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Structural Change, Industrial Upgrading and Middle Income Trap

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Abstract

Motivated by several stylized facts about middle-income trap, we develop a simple multi-sector general equilibrium model of structural change and industrial upgrading. The model features the distinction between production service and consumption service and the input-output linkages between different sectors. We show that the role of production service is asymmetric at different levels of development. Whereas an underdeveloped sector of production service is not a binding obstacle for development (sometimes even beneficial) at an early stage of development, it becomes a key bottleneck when the economy reaches a middle-income status. To escape the middle-income trap, government intervention is needed to prevent premature de-industralization and facilitate beneficial industrial upgrading. Moreover, it also requires a timely reduction of entry barrier to the production service and improvement in its productivity. These theoretical findings are shown to be consistent with the stylized facts and also useful to China. The analysis provides a justification for the government's strategic use of industrial policies to avoid middle-income trap.

Key Words: Structural Change, Industrial Policies, Middle-Income Trap, Chinese Economy, Economic Growth

JEL Codes: O11, O14, O33, O41

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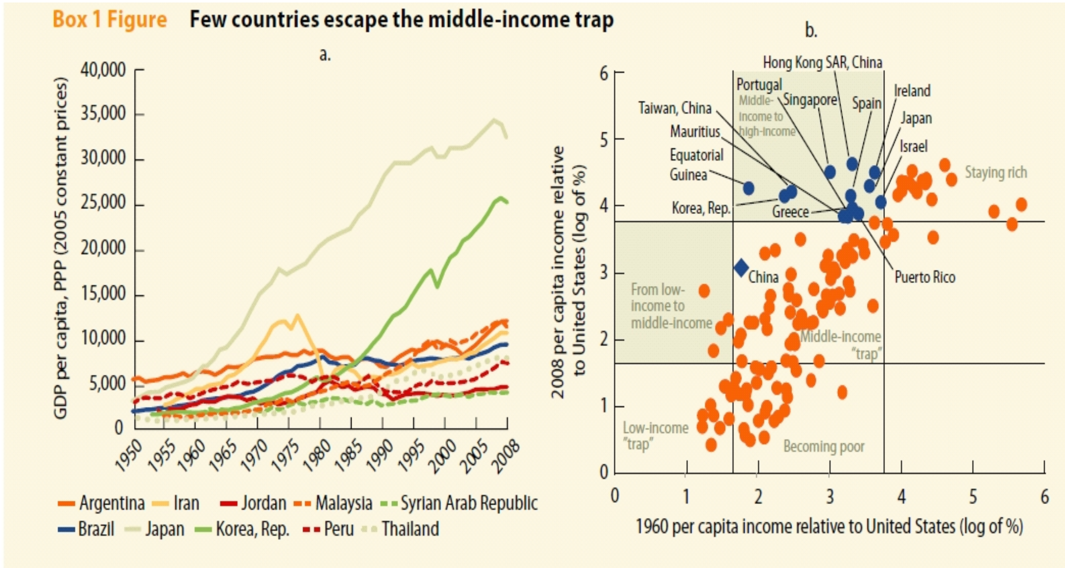
1 Introduction

The primary purpose of this paper is to study the "middle-income trap", its causes and how to escape it. An economy is classified by the World Bank as an middle-income economy if its GDP per capita is between approximately one thousand and twelve thousand USD. Based on this criterion, there were a total of one hundred and one middle-income economies in 1960 and only thirteen of them managed to graduate from the middle-income status and upgraded to the high-income status by 2008 (Agenor, Canuto, and Jelenic 2012). Alternatively speaking, more than 87% of the middle-income economies failed to grow fast enough to join the club of rich economies. This striking phenomenon is referred to as the "middle-income trap", a term first coined by Gill and Kharas (2007).

Figure 1 is taken from *China 2030*, a report jointly prepared by the World Bank and the Research Center of the State Council of China in 2008 (World Bank, 2008). On the left panel, it shows that the absolute growth performances of Japan and Korea were much better than the rest of a selected subset of economies. On the right panel, each of the thirteen successful escapers of the middle-income trap is explicitly labelled, and it is clear that their gap relative to the US in terms of GDP per capita was significantly reduced from 1960 to 2008. Despite the huge progress of modern growth theories that aim to explain why rich countries are persistently rich while poor countries remain persistently poor (see, for example, Lucas 1988), we are still lack of sufficient understanding why and how only these thirteen economies managed to outperform others and achieved economic prosperity (Commission on Growth and Development, 2008). Standard convergence theories based on the premises of diminishing return to physical capital or international technological diffusion cannot fully explain the non-convergence behaviors of the middle-income economies (Barro and Sala-i-Martin (1990, 2004)).

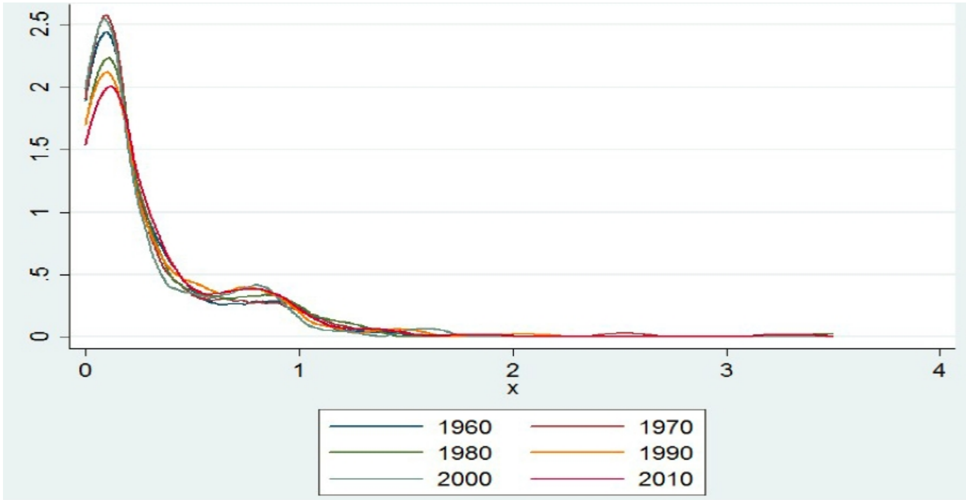


Source: Heston, Summers, and Aten 2011.

Source: Maddison database.

Figure 1: Middle-Income Trap

Figure 2 plots how the world relative income distribution changes in every past decade from 1960 to now. The horizontal axis is the real GDP per capita relative to the level of the US and the vertical axis is the empirical density. Different curves refer to density functions in different years. The figure shows that the relative income distribution exhibits a bimodal pattern, which is fairly stable over the past sixty years. This phenomenon of "twin peaks" has helped motivate further investigations that go beyond the standard convergence theories (see, for example, Danny Quah (1996), Acemoglu and Ventura (2002)). Surprisingly, however, few attempts have ever been made to focus on the growth performance of middle-income economies as distinct from that of the low-income countries. Growth challenges and bottlenecks that China faced in 1978 with its GDP per capita less than one third of that of the South Africa Sub Sharah region are unambiguously different from those it faces today as a middle-income country with GDP per capita close to USD 10,000. Unfortunately, existing theories have largely treated middle-income countries as qualitatively identical to those low-income ones under the same category - developing countries. But poverty trap is clearly not identical to "middle-income trap". The one hundred and one middle-income economies in 1960 have all managed to escape the poverty trap, but why so few of them have succeeded in their endeavours to escape the middle-income trap?



Graph (4): the world income distribution for the years 1960 to 2010

Figure 2: The World Relative Income Distribution in the Past Six Decases. Source: Authors' calculation.

Given that 85% of the world's population lives in low- or middle-income countries, the importance of the middle-income trap is self-evident. In fact, since the term was coined, it has been immediately attracting enormous attentions from almost all leading international policy institutions such as the World Bank, IMF, and Asian Development Bank as well as governments of almost all developing countries. See Agenor, Canuto and Jelenic (2012), Aiyar, et al (2013), Felipe, Kumar and Galope (2014), just to name a few. Although researchers may disagree on how to empirically categorize as middle-income countries and how to define the middle-income trap (some economists even deny the existence of middle-income trap based on their definitions, see for example, Im and Rosenblatt (2013)), it is widely accepted that economic convergence is only conditional (Barro and Sala-I-Martin (2004)), the relative distribution of world income remains stable (see Figure 2), and it is useful to understand why the thirteen successful economies can outperform the majority of economies with similar initial levels of GDP per capita after they all move out of the poverty trap. Given the widely accepted fact of unconditional non-convergence for both low-income and middle-income countries, we ask whether the mechanisms behind their non-convergence (and therefore the policy suggestions) could be qualitatively different and stage-dependent. In particular, in this paper we propose a potential mechanism and its policy implications that are specific to the development stage of middle-income countries.

Most of the existing academic papers attempt to sort out empirical differences between the economies in the middle-income trap and those successful escapers (see, for instance, Eichengreen, Park and Shin (2013), Han and Wei (2017)). Surprisingly, however, very few theoretical models have been proposed to analyze the causes and mechanisms of middle-income trap. One possible reason is that such models have to explain why certain growth mechanisms work for the low-income status but may lose their effectiveness once it reaches a middle-income level of development, which in turn requires us to go beyond quantitative differences and explicitly explore the structural differences between less developed countries at different stages of economic development, a perspective that has been advocated in New Structural Economics but largely ignored by most of existing theories[1] (Lin and Wang, 2019). There may be varieties of plausible mechanisms that could all result in middle-income traps, but in this paper we take the approach of New Structural Economics by focusing on the role of industrial upgrading and structural change, because successful economic development is impossible without continuous labor productivity-enhancing industrial upgrading and structural change (Lin, 2009). More specifically, we explore what may disturb the process of industrial upgrading from basic products to high-quality ones within the manufacturing sector and the process of structural change from manufacturing to service, both of which are essential to move from middle income up to high income. We divide service into three different categories: production service, consumption service and social service, the empirical classification criteria of which will be provided in Section 2.

Using cross-country data, we first document three stylized facts about industrial upgrading and structural change. First, production service is more intensively used in consumption service (CS) and high-quality consumption manufacturing (CH) than in basic consumption manufacturing (CB). Second, production service as share of GDP is higher in those middle-income-trap escapers (ME) than those trapped ones (MT). Third, production service as share of GDP is lower in those low-income-trap escapers (LE) than those trapped ones (LT).

Motivated by these facts, we develop a multi-sector general equilibrium model to simultaneously explore structural change from manufacturing (tradable) to service (non-tradable) and industrial upgrading from basic manufacturing to high-quality manufacturing. The model has two parts: autarky and trade. It features the distinction between production service and consumption service. It also highlights the input-output linkage across different sectors, as summarized in the Fact 1.

We show that in autarky multiple equilibria may arise due to the endogenous strategic complementarity between potential investors in the production service sector, and this strategic complementarity is reinforced by consumers' non-homothetic preference and the input-output linkage between production service and downstream modern sectors. To improve welfare, government coordination is desirable. Moreover, even the Pareto superior laissez-faire market equilibrium can be still inefficient due to the pecuniary externality caused by the non-competitive market structure in the production service sector augmented by the input-output linkage in the two interactive processes of structural change and industrial upgrading. Contrasted with the socially optimal allocation, these two processes could be either premature or delayed in the Laissez-faire market equilibrium, so welfare could be further enhanced if appropriate policy

[1] New Structural Economics advocates the use of Neoclassical approach to study the determinants and impacts of structure and structural transformation in an economy (Lin, 2011)

interventions are engineered to rectify the market failure.

In particular, we show how production service may have an asymmetric impact on the level of GDP per capita (and convergence). Underdevelopment of production service (due to high entry barrier or low productivity) may not be a binding constraint for growth at the low-income level because at that stage the dominant industry is basic manufacturing, which does not rely too much on production service, but it can become a serious bottleneck when an economy reaches the middle-income stage, because demand for high-quality consumption manufacturing goods and consumption service becomes disproportionately higher thanks to the non-homothetic preference (Engle's law), and both of them require production service as important intermediate inputs (recall Fact 1). When international trade is allowed, reducing the entry barrier to production service or increasing the productivity of production service in a developing country may reduce or enlarge its income gap with its trade partner (a developed country), depending on the trade specialization pattern which is in turn endogenous to the development stages of the developing country. It is shown that better development of the production service sector results in convergence only when the developing country manages to upgrade its manufacturing sector.

