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Antonio Doblas-Madrid
Michigan State University

Kevin J. Lansing
Federal Reserve Bank of San Francisco

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CREDIT-FUELLED BUBBLES*

Antonio Doblas-Madrid†
Michigan State University

Kevin J. Lansing
Federal Reserve Bank of San Francisco

Abstract

In the context of recent housing busts in the United States and other countries, many observers have highlighted the role of credit and speculation in fueling unsustainable booms that lead to crises. Motivated by these observations, we develop a model of credit-fuelled bubbles in which lenders accept risky assets as collateral. Booming prices allow lenders to extend more credit, in turn allowing investors to bid prices even higher. Eager to profit from the boom for as long as possible, asymmetrically informed investors fuel and ride bubbles, buying overvalued assets in hopes of reselling at a profit to a *greater fool*. Lucky investors sell the bubbly asset at peak prices to unlucky ones, who buy in hopes that the bubble will grow at least a bit longer. In the end, unlucky investors suffer losses, default on their loans, and lose their collateral to lenders. In our model, tighter monetary and credit policies can reduce or even eliminate bubbles. These findings are in line with conventional wisdom on macro prudential regulation, and stand in contrast with those obtained by Galí (2014) in an overlapping generations context.

JEL Classification: G01, G12

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†Corresponding author. Address: Department of Economics 486W Circle Drive 110 Marshall-Adams Hall Michigan State University East Lansing, MI 48824-1038, USA. Fax: 517.432.1068. Tel: 517.355.7583. Email: doblasma@msu.edu

1 Introduction

Overinvestment and overspeculation are often important; but they would have far less serious results were they not conducted with borrowed money.

Irving Fisher, 1933, p.341.

Like Irving Fisher, many economists have highlighted the interplay of credit and speculation as an essential part of booms and crises (Kindleberger and Aliber 2005, Minsky 1986, Borio and Lowe 2002, Jordà, et al. 2016). Household leverage in many industrial countries increased dramatically during the mid-2000s. Countries with the largest increases in household debt relative to income tended to experience the fastest run-ups in house prices over the same period. The same countries tended to experience the most severe declines in consumption once house prices started falling (Glick and Lansing 2010, International Monetary Fund 2012).

Within the United States, house prices rose faster in areas where subprime and exotic mortgages were more prevalent (Mian and Sufi 2009, Barlevy and Fisher 2010, Pavlov and Wachter 2011, Berkovec, et al. 2012). In a similar vein, the report by the U.S. Financial Crisis Inquiry Commission (2011) emphasizes the effects of a self-reinforcing feedback loop in which an influx of new homebuyers with access to easy mortgage credit helped fuel an excessive run-up in house prices, encouraging lenders to ease credit further on the assumption that prices would continue to rise. While the foregoing studies focus on housing market booms, similar logic applies to other assets markets, such as stocks, foreign exchange, and derivatives, where margin trading is common.

Motivated by these observations, we develop a model of credit-fuelled bubbles to analyze the interplay between lending and speculation. Our starting point is the theory of bubbles proposed by Abreu and Brunnermeier (2003; AB henceforth), and in particular the fully rational version developed by Doblas-Madrid (2012; DM henceforth). We borrow two key ingredients from AB. The first is the notion of a bubble as an overreaction to a shock that is at first fundamental in nature. The second is the introduction of asymmetric information. In outline, the model's structure is as follows. A fundamental shock sets off a boom in asset prices. Initial price increases are justified by a catch-up to the underlying fundamental v alue. At some time

t_0 the price becomes just equal to the present value of expected dividends. However, this point in time is only imperfectly observed, with different investors observing different private signals about t_0. That is, different investors have different information regarding when the fundamental part of the boom ends and the bubble begins. Since signals are private, investors know their own signals, but not those of others. Signals order investors along a line from lowest-t_0 signal to highest-t_0 signal. This uncertainty is key to generate speculative behavior. Investors know that if they are early in line, they will sell at peak prices, but they will lose in the crash if they are late. Nevertheless, as long as price growth is rapid enough to compensate for crash risk, they are willing to take a chance and *ride the bubble*. Thus, investors in the model behave like short-term traders rather than like long-term value investors. Instead of selling as soon as the price surpasses (their estimation of) fundamental value, they continue to buy the asset in an attempt to time the market and sell to a *greater fool* right before the crash.[1]

Credit plays an essential role in the model because the theory requires that prices experience a sustained boom instead of a one-time jump. In AB, the sustained boom is due to the gradual, but accelerating arrival of 'irrationally exuberant' behavioral agents, whereas in DM, the lasting boom is made possible by exogenously growing endowments. In both cases, the source of such growing resources is ultimately unknown. We endogenize this gradual inflow via the interaction between collateralized credit and asset prices. Thus, in addition of being of interest in its own right, the addition of credit provides a plausible response to a question left unanswered by previous work.

In our model, each period is subdivided into two stages, an asset market stage where investors trade assets and a credit market where investors repay old loans and take new ones. Lenders competitively intermediate between the model economy and the rest of the world, where they borrow or lend at an exogenous risk-free rate. Investors pledge their shares of the risky (i.e., bubbly) asset as collateral. Because of limited liability, lenders endogenously adjust the size of the loans they extend depending on the expected liquidation price of the collateral. Moreover, in addition to any endogenous lending limits, we assume that lenders face an exogenous upper bound $\overline{\phi}$ on the loan-to-value (LTV) ratio, in which the value of collateral is defined using the most recent (as opposed to expected future) price. We interpret this as a regulatory

[1] Barlevy (2015) provides a review of the literature on "greater fool" theories of bubbles.

constraint, akin to down payment requirements for home purchases or margin requirement for stock purchases. Given the difficulty and subjectivity involved in forecasting future prices, such regulatory limits are in practice defined as a function of observable prices, or appraised values which typically take recent transactions into account. A link between credit and recent prices is documented by Dell'Ariccia, et al. (2012) and Goetzmann et al. (2012), who show evidence that—in US housing markets—recent price appreciation in a given area has a significant positive impact on subsequent loan approval rates.

