Asset prices and incentives for climate policy (Empower the young!)[a]

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April 28, 2021

Abstract

Climate policy changes the supply and demand for investment, changing the price of undepreciated assets. When the asset price is endogenous, asset markets potentially provide self-interested agents an incentive to reduce emissions. Functional assumptions in previous Integrated Assessment Models produce a fixed asset price, and therefore neglect this possibility. The effect of abatement on the endogenous asset price depends on the Elasticity of Intertemporal Substitution (EIS). Using both analytic and numerical methods, we find that self-interested incentives arising from an asset market can lead to substantial climate policy, although still much lower than the level chosen by a utilitarian planner.

Keywords: Climate externality, overlapping generations, climate policy, generational conflict, dynamic bargaining, Markov perfection, concave production possibility frontier.

JEL codes: E24, H23, Q20, Q52, Q54

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[a]This paper supersedes “Asset prices and climate policy”. It uses a different model and calibration, asks a larger set of questions, obtains new analytic results, uses improved numerics, and has a larger set of authors. We incorporate the many valuable comments we received on the old paper; we do not thank the people who made those comments by name at this time, because they have not read this paper. But we thank them collectively.

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1 Introduction

The standard climate narrative emphasizes that: (i) climate change will harm future generations or currently living young people later in their life, and (ii) climate policy requires more expensive production methods and therefore lowers current world income. This framing implies that altruism with regard to future generations, and to a lesser extent the current young generation's concern for its own future, are the primary motivators for climate policy. Although substantially correct, this view of climate policy is incomplete because it neglects the potential for asset markets to influence selfish agents' incentives. Climate damages lower capital and labor's future productivity; climate policy can protect that productivity. If asset markets transfer future changes in productivity to current asset prices, current asset owners obtain some of the benefit of current climate policy.

This mechanism straightforward, but formal policy analysis carried out with Integrated Assessment Models (IAMs) neglects it. Figure 1 illustrates our point. IAMs assume that output consists of a composite commodity that can be either consumed or invested (Stern, 2006; Nordhaus, 2008). The composite commodity model is equivalent to a linear production possibility frontier (PPF). The heavy curve in Figure 1 shows a strictly concave PPF, and the dashed tangent with slope -1 corresponds to a composite commodity model that fixes (by choice of units) the price of investment at 1. A change in investment, e.g. arising from climate policy, *cannot* change the price of investment in the composite commodity setting; it *might* change the price of investment with a strictly concave PPF. When old capital and new capital are equally productive (as IAMs assume), a change in the price of investment changes the value of undepreciated capital.

Basic economics recognizes that markets may give asset owners a stake in environmental protection, even when the owners' have no direct interest in the environment (Oates, 1972). Empirical evidence shows that environmental outcomes can affect asset prices, e.g. in real estate (Chay and Greenstone, 2005; Bushnell et al., 2013) and market portfolios (Bansal and Ochoa, 2011;

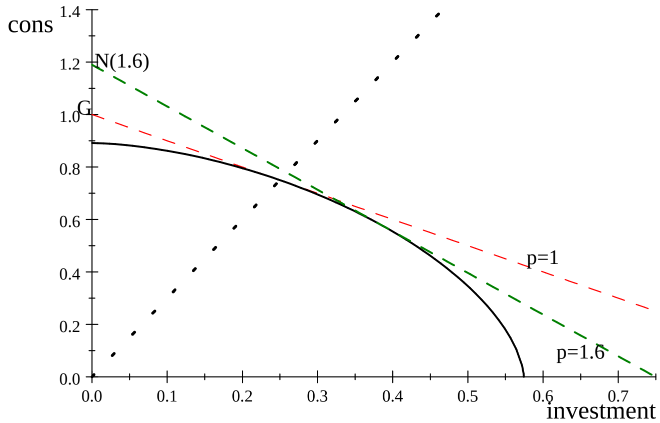

Figure 1: With a strictly concave PPF and $p = 1$ (the price of investment measured in consumption units) production occurs at the tangency shown by the dashed line labelled $p = 1$. This dashed line can also be interpreted as a PPF in the composite commodity setting. If investment increases, p must increase under the concave PPF (e.g. to $p = 1.6$). An increase in investment cannot change the price of investment in the composite commodity framework.

Bansal et al., 2015, 2016). Severen et al. (2018) find that U.S. land prices incorporate climate forecasts; Schlenker and Taylor (2019) show that weather markets reflect climate model predictions.[1]

Economists prefer IAMs over partial equilibrium models to study climate policy because the savings and climate decisions are potentially entwined. Climate policy can benefit future generations by leaving them a cleaner environment, but if this policy crowds out investment it might maker future generations poorer. In the composite commodity setting, all climate-induced adjustment to savings occurs via changes in quantity. A strictly concave PPF means that both prices and quantities adjust. This observation is the basis for the two broad questions that motivate our paper.

The first set of questions concerns the qualitative relations between climate

[1] Dietz et al. (2016) use DICE to estimate the “climate value at risk” at 1.8% of global financial assets, with much higher tail risks. Balvers et al. (2012) estimate the cost of past warming, as captured in asset prices, at 4.18% of wealth. The CEO of the asset management firm Blackrock announced that his firm would include environmental sustainability as an investment criterion (NY Times January 14 2020).

policy and the asset price and welfare. These relations provide a basis for understanding how asset markets affect self-interested incentives for policy. Abatement shifts in the current PPF, lowering national income at the pre-existing price, thereby lowering the demand for savings, putting downward pressure on the asset price. However, the cleaner (and more productive) future climate tends to raise capital's future productivity, increasing the demand for investment. The net effect of these offsetting forces depends on the Elasticity of Intertemporal Substitution (EIS).

If agents in a Diamond-style overlapping generations (OLG) model (Diamond, 1965) have logarithmic utility ($EIS = 1$), the income and substitution effects cancel. Here, climate policy lowers the asset price and lowers welfare for both the current old and young generations. Climate policy results in a lower stock of capital and a cleaner environment, with ambiguous welfare effects for the agent born in the next period. When $EIS > 1$, the current old and young generations' incentives are aligned: they both benefit from a policy that raises the asset price. In contrast, for $EIS < 1$ – the conventional choice for IAMs – a higher asset price harms the young generation but benefits the old agent.

To obtain sharper analytic results we specialize to a two-period setting and consider limiting cases. With $EIS \to \infty$ (linear preferences), the "first unit of abatement" increases the equilibrium asset price, strictly increasing welfare for currently living generations. The next generation inherits a cleaner environment and a larger stock of capital, so also has higher welfare. At the other extreme, $EIS = 0$ (Leontieff preferences) we find that the first unit of abatement lowers the equilibrium asset price, harming the current old generation but benefiting the current young generation.

The second set of questions concerns the quantitative significance (on incentives to undertake climate policy) of asset markets. If assets depreciate fully by the end of a period, as some models assume (Golosov et al., 2014), then the current asset owner has nothing to sell her successor. In that case, even if asset prices are endogenous, they could affect the incentives of current agents only via their effect on factor returns. Alternatively, if the PPF is almost linear, then the composite commodity framework is a good approximation, and asset markets have a negligible effect on incentives. We need a quantitative model to assess the significance of asset markets. We embed a model calibrated to DICE (Nordhaus, 2017) – but modified with a strictly concave PPF – in a dynamic game amongst a succession of *selfish* generations. We study the Markov Perfect equilibrium (MPE), where current generations can influence future actions via the condition of the climate and the stock of capital they leave the next generation. But they cannot choose future policy, and we exclude trigger strategies.

In MPE to this game, asset markets indeed influence incentives to undertake climate policy. In line with our theoretical results, the incentives tend to be more powerful for $EIS > 1$ than for $EIS < 1$, but they are not monotonic in this parameter; they are zero for $EIS = 1$ (logarithmic utility). For our baseline $EIS = 0.5$ (the conventional choice), climate policy lowers the asset price. Here, current equilibrium policy is significant only if the young generation has substantial influence in setting climate policy. Even for parameters that lead to very low levels of policy in the first period, in most cases cumulative policy significantly reduces climate damages. Thus, the model with selfish agents and endogenous asset prices predicts significant levels of long-run abatement, even if current effects are small.

Our analysis shows how currently living agents should set policy (i) if they have no concern for their successors and (ii) if they have solved the problem of international cooperation. Condition (i) is not ethically desirable, so the policy we study does not constitute a recommendation. Condition (ii), although desirable, is nowhere in evidence, so the policy we study is not a prediction. Our analysis sheds light on a type of incentive that has not previously been studied in the climate context. It shows how these incentives depend on agents' EIS and on the relative influence of the two generations in determining policy. Empowering the young leads to stronger climate policy.

As benchmarks, we compare outcomes in the dynamic game when currently living selfish agents choose the abatement level, having rational expectations about future policy, with the outcome under a standard social planner and under Business as Usual (zero abatement).

Additional literature and other considerations When considered in the climate setting, asset prices are typically discussed in the context of "stranded assets", whose value is reduced by climate policy. Fossil fuel companies, the common example of stranded assets, worth about 5 $T (trillion) (Bullard, 2014), constitute a large asset class, but a modest fraction of total world financial assets. The 2015 world stock market capitalization exceeded 69 $T, and the value of total financial assets exceeded 284 $T (Witkowski, 2015). Society at large, and people with a well-diversified portfolio, should be more concerned about the effect of climate policy on asset prices writ large.

Many OLG models examine environmental policy (Howarth and Norgaard, 1992; John and Pecchenino, 1994; Gerlagh and Keyzer, 2001; Schneider et al., 2012; Williams et al., 2015; Karp, 2017; Iverson and Karp, 2021). Karp and Rezai (2014) include asset prices in an OLG model with an endogenous resource stock but a fixed capital stock, $EIS = \infty$, and no climate component.[2]

Bovenberg and Heijdra (1998) show that the issuance of public debt can support Pareto-improving environmental policy by transferring the cost of policy, across time, to those who benefit from it. In a setting with 55 overlapping generations, Kotlikoff et al. (2021) find that a carbon tax beginning at $30/tCO_2$ and rising at 1.5% per year achieves a 0.73% uniform welfare increase over BAU consumption equivalents. The policymaker uses debt, taxing future generations at 8% of lifetime consumption, compensating current generations at 1.2% of their consumption. Anderson et al. (2020) show that a sequence of abatement rates, financed by debt, together with a distortionary labor tax, lead to Pareto improvements.

We exclude public debt, social security, and other means of intergenerational transfers from the future to the present: our research question concerns the *incentives to abate greenhouse gasses created by asset markets*, not the design of optimal climate policy. We do not allow the current generation to choose future policies: there is no commitment. In addition, we consider Markov Perfect Equilibria, thus excluding trigger strategies. In line with a

[2]Appendix B.4 discusses the relation between the models and the research questions in Karp and Rezai (2014) and in the current paper; neither is a special case of the other.

well-known Folk Theorem, Rangel (2003) show that trigger strategies can be used support intergenerational transfers as a subgame perfect equilibrium.

In the interest of tractability, we ignore several important issues in climate economics. First, we use a deterministic model, thus excluding questions arising from uncertainty about damages and growth (Lemoine and Traeger, 2014; Jensen and Traeger, 2014; Kelly and Tan, 2015; Lontzek et al., 2015) and specifically about the "climate beta"(Traeger, 2014; Giglio et al., 2021; Lemoine, 2021; Dietz et al., 2018). Second, in keeping with the tradition of IAMs, we use a single stock of capital. In fact, there are many asset classes, which will be affected differently depending on their sectoral and geographical location. This consideration is important, but a single-stock model provides the obvious place to begin, and is assumed in most IAMs; this stock serves as a proxy for a broad index of assets. Third, as with most IAMs, we ignore the cross-country free-riding problem. Asset markets potentially attenuate intergenerational free riding, without reducing cross-country free riding.

2 The Model

We describe the young agent's savings decision, discuss the technology, and then introduce preferences and define the decentralized equilibrium.

2.1 The savings decision

In each period, a cohort of constant size, $L \equiv 1$ is born.[3] Agents live two periods and maximize their lifetime welfare, Ω. Lifetime welfare is the discounted sum of utility, $U(\cdot)$, derived from consumption while young, c^y, and old, c^o: $\Omega_t^y = U(c_t^y) + \rho\, U(c_{t+1}^o)$ with ρ the constant utility discount factor and superscripts y and o denoting the young and old generation. Utility, $U(\cdot)$, satisfies the Inada conditions. The young agent receives labor income, w_t, but no inheritance, and spends c_t^y on consumption. With a depreciation rate δ, the amount of capital remaining at the end of period t is $(1-\delta)\, K_t$. Newly produced cap-

[3]Section 4 includes growth in both in TFP and population.

ital, I_t, and undepreciated old capital are equally productive; therefore, in equilibrium they have the same price, p_t. Our convention is that goods and undepreciated capital are sold at the end of the period; there is no discounting within a period, and p_t is the price of capital and the investment good at the end of the period. The young agent buys s_t shares of the old capital stock and I_t units of new capital at the cost $p_t\left[s_t(1-\delta)K_t+I_t\right]$. The rental rate on capital is r_t. When old in period $t+1$, the agent earns the factor payment $r_{t+1}(s_t(1-\delta)K_t+I_t)$, and obtains revenue from selling the end-of-period stock, $p_{t+1}(1-\delta)(s_t(1-\delta)K_t+I_t)$. Agents are selfish, so the old agent consumes all her income.

Agents take prices, w_t, r_t and p_t, as given and have rational point expectations of r_{t+1} and p_{t+1}. The young agent's maximization problem is

$$\max_{I_t,\ s_t,\ c_t^y,\ c_{t+1}^o} U(c_t^y)+\rho\, U(c_{t+1}^o) \text{ subject to}$$

$$\begin{aligned} c_t^y &\leq w_t-p_t\left[s_t(1-\delta)K_t+I_t\right] \\ c_{t+1}^o &\leq \left(r_{t+1}+p_{t+1}\left(1-\delta\right)\right)\left(s_t(1-\delta)K_t+I_t\right). \end{aligned} \tag{1}$$

The optimal decision to buy shares of existing capital, s_t, satisfies

$$\psi_t\equiv\frac{U'(c_t^y)}{\rho\, U'(c_{t+1}^o)}=\frac{r_{t+1}+p_{t+1}(1-\delta)}{p_t}, \tag{2}$$

which states that in equilibrium the marginal rate of intertemporal substitution equals the marginal rate of transformation. The right side in equation (2) gives the number of consumption units a young agent obtains in the next period by reducing consumption by 1 unit today and investing in capital instead. This ratio equals the marginal rate of intertemporal substitution, ψ_t, which equals 1 plus the endogenous interest rate between period t and $t+1$. In equilibrium, $s_t\equiv 1\ \forall t$, because the old generation has inelastic supply of undepreciated capital. Provided that $I_t>0$ for $t<H$ (as we hereafter maintain), the assumption that new and old capital are homogeneous implies

that the optimality condition for I_t is identical to equation (2).[4]

Rearranging the optimality condition (2) produces the asset price equation:

$$p_t = \frac{r_{t+1} + p_{t+1}(1-\delta)}{\psi_t} \text{ for } t < H, \tag{3}$$

where $H < \infty$ is the last period. The young generation in H only lives one period, so it does not accumulate capital, implying the asset price is zero, $p_H = 0$. (Section 4 further discusses this finite horizon assumption.)

2.2 Technology

Our technology, with strictly concave production possibility frontier (PPF), contains the "composite commodity" model (linear PPF) as a limiting case. World product is a function of inputs and policy, $\mathbf{z}_t = (K_t, L, E_t, \mu_t)$, where K_t is the stock of capital, L is the constant labor supply (normalized to 1), E_t, is the stock of atmospheric carbon in excess of pre-industrial levels ("Excess carbon"), and $\mu_t \in [0,1]$ is the abatement rate. The endogenously changing stocks, K_t and E_t, are the state variables, and the abatement rate is the policy variable. The consumption good, c_t, and the investment good, I_t, are joint outputs. We suppress time subscripts when the meaning is clear.

The equilibrium value of world output given the price of investment p is $N(p;\mathbf{z})$. The equilibrium value of world output *evaluated at* $p = 1$ is $G(\mathbf{z}) \equiv N(1;\mathbf{z})$. We treat $G(\mathbf{z})$ as a primitive, and adopt:

Assumption 1 *(i) $G(\mathbf{z})$ is increasing and concave in K, L; it is decreasing in (E,μ), convex in μ and convex in E for small E, and it is twice continuously differentiable in $\mathbf{z}$.*

(ii) The marginal cost of the first unit of abatement is zero: $\frac{\partial G}{\partial \mu}_{|\mu=0} = 0$.

(iii) ("DICE") $G = D(E)\Lambda(\mu) L^{\beta} K^{1-\beta}$, together with the other assumptions of Parts (i) and (ii).

[4]For example, we exclude the possibility that the end-of-period stock of undepreciated capital, $(1-\delta)K_t$, is large enough t produce an equilibrium with $I_t = 0$.

Assumption 1 (i) is self-explanatory. We use Part (ii) for the comparative static analysis, and Part (iii) to consider special cases and for our calibration.

To obtain a concave PPF we embed the function G in a constant elasticity of transformation (CET) function with elasticity of transformation $\infty > \sigma \geq 0$ and shape parameter a:[5]

$$\begin{aligned} c^{\frac{1+\sigma}{\sigma}} + aI^{\frac{1+\sigma}{\sigma}} &= \left(1 + a^{-\sigma}\right)^{-\frac{1}{\sigma}} G\left(\mathbf{z}\right)^{\frac{1+\sigma}{\sigma}} \Rightarrow \\ c &= \left(\left(1 + a^{-\sigma}\right)^{-\frac{1}{\sigma}} G\left(\mathbf{z}\right)^{\frac{1+\sigma}{\sigma}} - aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}}. \end{aligned} \tag{4}$$

Equation 4 is the PPF.[6] Given $p, \mathbf{z}$, world income, $N(p; \mathbf{z})$, is

$$N(p; \mathbf{z}) = \max_{c,I} c + pI \text{ s.t. equation 4.} \tag{5}$$

By construction, we have $G(\mathbf{z}) = N(1; \mathbf{z})$ for all σ, a and $\mathbf{z}$ (See Lemma 4 in Appendix A, where we collect technical details and proofs.) Figure 1 illustrates the model, using $G = 1$, $\sigma = 1.5$ and $a = 2.08$. A larger σ flattens the PPF and a larger G causes a radial expansion of the PPF.

Because the consumption good is the numeraire, the nominal and real factor prices are equal. The following lemma shows the effect of the output price on world income, investment, and factor returns.

Lemma 1 *Suppose that Assumption 1.i holds and that the joint outputs c, I*

[5] Powell and Gruen (1968) estimate supply functions that are approximately consistent with a CET PPF. The function $G(\mathbf{z})$ plays a role analogous to a fixed input, such as land, that appears in Computable General Equilibrium Models that use CET (van der Mensbrugghe and Peters, 2016). In models with monopolistic competition and Pareto distributions, the CET PPF arises endogenously (Feenstra, 2010; Constinot et al., 2016).

