The social value of offsets*

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Abstract

How much carbon should be stored in temporary and risky offsets to compensate an emission of 1 tonne of $CO2$? We answer this question by estimating the Social Value of Offsets (SVO). We show that the SVO is equal to the Social Cost of Carbon (SCC) multiplied by a correction factor reflecting the determinants of the SCC and offset-specific characteristics: impermanence, partial additionality and risk of failure. The key insight is that the SVO is positive because delaying emissions is socially valuable. Therefore, if their benefits (SVO) outweigh their costs, offsets could be part of an efficient net-zero portfolio. Offset offerings should therefore be accompanied by transparent information about their permanence, risks and additionality so that the SVO, which lies between \$0 and the SCC, can be calculated and a suitable net-zero investment portfolio determined. We provide a matrix of risk correction factors to calculate the SVO for this purpose.

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1 Introduction

The Paris Agreement states that temperature rises should be limited to 2C and proposed that the world should aim for an even more stringent target of 1.5C. In response, a number of governments have now committed to net zero carbon emissions by 2050 (e.g. UK, USA, France, Germany), and in the run-up to and aftermath of the COP 26, more countries are expected to follow suit and set out pathways towards net-zero. In the private sector, disclosure schemes for private companies and investment funds are now frequently

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benchmarked against the Paris targets (e.g. the Transition Pathway Initiative: TPI, and the Taskforce for Climate related Financial Disclosures: TCFD). Meeting the targets of the Paris Agreement will require concerted action in many aspects of the global economy, and the deployment of many different approaches to reduce carbon emissions. In the absence of significant technological advances in the medium term, net zero will require the use of offsetting of current emissions. The "net" in net zero implies this.

Unfortunately, there are considerable uncertainties associated with offsets due to the unregluated nature of the global offsets market, and the difficulties associated with establishing successful projects. Nature-based solutions in forestry are often seen as particularly risky options, often due to the absence of strong institutions on the ground to monitor, enforce and account for emissions sequestered. A frequently cited example is where offsets promise land-use change via avoided deforestation or reforestation in tropical forests, perhaps via REDD+ or related schemes. Indeed, empirical evidence suggests that the reported emissions reductions from such REDD+ projects are either vastly overstated (West et al., 2020), partial (Jayachandran et al., 2017) or minimal in relation to Nationally Defined Commitments (NDCs) (Groom et al. 2021). Overclaiming the efficacy of offsets is not confined to tropical countries either, with California forests also apparently over-crediting according (Badgley et al., 2021). As a result, either there is a risk that additionality of offsets is compromised, or the effective time horizon associated with nature-based (and indeed other) solutions to offsetting is shorter than those associated with extremely persistent emissions of carbon.

These observations raise the question as to the social value of offsets given their uncertainties and apparent imperfections, compared to emissions reductions or ideal carbon removal technologies that lock away carbon with certainty. The key insight is that a temporary absorption of CO2 is valuable because it will postpone warming and damages. An emission today which is offset by a temporary project can be thought of as a postponed emission, with the same warming effect when the project ends, but with less warming before the end of the project. Offsets therefore have a social value to the extent that they can delay emissions, even if they are impermanent, risky or only partially additional. Whether offsets are a worthwhile investment in any net-zero strategy will then depend on the comparison of the SVO with the social costs of provision. Whether offsets are more efficient than some other carbon removal technology will depend on how their Benefit-Cost Ratios (BCR) compare.

We derive a simple expression for the Social Value of an Offset (SVO) that will allow this cost-benefit analysis to take place. Intuitively, the expression for the SVO is the Social Cost of Carbon (SCC) today: the social value of a *permanent* removal of a ton of carbon from the atmosphere, multiplied by a correction factor that reflects both the secular macro-economic and climate dynamics factors that determine the SCC in the future, and the offset-specific characteristics reflecting the potentially transitory and uncertain way in which offsets remove carbon from the atmosphere. The relevant macro-economic and climatic factors are: 1) The time-path of CO2 emissions; 2) The resulting time-path of temperatures; 3) Expected growth in GDP; 4) The discount rate; and, 5) The climate damage function. The offset-specific factors are: 1) Impermanence: the offset may be of limited duration; 2) Risk of accidental loss/failure (e.g. through property rights failure or force majeure); and, 3) Additionality: reflecting the risk that emissions would have been reduced anyway, in the counterfactual world. The resulting correction factor means that the SVO is less than or equal to the SCC because it corrects for the risks and impermanence of offsets.

Our SVO formula can be easily operationalised using empirical evidence on offset-specific and macro/secular factors (growth, emissions etc.). We provide a matrix of correction factors using RCP2.6-8.5 emissions scenarios and hypothetical offset-specific risk factors. The formula shows that the SVO is not automatically zero simply because nature-based solutions are risky or impermanent, but rather the SVO takes values ranging from zero to the current SCC, depending on the severity of risk and impermanence. From the perspective of public appraisal of a net-zero strategy, comparing the SVO to the cost of provision establishes efficiency, and the Benefit (SVO) - Cost Ratio (BCR) will determine the optimal sequencing of offsets and other technologies in any net-zero strategy. As well as highlighting the risk and impermanence of offsets, the matrix of correction factors allows the calculation of how many offsets would on average be equivalent to 1 ton of carbon permanently removed. A correction factor of a half (0.5) for instance, implies that 2 such offsets would be required for equivalence.

Through the lens of the SVO it seems clear that the writing-off nature-based solutions compeletely could be a mistake. Hitting net zero requires a host of different approaches in the medium term and longer term. Nature-based solutions can form a cost effective part of a net-zero strategy in the medium term, before more expensive engineering solutions are adopted. When one considers the potential for co-benefits of nature-based solutions in the form of biodiversity conservation and ecosystem services, the case could be even stronger.

2 The effect of a temporary carbon offset on the climate

In principle, an offset has social value because it reduces CO2 in the atmosphere, which lowers temperatures and reduces economic damages. The Social Cost of Carbon (SCC) is an estimate of the social value of a permanent removal of a ton of CO2 from the atmosphere, and is usually calculated from integrated or analytical climate-economy models. Such approaches take into account the complex interactions between emissions, temperatures and economic damages. Due to factors such as impermanence and risk of failure, the Social Value of an Offset (SVO) will typically be less than the SCC. To estimate the SVO requires an understanding of the impact of a temporary reduction of CO2 and its complex relationship with temperatures and damages.

Consider the simple case of a temporary offset that removes a single ton of CO2 at time $t_1 = 0$, only to release it again at time t_2, where $t_2 - t_1 = \upsilon$. Figure 1 uses 16 climate models from the CIMP 5 ensemble (Joos et al., 2013; Geoffroy et al., 2013) to illustrate the complex impact on the climate system on emissons and temperatures of a 1GtCO2 reduction in 2020 for a period of υ =50 years: after 50 years the offset ends and the emissions are re-released, compared to a no-offset world. Firstly, Figure 1a reflects the baseline against which the offset's impact is evaluated: the pre-offset emissions and temperature (warming) path. Figure 1b shows the impact of the offset on CO2 concentration: i.e. the difference between offset and baseline scenarios. The shape of the response curves can be understood as follows. Atmospheric CO2 absorption by oceans and plants happens faster under higher CO2 concentration, so any difference in CO2 concentration between scenarios will fade out over time. The opposite is true for a negative pulse. In Figure 1b the immediate effect of 1GtCO2 removed in 2020 reduces over time, and the net effect is reduced over time. After 50 years, the effect is 60% of the initially absorbed quantity of CO2. After 50 years, 1 GtCO2 is re-released into the atmosphere as the offset ends, and atmospheric CO2 concentration is at first higher than the original concentation, but again this difference fades over time. Figure 1c show the impact on temperature, where the dynamics reflect recent findings that show that temperature responses to emissions pulses are relatively rapid and persistent (Ricke and Caldeira, 2014). The cooling effect occurs with a delay of 5 years due to the thermal inertia, after which the effect on temperature is more or less constant, reflecting the balancing of the countervailing effects of thermal inertia and absorption dynamics. After 50 years, when the GtCO2 is re-released, these dynamics are reversed, there is an overshoot of CO2, rapid energy forcing, which curtails the offset's cooling effect within 5 years without a large temperature overshoot. The overall effect of the offset on temperature resembles a step function.

Finally, note that although the marginal effect of the offset on temperature is constant, marginal damages will change over time because damages depend on the baseline temperature, which increases over time. The precise SVO will depend on the macro-baseline against which the offset is introduced: the emissions and temperature pathways, and the damages caused, as well as the specific characteristics of the offset: its duration and reliability. The SVO formula will reflect all of these aspects. Our SVO formula approximates these climate dynamics and damages.

