned to normal. Figure 1 documents these facts. In response to the crisis, the Federal Reserve began easing

Figure 1: Output, Employment, Expected Inflation, and Interest Rates during the U.S. Great Contraction of 2008

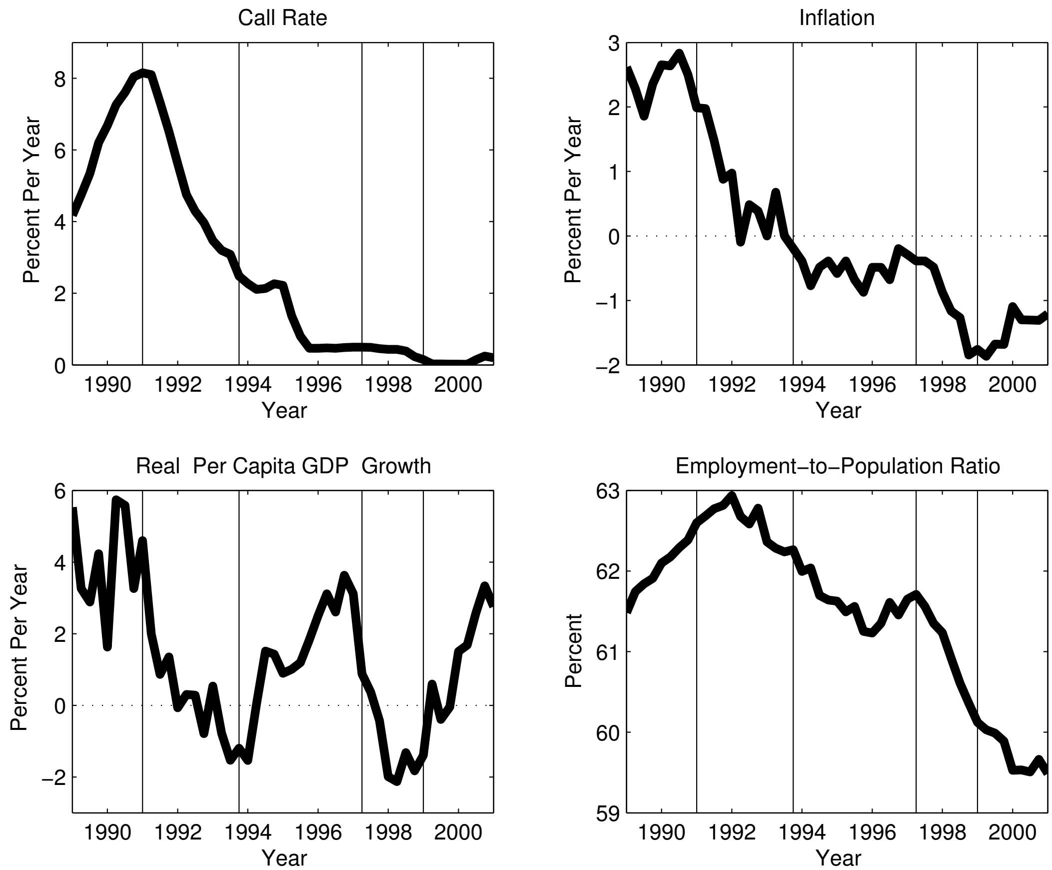


Note. Data sources are given in Appendix C. Vertical lines indicate the beginning and end of the great contraction according to the NBER.

in late 2007 and by 2008 had brought the federal funds rate to near zero. Even though the recession was officially declared over in 2009:Q2, interest rates have remained at zero. The reason the monetary authority has continued to set rates at their lowest possible level is that employment has not yet recovered. Prior to the contraction, the civilian-employment-to-population ratio was 62.5 percent. During the recession it fell by 4 percentage points to 58.5 percent. The recovery is regarded as jobless because this ratio has remained at roughly the level it reached at the trough of the recession.[1]

The experience of Japan in the 1990s provides an example of two recessions with zero nominal interest rates, declining inflation and jobless recoveries. Japan entered a severe recession in 1991Q1. As shown in figure 2 at that time the employment-to-population ratio

Figure 2: Japan during the 1990s



Note. Data sources are given in Appendix C. Vertical lines indicate the beginning and end of the 1991Q1-1993Q4 and the 1997Q2-1999Q1 recessions, respectively. Recession dates are taken from the Cabinet Office of the Government of Japan.

stood at 62.5 percent and the unemployment rate (not shown) was 2 percent. Employment fell during the recession but continued to fall after output growth recovered starting in 1993Q4. Japan entered a second recession in 1997Q2 that ended in 1999Q1. Again, the employment-to-population ratio continued to decline and unemployment continued to rise

[1] Erceg and Levin (2013) show that the observed 4 percentage point decline in the employment-to-population ratio is due to cyclical factors rather than demographic ones such as an aging population. In fact, these authors document that the labor force participation rate of people 65 and older has increased during the great recession.

even after the recession had ended. By the end of the decade the employment rate was only 59.5 percent, 3 percentage points lower than prior to the recession of 1991 and the unemployment rate was close to 5 percent, 3 percent higher than prior to the recession of 1991. In this sense both recoveries were jobless.

In response to the 1991 recession the Bank of Japan aggressively lowered the policy rate, or call rate, from 8 percent at the beginning of the 1991 recession to half a percent by 1995. During the recession of 1997 the call rate was lowered to zero, and Japan fell into a prolonged liquidity trap. Inflation declined continually during the 1991 recession and turned into deflation by the middle of the decade.

Motivated by the recent experiences of the United States and Japan, this paper presents a theoretical model that captures the joint occurrence of an economic contraction, a protracted liquidity trap, and a jobless recovery. The main elements of the model are downward nominal wage rigidity, a Taylor-type interest-rate feedback rule, the zero bound on nominal interest rates, and a negative confidence shock.

In the model, a negative confidence shock pushes the economy into a self-fulfilling deflationary spiral. By the Taylor rule, the declining path of inflation sets in motion a string of nominal interest rate cuts. At some point, the central bank runs into the zero lower bound and the economy is in a liquidity trap. Because of downward nominal wage rigidity, as the economy falls into the liquidity trap, declines in nominal wages fall short of declines in product prices. As a result, real wages become too high to be compatible with full employment. Once the economy is stuck in a liquidity trap, there is no inherent mechanism capable of bringing real wages down to their full-employment level. In this way, unemployment becomes a chronic phenomenon. Eventually, technological progress propels the recovery of output growth, but is unable to stimulate employment.

Our emphasis on the role of a confidence shock to explain the great contraction appears to be supported by the data. Aruoba and Schorfheide (2012) estimate that in 2008:Q4 the U.S. economy suffered a nonfundamental confidence shock and switched to a self-fulfilling liquidity trap equilibrium. Furthermore, these authors estimate that the economy was expected to stay in that equilibrium for several quarters.

The equilibrium dynamics implied by our model are quite different in response to fundamental shocks. When inflationary expectations are well anchored (i.e., in the absence of confidence shocks), inflationary expectations converge quickly to the central bank's intended inflation target as the negative fundamental shock fades away. As inflation converges to its target level, it erodes the real purchasing power of wages, fostering employment. Consequently, in our model the recovery from a contraction driven by fundamental shocks is characterized by both an increase in output growth and, more importantly, job creation. We illustrate how the economy responds to a fundamental shock by characterizing the effect of an exogenous fall in the natural rate. This shock is of interest because it has been extensively used in the recent literature on liquidity traps (e.g., Eggertsson and Woodford, 2003).

An important policy challenge is how to revive job creation in an economy that is stuck in a liquidity trap. Most academic and professional economists agree that an essential element to bring an economy out of a liquidity trap is to raise inflationary expectations (see, for example, Krugman, 1998; Eggertsson and Woodford, 2003; Woodford, 2012). However, what policy is able to raise inflationary expectations in a liquidity trap depends upon the nature of the shock that pushed the economy into the liquidity trap in the first place. Eggertsson and Woodford show that if the underlying shock is fundamental (in particular a fall in the natural rate), then inflationary expectations can be lifted by promising to keep nominal rates at zero for an extended period of time, even after the shock has dissipated. This prescription, however, could be counterproductive if the root cause of the slump is a nonfundamental confidence shock. In this situation, a promise of low interest rates for a prolonged period of time would only validate deflationary expectations and in this way perpetuate the slump.

The reason a policy of low interest rates for an extended period of time cannot generate expected inflation when the liquidity trap is the result of a confidence shock is that in these circumstances the negative relationship between nominal interest rates and inflationary expectations—a central economic relationship that economists and the general public alike hold as timeless truths—ceases to be valid and might indeed reverse sign. During normal times, that is, when the economy is not suffering from a lack-of-confidence shock, the primary effect of an increase in nominal interest rates is a decline in inflationary expectations via a fall in aggregate demand. Similarly, under normal circumstances, a reduction of the nominal interest rate tends to boost short-run inflationary expectations through an elevated level of aggregate spending. In sharp contrast, in a liquidity trap driven by lack of confidence, the sign is reversed. Low interest rates are not accompanied by high levels of inflation but rather by falling and even negative inflation. Moreover, because the economy is already inundated by liquidity, a fall in interest rates has no longer a stimulating effect on aggregate demand. The experience of Japan in the past two decades as well as the recent economic performance of the United States and other developed countries seems to suggest that zero nominal interest rates are not doing much to push expected inflation higher. (See the top right panels of figures 1 and 2 for the United States and Japan, respectively.) An insight that emerges from our model is that the reversal of sign in the relationship between interest rates and expected inflation also operates in the upward direction. That is, that in a liquidity trap caused by a confidence shock, an increase in nominal rates tends to raise inflationary expectations without further depressing aggregate spending. It follows from this insight that any policy that is to succeed in raising inflationary expectations during an expectations-driven liquidity trap must be associated with an increase in nominal rates.

Accordingly, this paper presents an interest-rate based strategy for escaping liquidity traps. Specifically, this strategy stipulates that when inflation falls below a threshold, the central bank temporarily deviates from the traditional Taylor rule by pegging the nominal interest rate at the target level. The paper shows that this policy, rather than exacerbating the recession as conventional wisdom would have it, can boost inflationary expectations and thereby lift the economy out of the slump.

The model developed in this paper builds on the work of Benhabib, et al. (2001) and Schmitt-Grohé and Uribe (2011). Benhabib et al. show that the combination of a Taylor rule and the zero lower bound on nominal rates can give rise to self-fulfilling deflations ending in a liquidity trap. However, their model does not generate involuntary unemployment. The theoretical framework developed in this paper generalizes the model of downward nominal wage rigidity and non-Walrasian labor markets of Schmitt-Grohé and Uribe (2011) by assuming that wages become more flexible as the rate of unemployment increases.

This paper is also related to a recent paper by Shimer (2012) who shows that in a real search model with real wage rigidity, recoveries can be jobless. Our model differs from Shimer's in two important aspects. First, our model assumes that nominal wages are downwardly rigid, but real wages are flexible. The assumption of nominal rather than real wage rigidity is motivated by an empirical literature suggesting that the former type of rigidity is pervasive in the United States (see, for instance, Gottschalk, 2005; Basu, Barattieri, and Gottschalk, 2010; Daly, Hobijn, and Lucking; 2012). Second, in real models of jobless recoveries, monetary policy plays no role by construction. By contrast, a central prediction of our formulation is that monetary policy plays a crucial role in determining whether a recovery is jobless or not. Indeed, there is empirical evidence showing that the stance of monetary policy does matter for labor-market outcomes in recoveries. Calvo, Coricelli, and Ottonello (2012) study recessions and recoveries from financial crises in developed and developing countries and find that recoveries tend to be jobless when inflation is low. Furthermore, they find that employment grows and real wages fall in high-inflation recoveries. Our model is consistent with both of these empirical findings.

Our analysis is also related to the work of Mertens and Ravn (2012). These authors study the size of fiscal multipliers in a version of the Benhabib et al. (2001) model. They find that the size of the fiscal multiplier associated with a particular fiscal instrument depends on the type of shock that pushed the economy into the liquidity trap. In particular, they show that when the liquidity trap is due to a nonfundamental shock, supply-side fiscal instruments have a large multiplier and demand-side fiscal instruments have a small multiplier. The reverse is true when the liquidity trap is caused by a fundamental shock. Aruoba and Schorfheide (2012) also find that demand-side fiscal instruments are less effective in stimulating the economy in a confidence-shock-induced liquidity trap.

The remainder of the paper is organized in six sections. Section 2 presents the model. Section 3 shows that the model possesses two steady states, one with zero unemployment and inflation equal to its target level, and one with involuntary unemployment and zero nominal rates. Section 4 shows that a lack-of-confidence shock leads to a recession, a liquidity trap, and a jobless recovery. Section 5 shows that in response to a fundamental decline in the natural rate, the recovery features job creation. Section 6 shows that raising nominal rates can lift the economy out of a confidence-shock induced liquidity trap without any initial costs in terms of output or unemployment. Section 7 concludes.

2 The Model

We develop a model of a closed economy with monetary nonneutrality due to downward nominal wage rigidity. The formulation of wage rigidity follows Schmitt-Grohé and Uribe (2011). This friction opens the door to involuntary unemployment in response to any shock that leads to a significant deceleration in the growth rate of the full-employment nominal wage. We model monetary policy as a Taylor-type interest-rate feedback rule with a lower bound at zero.

