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from a Macro-Finance Perspective**

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# The Bond Yield “Conundrum” from a Macro-Finance Perspective\*

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## Abstract

In 2004 and 2005, long-term interest rates remained remarkably low despite improving economic conditions and rising short-term interest rates, a situation that former Fed Chairman Alan Greenspan dubbed a “conundrum.” We document the extent and timing of this conundrum using two empirical no-arbitrage macro-finance models of the term structure of interest rates. These models confirm that the recent behavior of long-term yields has been unusual—that is, it cannot be explained within the framework of the models. Therefore, we consider other macroeconomic factors omitted from the models and find that some of these variables, particularly declines in long-term bond volatility, may explain a portion of the conundrum. Foreign official purchases of U.S. Treasuries appear to have played little or no role.

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*[L]ong-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience, which suggests that, other things being equal, increasing short-term interest rates are normally accompanied by a rise in longer-term yields... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.*

— Testimony of Fed Chairman Alan Greenspan to U.S. Senate, February 16, 2005

## 1 Introduction

As a broad empirical regularity, long-term interest rates tend to move month-by-month in the same direction as short-term rates, although by a lesser amount. In the United States, the simple correlation since 1984 between changes in short-term and long-term rates implies that a 1 percentage point increase in the monthly federal funds rate has been accompanied, on average, by about a 30-basis-point increase in the 10-year rate. However, in 2004 and 2005, long-term interest rates in many countries followed a very different pattern. For example, in the United States, while the federal funds rate steadily rose from 1 percent in June 2004 to 4.2 percent in December 2005, the rate on 10-year U.S. Treasury notes *fell* from 4.7 percent to 4.5 percent over that same period.<sup>1</sup> This directional divergence between U.S. short and long rates appears even more unusual given other pressures at the time, such as a robust economic expansion, rising energy prices, and a deteriorating federal fiscal situation, all of which have tended to boost long-term interest rates in the past. In this paper, we investigate the seemingly odd behavior of long-term interest rates over this recent episode, a development which former Fed Chairman Alan Greenspan has labeled a “conundrum.”<sup>2</sup>

Determining whether recent long-term interest rate movements truly represent a conundrum— as opposed to simply an extension of secular declines in long-term interest rates that have been taking place over the past 20 years—requires a rigorous theoretical framework that can

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<sup>1</sup> In addition, given the upward shifts in money market futures rates during 2005, the extent of the policy tightening appears to have been unanticipated.

<sup>2</sup> Although Greenspan referred to “world” bond markets, our focus is exclusively on U.S. markets. For discussion of recent bond rates in Japan from a macro-finance perspective, see Oda and Ueda (2005).

take into account the important factors that affect long-term rates. Our choice of a theoretical framework for this examination is guided by much recent research that suggests that a joint *macro-finance* modeling strategy provides the most comprehensive explanation of movements in long-term rates.<sup>3</sup> From a macroeconomic perspective, the short-term interest rate is a policy instrument under the direct control of the central bank, which adjusts that rate to achieve its economic stabilization goals. Therefore, financial market participants' understanding of central bank behavior will be an important element in the formation of their expectations of future short-term interest rates, which, in turn, will be a key component for the pricing of longer-term bonds. To illustrate this point, it is useful to contrast the recent behavior of U.S. interest rates to their behavior during the previous episode of extended monetary tightening, which occurred a decade earlier. Specifically, from January 1994 to February 1995, as the Federal Reserve raised the short-term federal funds rate by 3 percentage points, the 10-year rate also rose by 1.7 percentage points. This reaction was somewhat greater than the average response, and a common interpretation of this episode is that bond investors were especially worried that inflation pressures might not be counteracted by the Fed in a timely fashion. In contrast, there is a widespread view that long-term inflation expectations remained well contained in 2004 and 2005, which helped hold down long bond rates. Understanding the differential behavior of long-term interest rates during these two episodes likely requires an appreciation of the macroeconomic underpinnings of bond rate dynamics.

A finance perspective, which stresses the importance of changing investor perceptions of risk for bond pricing, is also likely to be a crucial element in assessing whether there is any bond rate conundrum. Indeed, many have suggested that a reduction in the risk premium is responsible for recent low levels of bond rates. Such a reduction may be attributable to changes in the amount of risk or to changes in the pricing of that risk, and numerous

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<sup>3</sup> The connection between the macroeconomic and finance views of the term structure has been a very fertile area for recent research including, among many others, Ang and Piazzesi (2003), Wu (2001), Dewachter and Lyrio (2006), Evans and Marshall (2001), Diebold, Rudebusch, and Aruoba (2006), Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2004, 2006), Kozicki and Tinsley (2001, 2005), and Diebold, Piazzesi, and Rudebusch (2005).

factors have been suggested that could have induced such changes. For example, a new appreciation of lower macroeconomic volatility or reduced monetary policy uncertainty could alter assessments of the amount of interest rate risk faced by investors. Alternatively, risk aversion may have been reduced as new global investors entered the market. Indeed, this is one interpretation of the explanation suggested by Bernanke (2005), who argued that “over the past decade a combination of diverse forces has created a significant increase in the global supply of saving—a global saving glut—which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today.” In any case, it is likely important to allow for time variation in risk premiums in understanding the recent behavior of the bond rate.

To conduct our analysis of the “conundrum,” we use two macro-finance models of the term structure from the literature. The first is a VAR-based model developed by Bernanke, Reinhart, and Sack (2005), denoted BRS, and the second is a New-Keynesian-based model developed by Rudebusch and Wu (2004), denoted RW. The BRS and RW models share several basic features. First, they are both factor models, so only a small number of sources of variation underlie the pricing of the entire term structure of interest rates. Second, both models impose the standard no-arbitrage restriction from finance (e.g., Duffie and Kan 1996), which ensures that, after accounting for risk, the dynamic evolution of yields over time and across states of nature is consistent with the cross-sectional shape of the yield curve at any one point in time. Finally, both models have important bi-directional linkages between interest rates and macroeconomic variables. However, despite their broad similarities, the BRS and RW models also have technical specifications that differ in important ways. Because these differences may lead to different results, we use both models in our analysis in order to provide us with additional perspective and robustness checks on our results.

By extending these models forward from their original samples to the 2004–2005 “conundrum” period, we can determine whether the fall in long-term bond yields over that period was unusual from the viewpoint of the models, as opposed to a continuation of secular declines in long-term interest rates and term premiums that have been taking place since

the early 1980s. Our analysis indicates that the level of long-term bond yields in 2004 and 2005 is, in fact, substantially lower than can be explained by either of these models—i.e., that this period does constitute a “conundrum” from a macro-finance perspective.

Having documented the existence of a conundrum within a rigorous econometric framework, we then turn to possible explanations for the conundrum—factors that necessarily lie outside of the two macro-finance models we have considered. We examine a number of popular explanations for the conundrum that have been promoted by financial analysts and find that one of them—lower volatility of long-term Treasury yields—seems to have substantial explanatory power. Interestingly, we find no support for the view that foreign official purchases of U.S. Treasuries have contributed to the low level of long-term yields, even though this factor is regarded by many financial market participants to have been the single most important factor holding down long-term U.S. Treasury yields.

Although there are numerous papers specifying finance and macro-finance models of the term structure, our paper fills a gap in the literature by applying these models to analyze the bond yield “conundrum.” The most closely related paper to the present one is Kim and Wright (2005), who use a pure finance three-factor model of the term structure to analyze the recent low level of long-term rates, and find that a declining term premium is the key factor underlying those low rates. By contrast, we use two macro-finance models of the term structure and find that, within the framework of these models, there is in fact a conundrum—that the model-implied term premiums from our two models are *unable* to explain the low level of long-term yields observed in 2004–2005, despite the fact that the models provide an otherwise excellent fit to the data over the previous 20 years.

The remainder of the paper proceeds as follows. In Section 2, we describe the BRS and RW macro-finance models of the term structure. In Section 3, we discuss the estimation of these models and use them to show that there was a bond yield conundrum in the 2004–2005 period. In Section 4, we investigate what other factors outside of the two models could potentially help to explain the bond yield conundrum episode. Section 5 concludes.

## 2 Macro-Finance Models of the Term Structure

To investigate whether and to what extent the low level of U.S. long-term interest rates can be explained by macroeconomic conditions, we will use two macro-finance models of the term structure, broadly following Ang and Piazzesi (2003), Piazzesi (2005), and many others. For analyzing the behavior of the yield curve, these macro-finance models offer a number of advantages over both pure finance models and pure macroeconomic models.

First, in contrast to standard finance models of the term structure (e.g., Dai and Singleton, 2000), which relate the yield curve to current and past interest rates, macro-finance models recognize that interest rates and macroeconomic variables evolve jointly over time, with feedback running from interest rates to macroeconomic variables and also from macroeconomic variables back to interest rates. The latter channel in particular is crucial for the behavior of short-term interest rates, which are determined in many countries by the central bank as a function of the state of macroeconomic variables such as the output gap and inflation.<sup>4</sup>

A second advantage of macro-finance models is that they allow the behavior of risk premiums to depend explicitly on macroeconomic conditions. Standard consumption-based models of asset returns imply that risk premiums are determined by the covariance of an asset's return with the marginal utility of consumption (e.g., Cochrane, 2001). Moreover, empirical studies of excess returns in bond markets and shorter-duration interest rate derivatives (Cochrane and Piazzesi, 2005, and Piazzesi and Swanson, 2004) find a strong countercyclical relationship between economic activity and excess returns on these securities, particularly during recessions. Macro-finance models of the term structure explicitly allow for a relationship between risk premiums and macroeconomic conditions.