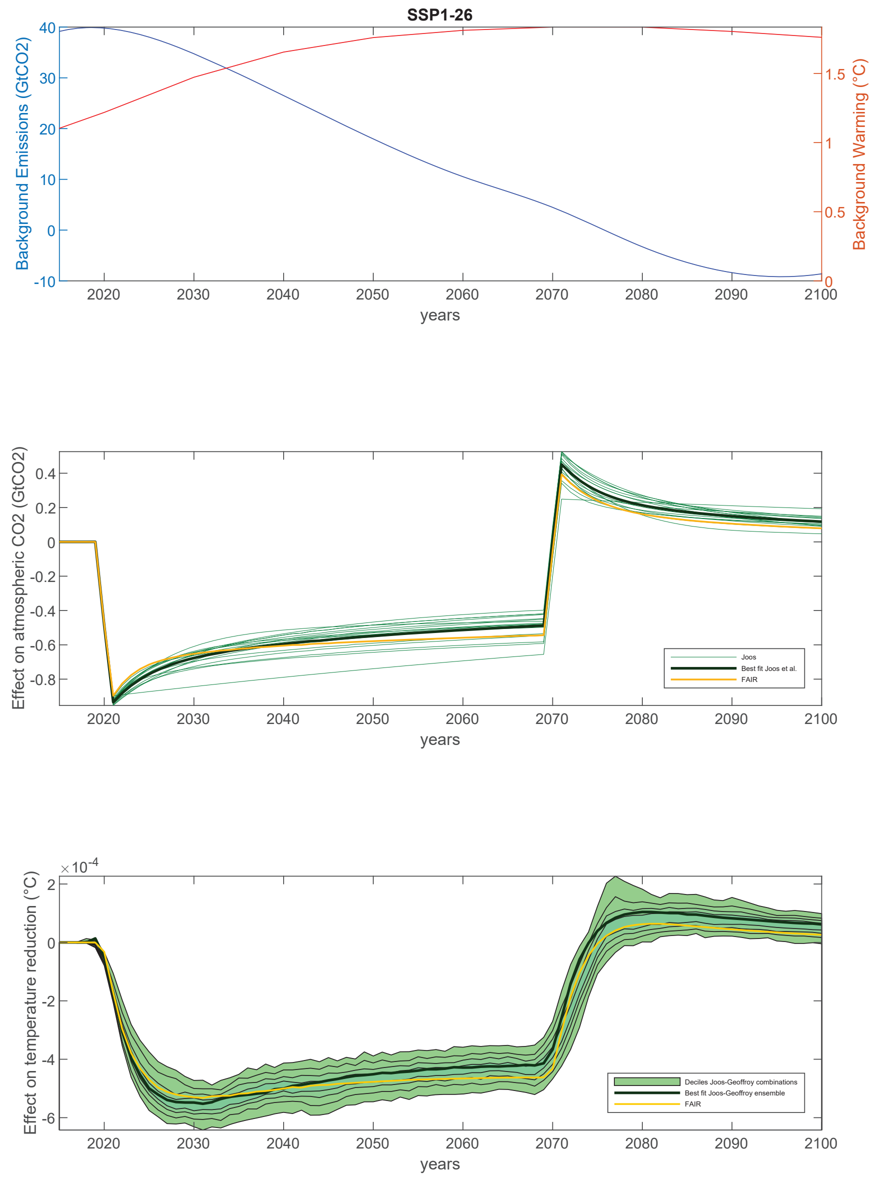

Figure 1: **The effect of an offset on atmospheric CO2 concentrations and on warming for the SSP1_26 background scenario.** Figure a shows the background emissions and temperature, following the SSP1-26 scenario. Figure b shows the difference between CO2 concentration of the background scenario and the scenario with a temporary removal project, intantaneously absorbing 1 GtCO2 in 2020 and reinjecting it in 2070. The 16 green lines correspond to 16 carbon absorption models in the CMIP 5 modeling ensemble described by Joos et al. (2013). The yellow line is the FAIR model, which is based on the the best fit of the CMIP 5 ensemble, but adds a carbon sink saturation feedback. Figure c shows the difference between the temperatures of the background scenario and the scenario with the removal project. The 16 absorption models are combined with 16 energy balance models from the CMIP 5 ensemble (as in Geoffroy et al., 2013) and the figure shows the deciles of the 256 possible combinations of models. The FAIR model uses the best fit of the CMIP5 energy balance models. The climate sensitivity of all energy balance models has been harmonized to 3.1°C. Impact response functions for other background scenarios are in the Appendix.

3 The Social Value of an Offset (SVO): A Cost-Benefit Framing

The Social Value of an Offset (SVO) depends on the damages prevented by the removal of CO2 from the atmosphere. The SVO is therefore closely related to the Social Cost of Carbon (SCC) which values the permanent removal of a ton of CO2 from the atmosphere. We assume that warming at time t (T_t) is proportional to cumulative emissions ($S_{t-\xi}$) between the pre-industrial time and time $t-\xi$: $T_t = \zeta S_{t-\xi}$, where ζ is known as the Transient Climate Response to cumulative Emissions (TCRE), and the temperature response in Figure 1c is approximated by a step-function with a delay of period ξ between absorption and the temperature effect.[1 2] The best fit for ξ is $\xi = 3$ years for the SSP1-RCP2.6 scenario. With respect to damages we use a damage function, $D(T)$, that is convex and increasing in temperature so that a unit of emissions at time τ will add a marginal damage $\zeta D_T(T_\tau)$ ($D_T(T_\tau) = \frac{d}{dT}D(T_\tau)$) from time $\tau+\xi$ onwards. In a warming world, the marginal damage as a result of an emission at time τ will increase over time. The SCC at time τ, SCC_τ, is defined as the integral of the present value of the marginal damage function from $\tau+\xi$ into the infinite future ∞:

$$SCC_\tau = \int_{t=\tau+\xi}^{\infty} exp\left(-r\left(t-\tau\right)\right)\zeta D_{T_t} dt \tag{1}$$

where r is the Social Discount Rate (SDR) and S_t is the stock of carbon in the atmosphere (See Supporting Information: *SI*, for the standard derivation).

If an offset were to remove 1 ton of CO2 from the atmosphere permanently at time τ, its social value would be SCC_τ. However, permanence and certainty are not characterisics of the typical offset offering, so the equivalence between SVO and SCC breaks down. We now consider the reasons for this lack of equivalence and generate an expression for the SVO in each case, before describing a general expression.

A temporary offset

Assume that an offset removes 1 ton of CO2 at time τ_1 and this lasts for υ years until emissions are re-released at time, τ_2. Figure 1 depicts the physical effect of such an offset, with an approximately constant effect on temperature between time $\tau_1+\xi$ and $\tau_2+\xi$. The SVO in this case is the present value (valued at date $t=0$) of the damages avoided

[1]Zickfeld et al. (2021) describe differences between positive and negative emissions, which are very small for small emission pulses.

[2]This is in line with the common assumption that warming is proportional to cumulative CO2 emissions $T = \zeta S$, with ζ the Transient Climate Response to cumulative Emissions (Dietz and Venmans, 2019; Zickfeld et al., 2016). It will allow us to write the social cost of carbon as a simple integral because the marginal effect of emissions on temperature does not depend on the entire emission history.

for time horizon $\tau_1 + \xi$ to $\tau_2 + \xi$:

$$SVO_{\tau_1\tau_2} = \int_{t=\tau_1+\xi}^{\tau_2+\xi} exp\left(-rt\right)\zeta D_{T_t} dt \tag{2}$$

For simplicity suppose that emissions, temperatures and damages evolve in the future so that SCC_τ grows exponentially at a rate x: $SCC_\tau = SCC_0 exp(x\tau)$, which is a reasonable assumption under certain circumstances.[3] In this case, Appendix 1 shows that $SVO_{\tau_1\tau_2}$ simplifies to:

$$SVO_{\tau_1\tau_2} = SCC_0 exp\left(\left(x-r\right)\tau_1\right)\left(1 - exp\left(\left(x-r\right)\nu\right)\right) \tag{3}$$

The $SVO_{\tau_1\tau_2}$ is simply a corrected version of SCC_0, where the correction factor reflects: i) the delay in implementation from today until τ_1 : $exp\left(\left(x-r\right)\tau_1\right)$; and, ii) the known truncation of the project after duration ν: $\left(1 - exp\left(\left(x-r\right)\nu\right)\right)$. Two characteristics of $SVO_{\tau_1\tau_2}$ are immediately obvious from the pricing formula in Equation 3. First, $SVO_{\tau_1\tau_2}$ depends on the trajectory over time of SCC_τ. Second, the $SVO_{\tau_1\tau_2}$ is be bounded between zero and SCC_0. In Section 6 an analytical formula is developed for the $SVO_{\tau_1\tau_2}$ for more general trajectories of the SCC. We first turn to two further offset-specific factors that affect the SVO: risk of failure and additionality.

An offset with failure risk

The analysis can be extended to take into account the likelihood that at any moment the offset technology could fail, e.g. reforestation or avoided deforestation is simply destroyed by force majeure, property rights failure or a change in land-use policy in situ. Suppose that in principle the offset remains temporary with a known fixed end date τ_2. Suppose also that an offset project is subject to the constant instantaneous hazard rate, ϕ, which reflects the instantaneous probability of an offset failing at time τ, conditional on having already survived until that date. By definition, the probability of the project surviving for τ years or longer is given by $P\left(t \geq \tau\right) = \exp\left(-\phi\tau\right)$. This means that at any future time τ the offset project continues to provide one ton of emissions reduction with probability $P\left(t \geq \tau\right) = \exp\left(-\phi\tau\right)$, or else has failed to offset with probability $1 - \exp\left(-\phi\tau\right)$. The duration of the offset is therefore uncertain, but ν is the maximum.[4] When the offset is

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[3] if x is the growth rate of global warming: celsius above preindustrial temperature, and climate damages are quadratic. In the case of the seminal model by Golosov et al. (2014) model or Traeger (2021), x corresponds to the growth rate of GDP. So the assumption of exponential growth of SCC is compatible with important Integrated Assessment Models.

[4] The hazard rate $\phi\left(t\right)$ is defined as $\phi\left(t\right) = \frac{f(t)}{S(t)}$, where $f(t)$ is the probability density function of the failure time, t, and $S\left(t\right)$ is the survivor which measures the probability of the project surviving longer than time t. $\phi\left(t\right)$ and $S\left(t\right)$ are consequently related as follows: $\phi\left(t\right) = -\frac{dlnS(t)}{dt}$, so $S\left(t\right) = \exp\left(-\int_0^t \phi\left(t\right)dt\right)$,which is equal to $S\left(t\right) = \exp\left(-\phi t\right)$ when $\phi\left(t\right) = \phi$.

subject to the hazard rate ϕ, the expected social value of the offset, $SVO^{\phi}_{\tau_1\tau_2}$ becomes:

$$SVO^{\phi}_{\tau_1\tau_2} = exp\left(-r\tau_1\right)\int\limits_{t=\tau_1}^{\tau_2} exp\left(-\left(r+\phi\right)\left(t-\tau_1\right)\right)\zeta D_{T_t}dt \qquad (4)$$

Given the definition of SCC_τ in Equation (1), and with marginal damages increasing in step with the SCC at rate x, the $SVO^{\phi}_{\tau_1\tau_2}$can again be written in terms of a corrected SCC (see SI, Derivations):[5]

$$SVO^{\phi}_{\tau_1\tau_2} = SCC_0 exp\left(\left(x-r\right)\tau_1\right)\left(1-exp\left(\left(x-r\right)\nu\right)\right)\frac{r-x}{r+\phi-x} \qquad (5)$$

where the correction factor now has three components. In addition to i) delay; and, ii) truncation/impermanence the pricing formula now corrects for: iii) the risk of failure: $\frac{r-x}{r+\phi-x}$. If the project starts immediately ($\tau_1 = 0$) and is expected to be permanent ($\tau_2 = \infty$), and the carbon stock is in a steady state ($x = 0$), then $SVO^{\phi}_{0\infty} = SCC_0\frac{r}{r+\phi}$ and the correction factor is simply $\frac{r}{r+\phi}$ (see SI Derivations). If after 100 years the likelihood of the offset still existing is 20%, so $P\left(t \geq 100\right) = exp\left(-\phi 100\right) = 0.2$, the implied hazard rate is $-ln(0.2)/100 = 1.6\%$. If the discount rate is $r = 3.5\%$ then the expected present value of the offset is reduced by 30%: $\frac{r}{r+\phi} = 0.7$. If $D_S\left(S\right)$ is increasing over time ($x > 0$) the risk of failure will have a larger effect because larger future damages are potentially not offset: if $x = 2\%$ then the value of the offset is halved due to the risk of failure: $\frac{r-x}{r+\phi-x} = 0.5$.

