Credit Search and Credit Cycles

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Abstract

The supply and demand of credit are not always well aligned and matched, as is reflected in the countercyclical excess reserve-to-deposit ratio and interest spread between the lending rate and the deposit rate. We develop a search-based theory of credit allocation to explain the cyclical fluctuations in bank reserves, the interest spread, as well as credit rationing. We show that search frictions in the credit market can not only naturally explain the countercyclical bank reserves, interest spread and credit rationing, but also generate endogenous business cycles driven primarily by the cyclical utilization rate of credit resources, as long conjectured by the Austrian school of the business cycle. In particular, we show that credit search can lead to endogenous increasing returns to scale and variable capital utilization in a model with constant returns to scale production technology and matching functions, thus providing a micro-foundation for the indeterminacy literature of Benhabib and Farmer (1994) and Wen (1998).

Keywords: Search Frictions, Credit Utilization, Credit Rationing, Self-fulfilling Prophecy, Business Cycles.

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1 Introduction

The role of financial intermediation and credit supply in driving and amplifying the business cycle has long been analyzed in the history of economic thoughts at least since the Austrian school. The Austrian theory of the business cycle emphasizes banks' issuance of credit as the cause of economic fluctuations, and asserts that the banking sector's excessive credit supply and low interest policy (a loanable funds rate below the natural rate) drive firms' investment boom, and its tight credit and interest rate policy (a loanable funds rate above the natural rate) generate economic slump.

The history of financial crisis seemed consistent with the Austrian theory. A notable feature of financial crisis in 2007, for example, is the coexistence of excessive demand for credit on the firm side (as reflected by the interest rate hick) but excess credit supply on the bank side (as reflected in the significantly increased excess reserve-to-deposit ratio). According to a survey on Chief Financial officers in 2008 by Campello, Graham and Harvey (2008), about 60 percent of U.S. CFOs states that their firms are financially constrained. Among them, 86% say that they have to pass attractive investment opportunities due to the inability to raise external financing. On the other hand, during the same period banks were building up their cash positions at unprecedented speed. The growth of bank loans plunged 37.17% from the second quarter of 2008 to the second quarter of 2010, while bank excessive reserves skyrocketed from 1.93 billion to 1043.30 billion during the same period. Corresponding to the tight credit supply was the high interest rate on bank loans. The opposite was true before the financial crisis. Namely, both the bank interest rate and the reserve-to-deposit ratio were excessively low in the economic boom periods leading to the financial crisis. Thus, we observe in Figure 1 a countercyclical movement in the excess reserve-to-deposit ratio (dashed line, re-scaled) and in Figure 2 a countercyclical movement in interest spread between the loan rate and the 3-month treasury bill rate (dashed line).

This paper provides a search-based financial intermediation theory to explain the observed countercyclical excess reserve-to-deposit ratio and the countercyclical interest spread in the data. Our main point is that in the real world there are always agents with savings (credit supply) and agents with investment projects (credit demand), but the demand side of the credit market (e.g., firms) and the supply side of the credit market (e.g., households and banks) need to overcome search frictions to be matched with each other. We show that such search frictions can lead not only to countercyclical excess reserve-to-deposit ratio and countercyclical interest spread, but also self-fulfilling business cycles driven by the effective utilization rate of the aggregate credit resources. Moreover, our calibration exercise reveals that an endogenous multiplier-accelerator propagation mechanism is empirically plausible despite the lack of production externalities and technological increasing returns to scale.[1]

Our framework also shed light on the issue of credit rationing. Credit rationing is not only of

[1] In contrast to Benhabib and Farmer (1994) and Wen (1998).

theoretical interest, but also plays a non-trivial role in real-world firm financing. As documented in Becchetti et al (2009), around 20.24% of firms are subject to credit rationing in the United States. However, the literature on credit rationing is extremely thin despite the seminal work of Stiglitz and Weiss (1983). Our search-theoretical approach provides a short cut to quantitatively study the business-cycle property credit rationing.

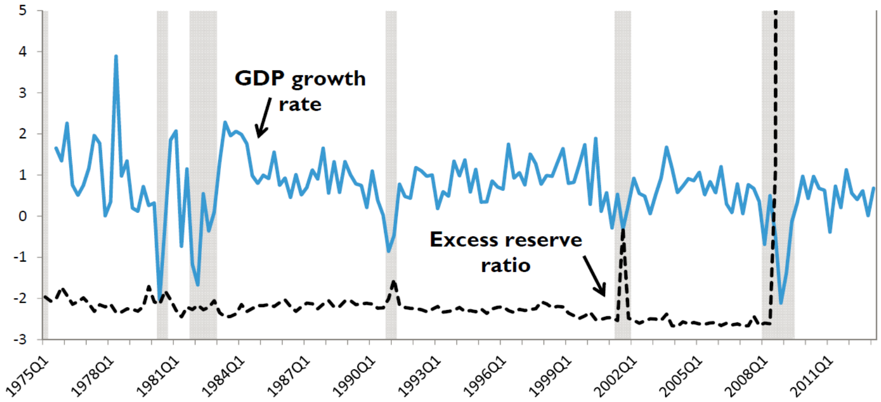

Figure 1. GDP Growth Rate and Excess Reserve Ratio.

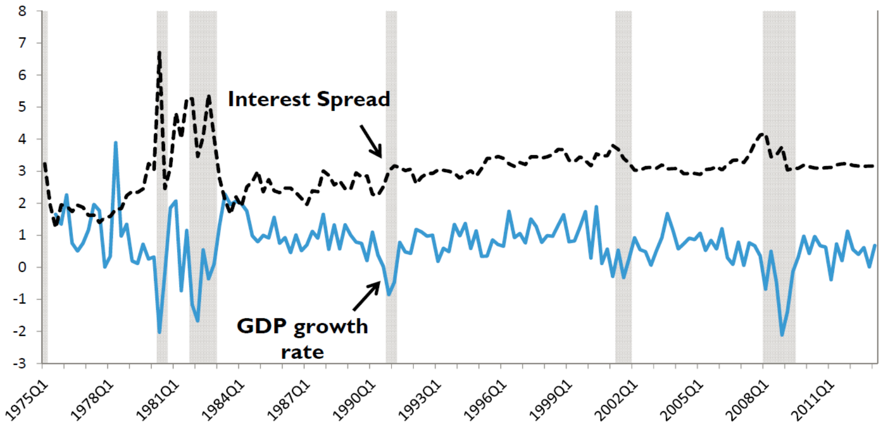

Figure 2. GDP Growth Rate and Interest Spread (Loan Rate minut Three-Month Treasury Bill)

Our framework is extremely simple. The benchmark model has three type of agents: a representative household with a continuum of ex anti identical members (depositors), a financial intermediary (bank) with a continuum of ex anti identical loan officers, and a continuum of firms. The bank accepts deposits from the household and then lend credit to firms. We assume there are search frictions between households and banks as well as between banks and firms, similar to that in a standard Diamond-Mortensen-Pissarides (DMP) search-and-matching model of unemployment. As in the DMP model, our model features aggregate matching functions that determine the number of credit relationships between depositors and banks, and between bank loan officers and firms. Such double search frictions create un-utilized savings and credit resources in equilibrium. For example, when bank deposits allocated to loan officers are not matched by firms, they become idle (excess reserves) in the banking system, while firms that are unmatched by loan officers are considered as being denied for credit. This simple matching friction and setup then explain the co-existence of "excess" reserves (analogous to unemployment) and credit rationing in the data. Since a booming economy encourages more firm-entry into the credit market to search for creditors (lenders), it increases the matching probability of credit resources. As a result, the reserve-to-deposit ratio is countercyclical over the business cycle. In addition, since the deposit rate facing the households is determined mainly by its time preference and the lending rate facing firms determined mainly by credit tightness and firms' credit demand, the spread between the loan rate and the deposit rate may also be countercyclical under aggregate shocks.

In addition, we show that such an endogenously elastic utilization rate of savings and credit resources due to search and matching can lead to endogenous aggregate increasing returns to scale (IRS) even though the underlying production and matching technologies both exhibit constant returns to scale (CRS). This endogenous source of IRS caused by procyclical credit utilization can lead to local indeterminacy and self-fulfilling credit cycles that feature a strong and powerful multiplier-accelerator propagation mechanism.