[6] This construction is an alternative to familiar two-sector models such as Ricardo-Viner or Heckscher-Ohlin-Samuelson, but it is much easier to work with. It also implies that the ratio of factor prices does not depend on current policy, a feature shared by the composite commodity model; thus, we avoid introducing an extraneous consideration: variable relative factor prices. The model has a cost-of-adjustment interpretation, but at the macro level. It also produces the pure endowment economy as a second limiting case, as $\sigma \to 0$.

satisfy equation 4. (i) For $\sigma < \infty$, the (real = nominal) factor prices are

$$w = \left(\tfrac{G}{(1+a^{-\sigma})c}\right)^{\frac{1}{\sigma}} G_L = \phi(p) G_L \text{ and } r = \left(\tfrac{G}{(1+a^{-\sigma})c}\right)^{\frac{1}{\sigma}} G_K = \phi(p) G_K$$
$$\text{with } \phi(p) \equiv \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{1}{\sigma+1}}. \tag{6}$$

The supply function for investment is

$$I = x(p) G(\mathbf{z}),\ x(p) \equiv \left(\frac{(1+a^{\sigma})^{-\frac{1}{\sigma}}}{a^{\sigma}p^{-(1+\sigma)}+1}\right)^{\frac{\sigma}{1+\sigma}},\ \text{with } x'(p) > 0 \tag{7}$$

(ii) For $\sigma < \infty$, an increase in the asset price p, holding $\mathbf{z}$ fixed, increases income and returns to factors

$$\frac{\partial N}{\partial p} = I > 0,\ \frac{\partial w}{\partial p} = \frac{wp^{\sigma}}{a^{\sigma}+p^{1+\sigma}} > 0 \text{ and } \frac{\partial r}{\partial p} = \frac{rp^{\sigma}}{a^{\sigma}+p^{1+\sigma}} > 0. \tag{8}$$

(iii) Assuming that the equilibrium $c > 0$, in the limit as $\sigma \to \infty$,

$$w = G_L \text{ and } r = G_K. \tag{9}$$

An increase in p draws factors of production from the consumption good sector, increasing factors' marginal product there. With the consumption good as the numeraire, the increase in the factors' marginal product also increases the value of their marginal product, thus increasing w and r (equation 8).

Abatement, μ, has two types of equilibrium effects. The "cost effect" arises because abatement forces the economy to use less polluting and therefore more expensive production methods, thus reducing $G(z)$. By Assumption 1 (ii), the cost effect of the first unit of abatement is zero. A "General Equilibrium (GE) effect" arises if abatement changes the asset price, thereby altering national income and factor payments. By equation 8, the GE effect of the first unit of abatement is proportional to $\frac{\partial p}{\partial \mu}_{|\mu=0}$.

With multiplicatively separable $G(z)$, the CET PPF – like the composite commodity model – implies that the relative factor price, $\frac{w}{r}$, is independent of abatement. However, by fixing $p = 1$, the composite commodity IAMs also eliminate the GE effect. Lemma 1 (iii) confirms that the CES PPF model contains the composite commodity model as a limiting case, $\sigma \to \infty$.

The equations of motion for the states (E, K) complete the description of the technology. With zero abatement, Business as Usual (BAU) emissions are an increasing continuously differentiable function of capital and labor, $\zeta F(K_t, L)$, with $\zeta > 0$ a scaling parameter. With the abatement rate μ_t, actual emissions equal $(1 - \mu_t)\,\zeta F(K_t, L)$. With constant decay rates δ for capital and ϵ for atmospheric carbon, the transition equations for the stock of atmospheric carbon and capital are[7]

$$
\begin{gathered}
E_{t+1} = (1-\epsilon)E_t + (1-\mu_t)\,\zeta F(K_t, L) \\
\text{and} \\
K_{t+1} = (1-\delta)K_t + I_t, \text{ with } E_0 \text{ and } K_0 \text{ given.}
\end{gathered}
\tag{10}
$$

2.3 Equilibrium and preferences

Section 4 presents the political economy setting that determines climate policy, the sequence of emissions standards, $\{\mu_{t+h}\}_{h=0}^{H-t}$. For the time being, we take this policy sequence as given, and define the conditional equilibrium under the assumption of positive investment in every period:

Definition 1 *A competitive equilibrium at t, with initial condition K_t and E_t, conditional on $\{\mu_{t+h}\}_{h=0}^{H-t}$, is a sequence of the carbon and capital stocks and asset price, $\{E_{t+h}, K_{t+h}, p_{t+h}\}_{h=0}^{H-t}$, satisfying: the asset market equilibrium (3) implied by the young agents' savings decision; the factor price conditions in equation (6); and the transition equations (10).*

Agents have constant elasticity single period utility, $U(c) = \frac{c^{1-\eta}-1}{1-\eta}$, with $\eta \geq 0$; η is the inverse of the elasticity of intertemporal substitution (EIS).

[7]Equation 41 in Appendix B.1 gives the formula for the carbon tax that supports a given level of μ. This tax has a slightly different appearance than the standard tax for two reasons. First, equation 10 assumes that unabated emissions are proportional to output $F(K, L)$, not to output net of damages, $D(E)F(K, L)$. Second, we have to take into account the asset price, p, which affects the value of output.

For $\eta = 1$, $U(c) = \ln c$. The old generation's welfare, Ω_t^o, equals its utility while old; the young generation's welfare, Ω_t^y, equals the discounted stream of utility in the current and the next period. Using the equilibrium savings rule, we have the following expressions for welfare.

Lemma 2 *For a given climate policy, equilibrium lifetime welfare in period t is Ω_t^y for the young agent and Ω_t^o for the old agent, with*

$$\Omega_t^y \equiv U(c_t^y) + \rho\, U(c_{t+1}^o) = \begin{cases} \frac{\left(c_t^y\right)^{-\eta}}{1-\eta} w_t - \frac{1}{1-\eta}\left(1+\rho\right) & \textit{for } \eta \neq 1 \\ \\ (1+\rho)\ln w_t - \ln(1+\rho) & \textit{for } \eta = 1, \end{cases} \tag{11}$$

and

$$\Omega_t^o \equiv U(c_t^o) = \begin{cases} \frac{\left[(r_t+(1-\delta)p_t)K_t\right]^{1-\eta}-1}{1-\eta} & \textit{for } \eta \neq 1 \\ \\ \ln\left[(r_t + (1-\delta)p_t)K_t\right] & \textit{for } \eta = 1. \end{cases} \tag{12}$$

3 Welfare effects of climate policy

Climate policy today diverts resources from current consumption and investment and reduces future carbon stocks. Beyond this feature, climate policy has qualitatively different effects under fixed versus endogenous asset prices.

With a fixed asset price (a composite commodity), climate policy harms the current old agent by reducing the current return to capital. Those agents have no selfish incentive to reduce emissions. Future agents benefit from a small level of abatement when the value of the reduction in the carbon stock exceeds the cost of a lower capital stock. The young agent suffers from the policy-induced reduction in consumption, but may benefit from a more productive future economy; the net effect of policy on this agent's welfare is ambiguous.

Matters are more complicated when the asset price is endogenous. By changing both the supply and demand for investment, policy can change the asset price. We first consider the current generations' alignment of incentives to abate. We then examine the price and welfare effects of policy when preferences are linear ($\eta = 0$), logarithmic ($\eta = 1$), or Leontieff ($\eta = \infty$).

3.1 The alignment of agents' incentives to abate

We consider the equilibrium welfare effect of a small level of current abatement ($\mu > 0$, $\mu \approx 0$), holding future abatement levels fixed.[8] The first unit of abatement has the same qualitative effect on the two generations' welfare if and only if $\eta < 1$. Their incentives are opposed if $\eta > 1$. Abatement raises the old agent's welfare if and only if it increases the asset price.

Proposition 1 *Assume that future abatement levels are fixed and Assumption 1 (i) and (ii) hold. (i) A small level of current abatement increases the old generation's welfare if and only if this policy raises the asset price:*

$$\frac{d\Omega_t^o}{d\mu}_{|\mu=0} > 0 \Leftrightarrow \frac{dp_t}{d\mu}_{|\mu=0} > 0.$$

(ii) For $\eta \neq 1$, welfare of the old and the young generations change in the same direction, due to a small level of abatement, if and only if $\eta < 1$:

$$\frac{d\Omega_t^y}{d\mu}_{|\mu=0} > 0 \Leftrightarrow (1-\eta)\,\frac{dp_t}{d\mu}_{|\mu=0} > 0.$$

The old agent benefits from a small level of abatement that increases the asset price, *even if she has no capital to sell at the end of the period* (i.e. if $\delta = 1$). If the first unit of abatement raises the asset price, it also raises the return to capital (equation **??**), benefiting the old agent (equation 12); if $\delta < 1$ the higher price increases the value of the old agent's end-of-period undepreciated assets, raising the agent's welfare. These two effects are both absent in the composite commodity framework.

Figure 2 provides intuition for Proposition 1 (ii). The line through point A with slope $-\psi_t$ graphs the budget constraint, $c_{t+1}^o = \psi_t\,(w_t - c_t^y)$, implied

[8] We approximate the welfare effect in the usual manner, using the first-order term of the Taylor expansion of welfare around $\mu = 0$. Our results do not change if, instead of considering the perturbation of only current policy, we consider the perturbation of a sequence of abatement levels. In that case, we begin with $\bar{\mu} \in \mathbb{R}^{H-t}$, $\bar{\mu}_h \geq 0$ and denote the sequence of climate policy as $\varepsilon\bar{\mu}$. The comparative statics is then with respect to ε, evaluated at $\varepsilon = 0$. Section 4 endogenizes the abatement decision, recognizing that future policy responds to current policy via changes in the state variable.

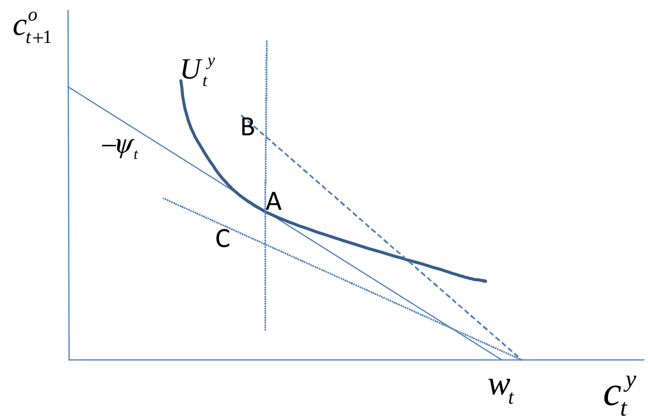

Figure 2: The initial equilibrium, with $\mu_t = 0$, is at point A on the indifference curve U_t^y; the slope of the budget constraint is $-\psi_t$ and the horizontal intercept is w_t. A small level of abatement, $\mu > 0$ that increases p_t raises w_t and decreases c_t^y. These changes are consistent with a movement of the consumption point toward B, and an increase in U_t^y, if and only if $\eta < 1$. They are consistent with a movement of the consumption point toward C, and a decrease in U_t^y, if and only if $\eta > 1$.

by the two lines in equation 1. At the initial equilibrium, with $\mu = 0$, the young agent at t consumes at point A and has lifetime welfare shown by the indifference curve U_t^y. Suppose that a small level of abatement increases p_t. In this case, w_t increases (by equation **??**), as represented by the rightward shift of the budget constraint's horizontal intercept, w_t. The increase in p raises the old agent's utility, and therefore must increase her consumption. The higher price also causes the production point to move down the PPF, leading to lower aggregate production of the consumption good. Market clearing therefore requires that c_t^y falls, e.g. from point A toward B (with higher welfare) or toward C (with lower welfare).

If the consumption point moves toward B, then the young agent's welfare increases. The increase in welfare combined with the decrease in c_t^y means that the budget constraint must have rotated clockwise, as shown by the line through point B, raising the opportunity cost of c_t^y. (Otherwise both the income and the substitution effect would have worked in the same direction, leading to an increase in c_t^y.) Therefore, the income and the substitution effects move in the opposite direction, and the substitution effect dominates. With iso-elastic utility, the substitution effect dominates the income effect if and only if $\eta < 1$, in line with Proposition 1.

If the consumption point moves toward point C, then the young agent's welfare decreases. The decrease in welfare implies that ψ_t has fallen. (If the abatement increased ψ_t, then welfare would have risen, because the higher p_t raises w_t.) The lower ψ_t encourages higher c_t^y via the substitution effect; the lower welfare encourages lower c_t^y via the income effect. If consumption moves toward point C then the income effect must dominate the substitution effect. Therefore, it must be the case that $\eta > 1$.

3.2 Special cases

We consider the effect of abatement where $\eta \in \{0, 1, \infty\}$: linear, logarithmic, and Leontieff preferences. We use the two-period setting ($H = 1$) for linear and Leontieff preferences, maintaining the $H \geq 1$-period setting under logarithmic preferences.

- For linear preferences, the first unit of abatement increases the equilibrium price and investment level, increasing welfare for all agents.

- For logarithmic preferences, the first unit of abatement has no effect on the equilibrium price or the current generations' welfare, but increases welfare for the agent born in the next period. A non-marginal level of abatement lowers the current price, current investment, and welfare for both the current old and the current young. It has an ambiguous effect on the welfare of the generation born in the next period.

- For Leontieff preferences, the first unit of abatement harms the current old generation, benefits the current young generation, and has an ambiguous effect on the welfare of the generation born in the next period.

The ambiguous welfare effect for the generation born in the next period occurs when abatement lowers the asset price. In these cases, the next generation benefits from a lower stock of GHGs and is harmed by a lower stock of capital.

3.2.1 Linear preferences

We emphasize the the two-period model ($H = 1$), where abatement increases welfare for all agents, the two currently living agents and the agent born in period 1. This result requires an endogenous asset price. In the limiting case $\sigma \rightarrow \infty$, where we obtain the composite commodity model and a fixed asset price, the first unit of abatement creates zero welfare effects for all agents; however, a strictly positive level of abatement lowers the welfare of the agents alive in period 0, and it has no effect on the welfare of the agent born in period 1. In the composite commodity framework, an endogenous reduction in investment offsets any reduction in the stock of GHGs, leaving the next-period wage unchanged. Thus, the endogenous asset price (which requires $\sigma < \infty$) creates the self-interested incentive to reduce emissions.

Results are not as crisp for the infinite horizon model, but that setting provides a simple means of showing that the endogeneity of the asset prices induces selfish agents to internalize some of the benefit of climate policy. The selfish agents act to protect the value of their asset, and in the process they benefit agents born in the future.

In the two-period model ($H = 1$), the agent born in the last period lives a single period, and consumes all its income. Equilibrium investment and abatement are both zero in the last period.

Proposition 2 *Suppose that Assumption 1 holds, with $\eta = 0$ and $H = 1$. (i) For $\sigma < \infty$, a small level of abatement in period 0 improves the welfare of all agents. (ii) For $\sigma = \infty$ (the composite commodity setting), period 0 abatement lowers the welfare of agents in period 0 and has no effect on the welfare of the agent born in period 1.*

The key to Part (i) of the proposition is the demonstration that a small level of abatement increases the period 0 asset price. By Proposition 1, the higher asset price raises the welfare of both the young and the old in period 0. By Lemma 1, equation 7, the increase in asset price increases investment. Therefore, abatement lowers the carbon stock and increases the capital stock in period 1, increasing the wage and thus increasing the welfare of the agent born in $t = 1$. In this setting, abatement induces the young agent at $t = 0$ to consume less and invest more. The higher investment benefits the old generation by increasing the asset price, and it benefits the agent born in the next generation – as do the reduced emissions.

Proposition 2.ii highlights the role of an endogenous asset price. For $\sigma = \infty$ we have the composite commodity framework, where the asset price is fixed at 1 in an interior equilibrium. The asset pricing equation here implies $p_0 = 1 = \rho r_1 = \rho G_K(K_1, E_1)$. This identity implies that investment changes to offset any abatement-induced change in the next-period stock of GHGs, leaving the period 1 return to capital unchanged. The combined change in the stocks of capital and GHGs also leaves the period 1 wage unchanged.

Thus, in the composite commodity setting ($\sigma = \infty$) when $\eta = 0$, the first unit of abatement creates a zero first order welfare effect for all agents, despite the fact that it leads to a positive first order reduction in the stock of carbon in the next period. A non-negligible level of abatement creates a welfare loss for the agents alive in period 0, because it reduces the wage and rental rate. However, this abatement has no effect on welfare of the agent born in the next period, because the loss due to reduced investment offsets the gain due to reduced GHGs. Thus, with $\sigma = \infty$, zero abatement is constrained Pareto efficient. With a concave PPF ($\sigma < \infty$), in contrast, a small level of abatement in period 0 *increases* equilibrium investment, while still reducing the next-period stock of GHGs. A non-trivial asset market creates non-trivial equilibrium effects.

We now consider the case $H = \infty$, where equation 2 implies $\psi_t = \frac{1}{\rho}$, so

$$p_t = \rho\left(r_{t+1} + p_{t+1}(1-\delta)\right) \Rightarrow p_t = \sum_{j=0}^{H} \left(\rho\left(1-\delta\right)\right)^j r_{t+1+j};$$

the current asset price equals the discounted stream of future rental rates, adjusted for depreciation. The current generations' joint welfare is

$$\Omega_t^o + \Omega_t^y = \left(r_t + (1-\delta)\, p_t\right) K_t + w_t,$$

equal to the value of world income, $N_t = r_t K_t + w_t$, plus wealth (the value of undepreciated end-of-period capital), $(1-\delta)\, p_t K_t$. A utilitarian planner's welfare criterion is

$$\Omega^{\text{planner}} \equiv \left(r_t + (1-\delta)\, p_t\right) K_t + w_t + \left(\sum_{j=0}^{H-1} \rho^{j+1} w_{t+j+1}\right),$$

the welfare of currently living generations plus the discounted stream of unborn generations' welfare (using equation 11 and ignoring the constant $1+\rho$). Both the selfish agents and the utilitarian planner care about the future utility stream incorporated into the value of the undepreciated capital stock; the planner also cares about the future stream of labor income.

For illustration, suppose that the per-period growth rate of the economy, g and labor's share, β, are fixed, so $w_{t+j} = \beta(1+g)^j N_t$. Define $\chi \equiv \frac{(1-\delta)p_t K_t}{N_t}$, the value of wealth as a share of world income in the current period, and let $H \to \infty$.[9] Denote as τ the fraction of the future stream of utility included in the selfish agents' welfare criterion:

$$\tau\left(\chi; \rho, \beta, g\right) \equiv \frac{(1-\delta)p_t K_t}{(1-\delta)p_t K_t + \sum_{j=0}^{H-1} \rho^{j+1} w_{t+j+1} = \frac{\chi}{\chi + \frac{(1+g)\rho\beta}{1-(1+g)\rho}}}.$$

If labor's share of income is $\beta = 0.6$, a period lasts for 35 years, the annual PRTP is 1%, and the annual growth rate is 0.5%, then $\rho = 0.7$, $g = 0.2$, and $\frac{(1+g)\rho\beta}{1-(1+g)\rho} = 3$. Thus, if wealth, $(1-\delta)\, p_t K_t$, is similar in magnitude to world output ($\chi \approx 1$), then $\tau \approx 0.25$. For this example, the asset market induces selfish agents to internalize a significant fraction of the effect of climate policy on aggregate welfare.