A third advantage of macro-finance models is that, in contrast to standard macroeconomic models, a substantial component of observed bond yields is allowed to reflect term or risk

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<sup>4</sup> While a reduced-form relationship between past interest rates and the future behavior of short-term interest rates exists, the Lucas critique suggests that it may be unstable in the face of changing monetary policy. A macro-finance approach that models monetary policy is arguably less subject to this criticism. Kim and Wright (2005), who examine the recent conundrum with a standard finance model, may alleviate this problem by using a fairly short estimation sample (since 1990).

premiums that may vary substantially over time. Indeed, as in a finance model, macro-finance models allow term premiums to evolve according to the estimated dynamics of the model in a way that is consistent with an absence of arbitrage opportunities in financial markets. Thus, while macro-finance models of the term structure retain the appealing macroeconomic features of a dynamic stochastic general equilibrium (DSGE) model or a vector autoregression (VAR), they do not impose the expectations hypothesis and can allow for a rich behavior of term premiums.

To explore the recent behavior of long-term bond rates, we use two macro-finance models from the literature, the VAR-based model developed in Bernanke, Reinhart, and Sack (2005) and the more structural New Keynesian-based model developed in Rudebusch and Wu (2004). We discuss each in turn.

## 2.1 Bernanke-Reinhart-Sack Model

The BRS model uses five observable macroeconomic variables, specified below, as factors with which to fit the yield curve. The dynamics of the five factors are governed by a VAR with four lags, which can be stacked into the 20-element vector  $X_t$  and described by a VAR(1) model:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{1}$$

where the stochastic shocks  $\varepsilon_t$  are *i.i.d.* over time and have a standard normal distribution. Consistent with the companion form specification in (1), only the uppermost blocks of  $\Phi$  and  $\Sigma$  are nontrivial. For simplicity, the prices of risk in the model, specified below, are assumed to load only on the five current values of the macroeconomic variables (i.e., the top five elements of  $X_t$ ).

Following Duffie and Kan (1996), Dai and Singleton (2000), and Ang and Piazzesi (2003), among others, we assume that the stochastic discount factor with which bonds in the model



are priced is conditionally log-normal with functional form:<sup>5</sup>

$$m_{t+1} = \exp(-i_t - \lambda_t' \lambda_t / 2 - \lambda_t' \varepsilon_{t+1}), \quad (2)$$

where  $\lambda_t$  is a 20-dimensional vector of market prices of risk associated with the innovations to the VAR,  $\varepsilon_t$ . Of course, because only the first five elements of  $\varepsilon_t$  are nonzero, we assume that only the first five elements of  $\lambda_t$  are nonzero, which is essentially a normalization. Following the papers cited above, the market prices of risk  $\lambda_t$  are assumed to be affine functions of the state variables of the economy:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \quad (3)$$

where  $\lambda_0$  is a 20-dimensional vector of constants and  $\lambda_1$  is a 20-by-20 matrix of risk price factor loadings on the state variables  $X_t$ . As mentioned above, we assume for simplicity that the prices of risk depend only on the current values of the variables in  $X_t$ , thus only the upper-left 5-by-5 block of  $\lambda_1$  is nonzero.

Bonds in the model are priced according to the standard relationship with the stochastic pricing kernel:

$$P_t^n = E_t m_{t+1} P_{t+1}^{n-1}, \quad (4)$$

where  $P_t^n$  denotes the price of an  $n$ -period zero-coupon bond at date  $t$ . Let  $y_t^n$  denote the corresponding continuously-compounded yield on the same zero-coupon bond. Then it follows from equations (1)–(4) that:

$$y_t^n = \frac{1}{n} (a_n + b_n' X_t), \quad (5)$$

where  $a_0 = 0$ ,  $b_0 = 0$ , and  $a_n$  (a scalar) and  $b_n$  (a 5-by-1 vector) are computed recursively according to:

$$\begin{aligned} a_{n+1} &= \delta_0 + a_n + b_n' (\mu - \Sigma \lambda_0) - b_n' \Sigma \Sigma' b_n, \\ b_{n+1} &= \delta_1 + (\Phi - \Sigma \lambda_1)' b_n, \end{aligned} \quad (6)$$

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<sup>5</sup> It is a well-known and important result from finance that the assumption of no arbitrage (or even more weakly, the law of one price) implies that a stochastic discount factor must exist, even in an incomplete market—see, e.g., Cochrane (2001) for discussion. The papers cited here place the additional functional form restriction on the pricing kernel for tractability.

and where  $\delta_0$  (a scalar) and  $\delta_1$  (a 5-by-1 vector) describe the one-period interest rate as a function of the state of the economy:  $y_t^1 \equiv i_t = \delta_0 + \delta_1' X_t$ .

## 2.2 Rudebusch-Wu Model

The RW model shares many features with the BRS model, since both models price bonds using an affine no-arbitrage framework. Thus, for expositional simplicity and to conserve notation, when concepts in the two models are the same, we will take the corresponding variable name from the BRS model exposition above and recycle it in the description of the RW model below (even though the number of factors, and hence the number of dimensions, in the two models are different). Note that, despite the similarity in the bond-pricing framework, the dynamics of the underlying macroeconomic factors in the RW model differ in important ways from BRS, with RW using a more structural New Keynesian macroeconomic framework to model these dynamic relationships as opposed to a VAR.

Bond pricing in the RW model is governed by the same affine no-arbitrage framework in (2)–(6) described above. However, in contrast to the BRS model, the RW model defines the one-period interest rate to be a sum of two latent factors,  $L_t$  and  $S_t$ :

$$i_t = \delta_0 + L_t + S_t, \tag{7}$$

where  $L_t$  can be thought of as the “level” of the yield curve and  $S_t$  as (the negative of) the yield curve “slope,” as discussed in Rudebusch and Wu (2004).<sup>6</sup> Intuitively, the Federal Reserve sets the one-period nominal interest rate in the model as the sum of a constant steady-state real interest rate ( $\delta_0$ ), a time-varying medium- to long-term inflation rate ( $L_t$ ), and a cyclically responsive component  $S_t$  that is given by:

$$S_t = \rho_S S_{t-1} + (1 - \rho_S)[g_y y_t + g_\pi(\pi_t - L_t)] + u_{S,t}, \tag{8}$$

$$u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}, \tag{9}$$

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<sup>6</sup> Rudebusch and Wu (2004) present charts comparing their estimated values for  $L_t$  and  $S_t$  to traditional principal components decompositions of the yield curve into “level” and “slope” factors, and show that the two sets of series line up very closely, justifying the use of the terminology.

where  $y_t$  is the output gap and  $\pi_t$  the inflation rate. Equation (8) is essentially a Taylor (1993) rule for the short-term rate  $i_t$  with both inertia (through the lag  $S_{t-1}$ ) and serially correlated errors (as described by Rudebusch 2002b).

The dynamics of  $L_t$  are given by:

$$L_t = \rho_L L_{t-1} + (1 - \rho_L)\pi_t + \varepsilon_{L,t}, \quad (10)$$

which says that the medium-term inflation goal is persistent but may depend also on the recent behavior of short-term inflation, as suggested by the evidence in Gürkaynak, Sack, and Swanson (2005).

The latent factors  $L_t$  and  $S_t$  are jointly determined with the macroeconomic variables  $y_t$  and  $\pi_t$  by a New Keynesian-type model (adjusted to apply to monthly data):

$$\pi_t = \mu_\pi L_t + (1 - \mu_\pi)[\alpha_{\pi_1}\pi_{t-1} + \alpha_{\pi_2}\pi_{t-2}] + \alpha_y y_{t-1} + \varepsilon_{\pi,t} \quad (11)$$

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y)[\beta_{y1}y_{t-1} + \beta_{y2}y_{t-2}] - \beta_r(i_{t-1} - L_{t-1}) + \varepsilon_{y,t}. \quad (12)$$

That is, inflation responds to the private sector’s expectation of the medium-term inflation goal ( $L_t$ ), two lags of inflation, and the output gap; the output gap, in turn, responds to the expected future output gap, two lags of the output gap, and a real interest rate. Equations (11) and (12) are “hybrid” New Keynesian equations in the sense that the expectational coefficients  $\mu_\pi$  and  $\mu_y$  are allowed to be less than unity when the model is fitted to the data.

### 3 Is There a Bond Yield “Conundrum”?

To investigate whether the recent low level of U.S. long-term bond yields is a conundrum from a macro-finance perspective, we use the BRS and RW models to fit the recent macroeconomic and interest rate data and examine the two model predictions for long-term bond yields.

## 3.1 Model Estimates

### 3.1.1 BRS Model Estimates

Our original inclination was to use the “off-the-shelf” BRS model parameter estimates for our analysis. However, after examining the data and model estimation code (kindly supplied by Brian Sack) and fine-tuning the nonlinear optimization procedure, we were able to obtain a better fit to the data than BRS report in their paper, with root-mean-squared errors that were about half as large even over the identical sample. Thus, we do not use the parameter values as estimated by BRS, since we have found that the model can fit the data better than originally reported. Moreover, because we are reestimating the parameters of the model, we take the opportunity to extend the BRS estimation period through the end of 2005, which has the added advantage of giving the model the best possible chance of fitting the recent low “conundrum” level of bond yields by fitting them in rather than out of the sample. In every other respect, however, we have followed BRS in their analysis.