While the addition of credit inevitably adds complexity relative to the growing-endowments model, our model nevertheless remains analytically tractable. Under appropriate parameter restrictions, the bubble growth rate—generated by a feedback loop between prices and credit—converges to a constant G. Moreover, we restrict attention to the case in which the LTV cap $\overline{\phi}$ is low enough that lenders always avoid credit losses when investors default. We refer to this as a no-risk-shifting condition, under which even in the equilibrium with the biggest bubble, the hapless late-signal investors who buy at peak prices remain 'above water' when the price crashes. These unlucky investors are forced to default on some of their loans when credit tightens in the crash, but these defaults do not saddle lenders with losses because the repossessed collateral is always valuable enough for full loan repayment. Moreover, since investors are not 'underwater' they have no incentive to 'walk away' from their loans in the event of default. This benchmark analysis gives us a parsimonious extension of DM, to the extent that we can apply characterization results derived therein with only minor adaptations. The relationship between bubble growth rate and bubble duration is the same as in the model with endowments. But these results are supported by with two additional results, one linking the bubble growth rate G to the degree of leverage $\overline{\phi}$ and one deriving the no-risk-shifting relationship between $\overline{\phi}$ and G and establishing compatibility between all the conditions needed to sustain equilibrium. Overall, our results are in line with macro-prudential conventional wisdom regarding interest rates, leverage and bubbles. The model predicts that looser lending standards (as measured by higher LTV ratios) imply faster growth in the price of the risky asset and therefore longer (and larger) bubbles. Low interest rates are also conducive to bubbles, for two reasons. First, because the price growth rate G is decreasing in R, as higher interest payments on borrowed funds slow down the growth of the funds that investors bring to the asset market. Second, low interest rates make the safe asset unattractive relative to the risky asset. As the G/R ratio increases, so does the size of the bubble, since investors are willing to risk a bigger crash in exchange for a bigger premium. In this regard, our results contrast with those of Galí (2014).

The no-risk-shifting condition allows us to isolate the speculative motive for riding of bubbles from the risk-shifting motive emphasized by Allen and Gale (2000) and Barlevy (2014), where investors use borrowed money to bid up prices in excess of expected dividends because lenders absorb part of the losses in bad states of the world. In our model, as in the growing-endowments model, investors bear the full costs and benefits of their speculative activity. Thus, willingness to pay above expected dividends for an asset is purely driven by the prospect of earning speculative profits, not the possibility of passing losses to third parties if caught in the crash. Another appealing—and related—feature of the no-risk-shifting analysis is that it is robust to any specification of lenders' beliefs about the bubble's start and bursting times. Since they cannot lose, lenders offer loans at the exogenous risk-free rate, regardless of their beliefs about t_0. In the context of the recent US housing boom and bust, the no-risk shifting assumption is reminiscent of the observation that many mortgage lenders bore little credit risk because of government guarantees or securitization—although a full analysis along these lines would require analyzing the beliefs of the securities' buyers.

In addition to AB, DM and risk-shifting models à la Allen and Gale (2000) and Barlevy (2014), our work relates to asymmetric information models by Allen, et al. (1993), Conlon (2004, 2015), and the work by Harrison and Kreps (1978), further extended by Scheinkman and Xiong (2003), where heterogenous beliefs/overconfidence play a central role. Our model, in addition to asymmetric information, emphasizes the role of binding constraints in gradually fuelling and riding bubbles. Finally, our model differs from the well-known 'rational bubbles' literature pioneered by Samuelson (1958) and Tirole (1985), and further developed by Martin and Ventura (2012), Galí (2014), and others. In this strand of literature, bubbles such as fiat money last indefinitely (either literally or in expectation) and typically have efficiency benefits by helping society overcome a shortage of stores of value. Our focus, by contrast, is on finite bubbles that are not efficiency enhancing.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we define equilibrium and propose a candidate strategy profile. In Section 4, we characterize equilibrium bubble duration in the most tractable, benchmark case. In Section 5, we discuss our assumptions, results, and possible extensions. In Section 6, we conclude.

2 The Model

Time is discrete and infinite with periods labeled $t \in \{..., -1, 0, 1, 2, ...\}$. The economy is populated by atomistic investors indexed by $i \in [0, 1]$ and a continuum of identical lenders. As in AB and DM, the timeline is split into three phases, a pre-boom phase for $t < 0$, a boom phase from $t = 0$ to the (endogenous) crash period t_c, and a post-crash phase for $t \geq t_c + 1$. The boom phase may in turn be subdivided into a fundamental part from $t = 0$ to $t = t_0$ and a bubble from $t_0 + 1$ to t_c. These phases, depicted in Figure 1, describe the evolution of p_t, the price of a risky asset which exists in unit supply and pays dividends $\{d_t\}_{t \in \mathbb{Z}}$. Besides the risky asset, there is a safe asset which yields an exogenous gross return R and can be turned into consumption at a one-to-one rate.

In the pre-boom phase $t < 0$, all agents expect dividends d_t to be zero for $t \leq 0$ and positive after period 0. Normalizing the expected value—discounted to time 0—of dividends to 1, the price p_t equals the expected dividend R^t while $t < 0$. Hence, in the pre-boom phase, the risky asset is not yet risky, but instead a perfect substitute of the safe asset.