2.1 Households

The economy is populated by a large number of infinitely-lived households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} e^{\xi_t} \beta^t U(C_t), \tag{1}$$

where C_t denotes consumption, ξ_t is an exogenous taste shock with mean zero, $\beta \in (0,1)$ is a subjective discount factor, and E_t is the expectations operator conditional on information available in period t. We assume that the period utility function takes the form

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma},$$

with $\sigma > 0$.

Households are assumed to be endowed with a constant number of hours, denoted $\bar{h}$, which they supply inelastically to the labor market. However, because of the presence of nominal wage rigidities, in general they are able to sell only $h_t \leq \bar{h}$ hours each period. Consequently, they take h_t as exogenously determined. In addition, households pay nominal lump-sum taxes in the amount T_t and receive nominal profits from the ownership of firms, denoted Φ_t. Households can purchase nominally risk-free government bonds, denoted B_t, which when held from period t to period $t+1$ earn a gross nominal interest rate of R_t. The budget constraint of the household is then given by

$$P_t C_t + B_t = W_t^n h_t + R_{t-1} B_{t-1} + \Phi_t - T_t, \tag{2}$$

where P_t denotes the nominal price level in period t and W_t^n denotes the nominal wage rate. In period t, the household takes the right-hand side of the budget constraint as exogenously given.

In each period $t \geq 0$, the optimization problem of the household consists in choosing C_t and B_t to maximize (1) subject to the budget constraint (2) and a no-Ponzi-game constraint of the form $\lim_{j\to\infty} E_t \left(\prod_{s=0}^{j} R_{t+s}^{-1} \right) B_{t+j+1} \geq 0$. The optimality conditions associated with this maximization problem are the budget constraint (2), the no-Ponzi-game constraint holding with equality, and

$$e^{\xi_t} U'(C_t) = \beta R_t E_t \left[e^{\xi_{t+1}} \frac{U'(C_{t+1})}{\pi_{t+1}} \right],$$

where $\pi_{t+1} \equiv P_{t+1}/P_t$ denotes the gross rate of inflation between periods t and $t+1$.

2.2 Firms

Consumption goods are produced using labor as the sole input via the technology

$$Y_t = X_t F(h_t),$$

Y_t denotes output, and X_t is a deterministic trend in productivity that grows at the constant gross rate $\mu \geq 1$, that is,

$$X_t = \mu X_{t-1}.$$

We assume that the production function takes the form

$$F(h) = h^{\alpha},$$

with $\alpha \in (0,1)$.

The firm operates in competitive product and labor markets. Profits, denoted Φ_t, are given by

$$\Phi_t = P_t X_t F(h_t) - W_t^n h_t.$$

The firm chooses h_t to maximize profits. The optimality condition associated with this problem is

$$X_t F'(h_t) = \frac{W_t^n}{P_t}.$$

This first-order condition implicitly defines the firm's demand for labor. It states that firms are always on their labor demand curve. Put differently, in this model firms never display unfilled vacancies nor are forced to keep undesired positions. As we will see shortly, this will not be the case for workers, who will sometimes be off their labor supply schedule and will experience involuntary unemployment.

2.3 Downward Nominal Wage Rigidity

Nominal wages are assumed to be downwardly rigid. Specifically, in any given period nominal wages can at most fall by a factor $\gamma(u_t) \geq 0$, where $u_t \equiv (\bar{h} - h_t)/\bar{h}$ denotes the aggregate rate of unemployment. The downward wage rigidity then takes the form

$$W_t^n \geq \gamma(u_t) W_{t-1}^n.$$

This setup nests the cases of absolute downward rigidity, when $\gamma(u_t) \geq 1$ for all u_t, and full wage flexibility, when $\gamma(u_t) = 0$ for all u_t. We impose the following assumption on the function $\gamma(.)$:

Assumption 1. *The function $\gamma(u_t)$ satisfies*

$$\gamma'(u_t) < 0,$$

and

$$\gamma(0) > \tilde{\beta}\mu,$$

where $\tilde{\beta} \equiv \beta\mu^{-\sigma}$.

The first condition in assumption 1 allows for nominal wages to become more flexible as unemployment increases. The second condition is necessary ensure the uniqueness of the full-employment steady state and the existence of a second steady state with unemployment.

In the simulations reported below, we assume that $\gamma(u)$ takes the form

$$\gamma(u) = \gamma_0(1-u)^{\gamma_1},$$

with $\gamma_0, \gamma_1 > 0$.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear at the inelastically supplied level of hours $\bar{h}$. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. Actual employment must satisfy

$$h_t \leq \bar{h}$$

at all times. Finally, we impose the following slackness condition on wages and employment

$$(\bar{h} - h_t)\left(W_t^n - \gamma(u_t)W_{t-1}^n\right) = 0.$$

This slackness condition implies that whenever there is involuntary unemployment, the lower bound on nominal wages must be binding. It also says that whenever the lower bound on nominal wages does not bind, the economy must be operating at full employment.

2.4 The Government

We assume that the government levies lump-sum taxes in the amount T_t, issues public debt, denoted B_t, and consumes no goods. The sequential budget constraint of the government is then given by

$$B_t + T_t = R_{t-1}B_{t-1}.$$

We assume that lump-sum taxes are chosen to ensure the government's solvency at all times and for any path of the price level. One such fiscal policy would be, for instance, to set T_t endogenously at a level such that $B_t = 0$ for all t.

Monetary policy takes the form of a Taylor-type feedback rule, whereby the gross nominal interest rate is set as an increasing function of inflation and the output gap.[2] Specifically, we assume that the interest-rate rule is of the form

$$R_t = \max\left\{1, R^* + \alpha_\pi\left(\pi_t - \pi^*\right) + \alpha_y \ln\left(\frac{Y_t}{Y_t^*}\right)\right\},$$

where π^* denotes the gross inflation target, and R^*, α_π, and α_y are positive coefficients. The

[2] There is a large literature showing that monetary policy in the United States as well as Japan can be described as having followed a Taylor-type interest rate feedback rule. See Taylor (1993) for evidence for the United States and Bernanke and Gertler (1999) for evidence for Japan.

variable Y_t^* denotes the flexible-wage level of output. That is,

$$Y_t^* = X_t \bar{h}^\alpha.$$

The interest-rate rule is bounded below by unity to satisfy the zero bound on nominal interest rates. We introduce the following assumption involving the parameters of the Taylor rule:

Assumption 2. *The parameters R^*, π^*, and α_π satisfy:*

$$R^* \equiv \frac{\pi^*}{\tilde{\beta}} > 1,$$

$$\alpha_\pi \tilde{\beta} > 1,$$

and

$$\pi^* > \frac{\gamma(0)}{\mu}.$$

The first two conditions are quite standard. The first one allows the inflation target, π^*, to be supported as a deterministic steady state equilibrium. The second one is known as the Taylor principle and guarantees local uniqueness of equilibrium in the neighborhood of a steady state with full employment and inflation at target. The third condition is needed for the existence of a unique full-employment steady-state equilibrium.

2.5 Equilibrium

In equilibrium, the goods market must clear. That is, consumption must equal production

$$C_t = X_t F(h_t).$$

To facilitate the characterization of equilibrium, we scale all real variables that display long-run growth by the deterministic productivity trend X_t. Specifically, let $c_t \equiv C_t/X_t$ and $w_t \equiv W_t^n/(P_t X_t)$. Then, the competitive equilibrium is defined as a set of processes $\{c_t, h_t, u_t, w_t, \pi_t, R_t\}_{t=0}^\infty$ satisfying

$$e^{\xi_t} c_t^{-\sigma} = \tilde{\beta} R_t E_t \left[e^{\xi_{t+1}} \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \tag{3}$$

$$c_t = F(h_t) \tag{4}$$

$$F'(h_t) = w_t \tag{5}$$

$$h_t \leq \bar{h} \tag{6}$$

$$w_t \geq \frac{\gamma(u_t)}{\mu}\frac{w_{t-1}}{\pi_t} \tag{7}$$

$$(\bar{h} - h_t)\left(w_t - \frac{\gamma(u_t)}{\mu}\frac{w_{t-1}}{\pi_t}\right) = 0 \tag{8}$$

$$u_t = \frac{\bar{h} - h_t}{\bar{h}} \tag{9}$$

and

$$R_t = \max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\pi_t - \pi^*\right) + \alpha_y \ln\left(\frac{F(h_t)}{F(\bar{h})}\right)\right\}, \tag{10}$$

given the exogenous process ξ_t and the initial condition w_{-1}. We next characterize steady-state equilibria.

3 Nonstochastic Steady-State Equilibria

Nonstochastic steady-state equilibria are equilibria in which all endogenous and exogenous variables are constant over time, that is, constant processes $\xi_t = 0$, $c_t = c$, $h_t = h$, $w_t = w$, $R_t = R$, $u_t = u$, and $\pi_t = \pi$ for all t satisfying

$$R = \frac{\pi}{\tilde{\beta}} \tag{11}$$

$$c = F(h) \tag{12}$$

$$F'(h) = w \tag{13}$$

$$h \leq \bar{h} \tag{14}$$

$$\pi \geq \frac{\gamma(u)}{\mu} \tag{15}$$

$$(\bar{h} - h)\left(1 - \frac{\gamma(u)}{\mu}\frac{1}{\pi}\right) = 0 \tag{16}$$

$$u = \frac{\bar{h} - h}{\bar{h}} \tag{17}$$

and

$$R = \max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\pi - \pi^*\right) + \alpha_y \ln\left(\frac{F(h)}{F(\bar{h})}\right)\right\}. \tag{18}$$

We next establish that the present economy possesses two distinct nonstochastic steady-state equilibria. In one, inflation equals the inflation target π^* and unemployment is nil. In the second steady state, the economy is in a liquidity trap with a zero nominal interest rate and perpetual unemployment. We refer to the former steady state as the full-employment steady state and to the latter as the unemployment steady state.

The following proposition establishes existence of a full-employment steady state. It further shows that there is a unique full-employment steady state.

Proposition 1 (Existence and uniqueness of a Full-Employment Steady State)**.** *Suppose assumptions 1 and 2 are satisfied. Then, there exists a unique full-employment steady state. Moreover, at the full-employment steady state the inflation rate equals the inflation target* π^**.*

Proof. See Appendix A.

This is the steady state around which the monetary authority wishes to stabilize the economy. We are interested, however, in the existence of a second, unintended, steady state featuring chronic unemployment, inflation below target, and a zero nominal interest rate. The following proposition establishes the existence of such a steady state. It further shows that the unintended steady state is unique.

Proposition 2 (Existence of an Unemployment Steady State)**.** *Suppose assumptions 1 and 2 are satisfied. Then, there exists a unique unemployment steady state* ($u = \bar{u} > 0$)*. Moreover, at the unemployment steady state the economy is in a liquidity trap* ($R = 1$ *and* $\pi = \tilde{\beta} < \pi^*$)*.*

Proof. See Appendix A.

The existence of two nonstochastic steady states, one in which the inflation rate equals the inflation target and one in which the economy is in a liquidity trap is reminiscent of the findings reported in Benhabib, Schmitt-Grohé, and Uribe (2001) with an important difference. In the present model, the unintended steady state features involuntary unemployment.

4 Great Contractions With Jobless Recoveries

In this section, we study equilibria driven by revisions in inflationary expectations. We are particularly interested in a situation in which, because of a loss of confidence, the rate of inflation is below the value agents had expected in the previous period. We will show that such a non-fundamental demand shock results in deflation, unemployment, and zero nominal interest rates. More importantly, the recovery is jobless in the sense that output growth returns to normal but unemployment lingers. The interest-rate rule in place, far from jolting the economy out of the slump, validates expectations of persistent future unemployment and zero nominal rates.

Consider a situation in which prior to period 0 the economy has been in a steady state with full employment, $u_{-1} = 0$, and an inflation rate equal to the policy target, $\pi_{-1} = \pi^*$. Furthermore, assume that in period -1, agents expected π_0 to equal π^*. Suppose that in period 0, a negative revision in agents' economic outlook causes the rate of inflation π_0 to fall below the expected level π^*. Assume that from period 0 on, inflationary expectations are always fulfilled and that there are no shocks to economic fundamentals.

We first show that a negative confidence shock depresses price growth. More precisely, the present model predicts that in response to a negative revision in expectations that results in an initial rate of inflation below target, inflation falls monotonically below a threshold given by $\gamma(0)/\mu < \pi^*$ and then remains below this threshold forever. In other words, the Taylor rule in place is unable to ensure the return of inflation to target. The following proposition formalizes this result.

Proposition 3 (Inflation Dynamics Under Lack of Confidence)**.** *Suppose assumptions 1 and 2 are satisfied and that $\xi_t = 0$ and deterministic for $t \geq 0$. Further, assume that $\pi_0 < \pi^*$. Then, in any perfect foresight equilibrium,*

$$\pi_{t+1} \begin{cases} < \pi_t < \pi^* & \text{if } \pi_t \geq \frac{\gamma(0)}{\mu} \\ < \frac{\gamma(0)}{\mu} < \pi^* & \text{if } \pi_t < \frac{\gamma(0)}{\mu} \end{cases}, \qquad \text{for all } t \geq 0.$$

Furthermore, there exists a finite integer $T \geq 0$ such that $\pi_T < \frac{\gamma(0)}{\mu}$.

