Pricing Climate Risk

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Abstract

Anthropogenic greenhouse gas emissions are changing the energy balance of our planet. Various climatic feedbacks make the resulting warming over the next decades and centuries highly uncertain. We derive a general analytic formula for the "risk premium" on the resulting climate policy. It clarifies the distinct roles of risk aversion, prudence, characteristics of the damage formulation, and future policy response. The formula generalizes simple precautionary savings analysis to more complex economic interactions and trains economic intuition about policy making under uncertainty. The formula relies on approximations. We compare the results to the optimal policy in a recursive stochastic dynamic programming implementation of DICE. Our analytic formula implies only small errors, clarifies the relative importance of the different risk channels and structural assumptions, and allows researchers to derive estimates of uncertainty's policy impact from deterministic models. Our quantitative model contributes estimates of the climate policy risk premium for a variety of scenarios.

JEL Codes: Q54, Q00, D90, C63
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1 Introduction

Economic activity relies heavily on fossil fuel use. The resulting emissions alter our planet's energy balance and impact future climate, economic activity, and human well-being. A variety of feedback processes make the temperature response to greenhouse gases highly uncertain. The World Economic Forum (2019) identifies the resulting increase in extreme weather events and the potential failure to address climate change through mitigation and adaptation as the top two global risks by likelihood, outranked only in impact by weapons of mass destruction. The present paper explains and quantifies the optimal policy response to structural climate change uncertainty and temperature stochasticity. We provide a general analytic formula for the climate risk premium and demonstrate its accuracy using a stochastic dynamic programming implementation of Bill Nordhaus' Nobel-awarded DICE model (Nordhaus & Sztorc 2013). We identify the different risk premium channels and the structural importance of various model assumptions. In particular, we show that structural uncertainty about the climate's sensitivity has different policy implications from temperature stochasticity, which is often used as a proxy.

The Intergovernmental Panel on Climate Change (IPCC) no longer agrees on a best-guess point estimate for the warming resulting from a doubling of carbon dioxide (CO_2) concentrations. The deterministic DICE model suggests a bit more than such a doubling along the optimal path. The temperature response to a doubling of atmospheric CO_2 is called the *climate sensitivity*.[1] The IPCC states that it is likely in the range of 1.5°C to 4.5°C, and many probabilistic estimates still place substantial mass on values several degrees higher (Stocker et al. 2013). How should we tax (or cap) greenhouse gases today acknowledging this climatic uncertainty? What mechanisms are driving of a potential risk premium? How do the different channels compare quantitatively? These are crucial questions we answer in this paper. More generally, we develop a new method how to simulate the optimal uncertainty response using a deterministic model and we extend common precautionary savings reasoning to a world where uncertainty propagates through economic production.

Reducing greenhouse gases is an investment into future capital. The capital is a

[1]The relation between the CO_2 concentration and global warming is logarithmic and climate sensitivity characterizes the absolute temperature increase resulting from a doubling of the CO_2 concentrations. We are currently some 70% toward a doubling w.r.t. pre-industrial concentrations, evaluating the different anthropogenic greenhouse gases by their 100 year global warming potential CO_2 equivalents.

climate favorable for economic activity and human welfare. Kimball (1990) taught us that a precautionary savings response to an uncertain capital evolution is not driven by risk aversion but by *prudence*, the third moment of our utility function: we can mitigate risk's harm by saving if our risk aversion decreases in wealth. In the typical precautionary savings setting, risk's harm directly links to the shock's utility cost. The present setting is more complex. Uncertainty propagates through the climate and economic productivity before translating into a utility impact. We introduce a convexity and a change-of-convexity measure for economic production, resembling Arrow-Pratt risk aversion and Kimball's (1990) prudence. The precautionary savings for climate depends on the interaction of these welfare and productivity measures.

We distinguish between the *structural uncertainty* about the climate's sensitivity to CO_2 and a merely stochastic evolution of global temperatures. Under uncertainty about the climate's sensitivity, risk aversion and convexity of production in temperature contribute directly. In contrast, prudence delivers a negligible contribution to precautionary climate savings. This difference to the usual precautionary savings model is driven by the fact that we have structural uncertainty about the way that each unit of CO_2 in the atmosphere translates into future warming. The biggest player for the climate risk premium is the convexity of production in temperatures. This contribution is absent in the response to a merely *stochastic climate* lacking the structural component. This finding is important because models and empirical papers frequently use stochastic temperature evolution as a proxy for structural uncertainty or even for deterministic long-run temperature change. Yet, the economic response to each of these scenarios differs substantially.

It is not possible to derive the risk premium's exact formula for a generic dynamic integrated assessment model of climate change. A crucial contribution of our paper is the following insight; (i) a simplification permits an analytic derivation of the risk premium *and* (ii) the quantitative implications of the assumption can easily be fixed ex-post. After the fix, our analytic formula matches the numeric risk premium up to an error of just a few percent for most scenarios and allows us to identify and quantify the different risk channels. The fix relies on deriving a crucial response elasticity from the deterministic base model.

To test our results, we rely on a stochastic dynamic programming implementation of DICE. It relates closely to Kelly & Kolstad (1999), a seminal paper that introduces quantitative stochastic modeling into the integrated assessment of climate change and was sligthly extended Leach (2007). These pioneering papers focus on the time to learn the climate's sensitivity. Our analysis focuses on today's optimal policy and distinguishes the policy contributions from temperature stochasticity and structural uncertainty governing climate policy. Kelly & Tan (2013) investigate uncertainty about climatic feedbacks in a numeric integrated assessment model with catastrophic damages resulting from a fat-tailed probability distribution. Their results suggest that the uncertainty effect derived from the fat tail is considerable in the first decade but wears off quickly as learning shrinks the tail. Millner et al. (2013) evaluate welfare loss under ambiguity about the climate sensitivity but do not analyze the impact on climate policy. Lemoine & Traeger (2014, 2016a, 2016b) model abrupt and irreversible changes in the climate's sensitivity resulting from crossing an unknown temperature threshold, using models with a single tipping point, permitting for a domino-style tipping interaction, and looking at the implications of ambiguity aversion. Ongoing work by Rudik et al. (2020) substantially refines the learning model of the earlier literature. Using a Smolyak grid to curtail the numeric "curse" of dimensionality, they introduce a more sophisticated model of the climate learning dynamics. A related literature has analyzed the consequences of damage uncertainty (Crost & Traeger 2014, Cai & Lontzek 2019, Rudik 2019), and has compared the effectiveness of different climate policy instruments under various uncertainties (Hoel & Karp 2001, Newell & Pizer 2003, Kelly 2005, Karp & Zhang 2006, Fischer & Springborn 2011, Karp & Traeger 2018, Pizer & Prest 2019). In particular, Karp & Zhang (2006) compare taxes and quotas under learning about the relation between greenhouse gas concentrations and economic damages in a stylized analytic integrated assessment model.

Golosov et al. (2014) paved the way for more complex closed-form integrated assessment models. Their paper assumes an effectively log-linear structure that eliminates non-trivial uncertainty effects, but it inspired the following research. Traeger's (2018) analytic integrated assessment model ACE introduces the nonlinear temperature response to emissions and a general degree of risk aversion, Hambel et al. (2018) develop a closely related model with a stylized economy-CO_2 response, and Van den Bremer & Van der Ploeg (2018) solve a related model using perturbation theory. These three papers use particular functional forms and only ACE introduces a simple form of structural uncertainty. Our paper will help to better understand and relate the different premia deriving from stochastic temperature change and structural uncertainty about climate sensitivity. Li et al. (2016), Anderson et al. (2014), and von zur Muehlen (2018) relax the von Neumann & Morgenstern's (1944) axioms and introduce a preference for robustness to escape the strong certainty equivalence of Golosov et al.'s (2014) framework and address model uncertainty and ambiguity.[2]

Several papers analyze the relation between growth uncertainty and optimal climate policy. Traeger (2013) analyzes the role of (potentially ambiguous) correlation between uncertain growth and mitigation or adaptation projects in a stylized two period from a discounting perspective. Jensen & Traeger (2014) study the SCC under growth uncertainty in a simple stochastic integrated assessment model, discussing the precautionary savings effect and the role of the damage function. Extending the analysis Epstein-Zin preferences, they explain why growth uncertainty's impact on the SCC turns negative for elasticities of intertemporal substitution exceeding unity as they are commonly applied in the long-run risk literature. In contrast, the present paper shows that climate uncertainty's risk premium remains positive also under these preference specifications. Jensen & Traeger's (2014) numeric results were recently confirmed in a full-complexity DICE implementation by Cai & Lontzek (2019) who also combine growth uncertainty with regime shifts in the damage function. Lemoine (2017) analyzes the role of growth uncertainty emphasizing the roles of precautionary savings and an insurance motive. Like Dietz et al. (2018) and Van den Bremer & Van der Ploeg (2018), the author relates his finding to a capital-asset-pricing-type reasoning, emphasizing different correlations. Dietz et al. (2018) study the "climate beta" analytically a simple two-period model and then Monte-Carlo-simulate a variety of uncertainties in DICE finding a positive "climate beta" that reduces the risk premium (though not necessarily the SCC). Van den Bremer & Van der Ploeg (2018) introduce a variety of "climate betas" corresponding to different, partly exogenous, risk channels. Bansal et al. (2019) enrich the consumption-based long-run-risk asset pricing model by temperature-triggered catastrophes, estimating the SCC from the capital market's response to temperature fluctuations. Daniel et al. (2019) find that the SCC would be decreasing over time under Epstein-Zin preferences in a much simplified seven period implementation of DICE with climate sensitivity uncertainty. We cannot confirm this finding in our somewhat more detailed implementation using an annual time step and an infinite time horizon.

[2]Anderson et al. (2014) explore a linear relation between the economic growth rate, temperature increase, and cumulative historic emissions. Both Li et al. (2016) and Anderson et al. (2014) combine a (simpler) analytic model with a more complex numeric IAM for quantitative simulation.

2 A Generic Integrated Assessment Model

This section introduces a simple and yet general integrated assessment model of climate change. It couples a growing world economy to a climate model through greenhouse gas emissions and economic impacts of climate change (see Figure 1). Section 2.1 introduces the functional structure and section 2.2 gives a heuristic derivation of the formula governing the optimal carbon tax, which we derive formally in Appendix A.

2.1 Analytic Model Structure

Figure 1 presents a schematic of our model. The right hand side characterizes a standard Ramsey growth model. World output is a function $F(K_t, T_t, E_t, t)$ of endogenous capital K_t, atmospheric temperature T_t, carbon dioxide emissions E_t, and a set of exogenous processes including labor and technology levels that depend on time t. The temperature increase T_t measured in degree Celsius above 1900 levels reduces the economy's productivity. Output is spent either on consumption C_t or on capital investment. We treat the economy's carbon dioxide emissions as an input into the production process; it is a reduced representation of an energy production sector whose carbon dioxide emissions trade off with capital and labor. We show the equivalence to the abatement setup employed in some integrated assessment model's like DICE in Appendix D.1. Emissions build up the stock of atmospheric carbon M_t. Atmospheric carbon together with other (exogenous) greenhouse gases trap our planet's outgoing infrared radiation. The resulting shift in the planet's energy balance causes an initial warming and induces further feedback processes.

The climate sensitivity s measures the medium to long-run temperature increase resulting from a doubling of pre-industrial atmospheric carbon dioxide concentrations. Its best guess lies currently around 3 degree Celsius. The approximate "climate change law" specifies that every further doubling (or fractions thereof) implies another 3 degree Celsius temperature increase (or fraction thereof).[3] It takes decades to

[3]Precisely, radiative forcing is logarithmic in the atmospheric CO_2 concentration, and warming is approximately proportional to radiative forcing. Climate sensitivity characterizes this proportionality constant. Recent model comparison studies of scientific climate change models suggest that the climate sensitivity is slightly lower for high than for low temperatures. A further assumption of the model is that also the short term temperature increase is proportional to climate sensitivity (though much smaller in magnitude). This assumption is a good approximation to models such as DICE and Maggic (Traeger 2014).

centuries to reach a new equilibrium temperature.[4] The change in temperature feeds back into economic production causing damages. Social welfare is the sum of per period utility $u_t(C_t)$, which is discounted at the pure time preference factor β.

In summary, the social planner solves the Bellman equation

$$V(K_t, M_t, T_t, t) = \max_{C_t, E_t} u_t(c_t) + \beta\, \mathbb{E}_t\, V(K_{t+1}, M_{t+1}, T_{t+1}, t+1) \tag{1}$$

subject to the equations of motion

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + F(K_t, T_t, E_t, t) - C_t \\ M_{t+1} &= M_{t+1}(M_t, E_t) \quad \equiv M(M_t, E_t, t) \\ T_{t+1} &= T_{t+1}(T_t, M_{t+1}) \quad \equiv T(T_t, M_{t+1}, s). \end{aligned} \tag{2}$$

At the present level of generality, each equation of motion can be subjected to stochastic shocks. We are interested in uncertainty generated by equation (2), as a result of both climate sensitivity s being uncertain and stochastic shocks. We spell out two alternative characterizations of this uncertainty in Assumptions 4 and 4'.

We assume that today's emission add directly to the next period's carbon stock: $\frac{\partial M_1}{\partial E_0} = 1$. Without this assumption, the SCC gains the additional factor $\frac{\partial M_1}{\partial E_0}$. We slightly violate notation in using M_t for the stock of carbon in the atmosphere and for the function $M(\cdot, t)$ describing its evolution over time. This notation enables us to abbreviate more intuitively the change of the carbon stock in period t as a result of a change of the carbon stock in period $\tau < t$ as

$$\frac{\partial M_t}{\partial M_\tau} = \prod_{i=\tau+1}^{t} \frac{\partial M_i}{\partial M_{i-1}} \equiv \prod_{i=\tau+1}^{t} \frac{\partial M(M_{i-1}, E_{i-1}, i-1)}{\partial M_{i-1}}.$$

We proceed analogously with temperature, abbreviating the change of the warming level in period t as a result of a change of the warming level in period $\tau < t$ as

$$\frac{\partial T_t}{\partial T_\tau} = \prod_{i=\tau+1}^{t} \frac{\partial T_i}{\partial T_{i-1}} \equiv \prod_{i=\tau+1}^{t} \frac{\partial T(T_{i-1}, M_i, i-1)}{\partial T_{i-1}}.$$

Finally, *we assume* that the optimization problem has a well-define solution and that the discounted marginal value at period τ of a change in atmospheric carbon or tem-

[4]The increasingly popular TCRE model Dietz & Venmans (2019) uses an approximate canceling of effects that stem from carbon removal in the atmosphere and warming delay to establish that temperature responds quickly to cumulative *emissions* (along some emission paths). Here, we point out that temperature adjustment to atmospheric *concentrations* takes time, which causes the major uncertainty governing climate sensitivity. If the TCRE model was literally true rather than an approximation, there would be no uncertainty about the climate sensitivity (it could be learned in the blink of an eye). Unfortunately, there is huge prevailing uncertainty governing climate sensitivity.

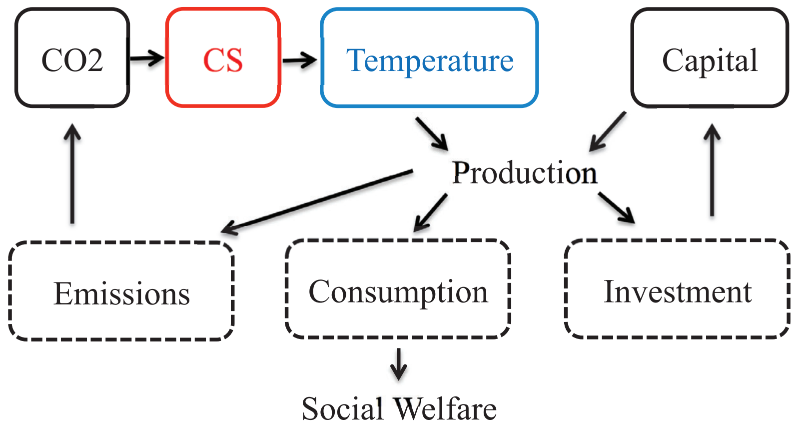

Figure 1: The main relations in the climate-economy model. Dashed rectangles represent control variables. Solid rectangles depict the main state variables of the system. Climate sensitivity ('CS') is uncertain. The model with learning represents climate sensitivity by a Bayesian prior (2 state variables). Temperature is stochastic.

perature at time $t < \tau$ approaches zero as τ approaches infinity: $\lim_{\tau\to\infty} \beta^{\tau-t}\, \mathbb{E}_t \frac{\partial V}{\partial M_\tau} \frac{\partial M_\tau}{\partial M_t}$ and $\lim_{\tau\to\infty} \beta^{\tau-t}\, \mathbb{E}_t \frac{\partial V}{\partial T_\tau} \frac{\partial T_\tau}{\partial T_t}$. Our analytic results rely only on this generic model structure, where Assumptions 4 and 4' in Section 3 restate the temperature's equation of motion (2) used for the derivation of the risk premium. Appendix D.1 summarizes the details of our numeric DICE implementation as a special case of this generic model structure.

2.2 The Social Cost of Carbon

An optimal mitigation policy equates the current benefits from releasing a ton of CO_2 with the present value of the future social damages from emitting a ton of CO_2. These damages are know as the social cost of carbon (SCC). We denote by $\frac{\partial M_\tau}{\partial E_0}$ the change of the carbon stock in period τ as a result of an additional ton of carbon emitted today. The carbon stock change in period τ causes a greenhouse effect (direct radiative forcing and feedbacks) increasing the temperature in subsequent periods $t > \tau$ by $\frac{\partial T_t}{\partial M_\tau}$. The resulting change in future atmospheric temperature impacts the output $Y_t = F_t(K_t, T_t, E_t, t)$, causing a marginal damage of $-\frac{\partial F_t}{\partial T_t}$. The output loss reduces period t welfare proportional to $u_t'(c_t)$. Summing the discounted welfare loss over an infinite time horizon and translating it into present day consumption units results in an analytic expression for the SCC.