At time 0, an unanticipated shock raises expected dividends. We assume for simplicity that a single dividend $d_{t_{pay}} = \overline{F} R^{t_{pay}}$ will be paid at the maturity date $t_{pay} > 0$. As of time 0, all agents know that $\overline{F}$ exceeds 1, but not precisely by how much. There is also uncertainty about t_{pay}, which is an increasing function of $\overline{F}$, so that the greater the fundamental gains from the shock, the later the payoff. The value of $\overline{F}$ will be observed at t_{pay}, but we will focus on situations where $\overline{F}$ becomes known before then.

A crucial ingredient in our model is the assumption that, due to borrowing constraints, prices cannot fully adjust to the shock by jumping to the new expected $\overline{F}$ at $t = 0$. Instead, the shock sets off a boom in which higher prices loosen borrowing constraints, allowing investors to bid prices even higher, and so on. Under certain conditions—which we will derive—the (gross) price growth rate p_{t+1}/p_t generated by this process exceeds R and converges over time to a constant $G > R$. Investors can predict the prices that will be observed as long as the boom lasts, but do not know the value of $\overline{F}$, and thus do not know how long the price boom will be justified by fundamentals. Following AB and DM, we denote by $t_0 \geq 0$ the number of periods it takes for the price to catch up with the present value of dividends. Specifically, we define

$$t_0 \equiv \{t < t_c | p_t = \overline{F}R^t\}.$$

If prices continue to boom past period t_0, a bubble will inflate.[2] Bubbly price gains are unjustified by fundamentals and bound to disappear when p_t crashes between periods t_c and $t_c + 1$. Following AB, we assume that nature draws t_0 from a geometric distribution with pdf

$$\psi(t_0) = (1 - \lambda)\lambda^{t_0}, \quad \text{for } t_0 = 0, 1, ... \tag{1}$$

and $\lambda > 0$. We also borrow from AB the crucial assumption that investors are asymmetrically informed as follows. At time 0, different investors observe different signals containing information about t_0.[3] Specifically, the signal function $\nu : [0, 1] \to \{t_0, ..., t_0 + N - 1\}$ divides investors into N types. The signal $\nu(i)$ reveals to investor i that the boom may only be justified by fundamentals up to period $\nu(i)$, but not longer. Since investors know that there are N types, investor i infers that t_0 cannot be below $\max\{0, \nu(i) - (N - 1)\}$ or above $\nu(i)$.[4] The distribution of t_0 conditional on $\nu(i)$ is thus given by

$$\psi(t_0|v(i)) = \begin{cases} \frac{\psi(t_0)}{\psi(\max\{0,\nu(i)-(N-1)\})+\cdots+\psi(\nu(i))} & \text{if } \max\{0, \nu(i) - (N - 1)\} \leq t_0 \leq \nu(i) \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Private signals about t_0 are equivalent to private signals about $\overline{F}$. In fact, signal $v(i)$ reveals that $\overline{F} \in \{p_{t_0}/R^{t_0} | \max\{0, \nu(i) - (N - 1)\} \leq t_0 \leq \nu(i)\}$ with the probabilities of each possible value of $\overline{F}$ given by (2).

Private signals order investors along a line from earliest (i.e., lowest) to latest (i.e., highest)

[2] In our discrete-time model—as well as in D-M—there has to be a 'coincidence' for any given price to exactly equal $\overline{F}R^t$. A more general assumption would be to define t_0 as the first period with $p_t \geq \overline{F}R^t$. However, this complicates formulae substantially without yielding additional insight.

[3] In AB and D-M, signals arrive over N periods of time, starting at t_0. Our assumption that all signals arrive at time 0 simplifies the exposition without affecting results.

[4] The $\max\{0, \cdot\}$ operator captures the fact that, in the special case with $t_0 < N - 1$, types with $\nu(i) < N - 1$ know that they cannot be last in line, since $\nu(i) - (N - 1) < 0$. For those types, the support of t_0 conditional on $\nu(i)$ is $\{0, ..., \nu(i)\}$ instead of $\{\nu(i) - (N - 1), ..., \nu(i)\}$.

signal, but they do not know their relative order in the line. Importantly, all investors—including those late in the line—assign positive probability to the event that they could be early.[5] As we will see, given the speed at which the bubble grows and the probability of a crash, investors will ride and fuel bubbles as long as prospective speculative gains in the event of being an early-signal investor outweigh the risky of losses in the event of being a late-signal investors. Thus, as in AB and DM, we model a bubble as a situation in which markets overreact to a fundamental shock, with the overreaction arising as the equilibrium of a market timing game played by investors. Compared to investors, lenders are relatively passive. Since we focus on equilibria without risk-shifting, we do not need to make assumptions about whether they observe any signals.

The boom continues as long as all types continue to invest in it, and ends as soon as one type exits the market anticipating a crash. To fix ideas, suppose that type-$\nu(i)$ investors plan to ride the bubble for $\tau^* \geq 0$ periods and then sell at $\nu(i) + \tau^*$. (We will later construct equilibria in which this is the case.) While $t < t_0 + \tau^*$, all types of investors are fully invested in the bubble. The crash arrives at $t_c = t_0 + \tau^*$ when investors of type $\nu(i) = t_0$ exit the market. Their sales affect the price, revealing to others that $t = t_c$ and that $t_0 = t_c - \tau^*$.[6] The boom concludes with investors of type $\nu(i) = t_0$ selling at the peak and all other types finding themselves in the position of the *greater fool*. Post-crash, the value of t_0 is common knowledge, and the risky asset trades at fundamental value $p_t = E[\overline{F}|t_0]R^t$ until the payoff date t_{pay}, falling to zero afterwards.

Keeping this sketch of the model in mind, we now proceed to a full description of the environment.