Proof: See Appendix A.

The significance of the inflation threshold $\gamma(0)/\mu$ is that once inflation falls below it, full employment becomes impossible. The reason is that because of the downward rigidity of nominal wages, the ratio $\gamma(0)/\pi_t$ represents a lower bound on real wage growth under full employment. If this ratio exceeds the growth rate of productivity, μ, then it must be that real wages are growing at a rate larger than μ. But under full employment, wages cannot grow at a rate exceeding μ, since $X_t F'(\bar{h})$ can grow at most at the rate μ. Therefore, if inflation falls below the threshold $\gamma(0)/\mu$, the economy must experience involuntary unemployment. Because the Taylor rule is unable to bring the rate of inflation above the threshold $\gamma(0)/\mu$, the presence of unemployment becomes chronic. We establish this result in the following proposition.

Proposition 4 (Chronic Involuntary Unemployment Under Lack of Confidence)**.** *Suppose assumptions 1 and 2 are satisfied and that $\xi_t = 0$ and deterministic for $t \geq 0$. Further, assume that $\pi_0 < \pi^*$. Then, in any perfect foresight equilibrium $u_t > 0$ for all $t \geq T$, where $T \geq 0$ is the finite integer defined in proposition 3.*

Proof: See Appendix A.

Given an initial rate of inflation $\pi_0 < \pi^*$, a perfect-foresight equilibrium can be shown to exist and to be unique. The following proposition formalizes this result.

Proposition 5 (Existence and Uniqueness of Chronic Unemployment Equilibria)**.** *Suppose assumptions 1 and 2 are satisfied and that $\xi_t = 0$ and deterministic for $t \geq 0$. Further, assume that $w_{-1} = F'(\bar{h})$. Then, given $\pi_0 < \pi^*$ there exists a unique perfect foresight equilibrium.*

Proof: See Appendix A.

The intuition for the existence of equilibria in which expectations of future increases in unemployment are self-fulfilling could be as follows. Suppose in period t agents expect unemployment in period $t+1$ to be higher. This change in expectations represents a negative income shock to the household as future labor income is expected to decline. This decline in income lowers desired consumption in all periods. Lower demand in period t leads to lower prices in period t. In turn, a decline in current inflation, by the Taylor rule, reduces the current nominal interest rate. And a lower nominal interest rate, as long as expected inflation does not fall by as much as the current nominal rate, causes the real interest rate to decline. The fall in the real interest rate induces a declining path in consumption. In this way, demand next period is weaker than demand today, validating the initial expectation of higher future unemployment.

Importantly, if the dynamics triggered by the initial revision in inflationary expectations converge, the convergence point is the unemployment steady state, characterized in proposition 2, featuring involuntary unemployment and a zero nominal interest rate. The following proposition states this result more formally.

Proposition 6 (Convergence To A Liquidity Trap With Unemployment)**.** *Suppose assumptions 1 and 2 are satisfied and that $\xi_t = 0$ and deterministic for $t \geq 0$. Further, assume that $\pi_0 < \pi^*$. Then, if inflation and unemployment converge, they converge to the unemployment steady state $(\pi, R, u) = (\tilde{\beta}, 1, \bar{u})$ of proposition 2.*

Proof: See Appendix A.

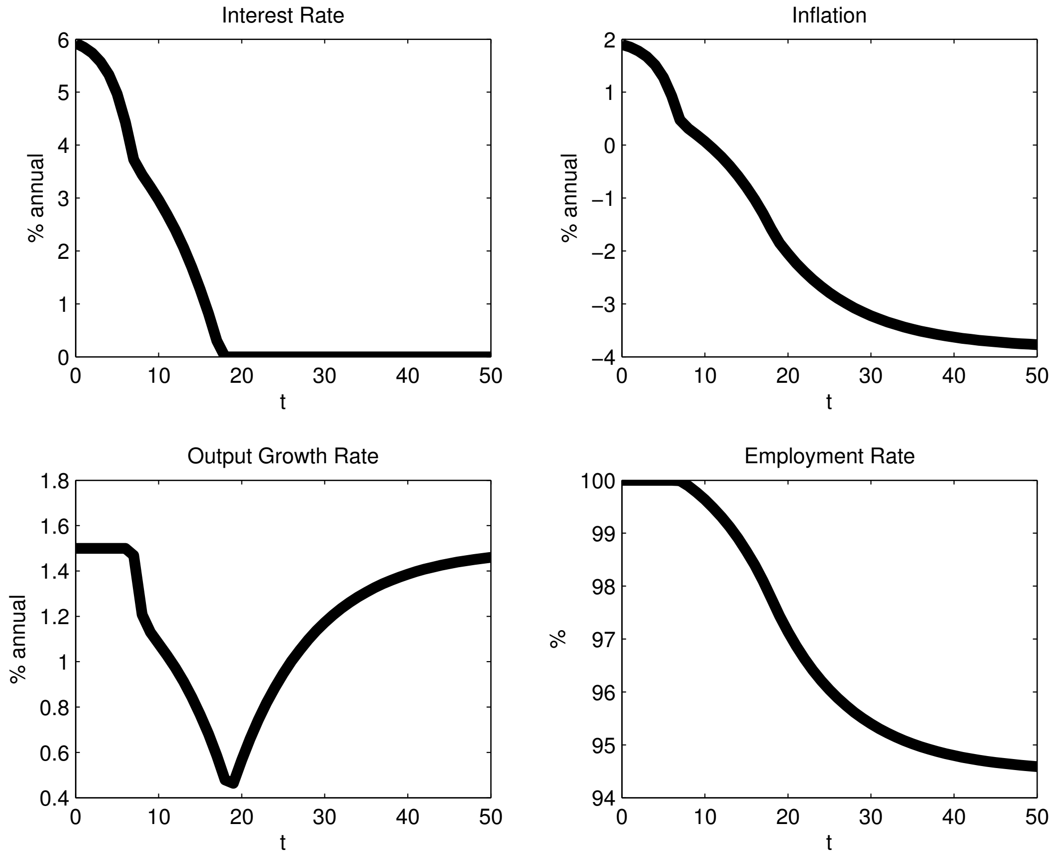
Clearly, if the economy converges to the unemployment steady state, then in the long run output grows at the rate μ. This is the same rate of growth as the one observed in the intended steady state. This means that output growth fully recovers whereas employment does not. It then follows immediately that the present model predicts a jobless recovery.

To illustrate the dynamics set in motion by a lack of confidence shock, we simulate a calibrated version of the model. The simulation also serves to confirm the possibility of convergence to the unemployment steady state. It is, however, not meant to match actual data, but simply as an illustration of the analytical results derived thus far. We assume that a time period is one quarter. We set $\sigma = 2$, which is a standard value in the business-cycle literature. We assume a labor share of 75 percent, which corresponds to setting $\alpha = 0.75$. We assign a value of $1.015^{1/4}$ to μ, to match the average growth rate of per capita output in the postwar United States. We set $\tilde{\beta} = 1.04^{-1/4}$, a value consistent with a long-run real interest rate of 4 percent per year. We normalize the time endowment to unity by setting $\bar{h} = 1$. Following standard parameterizations of Taylor rules in developed countries, we assume that the central bank has an inflation target of 2 percent per year ($\pi^* = 1.02^{1/4}$), and that the inflation and output coefficients of the interest-rate feedback rule take on the values suggested in Taylor (1993), that is, $\alpha_\pi = 1.5$ and $\alpha_y = 0.125$.

Two novel parameters of our model are γ_0 and γ_1 governing the degree of downward nominal wage rigidity. We set $\gamma_0 = \pi^*$. This implies that at the intended steady state, nominal wages are indexed at a rate at least as large as average inflation π^*. We note, however, that at the intended steady state this constraint is not binding because, due to the presence of productivity growth, nominal wages grow at the gross rate $\pi^*\mu > \pi^*$. To calibrate the parameter γ_1 governing the elasticity of wage rigidity with respect to unemployment, we assume that at an unemployment rate of 5 percent, nominal wages can fall frictionlessly by 2 percent per year, that is, we impose the restriction $0.98^{1/4} = \gamma(0.05)$. This calibration restriction is motivated by the observation that during the great contraction of 2008, the peak unemployment rate, reached in October 2009, was 10 percent, which represents an increase of 5 percent over the average unemployment rate observed in the pre-recession quinquennium. At the same time, during the great contraction of 2008 nominal hourly compensation costs in manufacturing did not fall. Our assumption that nominal wages can fall by 2 percent per year when unemployment is 5 percent lies, therefore, on the conservative side. The implied value of γ_1 is 0.1942.

Figure 3 displays the equilibrium dynamics triggered by a period-0 revision in expectations that results in an initial inflation rate 10 annual basis points below the target rate π^*, that is $\pi_0 = 1.019^{1/4}$. To compute the perfect-foresight equilibrium path, we follow the steps described in the proof of proposition 5. In particular, we solve for the exact dynamics of the nonlinear model. The figure shows that after the initial loss of confidence inflation starts drifting down. As a response, the monetary authority, following the dictum of the Taylor rule, lowers the nominal interest rate. Agents interpret the lowering of interest rates as a signal of lower future inflation, that is, the Fisher effect dominates in spite of the strong

Figure 3: A Great Contraction With A Jobless Recovery



Keynesian structure of the model. In turn, these expectations are validated, leading, via the Taylor rule, to a further round of easing. Initially, because the lower bound on nominal wages is not binding, the fall in inflation has no effect on the labor market and the economy continues to operate at full employment. However, after 8 quarters inflation falls below the threshold $\gamma(0)/\mu$. At this point the lower bound on nominal wage growth begins to bind and involuntary unemployment emerges. The presence of unemployment puts additional downward pressure on nominal rates through the output term of the Taylor rule. As time goes by, the fall in inflation tightens the wage constraint further causing more unemployment. At some point, the nominal interest rate hits its own lower bound of 0. From this point on, inflation continues to fall monotonically toward its long-run unintended steady-state value of $\tilde{\beta}$. In this deflationary environment, real wages continue to experience undesired growth causing further employment losses.[3] Consequently, the rate of unemployment in-

[3]This prediction of our model is consistent with the empirical findings of Daly, Hobijn, and Lucking (2012). These authors document that real wage growth stayed relatively solid during the 2008 U.S. recession. Furthermore, they show that real wage growth slowed substantially in the previous four recessions, making the behavior of real wages during the 2008 recession somewhat atypical.

creases monotonically and eventually converges to $\overline{u}$, which under the present calibration equals 5.5 percent.[4] Unlike the dynamics of inflation and unemployment, the dynamics of output growth are non-monotonic. Initially, because unemployment grows at an accelerating rate, output growth falls, reaching a trough in period 19, which coincides with the economy reaching the liquidity trap. As the rate of unemployment approaches its unintended steady-state level, output growth fully recovers to the rate of technological progress, μ, observed prior to the recession. However, this recovery is jobless, in the sense that unemployment remains high at its steady-state value of 5.5 percent. Comparing the dynamics of output growth, employment, the nominal interest rate, and expected inflation shown in figures 1 and 3 suggests that the predictions of our model are qualitatively in line with actual data. This finding gives some credence to the hypothesis that the great contraction of 2008 was at least in part the result of an unanchoring of inflationary expectations in the context of an economy with downward nominal wage rigidity, a Taylor rule, and a zero bound on nominal rates. To highlight the importance of nonfundamental loss of confidence shocks in generating jobless recoveries, we next analyze the dynamics triggered by fundamental shocks.

5 Contractions with Job-Creating Recoveries

In this section, we characterize unemployment and inflation dynamics when inflationary expectations are well anchored. By well-anchored inflationary expectations we mean environments in which agents expect inflation to converge toward its target level π^*. We show that when inflationary expectations are well anchored, a large negative fundamental demand shock, modeled as a decline in the natural rate of interest, causes deflation and unemployment on impact. More importantly the key distinguishing characteristic of the adjustment when inflationary expectations are well anchored is that recoveries feature both output and employment growth. This is in sharp contrast to the dynamics triggered by a negative confidence shock, studied in section 4, which are characterized by a jobless recovery and the expectation that the economy will continue to be afflicted by deflation and unemployment in the future.

As much of the recent related literature on liquidity traps (e.g., Eggertsson and Woodford, 2003), we focus on disturbances to the natural rate of interest. As in the previous section, to preserve analytical tractability, we limit attention to perfect foresight equilibria. We first characterize the response of the model economy to purely temporary negative shocks to the natural real rate of interest and later consider the response to more persistent shocks. In

[4] One can show analytically that if $R_t = 1$ and $u_t > 0$, then unemployment converges monotonically to its unintended steady state value $\overline{u}$.

our theoretical environment, the natural rate of interest, defined as the real interest rate that would prevail in the absence of nominal rigidities, is given by $\tilde{\beta}^{-1}e^{\xi_t-\xi_{t+1}}$. A purely temporary negative natural-rate shock is a situation in which at $t=0$, it is unexpectedly learned that $\xi_0-\xi_1<0$ and that $\xi_t=\xi_{t+1}$ for all $t\geq 1$. Without loss of generality, we model a temporary decline in the natural rate of interest by setting $\xi_0<0$ and $\xi_t=0$ for all $t\geq 1$. The path of the natural rate of interest is then given by $\tilde{\beta}^{-1}e^{\xi_0}<\tilde{\beta}^{-1}$ in period 0, and $\tilde{\beta}^{-1}$ for all $t>0$.