To understand the intuition, consider an anticipated increase in labor productivity by firms (in the absence of technology shocks). This would increase firms' labor demand and hence households' labor supply as well as household savings. The increase in household savings would then raise bank's credit supply and reduce the deposit rate, which would then lower the interest rate on loans and induce more firm to enter the credit market to search for loans. More firm entry in turn would increase the matching probability of bank loans, raising the effective capital stock used in firms' production, ratifying the initial optimistic expectation of higher labor productivity. Hence, a proportionate increase in household labor supply and savings would render firms' effective capital stock and aggregate production to increase more than proportionally, leading to endogenous IRS. Note that the IRS originate from a subtle pecuniary externality (based on firm entry and search) instead of technological externality. Also, as the matching probability of bank loans increases, the bank is able to pay a proportionately higher deposit rate relative to the loan rate, leading to countercyclical interest spread. This will increase the rate of return to saving even for those households who do not increase their saving rate and decrease the cost of credit (interest payments) even for those firms that do not increase their borrowings, further reinforcing the positive feedback loop among saving, credit, and investment, as emphasized by the Austrian school.

The endogenously arising IRS in our model are appealing for several reasons. First, aggregate demand shocks like preference shocks or government spending shocks are now able to generate positive business cycle comovements among aggregate consumption, investment, and output. Demand shocks are widely believed to be important sources of business cycles, yet in standard RBC models they generally produce a negative comovement between consumption and investment. Second, the standard RBC model has been criticized for requiring large technology shocks to produce realistic business cycles (see King and Rebelo (1999) for a survey of the literature). Thanks to the endogenous IRS in our model, small shocks (either demand or supply shocks) can generate large business cycle fluctuations. Third, our model can generate indeterminacy and self-fulfilling business cycles with hump-shaped output responses without productive externalities as in the model of Benhabib and Farmer (1994) and Wen (1998).

Our paper is related to several strands of literature. First, the search friction is in line with approaches proposed by Den Haan, Ramey and Waston (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013). These researchers have explored the implication of credit search on macroeconomy, but does not study the possibility of indeterminacy. For simplicity and tractability, they have assumed linear utility. In contrast, we incorporate the credit search friction into an otherwise standard RBC model. This allows us to study a richer set of economic variables of interest. Our paper is also inspired by search-theoretic model of asset trading such as Duffie, Gârleanu and Pederson (2005) and Lagos and Rocheteau (2009). Cui and Radde (2014) recently incorporate such line of research into a dynamic general equilibrium model and show it can explain the interesting flight-to-liquidity phenomena observed in great recession.

Our model also provides a micro foundation for the Benhabib-Farmer (1994) model with IRS and the Wen (1998) model with variable capital utilization rate. We show that search frictions in the credit market can generate an economic structure isomorphic to the Benhabib-Farmer-Wen model with both increasing returns and elastic capacity utilization, hence providing a micro foundation for this IRS-induced indeterminacy literature. Our paper is also closely related to several recent works on self-fulfilling business cycle due to credit market frictions, such as Gertler and Kyotaki (2014), Miao and Wang (2012), Benhabib, Miao and Wang (2014), Azariadis, Kaas and Wen (2014), Pintus and Wen (2013), Liu and Wang (2104), and Benhabib, Dong and Wang (2014).

Finally, our model is in the same spirit of Acemoglu (1996), who shows that search friction in labor market generate increasing returns to human capital accumulation in a two-period model. In his model, the workers have to make human capital investments before they enter the labor market. An increased in the average human capital investment induce more physical investments from firms. So even some of workers who have not increased their humane capital will earn a higher return on their human capital if matched with firms. In other words, search friction produce a positive pecuniary externality similar to the mechanism in our model. However, unlike Acemoglu (1996), we focus on search in credit market and explore its implication on indeterminacy and self-fulfilling expectation driven business cycles in an infinite-period model.

The rest of the paper is organized as follows. Section 2 lays out the baseline model and examines its key properties. Section 3 studies the model's business cycle implications under calibrated parameter values. Section 4 concludes the paper with remarks for future research.

2 The Benchmark Model

2.1 Environment

Time is continuous. The economy is populated by three types of agents: a representative household composed of a continuum of depositors, a representative and perfectly competitive bank (financial intermediary, FI) composed of a continuum of clerks or loan officers, and a continuum of firms. The household owns capital and firms, makes decision on labor supply and consumption, and deposits its savings into the bank, which then lends the deposits to firms. To break the conventionally assumed accounting identity between saving and investment (following a key idea of the Austrian school and Keynes), we assume search frictions among the three types of agents.

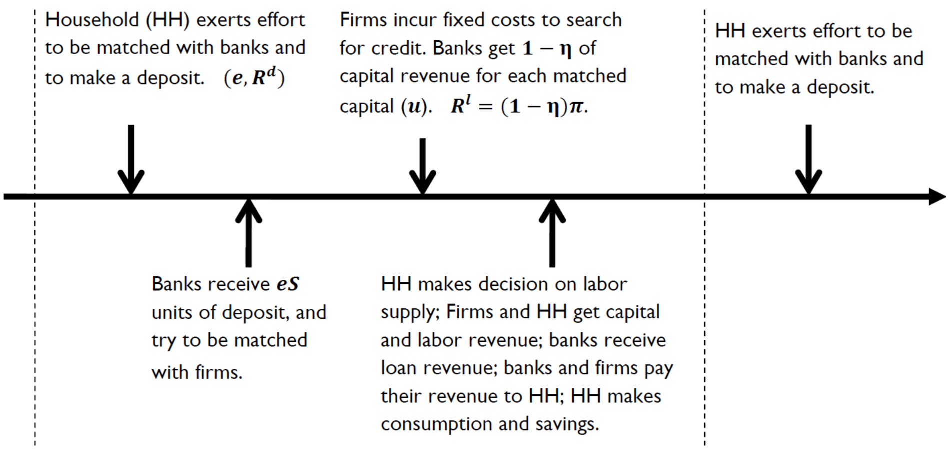

Figure 3. Timeline

The time line of events in a time interval between t and $t + dt$ is as follows. First, depositors from the representative household deposit the existing household savings (carried over from the last period) to the bank through search and match between depositors and bank clerks. Then the decentralized credit market opens, where the financial intermediary (loan officers) and firms randomly meet. In order to enter the credit market, a firm needs to pay a fixed cost. If matched with a loan, trading surplus is split between the firm and the bank and the credit relationship is then dissolved. The actual number of firms is determined by the free entry condition to the credit market, namely the expected surplus from a successful match equals to the fixed cost. Finally, the household pools wage income and profit incomes from the bank and firms and then makes decision on consumption and capital accumulation (next-period savings). The whole process is repeated again in the next time interval between $t+dt$ and $t+2dt$.

To facilitate the analysis, we can imagine that a continuum of depositors from the household exerting efforts to search and match with a continuum of bank clerks or loan officers. Denote S as the total savings of the household. Due to search frictions, only $\widetilde{S} < S$ units of savings are successfully matched and deposited into the banking system. After that, each loan officer is assigned with an equal fraction of the $\widetilde{S}$ units of deposits and goes out searching for potential borrows (firms).

We show that such a simple setup leads to a simple dynamic system that can generate (i) countercyclical excess reserve-to-deposit ratio, (ii) countercyclical interest spread between the loan rate and the deposit rate, and (iii) self-fulfilling business cycles with strong amplification and propagation mechanisms.