We estimate the BRS model on monthly macroeconomic and bond yield data over the period from January 1984 to December 2005.<sup>7</sup> Ideally, one would like to perform the estimation over the whole postwar period, but the preponderance of empirical evidence suggests that the relationships between interest rates and macroeconomic variables have not been stable over the past 40 years. In particular, many authors (e.g., Fuhrer, 1996, Bernanke and Mihov, 1998, and Clarida, Galí, and Gertler, 2000) have found that there was an important structural break in the conduct of monetary policy around 1980, when Paul Volcker became Chairman of the Federal Reserve.<sup>8</sup> As a result, macro-finance models estimated over the entire postwar period would likely be subject to the Lucas critique, so we avoid this issue by beginning our empirical estimation in January 1984, after the Volcker disinflation, after the structural breaks in monetary policy found by the authors above, and after the structural break in U.S. GDP volatility found by McConnell and Perez-Quiros (2000) and others.

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<sup>7</sup> Because the VAR has four lags, this requires using macroeconomic data back to September 1983.

<sup>8</sup> Evidence of a structural break in the conduct of monetary policy is also indirectly supported by the greater stability of inflation expectations that seem to have held in the U.S. in the 1990s and 2000s relative to the 1970s.

Following BRS, we use five macroeconomic variables as state variables of the model, which are the affine factors for pricing bonds: the federal funds rate, the deviation of employment from trend (where trend is measured using an HP filter), the year-on-year percentage change in the core personal consumption expenditures (PCE) deflator, the Blue Chip survey of inflation expectations for the upcoming year, and the rate on the eurodollar futures contract with four quarters to expiration. The last variable in particular is included to capture aspects of the stance of monetary policy that may not be adequately represented by the current and past federal funds rate, employment gap, and inflation alone, such as financial market expectations for the future path of policy over the next 12 months. For example, the “deflation scare” episode in the U.S. in the summer of 2003 led the Federal Reserve to include greater forward-looking language in its monetary policy announcements, such as the now-famous “considerable period” phrase,<sup>9</sup> which helped to shape financial market expectations for the future path of policy in a way that was perhaps different from what the past behavior of output, inflation, and interest rates would have suggested. The inclusion of the four-quarter-ahead eurodollar futures rate thus helps to capture changes in the expected future path of policy that may not be adequately accounted for by the lags of the VAR. Similarly, the inclusion of the Blue Chip inflation expectations survey helps to capture changes in the expected future course of inflation that may not be adequately captured by the VAR. Of course, the inclusion of these forward-looking expectational variables in the VAR creates a tension or internal inconsistency between the forecast data series that are included as variables and the forecasts that would be implied by the model itself based on the VAR. Although this technical issue can be addressed by estimating the model in such a way as to ensure consistency between the model and survey forecasts to the greatest extent possible (see Swanson, 2006), doing so would require moving away from standard VAR estimation procedures, which is beyond the scope of the present paper. Thus, we follow BRS and simply estimate a VAR on all five of the macroeconomic time series above, and leave the issue of

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<sup>9</sup> On August 12, 2003, the FOMC released a statement that, although it was maintaining its federal funds rate target at the level of 1 percent, the Committee judged “that, on balance, the risk of inflation becoming undesirably low is likely to be the predominant concern for the foreseeable future. In these circumstances, the Committee believes that policy accommodation can be maintained for a considerable period.”

consistency between model-based forecasts and forecast data to subsequent work.

Although the BRS model (and also the RW model) is linear in the state variables, the model's predictions for bond yields are highly nonlinear in the parameters (as are the impulse response functions in a VAR, for example).<sup>10</sup> Estimation of the bond-pricing implications of the model is thus highly nonlinear and can be tricky in practice, with a great many local minima. (Indeed, as noted above, by experimenting and exploring the parameter space, we were able to obtain a better fit to the data than BRS report in their paper.) To reduce the number of model parameters that must be estimated nonlinearly, we follow Ang, Piazzesi, and Wei (2006) and BRS and estimate the model in two stages: First, we estimate the VAR on the five macroeconomic time series above (with four lags) over the 1984–2005 period by ordinary least squares; and second, we take the VAR coefficients as given and estimate the stochastic pricing kernel factor loadings (which, between  $\lambda_0$  and  $\lambda_1$ , comprise 30 parameters) using nonlinear least squares to fit our bond yield data over the same period, 1984–2005. As in BRS, we estimate the model to match the 6-month and 1-, 2-, 3-, 4-, 5-, 7-, and 10-year yields as closely as possible, with equal weight on fitting each of those maturities. These yield curve data are continuously compounded zero-coupon yields estimated by the Federal Reserve Board from off-the-run Treasury coupon securities (for details, see Gürkaynak, Sack, and Wright, 2006); following BRS, we use the monthly average values of these yields.<sup>11</sup>

Our results from estimating the BRS model are reported in Tables 1, 2, and 3. The VAR parameter estimates in Table 1 show that each variable in the VAR has a coefficient near unity on its own first lag, with smaller coefficients on longer lags of itself and on lags of other variables. The sum of the coefficients on each variable's own lags is near unity. For the federal funds rate, the coefficients suggest a hump-shaped impulse response and autocorrelation functions, but the other variables exhibit impulse response and autocorrelation functions

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<sup>10</sup> Indeed, to compute the model-implied 10-year yield, we must project the monthly VAR and prices of risk forward 120 months. Thus, when fitting the model to the data, we are essentially trying to minimize a 120th-degree polynomial.

<sup>11</sup> The finance and macro-finance literatures typically use end-of-month yield data rather than month-average yields. However, the model is linear in the state variables, so it is completely consistent to view the model's equations as holding for the month-average data as well as the daily data and the end-of-month data. Moreover, our macro data are typically monthly averages, so using month-average yields is arguably more consistent with the macro data.

that are closer to a geometric decay.

The BRS model’s estimated risk factor loadings are reported in Table 2. Although these estimated loadings show the greatest coefficients on the Blue Chip inflation expectations variable, that variable also has a relatively lower variance, as can be seen in Table 1, so the net effect on bond prices is not as great as the factor loadings would suggest by themselves.<sup>12</sup>

Table 3 reports the quality of the model’s fit, in terms of root-mean-squared errors (RMSEs) for the eight points on the yield curve that the model was estimated against. Except at the shortest horizons, these root-mean-squared errors are less than half as large as those reported by BRS for their sample period. This difference is not simply due to the sample period: even when we restrict our sample to the one used by BRS, we are able to obtain RMSEs less than half as large by experimenting and exploring the parameter space.

### 3.1.2 RW Model Estimates

For the RW model, we take the original parameter values as estimated by Rudebusch and Wu (2004) from January 1988 to December 2000 using monthly data. In that estimation, the inflation rate was measured by the year-on-year change in the overall PCE deflator, the output gap was measured by capacity utilization, and interest rates were end-of-month data on five U.S. Treasury zero-coupon yields that have maturities of 1, 3, 12, 36, and 60 months (yields are unsmoothed Fama-Bliss data expressed at an annual rate in percent).

Since there are two underlying latent factors but five observable yields, RW follow the usual strategy and assume that the 3-, 12-, and 36-month yields are measured with *i.i.d.* error, as in Ang and Piazzesi (2003). (Note that this implies that, for a given set of parameter values, the latent factors  $L_t$  and  $S_t$  can be backed out perfectly from the observed 1-month and 60-month bond yields.) The estimated size of the measurement error that is required to fit the other yields is a common metric for the quality of the model’s fit. Also note that the 10-year rate is not used in the estimation, so examining the recent episode of low bond rates

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<sup>12</sup> It would be desirable to perform a variance decomposition of the 10-year Treasury term premium on the five stochastic shocks  $\varepsilon$ . However, the term premium at these longer horizons is highly nonlinear in the parameters of the model, so that a direct computation of the variance decomposition is impossible and the delta-method is likely to be a very poor approximation.

is an out-of-sample exercise in terms of both the estimation sample and bond maturity.

In contrast to the BRS model, all of the parameters of the RW model are estimated in a single step by maximum likelihood. Table 4 reports the RW model parameter estimates, from Rudebusch and Wu (2004). First, consider the dynamics of the factors. The factor  $L_t$  is very persistent, with a  $\rho_L$  estimate of 0.989, which implies a small but significant weight on actual inflation. The dynamics of  $S_t$  are also very persistent, but this persistence does not come from partial adjustment since the  $\rho_S$  estimate is a minuscule 0.026. Instead,  $S_t$  responds with only a very short lag to output and inflation shocks.

The persistence in  $S_t$  reflects the fact that the Fed adjusts the short rate promptly to various determinants—output, inflation, and other influences in the residual  $u_t$ —that are themselves quite persistent (e.g.,  $\rho_u = 0.975$ ).<sup>13</sup> The monetary policy interpretation of the slope factor is supported by the values of the estimated inflation and output response coefficients,  $g_\pi$  and  $g_y$ , which are 1.25 and 0.20, respectively. These estimates are similar to the usual single-equation estimates of the Taylor rule during this sample period (e.g., Rudebusch 2002b).