Proposition 1. *The social cost of carbon (SCC) of the generic integrated assessment model is*

$$SCC_0 = -\,\mathbb{E}_0 \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \underbrace{\beta^t \frac{u_t'(c_t)}{u_0'(c_0)}}_{\substack{\text{consumption}\\ \text{discount}\\ \text{factor}}} \underbrace{\frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial M_\tau} \frac{\partial M_\tau}{\partial E_0}}_{\substack{\text{marginal}\\ \text{emission}\\ \text{damage}}} . \tag{3}$$

Under certainty, the optimal carbon tax is the SCC evaluated along the optimal trajectory. In the stochastic setting, the optimal carbon tax evaluates equation (3) *on a decision tree where every period's policy on every branch is optimized conditional on past realizations.*

We obtain a double sum because we take into account the quantitatively relevant delay between initial radiative forcing and ultimate temperature response.[5] Numeric integrated assessment models usually calculate the SCC without relying on analytic expressions, either by perturbing emissions (e.g. FUND) or relying of the software's ability to compute shadow values (e.g. DICE). Appendix A derives equation (3) from the underlying Bellman equation. Our intuitive derivation kept policies constant. The derivation based on the Bellman equation takes into account the optimal future responses. The two results coincide (along the optimal trajectory) because of the envelope theorem; along the optimal path the decision maker's future response to the marginal ton of emissions does not change the program's value in first order and, thus, the marginal value of an emission unit remains the same.

3 The Price of Uncertainty

This section offers analytic insights into the optimal policy response to the uncertainty governing the climate system's response to greenhouse gas emissions. This uncertainty results from a variety of feedback processes: more heat implies more water vapor, which is itself an important greenhouse gas; higher temperatures melt glaciers and ice caps reducing the reflectivity of our planet's surface, further increasing its uptake of solar radiation; soils currently subject to permafrost will release large amounts of

[5] Gerlagh & Liski (2018) and Traeger (2018) illustrate particular examples of such a double sum and show that in special cases it can be represented in closed form inverting the transition matrices.

methane, a powerful greenhouse gas; other feedbacks change the vertical temperature profile in the atmosphere and cloud formation again feeding back onto our surface temperature. As a consequence of these feedbacks, the actual warming resulting from a given greenhouse gas concentration is highly uncertain.

As a result of this uncertainty, a deterministic SCC formula by itself is of limited practical use. The Interagency Working Group on the Social Cost of Carbon (2013) responded by simulating the underlying deterministic integrated assessment models for a wide range of climatic responses and reporting both the average and the 95^{th} percentile of the resulting SCC. Yet, the optimal response has to be a single value that anticipates the stochastic evolution of the future, and it generally differs from the average optimal response across different deterministic worlds. Connecting to the precautionary savings literature, section 3.1 reviews the concept of prudence and extends it to models with economic production. Section 3.2 discusses the different channels that increase (or decrease) the optimal carbon tax under uncertainty about the climate's sensitivity to greenhouse gas emissions. Section 3.3 shows the related but distinct impact of atmospheric temperature fluctuations on the SCC.

3.1 Prudence and the climate-economy interaction

Climate sensitivity impact. Climate sensitivity affects the optimal carbon tax by altering *climate, economic activity, and human well-being (utility).* We discuss initial to final impact following equation (1). First, a higher climate sensitivity s maps an *additional* unit of emissions into a larger *marginal temperature increase* $\frac{\partial T_t}{\partial M_\tau}(s)$. This effect tends to be linear in climate sensitivity (more below). Second, a higher climate sensitivity raises the temperature *level* resulting from the *prevailing concentration* of atmospheric greenhouse gases, i.e., $T_t(s)$ increases. This higher temperature level increases the *marginal production damage* resulting from an additional unit of temperature increase $\frac{\partial F_t}{\partial T_t}(T_t(s))$. Third, higher temperature and higher damages also imply a lower consumption level. Such a lower consumption level implies a higher *welfare loss* from a given marginal consumption reduction $u_t'(c_t(s))$.

By Jensen's inequality, uncertainty increases the SCC and the optimal carbon tax if the right side of equation (1) is convex in the uncertain climate sensitivity. Intuitively, convexity of the right side of equation (1) means that a high realization of the climate sensitivity increases the social cost of a marginal emission unit more than a low realization reduces this cost. Such reasoning might be most familiar from the precautionary savings literature. In difference to a simple precautionary savings model, climate sensitivity affects the right side of equation through a variety of different channels. We now introduce the "vocabulary" to discuss these uncertainty contributions.

Welfare response. We briefly review the basic intuition of the simple precautionary savings model of Leland (1968) as spelled out by Kimball (1990). Assume we introduce stochastic shocks to an agent's return to savings. The risk averse agent dislikes these shocks. But disliking shocks is not enough to change the behavior. Yet, the empirical literature finds that (absolute) risk aversion decreases in wealth. Then, the agent can reduce the (utility-) harm resulting from the stochastic shocks by increasing her savings and reaching a higher wealth level where she is less bothered by risk. Kimball (1990) characterizes the agent's decrease in risk aversion analogously to how Arrow (1965) and Pratt (1964) characterize risk aversion; we abbreviate their definitions as

$$\text{RRA} = -\frac{u''(c_t)}{u'(c_t)}c_t \quad \text{and} \quad \text{Prud} = -\frac{u'''(c_t)}{u''(c_t)}c_t.$$

Relative risk aversion (RRA) captures the curvature of the utility function, and (relative) prudence captures how risk aversion changes in the consumption level.

Economic response. In the precautionary savings example uncertainty only affects the marginal utility of consumption (direct well-being impact). To get a handle on the economic productivity impact, we define the second and third moment of the production function in temperature analogously to Arrow-Pratt's risk aversion and Kimball's prudence measures

$$\text{Dam}_2 = \frac{-\frac{\partial^2 F_t}{\partial T_t^2}}{-\frac{\partial F}{\partial T_t}}T_t \quad \text{and} \quad \text{Dam}_3 = \frac{-\frac{\partial^3 F_t}{\partial T_t^3}}{-\frac{\partial^2 F}{\partial T_t^2}}T_t.$$

Dam_2 measures the relative convexity of marginal damages $-\frac{\partial F}{\partial T}$ in temperature and Dam_3 measures its increase with higher temperatures. Our notation suppresses the various dependencies of the damage curvature measures. Similarly, RRA and Prud can vary over time with the changing consumption levels; our quantitative application uses constant relative risk aversion, which renders the preference measures independent of time-changing consumption levels.

Temperature dynamics and response. We take the following assumption merely for ease of exposition. The assumption follows almost by definition of climate sensitivity and is a stylized fact of our DICE-based numeric integrated assessment model (see equation 25 in Appendix D.2).

Assumption 1. *The temperature evolution is linear in climate sensitivity s:*

$$T_{t+1} = \gamma T_t + s\,\Gamma_1(M_{t+1}) + \Gamma_2(t) + \epsilon_t, \tag{4}$$

where s is uncertain and $(\epsilon_t)_{t\in\mathbb{N}}$ *is an iid process of mean zero temperature shocks.*

Climate sensitivity is the proportionality constant between temperature and the direct greenhouse effect (radiative forcing): the radiative forcing characterizes the additional heat influx as a result of the greenhouse gases that slowly heat our planet. The AR(1) process captures that heating takes time and that the temperature evolution is stochastic. Corollary 1 in Appendix B states the slightly more general case arising in the absence of Assumption 1. As a result of the *linearity of the temperature evolution in climate sensitivity* we *do not have to define convexity and prudence-type curvature measures* for this direct temperature impact channel.

Investment and emission response: a "radical" assumption. In deriving equation (1), the envelope theorem ensured that a marginal shift between consumption, investment, and abatement will not affect the welfare. Unfortunately, the impact of risk is non-marginal and there is no convenient analogue to the envelope theorem when employing Jensen's inequality to the dynamic equations. Instead, we take the following assumption enabling us to derive an analytic formula for the higher order risk effect on the optimal carbon tax.

Assumption 2. *The decision-maker sets investment and emissions to the deterministically optimal levels.*

The assumption limits the decision maker's optimal response to uncertainty. Without further adjustments, our formula in the next section would calculate the risk premium to the SCC when decision makers do not respond to uncertainty. Such a premium by itself is an interesting insight into the role of uncertainty. Yet, it does not characterize the optimal response to uncertainty nor the optimal carbon tax. We will show that the unadjusted formula based on Assumption 2 tends to overestimate the risk premium.[6]

[6] The reader will likely think that it is intuitive that such an approach would overestimate the optimal tax. For the purpose of understanding uncertainty implications we insert the following warning. It is intuitive that the approach would indeed overestimate the *costs* of uncertainty. Yet, we are not after the cost of climate uncertainty but after its policy impact, which depends on the asymmetry of the marginal cost impact.

We will evaluate and fix the quantitative implications of Assumption 2 in section 4. For now, we merely anticipate those results and claim that the assumption has little impact on the insights we are about to present.

Uncertainty conversion factors. We still have to translate the magnitude of climate uncertainty into the resulting magnitude of the uncertainty striking the economy and individual consumption. Starting with consumption, we define the sensitivity of the consumption level to a change in climate sensitivity by the elasticity

$$\epsilon_{c,s} = -\frac{dc}{ds}\,\frac{s}{c} > 0$$

and the sensitivity of global average temperature to climate sensitivity by

$$\epsilon_{T,s} = \frac{dT}{ds}\,\frac{s}{T} > 0.$$

Our notation suppresses the dependencies (including time) of these elasticities. Appendix B (page 43) states closed-form expressions for these elasticities under Assumption 2 (and a generalization). Yet, deriving these elasticities numerically from the deterministic model instead will be the most crucial fix countering Assumption 2.

3.2 Uncertain Climate Sensitivity

The present section presents the formula for the climate sensitivity risk premium. Here, we assume that the temperature stochasticity shocks ϵ_t in equation (4) are zero. The subsequent section derives the temperature stochasticity premium.

Introducing uncertainty, we fix the climate sensitivity's best guess and introduce a probability distribution over its true value. For our structural results the precise best guess is irrelevant. The uncertainty presents the subjective prior over a true underlying climate sensitivity distribution (epistemological uncertainty). This uncertainty affects the baseline emissions that are already in the atmosphere as well as the marginal unit released today.

Proposition 2. *Under Assumptions 1 and 2, uncertainty over climate sensitivity increases the social cost of carbon contribution from a given period t if and only if*

$$X_t(\cdot) \equiv \underbrace{\text{RRA}\,\epsilon_{c,s}\big[2}_{\substack{\textit{direct}\\ \textit{risk}\\ \textit{aversion}}} + \underbrace{\text{Prud}\,\epsilon_{c,s}}_{\substack{\textit{welfare}\\ \textit{prudence}}} + \underbrace{3\,\text{Dam}_2\,\epsilon_{T,s}\big]}_{\substack{\textit{welfare}\\ \textit{economy}\\ \textit{interaction}}} + \underbrace{\text{Dam}_2\,\epsilon_{T,s}\big[2}_{\substack{\textit{direct}\\ \textit{damage}\\ \textit{convexity}}} + \underbrace{\text{Dam}_3\,\epsilon_{T,s}\big]}_{\substack{\textit{economy}\\ \textit{prudence}}} \tag{5}$$

is greater than zero. Arguments and their period-dependence are suppressed. Moreover, under a small risk approximation, the climate uncertainty premium is

$$\Delta SCC_0 \approx \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \underbrace{\beta^t \frac{u_t'(c_t)}{u_0'(c_0)}}_{\substack{\text{consumption}\\ \text{discount}\\ \text{factor}}} \underbrace{\frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial M_\tau} \frac{\partial M_\tau}{\partial E_0}}_{\substack{\text{marginal}\\ \text{emission}\\ \text{damage}}} \underbrace{\frac{\mathbb{V}\text{ar}(s)}{2(\mathbb{E}\, s)^2}}_{\substack{\text{level of}\\ \text{uncertainty}\\ \text{(normalized)}}} X_t(\cdot) \tag{6}$$

The proof in Appendix B derives the result from the underlying Bellman equations and states as Corollary 1 the more general case when Assumption 2 is not met. The total SCC today ($t = 0$) is the sum of the deterministic contribution, equation (3) evaluated at the expected (best-guess) climate sensitivity, and the uncertainty premium ΔSCC_0. Equation (6) shows that the formula for the climate uncertainty premium ΔSCC_0 contains the same summation over future impacts as the deterministic SCC. Once again, we translate the future impacts into present day consumption equivalents evaluating the marginal impact of a ton of CO_2 released today onto the future carbon stocks, temperatures, production levels and marginal utility. The final two terms in equation (6) specify the uncertainty impact. First, the cost of uncertainty grows in the (expectation-normalized)[7] variance of the climate sensitivity s. Holding expected sensitivity fix, the contribution increases proportionally to the variance of our uncertainty about climate sensitivity, a result of our small risk approximation that neglects higher order terms. Second, the term abbreviated $X_t(\cdot)$ characterizes the core uncertainty contribution to climate policy per unit of the normalized uncertainty. Equation (5) spells out these contributions, where we suppress the period-dependence of the individual contributions for notational convenience.

Risk aversion. The first message of equation (5) is that risk aversion matters directly (2 RRA). This finding is an exception rather than the rule in precautionary savings. The contribution reflects not only decreasing marginal utility (RRA), but also of the structural uncertainty governing climate sensitivity. It results from the interaction of two simultaneous and mutually aggravating impacts of climate sensitivity governing both the *level* of warming resulting from the prevailing atmospheric CO_2 concentration and the *marginal* warming contributed by emitting an additional ton of carbon.

In detail, a high realization of climate sensitivity jointly increases the marginal warming *and* lowers the consumption level by increasing the base warming. As a re-

[7]The normalization by expected climate sensitivity is a result of expressing the core formula $X_t(\cdot)$ in terms of relative aversion, prudence, and damage measures as well as elasticities.

sult, the higher marginal warming strikes at a lower consumption level and, thereby, at increased marginal costs. For a low realization, the marginal warming reduction will be enjoyed at a higher consumption level, resulting in a lower marginal benefit. Such a lower marginal benefit from a low realization of climate sensitivity does not compensate for the costs of a high realization, creating a climate policy risk premium. The elasticity $\epsilon_{c,s}$ translates the high or low climate sensitivity realization into the corresponding consumption impact. The relative risk aversion measure RRA characterizes the difference in marginal consumption value between the good state (high consumption) and the bad state (low consumption). Our reasoning above employs Assumption 1, i.e., that climate sensitivity directly characterizes the warming factor in degree Celsius. A non-linear relation between climate sensitivity and warming slightly alters the reasoning (see Corollary 1 in Appendix B).

(Welfare-) Prudence. The second contribution in equation (3) is the common precautionary savings term. It operates directly through the climate sensitivity's impact on the base warming. The elasticity $\epsilon_{c,s}$ translates this variation into consumption. CO_2 abatement reduces climate sensitivity and increases the expected consumption level. A prudent decision maker's absolute risk aversion decreases in her consumption level. Thus, the prudent decision maker has an incentive to abate more CO_2, increasing the expected consumption level and reducing risk aversion and the welfare loss from uncertainty.

Damage convexity. Risk aversion and prudence explain how the welfare function makes a policy maker precautionary. The moments Dam_2 and Dam_3 characterize how economic production turns optimal policy precautionary. The term $2\,\text{Dam}_2$ in equation (5) captures a direct impact of the damage convexity and its intuition resembles that of the direct risk aversion contribution. In particular, it is not merely the convexity of damages giving rise to this contribution, but its interaction with the structural nature of the uncertainty.

Damage convexity captures that the first degree of warming is less damaging (sometimes even considered beneficial) than the second degree; additional degrees of warming are widely believed to become far more damaging. A high realization of climate sensitivity increases *both* the temperature *level* resulting from the prevailing atmospheric CO_2 and the *marginal temperature increase* from an additional ton released today. The larger marginal warming strikes production at a higher temperature level and, thus, causes a greater loss (convexity of damages). The low realization reduces the marginal warming but cannot compensate for the damages of a high realization because it relieves the economy at a low temperature level where the marginal warming is less harmful.

Economy-prudence. The "economy-prudence" term proportional to Dam_3 triggers a precautionary policy if the damage convexity increases in temperature. It is the real-economy analogue to the standard precautionary savings argument. The convexity of economic production in temperature implies an expected production loss under uncertainty. If this convexity increases in temperature, then a policy maker has an additional incentive to keep temperatures down. We note that damages are a bad and therefore an increasing convexity in damages plays the same role as a decreasing convexity of welfare.

It is a priori unclear whether the damage convexity increases in the temperature level. Physical damages are limited by total production or at least by the capital stock. Therefore, we expect the damage convexity to eventually fall and even turn concave. Already for optimal climate policy and corresponding low temperature levels we will find that the sign of Dam_3 changes even between different versions of the DICE model.

Welfare-economy-interaction. Finally, also the interaction of risk aversion with the damage convexity ($3\,\text{RRA}\,\text{Dam}_2$) results in a precautionary motive. As a result of convex damages, the policy maker faces an expected loss from uncertainty. The risk averse decision maker values this loss more severely at a lower consumption (high temperature) level than at a higher consumption (low temperature) level, which creates an additional incentive to reduce future temperatures.

In the case of **risk neutrality**, the risk premium contributions are limited to the last two terms in equation (5), i.e., those proportional to the damage convexity and its change in the temperature level.

3.3 Temperature Stochasticity

Temperatures fluctuate naturally from year to year and on larger time scales. The empirical literature employs such fluctuations as an approximation to climatic change to estimate economic damages. Propositions 1 and 2 establish how the fundamental drivers of the response to climate change differ between deterministic climate change and climate change uncertainty.[8] The present section explains why also the economic response to stochastic temperature fluctuations differs fundamentally from both of the

[8] Lemoine (2020) explains this distinction and develops a structurally inspired estimate of damages under a stochastic temperature evolution.

earlier responses. Climate change mitigation is an action that should be coordinated globally. Yet, also at the local level, farmers, businesses, and regional powers take precautionary actions in the face of known climate change, an uncertain future, or stochastic temperature fluctuations. Instead of (or in addition to) investing into the global capital "low CO_2", they invest into sea walls, infrastructure, irrigation, crop refinement, air conditioning, relocation, and more. In addition, they respond to uncertain policy. A similar message to the one we derive for the global mitigation response also holds at the local level; actors respond differently to deterministic and uncertain change, and they respond differently to stochastic fluctuations.