An offset with additionality risk

The time profile of additionality risk depends on the type of project. If a project removes CO2 from a baseline in which there was no removal, such as a reforestation project, there is a risk that in the absence of the project reforestation would have occurred anyway, e.g. if forested land becomes more productive than barren land. In this case the time profile of the risk is very similar to the risk of failure, as in Figure 1. Alternatively, conservation or preservation projects take as their baseline ongoing loss of forested land, and offsetting stems from avoided deforestation. The SVO in each case is different.

In the first case, the risk of zero additionality can be framed as a hazard rate φ leading to the probability $P\left(t \geq \tau\right) = \exp\left(-\varphi\tau\right)$ that the project has a causal effect at least until time τ. The expression is analogous to the case of a failure risk, leading to an SVO of the following form (see SI Derivations):

$$SVO^{\phi,\varphi}_{\tau_1\tau_2} = SCC_0 exp\left(\left(x-r\right)\tau_1\right)\left(1-exp\left(\left(x-r\right)\nu\right)\right)\frac{r-x}{r-x+\phi+\varphi} \qquad (6)$$

[5]It is shown in the SI (SI Derivations) that the rate of increase of marginal damages has to be the same as the rate of increase in the SCC, x, in the exponential case.

The offset correction factor now reflects the non-additionality risk via the hazard rate, and all that is required to evaluate the SVO is an estimate of φ. Note that our formula is also valid if ϕ and φ are both time dependent, but their sum is constant. This could happen if degradation of a forestry project were more likely early on, whereas reforestation in the baseline is more likely in the far future.

In the second case of a conservation/preservation project, the assumption is that in the baseline CO2 would have been emitted, but the project avoids this, as in the case of a forest preservation project. Here non-additionality occurs when there would have been no deforestation in the absence of the project. Assume that without the preservation project, there is a hazard rate $\tilde{\varphi}$ that the forest would have disappeared, making the offset additional. The that the project has an additional (or causal) effect at time τ is therefore: $P\left(t \leq \tau\right) = 1 - \exp\left(-\tilde{\varphi}\tau\right)$. Formula 6 is now replaced by:

$$SVO_{0\infty}^{\phi,\tilde{\varphi}} = SCC_0 exp\left(\left(x-r\right)\tau_1\right)\left(1-exp\left(\left(x-r\right)\nu\right)\right)\left(\frac{r-x}{r-x+\phi} - \frac{r-x}{r-x+\phi+\tilde{\varphi}}\right) \quad (7)$$

$SVO_{0\infty}^{\phi,\tilde{\varphi}}$ augments $SVO_{0\infty}^{\phi}$with an additional component in the correction term $-\frac{r-x}{r-x+\phi+\tilde{\varphi}}$. Note that if there is no risk whatsoever that the forest being preserved would have been deforested ($\tilde{\varphi}=0$),then $SVO_{0\infty}^{\phi,\tilde{\varphi}}=0$, reflecting the absence of additionality.

All the SVO expressions can be easily operationalised since the offset-specific parameters: τ_1, υ, ϕ, φ,and $\tilde{\varphi}$,for any given offset technology, and the macro-economic and climate parameters, r, SCC_0, x, can all be estimated.

4 The Social Value of Offsets on an arbitrary temperature path

The pricing formula for the Social Value of an Offset (SVO) in Equation (7) used an exponential growth path of marginal damages for analytical convenience to make the point that the SVO depends on the evolution of SCC over time. In the Supplementary Information (SI Derivations) we develop closed-form solutions for the SVO based on different assumptions concerning damages and emissions trajectories. The general formula allows for any temperature path, accommodates different trajectories for the SCC, and can embody more detailed project specific characteristics than the step-function used so far.

To get to the more general formula, assume that the damage function is proportional to production, Y, and quadratic in temperature $D = Y\frac{\gamma}{2}T^2$. This is the most common assumption in Integrated Assessment Models (Howard and Sterner, 2017). The marginal damage for a unit of CO2 emission at time t is therefore $\zeta D_T = \zeta\gamma YT$ and starts from time $t+\xi$ onwards. Using this damage function in equation 4 and adding the risk of non-additionality gives the following formula for the SVO:

$$SVO^{\phi,\varphi}_{\tau_1\tau_2} = \zeta\gamma exp\left(-r\tau_1\right)\int\limits_{t=\tau_1+\xi}^{\tau_2+\xi} exp\left(-\left(r+\phi+\varphi\right)\left(t-\tau_1\right)\right)Y_tT_tdt \tag{8}$$

Applying the same formula to a riskless permanent project will give the value of the SCC. Interestingly, the two most uncertain, hence difficult to parameterise, parameters, the TCRE, ζ, and the damage coefficient, γ, cancel out in this formula, and do not affect the offset correction factor for impermanence and risk:

$$\frac{SVO^{\phi,\varphi}_{\tau_1\tau_2}}{SCC_0} = \frac{exp\left(-r\tau_1\right)\int_{t=\tau_1+\xi}^{\tau_2+\xi} exp\left(-\left(r+\phi+\varphi\right)\left(t-\tau_1\right)\right)Y_tT_tdt}{\int_{t=\xi}^{\infty} exp\left(-rt\right)Y_tT_tdt} \tag{9}$$

Only the future temperature and GDP paths are needed to operatinalise this specific formula. The formula can also easily accommodate additional project specifics factors. For instance, time dependence can be introduced to the failure and non-additionality risks. Where conservation projects are concerned, the risk of non-additionality can be reflected by a factor $\left(1-exp\left(\tilde{\varphi}\left(t-\tau_1\right)\right)\right)$. The time profile of absorption and release, which will differ among projects, can be reflected by a factor, q_t, indicating the stock of carbon that is absorbed by the project at each moment in time. Such factors can be straighforwardly added to the integral in the numerator of Equation (1).

To illustrate the flexibility of Equation 1 the Supplementary Material provides a simple excel spreadsheet that calculates the adjustment factor for different temperature paths: the IPCC's RCP2.6, RCP4.5, RCP6 and RCP8.5 scenarios, and for different parameter values for project specific characteristics. Table 1 summarises the adjustment factors for a subset of parameters values and temperature paths. The supplementary material also provides closed form solutions for the SVO assuming linear and exponential temperature paths.

For any given emissions scenario, the conversion factor diminishes as the offset has shorter duration and a higher risk of failure or non-additionality. An offset of duration of 25 years with a 0.5% annual risk of failure or non-additionality has a correction factor of 24% in RCP 2.6 (1.8C), which drops to 11% in RCP 8.5 (5.1C), which has higher marginal damages in the future. Note that within each RCP scenario the conversion factor is independent of the damage function, due to the cancelling of terms ζ (TCRE) and γ (damages). Yet, the absolute value of an offset does depend on these parameters since they determine SCC_0. This means that in some cases the absolute dollar value of an offset may be higher even when the conversion factor is lower.

To the question of how much carbon should be held in offsets compared to alternative mitigation strategies, first note that a correction factor of x means that in order to offset the equivalent of 1 ton of carbon $1/x$ offsets would have to be purchased. Table 1 shows that this can mean anything from a near one-to-one relationship between offsets projects and permanent carbon removal, to a situation where 10 offsets, each claiming to offset 1 ton of carbon, would have to be purchased to be equivalent to a permanent emissions reduction. It is important to recognise that this equivalence is in the aggregate. Given uncertainty, some individuals will end up reducing emission by more than 1 ton in the end, others by less, but on average the overal impact would be a 1 ton emssions reduction per person. Table 1 makes this rate of conversion explicit. For a full picture of the efficiency of offsets compared to alternatives, a benefit-cost analysis is required, and interventions can be ordered in terms of there benefit-cost ratios (BCR).

5 The Social Value of an Offset: A Cost Effectiveness Framing

Climate change mitigation is frequently viewed in terms of cost-effectiveness. For instance, the carbon price in the UK reflects the marginal abatement cost of meeting a net zero target by 2050. Offsets can also be viewed as contributing to this target, with some caveats. Consider two approaches: 1) a project absorbing a tonne permanently; 2) a temporary project combined with a permanent project which starts immediately after the temporary projects ends, each aborbing a tonne of carbon. These approaches are equally effective in reducing emissions in the long-run. This yields a decision rule that favours appraoch 2) with the temporary project if it costs less:

$$C^P_{\tau_1,\infty} \geq C_{\tau_1,\tau_2} + e^{-r(\tau_2-\tau_1)} C^P_{\tau_2,\infty} \tag{10}$$

where that $C^P_{\tau_1,\infty}$ is the cost of a permanent project and the carbon price at time τ_1. Assuming that we know the rate at which the cost of permanent projects increases over time, x, we have the equivalent of equation 3 in the cost-effectiveness context, and the decision rule becomes:

$$C_{\tau_1,\tau_2} \leq \left(1 - e^{(x-r)(\tau_2-\tau_1)}\right) C^P_{\tau_1,\infty} \tag{11}$$

On an optimal trajectory, the cost of a project equals the social value: $C^P_{\tau_1,\infty} = SCC_{\tau_1,\infty}$, making the right hand side of Equation 11 the same as Equation 3.

However, in a non-optimal world, this approach is problematic. If intertemporal prices are not optimal, projects are ranked on the basis of prices that do not reflect their social value, and the prioritisation resulting from the decision rue will not maximise welfare over time.

To illustrate, consider a carbon price that follows a cost-effectiveness approach, i.e. it yields the lowest discounted cost to stay within a given temperature target. In this case the carbon price follows a Hotelling path, increasing at the rate of discount so that $x = r$. Cost-effectiveness, by its very nature, is indifferent to the timing of damages and this leads to a carbon price that starts too low today and ends up too high in the future compared to a target compatible optimal that takes into account both the costs and benefits of mitigation. With $x = r$, equation 11 indicates that a temporary project should only be realized if the cost is zero or negative. This criterion reflects the intuition that in a cost-effectiveness framework any temporary project that stops before the temperature constraint is hit makes no contribution to staying below that temperature. Yet, the expression for SVO_{τ_1,τ_2}in Equation 3shows that delaying emissions through offsetting can have a positive social value.