The estimated parameters describing the inflation dynamics also appear reasonable.<sup>14</sup> In particular, the estimated weight on explicit forward-looking expectations in determining inflation,  $\mu_\pi$ , is 0.074. Since this estimate is based on monthly data, with time aggregation, it implies a weight of about 0.21 on the interim inflation objective at a quarterly frequency. This estimate appears consistent with many earlier estimates obtained using a variety of different methods and specifications. For example, using survey data on expectations, Rudebusch (2002a) obtains a broadly comparable  $\mu_\pi$  estimate of 0.29, which is in the middle of the range of estimates in the literature. However, by using the yield curve to extract inflation expectations, our estimates bring new information to bear on this important macroeconomic

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<sup>13</sup> Thus, our estimate of  $\rho_S$  decisively dismisses the interest rate smoothing or monetary policy inertia interpretation of the persistence in the short rate.

<sup>14</sup> After taking into account time aggregation and the higher cyclical variability of capacity utilization compared with the output gap, the elasticity of inflation with respect to output ( $\alpha_y = 0.014$ ) appears about half the size of estimates that use the entire postwar sample of quarterly data, for example, Rudebusch (2002a). The estimate does appear more in line with estimates obtained in recent shorter samples (Rudebusch 2001).



question.

The estimated parameters describing the output dynamics also fall within reasonable ranges.<sup>15</sup> Specifically, the estimated value of  $\mu_y = 0.009$ , implies a negligible weight at a quarterly frequency on forward-looking output expectations in the determination of output behavior. This is very much in accord with the maximum likelihood estimation results reported by Fuhrer and Rudebusch (2004).

The risk price matrix ( $\lambda_1$ ) appears significant, and the model fits the 3-month, 12-month, and 36-month yields with measurement error standard deviations comparable to others in the literature. For instance, the standard deviation of the 3-month yield is 28 basis points (bp) (annualized), compared with 30 bp in the VAR-based “macro lag model” in Ang and Piazzesi (2003). The standard deviation of the 36-month yield is 13 bp, same as in Ang and Piazzesi (2003).

To consider the bond yield “conundrum” in 2004–2005, we must extend the RW model forward to include this period. We do so as follows. As mentioned above, we take the parameter values as estimated from Rudebusch and Wu (2004). We then extend the output gap and inflation data forward to the end of 2005. For consistency with our BRS model analysis, we use the continuously compounded zero-coupon yield curve data from the Federal Reserve Board for the period January 1988 to December 2005 (although we use the month-end values of these data rather than the month-average). From the 1-month and 5-year zero-coupon yields, we can back out the RW model’s level and slope factors,  $L_t$  and  $S_t$ , over the extended sample 1988–2005, as noted above. (Over the period 1988–2000, these agree closely with the original values for  $L_t$  and  $S_t$  in RW, but they are not identical.) Having obtained  $L_t$  and  $S_t$  over the full sample, and given the parameters of the model as estimated by RW, we can then compute the model’s implications for bond yields of all maturities from 1988–2005.

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<sup>15</sup> The interest rate sensitivity of output ( $\beta_r = 0.089$ ), after taking into account the time aggregation and the use of capacity utilization rather than the output gap, appears broadly in line with estimates that use the entire postwar sample of quarterly data.

### 3.2 Macro-Finance Model Analysis of the 10-year Treasury Yield

Before analyzing the recent episode of bond yields within the framework of the macro-finance models above, it may be useful to summarize the salient differences between the two models. Our estimates of the BRS model use monthly average interest rate data and include the 2004-2005 period in the estimation sample. The RW estimates use end-of-month interest rate data and are based on a sample that ends in 2000. (The in-sample/out-of-sample distinction is reinforced because the BRS model includes the 10-year yield in the estimation while the RW model does not.) The BRS model is a completely autoregressive, nonstructural specification (except for the no-arbitrage assumption), while the RW model is more tightly parameterized with identifiable, explicitly forward-looking aggregate demand, aggregate supply, and policy rule equations. As an indication of the differences in parameterization, the BRS model has 100 parameters describing the factor dynamics and 25 risk-pricing parameters, excluding constants, while the RW model has 13 and 4, respectively. Again, we view these differences between the two models as useful to the extent they can illuminate the robustness of our results.

To start our analysis, we plot the 10-year zero-coupon U.S. Treasury yield together with the decomposition of that yield into its constituent components, as implied by the BRS and RW macro-finance models, in Figures 1 and 2, respectively. We focus on the 10-year yield in particular because that yield was the benchmark for long-term interest rates in the U.S. over the period of interest and because discussions of the long-term bond yield “conundrum” by policymakers and in the popular press have often focused on the 10-year U.S. Treasury in particular.

In the top panel of Figure 1, we plot the zero-coupon U.S. Treasury yield from 1984 through 2005 together with the BRS model decomposition of that yield. The BRS model-implied risk-neutral rate (the blue line) is the model’s estimated yield on a riskless 10-year zero-coupon bond at each date  $t$  in a hypothetical world in which the prices of risk  $\lambda_t$  are always equal to zero and the state variables of the economy are governed by the VAR in Table 1. The BRS model-implied 10-year Treasury yield (the orange dashed line in Figure 1)

is the model’s estimated value of the same 10-year zero-coupon bond when the prices of risk  $\lambda_t$  are no longer zero, but are instead the estimated affine function of the macroeconomic variables given in Table 2. The BRS model-implied term premium (the red line) is the difference between the orange dashed line and the blue line, and can be interpreted as the model’s estimate of the risk or term premium on the 10-year zero-coupon bond at each date  $t$ . Finally, the BRS model does not match the data perfectly, so the model’s residuals—the difference between the model predictions (the orange dashed line) and the data (the black line)—are graphed in the bottom panel of Figure 1.

As can be seen in the top panel of Figure 1, the close fit of the BRS model to the data is striking. Not only does the model capture the general downward trend in the 10-year yield over this period, but even the high-frequency swings in this yield in the late 1980s and mid-1990s are matched extremely well. The fit is even more remarkable in light of the fact that the model was not optimized to fit the 10-year yield, but rather placed equal weight on eight maturities all along the term structure.

According to the model, both the risk-neutral 10-year yield and the term premium have fallen over our sample, with the fall in the term premium being somewhat more important. According to the model, the risk-neutral yield has fallen about 250 bp over this period while the term premium has fallen about 350 bp, and the term premium has fallen from about one-half of the total 10-year U.S. Treasury yield in the 1980s to about one-third of that yield more recently.

However, despite the model’s excellent fit to the data overall, the recent period of low 10-year yields is one episode that the model notably fails to fit. The model’s residuals from mid-2004 through the end of 2005 have typically been around 50 bp, and have been even greater over almost all of 2005. Although the model has failed to fit the data on a few other occasions as well—most notably at the end of 1984, in 1985, and from 1997 to 1999—these previous episodes were typically either much briefer (late 1984) or milder in size (1997 to 1999) than the latest episode. Indeed, the most recent episode’s residuals of 50 to 75 bp are even more remarkable in light of the very low level of long-term yields—thus, while the

model's residuals amounted to about one-tenth the level of the 10-year Treasury yield in 1984 and 1985, and about one-fifteenth in 1997–1998, the model's residuals have been about one-sixth the level of the 10-year Treasury yield in the most recent period.

In Figure 2, we present the analogous pair of graphs for the 10-year bond yield decomposition implied by the RW model. Again, the fit of the model to the data is excellent, and this is all the more remarkable given that the RW model was not optimized to fit the 10-year yield at all—indeed, RW in their paper chose the 5-year yield as the longest maturity in the estimation.

A striking observation from Figures 1 and 2 is that, although both the RW and BRS macro-finance models produce very high quality forecasts of the 10-year Treasury yield, the 2 models' implied decomposition of that yield into expected short rate and term premium components is very different. In the RW model, the term premium is relatively constant over the 1988–2005 period, hovering around the 2 percent level with very little high-frequency variation; instead, the RW model attributes most of the variation in the 10-year bond yield over time to changes in the expected future path of short rates. By contrast, the BRS model attributes most of the high-frequency variation in the 10-year yield to changes in the term premium component, with the risk-neutral component generally trending smoothly downward over time.

Why is the decomposition implied by the two models so different? Recall that the BRS model is estimated in two stages, with the macroeconomic VAR estimated first. The smoothly downward-trending risk-neutral rate in the BRS model is essentially a projection of the future path of short rates based on the VAR, so the VAR in the BRS model is implying a smoother path for the risk-neutral 10-year yield than is the case for the actual 10-year yield in the data. By contrast, in the RW model, the future path of short-term interest rates is affected greatly by the inflation “level” factor  $L_t$ . Since  $L_t$  can vary at high frequency in response to exogenous shocks to itself and to inflation, the RW model's specification allows these shocks to pass essentially directly through to the risk-neutral 10-year yield, leading to significant high-frequency variation in that variable.