Temperature stochasticity affects the level of warming in future periods but *not the marginal warming resulting from the additional ton of carbon* we release today. As a consequence, the followig proposition is closer to the usual case of precautionary savings. Here, the elasticity $\epsilon_{c,T}$ translates temperature change into its consumption impact. The subsequent proposition focuses on the *additional* contribution to the SCC from the stochastic evolution of global temperatures, keeping climate sensitivity fixed.

Proposition 3. *Under Assumptions 1 and 2, temperature stochasticity increases the social cost of carbon contribution from a given period if and only if*

$$Z_t(\cdot) \equiv \mathrm{RRA}\,\epsilon_{c,T}\Big[\underbrace{\mathrm{Prud}\,\epsilon_{c,T}}_{\substack{\textit{welfare}\\ \textit{prudence}}} + \underbrace{3\,\mathrm{Dam}_2}_{\substack{\textit{welfare}\\ \textit{economy}\\ \textit{interaction}}}\Big] + \underbrace{\mathrm{Dam}_2\,\mathrm{Dam}_3}_{\substack{\textit{economy}\\ \textit{prudence}}} \tag{7}$$

is greater than zero. Arguments and their period-dependence are suppressed. Moreover, under a small risk approximation, the temperature stochasticity contribution is

$$\hat{\Delta} SCC_0 \approx \sum_{t=1}^{\infty}\sum_{\tau=1}^{t} \underbrace{\beta^t \frac{u_t'(c_t)}{u_0'(c_0)}}_{\substack{\textit{consumption}\\ \textit{discount}\\ \textit{factor}}} \underbrace{\frac{\partial F_t}{\partial T_t}\frac{\partial T_t}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}}_{\substack{\textit{marginal}\\ \textit{emission}\\ \textit{damage}}} \underbrace{\frac{1-\gamma^{2t}}{1-\gamma^2}}_{\substack{\textit{intertemporal}\\ \textit{temperature}\\ \textit{correlation}}} \underbrace{\frac{\mathbb{V}\mathrm{ar}(\epsilon)}{2(\mathbb{E}\,T_t)^2}}_{\substack{\textit{level of}\\ \textit{uncertainty}\\ \textit{(normalized)}}} Z_t(\cdot). \tag{8}$$

Equation (7) shows that risk aversion and damage convexity do not play a direct role for the risk premium deriving from temperature stochasticity, eliminating the strongest contributor to the premium for uncertain climate sensitivity. First, (welfare) prudence continues to contribute to a precautionary climate policy. Second, the economy-prudence effect contributes, which will dominate the stochasticity premium in DICE under cubic damages. Third, the interaction of risk aversion and damage convexity continues to contribute, which will dominate the stochasticity premium in DICE for the base specification of quadratic damages.

The temperature shocks follow an iid process. While the shocks are independent, the temperature realizations in different periods are correlated because of temperature's autoregressive nature. As a result, the stochasticity premium picks up the new factor $\frac{1-\gamma^{2t}}{1-\gamma^2}$ in equation (8). It relates the variance of the shock-realization to the mean temperature realization in a given period. We note that, despite this apparent multiplier, the variance of temperature stochasticity and its contributions to the overall risk premium are small when compared to the variance and contributions of the uncertainty premium.

Proposition 3 offers interesting insights for reviewing some of the literature. The first generation of analytic integrated assessment models uses the risk-neutral linear-quadratic model building on Weitzman's (1974) prices versus quantities to evaluate policy instruments for stock pollutants Hoel & Karp (2002), Karp & Zhang (2006), Newell & Pizer (2008), Zhang (2012), Weitzman (2018), Karp & Traeger (2018), Pizer & Prest (2019), Heijmans & Gerlagh (2019). Propositions 2 and 3 emphasizes that their policy recommendations rely on a knife edge role of uncertainty. These models compare the welfare across different policy regimes relying on the principle of certainty equivalence; unlike welfare, the optimal policy level does not respond to uncertainty. First, these models assume risk neutrality eliminating the risk aversion and prudence contributions to the risk premium. Second, these models assume quadratic damages eliminating the "economy-prudence" effect that depends on the change of the damage convexity. Finally, they assume a stochastic evolution of future climate instead of structural uncertainty eliminating the direct contribution of the damage convexity (Proposition 3 instead of 2). As a result, uncertainty does not change the optimal climate policy levels. These models compare the welfare loss under uncertainty, which still depends on the damage convexity.

More recently, the second generation of analytic integrated assessment models builds on Golosov et al.'s (2014) breakthrough in modeling more complex economy-climate interactions. Golosov et al. (2014) introduce uncertainty, but their model is linear in all the relevant states so that uncertainty has no impact, neither on welfare nor on policy. Most of the literature following Golosov et al.'s (2014) do not include temperature. Two papers stand out. Van den Bremer & Van der Ploeg (2018) generalize the model and derive an approximate solution using perturbation theory. Embedded in our general analysis, their model drops the direct contributions from risk aversion and damage convexity because they model temperature stochasticity rather than structural uncertainty about climate sensitivity. Yet, they introduce a skewness of the temperature shocks that mimics the skewness of climate sensitivity. As a result, they obtain a contribution from the interaction of damage convexity and skewness of the shock. To explain the paper's results in our context, Corollary 2 in Appendix C generalizes Proposition 8 to a modified temperature evolution of the form

Assumption 1'. *The temperature evolution is:*

$$T_{t+1} = \gamma \; T_t + s \, \Gamma_1(M_{t+1}) + \Gamma_2(t) + \Gamma_3(\epsilon_t) \tag{9}$$

where s is uncertain and $(\epsilon_t)_{t\in\mathbb{N}}$ is an iid process of mean zero temperature shocks.

Equation (9) introduces the transformation $\Gamma_3(\cdot)$ of the shock ϵ_t. As we show in Corollary 2, under such a skewed shock distribution, equation (8) for the stochasticity premium picks up a normalized measure of the convexity of Γ_3 – resembling the Arrow-Pratt risk aversion and the damage convexity measures. This measure for skewness $\frac{\Gamma_3''(0)}{\Gamma_3'(0)} T_t$ contributes another term to the stochastic risk premium that interacts with risk aversion and damage convexity:

$$\frac{T_t \Gamma_3''(0)}{\Gamma_3'(0)} \left[\text{RRA}\, \epsilon_{c,T} + \text{Dam}_2\right].$$

In the absence of structural uncertainty, introducing such skewness and increasing the variance of temperature shocks can mimic climate sensitivity uncertainty. The approach delivers sizable risk premia even in the absence of structural uncertainty.

Traeger (2018) derives a closed-form solution for the optimal carbon tax in an analytic climate economy (ACE) model. In contrast to Golosov et al. (2014), the model includes temperature dynamics and uncertainty affects both policy and welfare. In addition to using a slightly skewed autoregressive gamma process to mimic climate sensitivity uncertainty, the paper also introduces a form of structural uncertainty by endogenizing the risk. Our equation (8) relies on a small risk approximation governing the shocks ϵ_t, employing only the first two moments of the distribution. Our numeric implementation employs normally distributed shocks making higher order moments irrelevant in the base calibration. For more convex damages, the effective skew after damages likely contributes to the formula's error. We partially address the issue by evaluating at least the temperature response ($\epsilon_{T,s}$) for a high and a low climate sensitivity realization (Gaussian quadrature). If one wants to shock temperature with a skewed distribution, then a non-linear transformation $\Gamma_3(\cdot)$ of a normal shock can help handle the additional complexity without losing the tractability of the small risk approximation.[9]

4 Quantification

The present section quantifies our formulas and compares the results to those of a recursive stochastic dynamic programming implementation of the wide-spread DICE model. Appendix D presents the details of our numeric implementation. Production, growth, benefits of emissions, and climate damages are all based on Nordhaus & Sztorc's (2013) DICE 2013 model.[10] DICE uses a quadratic damage function $D(T) = aT^2$ specifying the loss of (potential) world output as a result of a temperature increase T above preindustrial levels. Temperature follows equation (4) with Bayesian updating, and the CO_2 dynamics are slightly simplified w.r.t. DICE, also reducing the slight sluggishness of DICE's carbon cycle Traeger (2014), Van der Ploeg et al. (2020). Given our focus on uncertainty, our baseline scenario uses a relative risk aversion of RRA = 2 instead of Nordhaus' pick of RRA = 1.45 for a deterministic world. We estimate temperature stochasticity from GISTEMP (2019), and use a variance of 3 for our climate sensitivity prior, which is the rounded proxy to the variance of a large set of climate sensitivity estimates summarized in the Stocker et al. (2013) report. Section 4.1 presents a preliminary evaluation of our analytic formulas and motivates a simple adjustment of the temperature elasticity to climate sensitivity $\epsilon_{T,s}$. Section 4.2 discusses the total risk premium based on our formula and the stochastic model for different scenarios. Section 4.3 analyzes the different risk channels for selected scenarios.

[9] While it is easy to introduce skewness to the stochastic process, it is much harder to introduce skewness to the structural uncertainty about climate sensitivity and maintain a Bayesian learning model. Kelly & Tan (2013) have done so in a fat-tail setting with conjugate priors and approximate Bayesian updating, and ongoing work by Rudik et al. (2020) forgoes the conjugate prior setting by approximating a wider class of probability distributions and using a Smolyak-grid to handle the additional state variables.

[10] We prefer the 2013 vintage of DICE over the 2016 version because (i) Van der Ploeg et al. (2020) show that its carbon cycle performs substantially better compared to scientific models used by the Stocker et al. (2013) and (ii) we consider the other major update of DICE 2016, a sharp initial decline in the rate of technological progress descriptively unconvincing.

4.1 "Raw Result", Time Profile of Contributions, and Quantitative Adjustment in Response to Assumption 2

Formula, Unadjusted Result. Our unadjusted – or "raw" – formula-based risk premium evaluates the sum of equations (6) and (8). For this purpose we evaluate the formulas along the path where climate sensitivity and temperature stochasticity take their expected values of $3°C$ and $0°C$, respectively. We find an overall formula-based risk premium of 19.0 USD/tC, of which temperature stochasticity makes up for a mere 25 cents. We depict the composition of the contributions from climate sensitivity uncertainty (equation 5) in Figure 2. The left graph presents the contributions to the 2020 SCC from a given future period. The maximal contribution to the risk premium derives from the second half of the present century. The contributions increase because uncertainty unfolds over time, but they eventually decrease again because of discounting. The right graph depicts the cumulative contribution over the decision maker's time horizon. With our baseline discounting, the policy maker will not worry about the risk imposed onto a future that is more than a couple of centuries away. We note that discounting is not merely a consequence of the annual rate of pure time preference of 1.5%, but also of growth-based discounting because future generations will be richer. We will discuss and vary these assumptions in the section 4.2.

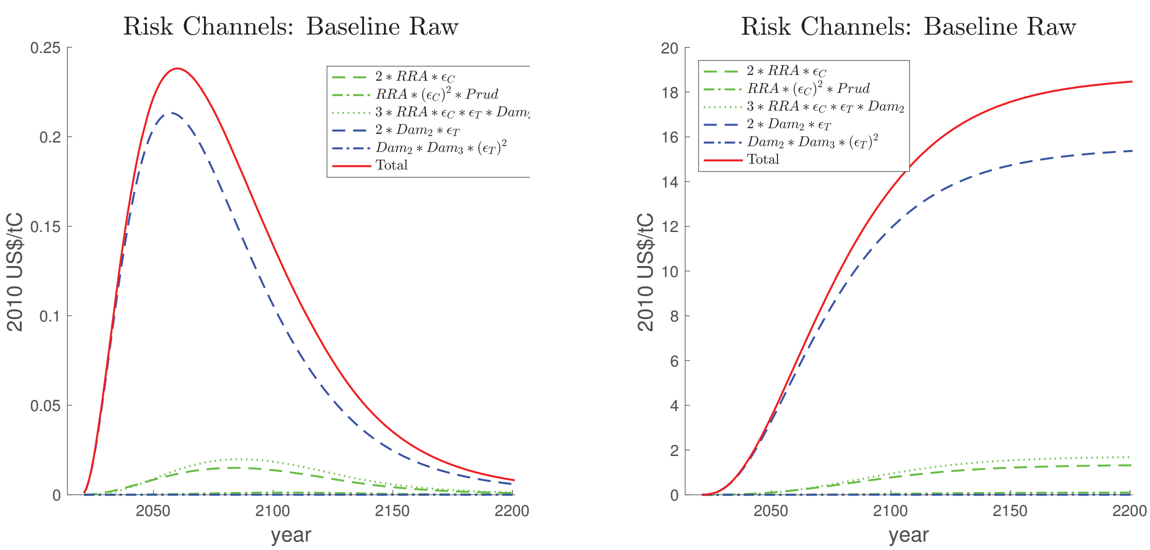

Figure 2: Per-unit-of-variance contributions of climate sensitivity uncertainty to the SCC based on equation (5) in Proposition 2 for our baseline calibration of DICE 2013. The left panel shows the contributions to the SCC in 2020 from a given year in the future, whereas the right panel shows the cumulative contributions corresponding to the depicted time horizon.

Dominant Channel. The striking insight from Figure 2 is that the contribution directly proportional to the convexity of damages contributes by far the most to the SCC's risk premium and a precautionary emission reduction. That term results from the structural nature of the uncertainty and is absent in the usual precautionary savings argument. This dominant contribution is absent in models that merely represent uncertainty about future temperatures by a stochastic shock process. It arises only if we take into account that uncertainty about climate sensitivity governs both the base warming resulting from the prevailing atmospheric CO_2 concentration as well as the marginal warming from a ton of carbon released today. The next biggest contributors are the direct risk aversion effect and the interaction of risk aversion and damage convexity. Both, the (welfare) prudence and the economy-prudence effect contribute essentially zero to the uncertainty premium.

Numeric Dynamic Programming Result. Comparing our risk premium to that of the recursive stochastic DICE implementation we find that we overestimate the premium. In contrast to the formula's 19 USD/tC, the stochastic numeric model finds a total premium of only 15.8 USD/tC (and 24 cents with only temperature stochasticity). Given our Assumption 2, this result might not be surprising. Our formula assumes that future policy makers do not adapt their optimal consumption and mitigation decisions to the presence of uncertainty. It therefore calculates the cost of carbon if policy responds to the expected change but not to actual future realizations. By adapting decisions to the presence of uncertainty, society can lower the welfare costs of uncertainty. Optimal policy depends on the uncertainty's impact on marginal rather than absolute welfare. It is therefore only intuitive but not obvious that this reasoning carries over to the risk premium on climate policy. We will now try to get a better handle on the uncertainty response of the decision variables.

Formula adjustment. Figure 2 shows that the contribution from risk aversion and prudence is small. Thus, the quantitative deviation of the formula is most likely driven by an interaction of limited policy response with the damage function related contributions. The terms Dam_2 and Dam_3 merely evaluate the moments of the damage functions. If these moments change substantially across the deterministically optimal path and the optimal policy under uncertainty this approximation could cause an error (see further below). Yet, much more likely is that the elasticity $\epsilon_{T,s} = \frac{dT}{ds}\frac{s}{T}$ causes the deviation. It currently depicts the temperature response to a change in climate sensitivity assuming that policy does not respond. Yet, if climate sensitivity turns out higher, optimal policy will strengthen and partially offset the

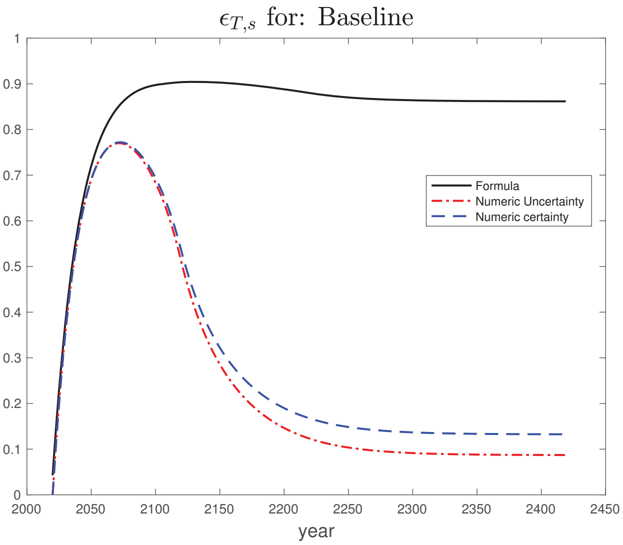

Figure 3: Elasticity of the temperature response to changes in climate sensitivity.
(i) solid line: without optimal policy response (Assumption 2);
(ii) blue-dashed line: including optimal policy response in the deterministic model;
(iii) red-dash-dotted line: including optimal response in the stochastic model.

warming from an increased climate sensitivity.