2.2 Deposit Search

We first consider search frictions between the household and the bank. The matching function between household members and bank clerks is $M^H(x_t H_t, B_t) = \gamma_H (x_t H_t)^{\varepsilon_H} B_t^{1-\varepsilon_H}$, where x_t is the effort choose by household depositors. There is unit measure of household depositors and bank clerks, *i.e.*, $H_t = B_t = 1$. Therefore, for each unit of savings, the expected revenue is

$$\max \left\{ e_t R_t^b - \phi^H x_t \right\} \cdot S_t \tag{1}$$

subject to

$$e_t = \gamma_H x_t^{\varepsilon_H}, \tag{2}$$

where e_t denotes the fraction of aggregate savings successfully deposited into the banking system. The first-order conditions (FOCs) are

$$x_t = \left[\left(\frac{\gamma_H \varepsilon_H}{\phi_H} \right) R_t^b \right]^{\frac{1}{1-\varepsilon_H}} \tag{3}$$

$$e_t = \gamma_H \left[\left(\frac{\gamma_H \varepsilon_H}{\phi_H} \right) R_t^b \right]^{\frac{\varepsilon_H}{1-\varepsilon_H}} \tag{4}$$

Notice that

$$\widetilde{S}_t = e_t S_t. \tag{5}$$

Denoting $\delta^0 = \frac{\phi_H(1+\kappa)}{\gamma_H^{1+\kappa}}$ and $\kappa = \frac{1}{\varepsilon_H} - 1$, we can derive a pseudo "depreciation" function of savings as

$$\delta(e_t) \equiv \phi^H \left(\frac{e_t}{\gamma_H} \right)^{\frac{1}{\varepsilon_H}} \equiv \delta^0 \left(\frac{e_t^{1+\kappa}}{1+\kappa} \right), \tag{6}$$

which is convex in the proportion (e_t) of savings the household puts into the bank. Then we can formulate the constrained optimization problem of the representative household. The life time utility of the representative household (consumer) is given by

$$\left\{ \int_0^{+\infty} e^{-\rho t} \left[log\left(C_t\right) - \psi \frac{N_t^{1+\xi}}{1+\xi} \right] \right\}, \tag{7}$$

where C_t is consumption, N_t is labor supply, $\rho > 0$ is the discount factor, $\psi > 0$ controls the utility weight on labor supply, and $\xi > 0$ is the inverse Frisch elasticity of labor supply. The household faces the following budget constraint,

$$C_t + \dot{S}_t = W_t N_t + \left[e_t R_t^d - \delta\left(e_t\right) \right] S_t + \Pi_t, \tag{8}$$

where S_t is the total capital stock of the households, W_t is the real wage, R_t^d is the deposit interest rate committed by the bank, and Π_t is the net profit from all firms and banks. The right hand of equation (8) is the total income of the households, which can be consumed, or used for capital accumulation. We use $\dot{S}_t$ to denote the change in the capital stock. The first order conditions of the household are given by

$$\frac{W_t}{C_t} = \psi N_t^{\xi}, \tag{9}$$

$$\frac{\dot{C}_t}{C_t} = e_t R_t^d - \delta\left(e_t\right) - \rho \tag{10}$$

$$R_t^d = \delta'\left(e_t\right) = \delta_0 e_t^{\kappa} \tag{11}$$

2.3 Loan Search

The credit market consists of a large number of credit lenders (loan officers) and borrowers (firms). More specifically, there are B_t number of loan officers and V_t number of firms. Each firm needs to pay a fixed cost ϕ_t in terms of capital to enter the credit market before it can search for a lender. Each loan officer carries $\frac{\widetilde{S}_t}{B_t}$ unit of capital and each firms demand 1 unit of capital. They randomly meet with each other. If they are matched, they can produce

$$y_t = A_t n_t^{1-\alpha} \tag{12}$$

units of output, where n_t is labor input of a matched firm. The search friction is captured by a matching technology $M\left(B, V\right)$, where B and V denote respectively the measure of loan officers and that of firms. As is standard in the search literature, we assume $M\left(B, V\right)$ is homogeneous of degree one in both augments. To make the results sharp and tractable, we also assume a Cobb-Douglas matching technology, $M(B,V) = \gamma B^{1-\varepsilon} V^{\varepsilon}$, with $\varepsilon \in (0,1)$. The probability that a firm can match with a credit supplier is

$$q \equiv \frac{M(B,V)}{V} = M(\theta, 1) = \gamma\theta^{1-\varepsilon}, \tag{13}$$

and the probability that a loan officer can match with a borrower (firm) is given by

$$u = p \equiv \frac{M(B,V)}{B} = M\left(1, \frac{1}{\theta}\right) = \gamma\theta^{-\varepsilon}. \tag{14}$$

Here $\theta \equiv \frac{B}{V}$ is the credit market tightness. Without loss of generality, we normalize the measure of loan officers to $B = 1$. Then we have

$$Vq = p \tag{15}$$

For simplicity, we assume that any matched credit relationship is dissolved at the end of each period.[2]

Given real wage w_t, the matching surplus can be determined by solving

$$\Pi_t = \max_{n_t \geq 0} \{y_t - W_t n_t\} = \max_{n_t \geq 0} \left\{A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha} - W_t n_t\right\}, \tag{16}$$

where $y_t = A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha}$. It is easy to show that

$$n_t = \left[A_t\left(\frac{1-\alpha}{W_t}\right)\right]^{\frac{1}{\alpha}} \widetilde{S}_t, \tag{17}$$

$$\Pi_t = \alpha A_t \left[A_t\left(\frac{1-\alpha}{W_t}\right)\right]^{\frac{1-\alpha}{\alpha}} \widetilde{S}_t \equiv \pi_t \widetilde{S}_t. \tag{18}$$

Notice that π_t depends only on the aggregate variables at time t. The surplus is split between the firm and the loan officer by Nash bargaining. More specifically the firm obtain $\eta \in [0,1]$ fraction of surplus. Denote R_t^l as the interest rate on loans or the returns to the bank, Nash bargaining then yields

$$R_t^l = (1-\eta)\,\pi_t. \tag{19}$$

The free entry condition for the firms is then given by

$$\phi_t = q_t \eta \Pi_t = q_t \eta \pi_t \widetilde{S}_t. \tag{20}$$

[2] See Dong, Wang and Wen (2014) for the theoretical and quantitative discussion on the dynamic credit relationship between firms and banks with search and matching frictions.

Then equation (13) can be rewritten as

$$q_t = M(\theta_t, 1) = \gamma\theta_t^{1-\varepsilon} = \frac{\phi_t}{\eta\pi_t\widetilde{S}_t}. \tag{21}$$

The above equation states that q_t must decrease with the match surplus. The intuition is as follows. A higher match surplus will induce more firms to enter, hence reduces the probability of each firm's match with credit suppliers.

The banking sector is perfectly competitive and thus makes zero profit in equilibrium. The bank needs to pay the depositors at the interest rate R_t^d, and earn the rate of return R_t^l (the lending interest rate) with probability p_t. The bank pools all profit income at the end of the day. Therefore the zero profit condition is given by

$$R_t^d = p_t R_t^l, \tag{22}$$

where p_t is given by equation (14). This equation captures the interest spread.

Finally the aggregate net profit income distributed to the household is given by

$$\Pi_t = \left(-R_t^d + p_t R_t^l\right)\widetilde{S}_t + \left(-\phi + q_t\eta\pi_t\right)V_t = 0, \tag{23}$$

where the second equality follows from the law of large number.

2.4 Equilibrium

Equilibrium is a collection of prices $\{W_t, R_t^d, R_t^l\}$ and quantities $\left\{C_t, S_t, N_t, \Pi_t, \pi_t, n_t, \widetilde{S}_t, K_t, e_t, p_t, q_t\right\}$ such that (i) given prices and aggregate profit income Π_t, the allocation $\{C_t, S_t, N_t\}$ solves household's utility maximization problem defined in (7); (ii) the surplus π_t and labor input n_t for a successfully matched firm are defined by (18) and (17); (iii) given the probability q_t of being matched with a bank loan officer, the equilibrium number of firms V_t is determined by the free entry condition (20); (iv) given the bank's probability of matching with a firm (p_t), the bank earns zero expected profit as characterized by (22); (v) the probability q_t and p_t are determined by (13) and (14), and all markets clear.

Since each match on the household side utilizes $\widetilde{S}_t = e_t S_t$ units of capital, and each match on the firm side utilizes $K_t = p_t\widetilde{S}_t$ units of capital, given the total initial available credit resources S_t, the fraction of aggregate credit resources being utilized in production is hence given by

$$u_t e_t \equiv \frac{K_t}{\widetilde{S}_t}\frac{\widetilde{S}_t}{S_t} = p_t e_t = (V_t q_t)\, e_t, \tag{24}$$

where we have used the normalization $B_t = 1$. As each matched firm employs n_t units of labor, the labor market equilibrium then requires

$$N_t = V_t q_t n_t = V_t q_t \left[A_t \left(\frac{1-\alpha}{W_t} \right) \right]^{\frac{1}{\alpha}} \widetilde{S}_t. \tag{25}$$

Finally, the total output produced by all firms is given by

$$Y_t = V_t q_t y_t = V_t q_t A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha} = V_t q_t A_t \widetilde{S}_t^{\alpha} \left(\frac{N_t}{V_t q_t} \right)^{1-\alpha} = A_t \left(V_t q_t \widetilde{S}_t \right)^{\alpha} N_t^{1-\alpha} = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{26}$$

where $K_t = u_t \widetilde{S}_t = u_t e_t S_t$. The surplus from a successful match is then given by

$$\pi_t \equiv \alpha A_t \left[A_t \left(\frac{1-\alpha}{W_t} \right) \right]^{\frac{1-\alpha}{\alpha}} = \alpha A_t \left(\frac{N_t}{V_t q_t \widetilde{S}_t} \right)^{1-\alpha} = \alpha \left(\frac{Y_t}{V_t q_t \widetilde{S}_t} \right) = \alpha \left(\frac{Y_t}{K_t} \right), \tag{27}$$

which equals to the marginal product of aggregate capital.