This incompatibility of a cost-based approach with welfare maximisation in the context

IPCC Scenario (Temp in 2100)	Risk at start $\tilde{\varphi}$	Risk at end $\phi+\varphi$	SVO Correction factors (max.duration, v) 25	50	100	∞	SCC ($\$/tCO_2$) Damages ($\gamma$) γ=0.0077	γ=0.0025
RCP 2.6	1000	0	24%	44%	70%	100%	109	35
(1.8°C)		0.25	23%	42%	63%	83%	109	35
		0.5	23%	40%	58%	71%	109	35
	0.5	0	23%	43%	69%	99%	109	35
		0.25	22%	40%	62%	82%	109	35
		0.5	21%	38%	56%	69%	109	35
	0.25	0	21%	41%	67%	97%	109	35
		0.25	20%	39%	60%	80%	109	35
		0.5	20%	36%	54%	68%	109	35
RCP 3.4	1000	0	19%	37%	66%	100%	142	46
(2.6°)		0.25	19%	35%	59%	81%	142	46
		0.5	18%	33%	53%	68%	142	46
	0.5	0	18%	36%	65%	99%	142	46
		0.25	18%	34%	58%	80%	142	46
		0.5	17%	32%	52%	67%	142	46
	0.25	0	17%	35%	63%	97%	142	46
		0.25	16%	33%	56%	79%	142	46
		0.5	16%	31%	51%	66%	142	46
RCP 6.0	1000	0	17%	34%	64%	100%	161	52
(3.1°C)		0.25	17%	32%	57%	81%	161	52
		0.5	16%	31%	51%	67%	161	52
	0.5	0	16%	33%	63%	99%	161	52
		0.25	16%	31%	56%	80%	161	52
		0.5	15%	30%	50%	66%	161	52
	0.25	0	15%	32%	61%	98%	161	52
		0.25	14%	30%	55%	78%	161	52
		0.5	14%	28%	49%	65%	161	52
RCP 8.5	1000	0	13%	29%	60%	100%	233	76
(5.1°C)		0.25	13%	27%	53%	79%	233	76
		0.5	12%	25%	47%	64%	233	76
	0.5	0	12%	28%	59%	99%	233	76
		0.25	12%	26%	52%	78%	233	76
		0.5	12%	24%	46%	64%	233	76
	0.25	0	11%	27%	58%	98%	233	76
		0.25	11%	25%	51%	77%	233	76
		0.5	11%	24%	45%	63%	233	76

Table 1: Adjustment factors for non-permanence and risk. We assume a quadratic damages proportional to GDP $exp\left(-\frac{\gamma}{2}T^2\right)$ with damage parameters of Howard and Sterner (2017) (Column 8) as well as Nordhaus (2017) (Column 9). Temperature pathways evolve according to SSP1-RCP2.6; SSP4-RCP3.4; SSP4-RCP6.0 and SSP5-RCP8.5 (Riahi et al. 2017, www.https://tntcat.iiasa.ac.at). Other parameters are $r = 3.2\%; \tau_1 = 3year; \zeta = 0.0006°C/GtCO_2; GDPgrowth = 2\%; T_0 = 1.2°C$. We use equation 9 . For $\tilde{\varphi}$ = [0.5 0.25] the likelihood that the project is additional after 5 years is 92% and 71% respectively. For $\varphi + \phi$ = [0.0025 0.005] the likelihood that the project is additional after 50 years is 78% and 88% respectively.

of offsets has important implications for some conventional approaches to valuing offsets. For instance, the formula of Carbon Plan (https://carbonplan.org/research/permanence-calculator-explainer) emerges after applying iterative substitution to equation 10, and allows a comparison of the cost of a permanent project, $C_{\tau_1,\infty}$, with an infinite stream of temporary projects, C_{τ_s,τ_t}:

$$e^{-r\tau_1}C_{\tau_1,\infty} \geq e^{-r\tau_1}C_{\tau_1,\tau_2} + e^{-r\tau_2}C_{\tau_2,\tau_3} + e^{-r\tau_3}C_{\tau_3,\tau_4} + ... \tag{12}$$

The Carbon Plan formula assumes that all temporary projects have the same duration and the cost of a forestry project does not change through time, we obtain

$$C_{0,\tau_1} = \frac{C_{0,\infty}}{\sum_{i=0}^{\infty} e^{-r\tau_i}} \tag{13}$$

The previous discussion of cost-effectiveness explains why this formula is problematic. On a welfar maximizing path, the cheapest offsetting projects are realized first and as the SCC rizes, more expensive projects are realized too. Therefore, a world were there are offsetting opportunities in the future at the same cost as today is a world where cost prices are not intertemporally optimized. This intertemporal inefficiency will lead to biased decision rules. Concretely, the hypothesis of cheap future offsetting opportunity is too optimistic, which leads to an underestimation of the adjustment factor.

6 Conclusion

A simple expression has been developed that provides the social value of an offset capturing its duration, likelihood of failure and its potential for non-additionality. While these factors do conspire to reduce the value of a ton of carbon sequestered via an offset, they do not make offsets valueless. In fact, the paper directs analysis towards the empirical questions associated with the time horizon, and the likelihoods of curtailed values from failure and non-additionality. Offsets have a role to play as long as they provide value for money and a sufficient benefit from their delaying of emissions. From the perspective of public sector appraisal offsets may well have an important role to place where their Benefit-Cost Ratio is higher than other alternatives. Despite the fact that SVO is less than the SCC offsets may still be competitive with other technologies where their costs of provision are low. Careful valuation of the SVO is required to make this decision, and offset providers should provide this information for each of their offerings, nature-based on otherwise.

Care is needed in public and private appraisal of offsets where a Cost Effectiveness Analysis (CEA) approach to carbon pricing is taken and the marginal abatement cost is used as the carbon price, as is the approach taken by the UK government. In the CEA approach delaying carbon emissions does not improve cost effectiveness because the efficient carbon price rises according to Hotelling's rule, at the rate of discount, so emissions reductions are costed equally in present value terms at all points in time prior to the target being met. Yet the conclusion that temporary offsets are 'valueless' is simply an artefact of CEA, which, other than the target itself, ignores the optimal scheduling of emissions and damages that is captured by the trajectory of the SCC (Dietz and Venmans, 2019). Including damages in the analysis of target-compatible carbon policy has recently been proposed by Stern and Stiglitz (2021), and the same logic applies to the public and private sector evaluations of offset schemes. In the case of offsets, ignoring climate damages will lead to the mistaken belief that offsets are worthless, and carbon sequestration will inevitably be the reserve of engineering solutions such as Carbon Capture and Storage or Geoengineering. All approaches have their place, and nature-based offsets are often low cost and have co-benefits in the form of biodiversity and ecosystem service provision. This contribution simply urges proper and transparent valuation of offsets in all contexts in terms of the practical risks associated with them to identify the offset correction factor. Indeed, this general principle should apply to all emissions reductions strategies.

References

Badgley, G., Freeman, J., Hamman, J. J., Haya, B., Trugman, A. T., Anderegg, W. R. and Cullenward, D. (2021). Systematic over-crediting in californias forest carbon offsets program. *Global Change Biology* n/a, doi:https://doi.org/10.1111/gcb.15943.

Dietz, S. and Venmans, F. (2019). The endowment effect, discounting and the environment. *Journal of Environmental Economics and Management* 97: 67–91.

Geoffroy, O., SAINT-MARTIN, D., OLIVIE, D. J. L., VOLDOIRE, A., BELLON, G. and TYTECA, S. (2013). Transient climate response in a two-layer energy-balance model. part i: Analytical solution and parameter calibration using cmip5 aogcm experiments. *Journal of climate* 26: 1841–1857.

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica* 82: 41–88.

Howard, P. H. and Sterner, T. (2017). Few and not so far between: A meta-analysis of climate damage estimates. *Environmental & resource economics* 68: 197–225.

Jayachandran, S., Laat, J. de, Lambin, E. F., Stanton, C. Y., Audy, R. and Thomas, N. E. (2017). Cash for carbon: A randomized trial of payments for ecosystem services to reduce deforestation. *Science (American Association for the Advancement of Science)* 357: 267–273.

Joos, F., Roth, R., Fuglestvedt, J. S., Peters, G. P., Enting, I. G., Bloh, W. von, Brovkin, V., Burke, E. J., Eby, M., Edwards, N. R., Friedrich, T., Frölicher, T. L., Halloran, P. R., Holden, P. B., Jones, C., Kleinen, T., Mackenzie, F. T., Matsumoto, K., Meinshausen, M., Plattner, G.-K., Reisinger, A., Segschneider, J., Shaffer, G., Steinacher, M., Strassmann, K., Tanaka, K., Timmermann, A. and Weaver, A. J. (2013). Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. *Atmospheric Chemistry and Physics* 13: 2793–2825, doi:10.5194/acp-13-2793-2013.

Nordhaus, W. D. (2017). Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences - PNAS* 114: 1518–1523.

Ricke, K. L. and Caldeira, K. (2014). Maximum warming occurs about one decade after a carbon dioxide emission. *Environmental research letters* 9: 124002.

Stern, N. and Stiglitz, J. E. (2021). The social cost of carbon, risk, distribution, market failures: An alternative approach .

Traeger, C. (2021). Ace - analytic climate economy. *CESIFO Working Paper* .

West, T. A. P., BÃ¶rner, J., Sills, E. O. and Kontoleon, A. (2020). Overstated carbon emission reductions from voluntary redd+ projects in the brazilian amazon. *Proceedings of the National Academy of Sciences - PNAS* 117: 24188–24194.

Zickfeld, K., Azevedo, D., Mathesius, S. and Matthews, H. D. (2021). Asymmetry in the climate carbon cycle response to positive and negative co2 emissions. *Nature climate change* .

Zickfeld, K., MacDougall, A. H. and Matthews, H. D. (2016). On the proportionality between global temperature change and cumulative co2 emissions during periods of net negative co2 emissions. *Environmental research letters* 11.