Our paper is not the first theoretical investigation on the plausible mechanisms of middle-income trap or mechanisms for other phenomena but relevant to middle-income trap. For instance, Acemoglu, Aghion and Zilibotti (2006) show that a non-convergence trap may arise if an economy fails to switch from investment-based growth mode to innovation-based growth model in time. Eeckout and Jovanovic (2012) develop an occupation choice model to show that trade may benefit the groups in the two extreme ends of a spectrum because of their big differences in comparative advantages but would hurt the middle-income group as their comparative advantage is weakest. This model has been tailored by Agenor, Canuto and Jelenic (2012) to explain the middle-income trap. Wang and Wei (2017) explore the middle-income trap by developing a three-country trade model to show how the middle-income country is sandwiched by the innovating North and the imitating South. Wang, Wong and Yip (2018) study the middle-income trap from the angle of technology assimilation and capital intensity. Our paper complements these research by proposing a different mechanism which focuses on the role of production service in the context of industrial upgrading and structural change.

Our paper also contributes to the literature of structural change by (1) differentiating production service from consumption service and dividing manufacturing into basic and high-quality ones, (2) highlighting the asymmetric role of production service via the input-output linkages across different subsectors, and (3) examining the two related processes of structural change (from manufacturing to service) and industrial upgrading (within manufacturing) both with and without international trade, whereas most of the existing literature either treats all service and all manufacturing each as a homogeneous sector (see, for example, Kongasmut, Rebelo and Xie (2001)), or ignores the asymmetric role of production service in the input-output linkages across different specific sectors (see, for example, Buera and Kaboski (2012), Buera, Kaboski and Rogerson (2018)), or remain agnostic about differences and/or interactions between structural change and industrial upgrading in autarky and in the open economy.

Whereas the deep root of market failure in our model is pecuniary externality caused by increasing returns to scale, which is similar to Murphy, Shleifer and Vishny (1989), our model differs in several important ways: Pecuniary externality is amplified through the channel of input-output linkages and further augmented through the channel of the non-homothetic preferences in the context of two processes: structural change and industrial upgrading, which are key features of our model mechanism, whereas neither of these two channels nor these two processes are simultaneously considered in Murphy, Shleifer and Vishny (1989). Moreover, Murphy, Shleifer and Vishny (1989) focuses on the market inefficiency with the symptom of delays in industrial upgrading or structural change, but our model shows that inefficiency may also come from premature upgrading and premature de-industrialization. It means that welfare-enhancing policy interventions may involve deterrence of premature industrial upgrading or undesirable structural change, a shift from high-value manufacturing to low-value service as observed in many developing countries (McMillan, Rodirk and Verduzco-Gallo, 2014). The same comments are also applicable to almost the entire existing literature on Marshallian externality and industrial policies, see Krugman (1987, 1991), Matsuyama (1991), Rodrik (1996), Rodriguez-Clare (2007), Ju, Lin and Wang (2011), Harrison and Rodrigues-Claire (2010), Wang and Xie (2014).

The rest of the paper is organized as follows. Section 2 documents the three stylized facts that motivate our approach in this paper. In Section 3, we develop a closed-economy model to illustrate the key ideas. Decentralized market equilibrium allocations and the socially efficient allocation are compared, based on which welfare-enhancing industrial policies are proposed. In Section 4, we extend the autarky model to an open economy and illustrate how international trade affects the key results. More discussions for China are provided in Section 5. The last section concludes. Technical proofs and some other extensions are in the Appendix.

2 Stylized Facts

In this section, we document three stylized facts using cross-country data. The main data set we use is the Input-Output tables from both OECD and WIOD for 49 economies.[2]

[2] These 49 economies are Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States, and Vietnam.

We divide manufacturing into three different categories: Consumption Basic Manufacturing (CB), Consumption High-quality Manufacturing (CH) and Production Manufacturing (PM). We divide services into three different categories: Consumption Service (CS), Production Service (PS) and Social Service (SS). The way how we categorize is the following: We follow the same method developed by Antras et al (2012) to compute the upstreamness indexes for all the sectors based on the Input-Output table. A sector with a higher upstreamness index implies that the sector is more upstream, that is, farther away from consumers. We choose the cutoff value 3.3 for the upstreamness index, below which we define as consumption goods or consumption service. If the index is higher than 3.3, we define the sector as production goods or production service[3]. Among the subsectors whose upstreamness index is below 3.3 within the manufacturing sector, we use Eurostat's high-technology classification to define whether the subsector is basic manufacturing or high-quality manufacturing. Different from consumption service (such as hotel, restaurant, tourism, entertainment *etc*) and production service (such as communication, financial intermediation, and R&D. For details, see Table 2 below), social service includes education, health care, pension, administrative services *etc*. For the main purpose of this paper, we mainly focus on four sectors: high-quality consumption manufacturing, basic consumption manufacturing, production service, and consumption service. The following table summarizes how we define these four sectors:

Industry category	Definition
Consumption Basic Manufacturing (CB)	Manufacturing with Medium-low-technology and Low-technology according to Eurostat's High-tech classification of manufacturing industries and upstreamness index ≤ 3.3
Consumption High-quality Manufacturing (CH)	Manufacturing with High-technology and Medium-high-technology according to Eurostat's High-tech classification of manufacturing industries and upstreamness index ≤ 3.3
Production Manufacturing (PM)	Manufacturing with upstreamness index > 3.3
Consumption Services (CS)	Services with upstreamness index ≤ 3.3
Production Services (PS)	Services with upstreamness index > 3.3
Unclassified	Primary industries including agriculture, hunting, forestry, fishing, mining, quarrying as well as miscellaneous industries including manufacturing n.e.c. & recycling (include Furniture); public admin., defence & compulsory social security; extra-territorial organisations & bodies

Table 1 : Sector Classifications

The following table lists the five subcategories for the production service.

[3]Production service has recently become an official category in various economies. For example, China's National Bureau of Statistics has provided the list of subsectors that are classified as production service since 2014. The list is almost identical to that with the upstreamness score higher than 3.3 based on our own calculation.

Acronym	Industry Category	Including
WEG	Water, electricity and gas	Production, collection and distribution of electricity, Manufacture of gas, distribution of gaseous fuels through mains, Steam and hot water supply, Collection, purification and distribution of water
TRAN	Transport	Land transport, transport via pipelines, Water transport, Air transport
COMM	Communication	Post & telecommunications
FINA	Financial intermediation	Finance & insurance
BUSS	Business services	Renting of machinery & equipment, Research & development, Other Business Activities

Table 2: Composition of Production Service

Based on these classifications, we establish the following three stylized facts.

Fact 1. *Production Service is more intensively used in consumption service (CS) and high-quality consumption manufacturing (CH) than in basic consumption manufacturing (CB).*

We specify the following regression

$$Xinputshare_{i,j,t} = a_i + \beta_1 \cdot CH\ _{i,j,t} + \beta_2 \cdot CS\ _{i,j,t} + \varepsilon_{i,j,t},$$

where the dependent variable $Xinputshare_{i,j,t}$ measures input share of X (for example, X may refer to production service (PS)) in the production of sector j in country i at year t. More precisely, we define

$$Xinputshare_{i,j,t} \equiv \frac{\text{Value of input X used by industry } j \text{ in country } i \text{ at year } t}{\text{Value of all inputs used by industry } j \text{ in country } i \text{ at year } t}. \tag{1}$$

We say industry A needs more X than B if X input share of A is higher than that of B. a_i is the country fixed effect for country i, $CH\ _{i,j,t}$ is a dummy variable, equal to 1 if sector j is CH in country i and year t, and equal to 0 otherwsie. Similarly, $CS\ _{i,j,t}$ is a dummy variable, equal to 1 if sector j is CS in country i and year t, and equal to 0 otherwsie. $\varepsilon_{i,j,t}$ is the error term.

The following table summarizes all the regression results:

	Dependent variable:						
	PM input share	WEG input share	TRAN input share	COMM input share	FINA input share	BUSS input share	PS input share
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
CH	0.267^{***}	-0.001	-0.004^{**}	0.004^{***}	0.006^{***}	0.013^{***}	0.018^{***}
	(0.012)	(0.002)	(0.002)	(0.001)	(0.001)	(0.003)	(0.005)
CS	0.019^{**}	0.008^{***}	0.020^{***}	0.023^{***}	0.040^{***}	0.065^{***}	0.156^{***}
	(0.010)	(0.003)	(0.003)	(0.001)	(0.003)	(0.007)	(0.009)
Constant	0.124^{***}	0.018^{***}	0.044^{***}	0.008^{***}	0.018^{***}	0.054^{***}	0.143^{***}
	(0.006)	(0.001)	(0.001)	(0.0004)	(0.001)	0.003)	(0.004)
Observations	2,091	2,091	2,091	2,091	2,091	2,091	2,091
Adjusted R^2	0.859	0.648	0.694	0.793	0.685	0.740	0.804
Note:							*p<0.1; **p<0.05; ***p<0.01

Table 2: Empircal Evidence for Fact 1 Data source: IO Tables from OECDs and WIOD for 49 countries

Column (1) shows that PM is significantly more intensively used in CH and CS than in CB. Column (7) shows that PS over all is significantly more intensively used in CH and CS than in CB. Columns (2)-(6) examine how intensively each of five subcategories of production service is used by downstream sectors. The regression results show that CS relies on each of the five production service more intensively than the other sectors. Moreover, β_1 is positive and significant in Columns (4) to (6), although it is negative in Column (3) and insignificant in Column (2). These empirical findings are generally summarized as Fact 1.

An alternative way to check Fact 1 is to directly compute the cross-country simple average of input share of PM and PS for CB, CH and CS at each year. More precisely,

$$\text{Average Share of Input } j \text{ at year } t = \frac{\sum_i Xinputshare_{i,j,t}}{\#i},$$

where $j \in$\{PM, PS\}, i stands for country, and $Xinputshare_{i,j,t}$ is defined in (1). We plot the cross-country average share of input PM and input PS in Figure 3. The right panel confirms Fact 1.

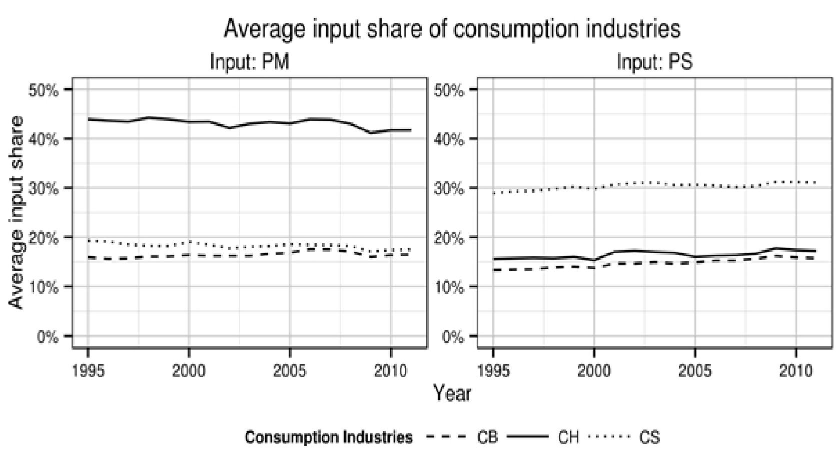
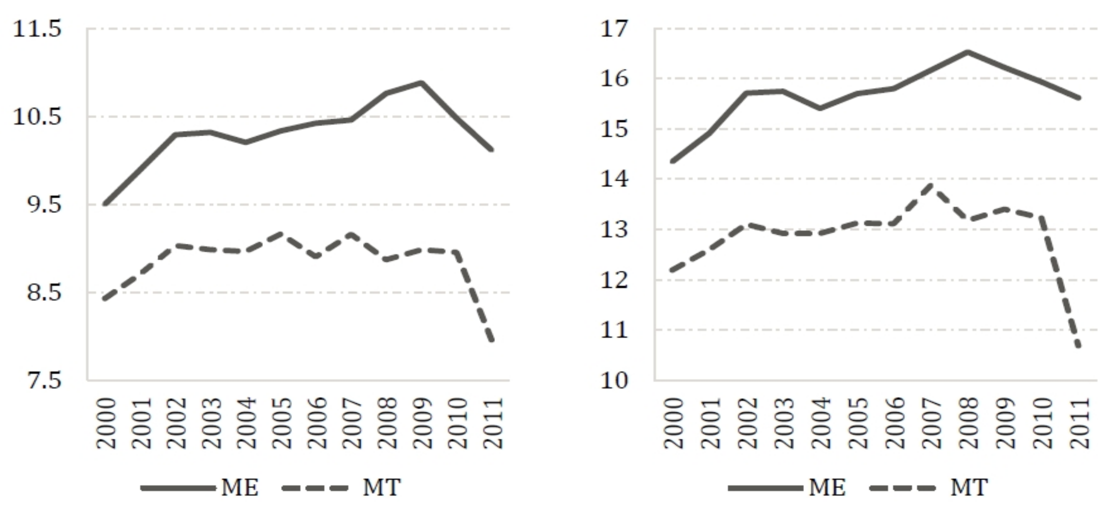


Figure 3. Average Input Share of PM and PS in CS and CH are higher than that in CB.