Equation (17) and (25) then yield

$$W_t = (1-\alpha) \left(\frac{Y_t}{N_t} \right). \tag{28}$$

The deposit rate is then given by

$$R_t^d = u_t R_t^l = u_t (1-\eta) \left[\alpha \left(\frac{Y_t}{K_t} \right) \right] = \alpha (1-\eta) \left(\frac{Y_t}{\widetilde{S}_t} \right). \tag{29}$$

The last equality is obtained by using $K_t = V_t q_t \widetilde{S}_t$ and $u_t = p_t = V_t q_t$. Since $K_t = e_t u_t S_t$, the aggregate production function can also be written as

$$Y_t = A_t (e_t u_t S_t)^{\alpha} N_t^{1-\alpha}. \tag{30}$$

When output increases, the demand for credit increases, hence it is easier for the credit suppliers to be matched with firms.

Finally, since $B_t = 1$, and $u_t = p_t = \gamma \theta_t^{-\varepsilon}$, the aggregate entry costs $V\phi$ satisfy

$$V\phi = \left(\frac{B}{\theta} \right) \phi = \left(\frac{u}{\gamma} \right)^{\frac{1}{\varepsilon}} \phi = \Delta(u) \equiv \Delta_0 \frac{u^{1+\lambda}}{1+\lambda}, \tag{31}$$

where $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$ and $\lambda \equiv \frac{1}{\varepsilon} - 1$.

2.5 Simplified Transition Dynamics

First, combining Equations (11), (22), (19) and (27) yields

$$\delta^0 e_t^{\kappa+1} = e_t R_t^d = e_t u_t R_t^l = (1-\eta)\, e_t u_t \pi_t = \alpha\,(1-\eta)\left(\frac{Y_t}{S_t}\right), \tag{32}$$

and thus

$$e_t = \left[\frac{\alpha\,(1-\eta)}{\delta_0}\left(\frac{Y_t}{S_t}\right)\right]^{\frac{1}{1+\kappa}}. \tag{33}$$

Since $\kappa = \frac{1}{\varepsilon_H} - 1$, and denoting $\widetilde{e} \equiv \left(\frac{\alpha(1-\eta)}{\delta_0}\right)^{\varepsilon_H}$, we then have

$$e_t = \widetilde{e}\left(\frac{Y_t}{S_t}\right)^{\varepsilon_H}, \tag{34}$$

where $\widetilde{e} \equiv \left(\frac{\alpha(1-\eta)}{\delta_0}\right)^{\frac{1}{1+\kappa}} = \gamma_H\left(\frac{\alpha(1-\eta)\varepsilon_H}{\phi_H}\right)^{\varepsilon_H}$, $\delta^0 = \frac{\phi_H(1+\kappa)}{\gamma_H^{1+\kappa}}$, and $\kappa = \frac{1}{\varepsilon_H} - 1$.

Secondly, as shown in Equation (31), firm's cost for credit search is $V\phi = \Delta(u) \equiv \Delta_0 \frac{u^{1+\lambda}}{1+\lambda}$. Meanwhile, the free entry condition (20) implies

$$V\phi = Vq\eta\pi\widetilde{S} = Vq\eta\left[\alpha\left(\frac{Y}{Vq\widetilde{S}}\right)\right]\widetilde{S} = \alpha\eta Y. \tag{35}$$

Combining Equation (**??**) and Equation (35) yields

$$u_t = \widetilde{u} Y_t^{\varepsilon}, \tag{36}$$

where $\widetilde{u} \equiv \left[\frac{\alpha\eta(1+\lambda)}{\Delta_0}\right]^{\frac{1}{1+\lambda}} = \gamma\left(\frac{\alpha\eta}{\phi}\right)^{\varepsilon}$, $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$ and $\lambda \equiv \frac{1}{\varepsilon} - 1$. Consequently, we can reduced the dynamics system of $\{C_t, S_t, N_t, W_t, R_t^d, R_t^l, \pi_t, K_t, e_t, u_t, q_t, \theta_t, Y_t, V_t\}$ to the following simpler system in $\{C_t, S_t, e_t, u_t, Y_t\}$:

$$\frac{\dot{C}_t}{C_t} = (1-\eta)\,\alpha\left(\frac{Y_t}{S_t}\right) - \delta(e_t) - \rho \tag{37}$$

$$\dot{S}_t = (1-\alpha\eta)\,Y_t - \delta(e_t)\,S_t - C_t \tag{38}$$

$$Y_t = A_t\,(e_t u_t S_t)^{\alpha}\, N_t^{1-\alpha} \tag{39}$$

$$e_t = \widetilde{e}\left(\frac{Y_t}{S_t}\right)^{\varepsilon_H} \tag{40}$$

$$u_t = \widetilde{u} Y_t^{\varepsilon} \tag{41}$$

$$\psi N_t^{\xi} = (1-\alpha)\left(\frac{Y_t}{N_t}\right)\left(\frac{1}{C_t}\right) \tag{42}$$

where $\delta(e_t) = \delta^0 \frac{e_t^{1+\kappa}}{1+\kappa}$. Then we have the following

Proposition 1 ***T****he aggregate production function in equation (39) exhibits IRS in household capital (S_t) and labor supply (N_t):*

$$Y_t = \bar{Y} A_t^{\tau} S_t^{\alpha_s} N_t^{\alpha_n}, \tag{43}$$

where

$$\bar{Y} \equiv (\widetilde{e}\widetilde{u})^{\alpha\tau} = \left[(1-\eta)^{\varepsilon_H} \left(\frac{\eta}{\varepsilon}\right)^{\varepsilon} \left(\frac{\alpha}{\delta^0}\right)^{\varepsilon_H} \left(\frac{\alpha\eta}{\Delta^0}\right)^{\varepsilon}\right]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}} \tag{44}$$

$$\tau \equiv \frac{1}{1-\alpha(\varepsilon+\varepsilon_H)} \tag{45}$$

$$\alpha_s \equiv \alpha(1-\varepsilon_H)\tau = \frac{\alpha(1-\varepsilon_H)}{1-\alpha(\varepsilon+\varepsilon_H)} \tag{46}$$

$$\alpha_n \equiv (1-\alpha)\tau = \frac{1-\alpha}{1-\alpha(\varepsilon+\varepsilon_H)} \tag{47}$$

and

$$\alpha_s + \alpha_n = \frac{1-\alpha\varepsilon_H}{1-\alpha(\varepsilon+\varepsilon_H)} > \frac{1}{1-\alpha\varepsilon} > 1. \tag{48}$$

Proof: Substituting Equation (34) and Equation (36) into Equation (30) yields Equation (43). ■

Notice that we obtain aggregate IRS in household capital S_t and labor supply N_t despite the lack of Benhabib-Farmer type production externalities. This is due to the endogenous feedback and reinforcing loop between the utilization rate of aggregate savings, aggregate credit resources, and firms' effective capital stock, as suggested by Equations (30) and (36). Meanwhile, we also obtain the amplification effect on productivity shock, *i.e.*,

$$\frac{\partial \log(Y_t)}{\partial \log(A_t)} = \frac{1}{1-\alpha(\varepsilon+\varepsilon_H)} > 1. \tag{49}$$

Our endogenous IRS model based on credit search is isomorphic to the models of Benhabib and Farmer (1994) and Wen (1998). Namely, our model also gives rise to local indeterminacy and self-fulfilling business cycles based on an endogenous amplification and propagation mechanism. To understand the intuition, consider a proportional increase in aggregate labor and capital supply from households. In a standard neoclassical model without credit search, such a proportional increase in labor and capital supply would only increase aggregate output one-for-one. However, in our model the increase of household savings leads to a higher credit supply in the banking system, which in turn would reduce the cost of borrowing and hence induce more firm to enter the credit market, which in turn increases the match probability of credit resources, raising the effective capital stock used in the production more than one-for-one and resulting in more than proportionate increase in aggregate output.