Figure 4.1 quantifies this reasoning. The solid black line depicts the elasticity assuming away a policy response (Assumption 2). The dashed blue line incorporates the policy response using the deterministic model; it calculates the actual model elasticity based on a small variation of climate sensitivity (a tenth of a degree) allowing the policy maker to respond optimally. The red dash-dotted line repeats this experiment in the stochastic model.[11] We find that, indeed, Assumption 2 overestimates the elasticity for solving the second half of the century and beyond, including the periods that are contributing most strongly to the risk premium. Conveniently for the practitioner, the actual elasticities hardly differ between the deterministic and the stochastic model. Thus, we can easily improve our formula by using the elasticity derived from a numeric simulation of the deterministic model without the need for the far more expensive stochastic model. We will henceforth use such an *adjusted formula*:

> *The adjusted formula uses the result in Proposition 2, but evaluates the temperature elasticity to changes in climate sensitivity* $\epsilon_{T,s} = \frac{dT}{ds}\,\frac{s}{T}$ *numerically, incorporating the policy maker's optimal response under certainty.*

Using this simple adjustment, our analytic formula results in a total risk premium of 15.7 USD/tC, less than a one percent error with respect to the stochastic dynamic

[11] Here, we calculate $\epsilon_{T,s} = \frac{dT}{ds}\,\frac{s}{T}$ by changing both expected and true climate sensitivity by a tenth of a degree and assuming that realized temperature shocks are zero. True climate sensitivity and actual shocks are unkown to the decision maker, see Appendix E.2 for details.

programming model. For our baseline run, evaluating the elasticity merely along the expected path works well. For scenarios, in particular those changing the damage function, we use the average response across two different (deterministic) scenarios to calculate $\epsilon_{T,s}$, one with a high and one with a low realization of climate sensitivity. Results increasing accuracy by averaging two response paths are marked by an asterix in Table 1.[12]

4.2 Quantifying the Risk Premium

Baseline. Table 1 presents the overall climate risk premium for our baseline calibration and for variations of the key parameters and functional forms. We find that our adjusted formula performs well for most scenarios. The climate risk premium of our baseline scenario is 16 USD/tC, which is about one quarter of its deterministic value. The low observed risk-free interest rates suggest that an annual rate of pure time preference of $\rho = 1.5\%$ might be a somewhat high pick. Reducing it to the median estimate of the recent expert survey by Drupp et al. (2018), $\rho = 0.5\%$, increases the risk premium to 22 USD/tC, but slightly decreases the risk premium as a fraction of the deterministic SCC. Without touching the damage convexity of the preference structure, we find a risk premium of 20-25%.

Damage Specification. The most substantial changes or the (relative) risk premium derive from changes in the damage function, which is in line with our initial quantification of the different risk channels in Section 4.1. DICE 2013's damage specification $D(T) = aT^2$ assumes that the world loses $a = 0.266\%$ of potential output at a 1°C warming (today) and nine times as much at a 3° warming (quadratic damages). Howard & Sterner's (2017)'s meta analysis of damage estimates suggests five times those damages, somewhat in agreement with a recent survey among climate scientists and economists by Pindyck (2020). Howard & Sterner's (2017)'s preferred estimate is a loss of $a = 1.15\%$ of potential output at a 1°C warming and, just as DICE, a quadratic increase in temperature. Under such an increase of the damage magnitude, the risk premium increases to approximately one third of the deterministic SCC. Several authors have suggested that damages might be much more convex that assumed in the quadratic specification (e.g. Weitzman 2012, Millner 2013). Changing

[12] We use the two Gaussian quadrature nodes and weights of the climate sensitivity distribution. Gaussian quadrature picks discrete nodes and weights to approximate the moments of a continuous distribution. The expectations calculated from two Gaussian nodes already match the first 4 moments of a distribution exactly. We only use them to calculate the elasticity $\epsilon_{T,s}$ from two different realizations of climate sensitivity in a deterministic model.

Scenario \ Risk Premium (USD/tC)	Analytic Formula Raw	Adjusted	Full Model	error	Stochastic Fraction	Fraction of Cert
RRA=2, $\rho = 1.5$, DICE13	19.1	15.8	15.8	0.2%	1.6%	26%
RRA = 1.45	29.8	21.3	21.4	0.7%	1.4%	21%
PRTP $\rho = 0.5$	34.3	22.6	23.0	2.1%	2.1%	21%
DICE 2007 Damages	16.0	13.4*	13.0	2.8%	0.4%	21%
Howard & Sterner Damages	100	74.4*	80.9	8.1%	6.7%	35%
Cubic Damages	70.8	46.8*	48.6	3.8%	8.7%	76%
Cubic Damages, $\rho = 0.5$	122	71.2*	70.1	1.5%	10.6%	63%
Epstein-Zin: $\eta = 2, RRA = 6$	26.4	21.8*	19.8	10.1%	3.7%	32%
Epstein-Zin: $\eta = \frac{2}{3}, RRA = 6$	87.7	57.5*	51.3	12.1%	5.7%	20%

Table 1: Total risk premium for formula without policy response, with policy response adjustment, and from the recursive dynamic programming model. The error column specifies the deviation of the adjusted formula from the numeric solution. The right column presents the relative risk premium, i.e., the risk premium relative to the deterministic SCC based on the full stochastic model. The asterix * denotes results using two Gaussian-quadrature nodes to calculate the temperature elasticity $\epsilon_{T,s}$ instead of merely using the expected climate sensitivity.

damage to a cubic formulation, $D^{cub}(T) = a^{cub}T^3$, we adjust the linear coefficient a^{cub} such that both functions imply the same damages at a 2.5°C warming, the main calibration point of Nordhaus DICE model. Thus, damages are initially smaller, but eventually larger, at least with some probability. In this scenario, the risk premium increases to 76% for our base specification and to 63% for the reduced time preference. Our simple formula adjustment would imply a an error of over 20%, substantially underestimating the risk premium. Here, evaluating the elasticity $\epsilon_{T,s}$ for a high and a low realization of the climate sensitivity substantially improves the formula's accuracy.

Risk Preferences. Finally, the table varies the decision maker's risk aversion. Merely reducing $RRA = 2$ back to DICE's original value of 1.45 reduces the relative premium to about 20%, but *increases* the absolute risk premium. In the standard economic model, the risk aversion RRA serves two purposes. First, it characterizes the decision maker's risk aversion. Second, it characterizes the decision maker's desire to smooth consumption over time. Because the world economy is growing over time, this second effect of RRA translates into growth discounting; the weight on the future increases as the desire to redistribute from the rich future to the poor present falls. As a result, the concern for climate change and the risk premium increases despite the reduction in risk aversion. Epstein & Zin (1989) preferences disentangle the concerns for consumption smoothing and for risk aversion. A large branch of asset pricing literature shows that such preferences also substantially improve the explanation of observed discount rates and risk premia on the asset markets (Vissing-Jørgensen & Attanasio 2003, Bansal & Yaron 2004, Bansal et al. 2010, Chen et al. 2013, Bansal et al. 2012, 2014, Collin-Dufresne et al. 2016, Nakamura et al. 2017).

Table 1's Epstein-Zin specification increases the risk aversion without simultaneously reducing the desire to smooth consumption over time. Based on the estimates cited above, we increase risk aversion to $RRA = 6$. In this specification, the increase in risk aversion also increases the relative risk premium, which is now over 30%. However, the cited literature on long-run risk also suggests that Epstein-Zin preferences should use a higher elasticity of intertemporal substitution (lower aversion, $\eta = \frac{2}{3}$), which reduces the risk premium back to 20%. The premium is large in absolute terms, but the higher intertemporal elasticity reduces the growth-based discounting and substantially increases the deterministic SCC. The large positive risk premium contrasts sharply with the impact of growth uncertainty on optimal climate policy studied in Jensen & Traeger (2014) and Cai & Lontzek (2019), who find a negative risk premium under the same increase in the elasticity of intertemporal substitution. The difference arises because of the "dual impact" of the structural climate sensitivity parameter. Our analytic formula performs worse for Epstein-Zin preferences because it does not account for the recursive evaluation structure of such preferences.[13]

Comment. We close this section by pointing out that we do not fully relax Assumption 2 with our adjusted formula. We merely relax it in one place where it is (i) easy to do and (ii) quantitatively most relevant. Relaxing the assumption in the full dynamic derivation would make the formula intractable and not particularly insightful. Yet, it turns out that our simple elasticity adjustment together with Proposition 2's formula does a perfectly adequate job in approximating the risk premium. We note that it is straight forward to calculate the consumption elasticity to climate sensitivity from the deterministic model in a similar way. Yet, given the small magnitude of the risk and prudence contributions, the corresponding error is negligible and we chose simplicity.[14]

[13]Our formula can disentangle risk aversion entering the risk premia $X_t(\cdot)$ in equation (5) and $Z_t(\cdot)$ in equation (7) from consumption smoothing entering the other terms of equations (6) and (8). We obtain the analytic formula only by omitting the recursive structure of Epstein-Zin preferences. The recursive evaluation can imply substantial premia for an early resolution of uncertainty. Epstein et al. (2014) show that these might be unreasonably high. A nice feature of the formula is that it does not involve such premia.

[14]The proof of Proposition 2 also discusses an even simpler adjustment of the consumption elas-

4.3 Quantifying the Risk Channels

Figure 4 quantifies the different uncertainty channels using the adjusted formula for selected scenarios. The top left panel depicts the contributions for the *baseline calibration*, which look similar to the original plot in Figure 2. The only notable difference is that the direct risk aversion contribution and the interaction term between risk aversion and the damage convexity are now truly indistinguishable in their contributions to the risk premium.

Time preference. The bottom left panel of Figure 4 identifies the contributing channels under the *reduction of the annual rate of pure time preference* from $\rho = 1.5\%$ to $\rho = 0.5\%$. We find that this reduction of the discount rate hardly changes the composition of the contributing factors (a tiny increase of the direct risk aversion contribution relative to the interaction term). Comparing the scales and the shape of the two l.h.s. panels, we find that the main contributions derive from the same periods as before, only weighted slightly higher. While the cumulative contributions flatten out a little slower, what happens after the year 2200 still hardly matters for the (SCC and its) risk premium.

Damage normalization. The right panels of Figure 4 change the shape of the DICE damage function and alter the contributing channels most notably. Nordhaus (2008) normalizes quadratic damages as $D(T) = \frac{aT^2}{1+aT^2}1$, which avoids that damages can exceed total production. Nordhaus & Sztorc (2013) and Nordhaus (2017) still use this formulation in the model description, but the actual implementations of DICE 2013 and DICE 2016 now use the simpler formulation $D(T) = aT^2$, which has the advantage that a directly characterizes the fraction of world GDP lost at a 1°C warming. The top right panel changes the damage function "back" to its normalized version. This minor change under certainty implies a notable distinction for the risk premium. The formulation bounding the potential loss implies a falling damage convexity. As a result, the economy prudence term contributes negatively; uncertainty's production loss caused by the damage convexity falls in temperature. The welfare prudence contribution is zero and the welfare-economy interaction contributes positively with a similar magnitude as the economy prudence effect. As a result, we only find a positive risk premium because of the structural uncertainty.

Cubic damages. The lower right panel of Figure 4 analyzes the channels for our cubic damage specification ($D^{cub}(T) = a^{cub}T^3$). We note that the scale of this lower

ticity to climate sensitivity to help bounding the error from a limited decision response, see the discussion of the additional parameter χ in $\epsilon_{T,s} = \chi \frac{dT}{ds} \frac{s}{T}$ in Appendix B.

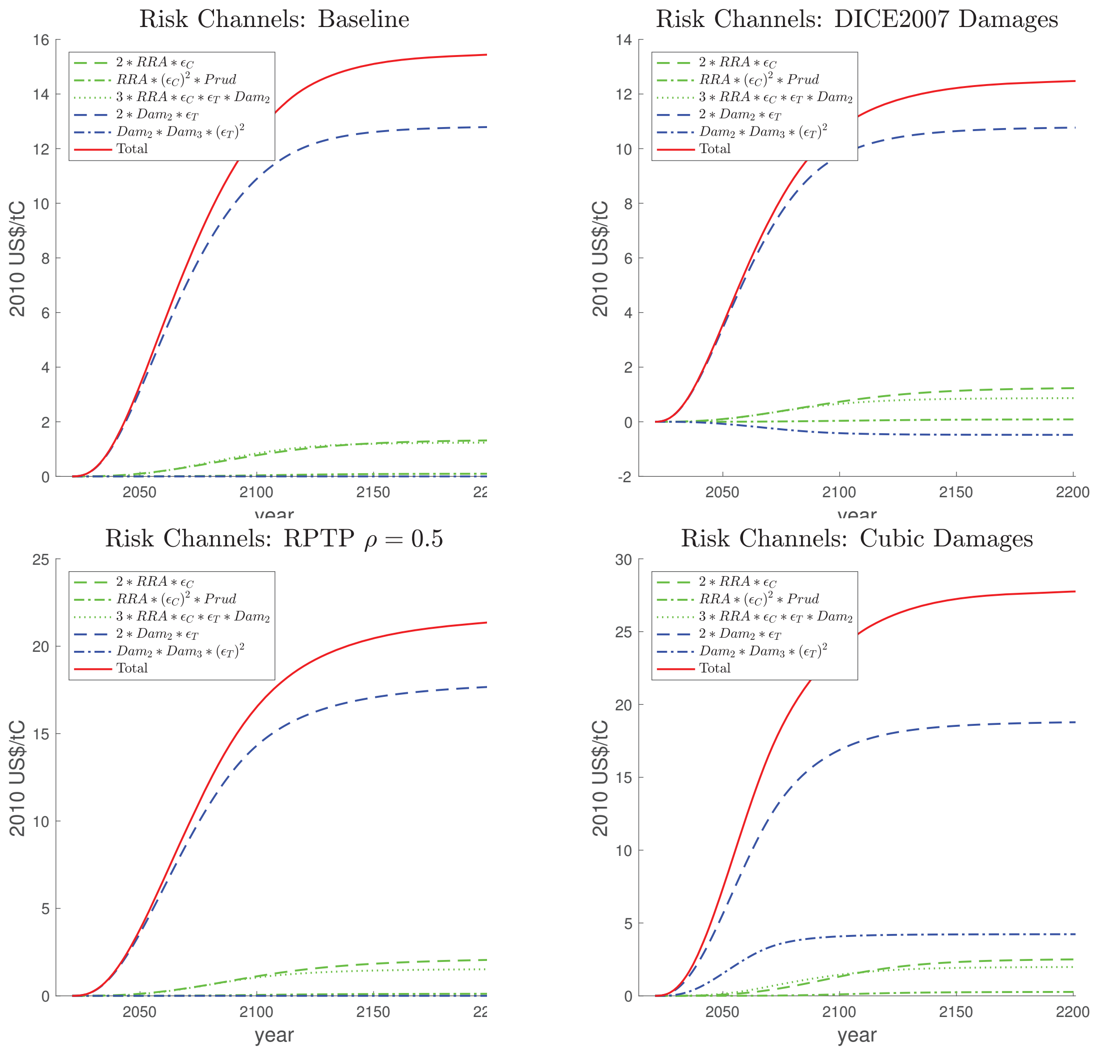

Figure 4: Uncertainty about climate sensitivity. Cumulative contribution to the SCC's risk premium in 2020 by channel. The base calibration (top left) uses a pure time preference of $\rho = 1.5\%$ and quadratic non-normalized damages, the other panels specify the change.

right panel is substantially higher. All non-zero contributions increase. In absolute terms, the damage convexity contribution increases the most. In relative terms, the economy-prudence channel catches up the most; it is zero or even negative in the other specifications and now contributes a substantial part of the risk premium. Also the welfare-economy interaction contribution grows together with the damage convexity.

Stochasticity premium. Figure 5 presents the corresponding graphs for the stochasticity premium. In the baseline (top left), the stochasticity premium is almost entirely driven by the welfare-economy interaction. The top right panel normalizes damages to never exceed production. As a result, the damage convexity falls in temperature and the economy prudence term almost cancels the positive contribution,

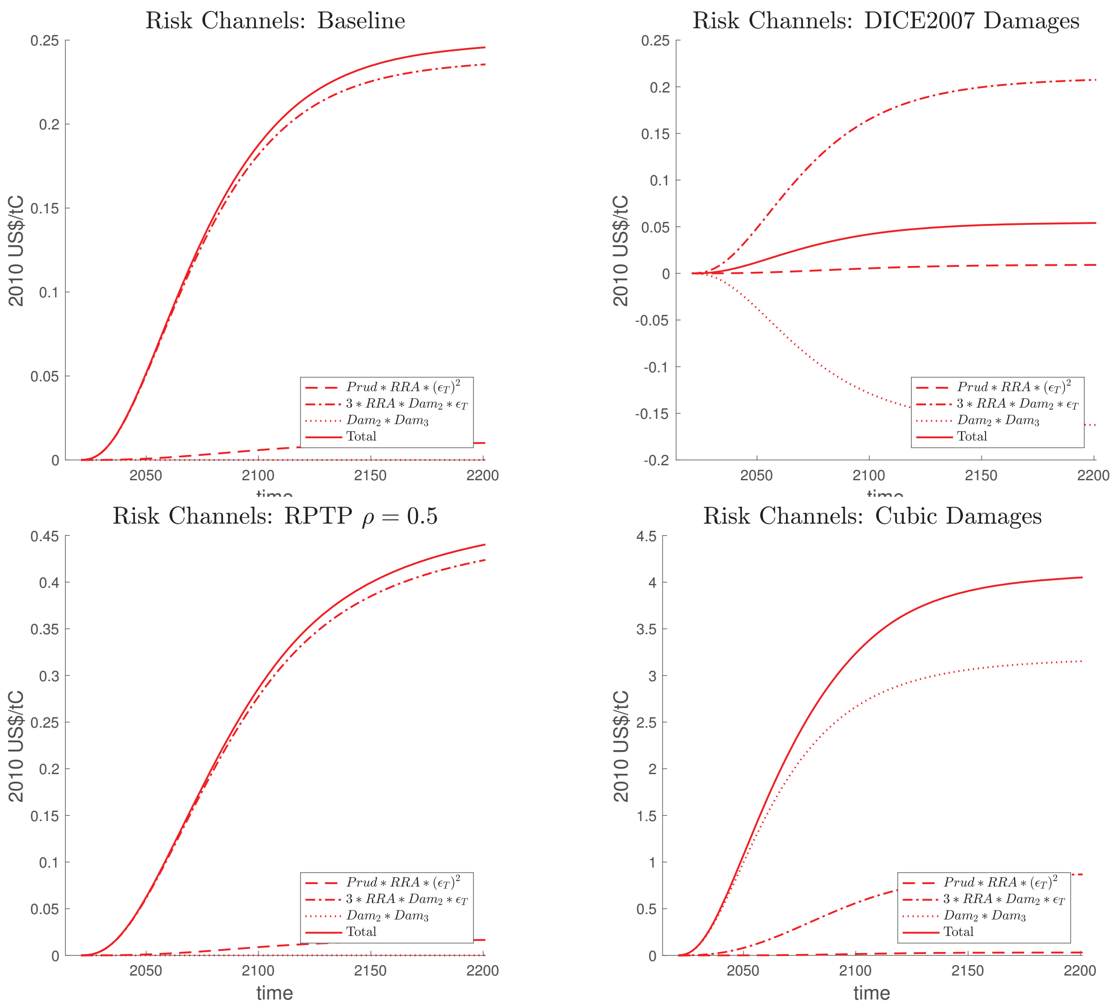

Figure 5: Stochastic temperature evolution. Cumulative contribution to the SCC's risk premium in 2020 by channel. The base calibration (top left) uses a pure time preference of $\rho = 1.5\%$ and quadratic non-normalized damages, the other panels specify the change.

leaving a negligible stochasticity premium. In contrast, the cubic damage specification results in a positive economy-prudence contribution that dominates the stochasticity premium, which now delivers a more notable contribution to the overall risk premium.