Supporting Material

The Optimal Trajectory of the Social Cost of Carbon

The dynamics of the social cost of carbon are explained in the context of a simple control problem of a stock pollutant. The damages associated with the pollutant (e.g. CO2 equivalents) are given by the function $D(S)$, where $S$is the stock and $\frac{\partial D(S)}{\partial S} = D_S(S) \geq 0$. The economics benefits of emitting the pollutant are goven by $B(R)$where R is the flow of emissions at any given point of time, and $\frac{\partial B(R)}{\partial R} = B_R(R) \geq 0$ and $\frac{\partial^2 B(R)}{(\partial R)^2} = B_{RR}(R) \leq 0$. The net benefits of economic activitiy that requires the emission of CO2e is therefore: $B(R) - D(S)$. This dynamics of the stock of carbon are given by the net effect of emissions R and the natural decay $G(S)$. for simplicty we assume that the decay is a simple linear function of the stock itself so that: $G(S) = -\gamma S$, where $\gamma > 0$. Given this simple set-up, the control problem is th emaximise the present value of the net benefits from emitting the stock pollutant taking into account the constraints on the stock dynamics, the technology associated with extraction of fossil fuels, the net benefits function, and the discount rate

r. The net benefits are measured in cash equivalents and so the appropriate discount rate is the consumption rate of discount, and for the purposes of the exposition, the discount rate is assumed to be invariant to the time horizon being evaluated. the control problem therefore takes the following form:

$$V = \max_{R} \int_{t=\tau}^{\infty} \exp\left(-r\left(t-\tau\right)\right)\left(B\left(R\left(t\right)\right) - D\left(S\left(t\right)\right)\right) dt \tag{14}$$

$$s.t.$$

$$\dot{S} = R - \gamma S$$

$$R, S \geq 0,$$

$$S\left(0\right) = S_0$$

The optimum path of extraction and stock accumulations can be solved using optimal control methods. We have omitted the fossil fuel stock for simplicity. A simple extension could reflect the dynamics of this stock, but without affecting the essential logic of the point we wish to make with regard to the Social Cost of Carbon and how this evolves over time. The solution stems from the Maximum Principle associated with the current value Hamiltonian:

$$H\left(R, S, \mu\right) = \left(B\left(R\left(t\right)\right) - D\left(S\left(t\right)\right)\right) + \mu\left(R - \gamma S\right) \tag{15}$$

where μ is the shadow value of the stock: the change in the value of the maximand in equation 14 as a result of a marginal change in the stock, S. the internal solution for this problem is given by:

$$\frac{\partial H}{\partial R} = B_R + \mu = 0 \tag{16}$$

$$-\frac{\partial H}{\partial S} = \dot{\mu} - r\mu = D_S\left(S\right) + \mu\gamma \tag{17}$$

$$\frac{\partial H}{\partial \mu} = R - \gamma S \tag{18}$$

$$\lim_{t\to\infty} \mu\left(t\right) \exp\left(-rt\right) S\left(t\right) = 0 \tag{19}$$

From 16 we know that the shadow price of the stock is negative because $B_R > 0$. This makes sense because the stock in this case is a pollutant, and so additional units of the stock are detrimental to net benefits, other things equal. Combining 16 and 17 leads to the following expression for the dynamics of the shadow price μ:

$$\frac{\dot{\mu}}{\mu} = \frac{D_S(S)}{\mu} + (r + \gamma) \tag{20}$$

which shows that the shadow price of the stock pollutant increases at a rate which is lower than the rate of discount, r, because $\mu < 0$. It remains to be shown that μ has the interpretation of the Social Cost of Carbon as presented in the main text in equation (1). Defining $\theta = -\mu$ and solving out the differential equation on 20 shows that (See Hoel 2016, p8-11):

$$\theta(\tau) = \int_{t=\tau}^{\infty} exp\left(-(r+\gamma)(t-\tau)\right) D_S(S(t))\, dt \tag{21}$$

which is identical to Equation 21 if one assumes that $\gamma = 0$. In an optimal control problem, the shadow price on the stock is the Social Cost of Carbon, the negative of which is the benefit of reducing the stock by a marginal ton. Given the persistence of CO2 in the atmosphere, the decay parameter γ is often argued to be very small in practice (Hoel 2016, Archer 2005). While we have illustrated the trajctory of the SCC in an optimal framework, showing that the trajectory of the SCC increases at a rate less than the rate of discount does not always require this optimal framework. Certainly the definition of the SCC is just that, a definition, meaning that it does not depend on the optimality of dynamic allocations.

Temperature and emission paths resulting in marginal damages with a constant growth rate x

The exposition of SVO in Section 3 has assumed for simplicity a marginal damage growing at a constant rate x. In this appendix, we look at conditions which are compatible with this assumption.

Consider the quadratic damage function in section 4 $D_T = \gamma YT$. Assume income growas at a constant rate g and temperature grows at constant rate y. As a result, marginal damges are $D_T = \gamma Y_0 S_0 e^{(g+y)t}$ and will grow at a constant rate $x = g + y$.

What if the damage function would not be quadratic? Assume that the damage function is a general power function of power θ, $D = \gamma YT^{\theta}$, that temperature raises at rate y and the economy at rate g. Then $D_T = -\gamma\theta YT^{(\theta-1)} = \theta\gamma\zeta Y_0 S_0 e^{(g+(\theta-1)y)t}$ and the growth rate of marginal damages is again constant and equal to $x = g + (\theta - 1)y$. With these assumptions, the SVO pricing formulas in Section 3 are appropriate.

Which emission paths will lead to a temperature path with a constant growth rate? Since emissions are the time derivative of cumulative emissions and using the approximation $T = \zeta S$, we can write $S_t = S_0 e^{yt} \Leftrightarrow E_t = \dot{S}_t = \underbrace{yS_0}_{E_0} e^{yt}$. Therefore, a temperature

Table 2: Mean growth rates of the SCC for different temperature paths and time frames. We assume a quadratic damage function, proportional to GDP, which increase at 2%. For a stable temperature, the SCC will incrrease at the growth rate of GDP. The discount rate is 3.2%. Since RCP scenarios are only defined until 2100 we assume a linear trend between 2095 and 2120 and constant temperatures thereafter.

	RCP2.6	RCP4	RCP6	RCP8.5
2020-2040	2.2%	2.4%	2.5%	2.8%
2020-2060	2.1%	2.3%	2.4%	2.7%
2020-2080	2.0%	2.3%	2.4%	2.6%
2020-2100	2.0%	2.2%	2.3%	2.5%

increasing at rate y requires emissions to increase at the same rate, with initial ($t = 0$) emissions $E_0 = yS_0$. With emissions in 2020 in the order of maginitude of 40GtCO2/y and cumulative emissions around 2000GtCO2, this is valid for y=2%.

Temperature paths are rising at a constant rate of more or less 2% until 2070 for the RCP8.5% scenario. For other RCP scenario's 2.6, 3.4 and 6.0 the growth rate of temperature starts at 2% but approaches 1% in 2030 2040 and 2045 respectively. If there is no risk involved, our formula 3 only requires a mean growth rate of the SCC, which are shown in Table 2.

Derivations and Proofs

Proof of Formula in Equation (3)

Multiplying Equation (1) by $exp\left(-r\tau_1\right)$outside the integral and by $exp\left(r\tau_1\right)$ inside integral, and adding and subtracting the same integral over $[\tau_2 + \xi, \infty]$ we obtain:

$$SVO_{\tau_1\tau_2} = exp\left(-r\tau_1\right) \int\limits_{t=\tau_1+\xi}^{\infty} exp\left(-r\left(t-\tau_1\right)\right)\zeta D_T dt - exp\left(-r\tau_2\right) \int\limits_{t=\tau_2+\xi}^{\infty} exp\left(-r\left(t-\tau_2\right)\right)\zeta D_{T_t} dt \tag{22}$$

Given the definition of SCC_τ in (1), $SVO_{\tau_1\tau_2}$ simplifies to:

$$SVO_{\tau_1\tau_2} = exp\left(-r\tau_1\right) SCC_{\tau_1} - exp\left(-r\tau_2\right) SCC_{\tau_2} \tag{23}$$

$SVO_{\tau_1\tau_2}$ is simply the difference between the present values of SCC_{τ_1} and SCC_{τ_2}. For simplicity suppose that emissions, temperatures and damages evolve in the future so that SCC_τ grows exponentially at a rate x: $SCC_\tau = SCC_0 exp(x\tau)$, which is a reasonable assumption under certain circumstances.[6] In this case, the $SVO_{\tau_1\tau_2}$ simplifies to Equation

[6]if x is the growth rate of global warming: celsius above preindustrial temperature, and climate damages are quadratic. In the case of the seminal model by Golosov et al. (2014) model or Traeger (2021), x corresponds to the growth rate of GDP. So the assumption of exponential growth of SCC is compatible with important Integrated Assessment Models.

3

Proof that if marginal damages increase at a constant rate *d*, the SCC increases at the same rate.

If the marginal damages increase exponentially at rate d, the SCC at time τ is:

$$SCC_\tau = \int_\tau^\infty e^{-r(t-\tau)} D_{S_\tau} e^{d(t-\tau)} dt$$

where $D_{S,\tau}$is the marginal damage at time τ. The SCC at time τ can then be re-written as:

$$SCC_\tau = D_{S_\tau}/(r-d)$$

from which it follows that:

$$SCC_\tau = D_{S_0} e^{d\tau} = SCC_0 e^{d\tau}$$

Derivation for steady state SVO.

The derivation of the SVO in the case of a steady state/equilibrium carbon stock: $S_t = \overline{S}$.for all t, follows from the definition of SCC_{τ_1}, which becomes:

$$SCC_{\tau_1} = \int\limits_{t=\tau_1}^{\infty} exp\left(-r\left(t-\tau_1\right)\right) D_{\overline{S}} dt = \frac{D_{\overline{S}}}{r} \tag{24}$$

Inserting into the Equation X for leads to the following expression for the $SVO^\phi_{\tau_1 \tau_2}$:

$$SVO^\phi_{\tau_1 \tau_2} = exp\left(-r\tau_1\right) \int\limits_{t=\tau_1}^{\tau_2} exp\left(-\left(r+\phi\right)\left(t-\tau\right)\right) D_{\overline{S}} dt = \frac{D_{\overline{S}}}{r+\phi} exp\left(r\tau_1\right)\left(1-exp\left(r\nu\right)\right) \tag{25}$$

which, since $SCC_\tau = SCC_0$for all τ in the steady state, can be written as:

$$SVO^\phi_{\tau_1 \tau_2} = SCC_0 exp\left(-r\tau_1\right) \frac{r}{r+\phi} \tag{26}$$

Derivation of SVO with hazard risk.