2.1 Preferences

Investors are risk neutral, and may be hit by preference shocks à la Diamond and Dybvig (1983), which induce an urgent need to consume. Specifically, every period a randomly chosen fraction $\theta \in [0, 1]$ of investors are hit by a shock that makes them impatient by setting their discount factor $\delta_{i,t}$ to 0, while the remaining mass $1 - \theta$ are patient and have discount factor $\delta_{i,t} = 1/R$.

[5] This specification of signals is simply a discretized version of that in AB. Moreover, it resembles the timing devised by Moinas and Pouget (2012), who conduct experiments in which participants are uncertain about the order in which they move relative to each other.

[6] As we will shortly see, the trading protocol is such that agents submit orders first, and observe the price second. Thus, unlike in a Walrasian setting, buyers get stuck with the bubble at t_c because they cannot change their orders after observing the price.

Preference shocks do not represent the investor's death. After being hit, investors stay in the model economy and can be hit again. Since shocks are i.i.d., the probability that $\delta_{i,t} = 0$ is always θ, regardless of past values $\delta_{i,s}$, $s < t$. Hence, investors who were recently impatient are just as likely to become impatient again as investors who have been patient for a long time.

Investor i learns the realization of her individual preference shock at the beginning of period t. At this point, she holds a portfolio $(b_{i,t}, h_{i,t}, l_{i,t})$, where $b_{i,t} \geq 0$ are balances of the safe asset, $h_{i,t} \geq 0$ shares of the risky asset and $l_{i,t} \geq 0$ a debt owed to a lender. Debt $l_{i,t}$ is collateralized by shares $h_{i,t}$. Due to limited liability, in the event of default lenders can seize $h_{i,t}$, but nothing else. Economy-wide aggregates of $b_{i,t}$, $h_{i,t}$ and $l_{i,t}$,are denoted by B_t, H_t and L_t.

A period is divided into two subperiods. In the first, investors visit an asset market. In the second, they refinance debt.

2.2 The asset market

Investor i enters the asset market with portfolio $(b_{i,t}, h_{i,t}, l_{i,t})$ and information $I_{i,t}$, which includes the price history up to $t-1$, denoted by $p^{t-1} = \{\ldots, p_{t-2}, p_{t-1}\}$, and her discount factor $\delta_{i,t}$. From time 0 onward, $I_{i,t}$ also includes the signal $\nu(i)$. Thus, $I_{i,t}$ is given by $\{p^{t-1}, \delta_{i,t}\}$ if $t < 0$ and by $\{p^{t-1}, \delta_{i,t}, \nu(i)\}$ if $t \geq 0$. At this stage, investors trade assets (as long as $t \leq t_{pay}$), and consume.

The risky asset is traded in the asset market while $t \leq t_{pay}$, i.e., before the dividend is paid. The market operates as a Shapley-Shubik trading post with two-step trading. In Step 1, investors submit orders to buy or sell as follows. In one bin, investors deposits risky shares they want to sell. The risky asset cannot be shorted, and thus the number of sales for sale must satisfy $s_{i,t} \in [0, h_{i,t}]$ for all i. In another bin, investors enter their bids, by depositing the amounts of the safe asset they wish to spend buying risky shares. Investor i bids $m_{i,t} \in [0, b_{i,t}]$, where some of $b_{i,t}$ may be borrowed funds, since the safe asset can be shorted by obtaining credit from lenders. The aggregates corresponding to $m_{i,t}$ and $s_{i,t}$ are denoted by M_t and S_t. In Step 2, bids and offers are combined and the price is determined as the ratio of the total bid M_t to total shares for sale S_t, i.e.,

$$p_t = \frac{M_t}{S_t}. \tag{3}$$

As long as $t \leq t_{pay}$, this ratio is well defined since there is always a mass θ of impatient sellers.

An important difference between this Shapley-Shubik/Cournot timing and Walrasian timing is that investors choose $m_{i,t}$ and $s_{i,t}$ before observing p_t, and cannot change their choices after observing it. This plays an important role, allowing one type of agents to sell at the crash time t_c, since buyers will not be able to revise their bids upon observing the price.[7]

After selling $s_{i,t}$ and buying $m_{i,t}/p_t$ shares, investor i holds

$$\widetilde{h}_{i,t} = h_{i,t} + \frac{m_{i,t}}{p_t} - s_{i,t} \tag{4}$$

risky shares. And, since debts backed by $s_{i,t}$ shares are settled upon sale, her liabilities $\widetilde{l}_{i,t}$ are given by

$$\widetilde{l}_{i,t} = \frac{h_{i,t} - s_{i,t}}{h_{i,t}} l_{i,t}. \tag{5}$$

From the initial $b_{i,t}$ safe balances investor i subtracts the bid $m_{i,t}$ and adds sales revenue, net of debt repayment. This net revenue is given by $\max\{p_t - l_{i,t}/h_{i,t}, 0\}s_{i,t}$, where the $\max\{\cdot, 0\}$ operator captures limited liability. (Since we focus on equilibria without risk-shifting, p_t will never fall below $l_{i,t}/h_{i,t}$ in the analysis that follows.) Of the resulting balance, investor i allocates a fraction $\xi_{i,t} \in [0, 1]$ to consumption, which is given by

$$c_{i,t} = \xi_{i,t} \left[b_{i,t} - m_{i,t} + \left(\max\left\{ 0, p_t - \frac{l_{i,t}}{h_{i,t}} \right\} \right) s_{i,t} \right], \tag{6}$$

and the rest to risk-free savings

$$\widetilde{b}_{i,t} = (1 - \xi_{i,t}) \left[b_{i,t} - m_{i,t} + \left(\max\left\{ 0, p_t - \frac{l_{i,t}}{h_{i,t}} \right\} \right) s_{i,t} \right]. \tag{7}$$

After her trading and consumption choices, investor i leaves the asset market stage with an interim portfolio $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t})$. Since it is common knowledge that the asset is worthless after the dividend is paid, $m_{i,t}$, $s_{i,t}$, p_t, and $l_{i,t}$ are zero for all $t > t_{pay}$.