The high-frequency variation in the risk-neutral 10-year yield that is allowed for (but not assumed) by the RW model is arguably one of its strengths—indeed, Gürkaynak, Sack, and Swanson (2005) found significant systematic variation in far-ahead forward nominal interest rates in response to macroeconomic news in a way that suggested changes in inflation expectations rather than changes in term premiums. Similarly, Kozicki and Tinsley (2001) found that statistical models that allow for a “moving endpoint” are able to fit interest rate and inflation time series much better than standard stationary or difference-stationary VARs. A weakness of the BRS model and many other macroeconomic models, according to both of these papers, is that those models assume long-run features of the economy, such as the steady-state real interest rate and rate of inflation, are too well-anchored in response to shocks. The RW model relaxes that restriction to a much greater extent than does the BRS model.<sup>16</sup>

## 4 Factors That May Underlie the Conundrum

According to the BRS and RW macro-finance models, the recent behavior of long-term interest rates does present us with something of a conundrum. In this section, we investigate whether there are any additional factors that lie outside of the two models that could potentially explain this episode of unusually low long-term bond yields. One could of course search for correlations between our macro-finance model residuals and *nonlinear* functions of the variables that are already included in the model, such as the output gap, inflation, and the federal funds rate. We focus instead on searching for variables outside of the models because we do not have particularly strong priors that nonlinearities in the above variables have played an important role in the U.S. long-term Treasury market recently, while there are many plausible candidate variables that have been omitted from the model (a number

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<sup>16</sup> The VAR in the BRS model is not restricted to be stationary, and thus the BRS model does allow for an unanchored steady-state to the extent that the model estimates a unit root. Nonetheless, the estimation of the VAR is based on the high-frequency dynamics of the system—using four lags of monthly data—rather than on the behavior of long-term bond yields, and the high-frequency dynamics of the model may not speak so strongly in favor of a unit root as do long-term bond yields or other more direct measures of the long-term implications of the model.

of which have been emphasized in the financial press).

We first briefly review various explanations for the conundrum that have been discussed in the literature, and then provide some empirical analysis based on the estimated macro-finance models in order to help assess the importance of the various factors.

## 4.1 Popular Explanations for Low Long-Term Bond Yields

A large number of possible candidate explanations for low bond yields have been discussed by financial analysts. A simple means to summarize this set of explanations (and to some extent limit consideration to the most important ones) is to examine a survey of bond traders, hedge fund managers, and business economists conducted by the firm Macroeconomic Advisers. Their survey was taken in early March 2005 and asked participants to provide their views on the importance of various factors that might explain the low level of the 10-year Treasury yield. Table 5 displays the seven most important factors that were identified as holding down bond yields as well as a rough estimate of how much each factor was judged by survey respondents to have reduced the bond yield (in basis points).<sup>17</sup> The largest effect by far was attributed to increased demand for U.S. longer-term securities from foreign central banks. Indeed, on average, the survey respondents thought that purchases by foreign central banks had lowered U.S. long-term rates by 21 bp.<sup>18</sup>

After foreign official purchases of U.S. Treasuries, survey respondents assigned about equal importance to the other factors in Table 5, with each factor accounting for 7 to 11 bp of lower long-term rates. For example, an increased demand for long bonds in light of the greater likelihood of future corporate pension fund reform in the United States, the United Kingdom, and elsewhere was assigned an importance of about 11 bp. Such reforms would likely encourage pension funds to better match the duration of their assets to their liabilities,

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<sup>17</sup> Several other factors (such as increased demand by holders of mortgage-backed securities) were often noted in the survey but were assessed to have an effect of only a few basis points and are excluded from the table.

<sup>18</sup> In the academic literature, Bernanke, Reinhart, and Sack (2005) and Warnock and Warnock (2005) have also stressed the importance of the recent increases in foreign central bank purchases, especially by East Asian countries. Wu (2005), however, notes that finding significant effects of such purchases on U.S. Treasury yields is sensitive to the sample period and can be quite elusive.

which is expected to boost demand for long-term bonds.<sup>19</sup> Although this factor has been widely cited (e.g., Bank for International Settlements 2005), it is difficult to quantify, and we do not include it in our empirical analysis below.

Similarly, we do not quantify two factors that appear to relate to shifts in investor appetite for risk. These include the factor, “reaching for yield,” which is shorthand for the view that high levels of liquidity had encouraged investors to reduce their aversion to risk (perhaps irrationally in an almost bubble-like manner). Also, a closely related factor is the view that excess global saving—the global saving glut hypothesis of Bernanke noted in the introduction—stemming perhaps from less home bias among foreign investors or rapid economic growth in countries with high saving rates, had boosted foreign demand for bonds generally (e.g., Warnock and Warnock 2005).

We do however employ some proxies that try to measure a possible decrease in the amount of risk that investors may feel they face. Indeed, the Macroeconomic Advisers survey respondents pointed to lower uncertainty as an important factor in three of their responses: minimal inflation risk, greater transparency of the Fed (which presumably would translate into lower short-term interest rate uncertainty), and low economic growth volatility. Taken together, these three factors suggest a very important role for reduced macroeconomic uncertainty in reducing the long-term bond rate.

## 4.2 An Empirical Assessment of Various Explanations

To examine some popular explanations for low long-term bond yields, we consider six variables excluded from the BRS and RW macro-finance models: three measures of financial market volatility, two measures of macroeconomic volatility, and one measure of international capital flows. For financial market volatility, we use the Merrill-Lynch MOVE Index to measure the implied volatility in the longer-term U.S. Treasury market;<sup>20</sup> we use the im-

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<sup>19</sup> These demands for duration would, of course, reach out much longer than a 10-year horizon, and indeed in 2005 there was strong demand for the revived 30-year U.S. Treasury bond, for a newly launched French 50-year bond issue, and for a 50-year British inflation-indexed bond.

<sup>20</sup> The Merrill Option Volatility Estimate (MOVE) Index reports the average implied volatility across a wide range of outstanding options on the 2-year, 5-year, 10-year, and 30-year U.S. Treasury securities (with

plied volatility from eurodollar options to measure uncertainty about the near-term path of monetary policy;<sup>21</sup> and we use the VIX measure of implied volatility from options on the S&P 500 index to measure uncertainty in the stock market. For macroeconomic uncertainty, we proxy for output uncertainty by using the eight-quarter trailing standard deviation of the growth rate of real GDP, interpolating between the resulting quarterly volatility measure to get a monthly series, and we proxy for inflation uncertainty by using the 24-month trailing standard deviation of core PCE deflator inflation. Finally, we proxy for foreign government and foreign central bank purchases of U.S. Treasury securities by using the 12-month change in the custodial holdings by the New York Fed for all foreign official institutions (and we normalize this series by the total stock of U.S. Treasury debt in the hands of the public).<sup>22</sup>

All of these series are natural candidates for omitted variables that could be impacting the level of long-term U.S. Treasury yields. For example, reduced volatility in long-term bond markets would tend to make those securities more attractive relative to other assets and drive long-term bond yields down, all else equal. Conversely, a reduced degree of uncertainty about stock returns would tend to drive the prices of stocks up and might require long-term interest rates to rise in order to keep fixed-income securities attractive by comparison. Reduced uncertainty about the future path of monetary policy may lower the riskiness of holding long-term bonds leading to a fall in long-term yields. Reduced uncertainty about inflation may make fixed income securities of all maturities more attractive, and reduced uncertainty about output may lower the risk premium and raise the prices of all risky assets in general. Finally, inelastic demand by foreign central banks and governments for long-term

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a total weight of 40 percent on the 10-year Treasury and total weights of 20 percent each on the other maturities). Note that the options underlying the MOVE Index have expiration dates of approximately one month; thus, the MOVE index measures the implied volatility of long-term yields over a relatively short horizon. Such short-term uncertainty about long-maturity yields may be related but is not the same as uncertainty about the path of short-term interest rates in the distant future.

<sup>21</sup> Specifically, we use the closest to at-the-money eurodollar option with expiration in six months' time to measure the implied volatility for the 90-day eurodollar rate.

<sup>22</sup> The idea that the available quantity of long-term U.S. Treasury securities matters for long-term yields suggests that we should also look at total *issuance* of marketable U.S. Treasury securities by the U.S. government as well as the purchases of those securities by foreign governments. In preliminary results not reported here, we do find evidence that greater total issuance of marketable U.S. Treasuries does lead to higher long-term interest rates (Bikbov and Chernov, 2006), also find a correlation between total issuance of U.S. Treasuries and their macro-finance model residuals). Even when we include total issuance as an explanatory variable in our regressions below, however, we find little evidence that foreign official purchases have had a significant impact on long-term Treasury yields.



U.S. Treasury securities has often been cited in the popular press (and in Warnock-Warnock, 2005) as a reason why yields on those securities have been so low recently.

Graphs of each of these six variables over the 1984–2005 sample are presented in Figure 3 (note that not all of the above series, particularly the implied volatility series for financial market data, are available before about 1989). It is immediately clear from the figure that many of these series show a marked dip in the last few years—thus, as readers of the financial press are well aware, there is no shortage of possible explanations for the conundrum!

In Table 6, we conduct a preliminary statistical analysis of the importance of these variables by regressing the *residuals* from the BRS model and the RW model on each of the six independent variables described above, in a set of univariate regressions. A significant correlation between one of these variables and the models’ residuals would suggest that that variable is perhaps an important determinant of long-term U.S. Treasury yields that has not been captured by the model.<sup>23</sup>

As can be seen in Table 6, and as was evident in Figure 3, many of the variables above are correlated with the models’ residuals. Indeed, five of the six variables are statistically significant at the 10 percent level for at least one of the two models’ residuals. Two of the six variables—implied volatility on long-term Treasuries and GDP volatility—are statistically significant at the 5 percent level or better for both models’ residuals. Interestingly, foreign official purchases of U.S. Treasuries—the variable that is most emphasized by the financial press and by the Macroeconomic Advisers survey respondents—is the only one of the six variables that is not significant at even the 10 percent level for at least one of the two sets of model residuals (and has a positive sign, rather than the negative correlation hypothesized by the press). This result is very similar to that of Wu (2005), who finds that, after controlling for macroeconomic determinants of bond yields, the correlation between foreign official

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<sup>23</sup> The purpose of these regressions is to help identify which variables might potentially be the most important to incorporate into the models going forward. Of course, just because a variable is significantly contemporaneously correlated with our macro-finance model residuals does not necessarily imply that it will remain significant when fully incorporated into a macro-finance framework, although we interpret a strong contemporaneous correlation as suggestive that it will be. Also, there is no guarantee that our macro-finance model residuals are orthogonal even to the variables included in the models because of the overidentifying restrictions those models impose. We interpret the regression results under the assumption that the overidentifying model restrictions are correct, although again, we consider both models in order to provide some assurance of robustness.

purchases and bond yields is significantly *positive* between 1987 and 2000, and negative only since 2002. Thus, the correlation that has been emphasized so much by the financial press is not one that has been consistent over recent history.