In addition to generating social IRS, the initial increase in household labor and capital supply can also become self-fulfilling. As the effective capital used in production increases, the returns to labor supply also increase for every household, reinforcing the initial increase in household labor supply. In addition, as the matching probability of bank increases, bank is able to pay a higher deposit rate. This will increase the returns to saving even for the households who do not increase their savings. Hence, the social increasing returns to scale originate from a subtle pecuniary externality that reinforces and multiplies itself in a positive feedback loop just like in the model of technological production externalities.

2.5.1 Hosios Condition and Welfare

Since we have used random search to characterize frictions in the credit market, it is natural for us to check whether the Hosios (1990) condition holds in our environment. Given (A_t, S_t, N_t), *i.e.*, if we control the technology and the supply of capital and labor, then the Hosios condition is obtained by solving the following constrained optimization problem of the social planner:

$$\max_{x_t, V_t} \{Y_t - \phi_H x_t S_t - \phi V_t\}, \tag{50}$$

subject to

$$Y_t = A_t (e_t u_t S_t)^\alpha N_t^{1-\alpha} \tag{51}$$

$$e_t = \gamma_H x_t^{\varepsilon_H} \tag{52}$$

$$u_t = \gamma \theta_t^{-\varepsilon} \tag{53}$$

$$\theta_t = \frac{B_t}{V_t} = \frac{1}{V_t}. \tag{54}$$

Using the notation adopted in the baseline model, we can rewrite the social planner's problem as

$$\max_{e_t, u_t} \{Y_t - \delta(e_t) S_t - \Delta(u_t)\}, \tag{55}$$

subject to

$$Y_t = A_t (e_t u_t S_t)^\alpha N_t^{1-\alpha} \tag{56}$$

$$\delta(e_t) = \delta^0 \frac{e_t^{1+\kappa}}{1+\kappa} \tag{57}$$

$$\Delta(u_t) = \Delta^0 \frac{u_t^{1+\lambda}}{1+\lambda}. \tag{58}$$

The FOCs are given by

$$\delta^0 e_t^{\kappa} = \frac{\alpha Y_t}{e_t S_t} \tag{59}$$

$$\Delta^0 u_t^{\lambda} = \frac{\alpha Y_t}{u_t} \tag{60}$$

Then we have

Corollary 1 *Given (A_t, S_t, N_t), The ratio of output in the decentralized economy to that in the social planner economy is*

$$\frac{Y^{DE}}{Y^{SP}} = \left[(1-\eta)^{\varepsilon_H}\left(\frac{\eta}{\varepsilon}\right)^{\varepsilon}\right]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}}, \tag{61}$$

which implies that

1. ***(Classic Hosios Condition)** If $\varepsilon_H > 0$ and $\varepsilon = 0$, i.e., there only exist search frictions between household and banks, then $\frac{Y^{DE}}{Y^{SP}} = \left(\frac{\eta}{\varepsilon}\right)^{\alpha\varepsilon}$, and*

$$Y^{DE} \gtreqless Y^{SP} \text{ if and only if } \eta \gtreqless \varepsilon. \tag{62}$$

2. ***(Modified Hosios Condition)** If $\varepsilon_H > 0$ and $\varepsilon > 10$, then $\frac{Y^{DE}}{Y^{SP}} < 1$ always holds, and*

$$\eta^* = \arg\max_{\eta\in[0,1]}\left(\frac{Y^{DE}}{Y^{SP}}\right) = \frac{\varepsilon}{\varepsilon+\varepsilon_H}. \tag{63}$$

Proof: Substituting Equations (30) and (30) into Equation (30) yields

$$Y^{SP} = \widetilde{Y}^{SP} A_t^{\tau} S_t^{\alpha_s} N_t^{\alpha_n}, \tag{64}$$

where $\widetilde{Y}^{SP} = \left[\left(\frac{\alpha}{\delta^0}\right)^{\varepsilon_H}\left(\frac{\alpha\eta}{\Delta^0}\right)^{\varepsilon}\right]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}}$. Dividing Equation (43) by Equation (64) yields Equation (61). ■

Three remarks are in order. First, if search frictions exist only between banks and firms, then we obtain the classic Hosios condition with $\eta = \varepsilon$. That is, the knife-edge condition exactly cancels the intra-group externality and the inter-group externality. Otherwise, there would be over-utilization of credit resources and over-entry of firms if firm's bargaining power is too high ($\eta > \varepsilon$), or under-utilization and under-entry if too low. Second and more interestingly, we contribute to the literature by detecting a general Hosios condition in the presence of dual search frictions. Intuitively, the increase of firm's bargaining power η delivers two competing effects. On the one hand, the increase of η intensifies firm's search for credit by inducing more firm entry. This in turn increases u, the utilization rate of credit in the second search-and-matching stage, and thus drives up output. On the other hand, a higher η dampens the profit share of the bank by lowering the loan rate, which translates into a lower deposit rate, and therefore discourages the search effort of household for the decision of deposit making. Therefore $\eta^* = \frac{\varepsilon}{\varepsilon+\varepsilon_H}$ strikes a balance between these two competing effects. In particular, η^* increases with ε (the matching elasticity of firms searching for credit) and decreases with ε_H (the matching elasticity of household searching for financial intermediation).

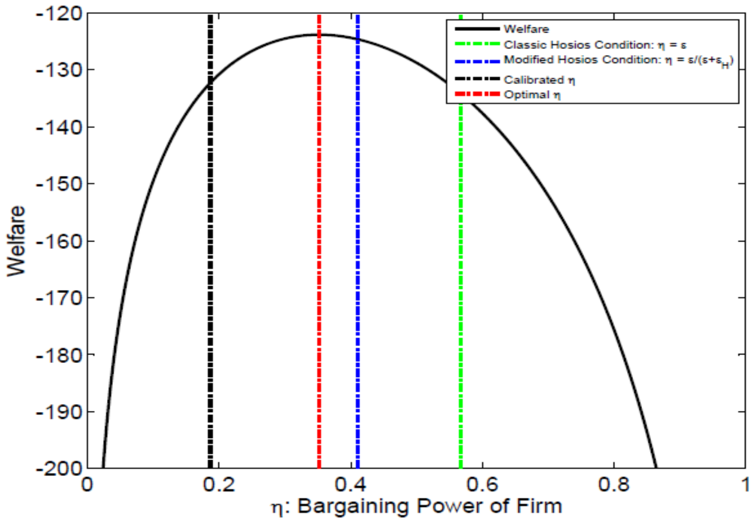

Figure 4. Hosios Conditions and Welfare.

Finally, in deriving the Hosios conditions we have so far followed the literature by holding the supply of labor and capital as fixed. This restriction is fine when it comes to the standard setup of macro labor economics *a la* DMP, which typically assumes inelastic labor supply and does not take into account capital accumulation, in addition to the assumption of risk neutral firms and workers. However, our paper has to address both of these two issues since the household in our model is allowed to make decision on labor supply as well as capital accumulation. Moreover, the household is risk averse when it comes to consumption. As a result, neither the classic nor the modified Hosios condition can guarantee a constrained optimum in welfare. Instead, we have to take into account the effect of η on both consumption and leisure decisions of the household over the lifetime horizon. To do so, we first obtain the steady state as follows:

$$R^d = \delta_0 = \frac{\rho(1+\kappa)}{\kappa} \tag{65}$$

$$\frac{S}{Y} = \frac{\alpha(1-\eta)\kappa}{\rho(1+\kappa)} \tag{66}$$

$$\frac{C}{Y} = \left(1-\frac{\alpha}{1+\kappa}\right)-\left(\frac{\kappa}{1+\kappa}\right)\alpha\eta \tag{67}$$

$$N = \left(\frac{1-\alpha}{\psi}\frac{1}{C/Y}\right)^{\frac{1}{1+\xi}} \tag{68}$$

$$Y = \left\{A\left[\gamma\left(\frac{\alpha\eta}{\phi}\right)^{\varepsilon}\right]^{\alpha}\left(\frac{S}{Y}\right)^{\alpha}N^{1-\alpha}\right\}^{\frac{1}{1-\alpha(1+\varepsilon)}} \tag{69}$$

Note that we set $\delta_0 = \frac{\rho(1+\kappa)}{\kappa}$ such that $e = 1$ in steady state, where $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. Consequently we know that the household welfare in the steady state is

$$\Omega = \frac{1}{\rho}\left[\log(C) - N\right] = \frac{1}{\rho}\left\{\log\left[\left(\frac{C}{Y}\right)Y\right] - N\right\}, \tag{70}$$

which is a function of η, the bargaining power of firms.