[9] Our calibration sections claims that the 2010 world stock of capital is approximately 200 USD ($T) and that *annual* output was roughly 64 ($T). With an annual depreciation rate of 6%, the value of χ at an *annual* scale is $\frac{(0.94)200}{63} \approx 3$. It is not clear how we should convert this to a 35 year period. I am pretty sure that it would not make sense to maintain the current stock estimate and merely change income and depreciation, which would lead to an estimate of $\frac{(0.12)200}{35(63)} \approx 0.01$.

3.2.2 Logarithmic preferences

For $\eta = 1$, zero abatement maximizes the currently living agents' welfare, regardless of future agents' actions. A non-negligible level of abatement lowers the stocks of both capital and GHGs inherited by the generation born in the next period, resulting in an ambiguous welfare change for that agent and a welfare loss for currently living agents. The first (marginal) unit of abatement has a zero first order effect on current generations' welfare and savings decision. However, by leaving the next generation with a lower stock of GHGs and an unchanged stock of capital, it improves that generation's welfare. In this setting, current generations would want to choose zero abatement, but a small level of abatement increases aggregate welfare.

Proposition 3 *Suppose that $\eta = 1$ and Assumption 1 holds. (i) There exists a unique stable equilibrium price $p(\mathbf{z})$ that is independent of future abatement policies. (ii) The marginal unit of abatement at $\mu = 0$ has zero first order effect on the asset price and welfare of both the young and the old agents alive at $t = 0$. This abatement creates a first order reduction in next-period stock of GHGs, without reducing the inherited stock of capital, thereby benefiting the agent born in the next period. (iii) For $\mu > 0$, an increase in abatement lowers welfare of the agents alive at $t = 0$, lowers the next-period stock of capital, and weakly lowers the asset price.*

For $\eta = 1$, young agents save a constant fraction of their income. The young agent's demand for physical capital depends on the asset price and labor income, but not on future abatement decisions.

The intuition for Part (ii) uses the fact that the first unit of abatement has zero first order effect on the PPF. Therefore, any abatement-induced change occurs because of a movement along the original PPF. If this abatement were to increase p, it would increase w, thereby increasing both agents' consumption in period 0. However, a higher p causes the production point to move south-east on the (original) PPF, reducing supply of the consumption good. Thus, a higher p is not consistent with market clearing for the consumption good. A parallel argument establishes that a lower p is not consistent with market clearing. Therefore, the first order effect of abatement on p is zero. The marginal reduction in current emissions reduces the next-period stock of carbon, without altering the next-period capital stock (because p is unchanged). The small level of current abatement consequently leads to a first order reduction in the next-period stock of GHGs, shifting out the PPF, raising the wage and welfare for the agent born in $t = 1$.

As is true for all η, an increase in abatement beginning at $\mu > 0$ leads to a first order inward shift in the PPF. For $\eta = 1$, the higher abatement weakly lowers the equilibrium asset price, leading to strict welfare loss for currently living agents and a reduction in savings.

3.2.3 Leontieff preferences

In the limit as $\eta \to \infty$, $U^y \to \min\left(c_t^y, \frac{c_{t+1}^o}{\rho}\right)$. The young agent saves to the point where $c_t^y = \frac{c_{t+1}^o}{\rho}$, implying the asset price equation

$$p_t = \frac{w_t}{K_{t+1}} - \frac{\left(r_{t+1} + (1-\delta)\, p_{t+1}\right)}{\rho}. \tag{13}$$

Defining $\varepsilon_{D,E} = \frac{D'(E)E}{D(E)}$, the elasticity of the damage function, we obtain

Proposition 4 *Under Assumption 1 and for $H = 1$ (the two-period case) The first unit of abatement in period 0 lowers the asset price, lowering welfare for the old agent alive in that period, lowering investment, and increasing welfare of the young agent. Abatement raises welfare for the agent born in the next period if and only if $\varepsilon_{D,E}\frac{1}{E_1}\frac{dE_1}{d\mu_0} > -\frac{1}{K_1}\frac{dK_1}{d\mu_0}$.*

The inequality in the last line means that the increase in next-period output arising from the lower stock of carbon exceeds the loss in output arising from a lower stock of capital. This inequality holds if it is efficient to reduce emissions.

It is instructive to compare results (for $H = 1$) in the models with a non-trivial asset market ($\sigma < \infty$) and the standard composite commodity ($\sigma = \infty$). With $\sigma = \infty$, the first unit of abatement creates a zero first order change in the $t = 0$ wage and return to capital (w_0 and r_0). However, the abatement creates a first order reduction in the next-period stock of GHGs, leading to a first order increase in next-period return to capital, r_1. To maintain the indifference relation $c_t^y = \frac{c_{t+1}^o}{\rho}$, the young agent at $t = 0$ increases consumption, c_0^y, saving a bit less, and moderating the increase in r_1. However, because c_0^y has increased, so has $r_1 K_1$. Because the two factor prices move in the same direction, w_1 also increases. Here, the current young and the agent born in the next period both obtain a first order increase in welfare, and the current old have a zero first order change, following the marginal unit of abatement.

In contrast, with $\sigma < \infty$, we saw that the marginal unit of abatement creates a first order welfare loss to the current old generation, due to the fall in the asset price. Here, the young agent needs a smaller (compared to the case $\sigma = \infty$) reduction in current savings to restore the indifference relation $c_t^y = \frac{c_{t+1}^o}{\rho}$. Thus, the first unit of abatement creates a smaller reduction in K_1 for $\sigma < \infty$ compared to $\sigma = \infty$. Abatement transfers welfare from the current old generation to the current young and to the generation born in the next period. The endogenous asset price increases the conflict between currently living generations' incentives to undertake climate policy.

The two-period model with low EIS shows how beliefs about future policies can affect current equilibrium policies. Suppose that a political economy equilibrium causes currently living generations to choose current climate policy to maximize a convex combination of their welfare. (Section 4 develops this idea.) Suppose also that the young generation has enough political power to produce a positive level of abatement at $t = 0$. Now add a previous period, $t = -1$, to this two-period model. The generation that is young at $t = -1$ understands that abatement will be positive at $t = 0$. That abatement benefits the young generation at $t = 0$, but it harms the old generation, i.e. it reduces c_0^o. Welfare for the young generation at $t = -1$ is $\min\left(c_{-1}^y, \frac{c_0^o}{\rho}\right)$. The reduction in c_0^o caused by the anticipated abatement in that period makes the young agent at $t = -1$ willing to transfer consumption from $t = -1$ to $t = 0$. The agent can make this transfer by saving a bit more, but abatement produces a more efficient transfer. Abating is not only more efficient in the standard sense of the word, but it also shifts some of the costs on to the old agent at $t = -1$.

Thus, in a world where the young agents have significant political power, the anticipation that abatement will be positive in the future, together with a low EIS, results in equilibrium abatement at $t = -1$.

4 Equilibrium abatement

We use a numerical model to study equilibrium policy when the asset price is endogenous. The equilibrium level of abatement is the solution to a dynamic game. In each period, abatement is chosen to maximize a convex combination of young and old agents' consumption-related welfare, $\xi\Omega_t^y + (1-\xi)\Omega_t^o$, with $0 \leq \xi \leq 1$.[10] The parameter ξ measures the influence of the young generation in the decision-making process. We refer to the fictitious agent who maximizes this function as the *planner* (as distinct from the *discounted utilitarian*).

The sequence of planners play a dynamic game. Apart from the logarithmic case (Proposition 3) planner t's optimal policy depends on future policies, via their effect on the asset price. The equilibrium abatement policies are not first-best; therefore, equilibrium levels of investment are also not efficient. Although the planner at t might want to change savings, we do not want a model in which climate policy is used to influence investment. Therefore we assume that planners take the level of investment as given:

Assumption 2 *(Nash) Planners take the current investment decision as given in choosing current abatement. Agents take prices and abatement policy as given in choosing investment.*

Planners understand that current abatement shifts the PPF inward. At a given asset price, abatement reduces the factor prices, w_t and r_t, because it increases production costs. The planner also understands that abatement can alter the asset price, changing the value of undepreciated assets (wealth) and

[10] There are several ways to motivate this criterion, e.g. using a probabilistic voting model in which voters care about their consumption-related welfare and about ideology (Lindbeck and Weibull, 1987; Persson and Tabellin, 2000). However, neither the microfoundations, nor the manner or implementing the policy (e.g. a tax or a quota) matter for our purposes.

further altering factor prices. However, by Assumption 2, the planner takes as given the equilibrium point on the investment supply function, $I = x(p)\,G(\mathbf{z}_t)$, rather than behaving as a monopsonist with respect to this supply function.

The directly payoff-relevant state variable is the triple (K, E, t); t picks up exogenous TFP and population growth. We study a Markov Perfect Equilibrium (MPE), where current policies and expectations concerning future policies are functions of the current state variable.[11] Equilibrium policies from periods $t+1$ onward induce an equilibrium price function, denoted $p_{t+1} = \Psi(K_{t+1}, E_{t+1}, t+1)$. The H-period model, $H < \infty$, uses the boundary condition $p_H = 0$; in the final period, abatement and investment are both 0.[12]

The planner at t understands that current abatement affects the stock of GHGs, changing both the next-period asset price and the rental rate. From the asset pricing equation 3, the planner therefore understands that current abatement potentially affects the current asset price, changing both the old agent's end-of-period wealth, $p_t(1-\delta)K_t$, and current factor prices via the general equilibrium effect (equation 6). Abatement also shifts in the PPF.

The planner chooses μ_t to maximize the convex combination of currently living agent's welfare, resulting in the equilibrium policy function, $M(K_, E, t)$:

$$M(K, E, t) = \underset{\mu_t}{\operatorname{argmax}}\, \xi\; L_t\; \Omega_t^y + (1-\xi)\; L_{t-1}\; \Omega_t^o, \tag{14}$$

using the definitions of welfare in equations 11 and 12, the factor prices in equation 6, the asset price equation 3, and the equations of motion 10. The investment supply function, equation 7, closes the model. Substituting the

[11] Hassler et al. (2003) and Conde-Ruiz and Galasso (2005) are early applications of MPE applied to games involving public goods. In our model, the public good is Earth's capacity to absorb carbon emissions.

[12] The finite horizon setting avoids the problem of the "incomplete transversality condition", a familiar source of multiplicity. We find no evidence of multiplicity in our finite horizon model. We choose $H = 14$ (490 years), which is long enough that a change in H has no discernible effect on policies in the first 200 years. We report equilibrium trajectories for the first 8 periods (280 years), long enough to capture the important dynamics, and short enough to avoid strong influence due to the approaching terminal time.

price and abatement functions into this equation gives equilibrium investment

$$I_t = x\left(\Psi\left(K_t, E_t, t\right)\right) G\left(K_t, L_t E_t, M\left(K_t, E_t, t\right), t\right). \tag{15}$$

We obtain numerical solutions to the equilibrium problem using the algorithm described in Appendix C.

Two benchmarks We compare the political economy equilibrium with two benchmarks. Under Business as Usual (BAU), abatement is zero in every period. The discounted utilitarian (DU) chooses investment and abatement to maximize the discounted sum of the welfare of aggregate consumption, using agents' pure rate of time preference: $\sum_{s=0}^{\infty} \rho^s U\left(c_t^y + c_t^o\right)$, with $c_t^y + c_t^0 = c_t$. The DU's dynamic programming equation is

$$J\left(K_t, E_t, t\right) = \max_{I_t, \mu_t} \left[P_t U\left(\frac{C_t}{P_t}\right) + \rho J\left(K_{t+1}, E_{t+1}, t+1\right)\right] \tag{16}$$

subject to transition equations (10); P_t is the population at t and C_t is aggregate consumption. As in the standard infinitely lived representative agent IAM, the resulting decision rules, $\mu_t = M\left(K_t, E_t, t\right)$ and $I_t = I\left(K_t, E_t, t\right)$, are first-best. The only difference here is that the PPF is concave.

4.1 Calibration

We use DICE-2016R (Nordhaus, 2017) to calibrate most of our model. Our baseline implies that BAU climate damages are small in the near future, eventually becoming large but never posing an existential threat. The calibration is also optimistic about technology, building in the assumption that eventually it will be inexpensive to undo damages caused by previous emissions.

These assumptions encourage purely selfish agents to defer, or perhaps never to undertake, policies that reduce climate change. Therefore, the fact that we find that in most equilibria selfish agents do undertake meaningful (but still inadequate) policy, cannot be ascribed to our having exaggerated the severity of the climate problem or the low cost (today) of ameliorating it. Our baseline uses moderate rates of capital depreciation and it gives each currently living generation equal influence in the decision-making process, $\xi = 0.5$

Assumption 3 summarizes the model's functional forms:

Assumption 3 *(Functional forms)* $G(K, E, t) = D(E)\Lambda(\mu)F(K, L, t)$ *with* $F(K_t, L_t, t) = K_t^{1-\beta}(A_t\ L_t)^{\beta}$ *and* $0 < \beta < 1$*;* $\Lambda(\mu_t) = 1 - \nu_{1,t}\,\mu_t^{\nu_2}$ *with* $\nu_2 > 1 > \nu_{1,0} > 0$*; and* $D(E_t)) = (1 + \iota\ E_t^2)^{-1}$ *with* $\iota > 0$.

Labor obtains the constant output share β. We replace physical capital K_t with capital in efficiency units $k_t = \frac{K_t}{A_t L_t}$, using exogenously changing L_t and A_t (Appendices B.2 and B.3). Reducing emissions to 0 ($\mu = 1$) reduces output by the fraction $\nu_{1,t}$, a decreasing function of time to reflect improved abatement technologies; ν_2 is the elasticity of abatement costs.

Table 4.1 collects parameter names and baseline values. The table shows the initial value of time-varying parameters; Appendix B.2 describes their exogenous dynamics, and also collects other information about our calibration. Agents live for 70 years, and one period lasts 35 years. The baseline elasticity of intertemporal substitution is 0.5, so $\eta = 2$, a conventional choice for IAMs. Agents discount future utility at $1\%/yr$, implying $\rho = 0.7$.

$\rho = 0.7$ discount factor	$\eta = 2$ inverse IES	$\beta = 0.6$ labor share	$\delta = 0.88$ capital depreciation
$\zeta_0 = 0.0957$ carbon intensity	$TCRE = 0.002$ transient climate response	$\iota = 9.44(10^{-9})$ damage parameter	$A_0 = 4,748$ initial labour productivity
$\sigma = 1.3210$ PPF elasticity	$a = 2.3636$ PPF shape parameter	$\nu_{1,0} = 0.074$, full abatement cost share in 2015	$\nu_2 = 2.6$ abatement cost elasticity

Table 4.1 Parameter names and baseline values

We scale nominal units by 10^{12} 2010 USD (Trillion). Capital stock, K_0, in 2015 is 223 $\$T$. With annual world output of 105.5 $\$T$, output during the first

35-year period is 35×105.5 $\$T \cong 3,692.5$ $\$T$ (Nordhaus, 2017). We calibrate the initial old and young population so that the total initial population equals 7.4 billion, and we fix the population dynamics so that the growth rate of the young population equals DICE's growth rate of total population. The stock of labor in our model is the population of the young, and in DICE it is the total population (Appendix B.2). Given the initial endowments of output, capital, and labor $L_0 = 5.1$, and using $\beta = 0.6$, initial labour productivity is calibrated to $A_0 = 4748$.[13] We set capital depreciation to $6\%/yr$ (implying $\delta = 0.88$), above the mean of $4\%/yr$ for 2010 of the Penn World Table and below the $10\%/yr$ used in DICE-2016R.

Units of the carbon stock, E, are GtC. By 2015, cumulative emissions since the pre-industrial period were 571 GtC (Allen et al., 2009), with 2015 emissions of 10.1 GtC (Nordhaus, 2017). Initial carbon intensity measured in GtC per $\$T$, ζ_0, is $\frac{10.1}{105.5} = 0.0957$. Following DICE, we impose a ceiling of 6000 GtC on cumulative emissions. Emissions in period t equal $e_t \equiv \zeta_t \left(1 - \mu_t\right) A_t K_t^{1-\beta} L_t^{\beta}$.[14] Period t emissions increase E_{t+1}, increasing next-period damages.[15]

We calibrate the damage parameter ι using the Nordhaus (2014) damage function and the Nordhaus (2017) estimate that a 2^oC temperature anomaly reduces output by 0.94%. We use the Transient Response to Cumulative Emissions model to convert a 2^oC temperature increase into cumulative emissions of 1000 GtC.[16] Our damage function implies a 0.94% output loss at $E = 1000$ (thus matching DICE), and an 8% loss at $E = 3000$. DICE estimates a 9^oC temperature change under BAU, where all 6000 GtC available units are eventually used, implying a 19% reduction in output. Our calibration implies asymptotic BAU damages of 25%, higher than in DICE, but still optimistic.

[13]With $\beta = 0.6$ and $\rho = 0.7$, the savings rate under log utility is 0.25. We do not calibrate on the savings rate, so different parameters produce different rates.

[14]DICE assumes that $e_t \equiv \zeta_t D(E_t) \left(1 - \mu_t\right) A_t K_t^{1-\beta} L_t^{\beta}$. There, climate-related damages reduce emissions, by reducing output.

[15]Ricke and Caldeira (2014) estimate that most of the warming effect of current emissions, and thus most of the temperature-related damage, occurs within a decade.

[16]The TRCE model is a linear approximation to a highly nonlinear system, and is not useful for predicting temperature changes corresponding to very high levels of cumulative emissions (IPCC, 2013; Dietz et al., 2021). Appendix B.2 explains our use of this model.

The DICE-2016R abatement cost elasticity is $\nu_2 = 2.6$; $\nu_{1,t}$ measures the share of GDP needed to abate all emissions ($\Lambda(1) = \nu_{1,t}$), based on a backstop technology. The initial backstop cost, $\$550/tCO_2$, declines over time, implying that eliminating emissions would cost 7.4% of output today, 1.0% in 100 years and 0.03% in 300 years. Rapid reductions in mitigation costs delay optimal abatement, leading to a climate "policy ramp" (Nordhaus, 2017).

As in DICE, we allow $\mu > 1$ to reflect the possibility that it eventually becomes possible to remove carbon from the atmosphere, thereby reducing damages. We can also interpret $\mu > 1$ as low-cost and safe geo-engineering. Our baseline uses the upper limit of $\mu = 1.7$. As noted at the beginning of this section, we consider these assumptions to be optimistic about the risks posed by climate change, thus encouraging selfish agents to defer climate policy.

We need the shape and elasticity parameters, a and σ (equation 4), to complete the description of technology. Define S as aggregate investment as a share of output; for 2015, we use the estimate $S = 0.243$.[17] Define κ as the elasticity of supply for the investment good with respect to its price. For our PPF, $\kappa = \sigma(1 - S)$ (equation 21); in addition, at $p = 1$ (the price in the composite commodity setting) the ratio of the equilibrium product mix, $\frac{1-S}{S}$, equals a^{σ} (equation 36). Goolsbee (1998) estimates $\kappa = 1$. With this value and $S = 0.243$, our calibration equations produce $\sigma = 1.321$ and $a = 2.3636$.