Fact 2. *Production service as share of GDP is higher in those middle-income-trap escapers (ME) than those trapped ones (MT).*

Trapped countries in middle income status are defined as those which stayed in the middle-income group during the time period of the data we have, while the escaped countries are defined as those which managed to move from middle income to high income during the sample period in the data. Based on GNI per capita, analytical classifications from World Bank data and production service share we calculated, we plot the two time series for the period of 2000 to 2011 on the same graph: One is the average production service share in GDP in those middle-income-trap escapers (ME) and the other is the share in those trapped ones (MT). As one may be concerned that financial and insurance service is too special, we make two different classifications on how to measure the production service: Type 1 excludes finance and insurance as production service whereas Type 2 includes it, and the other production services include professional, scientific, technical services, information and communication. Type 1 contains 43 countries and Type 2 contains 20 countries.

Figure 4 plots the value added share of production service as percentage of GDP. The left panel adopts the criterion of Type 1 and the right panel adopts that of Type 2. It shows that the pattern of Fact 3 is robust.[4]



[4]Ideally, it would be also useful to compare the value-added share of production service of ME and MT for the same per capita GDP level when both are in the middle-income status. However, such information is not sufficiently available in the current data set. Moreover, for a given GDP per capita level, the value added share of production service could be different in different years even for the same country, if it experiences big enough economic fluctuations. This is particularly likely during the 2008 Global Financial Crisis. We will leave this for future research.

Figure 4. Value-added Share of Production Service as percentage of GDP is Higher for ME than MT. The left panel uses Type 1 classification and the right panel uses Type 2.

Fact 3. *Production service as share of GDP is lower in those low-income-trap escapers (LE) than those trapped ones (LT).*

Following the similar approach for economies that escaped the low-income trap (LE) versus those trapped ones (LT), we obtain the Figure 5. Interestingly, the pattern for the low-income trap is exactly opposite to that for the middle-income trap. It suggests that higher shares of Production Service in GDP may not be necessarily better. This pattern is also robust to whether finance and insurance is counted as production service.

Figure 5: Value Added Share of Production Service in Total GDP is Higher in LT than in LE. Left Panel uses Type 1 and Right Panel uses Type 2.

In the next two sections, we take Fact 1 as given and explain Fact 2 and Fact 3. Section 3 studies a closed-economy model and Section 4 allows for international trade.

3 Autarky

3.1 Model Environment

Consider an economy populated by L identical households. Each household is endowed with one unit of labor, which is inelastically supplied. All households have equal equity shares for all the firms and hence share the profits equally. There are two broad sectors in the economy: a traditional sector which produces basic manufacturing consumption good b and a modern sector which consists of production service m, high-quality manufacturing consumption good h and consumption service s. The following graph illustrates the interactions between these different sectors in the economy:

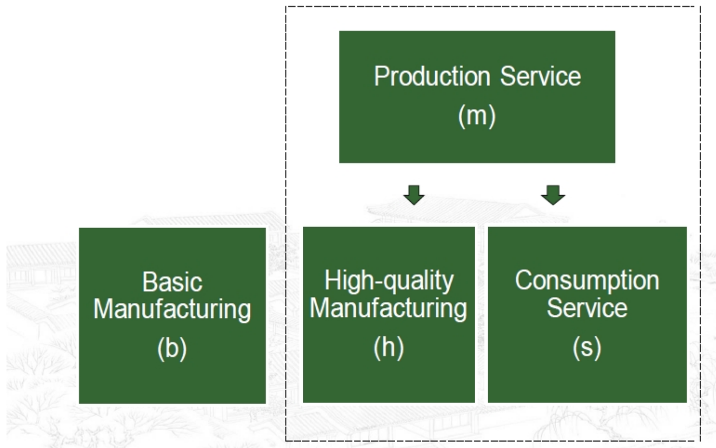


Figure 6. Relations Between Different Sectors

Preference All the agents have the same utility function

$$u(c) = c_h^{\theta} c_s^{1-\theta} + \frac{\epsilon}{\epsilon - 1} c_b^{\frac{\epsilon-1}{\epsilon}}, \ \epsilon > 1, \tag{2}$$

where c is a consumption vector composed of consumption for service c_s, high-quality manufacturing good c_h, and basic (low-quality) manufacturing good (denoted by c_b). The parameter ϵ is the price elasticity of demand for basic manufacturing good. All the three types of consumption must be non-negative. This quasi-linear utility function is to capture the Engle's law: consumers' demand for consumption service and high-quality goods increases disproportionately more as the income level increases.

Technology The production function for the basic manufacturing good is given by

$$Y_b = A_b l_b,$$

where A_b is the productivity and l_b is the labor input.

Both labor and production service m are required to produce high-quality manufacturing consumption good h and consumption service s. More specifically,

$$Y_h = A_h m_h^{\alpha} l_h^{1-\alpha},$$
$$Y_s = A_s m_s^{\beta} l_s^{1-\beta}.$$

The assumptions for the above three production functions are qualitatively consistent with Fact 1 established in Section 2, that is, high-quality manufacturing and consumption service require more production service as intermediate inputs than basic manufacturing. [5] The two arrows in Figure 6 reflect the input-output linkages between m and h and between m and s.

The production service m consists of n varieties of inputs as follows

$$Y_m = \left[\int_0^n m(i)^\sigma di\right]^{1/\sigma}, \sigma \in (0,1).$$

Each variety i is produced by a monopolist firm, which can enter the market after paying an entry cost F (in terms of labor). Firm i produces with the following technology:

$$m(i) = A_m(i) l_m(i),$$

where $l_m(i)$ is the labor input needed to produce variety i. Assume symmetry for all these varieties: $A_m(i) = A_m$, $\forall i \in [0,n]$. All the n firms are engaged in monopolistic competition. Entry is assumed free, so n will be endogenously determined and net profit for each firm is zero.

We investigate two processes simultaneously. One is the industrial upgrading process within the manufacturing sector, namely, upgrading from basic manufacturing b to high-quality manufacturing h. The other is the structural change process from manufacturing (b and h) to service (m and s). Since we mainly focus on middle-income countries, we abstract away the agriculture sector for simplicity.

Market Structure Except for monopolistic competition in the production service sector, all the other markets are perfectly competitive.

3.2 Decentralized Market Equilibrium

Household Problem A representative household maximizes (2) subject to the following budget constraint

$$p_h c_h + p_s c_s + p_b c_b = I = w, \tag{3}$$

where p_j and c_j denote price and per capita consumption of $j \in \{h,s,b\}$, I denotes per capita income, which is equal to the wage rate denoted by w. We require $c_j \geq 0$ for all $j \in \{h,s,b\}$. Without loss of generality, we normalize the price of $c_h^\theta c_s^{1-\theta}$ to unity (that is, $\frac{p_h^\theta p_s^{1-\theta}}{\theta^\theta(1-\theta)^{1-\theta}} = 1$), so welfare and income are in the same unit, which is convenient for welfare analyses.

When I is sufficiently large (to be more precise soon), we have

[5] For simplicity, physical capital is not explicitly modelled as one of the production factors. It is reserved for future research. For more discussions on the role of capital intensities, refer to Ju, Lin and Wang (2011, 2015) and Li, Liu and Wang (2016).

$$c_b = {p_b}^{-\epsilon}, \tag{4}$$

and

$$c_h = \frac{\theta\left[I - {p_b}^{1-\epsilon}\right]}{p_h}; c_s = \frac{(1-\theta)\left[I - {p_b}^{1-\epsilon}\right]}{p_s}.$$

Firms' problems In the production service sector, profit maximization implies that

$$p_m(i) = \frac{w}{\sigma A_m}, \forall i \in [0, n].$$

Therefore, the price of production service is given by

$$p_m = n^{1-\frac{1}{\sigma}} \frac{w}{\sigma A_m}. \tag{5}$$

The equilibrium total output of production service is given by

$$Y_m = n^{1/\sigma} \frac{\sigma A_m F}{1-\sigma}. \tag{6}$$

It is straightforward to show the following:

$$p_h = \frac{{p_m}^{\alpha} w^{1-\alpha}}{A_h \alpha^{\alpha}(1-\alpha)^{1-\alpha}}; p_s = \frac{{p_m}^{\beta} w^{1-\beta}}{A_s \beta^{\beta}(1-\beta)^{1-\beta}},$$

which, by revoking (5) and how numeraire is chosen, yields

$$w = \frac{n^{\chi}}{H(A_h, A_s, A_m)}, \tag{7}$$

where

$$H(A_h, A_s, A_m) \equiv \frac{\left[\theta\left(\sigma\alpha\right)^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right]^{-\theta}\left[\left(1-\theta\right)\left(\sigma\beta\right)^{\beta}\left(1-\beta\right)^{1-\beta}\right]^{-(1-\theta)}}{A_h^{\theta} A_s^{1-\theta} A_m^{\alpha\theta+\beta(1-\theta)}}, \tag{8}$$

$$\chi \equiv \left[\theta\alpha + (1-\theta)\beta\right]\frac{1-\sigma}{\sigma}. \tag{9}$$

Using the Shephard Lemma, we obtain the aggregate demand for production service m as follows

$$D_m = L\left\{\frac{\theta\alpha\left[I - \left(\frac{w}{A_b}\right)^{1-\epsilon}\right]}{p_m} + \frac{(1-\theta)\beta\left[I - \left(\frac{w}{A_b}\right)^{1-\epsilon}\right]}{p_m}\right\}, \tag{10}$$

where the two additive terms within the parenthesis on the right-hand side are the per capita demand for production service that comes from high-quality manufacturing production and from consumption service, respectively. c_h and c_s are strictly positive if and only if income I is sufficiently large, or equivalently,

$$1 > \left[Hn^{-\chi}\right]^{\epsilon} A_b^{\epsilon-1}. \tag{11}$$

Market Clearing The market clearing condition for production service m is $D_m = Y_m$. Combining (10) and (6), we obtain

$$\Gamma(n) = \Phi(n), \tag{12}$$

where

$$\Gamma(n) \equiv 1 - H^{\epsilon}(A_h, A_s, A_m)A_b^{\epsilon-1}n^{-\chi\epsilon}, \Phi(n) \equiv \frac{nF}{\sigma L\chi}.$$

The equilibrium number of firms in the production service sector n is determined by (12).

Lemma 1. *There are two distinct roots (denoted by n_1 and n_2 with $n_1 < n_2$) to equation (12) if and only if the following is true:*

$$HA_b^{\frac{\epsilon-1}{\epsilon}} \left[\frac{F}{L\sigma\chi}\right]^{\chi} \left[(\epsilon\chi)^{\frac{1}{\chi\epsilon+1}} + 1\right]^{\frac{\chi\epsilon+1}{\epsilon}} < 1. \tag{13}$$

There exists a unique root if and only if

$$HA_b^{\frac{\epsilon-1}{\epsilon}} \left[\frac{F}{L\sigma\chi}\right]^{\chi} \left[(\epsilon\chi)^{\frac{1}{\chi\epsilon+1}} + 1\right]^{\frac{\chi\epsilon+1}{\epsilon}} = 1, \tag{14}$$

in which case

$$n = \left(\frac{L\sigma\chi^2\epsilon H^{\epsilon}A_b^{\epsilon-1}}{F}\right)^{\frac{1}{\chi\epsilon+1}}. \tag{15}$$

No real solution exists otherwise.