Figure 4 indicates that in the presence of risk aversion, endogenous capital accumulation, and elastic labor supply, neither the standard Hosios condition (*i.e.*, $\eta = \varepsilon$) nor the modified Hosios condition (*i.e.*, $\eta = \frac{\varepsilon}{\varepsilon+\varepsilon_H}$) manage to maximize the true welfare function Ω.

2.5.2 Indeterminacy Analysis

Lemma 1 *Using the dynamic system established above, we can obtain the following simpler two-dimensional system:*

$$\begin{bmatrix} \dot{s}_t \\ \dot{c}_t \end{bmatrix} = J \cdot \begin{bmatrix} \widehat{s}_t \\ \widehat{c}_t \end{bmatrix}, \tag{71}$$

where

$$J \equiv \delta \cdot \begin{bmatrix} \left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)\lambda_s & \left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)(\lambda_c-1) \\ \kappa(\lambda_s-1) & \kappa\lambda_c \end{bmatrix}. \tag{72}$$

$\kappa \equiv \frac{1}{\varepsilon_H} - 1$, $\alpha_s \equiv \frac{\alpha(1-\varepsilon_H)}{1-\alpha(\varepsilon+\varepsilon_H)}$, $\alpha_n \equiv \frac{1-\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}$, *and*

$$\lambda_s \equiv \frac{\alpha_s(1+\xi)}{1+\xi-\alpha_n} \tag{73}$$

$$\lambda_c \equiv \frac{-\alpha_n}{1+\xi-\alpha_n}. \tag{74}$$

Proof: The FOCs indicate

$$\delta'(e_t) = (1+\kappa)\left(\frac{\delta(e_t)}{e_t}\right) = R_t^d, \tag{75}$$

and thus

$$\delta(e_t) = \frac{R_t^d e_t}{1+\kappa} = \varepsilon_H \alpha (1-\eta)\left(\frac{Y_t}{S_t}\right) \tag{76}$$

Using the transition dynamics in Section 5 yields

$$\begin{aligned}
\dot{c}_t &= (1-\varepsilon_H)\left(\frac{Y}{S}\right)(1+\widehat{y}_t-\widehat{s}_t)-\rho && (77)\\
\dot{s}_t &= [(1-\alpha\eta)-\varepsilon_H\alpha(1-\eta)]\left(\frac{Y}{S}\right)(1+\widehat{y}_t-\widehat{s}_t)-\left(\frac{C/Y}{S/Y}\right)(1+\widehat{c}_t-\widehat{s}_t) && (78)\\
\widehat{y}_t &= \alpha(\widehat{e}_t+\widehat{u}_t+\widehat{s}_t)+(1-\alpha)\widehat{n}_t && (79)\\
\widehat{e}_t &= \varepsilon_H(-\widehat{s}_t) && (80)\\
\widehat{u}_t &= \varepsilon\widehat{y}_t && (81)\\
(1+\xi)\widehat{n}_t &= (1-\alpha)(\widehat{y}_t-\widehat{c}_t) && (82)
\end{aligned}$$

Some algebraic manipulation yields Lemma 1. ■

The local dynamics around the steady state is then determined by the eigenvalues of J. If both eigenvalues of J are negative, then the model is indeterminate. As a result, the model can experience endogenous fluctuations driven by sunspots. The eigenvalues of J, x_1 and x_2, satisfy

$$\begin{aligned}
x_1+x_2 &= Trace(J) = \delta\left[\left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)\lambda_s+\kappa\lambda_c\right] && (83)\\
x_1x_2 &= Det(J) = \delta^2\left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{\kappa}{1-\eta}\right)(\lambda_s-\lambda_c-1), && (84)
\end{aligned}$$

We know that indeterminacy emerges if and only if $Trace\,(J)<0$ and $Det\,(J)>0$.

Proposition 2 *$Trace\,(J)<0$ and $Det\,(J)>0$ if and only if either of the following two conditions hold:*

1. *$\alpha\in\left(0,\frac{1}{2}\right)$, $\xi\in[0,\frac{\alpha}{1-2\alpha})$, $\varepsilon,\varepsilon_H\in[0,1]$ and*

$$\varepsilon+\varepsilon_H>\widetilde{\varepsilon}\equiv\left(\frac{1}{\alpha}\right)\left(\frac{\alpha+\xi}{1+\xi}\right)\in[1,2). \tag{85}$$

2. *$\alpha\in[\frac{1}{2},1)$, $\varepsilon_H\in[0,1]$ and $0\leq\varepsilon<\frac{1}{\alpha}-1$.*

Proof: We can prove that $Trace(J) < 0$ and $Det(J) > 0$ hold if and only if the following four conditions hold, in addition to the restriction that $\varepsilon, \varepsilon_H \in [0,1]$.

$$\varepsilon + \varepsilon_H < \frac{1}{\alpha} \tag{86}$$

$$\varepsilon + \varepsilon_H > \left(\frac{1}{\alpha}\right)\left(\frac{\alpha+\xi}{1+\xi}\right) \tag{87}$$

$$\varepsilon_H < 1 - \frac{(1-\eta)(1-\alpha)\kappa}{(1+\kappa-\alpha)(1+\xi)} \tag{88}$$

$$\varepsilon < \frac{1}{\alpha} - 1. \tag{89}$$

First, since $\varepsilon_H \in [0,1]$, then comparing Conditions (86) and (89) suggests that the former is never binding.

Secondly, note that $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. Thus the Condition (88) can be rewritten as

$$\varepsilon_H < \left[\frac{1+\xi-(1-\eta)(1-\alpha)}{1+\xi}\right]\left(\frac{1}{\alpha}\right). \tag{90}$$

Since $\xi \geq 0$,

$$\left[\frac{1+\xi-(1-\eta)(1-\alpha)}{1+\xi}\right]\left(\frac{1}{\alpha}\right) > [1-(1-\eta)(1-\alpha)]\left(\frac{1}{\alpha}\right) > 1, \tag{91}$$

so we know that Condition (88) is not binding.

Finally, if $\alpha \in [\frac{1}{2}, 1)$, then we know that $\frac{1}{\alpha} - 1 \in (0,1]$, and we must have $0 \leq \varepsilon < \frac{1}{\alpha} - 1$. Besides, we know that $\widetilde{\varepsilon} \equiv \left(\frac{1}{\alpha}\right)\left(1 - \frac{1-\alpha}{1+\xi}\right) > 2$ when $\alpha \in [\frac{1}{2}, 1)$. Therefore Condition (87) always holds in this case. In contrast, when $\alpha \in \left(0, \frac{1}{2}\right)$, we have $\frac{1}{\alpha} - 1 > 1 > \varepsilon$, and thus Condition (89) always holds. Meanwhile, since $\varepsilon + \varepsilon_H \leq 2$, to guarantee that Condition (87) can be satisfied, we must have $\widetilde{\varepsilon} \equiv \left(\frac{1}{\alpha}\right)\left(1 - \frac{1-\alpha}{1+\xi}\right) < 2$, *i.e.*, $\xi \in [0, \frac{\alpha}{1-2\alpha})$. ■

Suppose $\alpha = \frac{1}{3}$ and $\xi = 0$, then *indeterminacy arises if and only if*

$$\varepsilon + \varepsilon_H > 1, \text{ and } \varepsilon, \varepsilon_H \in (0,1). \tag{92}$$

This just a special case of Proposition 2.

2.6 With Search Only Between Firms and Banks

In our model, search between firms and banks leads to IRS isomorphic to the model of Benhabib and Farmer (1994), while adding search between households and banks generates a credit utilization function isomorphic to the capacity utilization model of Wen (1998). Hence, our credit search-based model provides a microfoundation for the indeterminacy literature pioneered by Benhabib and Farmer (1994) and Wen (1998).