4.2 Results

Figures 3 and 4 show trajectories of cumulative emissions and the carbon tax, $\$/tC$, under the different scenarios. (The taxes must be divided by 3.666 to convert to $\$/tCO_2$.) Table 4.2 reports the first-period carbon taxes and abatement levels. These results show that, when the young generation has significant representation in the policy decision, endogenous asset prices give selfish agents the incentive to engage in substantial abatement.

Cumulative emissions for the utilitarian planner remain under 1000 GtC for $\eta = 0.5$ and under 2000 GtC for $\eta = 2$. These and other carbon trajectories

[17] https://data.worldbank.org/indicator/NE.GDI.TOTL.ZS.

are nonmonotonic because of the assumption that abatement can exceed 100%; or geoengineering has an equivalent effect. Cumulative emissions under BAU reach 6000 GtC, the assumed upper limit. Investment, and therefore emissions, are higher for $\eta = 0.5$ compared to $\eta = 2$, so the BAU economy reaches the carbon ceiling earlier for $\eta = 0.5$. In the MPE, cumulative emissions are always lower when the young generation has more influence on policy (ξ is larger).

The lower panel, with $\eta = 2$, provides a striking illustration of the importance of ξ. For $\xi = 0.2$, where the young have little influence, cumulative emissions are nearly at the BAU level. For $\xi = 0.8$, where the young are dominant, cumulative emissions in the MPE are indistinguishable from the utilitarian's level for 160 years; beyond that point, the MPE planner removes carbon more aggressively compared to the utilitarian. When the two generations have equal influence, $\xi = 0.5$, the carbon trajectory is about half way between the BAU and utilitarian levels for the first 175 years, and thereafter remains closer to the utilitarian level.

The tax trajectories in Figure 4 illustrate the familiar DICE-style policy ramp. Apart from the utilitarian with $\eta = 0.5$, the tax trajectories begin quite low.[18] As Table 4.2 shows, the initial MPE policies are much smaller than the utilitarian levels. However, except for the case where $\eta = 2$ and the young have little influence ($\xi = 0.2$) the MPE taxes rise quickly enough to maintain cumulative emissions well below the BAU levels.

[18] The initial utilitarian tax with $\eta = 2$ is much smaller than estimates of the social cost of carbon, e.g. $\$40/tCO_2$. This "discrepancy" likely arises from a combination of our calibration's optimism about technology and the 35 year time time step, which creates a long lag between the costs and benefits of current action. Because our goal is to examine the incentives created by asset markets to undertake climate policy, not to recommend optimal policy, we care about the relation between MPE policy and the discounted utilitarian policy, not the level of either. Therefore, we chose to retain a familiar calibration, rather than adjusting it to make the discounted utilitarian's policy more closely match estimates of the social cost of carbon.

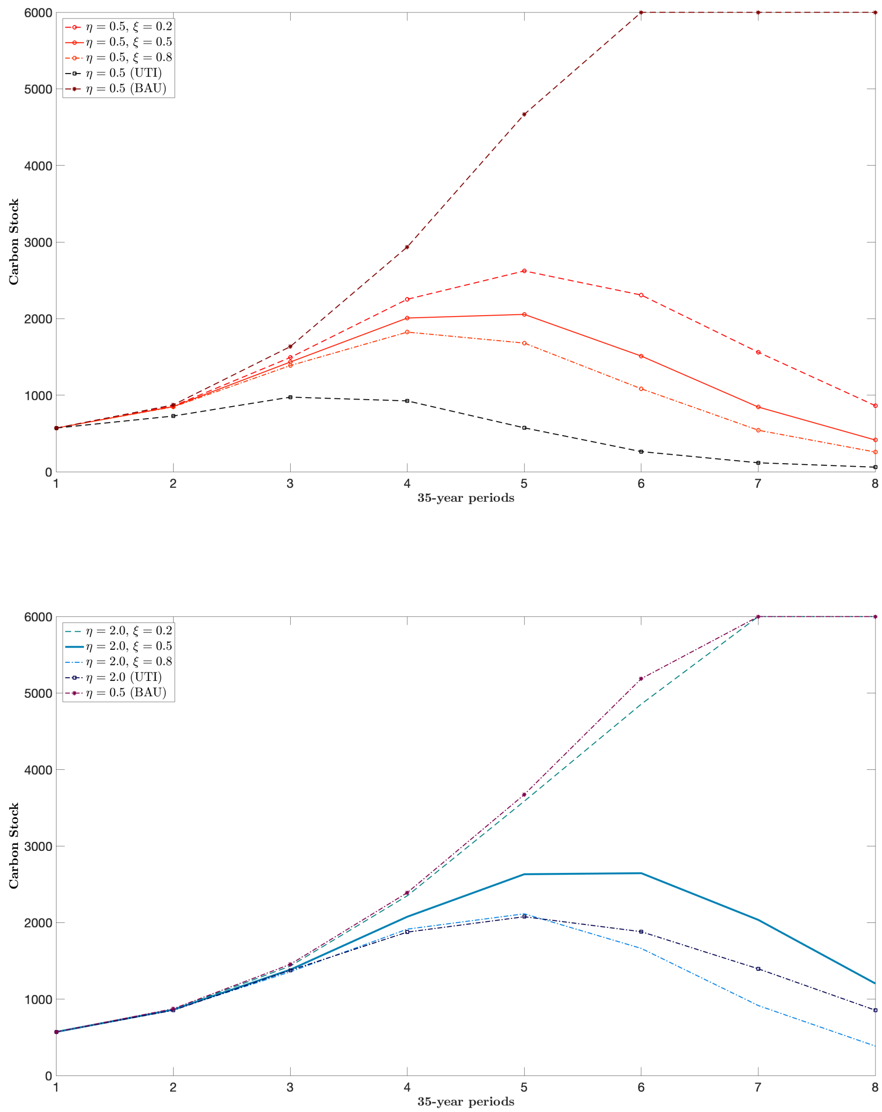

Figure 3: The equilibrium cumulative emissions (carbon stock) for $\eta = 0.5$ (top panel) and $\eta = 2$ (bottom panel), under the utilitarian, BAU, and in the MPE with different influence parameters, ξ.

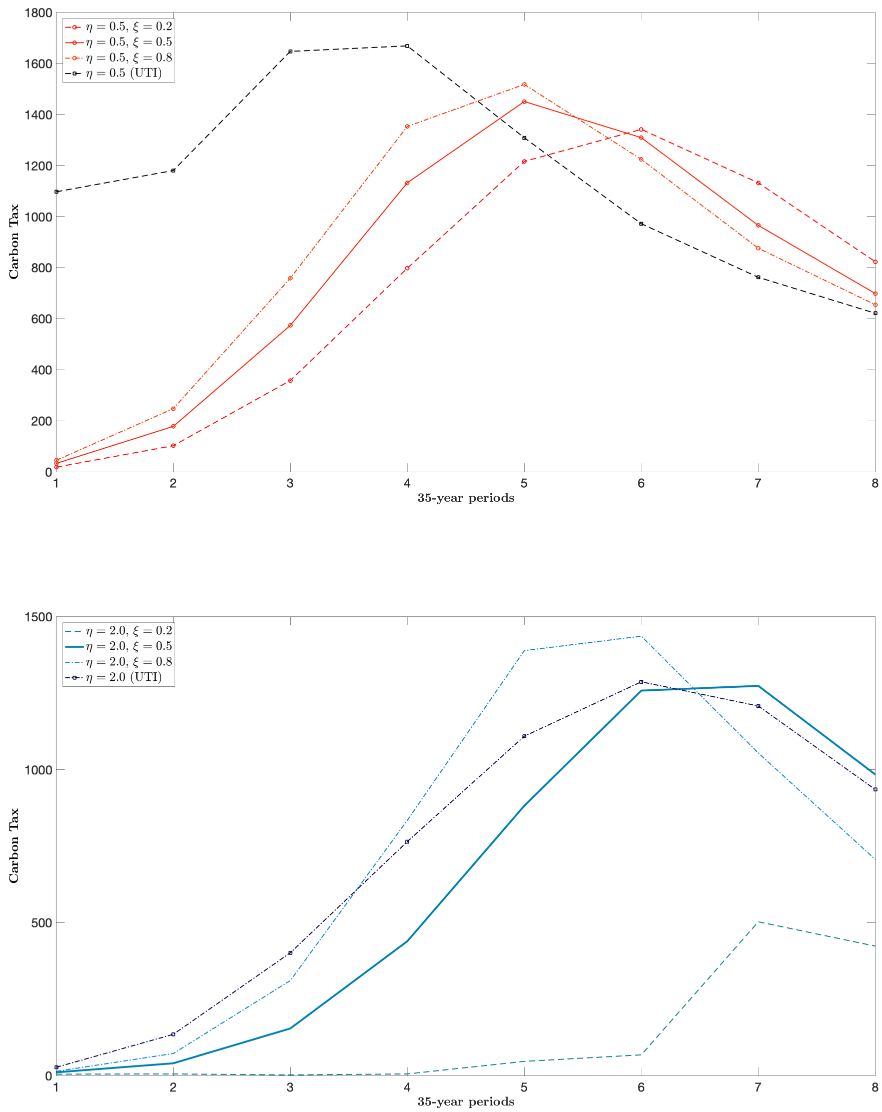

Figure 4: The equilibrium carbon taxes for $\eta = 0.5$ (top panel) and $\eta = 2$ (bottom panel), under the utilitarian and in the MPE with different influence parameters, ξ.

	$\eta = 0.5$ ($EIS = 2$)			$\eta = 2$ ($EIS = 0.5$)		
UTI	($\$1100, 48\%$)			($\$27, 7\%$)		
MPE	$\xi = 0.2$ ($\$18, 5\%$)	$\xi = 0.5$ ($\$33, 7\%$)	$\xi = 0.8$ ($\$45, 9\%$)	$\xi = 0.2$ ($\$5, 2\%$)	$\xi = 0.5$ ($\$11, 4\%$)	$\xi = 0.8$ ($\$14, 5\%$)

Table 4.2. First period carbon tax ($\$/tC$) and abatement (percent of BAU emissions) under the utilitarian and in the MPE.

5 Conclusion

We modified the standard IAM by replacing the composite commodity model with a strictly concave production possibility frontier, and replacing the infinitely lived agent with a Diamond-style OLG model. The first modification makes the price of the investment good (and thus of capital) endogenous, The second modification means that there is a buyer and seller of end-of-period undepreciated capital, so the price of capital matters. In this setting, climate policy can alter both the level of investment and its price; in the standard IAM, the level of investment is endogenous, but its price is fixed.

We used this new model to assess the possibility that asset markets give selfish agents an incentive to undertake meaningful climate policy. Previous empirical work establishes that asset markets can play this type of role in other contexts. However, the lack of binding climate policy until recently, and the slow-acting effect of policy, make it difficult to use econometrics to detect a relation between climate policy and asset markets. We therefore use a combination of comparative statics and numerical methods to investigate the relation. We also investigate the extent to which the young and the old generations' incentives, with regard to climate policy, are aligned.

Our numerical model assumes that each of a sequence of pairs of generations plays a dynamic game with their successors. The Markov Perfect equilibrium (MPE) excludes equilibrium outcomes supported by trigger strategies, and also the possibility of direct intergenerational transfers supported by debt. Intergenerational debt is important, but by excluding it we identify clearly the role of asset markets in generating incentives for climate policy.

The two generations' incentives are aligned if and only if the elasticity of intertemporal substitution (EIS) is less than one. The conventional choice in IAMs sets $EIS < 1$. Here, climate policy lowers the asset price, lowering the old generation's welfare. The MPE climate policy is substantial only if the young generation has significant representation in the political process.

Thus, under plausible conditions, endogenous asset prices provide a rationale to undertake meaningful climate policy, even when agents are completely selfish. However, the equilibrium level of policy is unlikely to reach the level of policy chosen by a standard discounted utilitarian. Moreover, asset markets do nothing to solve the problem of international free riding.

https://www.overleaf.com/project/60416fdce08835505cc76e3e

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A Appendix: Major Proofs

The proof of Lemma 1 is long and it contributes no new insights, so we delegate it, along with other intermediate results, to Referees' Appendix B.1.

Proof. (Lemma 2) The old generation consumes all of its income, so $c_t^o = (r_t + (1-\delta)\, p_t)\, K_t$, which implies equation 12.

We now establish the second part of equation 11; the first part is a definition. With isoelastic utility, $\psi_t = \frac{1}{\rho}\left(\frac{c_{t+1}^o}{c_t^y}\right)^{\eta}$. This result and equation 3 imply

$$p_t = \rho\left(\frac{c_{t+1}^o}{c_t^y}\right)^{-\eta}(r_{t+1} + (1-\delta)p_{t+1})\frac{K_{t+1}}{K_{t+1}} = \rho\frac{\left(c_{t+1}^o\right)^{1-\eta}}{\left(c_t^y\right)^{-\eta} K_{t+1}},$$

where the second equality follows from the second constraint in equation 1. Rearranging this equation gives

$$\left(c_{t+1}^o\right)^{1-\eta} = p_t\frac{\left(c_t^y\right)^{-\eta} K_{t+1}}{\rho}, \tag{17}$$

which holds for $\eta \in [0, \infty]$.

We use equation 17 to eliminate c_{t+1}^o from the expression of the young agent's lifetime welfare at t (the identity in equation 11) to write

$$\begin{aligned} \Omega_t^y &= \tfrac{1}{1-\eta}\left(\left(c_t^y\right)^{1-\eta} - 1 + \rho\left(\left(p_t\tfrac{\left(c_t^y\right)^{-\eta}K_{t+1}}{\rho} - 1\right)\right)\right) \\ &= \tfrac{\left(c_t^y\right)^{-\eta}}{1-\eta}\left(c_t^y + p_t K_{t+1}\right) - \tfrac{1}{1-\eta}\left(1+\rho\right) \\ &= \tfrac{\left(c_t^y\right)^{-\eta}}{1-\eta} w_t - \tfrac{1}{1-\eta}\left(1+\rho\right), \end{aligned} \tag{18}$$

where the last equality follows from the first constraint in equation 1. For $\eta \neq 1$, the last line of equation 18 produces the first line of equation 11.

For $\eta = 1$, we use equation 17 to obtain $c_t^y = \frac{p_t K_{t+1}}{\rho}$. The young agent's consumption plus investment equals the wage, i.e. $c_t^y + p_t K_{t+1} = \frac{p_t K_{t+1}}{\rho} +$

$p_t K_{t+1} = w_t$. This equality implies $p_t K_{t+1} = \frac{\rho}{1+\rho} w_t$ and $c_t^y = \frac{1}{1+\rho} w_t$, thus establishing the familiar result that for $\eta = 1$ the agent consumes a constant fraction of income and saves the rest. Now we substitute the equilibrium level of the young agent's consumption into the last line of equation 18 and use L'Hospital's Rule to take the limit as $\eta \to 1$:

$$\Omega_t^y = \lim_{\eta \to 1} \left(\frac{\left(\frac{1}{1+\rho} w_t\right)^{-\eta} w_t - (1+\rho)}{1-\eta} \right) = (1+\rho) \ln w_t - \ln(1+\rho),$$

thus confirming the second line of equation 11.

■

The following intermediate result simplifies the proof of Proposition 1

Lemma 3 *Using the definition $S \equiv \frac{pI}{c+pI}$ (the value of the investment good as a fraction of world income),*

$$\frac{c\sigma a^{-\sigma} p^{\sigma}}{1 + a^{-\sigma} p^{1+\sigma}} = \frac{\sigma c}{p} S. \tag{19}$$

Using equation 19 we can write the comparative static expressions for w and r in equation 8 as

$$\frac{\partial w}{\partial p} = \frac{wS}{p}, \text{ and } \frac{\partial r}{\partial p} = \frac{rS}{p}. \tag{20}$$

The change in investment due to a change in the price of investment, holding $\mathbf{z}$ *fixed, is*

$$\frac{\partial I}{\partial p}_{|\mathbf{z}\ fixed} = \frac{\sigma}{p^2} N (1-S) S. \tag{21}$$

Proof. (Proposition 1) Part (i). By Assumption 1.ii, the marginal cost of the first unit of abatement is zero. However, this abatement can affect welfare directly via its effect on the asset price, and indirectly via the effect of the asset price on the factor prices. Using equations 8 and 12, we obtain

$$\frac{\partial \Omega_t^o}{\partial \mu_t}_{|\mu=0} = ((r_t + (1-\delta) p_t) K_t)^{-\eta} K_t \left[\frac{r a^{-\sigma} p^{\sigma}}{1 + a^{-\sigma} p^{1+\sigma}} + 1 - \delta \right] \frac{\partial p_t}{\partial \mu_t}_{|\mu=0}.$$

Part (i) follows from the fact that the term in square brackets is positive.

Part (ii). For this part we assume $\eta \neq 1$. We drop the time index, $t = 0$, except where referring to c_0^y, which denotes the consumption of the agent who is young in period 0. We retain this time index to emphasize the distinction between c_0^y and aggregate consumption, c, in the same period.