If damages increase at an exponential rate d then the SCC becomes:

$$SCC_{\tau} = \int_{t=\tau}^{\infty} exp\left(-r\left(t-\tau\right)\right) D_{S,\tau} \exp\left(d\left(t-\tau\right)\right) dt$$

where $D_{S,\tau}$is the marginal damage at time τ. This leads to the following solution:

$$SCC_{\tau} = \frac{D_{S,\tau}}{r-d}$$

With the hazard rate the SVO becomes:

$$SVO^{\phi}_{\tau_1\tau_2} = exp\left(-r\tau_1\right) \int_{t=\tau_1}^{\tau_2} exp\left(-\left(r+\phi\right)\left(t-\tau_1\right)\right) D_S\left(S\left(t\right)\right) dt$$

$$= exp\left(-r\tau_1\right) \int_{t=\tau_1}^{\infty} exp\left(-\left(r+\phi\right)\left(t-\tau_1\right) D_S\left(S(t)\right) dt\right.$$

$$-exp\left(-r\tau_2\right) \int_{t=\tau_2}^{\infty} exp\left(-\left(r+\phi\right)\left(t-\tau_2\right) D_S\left(S(t)\right) dt\right.$$

but the integral:

$$\int_{t=\tau}^{\infty} exp\left(-\left(r+\phi\right)\left(t-\tau\right) D_S\left(S(t)\right) dt = \frac{D_{S,\tau}}{r+\phi-d}\right.$$

which can be re-written as:

$$SCC_{\tau} \frac{r-d}{r+\phi-d}$$

From here the formula in the text follows assuming that the SCC grows at a rate x, which has to be equal to d as shown above: the SCC must grow at the same rate as the marginal damages of marginal damages increase exponentially.

Concave increasing marginal damages

Assume marginal damages aproach a steady state marginal damages at a constant rate x. $D_S = D^*_S - \left(D^*_S - D^0_S\right) exp(-xt)$.

$$SVO_{\tau_1,\tau_2} = \int_{\tau_1}^{\tau_2} e^{-rt-\phi(t-\tau_1)} \left(D_S^* - \left(D_S^* - D_S^0\right) e^{-xt}\right) dt \tag{27}$$

$$= e^{\phi\tau_1} \left[\left[\frac{D_S^* e^{-(\phi+r)t}}{\phi + r} \right]_{\tau_1}^{\tau_2} - \left[\frac{\left(D_S^* - D_S^0\right) e^{-(\phi+r+x)t}}{\phi + r + x} \right]_{\tau_1}^{\tau_2} \right] \tag{28}$$

$$= e^{-r\tau_1} \left\{ \left[\frac{D_S^*}{\phi + r} \left(e^{-(\phi+r)\nu} - 1\right) \right] - e^{-x\tau_1} \left[\frac{\left(D_S^* - D_S^0\right)}{\phi + r + x} \left(e^{-(\phi+r+x)\nu} - 1\right) \right] \right\} \tag{29}$$

The above path for marginal damages can be compatible with several cumulative emissions paths. For example, marginal damages can be proportional to production $D_S = -\gamma Y S$ and cumulative emissions follow the path $S_t = \frac{D_S^* exp(-gt) - \left(D_S^* - D_S^0\right) exp(-(x+g)t)}{\gamma Y_0}$. As a result, emissions in the long run are negative and decrease at rate g, to offset the effect of increasing production on marginal damages $E_t = \frac{-g D_S^* exp(-gt) + (x+g)\left(D_S^* - D_S^0\right) exp(-(x+g)t)}{\gamma Y_0}$. For a simpler case, we can assume that marginal damages are γS and that cummulative emissions follow the path $S_t = S^* - (S^* - S_0)\, exp(-xt)$. As a result, emissions are exponentially decreasing $E = E_0 e^{-xt}$ with initial condition $E_0 = x(S^* - S_0)$. This leads to the following fomula for the social value of the offset

$$SVO_{\tau_1,\tau_2} = \gamma e^{-r\tau_1} \left\{ \left[\frac{S^*}{\phi + r} \left(e^{-(\phi+r)\nu} - 1\right) \right] - e^{-x\tau_1} \left[\frac{(S^* - S_0)}{\phi + r + x} \left(e^{-(\phi+r+x)\nu} - 1\right) \right] \right\}. \tag{30}$$

Quadratic hump-shaped marginal damages

Assume marginal damages follow a quadratic time path $D_{S_t} = D_{S_0} + a_1 t - a_2 t^2$. To make notation easier and associate an emissions path to this scenario, assume that marginal damges are γS (not proportional to production).[7] This implies a quadratic cumulative emissions path $S_t = S_0 + E_0 t - \frac{x}{2} t^2$and a linear decreasing emissions path $E_t = E_0 - xt$. Temperature peaks at time E_0/x, when emissions are zero.

The value of the project writes

$$SVO_{\tau_1,\tau_2} = \int_{\tau_1}^{\tau_2} e^{-rt-\phi(t-\tau_1)} \gamma \left(S_0 + E_0 t - \frac{x}{2} t^2\right) dt \tag{31}$$

Integrate by parts

[7]The general solution is obtained by setting $a_1 = \gamma E_0; a_2 = \frac{\gamma x}{2}$ in the equations below.

$$SVO_{\tau_1,\tau_2} = e^{\phi\tau_1}\left[\left[\gamma\left(S_0+E_0t-\frac{x}{2}t^2\right)\frac{e^{-(\phi+r)t}}{\phi+r}\right]_{\tau_1}^{\tau_2} - \int_{\tau_1}^{\tau_2} e^{-(r+\phi)t}\gamma\left(E_0-xt\right)dt\right] \tag{32}$$

Integrate by parts a second time

$$SVO_{\tau_1,\tau_2} = e^{\phi\tau_1}\gamma\left\{\left[\frac{e^{-(\phi+r)t}}{\phi+r}\left(\overbrace{S_0+E_0t-\frac{x}{2}t^2}^{S_t} - \frac{\overbrace{E_0-xt}^{E_t}}{\phi+r} - \frac{x}{(\phi+r)^2}\right)\right]_{\tau_1}^{\tau_2}\right\} \tag{33}$$

The social cost of carbon (the above formula with for period $0,\infty$) is not really meaningful because emissions on a linear path become ever more negative (and warming becomes negative in the very long run). Therefore, we will now assume that when emissions reach zero at time $t^* = E_0/x$,they remain zero. As a result, temperature peaks at $S^* = S_0 + \frac{E_0^2}{2x}$ and is stable thereafter. This gives the following social cost of carbon (using equation 33 between time zero and t^* and adding the present value cost of constant damages $\frac{e^{-rt^*}}{r}\gamma S^*$ thereafter)

$$SCC_0 = \gamma\frac{e^{-rt^*}}{r}\left(2S^* - \frac{x}{r^2}\right). \tag{34}$$

Substituting out γ allows to calculate the adjustement factor for impermanence and risk. In case the project stops before emissions are zero $\tau_2 \leq \frac{E_0}{x}$ this yields the following formula

$$SVO_{\tau_1,\tau_2} = SCC_0 e^{\phi\tau_1}\left[\frac{e^{-rt^*}}{r}\left(2S^* - \frac{x}{r^2}\right)\right]^{-1}\left[\frac{e^{-(\phi+r)t}}{\phi+r}\left(\overbrace{S_0+E_0t-\frac{x}{2}t^2}^{S_t} - \frac{\overbrace{E_0-xt}^{E_t}}{\phi+r} - \frac{x}{(\phi+r)^2}\right)\right]_{\tau_1}^{-\tau_2}. \tag{35}$$