Proof. Straightforward. Q.E.D.

The solutions to (12) can be graphically illustrated in Figure 7.

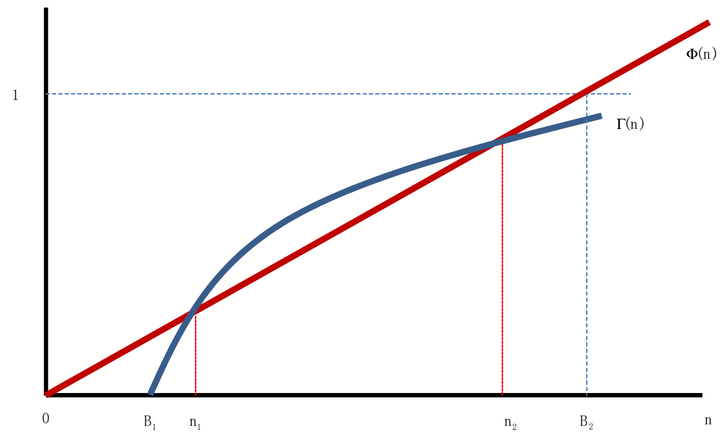


Figure 7. Equilibirum Number of Firms in Production Service

When $\frac{F}{L}$ or A_b is sufficiently small or when A_h, A_s, A_m are sufficiently large (that is, condition (13) is satisfied), ray $\Phi(n)$ and curve $\Gamma(n)$ have two distinct crossing points, n_1 and n_2, solutions to equation (12). Observe that the slope of ray $\Phi(n)$ is proportional to the ratio of entry cost to population $\frac{F}{L}$. When $\frac{F}{L}$ increases, ray $\Phi(n)$ rotates counter clockwise while curve $\Gamma(n)$ stays put, so n_1 increases while n_2 decreases (see Figure 8). In particular, when $\frac{F}{L}$ increases till condition (14) is satisfied, ray $\Phi(n)$ is tangent to curve $\Gamma(n)$, so equation (12) has a unique solution n given by (15). Observe that whenever ray $\Phi(n)$ and curve $\Gamma(n)$ cross, we must have $n \in (B_1, B_2)$, where $B_1 = \left[HA_b^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\chi}}$ and $B_2 = \frac{\sigma L \chi}{F}$. If $\frac{F}{L}$ increases further, then ray $\Phi(n)$ and curve $\Gamma(n)$ have no crossing point, which means that m is not produced ($n = 0$).

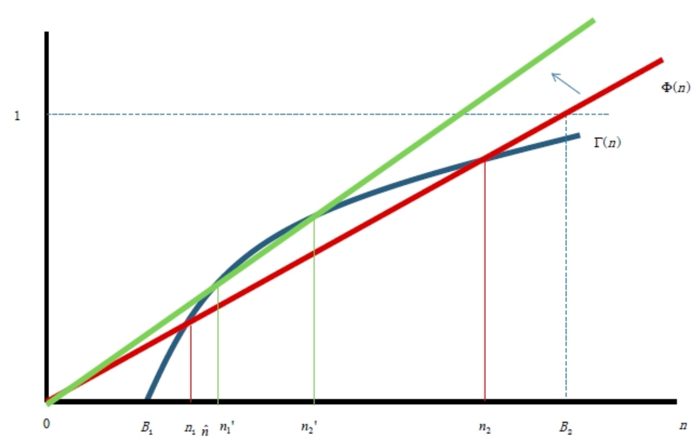


Figure 8. when F/Lincreases

When productivity A_h, A_s, or A_m decreases, or when A_b increases, curve $\Gamma(n)$ would shift downward, so n_1 increases while n_2 decreases. See Figure 9 below.

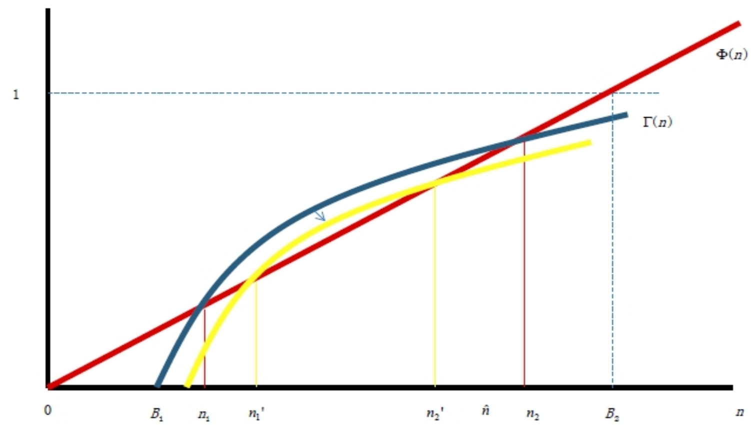


Figure 9. when A_h, A_s, A_mdecreases or when $A_b$increases

Proposition 1. *Neither structural change nor industrial upgrading would take place ($Y_m = Y_h = Y_s = 0$) in market equilibrium if and only if the entry cost to the production service sector is sufficiently high: $F > F_{\max}$,* where

$$F_{\max} \equiv \left[(\epsilon\chi)^{\frac{1}{\chi\epsilon+1}} + 1\right]^{-\frac{\chi\epsilon+1}{\chi\epsilon}} \left(HA_b^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{-1}{\chi}} L\chi\sigma, \tag{16}$$

where H and χ are defined in (8) and (9). In that case, only basic manufacturing b is produced and $u(c) = w = \frac{\epsilon}{\epsilon-1}A_b^{\frac{\epsilon-1}{\epsilon}}$.

Proof. Using the market clearing condition $c_b = A_b$. Q.E.D

This proposition says that production service sector does not exist when the entry cost F is high enough. Observe that $F_{\max}$ increases with L and productivities A_h, A_s, and A_m, but decreases with A_b. So an equivalent interpretation of this proposition is that, for any given entry cost F, only the basic manufacturing sector is active if the modern sector productivities (A_h, A_s, or A_m) are sufficiently small or when the traditional sector has sufficiently high productivity (A_b), because in this case the modern sector is unable to attract labor from the basic manufacturing sector. When the population size L is sufficiently small, the market size is too small to support firm entry to the production service sector due to positive entry cost because of the *de facto* increasing-returns-to-scale technologies in the production service sector.

Proposition 2. *There are two market equilibria if and only if $F < F_{\max}$, where $F_{\max}$ is given by (16). GDP per capita w is given by (7). In the high equilibrium, there are n_2 firms in the production service sector with the following properties:*

$$\frac{\partial n_2}{\partial A_h} > 0; \frac{\partial n_2}{\partial A_s} > 0, \frac{\partial n_2}{\partial A_m} > 0, \frac{\partial n_2}{\partial L} > 0; \frac{\partial n_2}{\partial A_b} < 0, \frac{\partial n_2}{\partial F} < 0, \tag{17}$$

and GDP per capita w_2 has the same comparative static properties as n_2. In the low equilibrium, there are fewer firms ($n_1 < n_2$) in the production service sector, and both GDP per capita and welfare are also lower. Moreover, the comparative static properties of n_1 and w are both exactly the opposite to (17).

Proof. Straightforward as seen in Figure 8 and Figure 9. For the comparative static analysis for w, use (7). Q.E.D

The reason why there exist multiple equilibria is the following. When investors hold optimistic belief that demand for production service m will be high, more firms enter that sector and competition drives down the price of aggregate production service. Consequently, both the consumption service and high-quality manufacturing good become cheaper while the price of the basic manufacturing good is unaffected if holding wage constant. It has both a substitution effect and an income effect. The substitution effect dictates that both consumption service c_s and high-quality manufacturing good c_h go up because their prices relative to the basic manufacturing good c_b become lower, which implies that the induced demand for production service m increases. On the other hand, the real income for each household increases as consumption prices decline, and notice that the income effect will only increase the demand for consumption service and high-quality manufacturing good because of the quasi-linear utility function (2), which in turn induces further demand for upstream production service m, reinforcing the substitution effect. Thus the initial belief that the demand for production service is high is indeed self-fulfilling. This supports the high equilibrium and vice versa for the low equilibrium.[6]

Comparative static properties in (17) are mostly natural for the high equilibrium. In particular, there is a positive scale effect ($\frac{\partial n_2}{\partial L} > 0$), suggesting that a larger market size (measured by L) encourages more entry in the production service sector because of the increasing-returns-to-scale technologies. It in turn implies a higher level of GDP per capita ($\frac{\partial w_2}{\partial L} > 0$) due to (7). A productivity increase in the basic manufacturing sector would reduce its price and hence

[6] Although pecuniary externality caused by increasing returns to scale exist both in our model and in Murphy, Shleifer and Vishny (1989), there are several crucial differences: First, in our model, pecuniary externality is amplified through the channel of input-output linkages and further augmented through the channel of the non-homothetic preferences in the context of two processes: structural change and industrial upgrading, whereas neither of these two channels nor these two processes are simultaneously considered in Murphy, Shleifer and Vishny (1989), as their model assumes that the traditional sector and the modern sector produce identical final goods and there is no sector producing intermediate goods. Second, the demand spillover mechanism highlighted in their model crucially relies on that the net profits of entering firms in the modern sector strictly increases with the number of entrants and must be strictly positive after sufficient entry, which results from the model assumption that the maximum number of entrants is exogenous and fixed. More profits imply higher income, and hence higher demand. In contrast, our model allows for free entry with no upper limit of firm entry, so the net profits of entering firms are always zero, independent of the number of entrants. This technical difference in modelling reflects that we highlight more on the supply side rather than the demand side: that is, more firm entry in the upstream sector enhances specialization and competition, which reduces the cost of intermediate inputs for downstream sectors, leading to a lower price of final consumption goods and a higher real income. Third, Murphy, Shleifer and Vishny (1989) focuses on the market inefficiency with the symptom of delays in industrial upgrading, but our model shows that inefficiency may also come from premature upgrading and premature de-industrialization, which will be explicitly explained later.

reduce the relative demand for the high-quality manufacturing and consumption service, which in turn reduces the market demand for the production service, leading to fewer entries into that sector ($\frac{\partial n_2}{\partial A_b} < 0$).

The comparative static properties for the low equilibrium appears counter-intuitive, but they can be explained as follows. To understand $\frac{\partial n_1}{\partial L} < 0$, it is important to observe two trade-offs. A larger population L implies a larger labor supply and hence a lower wage rate (per capita income), which in turn implies a lower indirect demand for production service, holding other things constant. On the other hand, a larger L implies a larger aggregate income for any given wage rate, which tends to boost the aggregate demand for production service. In addition to this trade off in terms of the aggregate income effect, there is another trade-off related to the substitution effect. On one hand, a lower wage reduces the entry cost and the production cost for the production service, which tends to increase the demand for production service. On the other hand, a lower wage may favor the basic manufacturing production because it is more labor intensive than all the other downstream sectors, so the aggregate demand for production service can be reduced due to the substitution effect. It turns out that the net effect on entry of an increase in L is negative in the low equilibrium because the basic manufacturing production gains more advantages from lower wages when the production service sector becomes less efficient due to the expected low entry, which in turn reduces wage even further because of the reduced demand for labor from the entering firms. It enhances the negative effect of lower aggregate income on production service.