Assumption 1.ii implies that μ has no first order effect on factor prices, apart from the effect arising from the response of p to a change in μ. Therefore, Assumption 1.ii and equation 20 imply that

$$\frac{\partial w}{\partial \mu} = \frac{\partial w}{\partial p}\frac{\partial p}{\partial \mu} = \frac{wS}{p}\frac{\partial p}{\partial \mu}. \tag{22}$$

Using equation 11, we have

$$\frac{d\Omega_0^y(\mu)}{d\mu} = \frac{1}{1-\eta}\left(\left(c_0^y\right)^{-\eta}\frac{\partial w}{\partial \mu} - \eta w \left(c_0^y\right)^{-\eta-1}\frac{\partial c_0^y}{\partial \mu}\right). \tag{23}$$

The young agent's budget constraint (using the normalization $L = 1$) is

$$\begin{gathered} c_0^y = w - p\left(\left(1-\delta\right)K + I\left(p\right)\right) \Rightarrow \\ \tfrac{\partial c_0^y}{\partial \mu} = \tfrac{\partial w}{\partial \mu} - \left(p\tfrac{\partial I}{\partial p} + \left(\left(1-\delta\right)K + I\right)\right)\tfrac{\partial p}{\partial \mu}. \end{gathered} \tag{24}$$

Using equation 24 in equation 23 we have

$$\begin{aligned} \tfrac{d\Omega_0^y(\mu)}{d\mu} &= \tfrac{1}{1-\eta}\left(\tfrac{\partial w}{\partial \mu}\left(c_0^y\right)^{-\eta} - \eta w\left(c_0^y\right)^{-\eta-1}\left[\tfrac{\partial w}{\partial \mu} - \left(p\tfrac{\partial I}{\partial p} + \left(\left(1-\delta\right)K + I\right)\right)\tfrac{\partial p}{\partial \mu}\right]\right) \\ &= \tfrac{1}{1-\eta}\left(c_0^y\right)^{-\eta}\left(\tfrac{\partial w}{\partial \mu}\left[1 - \eta w\left(c_0^y\right)^{-1}\right] + \eta w\left(c_0^y\right)^{-1}\left(p\tfrac{\partial I}{\partial p} + \left(\left(1-\delta\right)K + I\right)\right)\tfrac{\partial p}{\partial \mu}\right) \\ &= \tfrac{1}{1-\eta}\left(c_0^y\right)^{-\eta}\tfrac{\partial p}{\partial \mu}\digamma, \text{with} \end{aligned}$$

$$\digamma \equiv \tfrac{w}{p}S\left[1 - \eta w\left(c_0^y\right)^{-1}\right] + \eta w\left(c_0^y\right)^{-1}\left[\left(p\tfrac{\partial I}{\partial p} + \left(\left(1-\delta\right)K + I\right)\right)\right].$$

The last of the chain of equalities uses equation 22 to eliminate $\frac{\partial w}{\partial \mu}$. To complete the proof it is necessary and sufficient to show that $\digamma > 0$. Using equation 21 to eliminate $\frac{\partial I}{\partial p}$, we have

$$F = \frac{w}{p} S \left[1 - \eta w \left(c_0^y \right)^{-1} \right] + \eta w \left(c_0^y \right)^{-1} \left[\frac{\sigma}{p} N (1 - S) S + ((1 - \delta) K + I) \right] =$$

$$\frac{w}{p} S + \eta w \left(c_0^y \right)^{-1} \left[\frac{\sigma}{p} N (1 - S) S - \frac{w}{p} S + ((1 - \delta) K + I) \right] =$$

$$\frac{w}{p} S + \eta w \left(c_0^y \right)^{-1} N S \left[\frac{\sigma}{p} (1 - S) - \frac{w}{pN} + \frac{((1-\delta) K + I)}{pI} \right] =$$

$$\frac{w}{p} S + \eta w \left(c_0^y \right)^{-1} N S \left[\frac{\sigma}{p} (1 - S) + \frac{1}{p} \left(\frac{((1-\delta) K)}{I} + 1 - \frac{w}{N} \right) \right] > 0.$$

The penultimate equality uses $NS = pI$. The inequality uses the fact that $1 - \frac{w}{N} = 1 - \frac{wL}{N} \geq 0$, because labor's share of national income cannot exceed 1. Thus, the direction of change in the young agent's welfare, due to a small level of abatement is the same as the sign of $\frac{1}{1-\eta} \frac{\partial p}{\partial \mu}$. ■

Proof. (Proposition 2). Part (i) To conserve notation, we use $G(i)$ to denote the value of G evaluated at (K_i, E_i, μ_i), $i = 0, 1$ (the two periods); we use the same notation for the partial derivatives of G. We also define two new functions of parameters:

$$b_0 \equiv \rho \left(1 + a^{-\sigma} \right)^{\frac{-1}{\sigma+1}} \text{ and } b_1 \equiv a^{\sigma} \left(b_0 \right)^{-(1+\sigma)}.$$

In period 1 (the last period) investment and abatement are both zero. Using $p_1 = 0$ and $\phi(0) = (1 + a^{-\sigma})^{\frac{-1}{\sigma+1}}$, together with equation 6, we have $r_1 = (1 + a^{-\sigma})^{\frac{-1}{\sigma+1}} G_K(1)$. Using this relation, the asset pricing equation, and the definition of b_0, we have $p_0 = \rho (1 + a^{-\sigma})^{\frac{-1}{\sigma+1}} G_K(1) = b_0 G_K(1)$. Equation 7 and the definition of b_1 implies

$$\begin{aligned} I_0 = (1 + a^{\sigma})^{-\frac{1}{1+\sigma}} \left(a^{\sigma} (b_0)^{-(1+\sigma)} (G_K(1))^{-(1+\sigma)} + 1 \right)^{-\frac{\sigma}{1+\sigma}} G(0) = \\ (1 + a^{\sigma})^{-\frac{1}{1+\sigma}} \left(b_1 (G_K(1))^{-(1+\sigma)} + 1 \right)^{-\frac{\sigma}{1+\sigma}} G(0). \end{aligned} \tag{25}$$

Totally differentiating equation 25 gives

$$dI_0 = I_0 \frac{G_{\mu_0}(0)}{G(0)} d\mu_0 +$$
$$\sigma \frac{b_1\left((G_K(1))^{-(2+\sigma)}\right)}{\left(b_1(G_K(1))^{-(1+\sigma)}+1\right)} I_0 \left[G_{KK}(1)\, dI_0 + G_{KE}(1) \frac{dE_1}{d\mu_0} d\mu_0\right]$$

Collecting terms and rearranging produces

$$\frac{dI_0}{d\mu_0} = \frac{I_0 \left(\frac{G_{\mu_0}(0)}{G(0)} + \sigma \frac{b_1\left((G_K(1))^{-(2+\sigma)}\right)}{\left(b_1(G_K(1))^{-(1+\sigma)}+1\right)} \left[G_{KE}(1) \frac{dE_1}{d\mu_0}\right] \right)}{\left(1 - \sigma I_0 \frac{b_1\left((G_K(1))^{-(2+\sigma)}\right)}{\left(b_1(G_K(1))^{-(1+\sigma)}+1\right)} G_{KK}(1)\right)} \tag{26}$$

Assumption 1 (i) implies that that the denominator of the right side of this equation is positive. Assumption 1 (ii) (zero first order effect of abatement on abatement costs) implies that the first term of the numerator is zero. The second term is positive because abatement in the current period lowers the next-period stock of GHGs, increasing the next-period return to capital.

Because investment strictly increases in the price, we conclude that $\frac{dp_0}{d\mu_0} > 0$ for sufficiently small levels of abatement. By Proposition 1 we conclude welfare increases for both the young and the old in period 0. A small level of μ_0 increases K_1 and decreases E_1, so this abatement increases w_1, equal to the next generation's welfare.

Part (ii) The assumption that $\sigma < \infty$ is critical; if $\sigma = \infty$ (the linear PPF), the asset price is fixed at 1 in an interior equilibrium: $p_0 = 1 = \rho r_1 = \rho G_K(K_1, E_1)$. This identity implies that investment must change to offset an abatement-induced change in the next-period stock of GHGs:

$$D(E_1)(1-\beta) K_1^{-\beta} dK_1 + D'(E_1) K_1^{1-\beta} dE_1 = 0 \Rightarrow$$
$$\frac{dK_1}{dE_1} = -\frac{D'(E_1) K_1^{1-\beta}}{D(E_1)(1-\beta) K_1^{-\beta}}. \tag{27}$$

(Period-1, abatement is zero, so $\Lambda(\mu_1) = 1$; we also use the normalization $L = 1$.) The period-1 wage, equal to the welfare of the generation born in period 1, is $w_1 = \beta D(E_1) K_1^{1-\beta}$. The effect of the period-0 abatement on the period-1 wage is

$$\frac{dw_1}{d\mu_0} = \beta \left(D'\left(E_1\right) K_1^{1-\beta} + (1-\beta)\, D\left(E_1\right) K_1^{-\beta} \frac{dK_1}{dE_1} \right) \frac{dE_1}{d\mu_0} = 0.$$

The second equality uses equation 27.

Thus, the first (marginal) unit of abatement has zero welfare effect on all agents. However a larger level of abatement lowers the period 0 wage and rental rate, lowering welfare of the agents alive in period 0. Because equation 27 holds for all interior equilibria (not just the first marginal unit), abatement does not affect the welfare of the agent born in period 1.

■

Proof. (Proposition 3) We drop time subscripts where the meaning is clear. Part (i). Denote the young agent's demand for new capital as I^d. The proof of Lemma 2 confirms that for $\eta = 1$, the young agent's saves $\frac{\rho}{1+\rho} w = p\left(I^d + (1-\delta)K\right)$. Therefore, the demand for newly produced capital is $I^d(p;\mathbf{z}) = \frac{\rho}{1+\rho}\frac{w}{p} - (1-\delta)K$. From equation 20, using the definition $S = \frac{pI}{c+PI}$, with $S < 1$ (because consumption must be positive)

$$\frac{dI^d}{dp} = \frac{(S-1)w}{p^2} < 0.$$

Using equation 7, the supply of investment is $I^s(p;\mathbf{z}) = x(p)G(\mathbf{z})$ with $\frac{dx}{dp} > 0$. Equation 6 and Assumption 1.iii imply $w = \phi(p)\beta G(\mathbf{z})$. With these results, define excess demand as

$$\lambda(p;\mathbf{z}) = I^d - I^s = \Phi(p)G(\mathbf{z}) - (1-\delta)K. \tag{28}$$

The last part of equation 28 uses the definition

$$\Phi(p) \equiv \frac{\rho}{1+\rho}\frac{\phi(p)\beta}{p} - x(p).$$

Because demand decreases and supply increases with price, $\frac{d\lambda}{dp} < 0$. An equilibrium price, denoted $p(\mathbf{z})$, is a solution to $\lambda(p;\mathbf{z}) = 0$. An interior solution exists because λ is continuous in p and as $p \to \infty$ excess demand is negative, and as $p \to 0$ excess demand is positive. The equilibrium is unique and stable because the slope of the excess demand is negative. By inspection, the equilibrium depends on $\mathbf{z}$ but not on future policies.

Part (ii) Using equation 20 and the second line of equation 11, we have

$$\frac{\partial \Omega_t^y}{\partial \mu_0}_{|\mu_0=0} = \frac{(1+\rho)}{w}\frac{\partial w}{\partial p}\frac{\partial p}{\partial \mu} = \frac{(1+\rho)}{w}\frac{wS}{p}\frac{\partial p}{\partial \mu} = (1+\rho)\,\frac{S}{p}\frac{\partial p}{\partial \mu}.$$

This equation together with Proposition 1 implies that the young and the old agents' welfare levels respond in the same direction to a price change induced by a small level of abatement.

As noted above, the young agent uses the constant fraction $\frac{1}{1+\rho}$ of her wage on current consumption. Because the wage increases monotonically with the asset price, an increase in that price also increases the young agent's consumption. Because the rental rate increases monotonically in the asset price, an increase in that price also increases the old agent's consumption. Therefore if abatement were to increase p, it would lead to a increase in consumption by both agents in period 0. However, a higher p implies that the production point moves south-east on the PPF, resulting in reduced production of the consumption good. Thus, a higher p is not consistent with market clearing in the market for the consumption good. A parallel argument establishes that a lower p is not consistent with market clearing.

Therefore, the first order effect on p and also on the two agents' welfare of the first unit of abatement is 0. The first unit of abatement does not change the level of investment, but it lowers the next-period stock of GHGs, thus benefiting the agent born in the next period.

(Part iii) For $\mu > 0$ we differentiate the market clearing condition $\lambda(p, \mathbf{z}) = 0$ with respect to p and μ and rearrange the differential to obtain

$$\frac{dp}{d\mu} = -\frac{\lambda_\mu}{\lambda_p} > 0 \Leftrightarrow \lambda_\mu > 0.$$

The inequality follows from the fact that $\lambda_p < 0$ from Part (i) of the proof.

From equation 28 we have $\lambda_\mu = \Phi(p)G_\mu$; for $\mu > 0$, $G_\mu < 0$. Market clearing requires $\Phi(p)G(\mathbf{z}) = (1-\delta)K$. Thus, for $\delta = 1$, $\Phi(p) = 0$ in equilibrium. In this case, increased abatement does not alter the equilibrium asset price. However, it reduces the factor prices and the level of investment via the reduction in G. For $\delta < 1$, $\Phi > 0$, so increased abatement lowers the asset price, leading to an additional reduction in investment.

For fixed future policy, welfare of the agent born in the next period is proportional to $\ln w_{t+1}$, which increases in the inherited stock of capital and decreases in the stock of GHGs. This fact, together with the results shown above, establishes the Proposition's claims regarding the welfare effect for the next generation. ■

Proof. (Proposition 4). With $c_t^o = w_t - p_t K_{t+1}$ and

$$c_{t+1}^o = \left(r_{t+1} + (1-\delta)\, p_{t+1}\right) K_{t+1},$$

the optimal savings rule is $w_t - p_t K_{t+1} = \frac{(r_{t+1}+(1-\delta)p_{t+1})K_{t+1}}{\rho}$, which implies $w_t - \frac{(r_{t+1}+(1-\delta)p_{t+1})K_{t+1}}{\rho} = p_t K_{t+1}$. Rearranging this equation produces equation 13. For $H = 1$, investment and abatement are zero in period 1, so $p_1 = 0$ and $c_1^y = w_1$. Using Lemma 1 and Assumption 1.iii, $w_i = \phi\left(p_i\right) \beta D\left(E_i\right) \Lambda\left(\mu_i\right) K_i^{1-\beta}$ and $r_i = (1-\beta)\, \phi\left(p_i\right) D\left(E_i\right) \Lambda\left(\mu_i\right) K_i^{1-\beta}$. Normalize by setting $D\left(E_0\right) = 1$, and using $\mu_1 = 0$, the optimal savings rule becomes

$$\frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D\left(E_1\right) K_1^{2-\beta} + p_0 K_1 - \phi\left(p_0\right) \beta \Lambda\left(\mu_0\right) K_0^{1-\beta} = 0. \quad (29)$$

This equation gives the demand for investment, K_1^d as an implicit function of p_0, μ_0. Totally differentiating this equation and rearranging produces

$$dK_1^d = \frac{\left[\phi(p_0)\beta\Lambda'(\mu_0)K_0^{1-\beta} - \frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D'(E_1)K_1^{2-\beta}\frac{dE_1}{d\mu}\right]}{\left[\frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D(E_1)(2-\beta)K_1^{1-\beta} + p_0\right]} d\mu_0 \\ \frac{-\left[K_1 + \phi'(p_0)\beta\Lambda(\mu_0)K_0^{1-\beta}\right]}{\left[\frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D(E_1)(2-\beta)K_1^{1-\beta} + p_0\right]} dp_0 \quad (30)$$

The denominator of the right side of this equation is positive, so $\frac{dK_1^d}{dp_0} < 0$. The supply of investment (using equation 6) is $K_1^s = x(p_0)\Lambda(\mu_0)K_0^{1-\beta} + (1-\delta)K_0$. The differential of this equation is

$$dK_1^s = x'(p_0)\Lambda(\mu_0)K_0^{1-\beta}dp_0 + x(p_0)\Lambda'(\mu_0)K_0^{1-\beta}d\mu_0. \qquad (31)$$

Denote the excess demand function as $\Theta(p_0,\mu_0) = K_1^d(p_0,\mu_0) - K_1^s(p_0,\mu_0)$. The market clearing condition is $\Theta(p_0,\mu_0) = 0$. Using equations 30 and 31 $\Theta_p(p_0,\mu_0) = \frac{dK_1^d(p_0,\mu_0)}{dp} - \frac{dK_1^s(p_0,\mu_0)}{p} < 0$. Our assumption that an interior equilibrium exists implies that the unique equilibrium is stable.[19] Differentiating the market clearing condition $\Theta(p_0,\mu_0) = 0$ and rearranging the results produces $\frac{dp_0}{d\mu_0} = -\frac{\Theta_\mu(p_0,\mu_0)}{\Theta_p(p_0,\mu_0)}$. Thus, $sign\left(\frac{dp_0}{d\mu_0}\right) = sign\left(\Theta_\mu(p_0,\mu_0)\right)$. Using equations 30 and 31 (and $\Lambda'(\mu_0)_{|\mu_0=0} = 0$) we obtain

$$\Theta_\mu(p_0,\mu_0)_{|\mu_0=0} = \frac{-\frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D'(E_1)K_1^{2-\beta}\frac{dE_1}{d\mu}}{\frac{(1-\beta)}{\rho}\left(\frac{a^\sigma}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}} D(E_1)(2-\beta)K_1^{1-\beta} + p_0} < 0$$

Thus, the first unit of abatement lowers the asset price. By Proposition 1, this change lowers the welfare for the old agent at $t = 0$ and raises the welfare of the young agent.

Welfare of the agent born at $t = 1$ monotonically increases in $w_1 = \phi(0)\beta D(E_1)K_1^{1-\beta}$. The change in this agent's welfare

$$\frac{dw_1}{d\mu_0} = \phi(0)\beta K_1^{1-\beta}D(E_1)\left(\frac{D'(E_1)E_1}{D(E_1)}\frac{dE_1}{E_1 d\mu_0} + \frac{dK_1}{K_1 d\mu_0}\right), \qquad (32)$$

which implies the last part of Proposition 4. ■

[19] As $p \to \infty$ the demand for capital approaches 0 and the supply of new capital approaches a finite positive level, so there is excess supply. As $p \to 0$ the supply of new capital approaches 0 so the aggregate supply of capital is $(1-\delta)K_0$. For sufficiently large K_0 there is excess supply at a zero price. In this case, the equilibrium is on the boundary. Thus, the existence of an interior in this case requires that the initial stock of capital is not too large. Some calculations show that the condition for an interior equilibrium is $\left(\frac{1}{1-\delta}\right)^{2-\beta}\left(\frac{\rho\beta\Lambda(\mu_0)}{(1-\beta)D(E_1)}\right) > K_0$.

B Referees' Appendix (online publication)

This appendix collects additional proofs, provides calibration details, explains our use of efficiency units, discusses the numerical algorithm, and provides sensitivity results.

B.1 Additional proofs

We number the lemmas by their order of their appearance in the text (not by the order of their proof). This appendix begins with Lemmas 4, 5, and 6 that we use to prove Lemma 1.

Lemma 4 *Given the Constant Elasticity of Transformation PPF*

$$c^{\frac{1+\sigma}{\sigma}} + aI^{\frac{1+\sigma}{\sigma}} = B \Rightarrow c = \left(B - aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}}, \tag{33}$$

$N(1, \mathbf{z}) = G(\mathbf{z})$ *if and only if*

$$B(G(\mathbf{z});\sigma, a) \equiv \left(1 + a^{-\sigma}\right)^{-\frac{1}{\sigma}} G(\mathbf{z})^{\frac{1+\sigma}{\sigma}}. \tag{34}$$

Proof. (Lemma 4) Equation 33 implies that the slope of the tangent of a point on the PPF is

$$-p = \frac{dc}{dI} = -a\left(\frac{I}{c}\right)^{\frac{1}{\sigma}}. \tag{35}$$

Consider a ray through the origin with slope s, where $c = sI$. Using equation 35, the slope of the tangent at the intersection of the ray $c = sI$ and the PPF is $-p = -a\left(\frac{I}{c}\right)^{\frac{1}{\sigma}} = -a\left(\frac{1}{s}\right)^{\frac{1}{\sigma}}$, i.e.

$$s = \left(\frac{p}{a}\right)^{-\sigma}. \tag{36}$$

Thus, $p = 1 \Leftrightarrow s = a^{\sigma}$. At $p = 1$, the equilibrium value of national income is

$$c + I = (s+1)I = G \Rightarrow I = \frac{G}{1+a^{\sigma}} \text{ and } c = \frac{a^{\sigma}G}{1+a^{\sigma}}. \tag{37}$$

The equilibrium point lies on the PPF, so using equations 33 and 37 we have

$$\left[\left(\frac{a^{\sigma}}{1+a^{\sigma}}\right)^{\frac{1+\sigma}{\sigma}}+a\left(\frac{1}{1+a^{\sigma}}\right)^{\frac{1+\sigma}{\sigma}}\right]G^{\frac{1+\sigma}{\sigma}}=B. \tag{38}$$

The term in square brackets simplifies to

$$\left(\frac{a^{\sigma}}{1+a^{\sigma}}\right)^{\frac{1+\sigma}{\sigma}}+a\left(\frac{1}{1+a^{\sigma}}\right)^{\frac{1+\sigma}{\sigma}}=\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}. \tag{39}$$

Equations 38 and 39 produce equation 34. ■

The following lemma shows that the supply of each good and national income are multiplicatively separable in the asset price p and $\mathbf{z}$. These relations depend on the technology, but not on the demand side of the economy.