A primary problem with Table 6, of course, is that many of the variables show declines in 2004 and 2005, and thus univariate regression results may be double-counting the explanatory power of the variables for the conundrum. Table 7 thus reports results from multivariate regressions for the same variables over the May 1990 to December 2005 period (the longest sample for which we have data on all six explanatory variables).

The multivariate regression results in Table 7 are, for the most part, consistent with the univariate regression results in Table 6. The most significant and robust explanatory variable is the implied volatility on longer-term Treasuries. Macroeconomic volatility also seems to play a statistically significant role, with core PCE inflation volatility significant at the 5 percent level for both models' residuals, and GDP growth volatility significant for the BRS model residuals. The signs of these coefficients are also what one would typically expect, with lower implied volatility on Treasuries and lower realized volatility of output and inflation all being correlated with lower yields on long-term Treasury securities. Again, foreign official purchases is the only variable that is not statistically significant at the 10 percent level for at least one of the two models' residuals, and continues to have a positive sign.

### **4.3 Decomposition of the Bond Yield Conundrum**

How much of the bond yield conundrum can these additional variables explain? In Table 8, we perform a detailed decomposition of the decline in bond yields from June 2004 to June 2005, according to each of the BRS and RW models and the regression results from Table 7. We choose June 2004 as the starting point for the change in yields because that month, when the FOMC embarked on the removal of monetary accommodation, is most often cited as the beginning of the conundrum. During this period, long-term interest rates edged down as the Fed steadily tightened policy.

The first line of Table 8 reports the actual change in 10-year yields over the June 2004 to June 2005 period (which differs slightly across the BRS and RW models because the BRS model uses month-average yield data while the RW model uses end-of-month yields), amounting to a fall of about 90 bp. Both the BRS and RW models imply that only a small part of this change—about 20 to 25 bp—can be attributed to changes in the term premiums (line 3). Moreover, the BRS model even predicts that risk-neutral long-term yields should have *risen* about 13 bp (line 2), based on the improving outlooks for GDP and inflation. Thus, the decline in long-term yields from June 2004 to June 2005 is largely unexplained by both the BRS and RW models, with changes in the models' residuals (line 4) accounting for about 87 and 32 bp of the roughly 90 bp fall in yields from June 2004 to June 2005.

Lines 5–10 of Table 8 decompose the BRS and RW model residuals into the parts that are explained by each of the six variables in the previous section, according to the regressions in Table 7 multiplied by the change in each independent variable over the June 2004 to June 2005 period. Not surprisingly, the fall in implied volatility on longer-term Treasuries from June 2004 to June 2005 accounts for the greatest fraction of the two models' residuals, explaining a little over one-third of the residuals of each model. More surprisingly, the change in core PCE deflator inflation volatility over this period accounts for essentially none of the conundrum, even though it was statistically significant in Table 7. Apparently, the change in inflation volatility from 2004 to 2005 was close to zero and even slightly positive, so that the conundrum period played little role in the statistical significance of the results for that variable in Table 7. The fall in GDP volatility from 2004 to 2005 seems to have played a more important role, accounting for about one-eighth (12 bp) of the BRS model residuals and about one-tenth (3 bp) of the RW model residuals. The remaining three variables in Table 8 were not statistically significant in the previous section and account for relatively small changes in yields over the conundrum period.

As can be seen in the last line of Table 8, even the six omitted variables studied in the previous section can explain only about one-fourth to, at most, one-half of the BRS and RW model residuals. Thus, we have by no means found the missing link that explains the

bond yield conundrum, but we have found evidence that reductions in longer-term Treasury volatility probably have played an important role, and that foreign official purchases of U.S. Treasuries probably have not.

## 5 Conclusions

We draw a number of conclusions from the above analysis. First, the low level of long-term bond yields in the U.S. during the 2004–2005 period does appear to be a conundrum when viewed through a macro-finance lens. Specifically, neither of the two macro-finance empirical models we consider is able to explain the recent low level of, or the fall in, long-term bond yields. This finding is remarkable given that both models fit the earlier long-term yield data quite well. Therefore, the conundrum can likely only be explained with variables that lie outside of our baseline macro-finance models. Of the six such variables that we consider, it is declines in the (short-run implied) volatility of long-term Treasury yields that seem to have played the most important role. Even so, at best, almost two-thirds of the conundrum remains unexplained.

Interestingly, we find that the explanation for the conundrum emphasized by financial market participants—namely, large-scale purchases of long-term Treasuries by foreign central banks—has essentially *no* explanatory power for the conundrum episode. This discrepancy may reflect a difference between unconditional and conditional correlations. In particular, long-term Treasury yields have been declining steadily over time, and foreign official holdings and purchases of U.S. Treasuries have been rising steadily; thus, the unconditional correlation suggests a substantial negative effect. In contrast, a macro-finance econometric framework attributes the downward trend in long-term yields largely to declining current and future projected levels of inflation. After controlling for such factors, the residuals from our two baseline macro-finance models have no significant correlation with foreign official purchases of U.S. Treasuries.

Of course, this leaves us with about two-thirds of the bond yield conundrum yet to be

explained, to say nothing of the similar or perhaps even more extreme behavior of long-term yields in other countries, such as Germany and Japan. The resolution of these “conundrum” episodes, in the U.S. and abroad, presents a rich frontier for future research.

**Table 1: Bernanke-Reinhart-Sack VAR Parameter Estimates**

VAR coefficient estimates ( $\Phi$ )

	$y_t$	$\pi_t$	$\widehat{\pi}_t^{BC}$	$i_t$	$\widehat{i}_t^{ed}$
$y_{t-1}$	1.078	0.078	-0.080	0.027	0.266
$y_{t-2}$	0.171	0.138	0.228	0.123	0.254
$y_{t-3}$	-0.102	0.062	-0.055	-0.038	-0.283
$y_{t-4}$	-0.180	-0.249	-0.094	-0.103	-0.278
$\pi_{t-1}$	0.022	0.883	0.059	0.170	0.311
$\pi_{t-2}$	-0.036	-0.067	-0.012	-0.141	-0.033
$\pi_{t-3}$	-0.071	0.187	0.001	-0.000	-0.234
$\pi_{t-4}$	0.073	-0.016	0.026	-0.012	-0.068
$\widehat{\pi}_{t-1}^{BC}$	-0.094	0.083	1.027	0.078	-0.104
$\widehat{\pi}_{t-2}^{BC}$	0.273	-0.086	-0.118	-0.174	0.208
$\widehat{\pi}_{t-3}^{BC}$	-0.239	0.124	-0.021	0.341	0.222
$\widehat{\pi}_{t-4}^{BC}$	0.063	-0.114	0.005	-0.263	-0.190
$i_{t-1}$	0.010	-0.107	0.089	1.311	0.105
$i_{t-2}$	0.070	0.120	-0.130	-0.413	-0.310
$i_{t-3}$	-0.092	-0.106	0.098	0.177	0.367
$i_{t-4}$	0.018	0.093	-0.054	-0.115	-0.073
$\widehat{i}_{t-1}^{ed}$	0.029	-0.007	0.064	0.162	1.097
$\widehat{i}_{t-2}^{ed}$	-0.005	0.007	-0.071	-0.065	-0.264
$\widehat{i}_{t-3}^{ed}$	-0.020	-0.043	0.017	-0.029	0.139
$\widehat{i}_{t-4}^{ed}$	-0.003	0.036	-0.004	-0.034	-0.119
constant	-0.006	0.039	0.046	0.006	0.087

Cholesky-factored residual variance ( $\Sigma$ ) ( $\text{Var} = \Sigma\Sigma'$ )

.0867	0	0	0	0
.0034	.1618	0	0	0
-.0001	.0111	.0808	0	0
.0047	-.0105	.0168	.1539	0
.0800	.0309	.0283	.1150	.3826

**Table 2: Bernanke-Reinhart-Sack Model Risk Factor Loadings**

$\lambda_0$	$\lambda_1$				
-1.998	0.473	0.290	-1.292	2.517	0.209
-2.349	0.712	0.666	6.036	-2.022	0.256
-20.822	13.846	-0.323	1.028	4.941	12.002
3.366	-1.814	0.019	-0.287	-0.939	-1.720
2.525	-1.064	-0.465	-1.174	-0.030	-1.186

**Table 3: Bernanke-Reinhart-Sack Model Prediction Errors**

Treasury maturity	Model prediction error (RMSE, bp)
6 months	29.52
1 year	33.35
2 years	26.98
3 years	26.58
4 years	27.78
5 years	30.03
7 years	34.45
10 years	39.30
average	31.00

**Table 4: Rudebusch-Wu Model Parameter Estimates**

Factor dynamics					
$\rho_L$	0.989	(0.0068)	$g_\pi$	1.253	(0.0066)
$\rho_S$	0.026	(0.0111)	$g_y$	0.200	(0.0066)
$\rho_u$	0.975	(0.0062)			
Inflation dynamics					
$\mu_\pi$	0.074	(0.0113)	$\alpha_{\pi 1}$	1.154	(0.0525)
$\alpha_y$	0.014	(0.0074)	$\alpha_{\pi 2}$	-0.155	(0.0066)
Output dynamics					
$\mu_y$	0.009	(0.0066)	$\beta_{y1}$	0.918	(0.0604)
$\beta_r$	0.089	(0.0067)	$\beta_{y2}$	0.078	(0.0066)
Risk price ( $\lambda_1$ )					
		$L_t$		$S_t$	
$\Lambda_{L,t}$	-0.0045	(0.0068)	0.0168	(0.0068)	
$\Lambda_{S,t}$	-0.0223	(0.0064)	0.0083	(0.0067)	
Standard deviations					
$\sigma_L$	0.342	(0.0089)	$\sigma_\pi$	0.238	(0.0110)
$\sigma_S$	0.559	(0.0313)	$\sigma_y$	0.603	(0.0128)
Standard deviations of measurement error					
3-month	0.288	(0.0162)			
12-month	0.334	(0.0194)			
36-month	0.127	(0.0094)			

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Note: Standard errors of the estimates are in parentheses.