The following definition gives a precise meaning t the concept of perfect foresight equilibria with well anchored inflationary expectations in the present context.

Definition 1 (Equilibria With Well-Anchored Inflationary Expectations)**.** *Suppose that ξ_t is deterministic and that $\xi_t=0$ for all $t\geq T$, for some $T>0$. Then, a perfect-foresight equilibrium with well-anchored inflationary expectations is a perfect-foresight equilibrium in which π_t satisfies $\lim_{t\to\infty}\pi_t=\pi^*$.*

The present model displays strikingly different responses to negative natural rate shocks depending on their magnitude. The interest-rate feedback rule in place can preserve full employment in response to small negative natural rate shocks by an appropriate easing of the nominal interest rate. But if the negative natural rate shock is large, then the Taylor rule in place is unable to stabilize the economy and involuntary unemployment emerges.

The following proposition gives a lower bound for the set of natural rate shocks that can be fully neutralized, in the sense that they do not cause unemployment. We refer to natural rate shocks satisfying this bound as small. The proposition also shows that when inflationary expectations are well anchored, small negative natural rate shocks generate a temporary decline in inflation below its target level, π^*.

Proposition 7 (Full Employment Under Small Negative Natural Rate Shocks)**.** *Suppose that assumptions 1 and 2 hold, that $w_{-1}=F'(\bar{h})$, that*

$$1>e^{\xi_0}\geq\frac{\tilde{\beta}}{\pi^*}\max\left\{1,\frac{\pi^*}{\tilde{\beta}}+\alpha_\pi\left(\frac{\gamma(0)}{\mu}-\pi^*\right)\right\},\tag{19}$$

and that $\xi_t=0$ for all $t\geq 1$. Then, there exists a unique perfect foresight equilibrium with well-anchored inflationary expectations. Furthermore, the equilibrium features $u_t=0$ for all $t\geq 0$, $\frac{\gamma(0)}{\mu}\leq\pi_0<\pi^*$, and $\pi_t=\pi^*$ for all $t>0$.

Proof: See Appendix A.

The reason small negative shocks do not cause unemployment is that they can be fully accommodated by a downward adjustment in the nominal interest rate equal in size to the decline in the natural rate. Since the nominal interest rate cannot fall below zero, it follows immediately that one limit to accommodating negative shocks to the natural rate is the zero bound itself. But the present model delivers an additional limit to the ability of a Taylor rule to stabilize natural-rate shocks. Specifically, our model implies that inflation cannot fall below $\gamma(0)/\mu$ without causing unemployment. This threshold arises from the presence of downward nominal wage rigidity and may become binding before nominal interest rates hit the zero lower bound. If the inflation rate necessary to accommodate the exogenous decline in the natural rate is below $\gamma(0)/\mu$, then the real wage will rise above its market clearing level causing involuntary unemployment.

The maximum natural rate shock that the monetary authority can fully offset by its interest rate policy depends on the characteristics of the interest rate feedback rule, especially the inflation coefficient α_π and the inflation target π^*. If the monetary policy stance is aggressive, that is, if α_π is sufficiently large, then the monetary authority can lower the nominal interest rate down to zero without requiring current inflation below $\gamma(0)/\mu$, that is, without raising real wages in the current period. Under such monetary policy, the maximum natural rate shock the central bank can offset is one in which the natural rate is equal to the negative of the inflation target π^*. Hence, the larger is the inflation target, the larger is the range of negative shocks to the natural rate that the central bank can stabilize.

Consider now large negative shocks to the natural rate, that is, values of ξ_0 that violate condition (19). The following proposition shows that if the negative natural rate shock is large, the Taylor rule fails to preserve full employment.

Proposition 8 (Unemployment Due To Large Negative Natural Rate Shocks)**.** *Suppose that assumptions 1 and 2 hold, that* $w_{-1} = F'(\bar{h})$, *that*

$$e^{\xi_0} < \frac{\tilde{\beta}}{\pi^*} \max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi \left(\frac{\gamma(0)}{\mu} - \pi^*\right)\right\} \tag{20}$$

and that $\xi_t = 0$ *for all* $t \geq 1$. *Then, in any perfect-foresight equilibrium with well-anchored inflationary expectations the economy experiences unemployment in period 0, that is,* $u_0 > 0$.

Proof: See Appendix A.

To see why a large negative shock to the natural rate causes unemployment, it is of use to first understand how the optimal monetary policy (i.e., one that ensures full employment at all times) would react to such a shock. As in the case of small natural rate shocks, optimal policy calls for lowering the nominal interest rate in tandem with the decline in the natural rate. In this way, the real rate of interest can fall without igniting a future inflationary upward spiral. However, by the Taylor rule the easing of current nominal rates must be accompanied by a fall in the current inflation rate. The latter in turn, if sufficiently large, drives up real wages in the current period, causing involuntary unemployment. A second impediment to preserving full employment in response to large negative shocks to the natural rate is the zero bound on nominal interest rates. This is because the required decline in the nominal interest rate that keeps the real interest rate equal to the natural rate without causing a rise in expected inflation may imply a negative nominal interest rate. If this is the case, then unemployment must necessarily emerge.

A central prediction of our model is that when inflationary expectations are well anchored, the incidence of involuntary unemployment is transitory and that the recovery is accompanied by job creation. Specifically, after the shock unemployment converges monotonically back to zero in finite time. In other words, our model predicts that jobless recoveries are impossible when inflationary expectations are well anchored. The following proposition formalizes this result.

Proposition 9 (Recoveries With Job Creation)**.** *Suppose that assumptions 1 and 2 hold, that* $w_{-1} = F'(\bar{h})$*, that condition (20) holds, and that* $\xi_t = 0$ *for all* $t \geq 1$*. Then, in any perfect-foresight equilibrium with well-anchored inflationary expectations unemployment converges monotonically to zero in finite time. That is,* $u_{t+1} \leq u_t$ *for all* $t \geq 0$ *and there exists a date* $T > 0$ *such that* $u_{T+j} = 0$ *for all* $j \geq 0$*.*

Proof: See Appendix A.

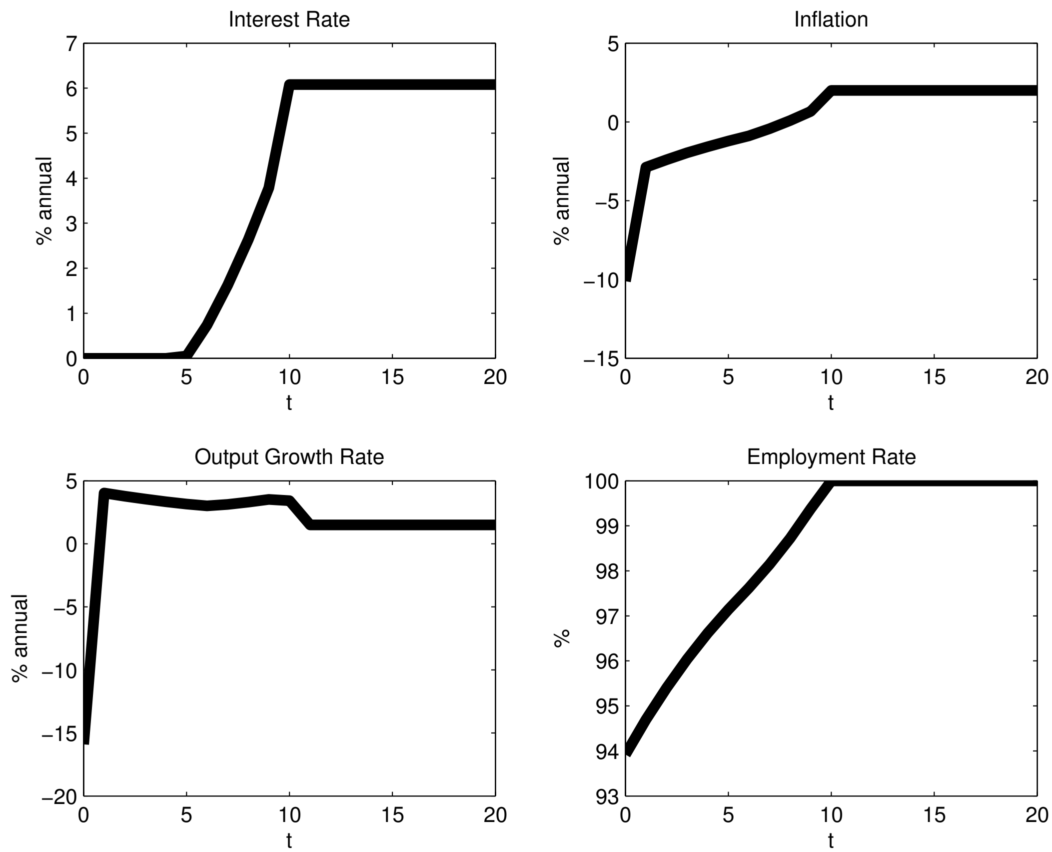
Furthermore, it is possible to show that the monetary authority lowers the nominal interest rate in the period in which the economy is hit by the large negative natural rate shock, but immediately thereafter raises it above its target level $R^*(\equiv \frac{\pi^*}{\beta})$. Remarkably, the tightening of policy occurs in an environment in which the economy has not yet fully recovered from the negative natural rate shock. During this transition, involuntary unemployment persists because the real wage is still above the level consistent with full employment. Because of downward nominal wage rigidity, the only way to reduce real wages quickly is to engineer temporarily higher price inflation. To this end, the central bank raises nominal rates to induce, through the Fisher effect, an elevation in the expected rate of inflation. The following proposition establishes these results.

Proposition 10 (Inflation and Interest Rate Dynamics)**.** *Suppose that assumptions 1 and 2 hold, that* $w_{-1} = F'(\bar{h})$*, that condition (20) holds, and that* $\xi_t = 0$ *for all* $t \geq 1$*. Then, in any perfect-foresight equilibrium with well-anchored inflationary expectations* $\pi_0 < \pi^*$*,* $\pi_t > \pi^*$ *for* $0 < t < T$ *and* $\pi_t = \pi^*$ *for all* $t \geq T$*, where* T *is defined in proposition 9. Further* $R_0 < R^*$*,* $R_t > R^*$ *for* $0 < t < T$*, and* $R_t = R^*$ *for* $t \geq T$*.*

Proof: See Appendix A.

We have fully characterized analytically the dynamics triggered by a purely temporary decline in the natural rate of interest and have established that the associated contraction features a recovery with job creation. A natural question, however, is whether this result is robust to allowing for persistence in the negative natural-rate shock. To shed light on this issue we perform a numerical simulation of the model. As in the simulation of the economy under a confidence shock, here we trace numerically the exact dynamics of the original nonlinear model. Figure 4 depicts the response of the model economy to a persistent decline in the natural rate of interest. Specifically, we assume that in period 0 the natural rate falls

Figure 4: A Contraction With A Job-Creating Recovery: Response To A Persistent Decline In the Natural Rate



from its long-run level of 4 percent per year to -2 percent per year and stays at that level for 10 quarters. At that point, the natural rate returns permanently to its steady-state level of 4 percent. We assume that the behavior of the natural rate is deterministic. Formally, we have that $\xi_t - \xi_{t+1} = \ln(0.98^{1/4}\tilde{\beta})$ for $t = 0, \ldots, 9$ and $\xi_t - \xi_{t+1} = 0$ for $t \geq 10$. We pick the size and duration of the natural-rate shock following Eggertsson and Woodford (2003).[5] All structural parameters of the model are as in the calibration presented in section 4.

The persistent natural-rate shock produces an initial reduction in output growth, involuntary unemployment, deflation, and interest rates up against the zero bound. However, contrary to what happens under a confidence shock, the recovery from the negative natural-rate shock features growth in both employment and output. That is, the recovery is characterized by job creation. Further, both output and employment begin to recover immediately after period 0. By contrast, the nonfundamental confidence shock generates a protracted slump.

6 Exiting The Slump: An Interest-Rate Peg

In this section, we consider a monetary policy that succeeds in re-anchoring inflationary expectations when the economy finds itself in a liquidity trap with high unemployment due to lack of confidence. Specifically, we argue that an interest rate peg that raises the interest rate from 0 to its intended target level can jump-start the economy and bring it quickly to the intended steady state with full employment.

A natural question is whether an interest-rate peg of this type would not make matters initially worse by pushing inflation further down and thereby creating more unemployment. We will show that the answer to this question is no. On the contrary, raising the interest rate from zero to its intended target lifts agents' expectations about future inflation. In turn, the expectation of a higher future rate of inflation erodes expected real wages, thereby facilitating employment growth.