Figures

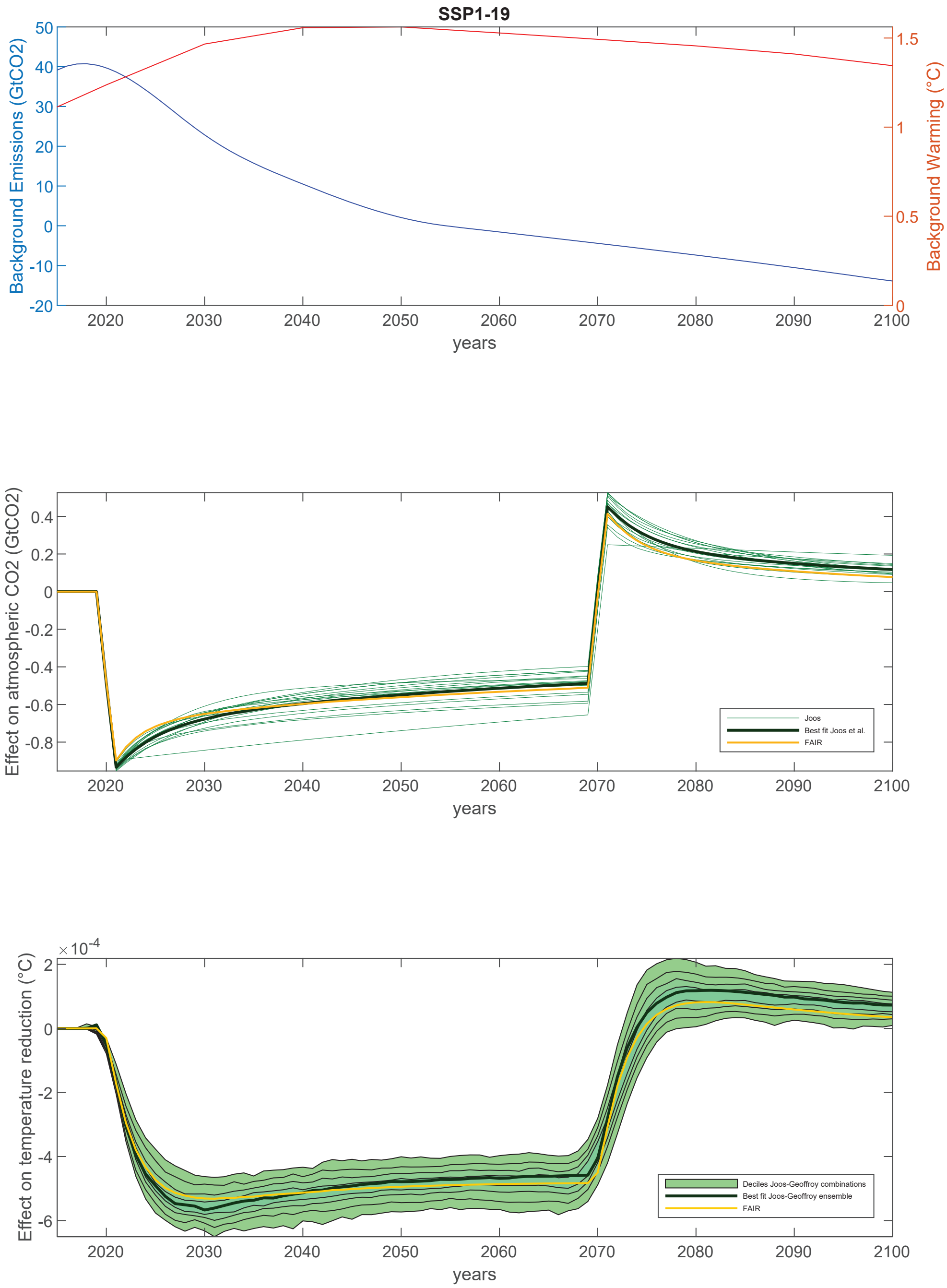

Figure 2: SSP 1-19

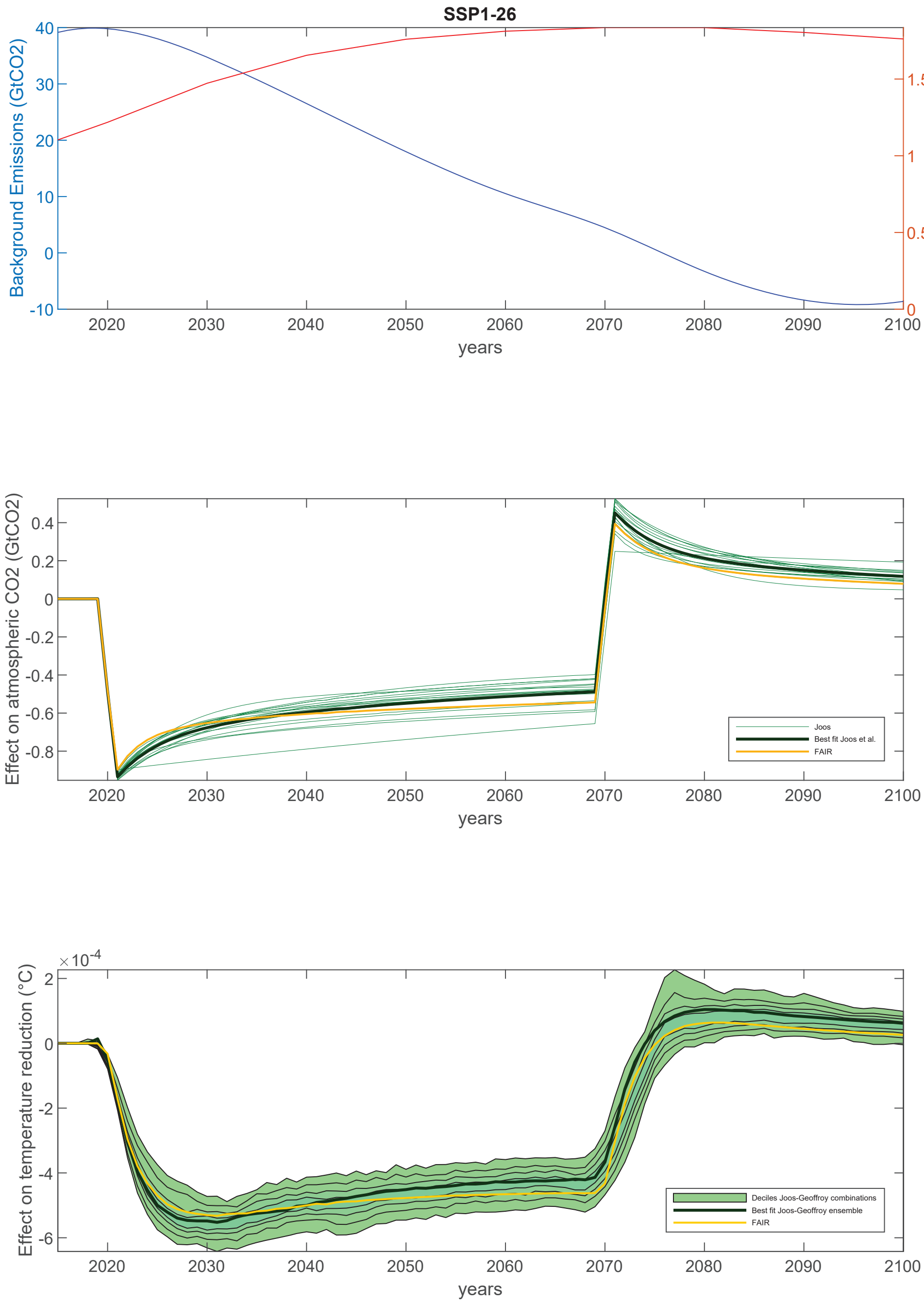

Figure 3: SSP 1-26

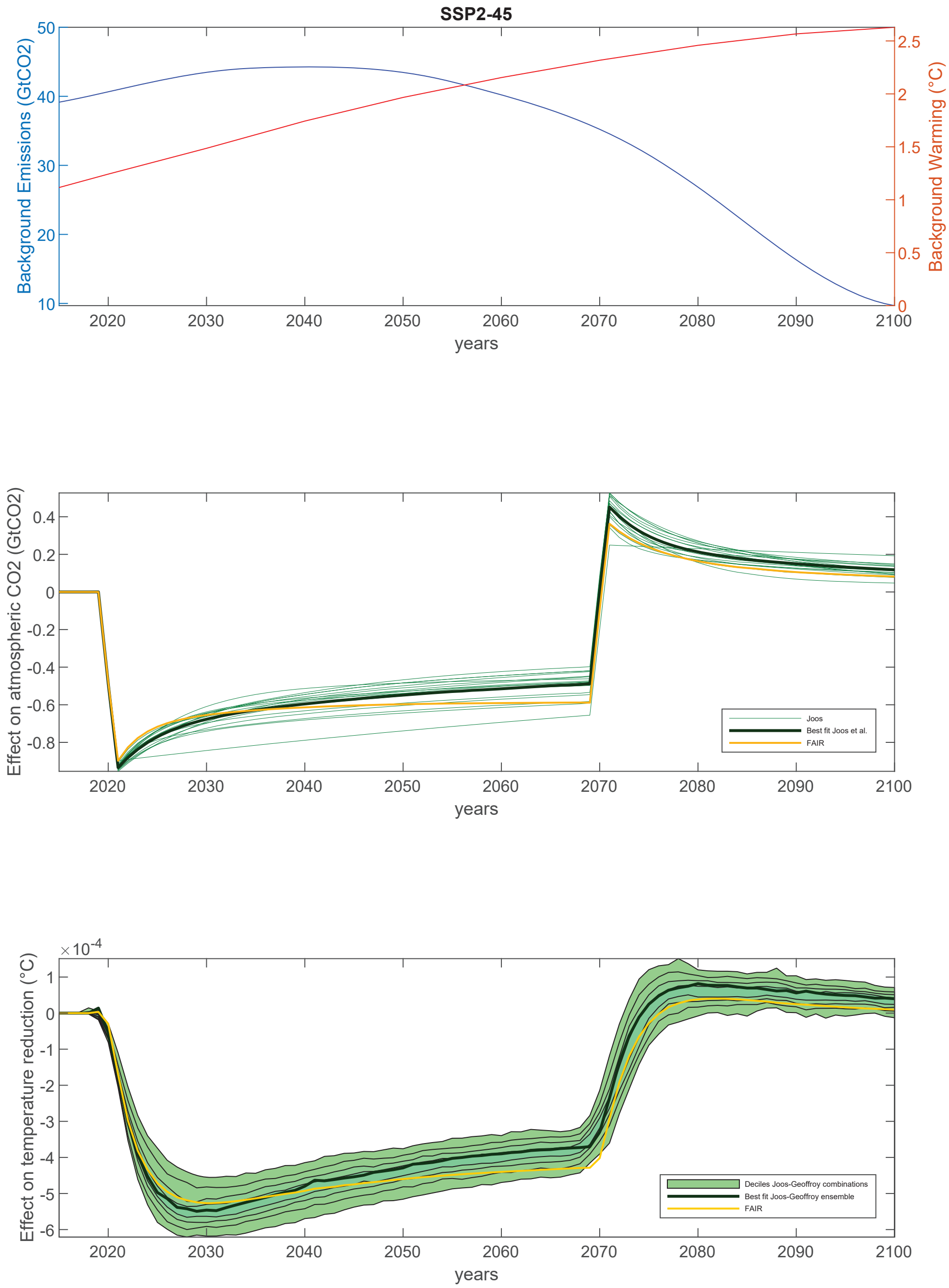

Figure 4: SSP 1-45