**Table 5: Survey Respondents' Assessments of Factors  
Holding Down the 10-Year Treasury Yield**

Factor affecting yields	Effect in basis points
Demand by foreign central banks	21
Increased demand by pension funds	11
“Reaching for yield”	10
Minimal inflation risk	10
Greater transparency of the Fed	8
Excess global savings	8
Low economic growth volatility	7

Source: A survey of clients by Macroeconomic Advisers as reported in the newsletter “Monetary Policy Insight’s Survey on Long-Term Interest Rates,” March 8, 2005.

**Table 6: Univariate Regressions of BRS and RW Model Residuals**

Independent variable	Model	Sample	Coeff.	t-stat.	$R^2$
Implied volatility on longer-term Treasury Securities (Merrill-Lynch MOVE Index)	BRS	4/88–12/05	1.10	7.83	.23
	RW	4/88–12/05	0.38	4.94	.10
Implied volatility on six-month-ahead eurodollar futures (from options, in bp)	BRS	1/89–12/05	.278	3.17	.05
	RW	1/89–12/05	.042	0.95	.00
Implied volatility on S&P 500 (VIX index)	BRS	1/90–12/05	.814	1.85	.02
	RW	1/90–12/05	.049	0.23	.00
Realized volatility of quarterly GDP growth (trailing 8-quarter standard deviation, in %)	BRS	1/88–12/05	20.3	4.25	.08
	RW	1/88–12/05	5.7	2.26	.02
Realized volatility of monthly core PCE price Inflation (trailing 24-mo. std. dev., in %)	BRS	1/88–12/05	316	2.33	.02
	RW	1/88–12/05	99	1.41	.01
Foreign official purchases of U.S. Treasury securities (trailing 12-mo. total, as % of U.S. debt in hands of public)	BRS	5/90–12/05	149	0.89	.00
	RW	5/90–12/05	47	0.58	.00

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Note: BRS and RW model residuals are in basis points. Sample period start dates differ owing to differences in data availability for each explanatory variable. Each regression also includes a constant term (not reported).

**Table 7: Multivariate Regressions of BRS and RW Model Residuals**

Independent variable	BRS Model		RW Model	
	Coeff.	(t-stat)	Coeff.	(t-stat)
Implied volatility on longer-term Treasury securities (Merrill-Lynch MOVE index)	1.203	(5.47)	0.490	(4.11)
Implied volatility on six-month-ahead eurodollar futures (from options, in bp)	-0.229	(-1.35)	-0.168	(-1.83)
Implied volatility on S&P 500 (VIX index)	-0.334	(-0.63)	-0.497	(-1.73)
Realized volatility of quarterly GDP growth (trailing 8-quarter standard deviation, in %)	15.4	(3.10)	3.9	(1.45)
Realized volatility of monthly core PCE price inflation (trailing 24-mo. std. dev., in %)	360	(2.18)	214	(2.39)
Foreign official purchases of U.S. Treasury securities (trailing 12-mo. total, as % of U.S. debt in hands of public)	147	(0.76)	38	(0.04)
	$R^2$			
	.30		.14	

Note: BRS and RW model residuals are in basis points. Sample: May 1990 to December 2005. Each regression also includes a constant term (not reported).

**Table 8: Decomposition of Long-Term Bond Yield Conundrum**

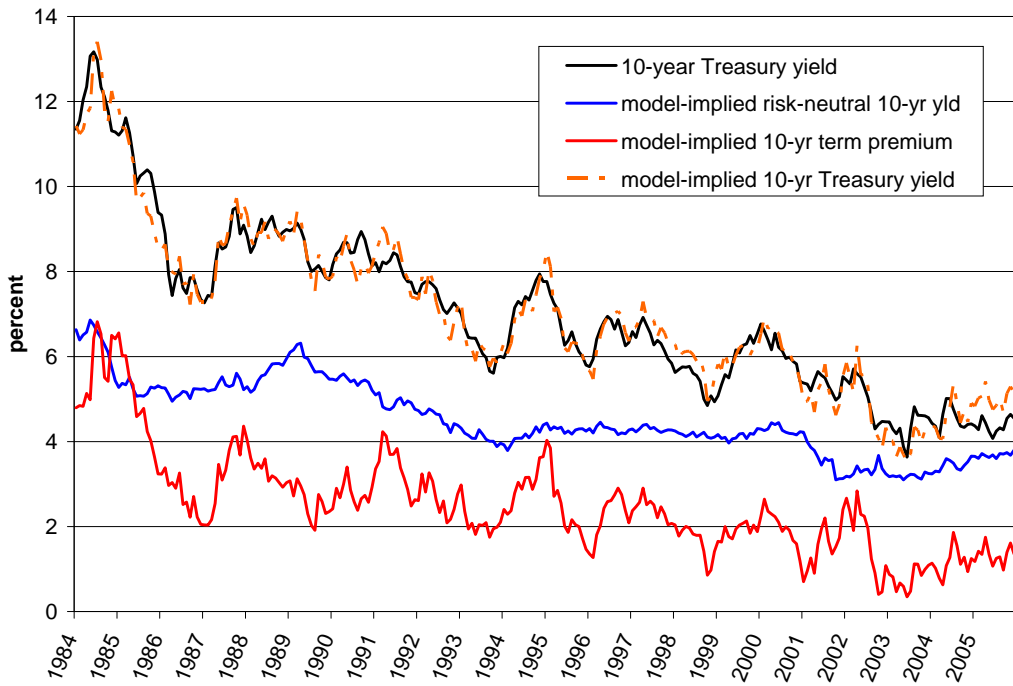
	BRS model	RW model
1. Observed change in 10-year yield (bp), 6/04–6/05	−93.3	−87.0
of which:		
2. model-implied change in risk-neutral 10-year yield	13.1	−29.6
3. model-implied change in term premium	−19.9	−25.2
4. change in model residuals	−86.5	−32.2
of which:		
5. change in implied volatility on longer-term Treasuries	−29.9	−12.2
6. change in realized volatility of core PCE inflation	1.1	0.7
7. change in realized volatility of GDP growth	−11.6	−2.9
8. change in implied volatility of eurodollar rate	7.0	5.1
9. change in implied volatility of S&P 500	1.2	1.7
10. change in foreign official purchases	−6.0	−1.6
11. unexplained by above	−48.6	−23.0

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Note: Line 1 denotes the change in observed zero-coupon 10-year Treasury yields from June 2004 to June 2005; the values differ across the BRS and RW models because the BRS model uses month-average yield data while the RW model uses end-of-month data. Lines 2–4 are the decompositions implied by the BRS and RW macro-finance models, graphed in figures 1 and 2. Lines 5–10 are the changes implied by the regression coefficients in Table 7 applied to the change in each independent variable from June 2004 to June 2005. Line 11 is the difference between line 4 and the sum of lines 5–10.