The interest-rate-based exit strategy we wish to consider is as follows. Let s_t be a binary variable that takes the value 1 if the nominal interest rate has fallen to zero in the past. Formally,

$$s_t = \begin{cases} 1 & \text{if } R_j = 1 \text{ for any } 0 \leq j < t \\ 0 & \text{otherwise} \end{cases}.$$

Then, the proposed interest-rate-based exit strategy is

$$R_t = \begin{cases} \max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi (\pi_t - \pi^*) + \alpha_y \ln\left(\frac{F(h_t)}{F(\bar{h})}\right)\right\} & \text{if } s_t = 0 \\ R^* & \text{otherwise} \end{cases}.$$

Our assumption of a permanent switch to an interest rate peg is made for simplicity. In

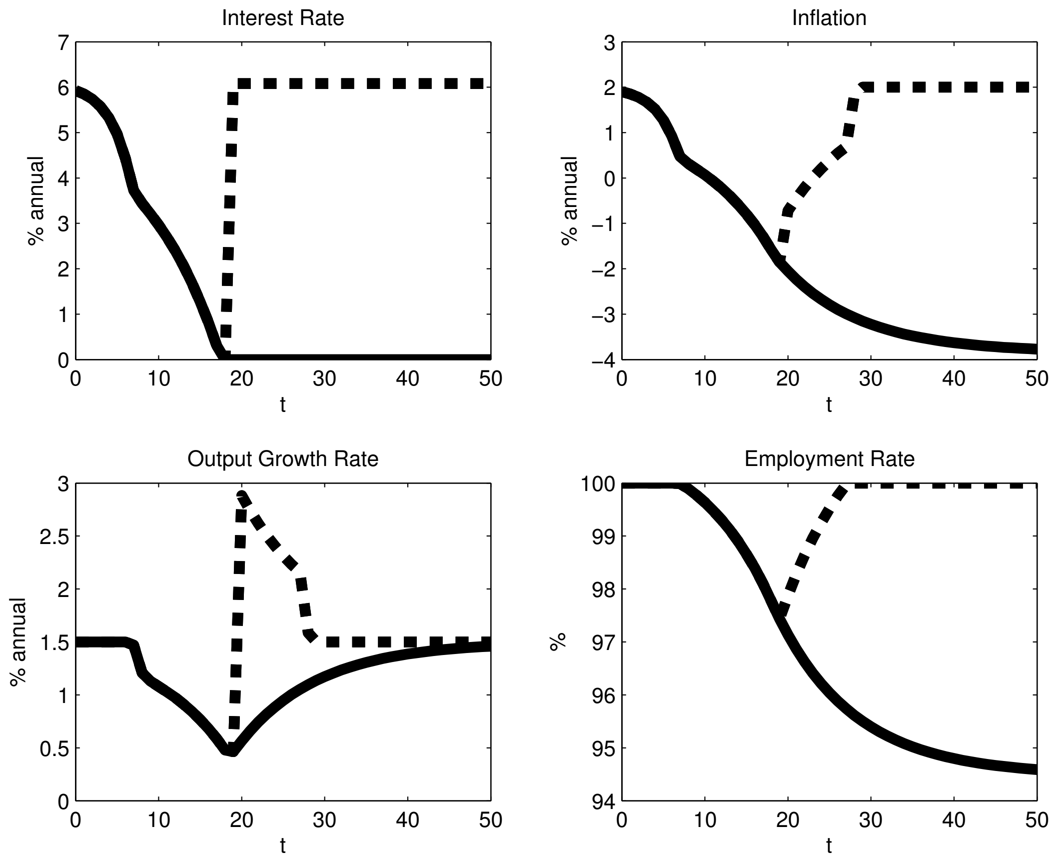
[5] These authors assume that the natural-rate shock is stochastic and has an average duration of 10 quarters and an absorbent state of 4 percent.

practice, the central bank could switch back to a Taylor rule once the intended steady state has been reached and inflationary expectations are well anchored again.[6]

6.1 The Exit Strategy And Confidence Shocks

As in section 4, we consider a lack of confidence shock that lowers the initial rate of inflation 10 annual basis points below the target rate π^*, that is, we assume that $\pi_0 = 1.019^{1/4}$. Figure 5 displays with dashed lines the equilibrium dynamics implied by this nonfundamental

Figure 5: Dynamics Effects of a Nonfundamental Shock Under the Exit Strategy



Solid Line: Taylor Rule Dashed Line: Exit Strategy

shock under the exit strategy. To facilitate comparison, the figure reproduces from figure 3

[6]Consider, for instance, the following alternative definition of s_t:

$$s_t = \begin{cases} 1 & \text{if } R_{t-1} = 1 \\ 0 & \text{if } \pi_{t-1} \geq \pi^* \\ s_{t-1} & \text{otherwise} \end{cases},$$

for $t \geq 0$, with $s_{-1} = 0$.

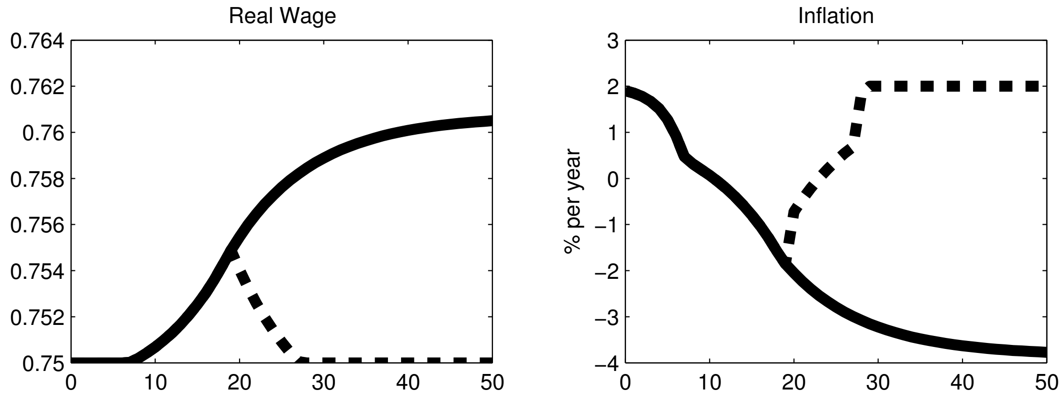
with solid lines the response of the economy when monetary policy is always dictated by the Taylor rule given in equation (10).

A remarkable feature of the equilibrium dynamics under the unconventional exit policy studied here is that in spite of the significant increase in interest rates (from 0 to 6 percent per annum) that takes place in period 19 and in spite of the fact that the underlying theoretical framework is highly Keynesian in nature, when the exit strategy is put in place, the economy suffers neither a drop in output nor an increase in unemployment. On the contrary, the economy starts to recover immediately. As policy switches to the interest-rate target, output growth jumps above its long-run rate and unemployment begins a monotonic decline to zero. The economy reaches the intended steady state ($\pi = \pi^*$ and $u = 0$) in finite time. That is, the exit strategy eliminates the liquidity trap and turns what would have been a jobless recovery into a job-creating one.

This finding suggests that monetary policy plays a crucial role in determining whether a recovery is jobless or not. Figure 5 shows two monetary polices, one associated with a jobless recovery and the other with a job-creating one. This is an important difference between the model of a jobless recovery presented in this paper and alternative explanations based upon real models with real wage rigidity (see, for instance, Shimer, 2012). By construction, in real models monetary policy plays no role. Existing empirical evidence suggests that the stance of monetary policy does matter for the occurrence of jobless recoveries. For example Calvo, Coricelli, and Ottonello (2012) study recession episodes that follow financial crises. Their sample includes 95 recession episodes in developed and developing countries. They show that inflation determines whether the recovery is jobless or not. Low inflation is associated with jobless recoveries whereas high inflation is not. Our model is consistent with this finding. The top right panel of figure 5 shows that the jobless recovery occurs in a low-inflation environment, whereas the job-creating recovery occurs in an environment with relatively higher inflation. Calvo, Coricelli, and Ottonello also report empirical evidence suggesting that high-inflation recoveries are associated with falling real wages, whereas low-inflation recoveries are not. The present model is also in line with this finding. Figure 6 displays the response of real wages and inflation to a lack-of-confidence shock under the Taylor rule (solid lines) and under the exit strategy (dashed lines). The recovery under the exit strategy features relatively higher inflation and lower real wage growth than the recovery under the Taylor rule. Under the Taylor rule, real wages rise by more than productivity throughout the recovery, which causes the recovery to be jobless. By contrast under the exit strategy real wages decline, thus ushering in employment growth.[7]

[7]The magnitude of the wage increases under the Taylor rule (about 1.5 percent), might seem small. Recall, however, that our assumed labor share of 75 percent ($\alpha = 0.75$) implies that the wage elasticity of

Figure 6: Response of Real Wages and Inflation to a Nonfundamental Shock Under the Exit Strategy



Solid Line: Taylor Rule   Dashed Line: Exit Strategy

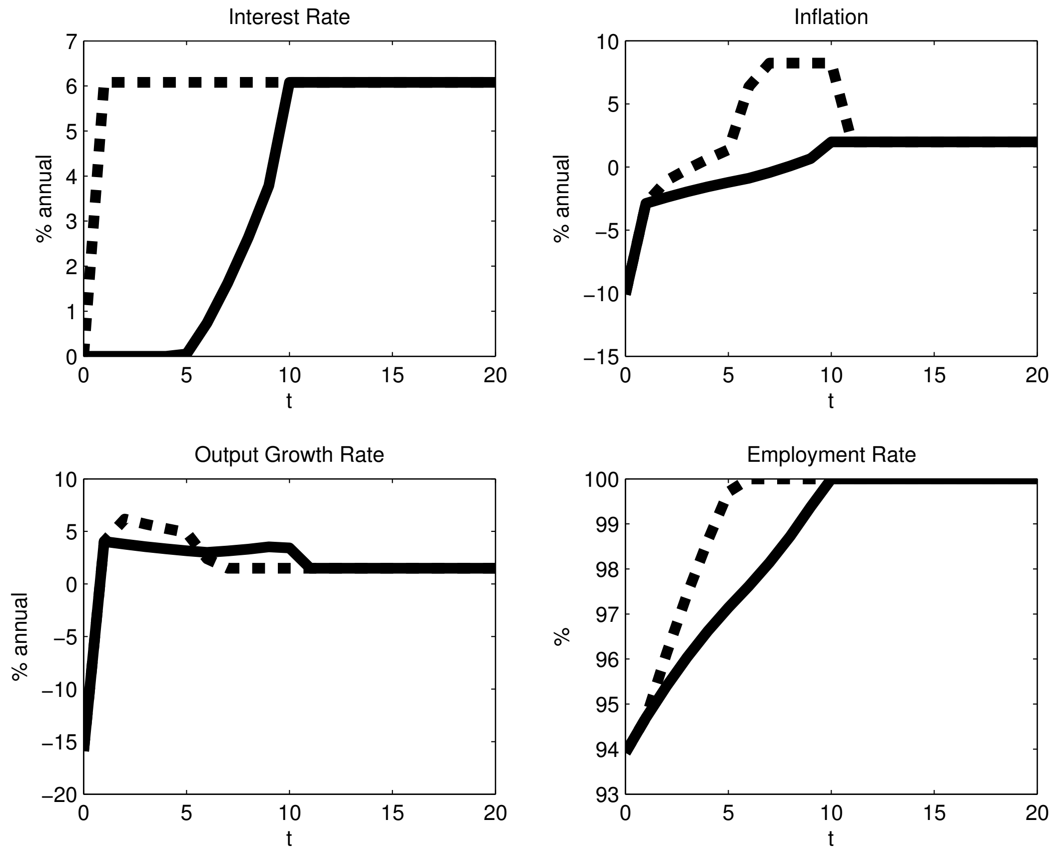
We wish to note that the economy can recover from a confidence-shock-induced liquidity trap without any change in monetary policy, i.e., with the Taylor rule in place. This would be the case if households' experienced a positive revision in inflationary expectations away from $\tilde{\beta}$ and toward the inflation target π^*. The peril posed by the Taylor rule, however, is that it leaves the door always open for expectations of a slump to be self-fulfilling. By contrast, the central property of the escape strategy considered here is that it forces people's inflationary expectations to be π^* thereby eliminating the possibility of protracted self-fulfilling liquidity traps.

6.2 The Exit Strategy And Natural-Rate Shocks

Finally, we consider the performance of the exit strategy studied here when the crisis is not caused by a nonfundamental lack-of-confidence shock but rather by a fundamental exogenous persistent fall in the natural rate identical to the one studied in section 5. When the shock is fundamental, applying the exit strategy has an advantage and a disadvantage relative to the Taylor rule. The advantage is that the exit strategy, by quickly raising inflationary expectations, can return the economy to full employment more rapidly. The disadvantage is that the interest-rate peg leaves the initial rates of inflation and unemployment indeterminate. To illustrate the point that the effect of a negative natural-rate shock need not be worse under the exit strategy than under the Taylor rule, figure 7 displays the response of the economy under the exit strategy assuming that π_0 takes the same value as under the Taylor rule. By design, the initial inflation and unemployment rates must be identical under both

labor demand is -4 (or $1/(\alpha - 1)$), which implies that an increase in the real wage of 1.5 percent above its market-clearing level causes an increase in the unemployment rate of 6 percent.

Figure 7: Dynamics Effects of a Fundamental Shock Under the Exit Strategy



Solid Line: Taylor Rule Dashed Line: Exit Strategy

policy regimes. However, under the exit strategy the inflation rate overshoots and converges to π^* from above. This brings real wages down contributing to a faster recovery in the labor market. By contrast, under the Taylor rule inflation converges to π^* from below, implying a slower decline in real wages, and hence a more prolonged period of unemployment. We conclude that the interest-rate based strategy to escape liquidity traps presented here may be beneficial even if the cause of the crisis is fundamental in nature.