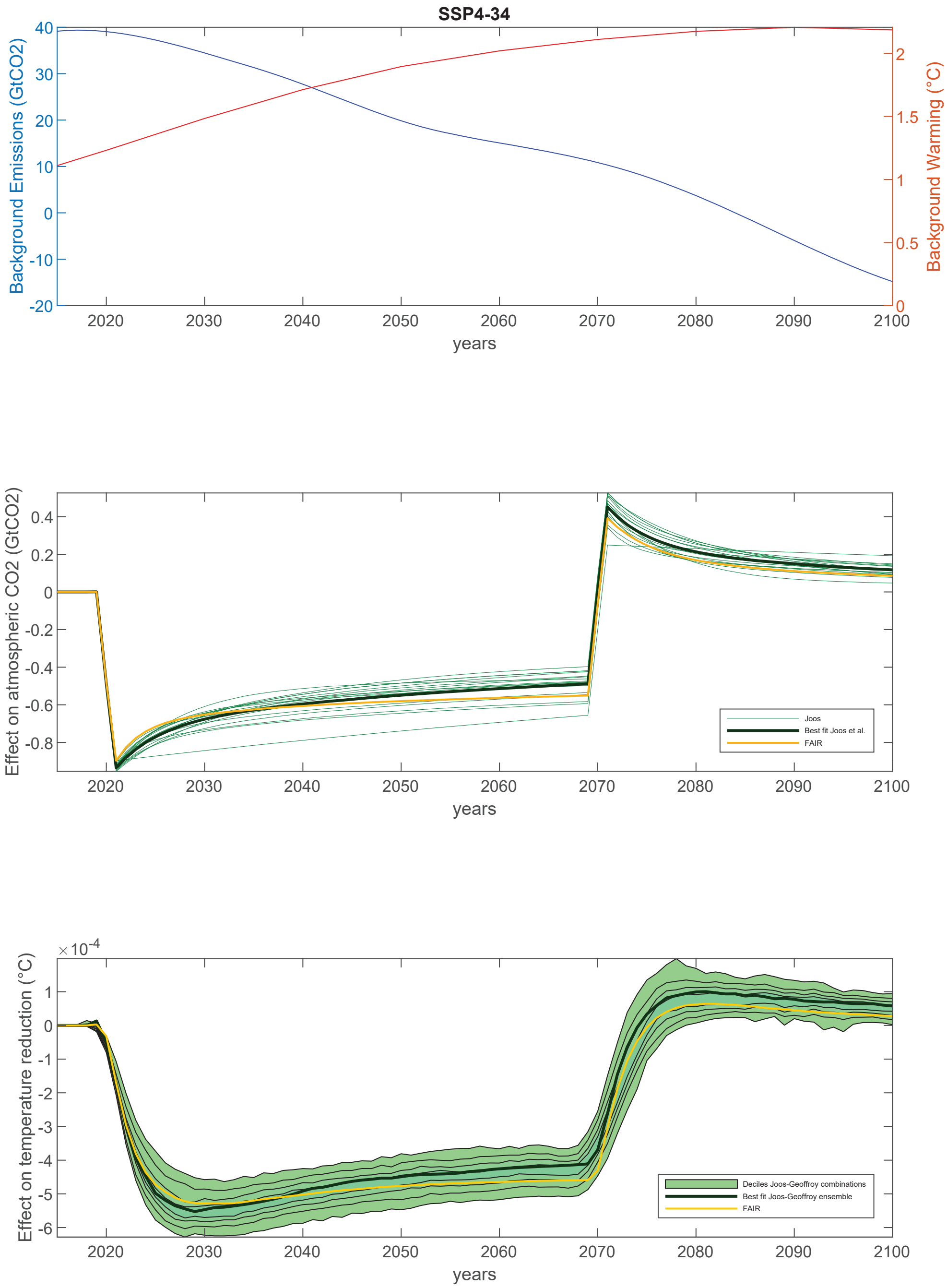

Figure 5: SSP4-34

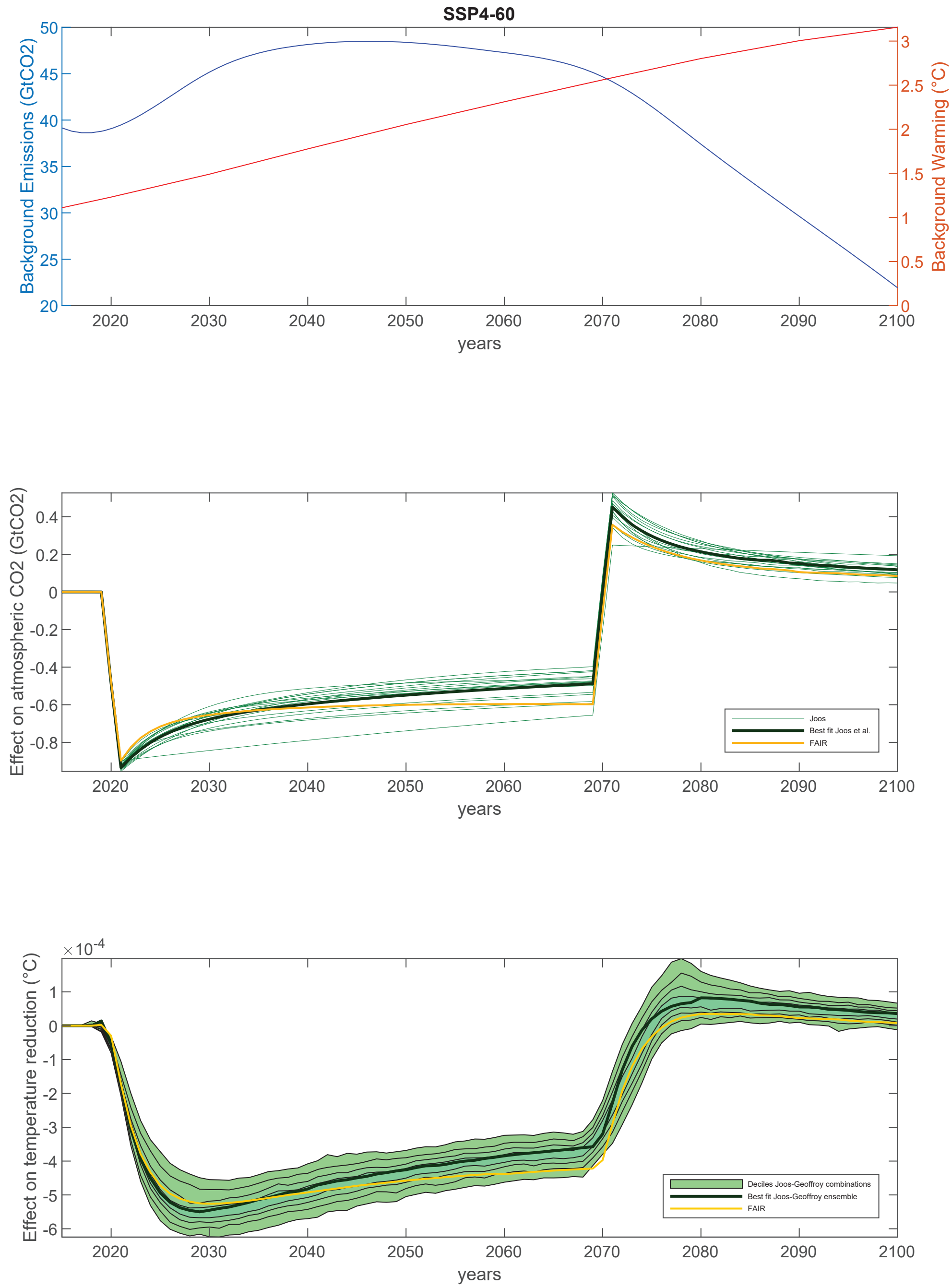

Figure 6: SSP 4-60

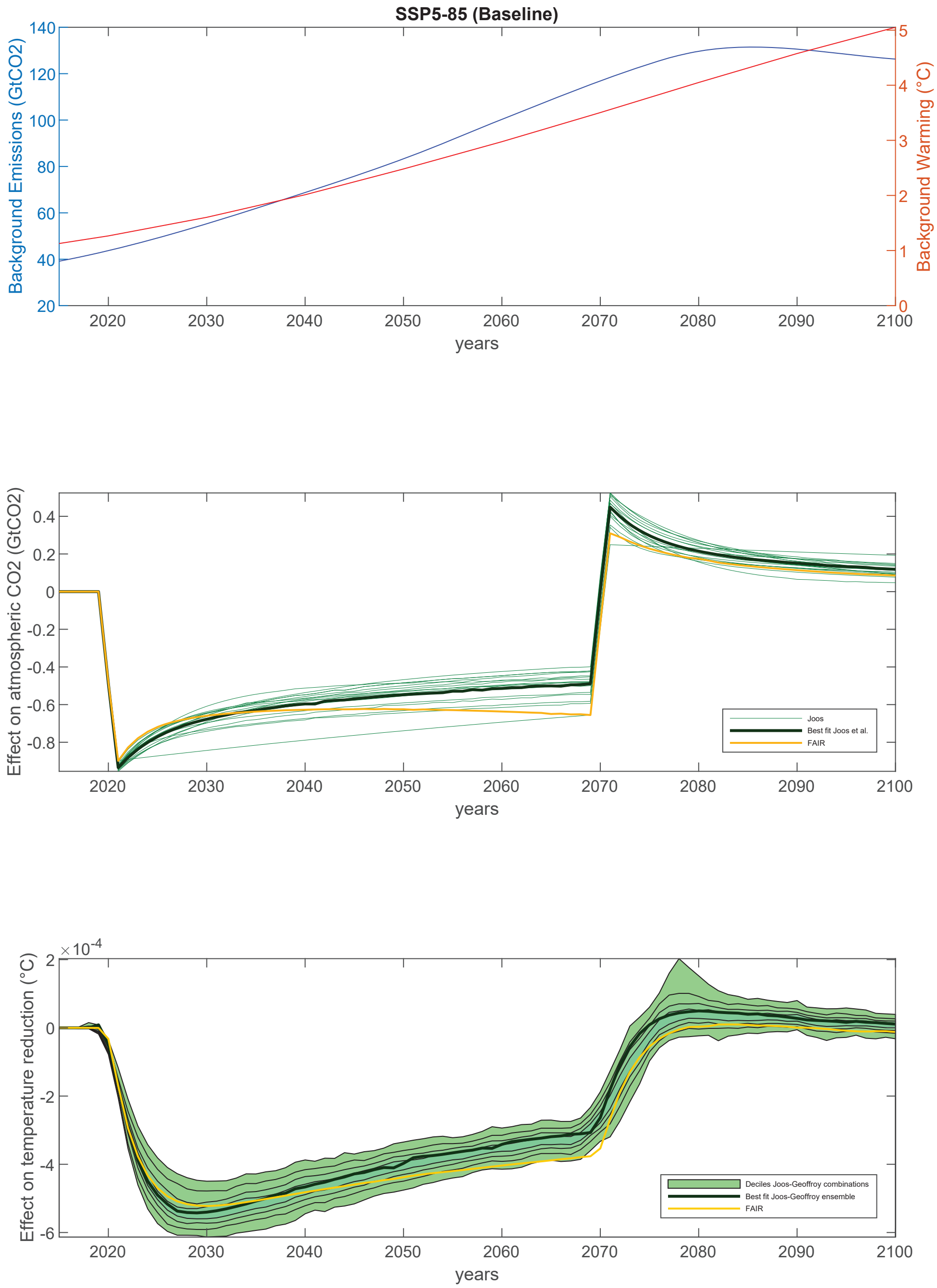

Figure 7: SSP 5-85 (Baseline)