Lemma 5 *The equilibrium supply function is given by equation 7 and national income equals*

$$\begin{aligned} N(p,\mathbf{z}) &= \Gamma(p)\,G(\mathbf{z}) \\ \Gamma(p) &\equiv \left(a^{\sigma}p^{-\sigma}+p\right)\left(\frac{\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}}{\left(\frac{p}{a}\right)^{-(1+\sigma)}+a}\right)^{\frac{\sigma}{1+\sigma}}=\left(a^{\sigma}p^{-\sigma}+p\right)x(p). \end{aligned} \tag{40}$$

Proof. (Lemma 5) Using equation 36, the tangent of the PPF with slope $-p$ is a point on the ray $c=sI$, with $s=\left(\frac{p}{a}\right)^{-\sigma}$. This fact and equations 33 and 34 imply

$$\left(\left(\frac{p}{a}\right)^{-\sigma}I\right)^{\frac{1+\sigma}{\sigma}}+aI^{\frac{1+\sigma}{\sigma}}=\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}G^{\frac{1+\sigma}{\sigma}}\Rightarrow$$

$$\left(\left(\left(\frac{p}{a}\right)^{-\sigma}\right)^{\frac{1+\sigma}{\sigma}}+a\right)I^{\frac{1+\sigma}{\sigma}}=\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}G^{\frac{1+\sigma}{\sigma}}\Rightarrow$$

$$I=\left(\frac{\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}}{\left(\frac{p}{a}\right)^{-(1+\sigma)}+a}\right)^{\frac{\sigma}{1+\sigma}}G=\left(\frac{\left(1+a^{\sigma}\right)^{-\frac{1}{\sigma}}}{a^{\sigma}p^{-(1+\sigma)}+1}\right)^{\frac{\sigma}{1+\sigma}}G.$$

which establishes the equality in 7. Taking the derivative, we establish $x'(p)>0$. A larger p supports higher production of I at given $\mathbf{z}$.

To establish equation 40 we use $c = sI$ and equation 36 to write world income as

$$N = c + pI = (s+p)\,x(p)\,G = \left(a^{\sigma}p^{-\sigma} + p\right)\left(\frac{(1+a^{\sigma})^{-\frac{1}{\sigma}}}{a^{\sigma}p^{-(1+\sigma)}+1}\right)^{\frac{\sigma}{1+\sigma}} G.$$

where

$$\Gamma(p) = \left(a^{\sigma}p^{-\sigma} + p\right)x(p)$$

■

We use equation 40, which contains the definition of Γ, to obtain the formula for the tax that supports a given level of abatement.

Corollary 1 *For the model with $G(\mathbf{z}) = D(E)\,\Lambda(\mu)\,AF(K,L)$, the carbon tax $\varpi = -\Gamma(p)\,\frac{D(E)\Lambda'(\mu)}{\zeta}$ supports the abatement level μ in a competitive equilibrium.*

Proof. (Corollary 1) Given $\mathbf{z}$, emissions are $e = (1-\mu)\,\zeta AF(K,L)$, so $\frac{d\mu}{de} = -\frac{1}{\zeta AF(K,L)}$. By emitting one more unit, firms increase the value of output by

$$\begin{array}{c} \varpi \equiv \frac{dN}{de} = \frac{\partial N}{\partial \mu}\frac{d\mu}{de} = \Gamma(p)\,G_{\mu}(\mathbf{z})\,\frac{d\mu}{de} = \\ -\Gamma(p)\,\frac{D(E)\Lambda'(\mu)AF(K,L)}{\zeta AF(K,L)} = -\Gamma(p)\,\frac{D(E)\Lambda'(\mu)}{\zeta}. \end{array} \tag{41}$$

Firms want to abate to the level that satisfies equation 41. ■

Note that for $p = 1$, $\Gamma = 1$. The functions Γ, D and Λ are fractions, and thus unit-free (as is Λ'). Thus the units of the tax, ϖ are the same as the units of $\frac{1}{\zeta}$, which are $\frac{\$10^{12}}{10^{9}tC} = \frac{\$10^{3}}{tC}$, thousands of dollars per ton of carbon. We multiply ϖ by 1000 to convert the tax to dollars per ton of carbon, and then divide by 3.666 if we want to express the tax in dollars per ton of $CO2$.

We also use Lemma 5 to establish the following:

Lemma 6 *For the CET techology, given the price of investment p, in equilibrium*

$$\phi(p) \equiv \left(\frac{G}{(1+a^{-\sigma})\,c}\right)^{\frac{1}{\sigma}} = \left(\frac{a^{\sigma}+p^{1+\sigma}}{1+a^{\sigma}}\right)^{\frac{1}{1+\sigma}} \tag{42}$$

Proof. (Lemma 6) Using equation 7 and the definition of s, we have (after simplification)

$$\frac{G}{c} = \frac{G}{I}\frac{I}{c} = \frac{1}{s(p)x(p)} = \frac{\left(\frac{\left(\frac{p}{a}\right)^{-(1+\sigma)}+a}{\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}}\right)^{\frac{\sigma}{1+\sigma}}}{a^{\sigma}p^{-\sigma}} =$$

$$\frac{1}{a^{\sigma}}p^{\sigma}\left(a^{-\sigma}+1\right)^{\frac{1}{\sigma+1}}\left(a+\frac{a^{\sigma+1}}{p^{\sigma+1}}\right)^{\frac{\sigma}{\sigma+1}}.$$

With this intermediate result we have (after simplification)

$$\frac{G}{\left(1+a^{-\sigma}\right)c} = \left(a^{\sigma}+1\right)^{\frac{-\sigma}{\sigma+1}}p^{\sigma}\left(1+a^{\sigma}p^{-(\sigma+1)}\right)^{\frac{\sigma}{\sigma+1}}.$$

The last equality implies

$$\left(\frac{G}{\left(1+a^{-\sigma}\right)c}\right)^{\frac{1}{\sigma}} = \left(\left(a^{\sigma}+1\right)^{\frac{-\sigma}{\sigma+1}}p^{\sigma}\left(1+a^{\sigma}p^{-(\sigma+1)}\right)^{\frac{\sigma}{\sigma+1}}\right)^{\frac{1}{\sigma}}.$$

Simplifying the right side of the last equation produces equation 42. ■

Proof. (Lemma 1) (i) Define $\Theta\left(I;B,\sigma,a\right) \equiv \left(B-aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}}$. Given the price p and using equation 4 and the definition of $\Theta\left(I;B,\sigma,a\right)$, the value of world output is

$$N\left(p;\mathbf{z}\right) = \max_{I}\left[\Theta\left(I;B,\sigma,a\right)+pI\right].$$

Using the envelope theorem, the factor returns are

$$\begin{aligned} w &= \frac{\partial N(p;\mathbf{z})}{\partial L} = \frac{\partial \Theta(I;B,\sigma,a)}{\partial B}B'G_L \\ r &= \frac{\partial N(p;\mathbf{z})}{\partial K} = \frac{\partial \Theta(I;B,\sigma,a)}{\partial B}B'G_K. \end{aligned} \tag{43}$$

Using the definition of $\Theta\left(I;B,\sigma,a\right)$ we have

$$\begin{aligned} &\frac{\partial\Theta}{\partial B} = \frac{\sigma}{1+\sigma}\left(B-aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}-1} = \\ &\frac{\sigma}{1+\sigma}\left(B-aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{-1}{1+\sigma}\frac{\sigma}{\sigma}} = \frac{\sigma}{1+\sigma}\left(B-aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}\frac{-1}{\sigma}} = \frac{\sigma}{1+\sigma}c^{\frac{-1}{\sigma}}. \end{aligned} \tag{44}$$

Using equation 34 we have

$$B'(G) = \frac{1+\sigma}{\sigma}\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}} G^{\frac{1}{\sigma}}. \tag{45}$$

Combining equations 43, 44 and 45 produces the first expressions for w and r in equation 6. To obtain the second expressions for w and r we apply Lemma 6. By Assumption 1 (iii), the pollution stock and the abatement rate do not affect relative factor prices, $\frac{w}{r}$. Lemma 5 establishes equation 7.

(ii) Equation 5 and the envelope theorem imply the first equality in 8. Taking the derivative of $w = \phi(p) G_L$ in equation 6, using the definition of $\phi(p)$, we have

$$\frac{\partial w}{\partial p} = \phi'(p) G_L = \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{-\sigma}{\sigma+1}} \tfrac{p^{\sigma}}{a^{\sigma}+1} G_L =$$

$$\left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{-\sigma}{\sigma+1}} \tfrac{p^{\sigma}}{a^{\sigma}+1} \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{-1}{\sigma+1}} \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{1}{\sigma+1}} G_L$$

$$= \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{-\sigma}{\sigma+1}} \tfrac{p^{\sigma}}{a^{\sigma}+1} \left(\tfrac{p^{(\sigma+1)}+a^{\sigma}}{a^{\sigma}+1}\right)^{\frac{-1}{\sigma+1}} w = \tfrac{p^{\sigma}}{a^{\sigma}+p^{\sigma+1}} w,$$

establishing the middle equation in 8. A parallel argument establishes the last equation in 8.

(iii) Using $\left(\frac{G}{c}\right)^{\frac{1}{\sigma}} = \exp\left[\ln\left(\left(\frac{G}{c}\right)^{\frac{1}{\sigma}}\right)\right] = \exp\left(\frac{1}{\sigma}\ln\frac{G}{c}\right)$ and the assumption that $c > 0$ (which implies that $\ln\left(\frac{G}{c}\right)$ remains bounded), we have $\lim_{\sigma\to\infty}\left(\frac{G}{c}\right)^{\frac{1}{\sigma}} = 1$. Similarly, using

$$\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}} = \exp\left(\frac{-1}{\sigma}\ln\left(1+a^{-\sigma}\right)\right),$$

we have $\lim_{\sigma\to\infty}\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}} = 1$. Therefore, $\lim_{\sigma\to\infty}\left(\frac{G}{(1+a^{-\sigma})c}\right)^{\frac{1}{\sigma}} = 1$. Using this fact in equation 6 produces equation 9. (We cannot simply take the limit of $\phi(p)$ as $\sigma \to \infty$ because in the limit it must be the case that $p \to 1$; otherwise, the equilibrium production point is at the boundary, i.e. either c or I approaches 0.) $\blacksquare$

Proof. (Lemma 3) Use $c = s(p)I$, with $s = \left(\frac{p}{a}\right)^{-\sigma}$, to write

$$\frac{\sigma c}{p}\frac{pI}{c+pI} = \sigma c \frac{1}{s+p} = \sigma c \frac{1}{\left(\frac{p}{a}\right)^{-\sigma}+p}$$

$$= \sigma \frac{c}{p^{-\sigma}a^{\sigma}+p}\left(\frac{a^{-\sigma}p^{\sigma}}{a^{-\sigma}p^{\sigma}}\right) = \sigma \frac{ca^{-\sigma}p^{\sigma}}{1+a^{-\sigma}p^{1+\sigma}},$$

thus establishing equation 19.

Next, we obtain an expression for $\frac{dc}{dp}$. Using equations 33 and 35 we write the equilibrium c as an implicit function of p:

$$c = \left(B - aI^{\frac{1+\sigma}{\sigma}}\right)^{\frac{\sigma}{1+\sigma}} = \left(B - c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{\frac{\sigma}{1+\sigma}}. \tag{46}$$

We have

$$\frac{d\left(B-c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{\frac{\sigma}{1+\sigma}}}{dc} = -\left(B - c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{-\frac{1}{1+\sigma}} c^{\frac{1}{\sigma}}a^{-\sigma}p^{1+\sigma}$$

$$= -\left(\left(B - c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{\frac{\sigma}{1+\sigma}}\right)^{\frac{1}{-\sigma}} c^{\frac{1}{\sigma}}a^{-\sigma}p^{1+\sigma} = -(c)^{\frac{1}{-\sigma}} c^{\frac{1}{\sigma}}a^{-\sigma}p^{1+\sigma} = -a^{-\sigma}p^{1+\sigma}.$$

and

$$\frac{d\left(B-c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{\frac{\sigma}{1+\sigma}}}{dp} = -\left(B - c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{1+\sigma}\right)^{-\frac{1}{1+\sigma}} \sigma c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{\sigma}$$

$$= -c^{\frac{-1}{\sigma}}\sigma c^{\frac{1+\sigma}{\sigma}}a^{-\sigma}p^{\sigma} = -c\sigma a^{-\sigma}p^{\sigma}.$$

Using the last two results, we obtain the differential of equation 46

$$\begin{gathered} dc = -a^{-\sigma}p^{1+\sigma}dc - c\sigma a^{-\sigma}p^{\sigma}dp \Rightarrow \\ \frac{dc}{dp} = -\frac{c\sigma a^{-\sigma}p^{\sigma}}{1+a^{-\sigma}p^{1+\sigma}} \end{gathered} \tag{47}$$

Using equation 19 in 8, we obtain equation 20.

We can also write the elaticities of c and I with respect to price as

$$\frac{\partial c}{\partial p}\frac{p}{c} = -\sigma S \text{ and } \frac{\partial I}{\partial p}\frac{p}{I} = \sigma\left(1-S\right). \tag{48}$$

Using equations 19 and 47 we obtain the elasticity of c with respect to p in equation 48. To calculate the elasticity of I with respect to p we differentiate the identity $c\left(p\right) = s\left(p\right)I\left(p\right) = a^{\sigma}p^{-\sigma}I\left(p\right)$ and convert to an elasticity:

$$\begin{gathered} \tfrac{dc}{dp} = -\sigma a^{\sigma}p^{-\sigma}Ip^{-1} + a^{\sigma}p^{-\sigma}\tfrac{dI}{dp} = -\sigma c p^{-1} + c\tfrac{1}{I}\tfrac{dI}{dp} \Rightarrow \\ \tfrac{dc}{dp}\tfrac{p}{c} = -\sigma + \tfrac{p}{I}\tfrac{dI}{dp} \Rightarrow \tfrac{p}{I}\tfrac{dI}{dp} = \tfrac{dc}{dp}\tfrac{p}{c} + \sigma = \sigma\left(1-S\right). \end{gathered} \tag{49}$$

For this analysis we take $\mathbf{z}$ as fixed, so the total and partial derivatives are equivalent.

To establish equation 21 we use

$$\begin{aligned} \tfrac{\partial I}{\partial p}_{|\mathbf{z}\text{ fixed}} &= \tfrac{\partial I}{\partial c}\tfrac{\partial c}{\partial p}_{|\mathbf{z}\text{ fixed}} = \tfrac{1}{p}\tfrac{\sigma c}{p}\tfrac{pI}{c+pI} \\ &= \tfrac{\sigma}{p^2}\tfrac{c}{N}N\tfrac{pI}{c+pI} = \tfrac{\sigma}{p^2}N\left(1-S\right)S. \end{aligned} \tag{50}$$

■

Remark 1 *As a consistency check, we confirm that if G is constant returns to scale in L, K, then $wL + rK = c + pI$. First we note that*

$$\frac{G}{c} = \frac{G}{I}\frac{I}{c} = \frac{1}{s\left(p\right)x\left(p\right)} = \frac{\left(\frac{\left(\frac{p}{a}\right)^{-(1+\sigma)}+a}{\left(1+a^{-\sigma}\right)^{-\frac{1}{\sigma}}}\right)^{\frac{\sigma}{1+\sigma}}}{a^{\sigma}p^{-\sigma}} \tag{51}$$

Using equation 6 and 51 we have

$$wL + rK = \left(\frac{G}{(1+a^{-\sigma})c}\right)^{\frac{1}{\sigma}} (G_L L + G_K K) = \left(\frac{G}{(1+a^{-\sigma})c}\right)^{\frac{1}{\sigma}} G$$

$$= \hat{\Gamma} G \text{ with } \hat{\Gamma} \equiv \left(\frac{1}{(1+a^{-\sigma})} \frac{\left(\frac{(\frac{p}{a})^{-(1+\sigma)}+a}{(1+a^{-\sigma})^{-\frac{1}{\sigma}}}\right)^{\frac{\sigma}{1+\sigma}}}{a^{\sigma}p^{-\sigma}} \right)^{\frac{1}{\sigma}}.$$

For example, with $\sigma = 4$, we have $\hat{\Gamma} = \Gamma = \frac{1}{\sqrt[5]{a^4+1}} \sqrt[5]{a^4 + p^5}$. The expressions for $\hat{\Gamma}$ and Γ do not simplify so neatly for non-integer values of σ. However, extensive numerical examples show that the two expressions are always equal.

B.2 Exogenously changing parameters

This Appendix reports the time profiles for time-varying parameters taken from DICE-2016R. DICE uses a 5-year and we use a 35-year time step, so we need to adapt the time scale. We first solve the difference equations for the parameters as a function of time and the initial condition and then evaluate at adjusted values for time. DICE contains the following time trajectories with τ time measured in 5 year steps:

- Population (in million): $p(\tau{+}1) = p(\tau)\left(\frac{\text{popasym}}{p(\tau)}\right)^{\text{popadj}}$ with $p(0) = \text{pop0}$.
- Total factor productivity growth: $al(\tau + 1) = al(\tau)/((1 - ga(\tau)))$ with $ga(\tau) = ga0e^{-dela\ 5\ \tau}$ and $al(0) = a0$.
- Carbon intensity of output: $\text{sigma}(\tau + 1) = \text{sigma}(\tau)e^{\tau\text{step gsig}(\tau)}$ with $gsig(\tau + 1) = gsig(\tau)(1 + dsig)^{\tau \text{ step}}$, $sigma(0) = sig0$, and $gsig(0) = gsigma1$.
- Cost of a back-stop technology (which determines the dynamics of mitigation costs): $\text{pbacktime}(\tau) = \text{pback}(1 - \text{gback})^{\tau}$.