7 Conclusion

This paper is motivated by the observation that the U.S. great contraction of 2008 was associated with a jobless recovery, declining inflation expectations, and a liquidity trap. The main contribution of this paper is to develop a model that captures these facts. The main elements of the model are downward nominal wage rigidity, a Taylor-type interest-rate rule, the zero bound on interest rates, and a lack-of-confidence shock.

In the model, a negative confidence shock leads agents to expect a slowdown in future inflation. Because nominal wages are downwardly rigid, this nonfundamental revision of expectations makes agents anticipate real wages to rise above the full employment level and unemployment to emerge. In turn, the expected rise in unemployment depresses current demand via a negative wealth effect. The weakness in current aggregate demand causes inflation to fall in the current period. The monetary authority, following the Taylor rule, reacts to the slowdown in current inflation by lowering interest rates. This policy action closes the circle, because, by the Fisher effect, the decline in the interest rate signals a fall in future inflation, validating agents' initial lack of confidence.

The dynamics triggered by fundamental shocks are quite different and, in particular, do not capture the empirical fact of a jobless recovery and a protracted liquidity trap. The paper shows that the behavior of output growth and unemployment inherit the stochastic properties of the underlying fundamental disturbance. This means that the recovery in output growth is accompanied by job creation. Put differently, in the proposed model fundamental shocks fail to generate a jobless recovery. The reason is that if inflationary expectations are well anchored (i.e., if agents expect the rate of inflation to converge to its intended target), as soon as the negative shock begins to fade away, inflation picks up driving real wages down to their full employment level. Another important prediction of the model is that when aggregate uncertainty is fundamental in nature, the liquidity trap, contrary to what was observed during the 2008 U.S. great contraction, is short lived. The reason is that when inflationary expectations are well anchored, inflationary expectations themselves and hence the nominal interest rate tend to gravitate toward their intended target levels.

Finally, the paper identifies an interest-rate-based strategy for escaping the liquidity trap and restoring full employment. It consists in pegging the nominal interest rate at its intended target level. The rationale for this strategy is the recognition that in a confidence-shock-induced liquidity trap the effects of an increase in the nominal interest rate are quite different from what conventional wisdom would dictate. In particular, unlike what happens in normal times, in an expectations driven liquidity trap the nominal interest rate moves in tandem with expected inflation. Therefore, in the liquidity trap an increase in the nominal interest rate is essentially a signal of higher future inflation. In turn, by its effect on real wages, future inflation stimulates employment, thereby lifting the economy out of the slump.

8 Appendix A: Proofs of Propositions

8.1 Proof of Proposition 1 (Existence and Uniqueness of the Full-Employment Steady State)

Set $u = 0$. Then, $h = \bar{h}$, $c = F(\bar{h})$, and $w = F'(\bar{h})$ are the unique solutions to (12), (13), and (17). Also, conditions (14) and (16) are satisfied. Combining (11) and (18), we have that π solves either

$$\frac{\pi}{\tilde{\beta}} = \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi(\pi - \pi^*)$$

or

$$\frac{\pi}{\tilde{\beta}} = 1.$$

By assumption 2, the first expression admits a unique solution given by $\pi = \pi^*$. The second expression also admits a unique solution given by $\pi = \tilde{\beta}$. By equation (15), π must satisfy $\pi \geq \frac{\gamma(0)}{\mu}$. According to assumptions 1 and 2, $\pi = \pi^*$ satisfies this condition but $\pi = \tilde{\beta}$ does not. When $\pi = \pi^*$, by assumption 2, $\max\{1, \pi^*/\tilde{\beta} + \alpha_\pi(\pi - \pi^*)\} = \pi^*/\tilde{\beta}$, which implies that $\pi = \pi^*$ and $R = \pi^*/\tilde{\beta}$ solve conditions (11) and (18).■

8.2 Proof of Proposition 2 (Uniqueness of the Unemployment Steady State)

Set $\bar{u} > 0$. From (17) we have that $h < \bar{h}$. Combining (11) and (18), we have that the rate of inflation is determined by the solution to either

$$\frac{\pi}{\tilde{\beta}} = \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\pi - \pi^*\right) + \alpha_y \ln\left(\frac{F(h)}{F(\bar{h})}\right)$$

or

$$\frac{\pi}{\tilde{\beta}} = 1.$$

The solution to the first expression must satisfy $\pi > \pi^*$. This follows directly from the assumption that $\alpha_\pi\tilde{\beta} > 1$ and the fact that $\alpha_y \ln\left(\frac{F(h)}{F(\bar{h})}\right)$ is negative. But $\pi > \pi^*$ cannot be the inflation rate at the unemployment steady state. To see this, note that by the slackness condition (16), condition (15) must hold with equality. That is, $\pi = \gamma(\bar{u})/\mu < \gamma(0)/\mu < \pi^*$. The last inequality follows from assumption 2. Therefore, π cannot exceed π^*. This implies that if an unemployment steady state exists, it must be a liquidity trap, $R = \pi/\tilde{\beta} = 1$. The unemployment rate in this steady state is determined by condition (15) holding with equality and evaluated at $\pi = \tilde{\beta}$. That is, $\bar{u}$ is given by the solution to $\tilde{\beta} = \gamma(\bar{u})/\mu$. The fact that $\bar{u} > 0$ follows from the assumptions that $\tilde{\beta} < \gamma(0)/\mu$ and that γ is decreasing in u. Finally, it remains to show that equation (18) holds when $\pi = \tilde{\beta}$. That is, it remains to show that $\max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\tilde{\beta} - \pi^*\right) + \alpha_y \ln\left(\frac{F(h)}{F(\bar{h})}\right)\right\} = 1$. To see this, note that

$$\begin{aligned}
\frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\tilde{\beta} - \pi^*\right) + \alpha_y \ln\left(\frac{F(h)}{F(\bar{h})}\right) &< \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\tilde{\beta} - \pi^*\right) \\
&= \left(\frac{\pi^*}{\tilde{\beta}} - 1\right)\left(1 - \alpha_\pi\tilde{\beta}\right) + 1 \\
&< 1.
\end{aligned}$$

This completes the proof.■

8.3 Proof of Proposition 3 (Inflation Dynamics Under Lack of Confidence)

In any period $t \geq 0$, the economy can be in one of the following five situations: (1) $u_{t+1} = 0$ and $R_t = 1$; (2) $u_{t+1} = 0$ and $R_t > 1$; (3) $u_{t+1} > 0$ and $u_{t+1} \geq u_t$; (4) $u_t > u_{t+1} > 0$ and $R_t = 1$; and (5) $u_t > u_{t+1} > 0$ and $R_t > 1$. The proof proceeds by establishing that for each of these situations, if the conditions of the proposition are satisfied, then $\pi_{t+1} < \pi^*$. Hence, in period $t+1$ the conditions of the proposition are again satisfied. This implies that $\pi_{t+j} < \pi^*$ for all $j > 0$.

Situation (1), $u_{t+1} = 0$ and $R_t = 1$: Because $u_{t+1} = 0$, it follows from conditions (6) and (9) that $h_{t+1} \geq h_t$ and by the assumed concavity of the utility function we have that $u'(F(h_{t+1}))/u'(F(h_t)) \leq 1$. Using (3) we have that π_{t+1} must satisfy $\pi_{t+1} = \tilde{\beta} R_t u'(F(h_{t+1}))/u'(F(h_t)) \leq \tilde{\beta} < \frac{\gamma(0)}{\mu} < \pi^*$, where the penultimate inequality follows from assumption 1 and the last one from assumption 2.

Situation (2), $u_{t+1} = 0$ and $R_t > 1$: Because $R_t > 1$, by (10), $\tilde{\beta} R_t - \pi_t = (\alpha_\pi \tilde{\beta} - 1)(\pi_t - \pi^*) - \alpha_y u_t < 0$. By (3), it then follows that $\pi_{t+1} = \tilde{\beta} R_t u'(F(h_{t+1}))/u'(F(h_t)) < \pi_t u'(F(h_{t+1}))/u'(F(h_t)) \leq \pi_t < \pi^*$, where the penultimate inequality follows from the fact that $h_{t+1} \geq h_t$ and the last inequality follows from the assumption that $\pi_t < \pi^*$.

Situation (3), $u_{t+1} > 0$ and $u_{t+1} \geq u_t$: The assumptions of situation (3) imply that $h_{t+1} \leq h_t$ and by the concavity of the production function that $F'(h_t)/F'(h_{t+1}) \leq 1$. Because $u_{t+1} > 0$, condition (7) must hold with equality so that $\pi_{t+1} = \frac{\gamma(u_{t+1})}{\mu} \frac{F'(h_t)}{F'(h_{t+1})} < \frac{\gamma(0)}{\mu} < \pi^*$, where the last inequality is implied by assumption 1.

Situation (4), $u_t > u_{t+1} > 0$ and $R_t = 1$: In this situation, $h_t < h_{t+1}$ and hence $F(h_t) < F(h_{t+1})$. This implies that $\frac{u'(F(h_{t+1}))}{u'(F(h_t))} < 1$. From equilibrium condition (3) we then have that $\pi_{t+1} = \tilde{\beta} R_t \frac{u'(F(h_{t+1}))}{u'(F(h_t))} < \tilde{\beta} < \frac{\gamma(0)}{\mu} < \pi^*$, where the penultimate inequality follows from assumption 1 and the last one from assumption 2.

Situation (5), $u_t > u_{t+1} > 0$ and $R_t > 1$: As in situation (4), we have that $h_t < h_{t+1}$ and hence $\frac{u'(F(h_{t+1}))}{u'(F(h_t))} < 1$. And as in situation (2), we have $R_t > 1$ and thus (10) implies that $\tilde{\beta} R_t < \pi_t$. Then use (3) to obtain $\pi_{t+1} = \tilde{\beta} R_t \frac{u'(F(h_{t+1}))}{u'(F(h_t))} < \tilde{\beta} R_t < \pi_t < \pi^*$.

It remains to show that there exists a finite T such that $\pi_T < \gamma(0)/\mu$. Suppose that for some $t \geq 0$, $u_t = 0$ and $u_{t+1} > 0$. Then, (7) implies that $\pi_{t+1} = \frac{\gamma(u_{t+1})}{\mu} \frac{F'(\bar{h})}{F'(h_{t+1})} < \frac{\gamma(u_{t+1})}{\mu} < \frac{\gamma(0)}{\mu}$. This shows the existence of a T such that $\pi_T < \gamma(0)/\mu$ for the case of a switch from full employment to unemployment. Suppose now that $u_t = 0$ for all t. Then, by (3) $R_t = \pi_{t+1}/\tilde{\beta}$. Using this expression to eliminate R_t from (10) yields $\pi_{t+1} = \max\{\tilde{\beta}, \pi^* + \alpha_\pi \tilde{\beta}(\pi_t - \pi^*)\}$. Because $\pi_t < \pi^*$ and because $\alpha_\pi \tilde{\beta} > 1$, we have that π_t reaches $\tilde{\beta} < \gamma(0)/\mu$ in finite time. This shows the existence of a T such that $\pi_T < \gamma(0)/\mu$. Finally, we must consider the case that $u_t > 0$ for all t. In this case, we prove the existence of a period T such that $\pi_T < \gamma(0)/\mu$ by contradiction. Suppose that $\pi_t \geq \gamma(0)/\mu$ for all t. We have already shown that under the assumptions of the proposition $\pi_{t+1} < \max\{\pi_t, \gamma(0)/\mu\}$. This means that as long as $\pi_t \geq \gamma(0)/\mu$, π_t must be monotonically decreasing over time. Since, by assumption π_t is bounded below by $\gamma(0)/\mu$, we have that π_t must converge. Let $\lim_{t\to\infty} \pi_t \equiv \underline{\pi} \geq \gamma(0)/\mu$. Since, by assumption, $u_t > 0$ for all t, (7) must hold with equality at all times, that is, $\pi_t = \frac{\gamma(u_t)}{\mu} \frac{F'(h_{t-1})}{F'(h_t)}$. Because $\pi_t \geq \gamma(0)/\mu > \gamma(u_t)/\mu$, we have that $F'(h_{t-1}) > F'(h_t)$. This means, by the fact that $u_t = (\bar{h} - h_t)/\bar{h}$ and by the concavity of F, that u_t is decreasing in t. Since u_t is bounded below by 0, we have that u_t must converge. Let $\underline{u} \equiv \lim_{t\to\infty} u_t$. Clearly, $0 \leq \underline{u} < 1$. The fact that $\underline{u} < 1$ implies that $\lim_{t\to\infty} F'(h_t)$ exists and is finite. Taking limits of (7) we obtain $\underline{\pi} = \lim_{t\to\infty} \frac{\gamma(u_t)}{\mu} \frac{F'(h_{t-1})}{F'(h_t)} = \frac{\gamma(\underline{u})}{\mu} \lim_{t\to\infty} \frac{F'(h_{t-1})}{F'(h_t)} = \frac{\gamma(\underline{u})}{\mu}$. Since $\underline{\pi} \geq \gamma(0)/\mu$ and since $\gamma(.)$ is strictly decreasing in u_t, we have that $\underline{u} = 0$. Using (10), we have that $\lim_{t\to\infty} \tilde{\beta} R_t = \max\{\tilde{\beta}, \pi^* + \alpha_\pi \tilde{\beta}(\underline{\pi} - \pi^*)\}$. Now by (3), we have that $\tilde{\beta} \lim_{t\to\infty} R_t = \underline{\pi} \lim_{t\to\infty} \frac{u'(F(h_t))}{u'(F(h_{t+1}))} = \underline{\pi}$. Combining these two expressions, we obtain $\underline{\pi} = \max\{\tilde{\beta}, \pi^* + \alpha_\pi \tilde{\beta}(\underline{\pi} - \pi^*)\}$. This expression has two solutions for $\underline{\pi}$, namely, $\tilde{\beta}$ and π^*. Because $\tilde{\beta} < \gamma(0)/\mu$ and because we have already shown that $\pi_t < \pi^*$ for all t, both of these solutions yield a contradiction.■