To recapitulate, investor i's asset market choices are given by $a_{i,t} = (m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t})$. Because they depend on the price, which is unknown when investor i moves, $\widetilde{b}_{i,t}$ and $\widetilde{h}_{i,t}$ are not part of

[7] In D-M, the fraction θ is noisy, causing random fluctuations in the price. Under certain conditions, this allows one type to exit the market without necessarily being noticed by other types, who cannot distinguish whether a low price is due to sales of the first type or simply a high realization of θ. Here, we simplify this dimension of the model and assume a deterministic θ.

$a_{i,t}$—although investors understand that $\widetilde{b}_{i,t}$ and $\widetilde{h}_{i,t}$ depend on $m_{i,t}$ and $s_{i,t}$ via (7) and (4). To preview of what investors will do, note that impatient investors always liquidate their assets, setting ($m_{i,t} = 0$, $s_{i,t} = h_{i,t}$, $\widetilde{h}_{i,t} = 0$, $\xi_{i,t} = 1$) to consume $c_{i,t} = b_{i,t} + \max\{p_t h_{i,t} - l_{i,t}, 0\}$. Patient investors do not consume, and move in and out of the risky asset as follows.[8] Pre-boom and post-crash, they are indifferent between both assets and find it (weakly) optimal to bid whatever amount equates price to expected dividend. In the boom, all patient investors are fully invested in the risky asset, i.e., they choose ($m_{i,t} = b_{i,t}$, $s_{i,t} = 0$, $\widetilde{l}_{i,t} = l_{i,t}$, $\xi_{i,t} = 0$), except for investors of type $\nu(i) = t_0$ at time t_c. These are the lucky investors who succeed in timing the market by setting ($m_{i,t_c} = 0$, $s_{i,t_c} = h_{i,t_c}$, $\widetilde{l}_{i,t_c} = 0$, $\xi_{i,t_c} = 0$) in anticipation of the crash.

2.3 Refinancing debt

Investor i enters the debt refinancing stage with portfolio $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t})$ and information $\widetilde{I}_{i,t}$. In addition to $I_{i,t}$, the information set $\widetilde{I}_{i,t}$ includes the price p_t observed in the asset market. At this stage, investors repay or default on their old debt, and take new loans.

We first describe refinancing in pre-dividend periods $t < t_{pay}$. For investor i, old loans $\widetilde{l}_{i,t}$ are due. These loans were originated at $t-1$ and collateralized by $h_{i,t} - s_{i,t}$. If investor i repays a fraction $r_{i,t}$ of these loans, she disburses $r_{i,t}\widetilde{l}_{i,t}$ units of the safe asset to the lender. If $r_{i,t} < 1$, investor i defaults on some loans. For simplicity, we assume that defaults are resolved on the spot, with the lender seizing $(1 - r_{i,t})(h_{i,t} - s_{i,t})$ risky shares and rebating any remaining equity $e_{i,t}$ to the defaulting borrower. Since the lender can sell the repossessed shares in next period's asset market, and since there is limited liability, the per-share equity rebate is given by

$$e_{i,t} = \max\left\{0, \frac{E[p_{t+1}|\widetilde{I}_t^L, r_{i,t} < 1]}{R} - \frac{l_{i,t}}{h_{i,t}}\right\}, \tag{8}$$

where next period's expected price $E[p_{t+1}|\widetilde{I}_t^L, r_{i,t} < 1]$ is conditional on lenders' information at the asset market stage $\widetilde{I}_t^L$ and on investor i choosing $r_{i,t} < 1$. (Since we focus on equilibria without risk-shifting, the repossessed collateral will always be valuable enough to recover the debt, and thus, the $\max\{0, \cdot\}$ operator will play no role in the analysis.) While lenders can sell repos-

[8] When a patient investor expects the risky asset to appreciate faster than R, consuming zero is the only optimal choice. When she expects the risky asset to appreciate at the rate R, she is indifferent between current-period and next-period consumption. We then assume zero consumption as a tie-breaking rule.

sessed shares, we did not include their orders in our description of the asset market because we assume that they hold repossessed shares shares until maturity. As we will see, in the equilibria we will consider, defaults only happen when the bubble bursts and $\overline{F}$ is common knowledge, at which point lenders are indifferent between selling a share or holding it until t_{pay} to collect the dividend.[9]

Investor i's ability to repay old loans depends on her ability to obtain new ones. Old and new loans are linked via the refinancing constraint

$$\frac{b_{i,t+1}}{R} + r_{i,t}\widetilde{l}_{i,t} = \widetilde{b}_{i,t} + y_t + (1 - r_{i,t})(h_{i,t} - s_{i,t})e_{i,t} + \frac{l_{i,t+1}}{R}. \tag{9}$$

Under this constraint, next period's safe balances $b_{i,t+1} \geq 0$ and repayment $r_{i,t}\widetilde{l}_{i,t}$ must be financed out of safe balances carried over from the asset market $\widetilde{b}_{i,t}$, the endowment y_t, equity rebate $(1 - r_{i,t})(h_{i,t} - s_{i,t})e_{i,t}$ and new loans $l_{i,t+1}/R$. We assume that endowments do not grow faster than the risk-free rate, i.e., $y_{t+1} \leq y_t R$ is assumed for all t. Given our focus on equilibria without risk-shifting, the interest rate on new loans $l_{i,t+1} \geq 0$ is the risk-free rate R. The amount investors can borrow is limited by their collateral and by a maximum loan-to-value (LTV) ratio $\overline{\phi}_t$, chosen by lenders. This gives rise to the borrowing constraint

$$l_{i,t+1} \leq \overline{\phi}_t p_t h_{i,t+1}, \tag{10}$$

where investor i's (post-default) risky shares $h_{i,t+1}$ are given by

$$h_{i,t+1} = \widetilde{h}_{i,t} - (1 - r_{i,t})(h_{i,t} - s_{i,t}) = \frac{m_{i,t}}{p_t} + r_{i,t}(h_{i,t} - s_{i,t}). \tag{11}$$