Since the high equilibrium Pareto strictly dominates the low one, the government could improve social welfare by coordinating all the potential investors in the production service sector to the high equilibrium. The strategic complementarity between potential investors makes it easier for government coordination because firms have incentives to be coordinated and move in the same direction. Possible policy instruments for such government interventions include provision of investment subsidies to production service and/or merely fueling optimism in growth forecast for the economy in the public. The existence of multiple equilibria in the model may to some extent explain why some countries manage to escape the middle-income trap but others with similar conditions do not: differences in pure luck or the availability of pro-active government coordination. We believe that the difference is most likely not because of pure luck but rather whether or not timely government coordination is available, as Commission on Growth and Development (2008) found that 13 super growth performers all have a "committed, credible & capable government". Concrete examples of effective government coordinations in these economies are provided and intensively discussed in Johnson (1982), Amsden (1989), Wade (1990), Amsden and Chu (2003), Canda (2006) and Lin (2009).[7]

[7] It is both interesting and challenging to empirically validate the positive role of effective government coordinations, which is beyond the scope of this paper and left for future research. Be reminded that multiple market equilibria are not the only main point of this model. In fact, we will show that even the high market equilibrium is not Pareto efficient and international trade also plays key roles.

By revoking (10), we derive the value-added share of production service in GDP[8]:

$$\frac{p_m D_m}{wL} = [\theta\alpha + (1-\theta)\beta]\left[1 - \left(\frac{1}{A_b}\right)^{1-\epsilon}\frac{1}{w^{\epsilon}}\right],$$

which increases with GDP per capita w, holding other things constant. This theoretical prediction is consistent with Fact 2: Those middle-income trap escapers (ME) by definition have higher per capita GDP than those trapped (MT), and the value-added share of production service in GDP in the former is higher than that in the latter.

In addition to the difference in luck or availability of correct government coordination, Proposition 2 also suggests that another possible difference between ME and MT could be that, although they both end up with the high market equilibrium, yet the productivities in the modern sector (A_m, A_h or A_s) in ME are higher than those in MT, or entry cost F is lower in ME than MT.

The following lemma shows that there may exist premature structural change and industrial upgrading in the laissez-faire market equilibrium, in which case even the high equilibrium is strictly Pareto dominated by the allocation with no structural change or industrial upgrading.

Lemma 2. *A laissez-faire market equilibrium allocation with positive production service is Pareto dominated by the allocation with $l_b = L$ if and only if firm entry to the production service sector is sufficiently small: $n < \widehat{n} \equiv \frac{L\sigma\chi}{F}\left[1 - \left(\frac{\epsilon-1}{\epsilon}\right)^{\epsilon}\right]$.*

Proof. Based on the previous two propositions, the welfare level of a representative household in a market equilibrium with positive entry to the production service sector is strictly larger than that in a no-structural-change situation if and only if the following is true

$$\frac{n^{\chi}}{H(A_h, A_s, A_m)} = A_b^{\frac{\epsilon-1}{\epsilon}}\left[1 - \frac{F}{L\sigma\chi}n\right]^{-\frac{1}{\epsilon}} > \frac{\epsilon}{\epsilon-1}A_b^{\frac{\epsilon-1}{\epsilon}},$$

where the equality comes from (12). The above inequality is reduced to $n > \widehat{n}$. Q.E.D

Suppose initially $\widehat{n} \in (n_1, n_2)$ as shown in Figure 8. Lemma 2 implies that the high market equilibrium allocation Pareto dominates the no-structural-change allocation, which in turns Pareto dominates the low market equilibrium allocation. Now suppose $\frac{F}{L}$ increases, so $\widehat{n}$ decreases whereas n_1 increases to $n_1' > \widehat{n}$. Then new market allocations in both the low and high equilibria Pareto dominate the no-structural-change allocation. Consider another scenario where initially $\widehat{n} \in (n_1, n_2)$ as depicted in Figure 9. Now suppose A_h, A_s, or A_m decreases or A_b increases. $\widehat{n}$ does not change, but the market equilibria change to n_1' and n_2' such that $n_2' < \widehat{n}$. It means that now even the high equilibrium allocation is Pareto dominated by the no-structural-change allocation. In other words, structural change and industrial upgrading are

[8] It can be shown that the value added share of production service in total service is $\frac{(1-\theta)(1-\beta)}{\theta\alpha+(1-\theta)}$,

premature in the market equilibrium.

The economic intuition for why premature structural change and industrial upgrading could happen is that, as explained earlier, the input-output linkage across sectors plus the non-homothetic preference makes supermodularity sometimes over strong among individual investors in the production service sector. Whereas the root of market failure is quite common: the market structure in production service is not perfectly competitive and the *de facto* technology is of increasing returns to scale due to the entry cost, the precise mechanism how market fails is different from the well-known and standard mechanism in Murphy, Shleifer and Vishny (1989). In their model, the constant-returns-to-scale traditional sector and the increasing-returns-to-scale modern sector produce the same good and there is input-output linkages across sectors, so the pecuniary externality only comes from the positive demand spillover due to monopolistic competition, whereas in our model pecuniary externality comes from the fact that firms in the production service sector make individual decisions without taking into account the influence imposed on the downstream producers through the input-output linkage and ultimately also on firms in the same sector through consumers' behaviors augmented by the non-homothetic preference in a general equilibrium fashion[9].

Next we characterize the first-best allocation by solving an artificial benevolent social planner problem.

3.3 Pareto Efficient Allocation

The social planner maximizes a representative household's welfare as follows:

$$\max_{l_b,l_m,l_h,l_s,n,m_h,m_s,c_h,c_s,c_b} c_h^{\theta} c_s^{1-\theta} + \frac{\epsilon}{\epsilon-1} c_b^{\frac{\epsilon-1}{\epsilon}}$$

subject to the non-negativity constraints for all choice variables and the following feasibility constraints:

$$\begin{aligned} c_b L &= A_b l_b; \\ c_h L &= A_h m_h^{\alpha} l_h^{1-\alpha}; \\ c_s L &= A_s m_s^{\beta} l_s^{1-\beta}; \\ m_h + m_s &= n^{\frac{1}{\sigma}-1} A_m l_m; \\ l_b + l_m + l_h + l_s + nF &= L, \end{aligned}$$

where the five equations above require demand equals supply for basic manufacturing, high-quality manufacturing, consumption service, production service and labor, respectively. Notice

[9] Appendix 2 characterizes what happens if production service is perfectly competitive without entry cost ($\sigma = 1$ and $F = 0$).

that the entry cost is still paid in this social planner problem.

Solving the above problem yields the following solution: When entry cost is sufficiently small ($F < \widehat{F}_{\max}$)[10], the Pareto efficient allocation is as follows:

$$n = \frac{\frac{1-\sigma}{\sigma}}{F}\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}l_s, \tag{18}$$

$$l_h = \frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}l_s, \tag{19}$$

$$l_m = \left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}l_s, \tag{20}$$

$$l_b = \left[\frac{L^{\frac{1}{\epsilon}}A_b^{\frac{\epsilon-1}{\epsilon}}A_m\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\beta}{\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}(1-\theta)(1-\beta)^2}\right]^{\epsilon} l_s^{\{2-[\theta(1-\alpha)+(1-\theta)(1-\beta)]\}\epsilon}, \tag{21}$$

$$Y_m = \left(\frac{\theta}{1-\theta}\frac{\alpha}{1-\beta}+\frac{\beta}{1-\beta}\right)\left[\frac{1-\sigma}{\sigma F}\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}\right]^{\frac{1-\sigma}{\sigma}} A_m l_s^{\frac{1}{\sigma}},$$

where l_s is uniquely determined by

$$\begin{aligned} L = {} & \left[\frac{1}{\sigma}\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}+\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}+1\right]l_s \\ & + LA_b^{\epsilon-1}A_m^{\epsilon}\left[\frac{\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\beta}{\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}(1-\theta)(1-\beta)^2}\right]^{\epsilon} l_s^{\epsilon\{2-[\theta(1-\alpha)+(1-\theta)(1-\beta)]\}}. \end{aligned} \tag{22}$$

The real output per capita and welfare of a representative household can be also uniquely determined (refer to Appendix 1 for more details).

When $F > \widehat{F}_{\max}$, the Pareto efficient allocation is that only basic manufacturing is produced:

$$\begin{cases} l_b = L, l_m = l_h = l_s = 0, \\ n = 0, m_h = m_s = 0, \\ c_b = A_b, c_h = c_s = 0, \end{cases} \tag{23}$$

and both the real GDP per capita and welfare of a representative household are equal to $w = \frac{\epsilon}{\epsilon-1}A_b^{\frac{\epsilon-1}{\epsilon}}$.

When $F = \widehat{F}_{\max}$, there are two different Pareto efficient allocations that give identical levels of welfare and per capita GDP ($w = \frac{\epsilon}{\epsilon-1}A_b^{\frac{\epsilon-1}{\epsilon}}$). One is that only basic manufacturing is produced as characterized by (23) and the other one has strictly positive labor allocations in all the three modern subsectors as characterized by (19)-(22).

We are now ready to make comparisons between the *laissez-faire* market equilibrium allo-

[10] $\widehat{F}_{\max}$ is endogenously determined. Refer to the appendix to see the equation that uniquely determines $\widehat{F}_{\max}$.

cation and the first best.

3.4 Comparison and Policy Implications

First of all, when $F > \max\{F_{\max}, \widehat{F}_{\max}\}$, there is a unique *Laissez-faire* market equilibrium, in which only basic manufacturing is produced, and it is socially efficient.

Secondly, when $\widehat{F}_{\max} < F_{\max}$ and $F \in (\widehat{F}_{\max}, F_{\max})$, the first best allocation is given by (23), whereas immature structural change and industrial upgrading must occur in both of the *Laissez-faire* market equilibria. This may help us understand Fact 3 established in Section 2, namely, the value added share of production service in total GDP is higher in those countries trapped in the low-income trap than those escapers. If a low-income country prematurely deindustralizes by rushing into services in the *Laissez-faire* market because of the market failure explained before, it is more likely to fall into the low-income trap. This is in fact exactly the case observed in Africa, see MacMillan, Rodrik and Versuzco-Galolo (2014). However, for the same country, if appropriate government interventions are implemented to deter this premature deindustralization by providing for example tax incentives to the investment in the manufacturing sector, the economy could stay in the traditional sector focusing on the production of basic manufacturing, which is consistent with its comparative advantage, and it achieves the first-best allocation and hence more likely to escape the low-income trap.

Thirdly, when $\widehat{F}_{\max} > F_{\max}$ and $F \in (F_{\max}, \widehat{F}_{\max})$, there is a unique *Laissez-faire* market equilibrium, in which only basic manufacturing is produced, whereas all the three modern subsectors produce in the first-best allocation, that is, both structural change and industrial upgrading should occur. This may help us understand Fact 2 established in Section 2. If a middle-income country fails to upgrade its manufacturing sector or fails to develop its service sector in time because of the pecuniary externality, it is more likely to fall into the middle-income trap. However, for the same country, if appropriate government interventions are implemented to facilitate industrial upgrading and structural change, for example, by subsidizing the modern sector or taxing the traditional sector, the economy could move closer to the first-best allocation and hence more likely to escape the middle-income trap.

Lastly, when $F < \min\{F_{\max}, \widehat{F}_{\max}\}$, output of production service is strictly positive both in the *Laissez-faire* market equilibria and in the first best allocation. By Proposition 2, the high *Laissez-faire* market equilibrium Pareto dominates the low one, so we only need to compare the high market equilibrium allocation with the first best. Since these two allocations are never identical, so there is still room for welfare-enhancing government interventions. The following are concrete examples how differently the first-best allocation and the high market equilibrium allocation would change when A_m, F, A_h or A_s changes.

When the productivity of production service A_m increases, the first-best (denoted with superscript FB) number of firms n^{FB} would decrease (that is, $\frac{\partial n^{FB}}{\partial A_m} < 0$) to lower the deadweight loss caused by the entry cost, the employment in all the three subsectors in the modern sector would decrease ($\frac{\partial l_x^{FB}}{\partial A_m} < 0, \forall x \in \{m, h, s\}$) because the labor-saving effect (due to higher A_m) dominates the income effect which tends to increase the consumption demand for h and s and hence the demand for the total employment from the modern sector, and increase the employment in the basic manufacturing ($\frac{\partial l_b^{FB}}{\partial A_m} > 0$). All these results are exactly the opposite for the high decentralized market equilibrium because higher A_m would incentivize more firms to enter the production service and increase employment in production service, so the production service becomes cheaper, which in turn helps increase the production scale and hence employment of downstream consumption service and high-quality manufacturing. The employment in the basic manufacturing sector must decrease because the total employment in the modern sector and labor used for entry cost nF both increase.