The analytical solutions for these difference equations are:

- $p(\tau) = \mathrm{pop0}^{(1-\mathrm{popadj})^{\tau}} \mathrm{popasym}^{1-(1-\mathrm{popadj})^{\tau}}$,
- $\mathrm{al}(\tau) = \frac{\mathrm{a0}\left(e^{5\mathrm{dela}}\right)^{\frac{1}{2}(\tau-1)\tau(-\mathrm{ga0})^{-\tau}}}{\mathrm{QPochhammer}\left(\frac{1}{\mathrm{ga0}};e^{5\ \mathrm{dela}}\right)_{\tau}}$,
- $\mathrm{sigma}(\tau) = \mathrm{sig0} \exp\left(\frac{5\ \mathrm{gsigma1}\left(\left((\mathrm{dsig}+1)^{5}\right)^{\tau}-1\right)}{\mathrm{dsig}(\mathrm{dsig}(\mathrm{dsig}(\mathrm{dsig}(\mathrm{dsig}+5)+10)+10)+5)}\right)$,
- $\mathrm{pbacktime}\ (\tau) = \mathrm{pback}(1-\mathrm{gback})^{\tau}$

with QPochhammer defined as $(a;q)_n = \frac{(a;q)_\infty}{(a\ q^n;q)_\infty}$ with $(a;q)_\infty = \prod_{k=0}^{\infty}\left(1-aq^k\right)$ and the following parameter values pop0 = 7403, popadj = 0.134, popasym = 11500, a0 = 5.115, ga0 = 0.076, dela = 0.005, gsigma1 = −0.0152, dsig = −0.001, expcost2 = 2.6, pback = 550, gback = 0.025, sig0 = 0.35032.

We adjust these time-dependent variables to the 35-year time step in our model using $t = 7\tau$ and the following conversion:

- Young generation (in billion): Population in 35-year time step is $p(7\tau) = \mathrm{pop0}^{(1-\mathrm{popadj})^{7\tau}} \mathrm{popasym}^{1-(1-\mathrm{popadj})^{7\tau}}$. We assume that the size of the young generation, L_t, grows at the same rate as population, p_t: $\frac{L_t}{L_{t-1}} = \frac{p_t}{p_{t-1}}$. We determine the size of the initial young and old generations using the additional condition that their sum equals the initial size of overall population: $L_{-1} + L_0 = p_0$. This gives $L_{-1} = 2.35033$ bn and $L_0 = 5.05267$ bn. With this calibration, the asymptotic total population in our model is 15.7 billion, compared to 11.5 billion in DICE. An alternative calibration requires the asymptotic population in our model to equal 11.5 billion. This alternative causes the growth rate of the young population (the number of workers) to differ from the growth rate in DICE. We do not use this alternative because we think that estimates of near-term growth rates are more reliable than estimates of asymptotic population.
- Carbon intensity, ζ_t, is $sigma(7\tau)/3.6666$ to convert GtCO2 into GtC.
- Cost of 100% emission reduction, $\nu_{1,t}$, is $(pbacktime(t) * sigma(t))/(1000\ expcost2)$ which follows directly from the DICE model. No additional adjustment as ν_{1t} is a unit-free number.

- Labour-augmenting technical progress, A_t, is $A_0\left(\frac{al(7\tau)}{al(0)}\right)^{1/\beta}$ with $A_0 =$ 4748 calibrated to initial 35-year output in $k_0^{1-\beta}(A_0\ L_0)^\beta = 35\ y_0$ with $k_0 = 223$, $L_0 = 5.05267$, $\beta = 0.6$, and $y_0 = 105.5$. We need to convert from total factor productivity (tfp) to labour-augmenting productivity (lap). A_t is lap_t in the notation above. We set $\mathrm{tfp}_t\ k_t^{1-\beta}(L_t)^\beta = k_t^{1-\beta}(\mathrm{lap}_t\ L_t)^\beta$, which gives $\mathrm{tfp}_t = (\mathrm{lap}_t)^\beta$ or $\mathrm{lap}_t = (\mathrm{tfp}_t)^{1/\beta}$. The initial value of lap_0 is the solution to $k_0^{1-\beta}(\mathrm{lap}_0\ L_0)^\beta = 35\ y_0$, $\mathrm{lap}_0 = 3240$. We take the growth rate of lap from tfp, setting $\frac{\mathrm{lap}_t}{\mathrm{lap}_0} = \left(\frac{\mathrm{tfp}_t}{\mathrm{tfp}_0}\right)^{1/\beta}$ or $\mathrm{lap}_t = \mathrm{lap}_0\left(\frac{\mathrm{tfp}_t}{\mathrm{tfp}_0}\right)^{1/\beta}$.
- The text discusses our damage calibration, using the DICE-2013R function, $D(T) = 1/(1+\hat{\imath}T^2)$, together with the Nordhaus (2017) estimate that $T = 2^oC$ reduces output by 0.94%, which implies $\hat{\imath} = 0.0023723$. Using the TCRE model with parameter $\tau = 0.002$ (a mid-range estimate) gives the calibration equation $1/(1+\hat{\imath}T^2) = 1/(1+\hat{\imath}(\tau E)^2) = 1/(1+\hat{\imath}\tau^2E^2) = 1/(1+\iota E^2)$, implying $\iota = \hat{\imath}\tau^2 = 9.44(10)^{-9}$. Note that we use the TCRE model only for calibration at low temperatures.

B.3 Efficiency units

Our analytic results in Sections 2 and 3 use a stationary model, one without labor or TFP growth. Our numerical results in Section 4 include exogenous growth in labor and technology. This appendix explains our use of efficiency units. K_t and L_t denote aggregate capital and labor. Using multiplicative damages and abatement costs, we replace $G(z)$ with $G(z,t)$:

$$G(z,t) = D(E_t)\,\Lambda(\mu_t,t)\,F(K_t, A_tL_t)\,.$$

Time enters explicitly via exogenous changes in population and labor-augmenting technical change. We obtain efficiency units by dividing variables by A_tL_t, defining

$$l_t \equiv \frac{L_t}{A_tL_t} \text{ and } k_t \equiv \frac{K_t}{A_tL_t} \tag{52}$$

With the Cobb Douglas form $F\left(K_t, A_tL_t\right) = \left(A_tL_t\right)^{\beta} K_t^{1-\beta}$, we have

$$f\left(k_t\right) \equiv F\left(\frac{K_t}{A_tL_t}, \frac{A_tL_t}{A_tL_t}\right) = k_t^{1-\beta}.$$

Defining $g\left(k_t, E_t, \mu_t\right) \equiv \frac{G(z,t)}{A_tL_t}$, we have

$$\begin{gathered}G\left(z,t\right) = D\left(E_t\right)\Lambda\left(\mu_t, t\right)F\left(K_t, A_tL_t\right) = \\ D\left(E_t\right)\Lambda\left(\mu_t, t\right)A_tL_tF\left(\tfrac{K_t}{A_tL_t}, \tfrac{A_tL_t}{A_tL_t}\right) = \\ D\left(E_t\right)\Lambda\left(\mu_t, t\right)A_tL_tk_t^{1-\beta} = A_tL_tg\left(k_t, E_t, \mu_t, t\right) \Rightarrow \\ g\left(k_t, E_t, \mu_t, t\right) = D\left(E_t\right)\Lambda\left(\mu_t, t\right)k_t^{1-\beta}.\end{gathered} \tag{53}$$

As in the text, we use w_t and r_t to denote the factor prices as functions of the physical units. From equation 6, the return to capital is:

$$\begin{gathered}r_t = \phi\left(p_t\right)G_K = \phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)F_K\left(K_t, A_tL_t\right) = \\ \left(1-\beta\right)\phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)\left(A_tL_t\right)^{\beta}K_t^{1-\beta}\tfrac{1}{K_t}\left(\tfrac{A_tL_t}{A_tL_t}\right) = \\ \left(1-\beta\right)\phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)k_t^{1-\beta}\tfrac{1}{k_t} = \left(1-\beta\right)\phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)k_t^{-\beta}.\end{gathered} \tag{54}$$

We define $\hat{w}_t \equiv \beta\phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)k_t^{1-\beta}$. From equation 6 we have the return to labor:

$$\begin{gathered}w_t = \phi\left(p_t\right)G_L = \phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)F_L\left(K_t, A_tL_t\right) = \\ \phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)\tfrac{\beta}{L_t}\left(A_tL_t\right)^{\beta}K_t^{1-\beta}\left(\tfrac{A_tL_t}{A_tL_t}\right) = \\ \phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)\tfrac{\beta}{L_t}\left(\tfrac{A_tL_t}{A_tL_t}\right)^{\beta}\left(\tfrac{K_t}{A_tL_t}\right)^{1-\beta}\left(A_tL_t\right) = \\ A_t\beta\phi\left(p_t\right)D\left(E_t\right)\Lambda\left(\mu_t, t\right)k_t^{1-\beta} = A_t\hat{w}_t.\end{gathered} \tag{55}$$

The cost of hiring a physical unit of labor is w_t, so the cost of hiring $\frac{1}{A_t}$ units of labor (equal to one efficiency unit of labor) of labor is $\hat{w}_t = \frac{w_t}{A_t}$.

The young generation's aggregate income in period t is $w_t L_t = A_t L_t \hat{w}_t$ and their aggregate savings is $p_t K_{t+1} = p_t A_{t+1} L_{t+1} k_{t+1}$ so *per capita* consumption of the young agent in period t is[20]

$$\begin{aligned} c_t^y = \frac{1}{L_t}\left(A_t \hat{w}_t L_t - p_t A_{t+1} L_{t+1} k_{t+1}\right) = A_t\left(\hat{w}_t - p_t \frac{A_{t+1}L_{t+1}}{A_t L_t} k_{t+1}\right) \\ = A_t\left(\hat{w}_t - p_t n_t k_{t+1}\right), \end{aligned} \tag{56}$$

where we use the definition $n_t \equiv \frac{A_{t+1}L_{t+1}}{A_t L_t}$.

The old agent's per capita consumption in period $t+1$ is

$$\begin{aligned} c_{t+1}^o = \frac{1}{L_t}\left(r_{t+1} + p_{t+1}\left(1-\delta\right)\right) A_{t+1} L_{t+1} k_{t+1} = \\ A_t\left(r_{t+1} + p_{t+1}\left(1-\delta\right)\right) \frac{A_{t+1}L_{t+1}}{A_t L_t} k_{t+1} = A_t\left(r_{t+1} + p_{t+1}\left(1-\delta\right)\right) n_t k_{t+1}. \end{aligned} \tag{57}$$

The ratio of discounted marginal utility, defined in equation 2, given iso-elastic utility, is

$$\begin{aligned} \psi_t \equiv \frac{U'(c_t^y)}{\rho\ U'(c_{t+1}^o)} = \frac{\left[A_t(\hat{w}_t - p_t n_t k_{t+1})\right]^{-\eta}}{\rho\left[A_t(r_{t+1} + p_{t+1}(1-\delta)) n_t k_{t+1}\right]^{-\eta}} \\ = \frac{\left[(\hat{w}_t - p_t n_t k_{t+1})\right]^{-\eta}}{\rho\left[(r_{t+1} + p_{t+1}(1-\delta)) n_t k_{t+1}\right]^{-\eta}}. \end{aligned} \tag{58}$$

Using the equality in equation 2 produces the asset price equation in the non-stationary model

$$\frac{\left[(\hat{w}_t - p_t n_t k_{t+1})\right]^{-\eta}}{\rho\left[\left(r_{t+1} + p_{t+1}\left(1-\delta\right)\right) n_t k_{t+1}\right]^{-\eta}} = \frac{r_{t+1} + p_{t+1}(1-\delta)}{p_t}. \tag{59}$$

[20]Note: c_t^y is per capita consumption. Aggregate consumption of the young generation is $C_t^y \equiv w_t L_t - p_t K_{t+1}$, and $c_t^y = \frac{C_t^y}{L_t}$. It is not $\frac{C_t^y}{A_t L_t}$. We could express c in efficiency units instead of per capita units, but then we have to work harder to write the asset price equation and the maximand for the political economy planner.

The equation of motion for capital, expressed in efficiency units, is

$$
\begin{gathered}
K_{t+1} = (1-\delta)\, K_t + I_t \Rightarrow \\
\tfrac{K_{t+1}}{A_t L_t} = A_{t+1} L_{t+t} \tfrac{k_{t+1}}{A_t L_t} = (1-\delta)\, k_t + i_t \Rightarrow \\
k_{t+1} = \tfrac{(1-\delta) k_t + i_t}{n_t}.
\end{gathered}
\tag{60}
$$

with

$$
I_t = x\,(p_t)\, G\,(\mathbf{z}_t) \Rightarrow i_t = x\,(p_t)\, g\,(k_t, E_t, \mu_t, t) \tag{61}
$$

Using equation 10 and setting $\epsilon = 0$ (because we are using the TCRE model), the equation of motion for carbon expressed in efficiency units is

$$
E_{t+1} = E_t + (1-\mu_t)\zeta_t\, (A_t L_t)^{\beta}\, K_t^{1-\beta} = E_t + (1-\mu_t)\zeta_t A_t L_t k_t^{1-\beta}. \tag{62}
$$

B.4 Relation to Karp and Rezai (2014)

Karp and Rezai (2014) use a Lucas tree model with zero investment to show how asset markets can create incentives to conserve a generic resource (e.g. a fishery). There, with a fixed capital stock, all adjustment occurs via asset prices. In the composite commodity setting, with a fixed asset price, all adjustment occurs via quantity. In the current paper, both investment and the asset price are endogenous.

Karp and Rezai assume that agents have infinite EIS. As noted in the text, the two current generations' incentives to reduce emissions, and their (lack of) alignment of interests, are sensitive to this parameter, which here takes any positive value. For $EIS \to \infty$ we find that the two generations' incentives are aligned (Proposition 1).

Finally, Karp and Rezai do not include a climate-related stock (e.g. atmospheric carbon, or temperature). These restrictions make their model unrelated to climate change. Moreover, that paper uses a numerical example not calibrated to a specific real-world stock. In contrast, the current paper uses a carefully calibrated climate model.

However, Karp and Rezai include two consumption goods, produced using mobile labor and sector-specific factors: the fixed stock of capital in the Manufacturing sector, and the endogenously changing resource stock in the Resource sector. In this setting, the relative commodity price affects agent's incentive to conserve the resource. Our paper also has two sectors, the consumption good and the investment good, where all factors are mobile. Here, the price of investment relative to the numeraire consumption is central. Thus, neither paper is a special case of the other.

C Computational Algorithm

This section discusses the computational algorithm used to solve the equilibrium described in (1) under: *(i)* discounted utilitarian planner (Eq. (16), Appendix C.1); *(ii)* Business as Usual (BAU) planner (Appendix C.3); and *(iii)* dynamic game between planners (Eq. (14), Appendix C.2), under Assumption (2).

Let $t = 1, \ldots, H$, we set the horizon of the H-periods model to $H = 14$ (490 years), which is long enough that a change in H has no discernible effect on policies in the first 200 years. All cases are solved by backward induction.

We discretize the state of capital per effective worker $\mathbf{K}$ and stock of atomospheric carbon $\mathbf{E}$ in grids of 32 points. Grid points of $k \in \mathbf{K} \equiv [k^{\min}, k^{\max}]$ are linearly spaced between with $k^{\min} = 0.0001$ and $k^{\max} = 0.12$. For reference, the capital per effective worker in the first period is k_1=0.0093. Hence, we allocate points over the stock of atomospheric carbon in $\mathbf{E} \equiv [E^{\min}, E^{\max}] = [0, 6000]$. The upper bound $E^{\max} = 6000$, coincides with a 12 degree Celsius increase in temperature and a loss in output of XX%. We allocate points on the $\mathbf{E}$-grid using the polynomial rule $x_i = (i/32))^{\theta}\, E^{\max}$ described in Maliar et al. (2010). This rule parsimoniously handles the allocation of (very) low and high level of stock of atmospheric carbon and provides a numerically stable way of addressing the incentives of our sophysticated planners to remove the stock of carbon from the atmosphere when it becomes technologically cheaper to do so. We find $\theta = 4$ to work well for our baseline calibration.

When the level of abatement and prices imply levels of k' (Eq. (60)) and E' (Eq. (62)) that fall within grid points, we use cubic splines interpolation to compute relevant numerical objects. When k' and E' falls outside the intervals $\mathbf{K}$ and $\mathbf{E}$, we cap these values at the boundary levels. For internal consistency, we add penalty functions to the equilibrium conditions to rule out these cases.

To validate interpolation scheme and grid choices, we produce results under our baseline calibration for different grid sizes (8, 16 and 32 grid points).

Root-finding algorithm constraints. We impose the following boundaries on the root-finding algorithms. First, we restrict the global search to positive level of abatement and prices. We also require the level of abatement to be less than 1.7. Finally, we set the minimum level of abatement to be weakly greater than 1 when $E = 6000$. This constraint disciplines the behaviour of our model economy under BAU planner, which would otherwise let the temperature rise at implausibly high levels.

The next sections discuss the details of the solution algorithm that are planner specific.

C.1 Discounted Utilitarian

At each state (k_t, E_t, t) the discounted utilitarian planner chooses investment per effective worker i and abatement μ to solve (16). We solve this bellman equation via value function iteration. We set the boundary condition, $J(k_{H+1}, E_{H+1}, H+1)$, $\forall (k_{H+1}, E_{H+1}) \in \mathbf{K} \times \mathbf{E}$. We determine the policy functions $(i^*(k_t, E_t, t), \mu(k_t, E_t, t))$ in a two-step approach. First, we perform a grid search over 200 grid points on investment per effective workers (linearly distributed between 0 and $k^{\max}$) and abatement (linearly distributed between 0 and 1.7). Second, we use a constrained nonlinear optimization algorithm[21] to perform a local search in the neighborhood of the grid-search solution. This solution method guarantees the solution is a global optimum and alleviates numerical instabilities arising from the flatness of the return function at later

[21] The Matlab *fmincon* routine, with 'interior-point' algorithm. For more information: https://www.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.htmlbrnpd5f.

horizons. To improve the numerical stability, we adopt only the grid search for any periods after $T^* = 8$.

C.2 Markov Perfect Equilibrium Algorithm

This section discusses the numerical solution of the Markov Perfect Equilibrium (defined in 1) under (Nash-)bargaining assumption (2), presented in Section 4. At each state (k_t, E_t, t), agents choose the level of investment per effective unit that satisfies the asset pricing equation (3). At each state (k_t, E_t, t), the sophysticated planner choose abatement μ to solve (14), taking as given the investment per effective unit.

Solution Algorithm. After setting the boundary conditions $(p(k_{H+1}, E_{H+1}, H+1) = \mu(k_{H+1}, E_{H+1}, H+1) = 0$, we implement the following backward-induction solution algorithm.

1. Start at $t = H$. For each (k_t, E_t, t), $(p(k_t, E_t, t), \mu(k_t, E_t, t))$ solve the following system in two equations and two unknowns

$$\begin{aligned} &F_1(p(k_t, E_t, t), \mu(k_t, E_t, t), k_t, E_t, t) = 0 \qquad \text{(Asset Pricing Equation)} \\ &F_2(p(k_t, E_t, t), \mu(k_t, E_t, t), k_t, E_t, t) = 0 \qquad \text{(FOC Planner)} \end{aligned} \tag{63}$$

   as described in the next section.

2. Iterate backward and implement the solution step at $t = H - 1$. Proceed in the same fashion until $t = 1$.

C.2.1 Solution Step.