Investor i chooses the repayment fraction $r_{i,t}$ from the interval $[0, \overline{r}_{i,t}]$, where $\overline{r}_{i,t} \leq 1$ is the highest fraction she can afford to repay. If credit does not tighten too much, full repayment $\overline{r}_{i,t} = 1$ is feasible. To see this, substitute $r_{i,t} = 1$ in (9), (10) and (11), and note that, for full repayment, investor i has to borrow $\widetilde{l}_{i,t} - (y_t + \widetilde{b}_{i,t})$, pledging up to $\widetilde{h}_{i,t}$ shares, and obtaining up to $\overline{\phi}_t p_t/R$ per share. If she can obtain the credit she needs, $\overline{r}_{i,t} = 1$. Otherwise, $\overline{r}_{i,t} < 1$ is found by substitut-

[9]Without this assumption, we would have to keep track of each investor's defaulted loans and reposessed assets. This could potentially be very complex, since debt levels are typically different across periods. We discuss these assumptions and their implications later in the paper.

ing (5), (10) with equality and (11) into (9), and solving for $r_{i,t}$. In sum, the maximum repayable fraction $\overline{r}_{i,t}$ is given by

$$\overline{r}_{i,t} = \begin{cases} \frac{y_t + \widetilde{b}_{i,t} + (h_{i,t} - s_{i,t})e_{i,t} + \frac{m_{i,t}}{p_t}\frac{\overline{\phi}_t p_t}{R}}{(h_{i,t} - s_{i,t})\left[\frac{l_{i,t}}{h_{i,t}} + e_{i,t} - \frac{\overline{\phi}_t p_t}{R}\right]} & \text{if } \frac{\overline{\phi}_t p_t}{R}\widetilde{h}_{i,t} < \widetilde{l}_{i,t} - (y_t + \widetilde{b}_{i,t}) \\ 1 & \text{if } \frac{\overline{\phi}_t p_t}{R}\widetilde{h}_{i,t} \geq \widetilde{l}_{i,t} - (y_t + \widetilde{b}_{i,t}). \end{cases} \tag{12}$$

At $t = t_{pay}$, investor i collects dividends $d_{t_{pay}} = FR^{t_{pay}}$ for each of her $\widetilde{h}_{i,t_{pay}}$ shares, and settles her debt $\widetilde{l}_{i,t_{pay}}$ by repaying $\min\{d_{t_{pay}}, l_{i,t_{pay}}/h_{i,t_{pay}}\}(h_{i,t_{pay}} - s_{i,t_{pay}})$. (This repayment will be the full $(h_{i,t_{pay}} - s_{i,t_{pay}})l_{i,t_{pay}}/h_{i,t_{pay}} = \widetilde{l}_{i,t_{pay}}$ given our focus on equilibria without risk-shifting.) Moreover, since the risky asset is worthless after t_{pay}, there is no new borrowing and the refinancing constraint becomes

$$\frac{b_{i,t_{pay}+1}}{R} = \widetilde{b}_{i,t_{pay}} + y_{t_{pay}} + d_{t_{pay}}\frac{m_{i,t_{pay}}}{p_{t_{pay}}} + \max\left\{d_{t_{pay}} - \frac{l_{i,t_{pay}}}{h_{i,t_{pay}}}, 0\right\}(h_{i,t_{pay}} - s_{i,t_{pay}}). \tag{13}$$

After t_{pay}, this refinancing constraint reduces to $b_{i,t+1} = R(\widetilde{b}_{i,t} + y_t)$. Finally, for all $t \geq t_{pay}$, $\overline{r}_{i,t} = 1$, $h_{i,t+1} = \widetilde{h}_{i,t}$ and $l_{i,t+1} = 0$.

In sum, investor i's choices at the refinancing stage are given by $\widetilde{a}_{i,t} = (r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})$. To preview what they will do in equilibrium, note that, since the lending rate equals R, there is no loss of generality in assuming that (10) holds with equality. Investors thus borrow the highest possible $l_{i,t+1}$ and—if they do not wish to buy risky shares—simply hold the safe asset at no cost. Regarding $r_{i,t}$, investors will fully repay debts at all times, except when the bubble bursts, which tightens credit and forces many to default. As mentioned above, however, in the event of default collateral will adequately protect lenders against losses.

2.4 Lenders

Lenders costlessly intermediate between the model economy and the rest of the world, where they borrow at the rate R. Their information entering the debt refinancing stage is given by $\widetilde{I}_t^L$. Regardless of whether lenders observe signals or not, this information includes the price history p^t. Every period before t_{pay}, lenders collect repayment—or repossess collateral—for old loans and make new ones. Their only choice is to set $\overline{\phi}_t$, the maximum LTV ratio they offer. Since the risky asset is worthless after t_{pay}, $\overline{\phi}_t = 0$ for all $t \geq t_{pay}$.

The highest LTV offered $\overline{\phi}_t$ depends on repayment expectations, but is also subject to an exogenous upper bound

$$\overline{\phi}_t \leq \overline{\phi}. \tag{14}$$

As discussed in the introduction, we interpret this as a regulatory constraint, akin to a down payment requirement for a mortgage, or margin requirements for stock traders. Our assumption is motivated by the observation that, lending caps are in practice expressed as a function of current prices, rather than expected future prices, because the former can be observed or appraised more easily and objectively. Although we do not model regulators, this kind of leverage constraint is often part of banking, or macro-prudential regulations, aimed at reducing the frequency and severity of financial crises.