8.4 Proof of Proposition 4 (Chronic Involuntary Unemployment Under Lack of Confidence)

The proof is by contradiction. Suppose that for some $t \geq T$, $u_t = 0$. Then, by condition (7) $\pi_t \geq \frac{\gamma(0)}{\mu} \frac{F'(h_{t-1})}{F'(\bar{h})} \geq \frac{\gamma(0)}{\mu}$. This is a contradiction, because, by proposition 3, $\pi_t < \gamma(0)/\mu$.■

8.5 Proof of Proposition 5 (Existence and Uniqueness of Chronic Unemployment Equilibria)

Consider the equilibrium conditions (3)-(10). Set $\xi_t = 0$ for all t. Use (4) to eliminate c_t from (3) and (5) to eliminate w_t from (8) and (7). Then the set of perfect-foresight equilibrium conditions involves only deterministic sequences for $\{\pi_{t+1}, h_t, u_t, R_t\}_{t=0}^{\infty}$ given π_0 and $h_{-1} = \bar{h}$. The proof is by construction. Suppose first that the initial value for π_0 satisfies $\gamma(0)/\mu \leq \pi_0 < \pi^*$. First we show that in this case $u_0 = 0$. Suppose the contrary, that is, $u_0 > 0$. Then by (9) $h_0 < \bar{h}$, and by (7) $\pi_0 = \frac{\gamma(u_0)}{\mu} \frac{F'(\bar{h})}{F'(h_0)} < \frac{\gamma(u_0)}{\mu} < \frac{\gamma(0)}{\mu}$, where the penultimate inequality follows from the concavity of the function $F(.)$ and from $h_0 < \bar{h}$ and the last inequality follows from assumption 1. But this is a contradiction. It follows that $u_0 = 0$ and $h_0 = \bar{h}$. Then R_0 is uniquely determined by (10). We have therefore shown that there are unique values for h_0, u_0, and R_0 if $\gamma(0)/\mu \leq \pi_0 < \pi^*$. To find π_1 proceed as follows: Note that by (3) $\pi_1 = \tilde{\beta} R_0 \frac{u'(F(h_1))}{u'(F(\bar{h}))} \geq \tilde{\beta} R_0$. Now distinguish two cases: (i) $\tilde{\beta} R_0 \geq \gamma(0)/\mu$ and (ii) $\tilde{\beta} R_0 < \gamma(0)/\mu$. In case (i), we have $\pi_1 \geq \gamma(0)/\mu$. And as we just have shown if in any period t we have $h_{t-1} = \bar{h}$ and $\pi_t \geq \gamma(0)/\mu$, then the period t perfect foresight solution is unique and features $u_t = 0$. In case (ii)$\tilde{\beta} R_0 < \gamma(0)/\mu$. To rule out $u_1 = 0$, note that if u_1 were 0, then by (3) $\pi_1 = \tilde{\beta} R_0 < \gamma(0)/\mu$. At the same time by (7) $\pi_1 \geq \gamma(0)/\mu$. This results in a contradiction and it follows if a perfect foresight equilibrium exists, it must feature $u_1 > 0$. Next we show that in this case there exists a unique $u_1 > 0$. Combining (3) and (7) holding with equality yields $\frac{\gamma(u_1)}{\mu} = \tilde{\beta} R_0 \frac{u'(F(h_1))F'(h_1)}{u'(F(h_0))F'(h_0)}$. Notice that the left hand side of this expression is monotonically decreasing in u_1 and that at $u_1 = 0$ it takes the value $\gamma(0)/\mu$. The right hand side is monotonically increasing in u_1 and at $u_1 = 0$ it takes the value $\tilde{\beta} R_0 < \gamma(0)/\mu$. Hence at $u_1 = 0$ the left hand side exceeds the right hand side. Further as u_1 approaches 1, the right hand side approaches infinity, so that as u_1 approaches one, the left hand side is smaller than the right hand side. It follows that there is a single crossing, and therefore there exists a single value u_1 that solves this expression. Given u_1 use (9) to find h_1, (3) to find π_1, use (10) to find R_1. This shows that also in case (ii), there exist unique values for π_1, R_0, u_0, h_0 that satisfy conditions (3)-(10), given $\gamma(0)/\mu \leq \pi_0 < \pi^*$ and $h_{-1} = \bar{h}$.

To find values for π_{t+1}, R_t, u_t, h_t given h_{t-1} and π_t for $t \geq 1$, note that if $h_{t-1} = \bar{h}$ and if $\gamma(0)/\mu \leq \pi_t < \pi^*$, then we can use the same arguments as those used above to construct the perfect foresight equilibrium for period t. It follows that for $t = 1$, because we have already shown that $h_0 = \bar{h}$ and $u_0 = 0$, we only need to check whether $\pi_1 \geq \gamma(0)/\mu$. If so, then we can proceed with the construction of the equilibrium as above, that is, we obtain $u_1 = 0$, $h_1 = \bar{h}$, and R_1 and π_2 in the same fashion as described above.

If however, $\pi_t < \gamma(0)/\mu$, construct the perfect-foresight equilibrium as follows. By proposition 4, in this case $u_t > 0$. Given π_t use (7) holding with equality to find a unique value for u_t. To see that this condition has a unique solution write it as: $\pi_t F'(h_t)/F'(h_{t-1}) = \gamma(u_t)/\mu$. The left-hand side is increasing in u_t and converges to infinity as u_t approaches unity. At $u_t = 0$ it is less or equal to π_t and hence less than $\gamma(0)/\mu$. The right-hand side is monotonically decreasing. At $u_t = 0$ it is equal to $\gamma(0)/\mu$. Thus at $u_t = 0$ the left-hand side is smaller than the right hand side. Consequently, we must have a single crossing implying a unique value for u_t. With u_t in hand and π_t given, use (10) to find R_t. From proposition (3) we have that $\pi_{t+1} < \gamma(0)/\mu$ and from proposition 4 that $u_{t+1} > 0$. Combine (3) and (7) holding with equality to obtain $\frac{\gamma(u_{t+1})}{\mu} = \tilde{\beta} R_t \frac{u'(F(h_{t+1}))F'(h_{t+1})}{u'(F(h_t))F'(h_t)}$. Use this expression to find u_{t+1}. The left hand side of this expression is decreasing and at $u_{t+1} = 0$ equal to $\gamma(0)/\mu$. The right hand side is monotonically increasing in u_{t+1}. At $u_{t+1} = u_t > 0$ it is equal to $\tilde{\beta} R_t$. Notice that $\tilde{\beta} R_t \leq \max\{\tilde{\beta}, \pi_t\} < \gamma(0)/\mu$. Therefore, at $u_{t+1} = 0$, the right-hand side must be less than the left-hand side. As u_{t+1} approaches unity the right-hand side becomes infinity. Therefore, there exists a single crossing. This shows that u_{t+1} is unique. With u_{t+1} in hand use (3) to find π_{t+1}. We have therefore shown how to construct values for u_t, h_t, R_t, π_{t+1} in the case that $\pi_t < \gamma(0)/\mu$ for an arbitrary h_{t-1} and that the so constructed values are unique. We can proceed in this fashion to obtain $\{h_t, u_t, R_t, \pi_{t+1}\}_{t=0}^{\infty}$. ▪

8.6 Proof of Proposition 6 (Convergence To A Liquidity Trap With Unemployment)

By proposition 3, inflation is always strictly below its target level π^*. This means that the economy cannot converge to the full-employment steady state $(\pi, R, u) = (\pi^*, R^*, 0)$. Since the only remaining steady state is the unemployment steady state. The claim of the proposition must hold.▪

8.7 Proof of Proposition 7 (Full Employment Under Small Negative Taste Shocks)

First, we show that if condition (19) is satisfied, then an equilibrium with $u_t = 0$ for all $t \geq 0$, $\pi_0 < \pi^*$, and $\pi_t = \pi^*$ for $t > 0$ exists. The proof proceeds by showing that the proposed equilibrium path satisfies the complete set of equilibrium conditions, given by expressions (3)-(10) for all $t \geq 0$. Setting $h_t = \bar{h}$ for all $t \geq 0$ ensures that (6), (8), and (9) hold for all t. Then setting $c_t = F(\bar{h})$ for all $t \geq 0$ and $w_t = F'(\bar{h})$ ensures the satisfaction of (4) and (5) for all $t \geq 0$. Setting $R_0 = e^{\xi_0} \frac{\pi^*}{\tilde{\beta}}$ and $R_t = \frac{\pi^*}{\tilde{\beta}}$ for all $t > 0$ implies that (3) holds for all $t \geq 0$. Condition (19) guarantees that $R_0 \geq 1$. To ensure that (10) holds in period 0, set π_0 at a value satisfying $e^{\xi_0}\pi^*/\tilde{\beta} \geq \pi^*/\tilde{\beta} + \alpha_\pi(\pi_0 - \pi^*)$. That is, pick π_0 such that $\pi_0 \leq \pi^* + (e^{\xi_0} - 1)\frac{\pi^*}{\tilde{\beta}\alpha_\pi}$. Clearly, $\pi_0 < \pi^*$. At the same time, for (7) to be satisfied at $t = 0$, π_0 must take values in the range $\pi_0 \geq \gamma(0)/\mu$. Combining the above two inequalities we obtain that $\pi^* + (e^{\xi_0} - 1)\frac{\pi^*}{\tilde{\beta}\alpha_\pi} \geq \pi_0 \geq \gamma(0)/\mu$. Condition (19) guarantees that this interval is nonempty. Finally, assumption 2 guarantees that the proposed equilibrium satisfies (7) and (10) for all $t > 0$. This completes the proof of existence of an equilibrium with well-anchored inflationary expectations.

The proof of uniqueness proceeds by first establishing that any equilibrium with well-anchored expectations must feature $\pi_t \geq \pi^*$ for any $t > 0$. This result is a direct implication of Proposition 3, which shows that if for any $t > 0$, $\pi_t < \pi^*$, then $\pi_{t+j} < \pi^*$ for all $j > 0$. By definition, an equilibrium with such a path for inflation is not one with well-anchored inflationary expectations. The next step is to show that under the conditions given in the proposition $u_0 = 0$. This result is implied by lemma 1 in Appendix B. It also follows that $h_0 = \bar{h}$ and that $w_0 = F'(\bar{h})$. Then lemmas 2 and 3 in Appendix B show that in any equilibrium with well-anchored inflationary expectations $u_t = 0$ for all $t > 0$ and that $\pi_t = \pi^*$ for all $t > 0$, respectively. It remains to be shown that π_0 is unique. Evaluating (3) at $t = 0$ yields a unique value of R_0, given by $R_0 = e^{\xi_0}\pi^*/\tilde{\beta}$. Since the event $e^{\xi_0}\pi^*/\tilde{\beta} = 1$ is of measure zero, we have, by condition (19), that $R_0 > 1$ in general. Then use this expression to eliminate R_0 from (10) to obtain $e^{\xi_0}\pi^*/\tilde{\beta} = \pi^*/\tilde{\beta} + \alpha_\pi(\pi_0 - \pi^*)$, which uniquely determines π_0.[8]∎

8.8 Proof of Proposition 8 (Unemployment Due To Large Negative Taste Shocks)

The proposition follows directly from lemmas 3 and 4 in Appendix B. Lemma 4 establishes that if $u_0 = 0$, then π_1 must be greater than π^*. Lemma 3 establishes that no equilibrium with well-anchored inflationary expectations exists if $u_0 = 0$ and $\pi_1 > \pi^*$.∎

8.9 Proof of Proposition 9 (Recoveries With Job Creation)

We first show that $u_{t+1} \leq u_t$ for all $t \geq 0$. If $u_{t+1} = 0$, then this condition is satisfied. If $u_{t+1} > 0$, then (7) states that $\pi_{t+1} = \frac{\gamma(u_{t+1})}{\mu}\frac{F'(h_t)}{F'(h_{t+1})} < \pi^*\frac{F'(h_t)}{F'(h_{t+1})}$. Because, proposition 3, $\pi_{t+1} \geq \pi^*$, the above expression implies that $h_{t+1} > h_t$ or that $u_{t+1} < u_t$. To show that

[8]Even in the event that $e^{\xi_0}\pi^*/\tilde{\beta} = 1$, this last expression would uniquely determine π_0 provided the parameter configuration is such that $\frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\frac{\gamma(0)}{\mu} - \pi^*\right) \geq 1$.