Notice that, if $\varepsilon_H = 0$, *i.e.*, there is no household search, then $e_t = 1$ and we obtain

$$Y_t = A\left(u_t S_t\right)^{\alpha} N_t^{1-\alpha}, \tag{93}$$

$$u_t = \gamma\left(\frac{\alpha\eta Y_t}{\phi}\right)^{\varepsilon}. \tag{94}$$

In turn, the aggregate production becomes

$$Y_t = \left[\gamma\left(\frac{\alpha\eta}{\phi}\right)^{\varepsilon}\right]^{\alpha_s} A_t^{\tau} S_t^{\alpha_s} N_t^{\alpha_n}, \tag{95}$$

where $\alpha_s \equiv \alpha\tau$, $\alpha_n \equiv (1-\alpha)\tau$, $\tau \equiv \frac{1}{1-\alpha\varepsilon}$, and

$$\alpha_s + \alpha_n = \frac{1}{1-\alpha\varepsilon} > 1; \tag{96}$$

which is isomorphic to the Benhabib-Farmer model. On the other hand, if $\varepsilon_H > 0$, then we obtain the Wen (1998) model with $Y_t = \tilde{Y} A_t^{\tau}\left(e_t S_t\right)^{\alpha_s} N_t^{\alpha_n}$. In the absence of household search, the depreciation rate is exogenously given. In turn, the dynamics are accordingly modified as

$$\frac{\dot{C}_t}{C_t} = (1-\eta)\,\alpha\left(\frac{Y_t}{S_t}\right) - \delta - \rho \tag{97}$$

$$\dot{S}_t = (1-\alpha\eta)Y_t - \delta S_t - C_t \tag{98}$$

$$Y_t = A_t\left(u_t S_t\right)^{\alpha} N_t^{1-\alpha} \tag{99}$$

$$u_t = \gamma\left(\frac{\alpha\eta Y_t}{\phi}\right)^{\varepsilon} \tag{100}$$

$$\psi N_t^{\xi} = \left(\frac{1}{C_t}\right)\left[(1-\alpha)\left(\frac{Y_t}{N_t}\right)\right]. \tag{101}$$

As we have already shown, when $\varepsilon_H = 0$, then indeterminacy is not possible although we have endogenous IRS. Hence, household search is necessary to generate indeterminacy, analogous to Wen's (1998) finding that variable capacity utilization can significantly reduce the required degree of IRS in the Benhabib-Farmer model for indeterminacy.

3 Quantitative Exercise

3.1 Calibration

We calibrate our model to quarterly frequency. The time discounting factor is $\rho = \frac{1}{\beta} - 1 = 0.01$, where $\beta = 0.99$ denotes the standard discount factor in discrete time models. We set the capital's share $\alpha = 0.33$, the coefficient of labor disutility $\psi = 1.75$ and the inverse Frisch elasticity of labor supply $\xi = 0$ (indivisible labor).

Now we have to calibrate the values of $(\varepsilon_H, \phi, \eta, \gamma, \varepsilon)$, which are specific to our model. First, as proved in previous section, $R^d = \frac{\rho(1+\kappa)}{\kappa}$where $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. Since $R^d = 5.4\%$, we immediately know that $\varepsilon_H = 0.82$ and $\kappa = 0.23$. In addition, our model implies $\delta = \frac{\rho}{\kappa} = 4.3\%$, which is basically in line with the standard calibration of $\delta = 3.5\%$. Second, we have shown that $\frac{S}{Y} = \frac{\alpha(1-\eta)}{R^d}$. Given a capital-output ratio of 5 and a deposit rate of 5.4%, the bargaining power of firm can be obtained as $\eta = 1 - \left(\frac{R^d}{\alpha}\right)\left(\frac{S}{Y}\right) = 0.19$. Third, we interpret ϕ as the cost of intermediation to finance firm investment. Therefore, we set $\phi = 0.1$ according to Chen and Ritter (2000). Finally, we use the following moments to jointly determine (γ, ε). On the one hand, $u = \frac{R^l}{R^d} = 67\%$ and we also know that u is related to (γ, ε). Thus we obtain one constraint on (γ, ε). On the other hand, Becchetti et al (2009) show that the proportion of firms subject to credit rationing is around 20.42%, and we know that $q = \gamma\theta^{1-\varepsilon} = 79.58\%$. Since θ is also related to (γ, ε), the moment on bank's credit utilization and that on firm's credit rationing jointly implies that $\gamma = 0.79$ and $\varepsilon = 0.57$. Our calibration exercise shows that $\varepsilon_H + \varepsilon > 1$. Consequently, indeterminacy due to credit search is empirically plausible. The calibrated parameter values are summarized in Table 1.

Table 1. Calibration

Parameter	Value	Description
ρ	0.01	Discount factor
A	1	Normalized aggregate productivity
α	0.33	Capital income share
ψ	1.75	Coefficient of labor disutility
ξ	0	Inverse Frisch elasticity of labor supply
ε_H	0.82	Matching elasticity in 1st Stage Search
δ	0.04	Depreciation rate
η	0.19	Firm's bargaining power
ϕ	0.1	Vacancy cost to search for credit.
γ	0.79	Matching efficiency in 2nd stage search
ε	0.57	Matching elasticity in 2nd stage search

3.2 Comparative Statics

Figure 5 shows, in terms of comparative statics, that our model is capable of explaining the countercyclical reserve-to-deposit ratio and interest spread between the loan rate and the deposit rate under either aggregate TFP changes or the credit-market matching efficiency changes (γ). For example, the top panel shows from left to right shows, respectively, (i) a positive correlation between log output and TFP, (ii) a positive correlation between the utilization rate of credit resources and TFP, hence a negative correlation between "excess" reserve-to-deposit ratio and TFP, and (iii) a negative correlation between the interest spread and TFP. Similarly, the bottom panel shows from left to right (i) a positive correlation between log output and matching efficiency, (ii) a positive correlation between the utilization rate of credit resources and matching efficiency, or a negative correlation between the reserve-to-deposit ratio and matching efficiency, and (iii) a negative correlation between the interest spread and matching efficiency. These qualitative predictions obtain regardless of the economy's steady state being determinate or not. Our qualitative results are robust to the calibrations.

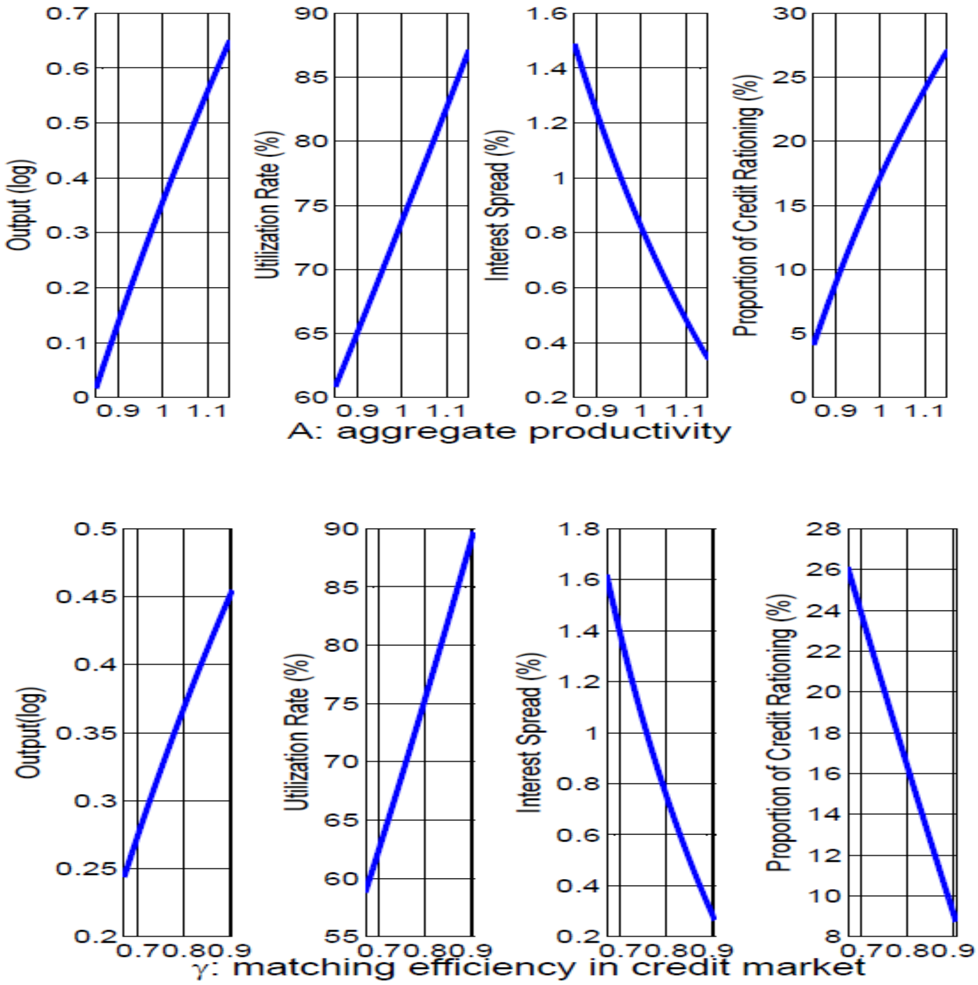

Figure 5. Comparative Statics w.r.t TFP (A) and Credit Matching Efficiency (γ).