The impact of changes in entry cost F is also different. In the first best allocation, an increase in F has no effect on effect on the labor allocation across the four subsectors ($\frac{\partial l_x^{FB}}{\partial F} = 0; \forall x \in \{m, h, s, b\}$), nor does it affect the total labor used to pay the entry cost as implied by the feasibility constraint for labor. The key intuition is that F is a fixed cost and does not alter any marginal rate of transformation or marginal rate of substitution. However, there are fewer firms and less total output in the production service sector ($\frac{\partial Y_m^{FB}}{\partial F} < 0$), although each firm employs more labor and produces more. In contrast, in the decentralized (high) market equilibrium, an increase in entry cost increases the basic manufacturing employment but reduces the employment in each of the three subsetors in the modern sector ($\frac{l_b}{\partial F} > 0; \frac{\partial l_y}{\partial F} < 0, \forall y \in \{m, h, s\}$), and the total labor used for entry cost nF also decreases (refer to Figure 8).

What happens when the productivities in the modern downstream sectors change? In the first best allocation, changes in A_h or A_s have no impact on labor allocation across sectors ($\frac{\partial l_x^{FB}}{\partial A_h} = \frac{\partial l_x^{FB}}{\partial A_s} = 0, \forall x \in \{b, m, h, s\}$), nor do they affect firm entries or output in the production service sector ($\frac{\partial n^{FB}}{\partial A_h} = \frac{\partial n^{FB}}{\partial A_s} = 0; \frac{\partial Y_m^{FB}}{\partial A_h} = \frac{\partial Y_m^{FB}}{\partial A_s} = 0$). To compare, in the decentralized (high) market equilibrium, an increase in A_h or A_s raises the employment in each of the three modern subsectors but reduces the employment in the basic manufacturing sector (that is, $\frac{\partial l_y}{\partial A_h} > 0; \frac{\partial l_y}{\partial A_s} > 0. \forall y \in \{m, h, s\}; \frac{\partial l_b}{\partial A_h} < 0; \frac{\partial l_b}{\partial A_s} < 0$). In addition, an increase in A_h or A_s raises both firm entries and output in the production service sector (that is, $\frac{\partial n_2}{\partial A_h} > 0; \frac{\partial n_2}{\partial A_s} > 0; \frac{\partial Y_m}{\partial A_h} > 0; \frac{\partial Y_m}{\partial A_s} > 0$).

All the three comparative static analyses above suggest that the first best allocation and market equilibrium allocations would change in exactly the opposite directions when most of sectorial productivities change. This may again shed light on the middle-income trap: When the entry cost of production service is low enough, the market-supported industrial upgrading and structural change can achieve higher levels of GDP and welfare than the traditional-sector-only allocation, but there is still room for optimal industrial policies to further increase real output per capita and welfare, which reduces the likelihood of falling into the middle-income trap.

To summarize, the *Laissez-faire* market equilibrium is often inefficient because of the pecuniary externality caused by the combination of the increasing-returns-to-scale nature of the production service sector and the input-output linkage across sectors. Government may improve the market performance by providing coordination, deterring immature deindustrialization, overcoming delays in structural change and industrial upgrading, depending on the specific scenarios. The availability of optimal government intervention can help a developing economy escape the middle-income trap or low-income trap.

Another implication we can draw from the above analyses is that, an increase in the productivity of the production service sector A_m and/or a reduction in the entry cost to that sector F may have positive, negative or no impact on the GDP per capita and welfare in market equilibrium (equilibria), depending on productivities of all the other sectors and the population. In other words, a more efficient production service sector can be a double-edge sword for growth and welfare. It turns out that this result is robust even when we consider an open economy with international trade.

Next, we extend the autarky model to the open economy. Apart from being more realistic, introducing international trade also allows us to examine how differences in tradability between manufacturing and service may affect industrial upgrading and structural change. Another benefit of extending to trade is that we can discuss endogenous interactions between developing and developed countries and their GDP gap. Middle-income trap can be cast as a phenomenon of divergence from rich countries instead of a low level of absolute GDP per capita.

4 International Trade

Suppose that there are two countries in the world: Home and Foreign. The home country is the same as the economy described in Section 4. In the Foreign country there are L^* identical households, each of whom is endowed with one unit of labor. Denote all the variables in Foreign with asterisks. Foreign households share the same preference as (2). Our main focus is on Home, so we simplify away the vertical structure in Foreign (*i.e.*, no production service is needed in producing anything in Foreign). The technologies in Foreign are specified as follows:

$$Y_b^* = A_b^* l_b^*, Y_h^* = A_h^* l_h^*, Y_s^* = A_s^* l_s^*.$$

Only high-quality manufacturing good (h) and basic manufacturing good (b) are tradable. Consumption service (s) is not tradable. Production service (m) is not needed in Foreign so it is not traded. Trade is free. All markets in foreign are perfectly competitive. Let w^* denote foreign wage rate or its GDP per capita.

Foreign is a developed economy and has comparative advantage in high-quality manufacturing. Home is a less developed economy and has comparative advantage in basic manufacturing.

Obviously, when F, entry cost to the production service sector in Home, is sufficiently large, no production service m will be produced in Home. Only basic manufacturing b will be produced. Moreover, there will be no market demand for high-quality manufacturing h because consumption service s is unavailable in Home (recall utility function (2)). Consequently, there will be no trade between Domestic and Foreign no matter how cheap h could be in Foreign. In this case, Proposition 1 applies and $w = \frac{\epsilon}{\epsilon-1} A_b{}^{\frac{\epsilon-1}{\epsilon}}$.

Proposition 3. *When basic manufacturing b is produced in both countries, we have*

$$\frac{w^*}{w} = \frac{A_b^*}{A_b}; \tag{24}$$

When H and F are completely specialized in b and h, there exists a unique market equilibrium, in which

$$\frac{\partial}{\partial F}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial A_m}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_h}\left(\frac{w^*}{w}\right) = 0; \tag{25}$$

When H produces both b and h while F only produces h, there is a unique equilibrium, in which

$$\frac{\partial}{\partial F}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_m}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial A_h}\left(\frac{w^*}{w}\right) < 0 \tag{26}$$

Proof. For the explicit necessary and sufficient conditions for each of the three different scenarios, please refer to the appendix. **Q.E.D**

This proposition shows that, the determinants of per capita GDP gap between Home and Foreign are different when the trade specialization patterns are different.

More concretely, when the Home country is at a very low stage of development stage so that Home only produces basic manufacturing b whereas Foreign produces both b and high-quality manufacturing h, the gap in GDP per capita between these two countries is solely determined by their relative labor productivities in basic manufacturing, see (24). Conditional on this trade specialization pattern, entry cost of upstream sector F or upstream productivity A_m or high-quality manufacturing productivity A_h has no impact on the GDP per capita gap. In other words, neither the production service sector nor the high-quality manufacturing, even very inefficient, is not a binding constraint for Home to converge to Foreign. This case may be relevant for most low-income countries such as those in Sub Sahara African countries, which barely industrialize.

When Home country is more developed than the previous case but still at a quite low income status, it specializes in basic manufacturing and Foreign specializes in high-quality manufacturing, an increase in entry cost F, or a decrease in upstream productivity A_m would lead to a smaller GDP gap between Foreign and Home (see (25)). This seemingly counter-intuitive results can be understood in the following way. Home country now imports h from Foreign, which yields positive utility if and only if consumption service s is provided domestically as it is non-tradable. When F increases and/or A_m decreases, consumption service s becomes more expensive as intermediate input m is now more costly. This would lower the marginal utility of high-quality manufacturing good h because of the complementarity between h and m as implied by (2). Consequently, Home will import less from Foreign, which narrows the per capita GDP gap (convergence). In this trade specialization pattern, a marginal increase in A_h has no impact on the GDP gap because h is not produced domestically. This case may be relevant for countries in the middle-income trap such as Mexico or Argentina. These countries prematurely de-industralize and the service sector is over developed for their stage of development.

When Home country is more developed in the sense that it becomes sufficiently more competitive in producing h such that it produces both h and b whereas Foreign only produces h, the results are diametrically opposite to the previous case. As (26) shows, in this case, an increase in entry cost F or a decrease in upstream productivity A_m would result in a larger GDP gap between Foreign and Home (divergence), because domestic production cost of h would increase and Home would have to import more from Foreign. Moreover, an increase in A_h in Home will result in convergence because domestic production cost of h decreases. This case may be relevant for the middle-income countries such as China, which must lower the entry barrier to the production service sector and increase productivities in both high-quality manufacturing and production service in order to escape the middle-income trap.

As is clear from this proposition, the role of production service m is asymmetric at different development stages. A more productive production service sector in the developing country does not necessarily imply a smaller GDP gap from developed countries. Only when the developing country becomes sufficiently effective in producing high-quality manufacturing would the GDP per capita gap be reduced by having a more efficient production service sector.

5 Discussion for China

Mainland China has achieved a miraculous GDP growth at an average annual rate of 9.4% in the past forty years and it is now a middle-income economy. Its per capita GDP was USD 9,780 in 2018. If mainland China manages to escape the middle-income trap, it will become the third economy that successfully moves from a low-income status to a high-income status in the past century (the other two economies are Taiwan and South Korea) and the percentage of world population that lives in high-income economies will more than double (from 15% to 34%).

Whereas it is increasingly becoming a consensus that China will be able to escape the middle-income trap with high probabilities, there are still concerns about various alarming issues that could hurt its growth potentials such as ageing, lagged reforms in land, capital and labor markets, unfinished State-Owned Enterprises (SOEs) reforms, deteriorating income inequality, corruption, systemic financial risks, insufficient innovation capabilities, or, the recent trade war and overall tensions between China and US, and so on. Many of those issues are explicitly studied in the report *China 2030* (World Bank, 2008). In particular, underdevelopment of the service sector is another phenomenon that is often viewed as a symptom of the Chinese economy. Figure 10 plots the share of service in GDP for the economies with real GDP per capita between 3,000 and 20,000 US dollars for year 2014.

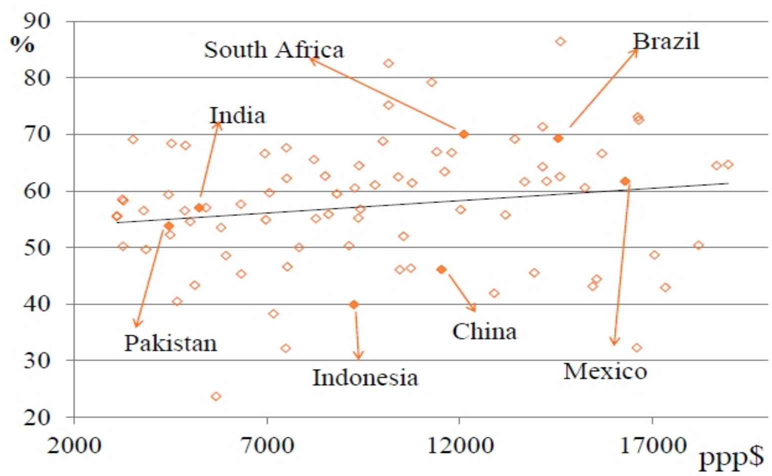


Figure 10. Service Shares in GDP

As we can see, China's service-to-GDP ratio is lower than the average for economies with similar levels of GDP per capita. A further decomposition analysis of China's service sector shows that its consumption service (downstream service) such as hotel service, restaurants, entertainment is liberalized and competitive, subject to low entry barriers, however, production service (upstream service) including financial service and business service such as telecommunications is still facing high entry barriers. More specifically, Li, Liu and Wang (2016) document a "vertical structure" observed in the Chinese economy, namely, some key upstream sectors (especially production service) are dominated by SOEs and there exist huge entry barriers due to administrative regulations (different from natural monopoly power, which would impose entry cost for merely technological reasons instead of red tape costs), whereas the downstream sectors including manufacturing of consumption goods and consumption service are largely liberalized (SOEs in downstream sectors have mostly lost policy protections and, as a consequence, most of inefficient SOEs exited the downstream sectors), especially after the SOE reforms were undertaken in the late 1990s and China joined the World Trade Organization in 2001.