5 Conclusions

We analyze the climate policy risk premium, focusing on the uncertainty governing the temperature's response to greenhouse gas emissions (climate sensitivity). Using a simple but general integrated assessment model of climate change, we present an analytic formula approximating the risk premium and characterizing the different risk channels. Our initial analytic derivation relies on shutting down the decision maker's future response to uncertainty. In a DICE-based application, we show that a lack of future responsiveness substantially overestimates the climate risk premium, and we motivate a simple "fix" that derives a crucial response elasticity from the underlying deterministic model. The adjusted formula closely matches the risk premium of the stochastic dynamic programming model.

The familiar precautionary savings motive is driven by prudence. Prudence characterizes how risk aversion responds to the consumption level; as we become richer, (absolute) risk aversion falls and, thus, we have an additional incentive to save under risk. Integrated assessment of climate change augments this reasoning in two fundamental ways, giving rise to two additional sets of risk channels. First, future temperatures have a non-linear impact on the economy that interacts with the marginal appreciation for consumption. As a result, we identify the additional economy-prudence and welfare-economy interaction channels. Climate damages are convex and this convexity makes uncertainty costly. If this costly convexity increases in temperature, then policy has an additional incentive to keep temperature down under uncertainty, establishing the economy-prudence effect. We show that the economy-prudence effect is either negative or zero in DICE, depending on the model version, and that it delivers a positive premium if damages are more convex (cubic rather than quadratic) that is substantially larger than the welfare prudence effect ("standard precautionary savings motive"). The welfare-economy interaction channel is always positive for a risk averse decision maker and convex climate damages. This interaction channel dominates the risk premium for a merely stochastic evolution of temperature. However, it is still small compared to the risk premium resulting from climate sensitivity uncertainty.

The second set of novel channels arises from the fact that climate sensitivity is a structural parameter that governs both the *marginal* impact of CO_2 released today and the long-run temperature *level*. We show that, (only!) as a result of the structural parameter's "dual impact", risk aversion and damage convexity have a direct impact on the risk premium. Quantitatively, the damage convexity channel dominates in all of our scenarios. In contrast to common belief, the welfare based prudence and risk aversion effects contribute only a small fraction to the climate policy's risk premium. The empirical literature and some of the modeling literature uses stochastic temperature evolution as a proxy for uncertain future climate change. Our results raise awareness that the optimal response to an uncertain climate differs fundamentally from the response to a merely stochastic evolution of the climate. While our paper quantifies the impact for a global "mitigation investment", the basic insight carries over to adaptation investments.

We find a risk premium of 20-25% in our base calibration, varying alternatively time preference, risk aversion, and whether we use simple or normalized DICE-damages. Changing DICE's quadratic damage function to a cubic form substantially increases the risk premium to around 65-75% of its deterministic value (depending on time preference). Epstein-Zin preferences disentangle risk aversion from the intertemporal elasticity of substitution. They allow us to increase risk aversion more drastically (RRA=6) without excessively discounting the future. This increase in risk aversion raises the baseline premium to over 30%. However, the long-run risk literature also suggests that Epstein-Zin preferences should use a higher elasticity of intertemporal substitution, which reduces the risk premium back to 20%. The premium is large in absolute terms, but the higher intertemporal elasticity reduces the growth-based discounting and (even more) substantially increases the deterministic SCC. This large positive premium contrasts sharply with the impact of growth uncertainty on optimal climate policy studied in Jensen & Traeger (2014) and Cai & Lontzek (2019), who find a negative risk premium for the same increase in the elasticity of intertemporal substitution. The difference arises because yet again because climate sensitivity is a structural parameter with "dual impact".

Our study emphasizes the analytic drivers of the risk premium and quantifies the premium for a variety of scenarios. We use a simple climate change model with reasonable performance. We leave more detailed climate change models that accurately generate higher order moments of the climate sensitivity distribution to future and paralleling research. We focus on climate sensitivity uncertainty. Several exciting papers are quantifying the climate policy impact of multiple interacting uncertainties and we hope that the concepts developed here will also help to better understand the interactions.

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Appendix

A Proof of Proposition 1

The social planner solves the Bellman equation (1)

$$V(K_t, M_t, T_t, t) = \max_{C_t, E_t} u_t(C_t) + \beta\, \mathbb{E}_t\, V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)$$

subject to the equations of motion

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + F(K_t, T_t, E_t, t) - C_t \\ M_{t+1} &= M_{t+1}(M_t, E_t) \quad \equiv M(M_t, E_t, t) \\ T_{t+1} &= T_{t+1}(T_t, M_{t+1}) \quad \equiv T(T_t, M_{t+1}, t). \end{aligned}$$

We assume that today's emission add directly to the next period's carbon stock: $\frac{\partial M_1}{\partial E_0} = 1$. Without this assumption, the derived formula for the social cost of carbon will merely be multiplied by the factor $\frac{\partial M_1}{\partial E_0}$ if it differs from unity. We slightly violate notation in using M_t for the stock of carbon in the atmosphere and for the function $M(\cdot, t)$ describing its evolution over time. This notation enables us to abbreviate more intuitively the change of the carbon stock in period t as a result of a change of the carbon stock in period $\tau < t$ as $\frac{\partial M_t}{\partial M_\tau} = \prod_{i=\tau+1}^{t} \frac{\partial M_i}{\partial M_{i-1}} \equiv \prod_{i=\tau+1}^{t} \frac{\partial M(M_{i-1}, E_{i-1}, i-1)}{\partial M_{i-1}}$. We proceed analogously with temperature, abbreviating the change of the warming level in period t as a result of a change of the warming level in period $\tau < t$ as $\frac{\partial T_t}{\partial T_\tau} = \prod_{i=\tau+1}^{t} \frac{\partial T_i}{\partial T_{i-1}}$.

The first order conditions for consumption and emissions imply

$$u_t'(C_t) = \beta\, \mathbb{E}_t \frac{\partial V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)}{\partial K_{t+1}} \tag{10}$$

and

$$\begin{aligned} &\beta\, \mathbb{E}_t \frac{\partial V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)}{\partial K_{t+1}} \left(-\frac{\partial F(K_t, T_t, E_t, t)}{\partial E_t} \right) \\ &= \beta\, \mathbb{E}_t \left(\frac{\partial V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)}{\partial M_{t+1}} + \frac{\partial V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)}{\partial T_{t+1}} \frac{\partial T_{t+1}}{\partial M_{t+1}} \right) \frac{\partial M(M_t, E_t, t)}{\partial E_t} \end{aligned} \tag{11}$$

The term $-\frac{\partial F(K_t, T_t, E_t, t)}{\partial E_t}$ denotes the benefits from another emission unit. In period t, this emission benefit is deterministic, and the remaining stochastic term on the left hand side, $\beta \frac{\partial V(K_{t+1}, M_{t+1}, T_{t+1}, t+1)}{\partial K_{t+1}}$, equals by equation (10) the marginal utility in period t. Thus, making use of equation (10) when solving equation (11) for the optimality condition of the marginal benefit from another emission unit in the present ($t = 0$) delivers the expression

$$-\frac{\partial F}{\partial E_0} = \underbrace{\frac{1}{u_0'(C_0)}\beta\,\mathbb{E}_0\left(\frac{\partial V}{\partial M_1} + \frac{\partial V}{\partial T_1}\frac{\partial T_1}{\partial M_1}\right)\frac{\partial M(M_0, E_0, 0)}{\partial E_0}}_{\equiv SCC_0}\,. \tag{12}$$

In this social optimum, the marginal benefit from emitting another ton of carbon $-\frac{F(K_t,T_t,E_t,t)}{\partial E_t}$ has to equal the social cost of carbon given by the expression on the right hand side.

By the envelope theorem, the Bellman equation (1) delivers

$$\begin{aligned}\frac{\partial V}{\partial T_t} &= \beta\,\mathbb{E}_t\left(\frac{\partial V}{\partial K_{t+1}}\frac{\partial F}{\partial T_t} + \frac{\partial V}{\partial T_{t+1}}\frac{\partial T(T_t, M_{t+1}, t)}{\partial T_t}\right) \\ &= u_t'(C_t)\frac{\partial F}{\partial T_t} + \beta\,\mathbb{E}_t\,\frac{\partial V}{\partial T_{t+1}}\frac{\partial T(T_t, M_{t+1}, t)}{\partial T_t}\end{aligned} \tag{13}$$

where we obtain the second equality using that $\frac{\partial F}{\partial T_t}$ is known in period t so that we can employ equation (10).[15] Advancing equation (13) by one period and reinserting it into equation (13) delivers

$$\begin{aligned}\frac{\partial V}{\partial T_t} &= u_t'(C_t)\frac{\partial F}{\partial T_t} + \beta\,\mathbb{E}_t\,u_{t+1}'(C_{t+1})\frac{\partial F}{\partial T_{t+1}}\frac{\partial T(T_t, M_{t+1}, t)}{\partial T_t} \\ &\quad+\beta\,\mathbb{E}_t\,\beta\,\mathbb{E}_{t+1}\,\frac{\partial V}{\partial T_{t+2}}\frac{\partial T(T_{t+1}, M_{t+2}, t+1)}{\partial T_{t+1}}\frac{\partial T(T_t, M_{t+1}, t)}{\partial T_t} \\ &= u_t'(C_t)\frac{\partial F}{\partial T_t} + \beta\,\mathbb{E}_t\,u_{t+1}'(C_{t+1})\frac{\partial F}{\partial T_{t+1}}\frac{\partial T_{t+1}}{\partial T_t} + \beta^2\,\mathbb{E}_t\,\frac{\partial V}{\partial T_{t+2}}\frac{\partial T_{t+2}}{\partial T_t},\end{aligned} \tag{14}$$

where we made use of the law of iterated expectations (tower rule) and the notation $\frac{\partial T_t}{\partial T_\tau} \equiv \prod_{i=\tau+1}^{t}\frac{\partial T_i}{\partial T_{i-1}}$ for $\tau < t$. For simplicity in subsequent notation we "define" $\frac{\partial T_t}{\partial T_t} = 1$. Repeating the step of advancing equation (13) and reinserting it into equation (14) eventually delivers

$$\frac{\partial V}{\partial T_t} = \mathbb{E}_t\sum_{\tau=t}^{\infty} u_\tau'(C_\tau)\frac{\partial F}{\partial T_\tau}\beta^{\tau-t}\frac{\partial T_\tau}{\partial T_t} + \lim_{\tau\to\infty}\beta^{\tau-t}\,\mathbb{E}_t\,\frac{\partial V}{\partial T_\tau}\frac{\partial T_\tau}{\partial T_t}.$$

We assume that the dynamic system is well-defined so that the shadow value of the temperature increase $\frac{\partial V}{\partial T_\tau}$ grows sufficiently slow along the optimal path to make

[15] Let $(\Omega, \mathcal{F}, \mathcal{P})$ denote the underlying probability space. The equations of motions are conditional on the events $\omega \in \Omega$. Let $\{\mathcal{F}_t\}_{t\geq 0}$ denote the filtration generated by the underlying stochastic processes. Then, $\frac{\partial F}{\partial T_t}$ is measurable with respect to the sigma algebra $\mathcal{F}_t$.

the limit approach zero (which should hold along any somewhat reasonable policy scenarios).

By the envelope theorem, the Bellman equation (1) also delivers

$$\frac{\partial V}{\partial M_t} = \beta\, \mathbb{E}_t \left(\frac{\partial V}{\partial M_{t+1}} + \frac{\partial V}{\partial T_{t+1}} \frac{\partial T(T_t, M_{t+1}, t)}{\partial M_{t+1}} \right) \frac{\partial M(M_t, E_t, t)}{\partial M_t} \tag{15}$$

$$= \mathbb{E}_t \left(\beta \frac{\partial V}{\partial M_{t+1}} + \beta \sum_{\tau=t+1}^{\infty} u'_\tau(C_\tau) \frac{\partial F}{\partial T_\tau} \beta^{\tau-(t+1)} \frac{\partial T_\tau}{\partial T_{t+1}} \frac{\partial T_{t+1}}{\partial M_{t+1}} \right) \frac{\partial M_{t+1}}{\partial M_t}$$

making use of equation (14). Using the definitions $\frac{\partial M_t}{\partial M_\tau} \equiv \prod_{i=\tau+1}^{t} \frac{\partial M_i}{\partial M_{i-1}}$ and $\frac{\partial M_t}{\partial M_t} = 1$ and repeatedly advancing equation (15) and reinserting it into itself yields

$$\frac{\partial V}{\partial M_t} = \mathbb{E}_t \sum_{j=t+1}^{\infty} \beta^{j-t} \sum_{\tau=j}^{\infty} u'_\tau(C_\tau) \frac{\partial F}{\partial T_\tau} \beta^{\tau-j} \frac{\partial T_\tau}{\partial T_j} \frac{\partial T_j}{\partial M_j} \frac{\partial M_j}{\partial M_t} + \lim_{\tau\to\infty} \beta^{\tau-t}\, \mathbb{E}_t \frac{\partial V}{\partial M_\tau} \frac{\partial M_\tau}{\partial M_t}.$$

We assume that also the shadow value of carbon (weighted with its persistence) does not outgrow the exponential damping of the discount factor and that the limit approaches zero.

Reinserting our findings into equation (12) implies

$$-\frac{\partial F}{\partial E_0} = SCC_0 = \frac{1}{u'_0(C_0)}\, \mathbb{E}_0 \sum_{j=1}^{\infty} \sum_{\tau=j}^{\infty} \beta^\tau u'_\tau(C_\tau) \frac{\partial F}{\partial T_\tau} \frac{\partial T_\tau}{\partial T_j} \frac{\partial T_j}{\partial M_j} \frac{\partial M_j}{\partial M_1} \frac{\partial M_1}{\partial E_0}\ . \tag{16}$$

We assumed that $\frac{\partial M_1}{\partial E_0} = 1$. The indices τ of the second sum are always larger than the j-index of the first sum. For every τ the double-sum runs through the j indices lower or equal to τ, capturing the marginal emission unit's impact on warming in all previous periods. Thus, we can reorder the sum to the form

$$-\frac{\partial F}{\partial E_0} = SCC_0 = \frac{1}{u'_0(C_0)}\, \mathbb{E}_0 \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \beta^t u'_t(C_\tau) \frac{\partial F}{\partial T_t} \frac{\partial T_t}{\partial T_\tau} \frac{\partial T_\tau}{\partial M_\tau} \frac{\partial M_\tau}{\partial M_1} \tag{17}$$

where we relabled $\tau \to t$ and $j \to \tau$ with respect to equation (16). In the main text, we abbreviated $\frac{\partial T_\tau}{\partial M_j} \equiv \frac{\partial T_\tau}{\partial T_j} \frac{\partial T_j}{\partial M_j}$.

B Proof of Proposition 2

Under Assumption A2, the emission and investment levels do not respond to the realizations of climate sensitivity. As a consequence, future capital levels remain independent of the climate sensitivity realizations, and all damage to production translates straight into consumption and welfare. By the envelope theorem we know that this assumption does not affect the first order calculations and the derivation of the social cost of carbon in section A. However, the assumption will matter further below when we take higher order derivatives of the social cost of carbon. As a result of the assumption, the impact of climate sensitivity on the social cost of carbon is entirely transmitted through temperature, production increase, and consumption loss. Marking these, and only these dependencies restates equation (17) as

$$SCC_0 = \frac{1}{u_0'(C_0)} \, \mathbb{E} \left[\sum_{t=1}^{\infty} \beta^t \underbrace{u_t'\big(C_t(T_t(\mathrm{s}))\big) \left[-\tfrac{\partial F_t}{\partial T_t}(T_t(\mathrm{s})) \right] \left[\sum_{\tau=1}^{t} \tfrac{\partial T_t}{\partial M_\tau}(\mathrm{s}) \tfrac{\partial M_\tau}{\partial E_0} \right]}_{\equiv z_t(\mathrm{s})} \right].$$

For now, we analyze the period t contribution individually. By Jensen's inequality, the uncertainty over climate sensitivity will increase the SCC's period t contribution if and only if the term $z_t(\mathrm{s})$ is convex. Moreover, the variance based small-risk approximation adds a contribution to the deterministic $z_t(\mathbb{E}\, s)$ that is proportional to the climate sensitivity's variance and to $z_t''(s)$:

$$\begin{aligned} \mathbb{E}\, z_t(s) &\approx z_t(\mathbb{E}\, s) + \mathbb{E}\, z_t'([\mathbb{E}\, s])(s - \mathbb{E}\, s) + \frac{1}{2}\, \mathbb{E}\, z_t''([\mathbb{E}\, s])(s - \mathbb{E}\, s)^2 \\ &= z_t(\mathbb{E}\, s) + \frac{1}{2} z_t''([\mathbb{E}\, s])\, \mathbb{V}\mathrm{ar}(s). \end{aligned}$$

Because s is a common uncertain parameter for all periods, the climate sensitivity's variance simultaneously multiplies the sum over per period contributions: $\mathbb{V}\mathrm{ar}(s) \sum_{t=1}^{\infty} \beta^t z_t''([\mathbb{E}\, s])$. Thus, for small risks, the present SCC's risk premium per unit of variance is

$$\frac{\Delta SCC_0}{\mathbb{V}\mathrm{ar}(s)} = \frac{1}{u_0'(C_0)} \sum_{t=1}^{\infty} \beta^t \frac{\mathbb{E}\, z_t(s) - z_t(\mathbb{E}\, s)}{\mathbb{V}\mathrm{ar}(s)} \approx \frac{1}{2u_0'(C_0)} \sum_{t=1}^{\infty} \beta^t z_t''([\mathbb{E}\, s]). \tag{18}$$

In the following we calculate the second order derivative of the term $z_t(s)$, abbreviatig under slight violation of notation $\mathbb{E}\, s$ by s. Our assumption that investment remains constant while consumption absorbs the climate sensitivity uncertainty translates into $C_t(T_t(\mathrm{s})) = F(T_t(\mathrm{s})) - I_t$, where investment $I_t = K_{t+1} - (1-\delta)K_t$ does not respond to climate sensitivity. As a consequence we have $\frac{\partial C_t(T_t(\mathrm{s}))}{\partial s} = \frac{\partial F(T_t(\mathrm{s}))}{\partial s}$ and consumption absorbs all the uncertainty.