To see how lenders set borrowing limits, consider a lender's expected profit when making loan $l_{i,t+1}$ at time t, where $t < t_{pay} - 1$. At time t, given information $\widetilde{I}_t^L$, the lender disburses $l_{i,t+1}/R$ to investor i, who pledges shares $h_{i,t+1}$ as collateral. If investor i sells $s_{i,t+1}$ shares at $t+1$, the lender is repaid $\min\{p_{t+1}, l_{i,t+1}/h_{i,t+1}\}$ for each sold share. For the remaining debt $\widetilde{l}_{i,t+1}$, the lender collects repayment $r_{i,t+1}\widetilde{l}_{i,t+1}$, and, if $r_{i,t+1} < 1$, repossesses $(1 - r_{i,t+1})(h_{i,t+1} - s_{i,t+1})$ shares, where each seized share will be valued at $E[p_{t+2}|\widetilde{I}_{t+1}^L, r_{i,t+1} < 1]/R$. After rebating remaining any equity $e_{i,t+1}$—given by (8)—to investor i, the lender expects to recover a net amount $\min\{E[p_{t+2}|\widetilde{I}_{t+1}^L, r_{i,t+1} < 1]/R, l_{i,t+1}/h_{i,t+1}\}$.[10] In sum, expected profit from lending $l_{i,t+1}$ is given by

$$\begin{aligned} E\left[\pi_{t+1}(l_{i,t+1})|\widetilde{I}_t^L\right] = -l_{i,t+1} + E\left[\min\left\{p_{t+1}, \tfrac{l_{i,t+1}}{h_{i,t+1}}\right\} s_{i,t+1}|\widetilde{I}_t^L\right] + \\ +E\left[(h_{i,t+1} - s_{i,t+1})\left(r_{i,t+1}\tfrac{l_{i,t+1}}{h_{i,t+1}} + (1 - r_{i,t+1})\min\left\{\tfrac{E[p_{t+2}|\widetilde{I}_{t+1}^L, r_{i,t+1}<1]}{R}, \tfrac{l_{i,t+1}}{h_{i,t+1}}\right\}\right)|\widetilde{I}_t^L\right]. \end{aligned} \tag{15}$$

At the lending rate R, expected profit is zero only if there can be no credit losses. This is the case under two conditions. The first is that, given $\widetilde{I}_t^L$, $p_{t+1} \geq l_{i,t+1}/h_{i,t+1}$ with probability 1, so that it is a certainty that if shares are sold at $t+1$, the price suffices to repay the debt collateralized by the shares. The second is that, given $\widetilde{I}_t^L$, $E[p_{t+2}|\widetilde{I}_{t+1}^L, r_{i,t+1} < 1]/R \geq l_{i,t+1}/h_{i,t+1}$ with probability 1, so

[10] As stated earlier, we assume that lenders actually hold reposessed shares until maturity, instead of selling them at $t+1$. In equilibrium, they will be willing to do this, since repossessions will only happen at t_c, at which point $\overline{F}$ becomes common knowledge and the price becomes $\overline{F}R^t$ from $t_c + 1$ until t_{pay}.

that if shares are not sold and the borrower defaults, it is certain that the collateral will suffice to recover the debt. Given the borrowing constraint $l_{i,t+1}/h_{i,t+1} \leq \overline{\phi}_t p_t$, to maximize profit—which absent risk-shifting means earning zero profit—the LTV cap $\overline{\phi}_t$ must satisfy

$$\Pr\left[\overline{\phi}_t \leq \min\left\{\frac{p_{t+1}}{p_t}, \frac{E[p_{t+2}|\widetilde{I}^L_{t+1}, r_{i,t+1} < 1]}{Rp_t}\right\}|\widetilde{I}^L_t\right] = 1. \tag{16}$$

Absence of risk-shifing will depend crucially on the upper bound $\overline{\phi}$, which will limit price growth in the boom. If $\overline{\phi}$ is below a threshold—which we will derive—lenders will avoid credit losses even on loans made right before the crash, at $t_c - 1$. Most borrowers will spend these borrowed funds buying risky shares at time t_c, only to realize as soon as they observe the price p_{t_c} that the bubble has burst. Since p_{t_c+1} will fall to equal fundamental value, lenders will choose $\overline{\phi}_{t_c} < \overline{\phi}$, i.e., (14) will not bind at time t_c. From period $t_c + 1$ until $t_{pay} - 1$, (14) will bind again.

It remains to discuss loans made at time at $t_{pay} - 1$. These loans are settled either when shares are sold in the asset market at t_{pay}, or when shares are redeemed for dividends in the debt refinancing stage at t_{pay}. Overall, lender profit is given by

$$\pi_{t_{pay}}(l_{i,t_{pay}}) = \min\left\{p_{t_{pay}}, \frac{l_{i,t_{pay}}}{h_{i,t_{pay}}}\right\} s_{i,t_{pay}} + \min\left\{d_{t_{pay}}, \frac{l_{i,t_{pay}}}{h_{i,t_{pay}}}\right\}\left(h_{i,t_{pay}} - s_{i,t_{pay}}\right) - l_{i,t_{pay}}, \tag{17}$$

This profit is zero if $p_{t_{pay}}$ and $d_{t_{pay}}$ exceed $l_{i,t_{pay}}/h_{i,t_{pay}}$.

Having concluded the exposition of the environment, we summarize the within-period timing of shocks, as well as choices by investors and lenders in Figure 2.