As documented in Ruckes (2004) and the references therein, the lending standards are counter-cyclical. It in turn implies that credit rationing is also counter-cyclical. As shown in the last column in Figure 6, TFP shocks and matching efficiency shocks generate the opposite predictions for the correlation between output and credit rationing, thus may help quantitatively identify the relative importance of various business cycle shocks.

3.3 Impulse Responses

This subsection investigates the dynamic effect of TFP shocks and matching efficiency shocks (γ_t) on aggregate output, the interest spread, the utilization rate of credit (the opposite of the reserve-to-deposit ratio), and credit rationing. Figure 6A shows that under a 1% TFP shock, both the reserve-to-deposit ratio (the negative of the top-right panel) and interest spread (lower-left panel) are countercyclical, consistent with the data. However, the tightness of credit rationing (lower-right panel), or the fraction of firms denied for credit, is procyclical. Figure 6B shows that a 1% credit matching efficiency shock can also generate countercyclical reserve-to-deposit ratio and interest spread. In addition, it also generates a countercyclical credit rationing (i.e., negative response of the fraction of firms denied for credit), consistent with data.

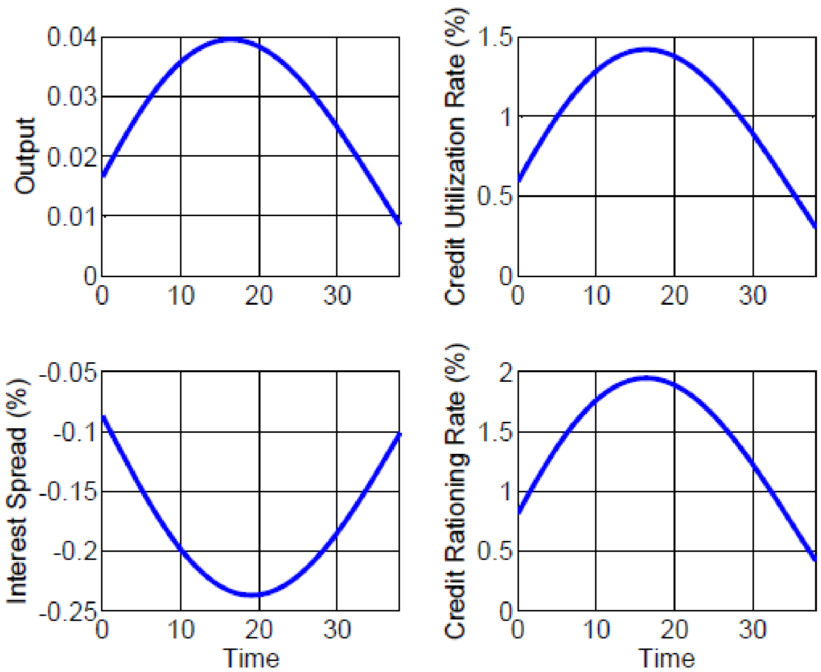

Figure 6A. Impulse Responses to TFP shock.

Also notice the more than proportionate increase (the multiplier effect) and the hump-shaped pattern (the accelerator effect) of the impulse responses in output and other variables. This endogenous propagation mechanism or multiplier-accelerator effect is the consequence of the endogenous increasing returns to the utilization rate of credit resources due to search and matching. Hence, under a favorable aggregate shock, as more and more credit resources are unleashed from the banking sector into the production sector, the economy goes through a long period of sustained boom featuring excessively low reserve-to-deposit ratio and interest spread between the loan rate and the deposit rate. However, the credit boom also plants the seed for its future bust. As the credit resources in the banking sector become scarcer, the loanable funds rate rises more than proportionately than does the deposit rate, which will soon or later chock off both credit supply and demand, and generate an investment slump.

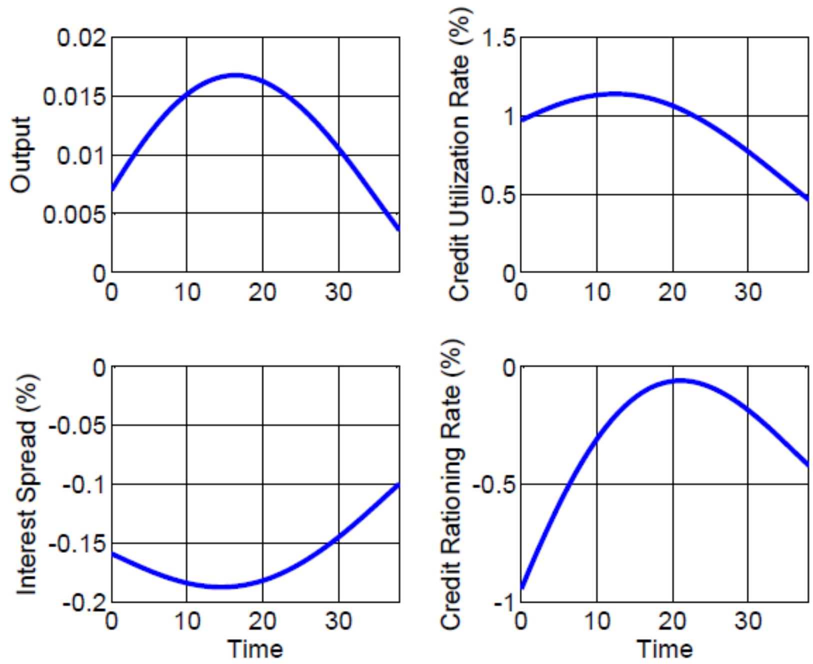

Figure 6B. Impulse Responses to Matching Efficiency Shock (γ).

4 Conclusion

The critical role that credit supply and financial intermediation play in generating and amplifying the business cycle has long been noted by economists at least since the Austrian school, as manifested in the countercyclical excess reserve-to-deposit ratio, the countercyclical interest spread between the loan rate and the deposit rate, as well as the countercyclical proportion of firms subject to credit rationing. This paper provides a framework to rationalize the Austrian theory and the observed credit cycles. Our framework is based on a simple idea. In an industrial economy with the division of labor and segregation between demand and supply, savers (lenders) with "idle" credit resources need to search and be matched with investors (borrowers) to utilize the available saving/credit resources and make them productive. But search and matching are costly due to information frictions and transaction costs and it requires efforts and bilateral coordination between borrowers and lenders. Hence, in equilibrium, credit resources are not always fully utilized, creating an important margin for elastic credit supply due to endogenous utilization rate of available credit resources. Meanwhile, the under-utilization of credit resources coexists with the prevalence of credit rationing to firms. Therefore our work offers a dynamic framework to address credit frictions on both the supply and demand sides at the same time. We demonstrate that aggregate shocks to matching efficiency in the credit market appear to be most important for the counter-cyclicality of excess reserves, interest spread, as well as credit rationing. Finally, we show that such a margin of elastic credit supply turns out critical not only in understanding the credit cycle, but also in leading to endogenous social increasing returns to the utilization rate of capital, hence providing a micro-foundation for the powerful amplification and propagation mechanism underling the endogenous business cycle literature studied by Benhabib and Farmer (1994) and Wen (1998).

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