**Figure 1: 10-year Treasury Yield and BRS Model Implied Values**

(a) 10-year Treasury Yield and BRS Model Decomposition



(b) BRS Model Residuals

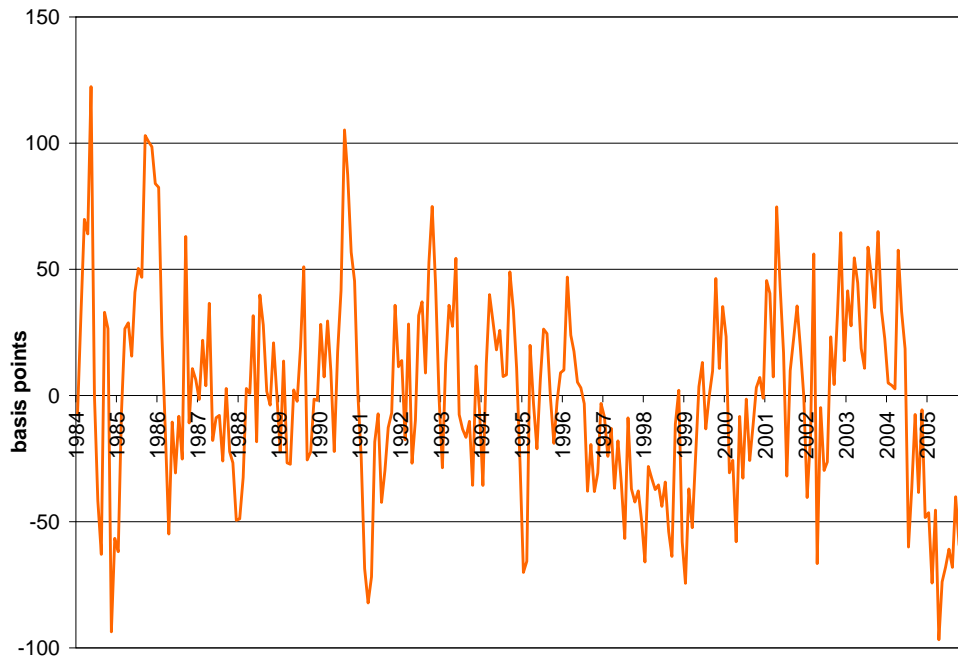
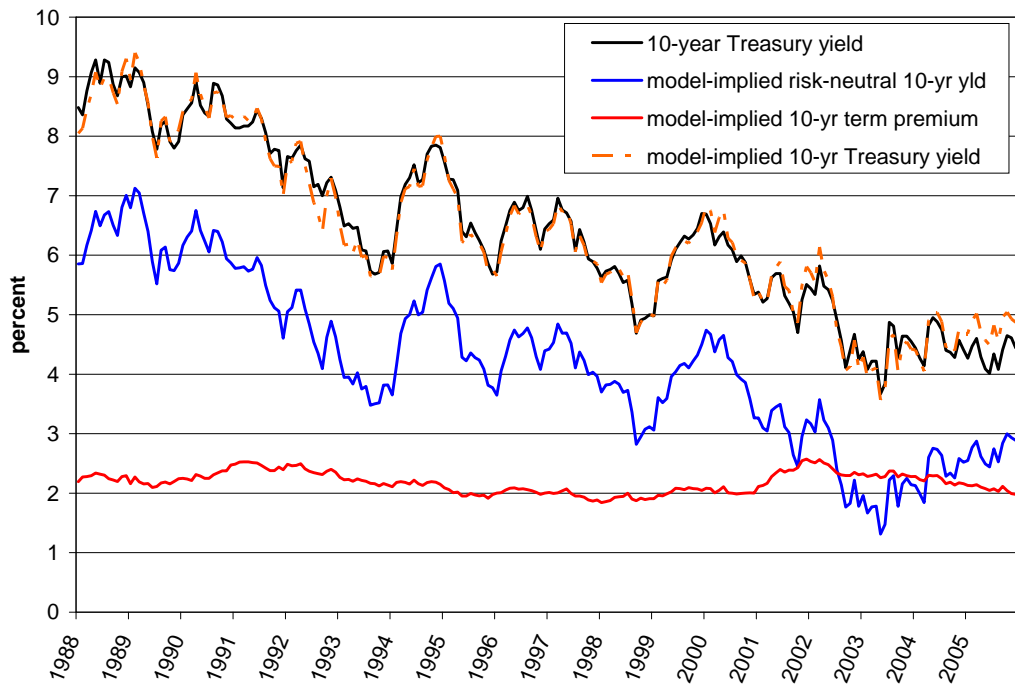
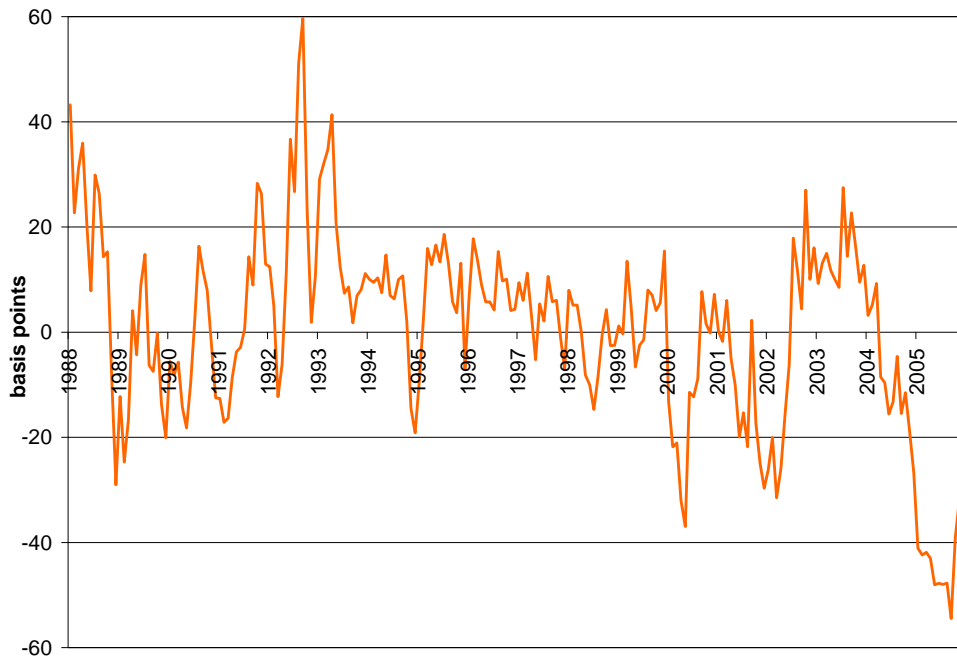


Figure 2: 10-year Treasury Yield and RW Model Implied Values

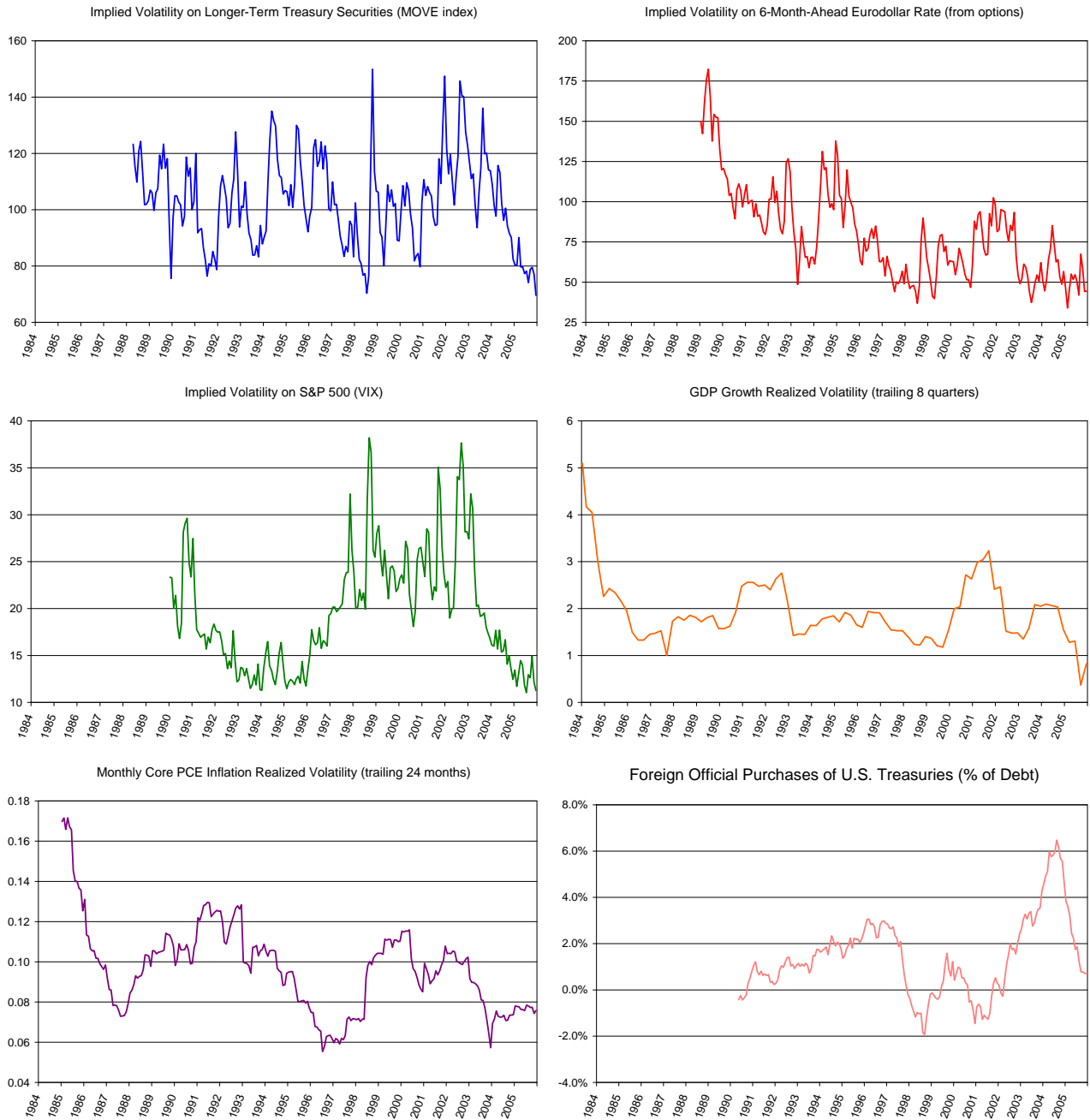
(a) 10-year Treasury Yield and RW Model Decomposition



(b) RW Model Residuals



**Figure 3: Explanatory Variables for BRS and RW Model Residuals**



Note: Not all data series available back to 1984.

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