Without Assumption A2, the optimal policy would spread the uncertainty between consumption and investment, slightly reducing the cost of uncertainty. In contrast, Assumption A2 slightly exaggerates the consumption response to uncertainty. Alternatively, we can assume that only a fraction χ of the production impact translates into consumption change and assume

$$\frac{\partial C_t(T_t(\mathrm{s}))}{\partial s} = \chi \frac{\partial F(T_t(\mathrm{s}))}{\partial s},$$

$0 < \chi \leq 1$. Picking χ as the consumption rate we would more appropriately approximate the immediate cost of uncertainty for consumption, but neglect the consequences for investment that would respond proportional to $1-\chi$. This approach would undervalue the costs of uncertainty. We will derive the subsequent analysis with general χ, and Proposition B corresponds to the case where $\chi = 1$. Then

$$\begin{aligned} z_t(s) &= -u_t'\big(F(T_t(\mathrm{s})) - I_t\big)\ F_t'(T_t(\mathrm{s}))\ \left[\sum_{\tau=1}^{t} \tfrac{\partial T_t}{\partial M_\tau}(\mathrm{s})\tfrac{\partial M_\tau}{\partial E_0}\right] \\ z_t'(s) &= -u_t''\big(F(T_t(\mathrm{s})) - I_t\big)\,\chi\, F_t'(T_t(\mathrm{s}))^2\, T_t'(\mathrm{s})\ \left[\sum_{\tau=1}^{t} \tfrac{\partial T_t}{\partial M_\tau}(\mathrm{s})\tfrac{\partial M_\tau}{\partial E_0}\right] \\ &\quad -u_t'\big(F(T_t(\mathrm{s})) - I_t\big)\ F_t''(T_t(\mathrm{s}))\, T_t'(\mathrm{s}) \left[\sum_{\tau=1}^{t} \tfrac{\partial T_t}{\partial M_\tau}(\mathrm{s})\tfrac{\partial M_\tau}{\partial E_0}\right] \\ &\quad -u_t'\big(F(T_t(\mathrm{s})) - I_t\big)\ F_t'(T_t(\mathrm{s}))\ \left[\sum_{\tau=1}^{t} \tfrac{\partial^2 T_t}{\partial M_\tau \partial s}(\mathrm{s})\tfrac{\partial M_\tau}{\partial E_0}\right]. \end{aligned}$$

Defining $A_t(s) = \sum_{\tau=1}^{t} \frac{\partial T_t}{\partial M_\tau}(\mathrm{s})\frac{\partial M_\tau}{\partial E_0}$ we obtain the second order derivative

$$\begin{aligned} z_t''(s) = &-u_t'''(\cdot)\ \chi^2\ F_t'(\cdot)^3\, T_t'(\cdot)^2 A_t(s) - 2u_t''(\cdot)\ \chi\ F_t'(\cdot)F_t''(\cdot)\, T_t'(\cdot)^2 A_t(s) \\ &-u_t''(\cdot)\ \chi\ F_t'(\cdot)^2\, T_t''(\cdot) A_t(s) - u_t''(\cdot)\ \chi\ F_t'(\cdot)^2\, T_t'(\cdot) A_t'(s) \\ &-u_t''(\cdot)\ \chi\ F_t'(\cdot)F_t''(\cdot)T_t'(\cdot)^2 A_t(s) - u_t'(\cdot)F_t'''(\cdot)T_t'(\cdot)^2 A_t(s) \\ &-u_t'(\cdot)F_t''(\cdot)T_t''(\cdot)A_t(s) - u_t'(\cdot)F_t''(\cdot)T_t'(\cdot)A_t'(s) \\ &-u_t''(\cdot)\ \chi\ F_t'(\cdot)^2\, T_t'(\cdot)A_t'(s) - u_t'(\cdot)\ F_t''(\cdot)T_t'(\cdot)A_t'(s) - u_t'(\cdot)\ F_t'(\cdot)A_t''(s) \end{aligned}$$

Collecting terms delivers

$$\begin{aligned} z_t''(s) = &-u_t'''(\cdot)\ \chi^2\ F_t'(\cdot)^3\, T_t'(\cdot)^2 A_t(s) - 3u_t''(\cdot)\ \chi\ F_t'(\cdot)F_t''(\cdot)\, T_t'(\cdot)^2 A_t(s) \\ &-u_t''(\cdot)\ \chi\ F_t'(\cdot)^2\, T_t''(\cdot)A_t(s) - 2u_t''(\cdot)\ \chi\ F_t'(\cdot)^2\, T_t'(\cdot)A_t'(s) - u_t'(\cdot)F_t'''(\cdot)T_t'(\cdot)^2 A_t(s) \\ &-u_t'(\cdot)F_t''(\cdot)T_t''(\cdot)A_t(s) - 2u_t'(\cdot)F_t''(\cdot)T_t'(\cdot)A_t'(s) - u_t'(\cdot)F_t'(\cdot)A_t''(s). \end{aligned}$$

Dividing by $u_t'(C_t)$, $-F'(T_t(s)) > 0$, and A_t results in

$$
\begin{aligned}
&\frac{z_t''(s)}{u_t'(\cdot)(-F'(\cdot))A_t} \\
&= -\frac{u_t'''(\cdot)}{u_t''(\cdot)}C_t \frac{-F_t'(\cdot)\,T_t'(\cdot)}{C_t}\left(-\frac{u_t''(\cdot)}{u_t'(\cdot)}C_t\right)\frac{-F_t'(\cdot)\,T_t'(\cdot)}{C_t}\,\chi^2 - 3\frac{u_t''(\cdot)}{u_t'(\cdot)}C_t\frac{-F_t'(\cdot)\,T_t'(\cdot)}{C_t}\frac{-F_t''(\cdot)}{-F_t'(\cdot)}T_t\frac{T_t'(\cdot)}{T_t}\,\chi \\
&\quad -\frac{u_t''(\cdot)}{u_t'(\cdot)}C_t\frac{-F_t'(\cdot)T'(\cdot)}{C_t}\frac{T_t''(\cdot)}{T_t'(\cdot)}\,\chi - 2\frac{u_t''(\cdot)}{u_t'(\cdot)}C_t\frac{-F_t'(\cdot)T_t'(\cdot)}{C_t}\frac{A_t'(\cdot)}{A_t}\,\chi + \frac{-F_t'''(\cdot)}{-F_t''(\cdot)}T_t\frac{T_t'(\cdot)}{T_t}\frac{-F_t''(\cdot)}{-F_t'(\cdot)}T_t\frac{T_t'(\cdot)}{T_t} \\
&\quad + \frac{-F_t''(\cdot)}{-F_t'(\cdot)}T_t\frac{T_t'(\cdot)}{T_t}\frac{T_t''(\cdot)}{T_t'(\cdot)} + 2\frac{-F_t''(\cdot)}{-F_t'(\cdot)}T_t\frac{T_t'(\cdot)}{T_t}\frac{A_t'(\cdot)}{A_t} + \frac{A_t''(\cdot)}{A_t'(\cdot)}\frac{A_t'(\cdot)}{A_t} \\
&= \text{Prud}\,\hat\epsilon_{c,s}\,\text{RRA}\,\hat\epsilon_{c,s} + 3\,\text{RRA}\,\hat\epsilon_{c,s}\,\text{Dam}_2\,\hat\epsilon_{T,s} + \text{RRA}\,\hat\epsilon_{c,s}\frac{T_t''(\cdot)}{T_t'(\cdot)} + 2\,\text{RRA}\,\hat\epsilon_{c,s}\frac{A_t'(\cdot)}{A_t} \\
&\quad + \text{Dam}_3\,\hat\epsilon_{T,s}\,\text{Dam}_2\,\hat\epsilon_{T,s} + \text{Dam}_2\,\hat\epsilon_{T,s}\frac{T_t''(\cdot)}{T_t'(\cdot)} + 2\,\text{Dam}_2\,\hat\epsilon_{T,s}\frac{A_t'(\cdot)}{A_t} + \frac{A_t''(\cdot)}{A_t'(\cdot)}\frac{A_t'(\cdot)}{A_t} \\
&= \text{RRA}\,\hat\epsilon_{c,s}\left[2\frac{A_t'(\cdot)}{A_t} + \text{Prud}\,\hat\epsilon_{c,s} + 3\,\text{Dam}_2\,\hat\epsilon_{T,s}\right] + \text{Dam}_2\,\hat\epsilon_{T,s}\left[2\frac{A_t'(\cdot)}{A_t} + \text{Dam}_3\,\hat\epsilon_{T,s}\right] \\
&\quad + \frac{T_t''(\cdot)}{T_t'(\cdot)}\left[\text{RRA}\,\hat\epsilon_{c,s} + \text{Dam}_2\,\hat\epsilon_{T,s}\right] + \frac{A_t''(\cdot)}{A_t'(\cdot)}\frac{A_t'(\cdot)}{A_t} \\
&= \frac{1}{s^2}\Bigg\{\text{RRA}\,\epsilon_{c,s}\left[2\frac{sA_t'(\cdot)}{A_t} + \text{Prud}\,\epsilon_{c,s} + 3\,\text{Dam}_2\,\epsilon_{T,s}\right] \\
&\qquad + \text{Dam}_2\,\epsilon_{T,s}\left[2\frac{sA_t'(\cdot)}{A_t} + \text{Dam}_3\,\epsilon_{T,s}\right]\Bigg\} \\
&\quad + \frac{1}{s^2}\left\{\frac{sT_t''(\cdot)}{T_t'(\cdot)}\left[\text{RRA}\,\epsilon_{c,s} + \text{Dam}_2\,\epsilon_{T,s}\right] + \frac{sA_t''(\cdot)}{A_t'(\cdot)}\frac{sA_t'(\cdot)}{A_t}\right\}
\end{aligned} \tag{19}
$$

where we used the definitions $\text{RRA} = -\frac{u_t'''(\cdot)}{u_t''(\cdot)}c$, $\text{Prud} = -\frac{u_t'''(\cdot)}{u_t''(\cdot)}c$, $\text{Dam}_2 = \frac{-F_t''(\cdot)}{-F_t'(\cdot)}T$, $\text{Dam}_3 = \frac{-F_t'''(\cdot)}{-F_t''(\cdot)}T$, the semi-elasticities $\hat\epsilon_{c,s} = -\frac{dc}{ds}/c = -\chi\,F_t'(\cdot)T_t'(\cdot)\frac{1}{c}$ and $\hat\epsilon_{T,s} = \frac{dT}{ds}/T$, and the regular elasticities $\epsilon_{c,s} = -\frac{dc}{ds}/\frac{c}{s} = -\chi\,F_t'(\cdot)T_t'(\cdot)\frac{s}{c}$ and $\epsilon_{T,s} = \frac{dT}{ds}/\frac{T}{s}$.

Under Assumption A2 we have

$$
\begin{aligned}
T_{t+1} &= \gamma\,T_t + s\,\Gamma_1(M_{t+1}) + \Gamma_2(t) + \epsilon_t \\
&= \gamma\big(\gamma T_{t-1} + s\,\Gamma_1(M_t) + \Gamma_2(t-1) + \epsilon_{t-1}\big) \\
&\qquad + s\,\Gamma_1\big(M_{t+1}(M_t, E_t)\big) + \Gamma_2(t) + \epsilon_t \\
&= \gamma^{t+1}T_0 + s\sum_{\tau=1}^{t+1}\gamma^{t+1-\tau}\Gamma_1(M_\tau(\cdot)) + \sum_{\tau=0}^{t}\gamma^{t-\tau}\Gamma_2(\tau) + \sum_{\tau=0}^{t}\gamma^{t-\tau}\,\tilde\epsilon_\tau.
\end{aligned} \tag{20}
$$

Equation (25) in Appendix D.2 states the details of this relation for our numeric DICE implementation. Because the equation is linear in climate sensitivity we find that $\frac{sT_t''(\cdot)}{T_t'(\cdot)}=0$. Moreover, from $A_t(s)=\sum_{\tau=1}^{t}\frac{\partial T_t}{\partial M_\tau}(\mathrm{s})\frac{\partial M_\tau}{\partial E_0}$ we have

$$\frac{sA_t'(\cdot)}{A_t}=\frac{s\sum_{\tau=1}^{t}\frac{\partial^2 T_t}{\partial s\partial M_\tau}(\mathrm{s})\frac{\partial M_\tau}{\partial E_0}}{\sum_{\tau=1}^{t}\frac{\partial T_t}{\partial M_\tau}(\mathrm{s})\frac{\partial M_\tau}{\partial E_0}}=\frac{s\sum_{\tau=1}^{t}\frac{\partial \Gamma_1(M_\tau)}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}}{\sum_{\tau=1}^{t}s\frac{\partial \Gamma_1(M_\tau)}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}}=1$$

$$\frac{sA_t''(\cdot)}{A_t'(\cdot)}=0.$$

Thus, under Assumption A2, equation (19) simplifies to

$$\frac{s^2\, z_t''(s)}{u_t'(\cdot)(-F'(\cdot))A_t(\cdot)} = \underbrace{\mathrm{RRA}\,\epsilon_{c,s}\left[2+\mathrm{Prud}\,\epsilon_{c,s}+3\,\mathrm{Dam}_2\,\epsilon_{T,s}\right]+\mathrm{Dam}_2\,\epsilon_{T,s}\left[2+\mathrm{Dam}_3\,\epsilon_{T,s}\right]}_{\equiv X_t(\cdot)}. \tag{21}$$

Combining equations (18) and (21), and properly writing $\mathbb{E}\,s$ again for the expected climate sensitivity, we obtain the approximate risk premium to the SCC today in consumption equivalents as

$$\frac{\Delta SCC_0}{\mathbb{V}\mathrm{ar}(s)}\approx\frac{1}{2u_0'(C_0)}\sum_{t=1}^{\infty}\beta^t z_t''([\mathbb{E}\,s])=\frac{1}{2(\mathbb{E}\,s)^2}\sum_{t=1}^{\infty}\beta^t\frac{u_t'(C_t)}{u_0'(C_0)}(-F'(\cdot))A_t(\cdot)X_t(\cdot)$$
$$=\sum_{t=1}^{\infty}\sum_{\tau=1}^{t}\beta^t\frac{u_t'(c_t)}{u_0'(c_0)}\frac{\partial F_t}{\partial T_t}\frac{\partial T_t}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}\frac{1}{2(\mathbb{E}\,s)^2}X_t(\cdot),$$

which concludes the proof of the proposition.

Combining equations (18) and (19) gives generalizes the statement of Proposition 2 by abandoning Assumption 1.

Corollary 1. *Let $T_t(s)$ denote future temperature as a function of climate sensitivity (given emissions and today's state) and define $A_t(s)=\frac{\partial T_t}{\partial E_0}(s)\equiv\sum_{\tau=1}^{t}\frac{\partial T_t}{\partial M_\tau}(\mathrm{s})\frac{\partial M_\tau}{\partial E_0}$. Under Assumption A2, uncertainty over climate sensitivity increases the social cost of carbon contribution from a given period if and only if*

$$X_t^*(\cdot)\equiv\mathrm{RRA}\,\epsilon_{c,s}\left[2\frac{sA_t'(\cdot)}{A_t}+\mathrm{Prud}\,\epsilon_{c,s}+3\,\mathrm{Dam}_2\,\epsilon_{T,s}\right]$$
$$+\frac{sT_t''(\cdot)}{T_t'(\cdot)}\left[\mathrm{RRA}\,\hat\epsilon_{c,s}+\mathrm{Dam}_2\,\hat\epsilon_{T,s}\right]+\frac{sA_t''(\cdot)}{A_t'(\cdot)}\frac{sA_t'(\cdot)}{A_t}$$

is greater than zero. Arguments and their period-dependence are suppressed. Moreover, under a small risk approximation, the climate uncertainty premium is

$$\Delta SCC_0 \approx -\mathbb{E}_0 \sum_{t=1}^{\infty} \underbrace{\beta^t \frac{u_t'(C_t)}{u_0'(C_0)}}_{\substack{\textit{consumption}\\ \textit{discount}\\ \textit{factor}}} \underbrace{\frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial E_0}}_{\substack{\textit{marginal}\\ \textit{emission}\\ \textit{damage}}} \frac{\mathbb{V}\mathrm{ar}(s)}{2(\mathbb{E}\, s)^2} X_t^*(\cdot).$$

C Proof of Proposition 3

The proof is analogous to that of Proposition 2 and we focus on the steps that are different. Here, the relevant uncertainty is the shock process $(\epsilon_t)_{t\in\mathbb{N}}$ to temperature rather than climate sensitivity s. We spell out equation (17) suppressing all dependencies but the one of the stochastic shock process

$$SCC_0 = \frac{1}{u_0'(C_0)} \, \mathbb{E} \left[\sum_{t=1}^{\infty} \beta^t \underbrace{u_t'\big(C_t(T_t(\varepsilon_1, ..., \varepsilon_{t-1}))\big) \left[-\tfrac{\partial F_t}{\partial T_t}(T_t(\varepsilon_1, ..., \varepsilon_{t-1})) \right] \left[\sum_{\tau=1}^{t} \tfrac{\partial T_t}{\partial M_\tau} \tfrac{\partial M_\tau}{\partial E_0} \right]}_{\equiv v_t(\varepsilon_1, ..., \varepsilon_{t-1})} \right].$$

The main difference to the case of climate sensitivity uncertainty is that the temperature response to CO_2, $\frac{\partial T_t}{\partial M_\tau}$, is independent of the temperature shock ε_t and that we face a sequence of stochastic shocks.