Based on the vertical structure documented in Li, Liu and Wang (2016) and the theoretical analyses shown above in this paper, we can infer that China should further improve its upstream production service sector by lowering entry barriers, encouraging more entries and enhancing market competition. Only by doing so can China develop an efficient enough production service sector that can facilitate industrial upgrading within its manufacturing sector and healthy structural change into the service sector. This is particularly true in a globalized world, as analyzed in Proposition 3.[11]

In addition, our model also alerts that China should be cautious to curb premature de-industrialization, especially in those regions where the real estate bubbles deincentivize entrepreneurs in the manufacturing sector to stay in the real sector and upgrade to high-quality manufacturing.

6 Conclusion

In this paper, motivated by several stylized facts, we develop a very simple multi-sector general-equilibrium model to explain why only a minority of middle-income countries manage to escape the middle-income trap. Our model has three key structural features. First, it examines two processes and their interactions: industrial upgrading from the basic manufacturing to high-quality manufacturing and structural change from manufacturing to service. Second, it considers the different input-output linkages between the upstream production service and downstream modern sectors vis-a-vis the traditional sector. Third, manufacturing is tradable whereas service is not, so they are asymmetrically affected by international trade.

Among all the interesting theoretical findings, two novel results deserve special attention. First, there may exist multiple equilibria in the market because of the endogenous supermodularity among upstream production service firms, which results from the increasing-return-to-scale nature in that sector and is also amplified by the input-output linkage across sectors in the presence of non-homothetic preferences. The resulting pecuniary externality implies that the *Laissez-faire* market equilibrium (or equilibria) is inefficient in most cases, so it is desirable for the government to use industrial policy strategically to improve the market performance by providing coordination, deterring premature de-industrialization or overcoming delays in structural change and industrial upgrading. The industrial policy used in an appropriate way can help a developing economy escape the middle-income trap or low-income trap. Second, the role of production service is asymmetric at different levels of development. Whereas an underdeveloped sector of production service is not a binding obstacle for development (sometimes even beneficial) at an early stage of development, it becomes a key bottleneck when the

[11] Other factors such as imperfections in labor market (Hukou system) and associated regulations in the social service sector (including schooling, medicare, pension etc) may also account for underdevelopment of the service sector, which is beyond the scope of this paper and deserves separate explorations in the future.

economy reaches a middle income status because it hampers both structural change and industrial upgrading. It helps increase the probability of escaping the middle-income trap by using industrial policies to ensure a timely reduction of entry barrier to the production service and improvement in its productivity. These theoretical findings are shown to be consistent with the stylized facts. In particular, we show how this model could help us provide policy suggestions for China to avoid the middle-income trap.

Several directions for future research seem appealing. Whereas the model can be made dynamic by assuming exogenous productivity growth and all the static results remain valid, it is desirable to extend the model to a truly dynamic one by either adding capital accumulation or incorporating endogenous technical changes. Another interesting direction is to endogenize the level of entry cost F to the production service sector and explore the deregulation process. How to conduct a more thorough quantitative exercise (including counterfactual analyses) based on the current model and evaluate its empirical performance is certainly another option.

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7 Appendices

7.1 Appendix 1

It is easy to show that $\widehat{F}_{\max}$, the cutoff value for entry cost in the social planner problem, is uniquely determined by

$$A_h^{\theta}A_s^{1-\theta}\widehat{F}_{\max}{}^{-\frac{1-\sigma}{\sigma}[\alpha\theta+\beta(1-\theta)]}A_m^{\alpha\theta+\beta(1-\theta)}l_s^{\theta(1-\alpha)+(1-\theta)(1-\beta)+\frac{1}{\sigma}[\alpha\theta+\beta(1-\theta)]}\left(\frac{1-\sigma}{\sigma}\right)^{\frac{1-\sigma}{\sigma}[\alpha\theta+\beta(1-\theta)]}$$
$$\cdot\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}\right)^{\theta\alpha}\left\{\frac{\beta}{1-\beta}\left[\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}\right]^{\frac{1-\sigma}{\sigma}}\right\}^{\alpha\theta+\beta(1-\theta)}$$
$$=\frac{\epsilon}{\epsilon-1}L\left\{A_b{}^{\frac{\epsilon-1}{\epsilon}}-\left[\frac{\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\beta A_bA_ml_s^{2-[\theta(1-\alpha)+(1-\theta)(1-\beta)]}}{\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}(1-\theta)(1-\beta)^2}\right]^{\epsilon-1}\right\},$$

where l_s is uniquely determined by (22). When $F\leq\widehat{F}_{\max}$, it can be shown that the real output per capita and welfare of a representative household is given by

$$w=A_h^{\theta}A_s^{1-\theta}\frac{F^{-\frac{1-\sigma}{\sigma}[\alpha\theta+\beta(1-\theta)]}}{L}A_m^{\alpha\theta+\beta(1-\theta)}l_s^{\theta(1-\alpha)+(1-\theta)(1-\beta)+\frac{1}{\sigma}[\alpha\theta+\beta(1-\theta)]}\left(\frac{1-\sigma}{\sigma}\right)^{\frac{1-\sigma}{\sigma}[\alpha\theta+\beta(1-\theta)]}$$
$$\cdot\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}\right)^{\theta\alpha}\left\{\frac{\beta}{1-\beta}\left[\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\frac{\beta}{1-\beta}\right]^{\frac{1-\sigma}{\sigma}}\right\}^{\alpha\theta+\beta(1-\theta)}$$
$$+\frac{\epsilon}{\epsilon-1}\left[\frac{\left(\frac{\theta}{1-\theta}\frac{\alpha}{\beta}+1\right)\beta A_bA_ml_s^{2-[\theta(1-\alpha)+(1-\theta)(1-\beta)]}}{\left(\frac{\theta}{1-\theta}\frac{1-\alpha}{1-\beta}\right)^{\theta(1-\alpha)}(1-\theta)(1-\beta)^2}\right]^{\epsilon-1},$$

where l_s is uniquely determined by (22).

7.2 Appendix 2

Suppose $F=0$ and $\sigma=1$, then we are in the perfectly competitive market environment with constant returns to scale technologies. Both the First and Second Welfare Theorems apply. The decentralized market equilibirum allocation is Pareto efficient and is identical to the solution to the following artifical social planner problem:

$$\max_{l_b,l_m,l_h,l_s,m_h,m_s,c_h,c_s,c_b} c_h^{\theta}c_s^{1-\theta}+\frac{\epsilon}{\epsilon-1}c_b{}^{\frac{\epsilon-1}{\epsilon}}$$

subject to

$$\begin{aligned} c_b L &= A_b l_b; \\ c_h L &= A_h m_h^{\alpha} l_h^{1-\alpha}; \\ c_s L &= A_s m_s^{\beta} l_s^{1-\beta}; \\ m_h + m_s &= A_m l_m; \\ l_b + l_m + l_h + l_s &= L, \end{aligned}$$

and no-negativity conditions for all relevant variables.

Define $\widetilde{H}(A_m, A_h, A_s)$ as the same function $H(A_m, A_h, A_s)$ given by (8) except that σ is substituted out with unity and $\chi = 0$ based on (9). The socially efficient allocation is as follows: When $1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s) > 0$, or equivalently, when (11) is satisfied with $\sigma = 1$, we have

$$\begin{aligned} l_b &= A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)L \\ l_m &= [\alpha\theta + \beta(1-\theta)]\left[1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)\right]L \\ l_s &= (1-\theta)(1-\beta)\left[1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)\right]L \\ l_h &= (1-\alpha)\theta\left[1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)\right]L \end{aligned}$$

$$\begin{aligned} m_h &= \alpha\theta A_m\left[1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)\right]L \\ m_s &= \beta(1-\theta)A_m\left[1 - A_b^{\epsilon-1}\widetilde{H}^{-\epsilon}(A_m, A_h, A_s)\right]L. \end{aligned}$$

There are strictly positive entries to the production service sector but the firm number n is indeterminate because of the constant-returns-to-scale technology plus free entry with $F = 0$.

7.3 Appendix 3

7.3.1 H and F are completely specialized in b and h.

It is easy to show that in equilibrium, we have

$$p_b = \frac{w}{A_b}; p_m = n^{1-\frac{1}{\sigma}} \frac{w}{\sigma A_m}; p_s = \frac{p_m^\beta w^{1-\beta}}{A_s \beta^\beta (1-\beta)^{1-\beta}},$$
$$p_h^* = \frac{w^*}{A_h^*}; p_s^* = \frac{w^*}{A_s^*}.$$

The demand functions for b, h, s are as follows when both b and h are consumed in both countries:

$$\begin{aligned}
c_b &= \left(\frac{p_h^{*\theta} p_s^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta} p_b} \right)^\varepsilon \\
c_h &= \frac{\theta \left[w - p_b \left(\frac{p_h^{*\theta} p_s^{1-\theta}}{p_b \theta^\theta (1-\theta)^{1-\theta}} \right)^\varepsilon \right]}{p_h^*} \\
c_s &= \frac{(1-\theta) \left[w - p_b \left(\frac{p_h^{*\theta} p_s^{1-\theta}}{p_b \theta^\theta (1-\theta)^{1-\theta}} \right)^\varepsilon \right]}{p_s} \\
c_b^* &= \left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b \theta^\theta (1-\theta)^{1-\theta}} \right)^\varepsilon \\
c_h^* &= \frac{\theta \left[w^* - p_b \left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b \theta^\theta (1-\theta)^{1-\theta}} \right)^\varepsilon \right]}{p_h^*} \\
c_s^* &= \frac{(1-\theta) \left[w^* - p_b \left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b \theta^\theta (1-\theta)^{1-\theta}} \right)^\varepsilon \right]}{p_s^*}
\end{aligned}$$

Market clearing conditions:

$$\begin{aligned}
Lc_b + L^* c_b^* &= A_b l_b \\
Lc_h + L^* c_h^* &= A_h^* l_h^* \\
Lc_s &= Y_s = A_s m_s^\beta l_s^{1-\beta} \\
L^* c_s^* &= Y_s^* = A_s^* l_s^* \\
D_m &= Y_m \Rightarrow m_s = A_m l_m \\
l_b + l_s + l_m + nF &= L \\
l_h^* + l_s^* &= L^*
\end{aligned}$$

We always have

$$Y_m = n^{1/\sigma} \frac{\sigma A_m F}{1-\sigma} \tag{27}$$

H does not produce h, so m is used for producing s only. Using Shephard's Lemma, we can obtain the aggregate demand for m

$$m_s = D_m = D_s \frac{\partial p_s}{\partial p_m} = L \frac{\beta c_s p_s}{p_m}$$

Since $Y_m = D_m$, we have $n^{1/\sigma} \frac{\sigma A_m F}{1-\sigma} = L \frac{\beta c_s p_s}{p_m}$, which implies

$$\begin{aligned} & \frac{nF}{(1-\sigma) L \beta (1-\theta)} \\ & = 1 - A_b^{\epsilon-1} n^{\left(1-\frac{1}{\sigma}\right)\beta(1-\theta)\epsilon} \left(\left[\frac{w^*}{w\theta A_h^*} \right]^{\theta} \left[\frac{1}{(1-\theta)(\sigma A_m)^{\beta} A_s \beta^{\beta} (1-\beta)^{1-\beta}} \right]^{1-\theta} \right)^{\epsilon} \end{aligned}$$