Given time t, the solution algorithm finds for every state (E_t, k_t) the equilibrium functions $(p(k_t, E_t, t), \mu(k_t, E_t, t))$ as the solution to the previous system of two equations in two unknowns. To save notation, we drop the dependence of the equilibrium functions on the state (E_t, k_t), and rewrite

$$F_1(p_t^*, \mu_t^*) = 0$$

$$F_2(p_t^*, \mu_t^*) = 0$$

where $p_t^* = p(k_t, E_t, t)$ and $\mu^* = \mu(k_t, E_t, t)$. The algorithm proceeds as follows.

1. Guess (p_t^0, μ_t^0)
2. If $F_1(p_t^0, \mu_t^0) \neq 0$ and $F_2(p_t^0, \mu_t^0) \neq 0$ then pick a new guess (p_t^0, μ_t^0). Otherwise exit.

We find the solution using a system of nonlinear equation solver.[22] The algorithm checks for corner solutions, computed setting $\mu = 0$ and letting agents optimize (i.e. $F_1(p, 0) = 0$). Given the guess of p_t^*, we use

$$I_t = x\left(p_t^*\right) G\left(\mathbf{z}_t\right) \Rightarrow i_t = x\left(p_t^*\right) g\left(k_t, E_t, \mu_t, t\right) \tag{64}$$

to obtain the equilibrium investment per effective unit $i(k_t, E_t, t)$.

C.2.2 Equilibrium Conditions.

This section details the equilibrium conditions. Given (p_t^0, μ_t^0)

- **Asset Pricing,** $F_1 = 0$. The first equilibrium condition is the asset pricing equation (3).

$$F_1(p_t^0, \mu_t^0) = p_t^0 - \frac{p(k_{t+1}(\mu_t^0, p_t^0), E_{t+1}(\mu_t^0, p_t^0), t+1)(1-\delta) + r_{t+1}(E_{t+1}(\mu_t^0, p_t^0), k_{t+1}(\mu_t^0, p_t^0)}{\psi_t(\mu_t^0, p_t^0)}$$

- **Social Planner Optimization,** $F_2 = 0$.

  The planner's objective in the political economy setting is

$$\max \tfrac{1}{1-\eta} L_t \left[\xi \left(\left(c_t^y\right)^{1-\eta} + \rho \left(c_{t+1}^o\right)^{1-\eta}\right) + (1-\xi) \tfrac{L_{t-1}}{L_t} \left(c_t^0\right)^{1-\eta}\right]. \tag{65}$$

  We use Lemma 2 to rewrite the planner's objective as

$$\max_{\mu} \left[\xi L_t \frac{\left(c_t^y\right)^{-\eta}}{1-\eta} w_t + (1-\xi) L_{t-1} \frac{\left[c_t^o\right]^{1-\eta} - 1}{1-\eta}\right] \tag{66}$$

[22] The Matlab *fsolve* routine, with 'trust-region dogleg' algorithm. For more information: https://www.mathworks.com/help/optim/ug/fsolve.html.

Hence we simplify the objective function

$$\xi(c_t^y)^{-\eta}w_t + (1-\xi)\,\frac{L_{t-1}}{L_t}(c_t^o)^{1-\eta}$$

and we take first order condition with respect to abatement:

$$\xi\left[-\eta(c_t^y)^{-\eta-1}\frac{\partial c_t^y}{\partial \mu_t}w_t + (c_t^y)^{-\eta}\frac{\partial w_t}{\partial \mu_t}\right] + (1-\xi)\,\frac{L_{t-1}}{L_t}(1-\eta)(c_t^o)^{-\eta}\frac{\partial c_t^o}{\partial \mu_t} = 0$$
$$(c_t^y)^{-\eta}\xi\left[-\eta\frac{1}{c_t^y}\frac{\partial c_t^y}{\partial \mu_t}w_t + \frac{\partial w_t}{\partial \mu_t}\right] + (1-\xi)\,\frac{L_{t-1}}{L_t}(1-\eta)(c_t^o)^{-\eta}\frac{\partial c_t^o}{\partial \mu_t} = 0$$

Let us recover the partial derivatives:

$$\frac{\partial \Lambda(\mu_t,t)}{\partial \mu_t}(p_t^0,\mu_t^0) = -\nu_1(t)\nu_2\cdot\mu^{\nu_2-1}$$
$$\frac{\partial \phi(p_t)}{\partial p_t}(p_t^0,\mu_t^0) = \left(\frac{1}{a^\sigma+1}\right)^{\frac{1}{\sigma+1}}\left(p^{(\sigma+1)}+a^\sigma\right)^{\frac{-\sigma}{\sigma+1}}p^\sigma$$
$$\frac{\partial w_t}{\partial \mu_t}(p_t^0,\mu_t^0) = A_t\beta D(E_t)k_t^{1-\beta}\left[\frac{\partial \phi(p_t)}{\partial p_t}\mathbf{I}_\phi\frac{\partial p_t}{\partial \mu_t}\Lambda(\mu_t,t) + \phi(p_t)\frac{\partial \Lambda(\mu_t,t)}{\partial \mu_t}\right]$$
$$\frac{\partial c_t^y}{\partial \mu_t}(p_t^0,\mu_t^0) = \frac{\partial w_t}{\partial \mu_t} - A_t n_t k_{t+1}\frac{\partial p_t}{\partial \mu_t}$$
$$\frac{\partial r_t}{\partial \mu_t}(p_t^0,\mu_t^0) = (1-\beta)D(E_t)k_t^{-\beta}\left[\frac{\partial \phi(p_t)}{\partial p_t}\mathbf{I}_\phi\frac{\partial p_t}{\partial \mu_t}\Lambda(\mu_t,t) + \phi(p_t)\frac{\partial \Lambda(\mu_t,t)}{\partial \mu_t}\right]$$
$$\frac{\partial c_t^o}{\partial \mu_t}(p_t^0,\mu_t^0) = A_{t-1}n_{t-1}k_t\left(\frac{\partial r_t}{\partial \mu_t} + (1-\delta)\frac{\partial p_t}{\partial \mu_t}\right)$$

and substitute the partial derivatives in the FOC:

$$(c_t^y)^{-\eta}\xi\left[-\eta\frac{w_t}{c_t^y}\frac{\partial c_t^y}{\partial\mu_t}+\frac{\partial w_t}{\partial\mu_t}\right]+(1-\xi)\frac{L_{t-1}}{L_t}(1-\eta)(c_t^o)^{-\eta}\frac{\partial c_t^o}{\partial\mu_t}=0$$

$$(c_t^y)^{-\eta}\xi\left[\left(1-\eta\frac{w_t}{c_t^y}\right)\frac{\partial w_t}{\partial\mu_t}+\eta\frac{w_t}{c_t^y}A_tn_tk_{t+1}\frac{\partial p_t}{\partial\mu_t}\right]$$
$$+(1-\xi)\frac{L_{t-1}}{L_t}(1-\eta)(c_t^o)^{-\eta}A_{t-1}n_{t-1}k_t\left(\frac{\partial r_t}{\partial\mu_t}+(1-\delta)\frac{\partial p_t}{\partial\mu_t}\right)=0$$

Let

$$\Gamma_1=(c_t^y)^{-\eta}\xi\left(1-\eta\frac{w_t}{c_t^y}\right)$$
$$\Gamma_2=(c_t^y)^{-\eta}\xi\eta\frac{w_t}{c_t^y}A_tn_tk_{t+1}$$
$$\Gamma_3=(1-\xi)\frac{L_{t-1}}{L_t}(1-\eta)(c_t^o)^{-\eta}A_{t-1}n_{t-1}k_t$$
$$\Gamma_4=A_t\beta D(E_t)k_t^{1-\beta}\frac{\partial\phi(p_t)}{\partial p_t}\mathbf{I}_\phi\Lambda(\mu_t,t)$$
$$\Gamma_5=A_t\beta D(E_t)k_t^{1-\beta}\phi(p_t)\frac{\partial\Lambda(\mu_t,t)}{\partial\mu_t}$$
$$\Gamma_6=(1-\beta)D(E_t)k_t^{-\beta}\frac{\partial\phi(p_t)}{\partial p_t}\mathbf{I}_\phi\Lambda(\mu_t,t)$$
$$\Gamma_7=(1-\beta)D(E_t)k_t^{-\beta}\phi(p_t)\frac{\partial\Lambda(\mu_t,t)}{\partial\mu_t}$$

Rewrite the FOC as follows

$$\Gamma_1\frac{\partial w_t}{\partial\mu_t}+\Gamma_2\frac{\partial p_t}{\partial\mu_t}+\Gamma_3\frac{\partial r_t}{\partial\mu_t}+\Gamma_3(1-\delta)\frac{\partial p_t}{\partial\mu_t}=0$$

$$\Gamma_1\left\{A_t\beta D(E_t)k_t^{1-\beta}\left[\frac{\partial\phi(p_t)}{\partial p_t}\mathbf{I}_\phi\frac{\partial p_t}{\partial\mu_t}\Lambda(\mu_t,t)+\phi(p_t)\frac{\partial\Lambda(\mu_t,t)}{\partial\mu_t}\right]\right\}+\Gamma_2\frac{\partial p_t}{\partial\mu_t}+$$
$$\Gamma_3\left\{(1-\beta)D(E_t)k_t^{-\beta}\left[\frac{\partial\phi(p_t)}{\partial p_t}\mathbf{I}_\phi\frac{\partial p_t}{\partial\mu_t}\Lambda(\mu_t,t)+\phi(p_t)\frac{\partial\Lambda(\mu_t,t)}{\partial\mu_t}\right]\right\}+\Gamma_3(1-\delta)\frac{\partial p_t}{\partial\mu_t}=0$$

$$\Gamma_1\Gamma_4\frac{\partial p_t}{\partial\mu_t}+\Gamma_1\Gamma_5+\Gamma_2\frac{\partial p_t}{\partial\mu_t}+\Gamma_3\Gamma_6\frac{\partial p_t}{\partial\mu_t}+\Gamma_3\Gamma_7+\Gamma_3(1-\delta)\frac{\partial p_t}{\partial\mu_t}=0$$

Then, given (p_t^0, μ_t^0), $\frac{\partial p_t}{\partial \mu_t}$ is the solution of a linear equation in one unknown:

$$\frac{\partial p_t}{\partial \mu_t}(p_t^0, \mu_t^0) = -\frac{\Gamma_1\Gamma_5 + \Gamma_3\Gamma_7}{\Gamma_1\Gamma_4 + \Gamma_2 + \Gamma_3\Gamma_6 + \Gamma_3(1-\delta)} \equiv LHS(\mu_t^0, p_t^0) \tag{67}$$

Since the asset pricing equation (3) holds in every period t, we have that $\frac{\partial p_t}{\partial \mu_t}$ is also equal to

$$\frac{\partial p_t}{\partial \mu_t} = \frac{\partial \left[(p_{t+1}(1-\delta) + r_{t+1}(\mu_t^0, p_t^0))/\psi_t \right]}{\partial \mu_t} \equiv RHS(\mu_t^0, p_t^0) \tag{68}$$

So, we can express

$$F_2(p_t^0, \mu_t^0) = LHS(\mu_t^0, p_t^0) - RHS(\mu_t^0, p_t^0)$$

Let us now determine $RHS(\mu_t^0, p_t^0)$. Using equation

$$p_t = \frac{\rho}{A_t n_t k_{t+1}} \left(c_{t+1}^o\right)^{1-\eta} \cdot \left(c_t^y\right)^\eta$$

we can rewrite

$$\begin{aligned}
RHS(\mu_t^0, p_t^0) &= \frac{\rho}{A_t n_t k_{t+1}} \frac{\partial \left(\left(c_{t+1}^o\right)^{1-\eta} \cdot \left(c_t^y\right)^\eta \right)}{\partial \mu_t} \\
&= \frac{\rho}{A_t n_t k_{t+1}} \left[\left(c_t^y\right)^\eta \frac{\partial \left(c_{t+1}^o\right)^{1-\eta}}{\partial \mu_t} + \left(c_{t+1}^o\right)^{1-\eta} \frac{\partial \left(c_t^y\right)^\eta}{\partial \mu_t} \right] \\
&= \frac{\rho}{A_t n_t k_{t+1}} \left[\left(c_t^y\right)^\eta (1-\eta) \left(c_{t+1}^o\right)^{-\eta} \frac{\partial c_{t+1}^o}{\partial \mu_t} + \left(c_{t+1}^o\right)^{1-\eta} \eta \left(c_t^y\right)^{\eta-1} \frac{\partial c_t^y}{\partial \mu_t} \right] \\
&= \frac{1}{A_t n_t k_{t+1}} \frac{1}{\psi_t} \left[(1-\eta) \frac{\partial c_{t+1}^o}{\partial \mu_t} + \eta \cdot \frac{c_{t+1}^o}{c_t^y} \cdot \frac{\partial c_t^y}{\partial \mu_t} \right]
\end{aligned}$$

We determine $\frac{\partial c_t^y}{\partial \mu_t}$ and $\frac{\partial c_{t+1}^o}{\partial \mu_t}$, by collecting the partial derivatives

$$\frac{\partial D(E_{t+1})}{\partial E_{t+1}} = -2 * TCRE^2 \frac{beeE_{t+1}}{(1 + bee(TCRE\, E_{t+1})^2)^2}$$

$$\frac{\partial E_{t+1}}{\partial \mu_t} = -\zeta A_t L_t \cdot k_t^{1-\beta}$$

$$\frac{\partial p_{t+1}}{\partial \mu_t}(k_{t+1}, E_{t+1}) = \underbrace{\frac{\partial p_{t+1}}{\partial E_{t+1}}}_{\text{Numerical}} \frac{\partial E_{t+1}}{\partial \mu_t} + \frac{\partial p_{t+1}}{\partial k_{t+1}} \underbrace{\frac{\partial k_{t+1}}{\partial \mu_t}}_{=0(\text{Nash})} = \underbrace{\frac{\partial p_{t+1}}{\partial E_{t+1}}}_{\text{Numerical}} |_{E_{t+1}(p_t^0,\mu_t^0),k_{t+1}(p_t^0,\mu_t^0)} \frac{\partial E_{t+1}}{\partial \mu_t}$$

$$\frac{\partial \mu_{t+1}}{\partial \mu_t}(k_{t+1}, E_{t+1}) = \underbrace{\frac{\partial \mu_{t+1}}{\partial E_{t+1}}}_{\text{Numerical}} |_{E_{t+1}(p_t^0,\mu_t^0),k_{t+1}(p_t^0,\mu_t^0)} \frac{\partial E_{t+1}}{\partial \mu_t}$$

$$\begin{aligned}\frac{\partial r_{t+1}}{\partial \mu_t}(p_t^0, \mu_t^0) = (1-\beta)\, k_{t+1}^{-\beta} \Big[& \frac{\partial D\,(E_{t+1})}{\partial E_{t+1}} \frac{\partial E_{t+1}}{\partial \mu_t} \Lambda\,(\mu_{t+1}, t+1)\, \phi\,(p_{t+1}) + \\ & D\,(E_{t+1}) \frac{\partial \Lambda\,(\mu_{t+1}, t+1)}{\partial \mu_{t+1}} \frac{\partial \mu_{t+1}}{\partial \mu_t} \phi\,(p_{t+1}) + \\ & D\,(E_{t+1})\, \Lambda\,(\mu_{t+1}, t+1)\, \frac{\partial \phi\,(p_{t+1})}{\partial p_{t+1}} \mathbf{I}_\phi \frac{\partial p_{t+1}}{\partial \mu_t} \mathbf{I}_{p'} \Big]\end{aligned}$$

$$\frac{\partial c_{t+1}^o}{\partial \mu_t}(p_t^0, \mu_t^0) = A_t n_t k_{t+1} \left[(1-\delta) \frac{\partial p_{t+1}}{\partial \mu_t} \mathbf{I}_{p'} + \frac{\partial r_{t+1}}{\partial \mu_t} \right]$$

$$\frac{\partial c_t^y}{\partial \mu_t}(p_t^0, \mu_t^0) = A_t \beta D(E_t) k_t^{1-\beta} \left[\frac{\partial \phi(p_t)}{\partial p_t} \mathbf{I}_\phi \frac{\partial p_t}{\partial \mu_t} \Lambda(\mu_t, t) + \phi(p_t) \frac{\partial \Lambda(\mu_t, t)}{\partial \mu_t} \right] - A_t n_t k_{t+1} \frac{\partial p_t}{\partial \mu_t}$$

C.3 Business-As-Usual

The solution of the equilibrium under business as usual, is obtained by setting $\mu(k_t, E_t, t) = 0$ for each state (k_t, E_t) and every t. and reducing the system of equations (63) to

$$F_1(p(k_t, E_t, t), \mu(k_t, E_t, t), k_t, E_t, t) = 0 \qquad \text{(Asset Pricing Equation)}$$

C.4 The Derivative of Price with Respect to Abatement

The derivative $\frac{\partial p_t}{\partial \mu_t}$ captures how the planner evaluates a change of mitigation on the asset price. This differs across planners. While in every equilibrium we have the asset price equation 3 holding, i.e. $p_t = (p_{t+1}(1-\delta)+r_{t+1}(\mu_t^0, p_t^0))/\varphi_t$, the derivative varies across planners:

1. Our **current sophisticated planner** takes investment as given (the Nash assumption) but recognizes that the asset price depends on the next-period stock of carbon. This planner has rational expectations insofar as she correctly anticipates the current level of investment but she is unaware of the supply function, $i_t = x\left(p_t\right) g\left(k_t, E_t, \mu_t, t\right)$, that fixes the relation between i and p (for a given μ_t). This means that the planner takes into account both expressions in the sum, i.e. the effect of mitigation on today's assets includes the change of mitigation on tomorrow's rental rate (but only via the lower stock of carbon, not via changes on investment and tomorrow capital price) and tomorrow's asset price.

$$\frac{\partial p_t}{\partial \mu_t} = \frac{\partial\left[(p_{t+1}(1-\delta)+r_{t+1}(\mu_t^0, p_t^0))/\varphi_t\right]}{\partial \mu_t} \equiv RHS(\mu_t^0, p_t^0)$$

a In order to solve the problem for this planner, we use the function $p_{t+1} = \Psi\left(k_{t+1}, E_{t+1}, t+1\right)$ obtained from iterating on the asset price equation up to the period $t+1$.

b We substitute this function into the asset price equation 3. Using the equations for $\hat{w}$ and r, and taking k_{t+1} as given (due to the Nash assumption) we obtain an equation for p_t as a function of μ_t, given k_t, k_{t+1}, and E_t and t; call this function $p_t = \Xi\left(k_t, k_{t+1}, E_t, \mu_t, t\right)$.

c The planner sets $\frac{\partial p_t}{\partial \mu_t}$ equal to $\frac{\partial \Xi}{\partial \mu_t}$ and we have

$$F_2(p_t^0, \mu_t^0) = LHS(\mu_t^0, p_t^0) - RHS(\mu_t^0, p_t^0)$$

2. Our **business-as-usual planner** takes both investment and (asset and factor) prices as given. As a result, $\frac{\partial p_t}{\partial \mu_t} = 0$ and equilibrium mitigation

is zero (at its lower bound). The LHS expression above will always be negative. The planner would like to choose negative mitigation levels.