Because temperature behaves as an autoregressive process we have $\frac{\partial T_t}{\partial \varepsilon_\tau} = \gamma^{t-\tau-1}$ for $\tau < t$. Therefore $\frac{\partial v_t}{\partial \varepsilon_\tau} = \gamma^{t-\tau-1} \frac{\partial v_t}{\partial \varepsilon_{t-1}}$. We find the small risk approximation

$$\begin{aligned}
&\mathbb{E} \sum_t \beta^t v_t(\varepsilon_1, ..., \varepsilon_{t-1}) \\
&\approx \sum_t \beta^t v_t(\mathbb{E}\,\varepsilon_1, ..., \mathbb{E}\,\varepsilon_{t-1}) + \mathbb{E} \sum_t \sum_{\tau<t} \beta^t \frac{\partial v_t(\mathbb{E}\,\varepsilon_1, ..., \mathbb{E}\,\varepsilon_{t-1})}{\partial\, \varepsilon_\tau} (\varepsilon_\tau - \mathbb{E}\,\varepsilon_\tau) \\
&\quad + \mathbb{E}\,\frac{1}{2} \sum_t \sum_{\tau<t} \sum_{k\le t} \beta^t \frac{\partial^2 v_t(\mathbb{E}\,\varepsilon_1, ..., \mathbb{E}\,\varepsilon_{t-1})}{\partial\, \varepsilon_\tau \,\partial\, \varepsilon_k} (\varepsilon_\tau - \mathbb{E}\,\varepsilon_\tau)(\varepsilon_k - \mathbb{E}\,\varepsilon_k) \\
&= \sum_t \beta^t v_t(0, ..., 0) + \frac{1}{2} \sum_t \sum_{\tau<t} \beta^t \gamma^{2(t-\tau-1)} \frac{\partial^2 v_t(\mathbb{E}\,\varepsilon_1, ..., \mathbb{E}\,\varepsilon_t)}{\partial {\varepsilon_{t-1}}^2} \mathbb{V}\mathrm{ar}(\varepsilon_\tau) \\
&\quad + \sum_t \sum_{\tau<t} \sum_{k<\tau} \beta^t \gamma^{t-\tau-1} \gamma^{t-k-1} \frac{\partial^2 v_t(\mathbb{E}\,\varepsilon_1, ..., \mathbb{E}\,\varepsilon_t)}{\partial {\varepsilon_{t-1}}^2} \mathrm{COV}(\varepsilon_\tau, \varepsilon_k).
\end{aligned}$$

Because the shocks are iid, the term triple sum in the last line vanishes ($\mathrm{COV}(\varepsilon_\tau, \varepsilon_k) = 0$) and $\mathbb{V}\mathrm{ar}\,\varepsilon \equiv \mathbb{V}\mathrm{ar}\,\varepsilon_\tau = \mathbb{V}\mathrm{ar}\,\varepsilon_t$. For small risks, the present day risk premium of the

SCC per unit variance is

$$
\begin{aligned}
\frac{\hat{\Delta} SCC_0}{\mathrm{VAR}(\varepsilon)} &= \frac{1}{u_0'(C_0)} \frac{\mathbb{E}\sum_{t=1}^{\infty}\beta^t v_t(\varepsilon_1,...,\varepsilon_t) - \sum_{t=1}^{\infty}\beta^t v_t(0,...,0)}{\mathrm{VAR}(\varepsilon)} \\
&\approx \frac{1}{2u_0'(C_0)} \sum_t \sum_{\tau\leq t} \beta^t \gamma^{2(t-\tau)} \frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2} \\
&= \frac{1}{2u_0'(C_0)} \sum_t \beta^t \frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2} \sum_{\tau\leq t} \gamma^{2(t-\tau-1)} \\
&= \frac{1}{2u_0'(C_0)} \sum_t \beta^t \frac{1-\gamma^{2t}}{1-\gamma^2} \frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2},
\end{aligned} \tag{22}
$$

where we used the closed-form expression for the geometric sum $\sum_{\tau<t}\gamma^{2(t-\tau-1)} = \sum_{l=0}^{t-1}\gamma^{2l} = \frac{1-\gamma^{2t}}{1-\gamma^2}$ (note that index τ starts at zero whereas index t starts at unity).

The evaluation of $\frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2}$ follows closely that of $z''(s)$ in the proof of Proposition 2. In contrast to climate sensitivity uncertainty, the temperature shocks ε_t leave the temperature response to CO_2, $\frac{\partial T_t}{\partial M_\tau}$, unchanged. As a consequence, $\frac{\partial^2 T_t}{\partial M_\tau \partial \epsilon_t} = 0$ and the contributions relating to $A'(s)$ in the case of climate sensitivity will be missing in the case of temperature stochasticity. A derivation analogous to equation (19) yields

$$
\begin{aligned}
\frac{\frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2}}{u_t'(\cdot)(-F'(\cdot))A_t} &= \tfrac{1}{T_t^2}\left\{\mathrm{RRA}\,\epsilon_{c,T}\left[\mathrm{Prud}\,\epsilon_{c,T} + 3\,\mathrm{Dam}_2\right] + \mathrm{Dam}_2\,\mathrm{Dam}_3\right\} \\
&\quad + \tfrac{1}{T_t^2} \underbrace{\left\{\frac{T_t T_t''(\cdot)}{T_t'(\cdot)}\left[\mathrm{RRA}\,\epsilon_{c,T} + \mathrm{Dam}_2\right]\right\}}_{\equiv Z_t(\cdot)},
\end{aligned} \tag{23}
$$

where $\epsilon_{c,T} = -\frac{dc}{dT}/\frac{c}{T} = F_t'(\cdot)\frac{T}{c}$ (and the other elasticity is unity). Under Assumption 2, we know from equation (20) that $T_t''(\cdot) = \frac{\partial^2 T_t(\cdot)}{\partial \epsilon_{t-1}^2} = 0$ and the second line vanishes.

Combining equations (22) and (23) (with $T_t''(\cdot) = 0$) delivers the stochasticity premium to the SCC today in consumption equivalents as

$$
\begin{aligned}
\frac{\hat{\Delta} SCC_0}{\mathbb{V}\mathrm{ar}(\epsilon)} &\approx \frac{1}{2u_0'(C_0)} \sum_{t=1}^{\infty} \beta^t \frac{1-\gamma^{2t}}{1-\gamma^2} \frac{\partial^2 v_t(0,...,0)}{\partial {\varepsilon_{t-1}}^2} \\
&= \frac{1}{2} \sum_{t=1}^{\infty} \beta^t \frac{1-\gamma^{2t}}{1-\gamma^2} \frac{u_t'(C_t)}{u_0'(C_0)} \frac{-F'(\cdot)A_t(\cdot)}{T_t} Z_t(\cdot) \\
&= \sum_{t=1}^{\infty}\sum_{\tau=1}^{t} \beta^t \frac{1-\gamma^{2t}}{1-\gamma^2} \frac{u_t'(C_t)}{u_0'(C_0)} \frac{\partial F_t}{\partial T_t} \frac{\partial T_t}{\partial M_\tau} \frac{\partial M_\tau}{\partial E_0} \frac{1}{2(T_t)^2} Z_t(\cdot),
\end{aligned}
$$

where the terms are evaluated at the expected shock values ($\epsilon_t = 0\ \forall t$). Emphasizing this evaluation for temperatures we write $\mathbb{E}\, T_t$ instead of T_t in the Proposition. Having proven the second part of Proposition 3, the first part follows as in the proof of Proposition 2 from Jensen's inequality and equation (23).

For an additional corollary we relax Assumption 1 in the last argument.

Assumption 1'. *The temperature evolution is:*

$$T_{t+1} = \gamma\ T_t + s\,\Gamma_1(M_{t+1}) + \Gamma_2(t) + \Gamma_3(\epsilon_t)$$

where s is uncertain and $(\epsilon_t)_{t\in\mathbb{N}}$ is an iid process of mean zero temperature shocks.

Then $T''_t(\cdot) = \Gamma''_3(\epsilon_t)$ no longer vanishes. Note that we maintain the autoregressive structure of temperature to keep the simplified sum in equation (22) over shocks in different periods. Then, combining equations (22) and (23) delivers the following corollary.

Corollary 2. *Under Assumptions A1' and A2, temperature stochasticity increases the social cost of carbon contribution from a given period if and only if*

$$Z^*_t(\cdot) \equiv \text{RRA}\,\epsilon_{c,T}\,[\text{Prud}\,\epsilon_{c,T} + 3\,\text{Dam}_2] + \text{Dam}_2\,\text{Dam}_3 + \frac{T_t\Gamma''_3(0)}{\Gamma'_3(0)}\,[\text{RRA}\,\epsilon_{c,T} + \text{Dam}_2]$$

is greater than zero. Arguments and their period-dependence are suppressed. Moreover, under a small risk approximation, the temperature stochasticity premium is

$$\hat{\Delta} SCC_0 \approx \sum_{t=1}^{\infty}\sum_{\tau=1}^{t} \underbrace{\frac{1-\gamma^{2t}}{1-\gamma^2}}_{\substack{\text{intertemporal}\\ \text{temperature}\\ \text{correlation}}} \underbrace{\beta^t\frac{u'_t(c_t)}{u'_0(c_0)}}_{\substack{\text{consumption}\\ \text{discount}\\ \text{factor}}} \underbrace{\frac{\partial F_t}{\partial T_t}\frac{\partial T_t}{\partial M_\tau}\frac{\partial M_\tau}{\partial E_0}}_{\substack{\text{marginal}\\ \text{emission}\\ \text{damage}}} \frac{\mathbb{V}\text{ar}(\epsilon)}{2(\mathbb{E}\,T_t)^2}\, Z^*_t(\cdot).$$

D The Quantitative Model

This section introduces the quantitative model. It is based on the DICE-2013 with modifications derived in Traeger (2014). These modification are helpful for a stochastic dynamic programming implementation, reducing the state space and interpolating exogenous processes in closed-form. Moreover, they allow us to fit the modified DICE in our generic IAM description presented in section 2.

D.1 Details of the quantitative model

The following model emulates DICE-2013. The three most notable differences are the annual time step (DICE-2013 features a five year time step), the infinite time horizon, and the replacement of the ocean feedbacks by exogenous processes. We adopt the latter simplification because the ocean carbon sink and ocean temperature would each require an own state variable in a recursive framework, which is computationally very costly and would further complicate the analytic formulas. Instead, we calibrate a decay rate for atmospheric carbon and a temperature difference between atmosphere and ocean which closely match the behavior of DICE's original carbon cycle. For a detailed description of the procedure and its (very good) performance relative to the original DICE model see Traeger (2014), who compares both to the MAGICC model used in the IPCC reports. That paper also shows how to reformulate the decision problem expressing capital stock and consumption in efficient labor units, which is better suited for the numeric implementation. All parameters are characterized and quantified in Table 2 on page 57.

Climate. Global average temperatures respond with a delay to the forcing from atmospheric carbon stocks M_t (above preindustrial level M_{pre}) and other non-CO2 forcing. Restating Equation (26) with climate sensitivity as a uncertain parameter

$$\tilde{T}_{t+1} = (1-\sigma_{forc})T_t + \sigma_{forc}\tilde{s}\left[\frac{\ln\frac{M_t}{M_{pre}}}{\ln 2} + \frac{EF_{t+1}}{\lambda}\right] - \sigma_{ocean}\Delta T_t + \tilde{\epsilon}_t \ .$$

In equation (26) we use the definitions

$$\begin{aligned}\chi_t(M_{t+1},t) &= \sigma_{forc}\frac{\log\frac{M_{t+1}}{M_{pre}}}{\log 2} + \frac{EF_{t+1}}{\eta_{forc}} \ , \quad \text{and}\\ \xi(T_t,t) &= (1-\sigma_{forc})T_t - \sigma_{ocean}\Delta T_t \ .\end{aligned}$$

The ocean temperature difference ΔT_t replicates the relation between oceanic and atmospheric temperatures in DICE. It follows the simple quadratic equation

$$\Delta T_t = \max\{0.7 + 0.02.*t - 0.00007.*t.^2 \ , \ 0\} \ .$$

Exogenous forcing EF_t from non-CO2 greenhouse gases, aerosols and other processes is assumed to follow the process

$$EF_t = EF_0 + 0.01(EF_{100} - EF_0) \times \max\{t, 100\} \ .$$

Note that it starts out slightly negatively. Carbon in the atmosphere accumulates according to

$$M_{t+1} = M_{pre} + (M_t - M_{pre})\,(1 - \delta_M(t)) + E_t \quad \text{with}$$
$$\delta_{M,t} = \delta_{M,\infty} + (\delta_{M,0} - \delta_{M,\infty})\exp[-\delta_M^* \; t] \; .$$

The stock of CO_2 (M_t) exceeding preindustrial levels (M_{pre}) decays exponentially at the rate $\delta_M(M, t)$. This decay rate falls exogenously over time to replicate the carbon cycle in DICE-2007, mimicking that the ocean reservoirs reduce their uptake rate as they fill up (see Traeger 2014).

Emissions. Yearly CO_2 emissions E_t, consisting of industrial emissions and emissions from land use change an forestry B_t are

$$E_t = (1 - \mu_t)\,\sigma_t A_t L_t k_t^\kappa + B_t \; .$$

Emissions from land use change and forestry fall exponentially over time

$$B_t = B_0 \exp[g_B \; t] \; .$$

Industrial emissions are proportional to gross production $A_t L_t k_t^\kappa$. They can be reduced by abatement μ_t. As in the DICE model, the carbon intensity of production falls at an exogenous rate of decarbonization σ_t

$$\sigma_t = \sigma_{t-1}\exp[g_{\sigma,t}] \qquad \text{with} \qquad g_{\sigma,t} = g_{\sigma,0}\exp[-\delta_\sigma \; t] \; .$$

Economy. We denote global net output in per effective labor units by $y_t = \frac{Y_t}{A_t L_t}$. Population grows exogenously as

$$L_{t+1} = \exp[g_{L,t}]L_t \quad \text{with} \quad g_{L,t} = \frac{g_L^*}{\frac{L_\infty}{L_\infty - L_0}\exp[g_L^* \; t] - 1} \; .$$

Here L_0 denotes the initial and L_∞ the asymptotic population. The parameter g_L^* characterizes the convergence from initial to asymptotic population. Similarly, technology grows exogenously as

$$A_{t+1} = A_t \exp\left[g_{A,t}\right] \quad \text{with} \quad g_{A,t} = g_{A,0} * \exp\left[-\delta_A t\right] \; .$$

The economy accumulates capital according to

$$k_{t+1} = \left[(1 - \delta_k)\,k_t + y_t - c_t\right]\exp[-(g_{A,t} + g_{L,t})] \; , \tag{24}$$

where δ_K denotes the depreciation rate and c_t denotes aggregate global consumption of produced commodities per effective unit of labor. The growth rates in equation (24) result from the effective unit per labor normalization.

Global net output results from the (reduced-form) Cobb-Douglas production (k_t^κ) less abatement costs and less damages

$$y_t = \big(1 - \Lambda(\mu_t)\big)\big(1 - D(T_t)\big)k_t^\kappa$$

changing to $y_t = \frac{1-\Lambda(\mu_t)}{1+D(T_t)}k_t^\kappa$ for our DICE 2007 damage specification. Abatement costs are

$$\Lambda(\mu_t) = \Psi_t \mu_t^{a_2}$$

and characterized as percent of output. They depend on the emission control rate $\mu_t \in [0, 1]$. The coefficient of the abatement cost function Ψ_t follows

$$\Psi_t = \frac{\sigma_t}{a_2} a_0 \left(1 - \frac{(1 - \exp[g_\Psi\, t])}{a_1}\right)$$

with a_0 denoting the initial cost of the backstop, a_1 denoting the ratio of initial over final backstop, and a_2 denoting the cost exponent. The rate g_Ψ describes the convergence from the initial to the final cost of the backstop.

Climate damage as a fraction of world output depends on the temperature difference T_t of current to preindustrial temperature and is

$$D(T_t) = b_1 T_t^{b_2} \ .$$

Nordhaus (2008) estimates $b_1 = 0.0028$ and $b_2 = 2$, implying a quadratic damage function with a loss of 0.28% of global GDP at a 1 degree Celsius warming.

We note that our DICE implementation indeed fits our generic integrated assessment model where emissions are treated as a production factor. For this purpose, we re-write the emission control rate $\mu_t = 1 - \frac{E_t}{E_t^{BAU}}$ as a function of actual and business as usual emissions and the DICE production function in non-effective-labor units as

$$F(K_t, T_t, E_t, t) = (A_t L_t)^{1-\kappa} K_t^\kappa \left(1 - D(T_t) - \Psi_t \Big[1 - \frac{E_t}{E_t^{BAU}(K_t)}\Big]^{2.8}\right)$$

where $E_t^{BAU}(K_t)$ is DICE's business as usual emission forecast as a function of the (endogenous) capital stock.