So an equilibrium exists only if the right hand side of the above equation is strictly positive. In particular, when there exist two roots: n_1 and n_2, with $n_1 < n_2$, the equation above impliesr that $n_2 = \Xi(\frac{w^*}{w})$ with $\Xi'(\frac{w^*}{w}) < 0$ and $n_1'(\frac{w^*}{w}) > 0$. We focus on the high equilibrium n_2. On the other hand, balanced trade implies

$$L c_h p_h^* = L^* c_b^* p_b$$

which is equivalent to

$$\begin{aligned} & A_b^{1-\epsilon} \left[A_h^{*\theta} \theta^{\theta} (1-\theta)^{1-\theta} A_s^{*(1-\theta)} \right]^{\varepsilon} \\ & = \frac{L^*}{L\theta} \left[\frac{w^*}{w} \right]^{\varepsilon} + \left[\frac{\Lambda^{\left(1-\frac{1}{\sigma}\right)\beta}}{(\sigma A_m)^{\beta} \beta^{\beta} (1-\beta)^{1-\beta}} \right]^{(1-\theta)\varepsilon} \left[\frac{w^*}{w} \right]^{\varepsilon\left[\theta + \epsilon\left(1-\frac{1}{\sigma}\right)\beta(1-\theta)\right]}, \end{aligned} \tag{28}$$

where

$$\Lambda \equiv \frac{L^* \frac{1-\sigma}{F} \frac{\beta(1-\theta)}{\theta}}{A_b^{1-\epsilon} \left[A_h^{*\theta} \theta^{\theta} (1-\theta)^{1-\theta} A_s^{*(1-\theta)} \right]^{\epsilon}}.$$

Therefore, $n = \Lambda \cdot \left[\frac{w^*}{w}\right]^{\varepsilon}$. Thus (28)implies that there exists a unique equilibrium when $\theta + \epsilon\left(1-\frac{1}{\sigma}\right)\beta(1-\theta) \geq 0$ because the right hand side strictly increases with $\frac{w^*}{w}$. It implies $n = \Psi(\frac{w^*}{w}) \equiv \Lambda \cdot \left[\frac{w^*}{w}\right]^{\varepsilon}$ with $\Psi'(\frac{w^*}{w}) > 0$.

$$\begin{aligned} & \frac{\partial}{\partial F}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial L^*}\left(\frac{w^*}{w}\right) ?0; \frac{\partial}{\partial A_b}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial L}\left(\frac{w^*}{w}\right) > 0; \\ & \frac{\partial}{\partial A_h^*}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_s^*}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_s}\left(\frac{w^*}{w}\right) = \frac{\partial}{\partial A_b^*}\left(\frac{w^*}{w}\right) = \frac{\partial}{\partial A_h}\left(\frac{w^*}{w}\right) = 0; \frac{\partial}{\partial A_m}\left(\frac{w^*}{w}\right) > 0 \end{aligned}$$

Specifically, when $\theta = 0$, no trade occurs. When $\theta = 1$, $\frac{w^*}{w} = \left[A_b^{1-\epsilon} A_h^{*\varepsilon} \frac{L}{L^*}\right]^{\frac{1}{\varepsilon}}$.

So $n_2 = \Xi(\frac{w^*}{w})$ and $n = \Psi(\frac{w^*}{w})$ must determine a unique solution, denoted by $\widetilde{n}(F, L, L^*, A_s, A_m, A_b, A_h^*, A_s^*)$ and $\widetilde{\frac{w^*}{w}}(F, L, L^*, A_s, A_m, A_b, A_h^*, A_s^*)$, if they cross. Moreover, it is easy to show that the following is true.

$$
\begin{aligned}
&\frac{\partial}{\partial F}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial L^*}\left(\frac{w^*}{w}\right) < 0; \frac{\partial}{\partial A_b}\left(\frac{w^*}{w}\right) < 0;\\
&\frac{\partial}{\partial L}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_h^*}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_s^*}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_s}\left(\frac{w^*}{w}\right) > 0; \frac{\partial}{\partial A_m}\left(\frac{w^*}{w}\right) > 0
\end{aligned}
$$

7.3.2 H and F both produce h and b.

In this case, we can show that

$$
\begin{aligned}
p_b &= \frac{w}{A_b}, p_m = n^{1-\frac{1}{\sigma}}\frac{w}{\sigma A_m}, p_h = \frac{p_m^\alpha w^{1-\alpha}}{A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}, p_s = \frac{p_m^\beta w^{1-\beta}}{A_s \beta^\beta (1-\beta)^{1-\beta}},\\
p_b^* &= \frac{w^*}{A_b^*}, p_h^* = \frac{w^*}{A_h^*}, p_s^* = \frac{w^*}{A_s^*}.
\end{aligned}
$$

In equilibrium:

$$
\begin{aligned}
p_b = p_b^* &\Rightarrow \frac{A_b^*}{A_b} = \frac{w^*}{w},\\
p_h = p_h^* &\Rightarrow \frac{\left[n^{1-\frac{1}{\sigma}}\frac{w}{\sigma A_m}\right]^\alpha w^{1-\alpha}}{A_h \alpha^\alpha (1-\alpha)^{1-\alpha}} = \frac{w^*}{A_h^*}.
\end{aligned}
$$

therefore,

$$
n = \left\{\sigma A_m \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}\right\}^{\frac{\sigma}{\sigma-1}}
$$

and

$$
\begin{aligned}
p_m &= \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}} w\\
p_h &= \frac{A_b^*}{A_b A_h^*} w; \;\; p_s = \frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta}{\alpha}} w}{A_s \beta^\beta (1-\beta)^{1-\beta}}.
\end{aligned}
$$

The equilibrium conditions are as follows:

$$
\begin{aligned}
l_m &= \frac{n\sigma F}{(1-\sigma)} = \frac{\sigma F}{(1-\sigma)} \left\{ \sigma A_m \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}} \right\}^{\frac{\sigma}{\sigma-1}} \\
n^{1/\sigma} \frac{\sigma A_m F}{1-\sigma} &= m_h + m_s \\
m_h &= L \frac{\alpha y_h p_h}{p_m} = L y_h \frac{\alpha \frac{A_b^*}{A_b A_h^*}}{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}}} \\
L y_h &= A_h m_h^a l_h^{1-\alpha} \\
m_s &= L \frac{\beta c_s p_s}{p_m} \\
&= \frac{L\beta (1-\theta)}{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}}} \left(1 - A_b^{-1} \left[\frac{A_b \left(\frac{A_b^*}{A_b A_h^*} \right)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \right]^\varepsilon \left(\frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{\beta}{\alpha}}}{A_s \beta^\beta (1-\beta)^{1-\beta}} \right)^{(1-\theta)\varepsilon} \right)
\end{aligned}
$$

$$
\begin{aligned}
m_h = {} & \left\{ \sigma A_m \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}} \right\}^{\frac{1}{\sigma-1}} \frac{\sigma A_m F}{1-\sigma} \\
& - \frac{L\beta (1-\theta)}{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}}} \left(1 - A_b^{-1} \left[\frac{A_b \left(\frac{A_b^*}{A_b A_h^*} \right)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \right]^\varepsilon \left(\frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{\beta}{\alpha}}}{A_s \beta^\beta (1-\beta)^{1-\beta}} \right)^{(1-\theta)\varepsilon} \right) \\
l_h = {} & \frac{(1-\alpha)}{\alpha} \left\{ \sigma A_m \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}} \right\}^{\frac{\sigma}{\sigma-1}} \frac{F}{1-\sigma} \\
& - \frac{(1-\alpha)}{\alpha} L\beta (1-\theta) \left(1 - A_b^{-1} \left[\frac{A_b \left(\frac{A_b^*}{A_b A_h^*} \right)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \right]^\varepsilon \left(\frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{\beta}{\alpha}}}{A_s \beta^\beta (1-\beta)^{1-\beta}} \right)^{(1-\theta)\varepsilon} \right)
\end{aligned}
$$

Thus $l_h > 0$ iff

$$
\frac{\left\{ \sigma A_m \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{1}{\alpha}} \right\}^{\frac{\sigma}{\sigma-1}} F}{L\beta (1-\theta)(1-\sigma)} > 1 - A_b^{-1} \left[\frac{A_b \left(\frac{A_b^*}{A_b A_h^*} \right)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \right]^\varepsilon \left(\frac{1}{A_s \beta^\beta (1-\beta)^{1-\beta}} \right)^{(1-\theta)\varepsilon} \left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*} \right]^{\frac{\beta}{\alpha}(1-
$$

Moreover,

$$c_b = \left(\frac{p_h^{\theta} p_s^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta} p_b}\right)^{\varepsilon} = \left(\frac{A_b\left(\frac{A_b^*}{A_b A_h^*}\right)^{\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}\left(\frac{\left[\frac{A_b^* A_h \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta}{\alpha}}}{A_s \beta^{\beta}(1-\beta)^{1-\beta}}\right)^{1-\theta}\right)^{\varepsilon}$$

$$c_h = \frac{\theta\left[w - p_b\left(\frac{p_h^{\theta} p_s^{1-\theta}}{p_b \theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon}\right]}{p_h} = \theta\left[\frac{A_b A_h^*}{A_b^*} - \frac{A_h^*}{A_b^*} c_b\right]$$

$$c_s = \frac{(1-\theta)\left[w - p_b\left(\frac{p_h^{\theta} p_s^{1-\theta}}{p_b \theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon}\right]}{p_s} = (1-\theta)\left[\frac{A_s \beta^{\beta}(1-\beta)^{1-\beta}}{\left[\frac{A_b^* A_h \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta}{\alpha}}}\left(1 - \frac{c_b}{A_b}\right)\right]$$

$$c_b^* = \left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b^* \theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon} = \left(\frac{A_b^*\left(\frac{1}{A_h^*}\right)^{\theta}\left(\frac{1}{A_s^*}\right)^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon}$$

$$c_h^* = \frac{\theta\left[w^* - p_b^*\left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b^* \theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon}\right]}{p_h^*} = \theta\left[A_h^* - \frac{A_h^*}{A_b^*} c_b^*\right]$$

$$c_s^* = \frac{(1-\theta)\left[w^* - p_b^*\left(\frac{p_h^{*\theta} p_s^{*1-\theta}}{p_b^* \theta^{\theta}(1-\theta)^{1-\theta}}\right)^{\varepsilon}\right]}{p_s^*} = (1-\theta)\left[A_s^* - \frac{A_s^*}{A_b^*} c_b^*\right]$$

$$\frac{\frac{(1-\alpha)}{\alpha}\left\{\sigma A_m\left[\frac{A_b^* A_h \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}\right\}^{\frac{\sigma}{\sigma-1}}\frac{F}{1-\sigma}}{\frac{(1-\alpha)}{\alpha} L\beta(1-\theta)\left(1 - A_b^{-1}\left[\frac{A_b\left(\frac{A_b^*}{A_b A_h^*}\right)^{\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}}\right]^{\varepsilon}\left(\frac{\left[\frac{A_b^* A_h \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta}{\alpha}}}{A_s \beta^{\beta}(1-\beta)^{1-\beta}}\right)^{(1-\theta)\varepsilon}\right)} > 1,$$

Market clearing conditions are

$$\begin{aligned} A_b l_b + A_b^* l_b^* &= L c_b + L^* c_b^* \\ L y_h + A_h^* l_h^* &= L c_h + L^* c_h^* \\ A_s m_s^{\beta} l_s^{1-\beta} &= L c_s \\ A_s^* l_s^* &= L^* c_s^* \\ l_b^* + l_h^* + l_s^* &= L^* \\ l_m + l_b + l_h + l_s + nF &= L, \end{aligned}$$

so

$$
\begin{aligned}
n^{1/\sigma}\frac{\sigma A_m F}{1-\sigma} &= m_h + m_s \\
&= L\left(\frac{\alpha y_h p_h}{p_m} + \frac{\beta c_s p_s}{p_m}\right) \\
\left\{\sigma A_m\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}\right\}^{\frac{1}{\sigma-1}}\frac{\sigma A_m F}{1-\sigma} &= L\left(\frac{\alpha y_h \frac{A_b^*}{A_b A_h^*}}{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}} + \beta c_s \frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta-1}{\alpha}}}{A_s \beta^\beta \left(1-\beta\right)^{1-\beta}}\right) \\
\left\{\sigma A_m\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}\right\}^{\frac{1}{\sigma-1}}\frac{\sigma A_m F}{1-\sigma} &= L\left(\frac{\alpha y_h \frac{A_b^*}{A_b A_h^*}}{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{1}{\alpha}}} + \beta c_s \frac{\left[\frac{A_b^* A_h \alpha^\alpha (1-\alpha)^{1-\alpha}}{A_b A_h^*}\right]^{\frac{\beta-1}{\alpha}}}{A_s \beta^\beta \left(1-\beta\right)^{1-\beta}}\right)
\end{aligned}
$$