unemployment disappears in finite time, note that if $u_{t+1} > 0$, then the above analysis yields $\frac{F'(h_t)}{F'(h_{t+1})} \geq \frac{\pi^*}{\left(\frac{\gamma(u_{t+1})}{\mu}\right)}$. Since $\frac{\gamma(u_{t+1})}{\mu}$ is bounded above by $\frac{\gamma(0)}{\mu} < \pi^*$, we have that $\frac{F'(h_t)}{F'(h_{t+1})}$ is bounded below by the constant $\frac{\pi^*}{\left(\frac{\gamma(0)}{\mu}\right)} > 1$. This means that $u_{t+1} - u_t$ is negative and bounded away from zero. Thus, u_{t+1} must reach zero in finite time.∎

8.10 Proof of Proposition 10 (Inflation and Interest Rate Dynamics)

It follows directly from proposition 3 that $\pi_t \geq \pi^*$ for $t > 0$ in any equilibrium with well-anchored inflationary expectations and under the assumptions of the proposition. To see that $\pi_0 < \pi^*$ notice that because, by proposition 8, $u_0 > 0$, condition (7) must hold with equality in period 0, that is, $\pi_0 = \frac{\gamma(u_0)}{\mu}\frac{F'(\bar{h})}{F'(h_0)} < \frac{\gamma(u_0)}{\mu} < \frac{\gamma(0)}{\mu} < \pi^*$. The first inequality follows from the fact that $h_0 < \bar{h}$ and the concavity of F, the second inequality follows from assumption 1, and the third follows from assumption 2. Because $u_0 > 0$ and $\pi_0 < \pi^*$, equation (10) implies that $R_0 < \pi^*/\tilde{\beta} \equiv R^*$. The result that $\pi_t \geq \pi^*$ for $t > 0$ together with Lemma 4 in Appendix B implies that $\pi_t = \pi^*$ for all $t > T$. To see that $\pi_T = \pi^*$, note that, by condition (3) and the facts that $u_T = u_{T+1} = 0$ and that $\pi_{T+1} = \pi^*$, $R_T = \pi^*/\tilde{\beta} > 1$. Using this expression to eliminate R_T from (10) and taking into account that $u_T = 0$, yields $\pi_T = \pi^*$. It remains to establish that $\pi_t > \pi^*$ for $0 < t < T$. By (3), we have that for $0 < t < T$ $R_t = \frac{\pi_{t+1}}{\tilde{\beta}}\frac{u'(F(h_t)}{F'(h_{t+1})} > \frac{\pi_{t+1}}{\tilde{\beta}} \geq \frac{\pi^*}{\tilde{\beta}} > 1$. The first inequality follows from the result that $h_{t+1} > h_t$ established in proposition 9. The second inequality follows from the result that $\pi_t \geq \pi^*$. Using this expression to eliminate R_t from (10) and taking into account that u_t is positive for $0 < t < T$, we obtain $\pi_t > \pi^*$ for $0 < t < T$.∎

9 Appendix B: Lemmas

Lemma 1. *Suppose that assumptions 1 and 2 hold, that $w_{-1} = F'(\bar{h})$, that condition (19) holds, and that $\xi_1 = 0$ with probability 1. Then $u_0 > 0$ implies that $\pi_1 < \pi^*$.*

Proof: If $u_0 > 0$, then $h_0 < \bar{h}$ and (7) must hold with equality, so that π_0 is given by $\pi_0 = \frac{\gamma(u_0)}{\mu}\frac{F'(\bar{h})}{F'(h_0)} < \frac{\gamma(0)}{\mu}$. Use (10) to obtain that the nominal interest rate in period 0 is given by $R_0 = \max\left\{1, \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\frac{\gamma(u_0)}{\mu}\frac{F'(\bar{h})}{F'(h_0)} - \pi^*\right) - \alpha_\gamma u_0\right\}$. Consider first the case that $R_0 > 1$. It follows then that $1 < R_0 < \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi\left(\frac{\gamma(0)}{\mu} - \pi^*\right)$. This expression together with condition (19) implies that $e^{\xi_0} > \frac{\tilde{\beta}}{\pi^*}R_0$. Solve (3) for e^{ξ_0} to obtain $e^{\xi_0} = \tilde{\beta}R_0\frac{u'(c_1)}{u'(c_0)\pi_1}$. Combining this expression with the above inequality yields $\tilde{\beta}R_0\frac{u'(c_1)}{u'(c_0)\pi_1} > \frac{\tilde{\beta}}{\pi^*}R_0$. Rearrange to obtain $\frac{u'(c_1)}{u'(c_0)} > \frac{\pi_1}{\pi^*}$. Suppose, contrary to the claim of the lemma, that $\pi_1 \geq \pi^*$. This and the previous inequality imply that $u'(c_1) > u'(c_0)$, or equivalently, that $c_1 < c_0$. In turn, $c_1 < c_0$ implies that $u_1 > u_0 > 0$ and that $h_1 < h_0$. Hence in period 1 (7) must hold with equality and $\pi_1 = \frac{\gamma(u_1)}{\mu}\frac{F'(h_0)}{F'(h_1)} < \frac{\gamma(u_1)}{\mu} < \frac{\gamma(0)}{\mu} < \pi^*$, which is a contradiction. We have therefore established that the statement of the lemma holds in the case in which $R_0 > 1$. Suppose now that $R_0 = 1$. Then, one can rewrite equation (3) as $\frac{e^{\xi_0}\pi^*}{\tilde{\beta}}\frac{\pi_1}{\pi^*} = \frac{u'(F(h_1)}{F'(h_0)}$. Suppose that, contrary to the statement of the lemma, $\pi_1 \geq \pi^*$. Then, by, condition (19), the left-hand side of the above expression must be greater than or equal to unity. This implies that so mat be the right-hand side, that is, $\frac{u'(F(h_1)}{F'(h_0)} \geq 1$, or $h_1 \leq h_0$. This means that $u_1 \geq u_0 > 0$. This means that (7) must hold with equality in period 1. That is $\pi_1 = \frac{\gamma(u_1)}{\mu}\frac{F'(h_0)}{F'(h_1)} \leq \frac{\gamma(u_1)}{\mu} < \frac{\gamma(0)}{\mu} < \pi^*$. But this is a contradiction. Therefore, it must be the case that $\pi_1 < \pi^*$ as claimed by the lemma.■

Lemma 2. *Suppose assumptions 1 and 2 hold and $w_{t-1} = F'(\bar{h})$. Assume further that $\xi_t = 0$ for all t with probability 1. Then, any equilibrium in which $\pi_t \geq \pi^*$ must feature $u_t = 0$.*

Proof: The proof is by contradiction. Suppose $\pi_t \geq \pi^*$ and $u_t > 0$. Then by (9) $h_t < \bar{h}$. Equations (8) and (7) together then imply that $\pi_t = \frac{\gamma(u_t)}{\mu}\frac{F'(\bar{h})}{F'(h_t)} < \frac{\gamma(0)}{\mu} < \pi^*$, which contradicts the maintained assumption that $\pi_t \geq \pi^*$. The first inequality follows from the assumption that $F(.)$ is strictly concave and from assumption 1, and the second inequality follows from assumption 2.■

Lemma 3. *Suppose assumptions 1 and 2 hold and $w_{t-1} = F'(\bar{h})$. Assume further that $\xi_t = 0$ for all t with probability 1. Then, any equilibrium in which $\pi_t > \pi^*$ features $\lim_{j\to\infty}\pi_{t+j} = \infty$.*

Proof: Suppose $\pi_t > \pi^*$. Given the conditions stated in the current lemma it follows from Lemma 2 that $u_t = 0$. Condition (10) then yields that $R_t = \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi(\pi_t - \pi^*) > \frac{\pi^*}{\tilde{\beta}} > 1$. Solve (3) for π_{t+1} to obtain $\pi_{t+1} = \tilde{\beta}R_t\frac{u'(F(h_{t+1}))}{u'(F(\bar{h}))}$. Because $u(.)$ and $F(.)$ are concave and because, by (6), $h_{t+1} \leq \bar{h}$, we have that $\pi_{t+1} \geq \tilde{\beta}R_t$. Use the above expression to eliminate R_t. This yields: $(\pi_{t+1} - \pi^*) \geq \tilde{\beta}\alpha_\pi(\pi_t - \pi^*)$. Because, by assumption 2, $\tilde{\beta}\alpha_\pi > 1$, we have that $\pi_{t+1} > \pi_t$. Since $w_t = F'(\bar{h})$ and $\pi_{t+1} > \pi^*$, the conditions of the lemma are also satisfied for period $t+1$, which means that by the above arguments $\pi_{t+2} - \pi^* \geq \tilde{\beta}\alpha_\pi(\pi_{t+1} - \pi^*)$. Continuing with this argument yields that $\lim_{j\to\infty}\pi_{t+j} = \infty$.■

Lemma 4. *If the taste shock is such that condition (19) is not satisfied and assumptions 1 and 2 hold and $w_{-1} = F'(\bar{h})$, any equilibrium featuring $u_0 = 0$ must also feature $\pi_1 > \pi^*$.*

Proof: The proof is in two steps, one corresponding to the case in which condition (19) is violated because its left hand side is less than the first argument on the right hand side, and the other corresponding to the case in which the left hand side is less than the second argument on the right-hand side.

Step 1: suppose $e^{\xi_0} < \tilde{\beta}/\pi^*$. Then, (3) implies that $R_0 = \frac{u'(F(\bar{h}))}{u'(F(h_1))}\frac{e^{\xi_0}\pi^*}{\tilde{\beta}}\frac{\pi_1}{\pi^*}$. Because $\frac{u'(F(\bar{h}))}{u'(F(h_1))} \leq 1$, $\frac{e^{\xi_0}\pi^*}{\tilde{\beta}} < 1$, and $R_0 \geq 1$, we have that $\pi_1 > \pi^*$ as claimed by the lemma.

Step 2: Suppose $e^{\xi_0} < \frac{\gamma(0)\alpha_\pi\tilde{\beta}}{\mu\pi^*} + 1 - \alpha_\pi\tilde{\beta}$. By (3), $R_0 = \frac{u'(F(\bar{h}))}{u'(F(h_1))}\frac{e^{\xi_0}\pi^*}{\tilde{\beta}}\frac{\pi_1}{\pi^*}$. From (10), $R_0 \geq \frac{\pi^*}{\tilde{\beta}} + \alpha_\pi(\pi_0 - \pi^*)$. Combine these two expressions and solve for π_0 to obtain $\pi_0 \leq \left[\frac{u'(F(\bar{h}))}{u'(F(h_1))}\frac{e^{\xi_0}\pi^*}{\tilde{\beta}}\frac{\pi_1}{\pi^*} - \frac{\pi^*}{\tilde{\beta}}\right]\alpha_\pi^{-1} + \pi^*$. From (7), $\pi_0 \geq \frac{\gamma(0)}{\mu}$. Combining these two inequalities, we obtain $\frac{u'(F(\bar{h}))}{u'(F(h_1))}\frac{e^{\xi_0}\pi_1}{\pi^*} \geq \frac{\gamma(0)}{\mu}\frac{\alpha_\pi\tilde{\beta}}{\pi^*} > e^{\xi_0}$. The last inequality follows from the assumption that the left-hand side of (19) is less than the second argument of its right-hand side. The above condition implies that $\pi_1 > \pi^*$.$\blacksquare$

Appendix C: Data Sources

1. Real Gross Domestic Product. Quarterly data, seasonally adjusted annual rates, GDP in billions of chained 2005 dollars. `www.bea.gov/national/xls/gdplev.xls`.

2. Civilian non-institutional population, 16 years and over, Labor Force Statistics from the Current Population Survey, BLS Series LNU00000000Q.

3. Employment-population ratio, individuals age 16 years and over, BLS Data Series LNS12300000, Labor Force Statistics from the Current Population Survey, Seasonally Adjusted, quarterly average of monthly data.

4. Effective Federal Funds Rate. Board of Governors of the Federal Reserve System. Table H.15 Selected Interest Rates. Unique identifier `H15/H15/RIFSPFF_N.M` Quarterly average of monthly data.

5. Ten-Year Expected Inflation. Federal Reserve Bank of Cleveland. Quarterly average of monthly data. `www.clevelandfed.org/research/data/inflation_expectations/`

6. Japan: Recession dates. Economic and Social Research Institute (ESRI), Cabinet Office, Government of Japan, `http://www.esri.cao.go.jp/en/stat/di/di2e.html`

7. Japan: Real GDP and GDP deflator. SNA (National Accounts of Japan), Economic and Social Research Institute (ESRI), Cabinet Office, Government of Japan. `http://www.esri.cao.go.jp/en/sna/data/sokuhou/files/2011/qe113/gdemenuea.html` The GDP deflator is at market prices and thus reflects consumption taxes. Following

the BOJ Report "Price Developments in Japan, A Review Focusing on the 1990s," Research and Statistics Department, Bank of Japan, October 6, 2000. Available online at http://www.boj.or.jp/en/research/brp/ron_2000/ron0010a.htm/, notes to Chart 6, we adjust the GDP deflator for the effects of the increase in the consumption tax in April 1997 from 3 to 5 percent by using a level-shift dummy.

8. Japan: Employed persons, labor force, not in labor force data is taken from the Labor Force Survey, Statistics Bureau, Ministry of Internal Affairs and Communications, http://www.stat.go.jp/english/data/roudou/lngindex.htm

9. Japan: Call rate is taken from the Bank of Japan.

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