D.2 Temperature is linear in climate sensitivity

This brief subsection shows that temperature is linear in climate sensitivity, irrespectively of how many periods ahead we look into the future, in line with our Assumption 4. Advancing the temperature equation by one period and inserting for the previous period's temperature gives

$$\tilde{T}_{t+1} = (1-\sigma_{forc})T_t + \tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \ln\left(\frac{M_{t+1}}{M_{pre}}\right) + \frac{\sigma_{forc}}{\lambda}\ EF_t - \sigma_{ocean}\Delta T_t + \tilde{\epsilon}_t\ ,$$

$$\begin{aligned}
\tilde{T}_{t+2} &= (1-\sigma_{forc})\left((1-\sigma_{forc})T_t + \tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \ln\left(\frac{M_{t+1}}{M_{pre}}\right) + \frac{\sigma_{forc}}{\lambda}\ EF_t - \sigma_{ocean}\Delta T_t + \tilde{\epsilon}_t\right) \\
&\quad + \tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \ln\left(\frac{M_{t+2}}{M_{pre}}\right) + \frac{\sigma_{forc}}{\lambda}\ EF_{t+1} - \sigma_{ocean}\Delta T_{t+1} + \tilde{\epsilon}_{t+1}\ , \\
&= (1-\sigma_{forc})^2 T_t + (1-\sigma_{forc})\tilde{\epsilon}_t + \tilde{\epsilon}_{t+1} \\
&\quad + (1-\sigma_{forc})\left(\tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \ln\left(\frac{M_{t+1}}{M_{pre}}\right)\right) + \tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \ln\left(\frac{M_{t+2}}{M_{pre}}\right) \\
&\quad + (1-\sigma_{forc})\left(\frac{\sigma_{forc}}{\lambda}\ EF_t - \sigma_{ocean}\Delta T_t\right) + \frac{\sigma_{forc}}{\lambda}\ EF_{t+1} - \sigma_{ocean}\Delta T_{t+1}\ .
\end{aligned}$$

Iteratively we arrive at the formula

$$\begin{aligned}
\tilde{T}_{t+\tau} = \tilde{s}\ \frac{\sigma_{forc}}{\ln 2}\ \sum_{i=1}^{\tau}(1-\sigma_{forc})^{\tau-i}\ln\frac{M_{t+i}}{M_{pre}}\ + \sum_{i=0}^{\tau-1}(1-\sigma_{forc})^{\tau-i-1}\ \tilde{\epsilon}_{t+i} \\
+ \sum_{i=0}^{\tau-1}(1-\sigma_{forc})^{i}\left(\frac{\sigma_{forc}}{\lambda}EF_{t+i} - \sigma_{ocean}\Delta T_{t+i}\right) + (1-\sigma_{forc})^{\tau}T_t\ .
\end{aligned} \tag{25}$$

In particular, we observe that temperature in any period $t+\tau$ depends linearly on climate sensitivity.

E Full Stochastic Dynamic Programming Model

The full stochastic model takes into account that future decision makers have more information on climate dynamics than present decision makers. Section E.1 specified the informational dynamics (Bayesian updating) and the Bellman equation.

E.1 Uncertainty and Bellman Equation

A dynamic model of climate sensitivity uncertainty naturally invites a Bayesian learning model; in contrast to our analytic formula, a full dynamic model has to explicitly specify how uncertainty resolves over time.[16] Our model follows Kelly & Kolstad (1999) and represents uncertainty over climate sensitivity as a Bayesian prior. The decision maker updates the prior annually based on stochastic temperature observations. Assuming a normal distribution for both the climate sensitivity prior and the temperature shocks (likelihood) results in conjugate priors; the updated posterior will be of the same class of distributions as the prior. This feature is extremely helpful as it allows us to track information merely through the mean and the variance of climate sensitivity, which fully characterize the normal distribution. Limiting the informational states to two helps substantially when solving the Bellman equation for an already state-rich integrated assessment model.

The social planner is uncertain about the value of climate sensitivity and holds the following initial prior $\Pi(s)$

$$\tilde{s}_0 \sim \Pi(s) = \mathcal{N}(\mu_{s,0}, \sigma^2_{s,0}) \qquad \text{with} \qquad \mu_{s,0} = 3 \qquad \sigma^2_{s,0} = 3 \; .$$

Better estimates of the climate sensitivity result in a skewed distribution, but to solve the stochastic dynamic programming problem we rely on conjugate priors using the normal. Given this limitation, $\sigma^2_{s,0} = 3$ is a rounded-up empirical approximation to the set of distributions found in Stocker et al. (2013)[17]

We can learn the value of climate sensitivity from observing the CO_2 stock and temperatures over time. Every period the decision maker foresees what a future realization of the temperature teaches her about climate sensitivity distribution and updates her prior accordingly, where temperature follows

$$\tilde{T}_{t+1} = \chi_t(M_{t+1})\tilde{s}_t + \xi_t(T_t) + \tilde{\epsilon}_t \; . \tag{26}$$

The factor χ_t captures the forcing from atmospheric CO_2 and other greenhouse gases. The term ξ_t reflects that atmospheric warming is a slow process and approximately

[16]Our formula merely approximates today's risk premium and we only have to model future uncertainty as viewed from today. A full stochastic recursive dynamic programming model derives the uncertainty as viewed from today from the actual uncertainty prevailing in future periods conditional on future information. It thereby requires a more explicit formulation of the stochastic processes.

[17]A normal distribution with mean 3 and variance 3 has the one-standard-deviation bands [1.27,4.73], which also mimics the IPCC's "likely" range [1.5,4.5].

governed by an AR(1) type process with a moving target. The structural parameter $\tilde{s}$ is the unknown climate sensitivity over which the decision forms her prior; the time index merely relates to the decision maker's subjective prior, the climate sensitivity itself does not change over time.

Each year random events $\tilde{\epsilon}$ shock temperature. These "weather fluctuations" are normally distributed with mean zero. For a *given* value of climate sensitivity, the next period's temperature is then normally distributed

$$\tilde{T}_{t+1} \sim \mathcal{N}(\mu_{T,t+1}(s), \sigma_T^2) \qquad \text{with} \qquad \sigma_T^2 = 0.042.$$

The variance σ_T^2 is exogenous. We estimate the annual volatility of global mean temperature as $\sigma_T^2 = 0.042$ using GISTEMP (2019).[18]

The policy maker's posterior in period t is the prior conditional on historic temperature realizations $\Pi(s|\hat{T}_1, ..., \hat{T}_t)$. This posterior also depends on the historic CO_2 stock information which we suppress for notational convenience. Given the current stock M_t, a realization of temperature $\hat{T}_{t+1}$ in the subsequent period results in the updated posterior $\Pi(s|\hat{T}_1, ..., \hat{T}_{t+1})$. In Appendix E.2 we show that the updated posteriors are again normally distributed so that at all times $\Pi(s|\hat{T}_1, ..., \hat{T}_t) = \mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$ for some $\mu_{s,t}$ and $\sigma_{s,t}^2$. Appendix E.2 also derives the following updating rules for expected climate sensitivity and its variance

$$\mu_{s,t+1} = \frac{\chi_t^2 \sigma_{s,t}^2 \frac{\hat{T}_{t+1} - \xi_t}{\chi_t} + \sigma_T^2 \mu_{s,t}}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \qquad \text{and} \qquad \sigma_{s,t+1}^2 = \frac{\sigma_T^2 \sigma_{s,t}^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}\ , \tag{27}$$

where $\hat{T}_{t+1}$ characterizes observed temperature, σ_T^2 is the variance of temperature stochasticity (ϵ_t-shocks), and χ_t and $\xi_t(M_t)$ abbreviate the relevant endogenous quantities of the DICE model affecting the updating equation. The updated expected value of climate sensitivity s is a weighted mean of the previous expected value and the inferred "climate sensitivity observation", $\frac{\hat{T}_{t+1} - \xi_t}{\chi_t}$. The weight on the new observation is proportional to the precision (the inverse of the variance) of the temperature and the magnitude of the multiplicative factor $\chi_t(M_t)$, which increases in the carbon stock. This stock dependence of the updates differs from the common normal-normal updating formula and introduces active learning: by increasing the carbon stock the decision maker can increase $\chi_t(M_t)$ and, thus, the learning speed.

[18]Kelly & Kolstad (1999) and Leach (2007) both use $\sigma_T^2 = 0.1$. Averaging temperatures over 174 countries and estimating yearly fluctuations with respect to a common trend over 109 years results in our lower $\sigma_T^2 = 0.042$.

The social planner maximizes her value function subject to the exogenous and endogenous equations of motion for the economy and the climate summarized in Appendix D.1 as well as the informational updating equation (27). The physical state variables describing the system are capital k, CO_2 stock M, and temperature T, and the informational state variables $\mu_{s,t}$ and $\sigma^2_{s,t}$ characterizing the climate sensitivity prior. We use the additional state time t to capture exogenously evolving processes, such as population, technology, and time changing climate processes. For numerical reasons, we express capital (and production and consumption) in effective labor units, i.e. $k_t = {}^{K_t}\!/_{A_t L_t}$ where A_t is technology level and L_t population at time t. The Bellman equation reads

$$V(k_t, M_t, t, T_t, \mu_{s,t}, \sigma^2_{s,t}) = \max_{c_t, \mu_t} \frac{(c_t)^{1-\hat{\eta}}}{1-\hat{\eta}} \qquad (28)$$
$$+\beta_t \, \mathbb{E}\left[V(k_{t+1}, M_{t+1}, t+1, \tilde{T}_{t+1}, \tilde{\mu}_{s,t+1}, \tilde{\sigma}^2_{s,t+1})\right],$$

where we marked one-step-ahead uncertainty by a tilde and the time index on the discount factor β once again incorporates time-varying population. See Appendix D.1 for the transformation of population weighted average utility to the above form.

We solve the dynamic programming equation (28) by function iteration, using the collocation method to approximate the value function. As basis functions we choose Chebychev polynomials with $35,200$ Chebychev nodes and coefficients. The normal distributions for temperature stochasticity and the climate sensitivity prior are approximated by Gauss-Legendre quadrature with 3 nodes.[19] Our convergence criterion is a change in the value function coefficients of less than 10^{-4}. The code is written in Matlab, we use the CompEcon toolbox by Miranda & Fackler (2002) to generate and evaluate the Chebychev polynomials, and we use KNITRO for the optimization at a given node.

E.2 Derivation of Updating Rules

This appendix derives the updating rules for the climate sensitivity prior and the predictive distribution for temperature. Let $l_t(x_{t+1}|s) = \mathcal{N}(\mu_{x,t+1}, \sigma^2_T|s, x_t, h_t)$ denote

[19]The results are robust to increasing Gauss-Legendre nodes and Chebychev nodes.

the likelihood function in period t. Then[20]

$$\Pi(s|\hat{T}_1, ..., \hat{T}_{t+1}) = \frac{l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, ..., \hat{T}_t)}{\int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, ..., \hat{T}_t)ds} \,.$$

We use $\propto$ to denote proportionality and suppress the normalization constants of the distributions, finding

$$\begin{aligned}
l_t(x|s)\ \Pi(s|\hat{T}_1, ..., \hat{T}_t) &\propto \exp\left(-\frac{(x-\mu_{x,t+1}(s))^2}{2\sigma_T^2}\right)\exp\left(-\frac{(s-\mu_{s,t})^2}{2\sigma_{s,t}^2}\right) \\
&\propto \exp\left(-\frac{(x-(s\chi_t+\xi_t))^2}{2\sigma_T^2} - \frac{(s-\mu_{s,t})^2}{2\sigma_{s,t}^2}\right) \\
&\propto \exp\left(-\frac{x^2-2x(s\chi_t+\xi_t)+(s\chi_t+\xi_t)^2}{2\sigma_T^2} - \frac{s^2-2s\mu_{s,t}+\mu_{s,t}^2}{2\sigma_{s,t}^2}\right) \\
&\propto \exp\left(-\frac{x^2-2xs\chi_t-2x\xi_t+s^2\chi_t^2+2s\chi_t\xi_t+\xi_t^2}{2\sigma_T^2} - \frac{s^2-2s\mu_{s,t}+\mu_{s,t}^2}{2\sigma_{s,t}^2}\right) \\
&\propto \exp\left(-\frac{1}{2}\left[s^2\left(\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}\right)-2s\left(\frac{(x-\xi_t)\chi_t}{\sigma_T^2}+\frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)+\frac{x^2-2x\xi_t+\xi_t^2}{\sigma_T^2}+\frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\
&\propto \exp\left(-\frac{1}{2}\left[s^2\left(\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}\right)-2s\left(\frac{(x-\xi_t)\chi_t}{\sigma_T^2}+\frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)+\frac{(x-\xi_t)^2}{\sigma_T^2}+\frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\
&\propto \underbrace{\exp\left(-\frac{1}{2}\left(\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}\right)\left(s-\frac{\frac{(x-\xi_t)\chi_t}{\sigma_T^2}+\frac{\mu_{s,t}}{\sigma_{s,t}^2}}{\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}}\right)^2\right)}_{\equiv\bar{\Pi}} \\
&\qquad \cdot\exp\left(-\frac{1}{2}\left[-\frac{\left(\frac{(x-\xi_t)\chi_t}{\sigma_T^2}+\frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)^2}{\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}}+\frac{(x-\xi_t)^2}{\sigma_T^2}+\frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\
&\propto \bar{\Pi}\cdot\exp\left(\frac{1}{2}\frac{\cancel{\left(\frac{(x-\xi_t)\chi_t}{\sigma_T^2}\right)^2}+2\frac{(x-\xi_t)\chi_t}{\sigma_T^2}\frac{\mu_{s,t}}{\sigma_{s,t}^2}+\cancel{\left(\frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)^2}-\cancel{\frac{(x-\xi_t)^2}{\sigma_T^2}\frac{\chi_t^2}{\sigma_T^2}}-\frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\frac{\chi_t^2}{\sigma_T^2}-\frac{(x-\xi_t)^2}{\sigma_T^2}\frac{1}{\sigma_{s,t}^2}-\cancel{\frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\frac{1}{\sigma_{s,t}^2}}}{\frac{\chi_t^2}{\sigma_T^2}+\frac{1}{\sigma_{s,t}^2}}\right)
\end{aligned}$$

[20] This simplified updating equation only using the latest prior and the latest observation is a consequence of our convenient choice of the conjugate prior.

$$\propto \bar{\Pi} \cdot \exp\left(-\frac{1}{2\sigma_T^2 \sigma_{s,t}^2} \frac{(x-\xi_t)^2 - 2(x-\xi_t)\chi_t \mu_{s,t} + \mu_{s,t}^2 \chi_t^2}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}} \right)$$

$$\propto \bar{\Pi} \cdot \exp\left(-\frac{1}{2} \frac{(x-\xi_t-\chi_t\mu_{s,t})^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \right) \; .$$

The following predictive distribution P_{t+1} governs the temperature realization in period $t+1$ incorporating stochasticity and parameter uncertainty

$$P_{t+1}(x) = \int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, ..., \hat{T}_t)ds \propto \exp\left(-\frac{1}{2} \frac{(x-\xi_t-\chi_t\mu_{s,t})^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \right) \; .$$

It is the normal distribution $\mathcal{N}(\chi_t \mu_{s,t}, \chi_t^2 \sigma_{s,t}^2 + \sigma_T^2)$. We find the posterior

$$\Pi(s|\hat{T}_1, ..., \hat{T}_{t+1}) = \frac{l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, ..., \hat{T}_t)}{\int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, ..., \hat{T}_t)ds}$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right) \left(s - \frac{\frac{(\hat{T}_{t+1}-\xi_t)\chi_t}{\sigma_T^2} + \frac{\mu_{s,t}}{\sigma_{s,t}^2}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}} \right)^2 \right) \; .$$

Thus, if $\Pi(s|\hat{T}_1, ..., \hat{T}_t)$ is distributed normally with expected value $\mu_{s,t}$ and variance $\sigma_{s,t}$, then the posterior in the subsequent period $\Pi(s|\hat{T}_1, ..., \hat{T}_{t+1})$ is also distributed normally with expected value

$$\mu_{s,t+1} = \frac{\frac{\chi_t^2}{\sigma_T^2}\frac{\hat{T}_{t+1}-\xi_t}{\chi_t} + \frac{1}{\sigma_{s,t}^2}\mu_{s,t}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}} = \frac{\chi_t^2 \sigma_{s,t}^2 \frac{\hat{T}_{t+1}-\xi_t}{\chi_t} + \sigma_T^2 \mu_{s,t}}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}$$

and variance

$$\sigma_{s,t+1}^2 = \left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2} \right)^{-1} = \frac{\sigma_T^2 \sigma_{s,t}^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \; .$$

Table 2: Parameters of the model

Economic Parameters		
η	2	intertemporal consumption smoothing and risk aversion
b_1	0.00284	damage coefficient
b_2	2	damage exponent
δ_u	1.5%	pure rate of time preference
L_0	6514	in millions, population in 2005
L_∞	8600	in millions, asymptotic population
g_L^*	0.035	rate of convergence to asymptotic population
K_0	137	in trillion 2005-USD, initial global capital stock
δ_K	10%	depreciation rate of capital
κ	0.3	capital elasticity in production
A_0	0.0058	initial labor productivity; corresponds to total factor productivity of 0.02722 used in DICE
$g_{A,0}$	1.31%	initial growth rate of labor productivity; corresponds to total factor productivity of 0.92% used in DICE
δ_A	0.1%	rate of decline of productivity growth rate
σ_0	0.1342	CO_2 emission per unit of GDP in 2005
$g_{\sigma,0}$	-0.73%	initial rate of decarbonization
δ_σ	0.3%	rate of decline of the rate of decarbonization
a_0	1.17	cost of backstop 2005
a_1	2	ratio of initial over final backstop cost
a_2	2.8	cost exponent
g_Ψ	-0.5%	rate of convergence from initial to final backstop cost
Climatic Parameters		
T_0	0.76	in °C, temperature increase of preindustrial in 2005
σ_T^2	$0.042, 0.2, 0.7$	temperature stochasticity
M_{pre}	596.4	in GtC, preindustiral stock of CO2 in the atmosphere
M_0	808.9	in GtC, stock of atmospheric CO_2 in 2005
$\delta_{M,0}$	1.7%	initial rate of decay of CO2 in atmosphere
$\delta_{M,\infty}$	0.25%	asymptotic rate of decay of CO2 in atmosphere
δ_M^*	3%	rate of convergence to asymptotic decay rate of CO2
B_0	1.1	in GtC, initial CO2 emissions from LUCF
g_B	-1%	growth rate of CO2 emisison from LUCF
$\mu_{s,0}$	3	climate sensitivity prior mean in $t = 0$
$\sigma_{s,0}^2$	3	climate sensitivity prior variance in $t = 0$
EF_0	-0.06	external forcing in year 2000
EF_{100}	.3	external forcing in year 2100 and beyond
σ_{forc}	3.2%	warming delay, heat capacity atmosphere
σ_{ocean}	0.7%	warming delay, ocean related