3 Equilibrium

The equilibrium concept is Perfect Bayesian Equilibrium (PBE), consisting of mutually consistent strategies and beliefs. Investor i's strategy is a plan describing for all t, asset market choices $a_{i,t} = \{m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t}\}$ given $(b_{i,t}, h_{i,t}, l_{i,t})$ and $I_{i,t}$, and debt refinancing choices $\widetilde{a}_{i,t} = \{r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1}\}$ given $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t})$ and $\widetilde{I}_{i,t}$. Investor i's beliefs $\mu_{i,t}$ and $\widetilde{\mu}_{i,t}$ are probability distributions over values of t_0, updated using Bayes' rule as prices are observed. The list of equilibrium objects also includes lenders' LTV limits $\overline{\phi}_t$, and lender beliefs $\widetilde{\mu}^L_t$. Finally, prices p_t complete the list of equilibrium objects.

Investors' beliefs are determined as follows. In the asset market, given $I_{i,t}$, the set of possible values of t_0 is *supp*$_{i,t}(t_0)$. As noted above, given signal $\nu(i)$, investor i has support *supp*$_{i,t}(t_0) = \{\max\{\nu(i)-(N-1), 0\}, \ldots, \nu(i)\}$. Moreover, if trading strategies depend on signals, agents continue to discard values from *supp*$_{i,t}(t_0)$ as they observe prices. Among the values in *supp*$_{i,t}(t_0)$, probabilities are distributed according to Bayes' rule as follows:

$$\mu_{i,t}(t_0) = \frac{\psi(t_0)}{\sum\limits_{\tau_0 \in supp_{i,t}(t_0)} \psi(\tau_0)}, \tag{18}$$

with the probability function ψ given by (1). Similarly, at the debt refinancing stage, $\widetilde{supp}_{i,t}(t_0)$ denotes the set of possible values of t_0 given $\widetilde{I}_{i,t}$, and $\widetilde{\mu}_{i,t}(t_0)$ assigns probabilities as in (18), summing over $\tau_0 \in \widetilde{supp}_{i,t}(t_0)$ in the denominator. Lenders' beliefs $\widetilde{\mu}_t^L$ are similarly constructed, with the set of possible values of t_0 given by $\widetilde{supp}_t^L(t_0)$.

Given $(b_{i,t}, h_{i,t}, l_{i,t})$ and $I_{i,t}$, investor i's asset market choices $a_{i,t}$ are best response to other agents' strategies. More precisely, $a_{i,t} = (m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t})$ solves

$$V_{i,t}(b_{i,t}, h_{i,t}, l_{i,t}) = \max_{(m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t})} E\left[c_{i,t} + \widetilde{V}_{i,t}(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t}) | I_{i,t}\right]$$

subject to

$$s_{i,t} \in [0, h_{i,t}], m_{i,t} \in [0, b_{i,t}], \xi_{i,t} \in [0,1], (4), (5), (6), (7), \text{ if } t \le t_{pay}$$

$$s_{i,t} = 0, m_{i,t} = 0, \xi_{i,t} \in [0,1], (4), \widetilde{l}_{i,t} = 0, (6), (7), \text{ if } t > t_{pay}.$$

At the debt refinancing stage, given $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t})$, $\widetilde{I}_{i,t}$ and the lending cap $\overline{\phi}_t$, $\widetilde{a}_{i,t} = (r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})$ solves

$$\widetilde{V}_{i,t}(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t}) = \max_{(r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})} \delta_{i,t} E\left[V_{i,t+1}(b_{i,t+1}, h_{i,t+1}, l_{i,t+1}) | \widetilde{I}_{i,t}\right]$$

subject to

$$b_{i,t+1} \ge 0, r_{i,t} \in [0, \overline{r}_{i,t}], (12), (9), (11) \text{ and } (10) \text{ if } t < t_{pay}$$

$$b_{i,t+1} \ge 0, r_{i,t} \in [0, \overline{r}_{i,t}], \overline{r}_{i,t} = 1, (13), h_{i,t+1} = \widetilde{h}_{i,t} \text{ and } l_{i,t+1} = 0 \text{ if } t = t_{pay}$$

$$b_{i,t+1} \ge 0, r_{i,t} \in [0, \overline{r}_{i,t}], \overline{r}_{i,t} = 1, b_{i,t+1} = R(\widetilde{b}_{i,t} + y_t), h_{i,t+1} = \widetilde{h}_{i,t} \text{ and } l_{i,t+1} = 0 \text{ if } t > t_{pay}.$$

Equilibrium also requires that $\overline{\phi}_t$ maximizes (15) subject to (14). Finally, prices must satisfy $p_t = M_t/S_t$ for $t \le t_{pay}$ and $p_t = 0$ for $t > t_{pay}$.

4 Bubbles without risk shifting

We focus on situations in which the LTV cap $\overline{\phi}$ is low enough to avoid risk-shifting. In this case, even the largest bubble and debt that can arise in equilibrium are low enough to keep borrowers 'above water' when the bubble bursts. Since the unlucky investors who buy at time t_c have positive equity after the crash, the collateral seized by lenders in the event of default is valuable enough to recover principal and interest on outstanding loans. Absence of credit losses allows us to greatly simplify the credit market. Since loans are repaid for any t_0, the probabilites that lenders assign to different values of t_0 are irrelevant, and the competitive interest rate function is a constant R for all LTV ratios offered. Investors' debt refinancing plans are also simple—borrow as much as possible, i.e., $l_{i,t+1} = \overline{\phi}_t p_t h_{i,t+1}$ at all times. The price dynamics that arise in this case converge to those in the growing-endowment environment of DM, allowing us to apply bubble characterization results subject to only minor modifications. Our model thus provides a parsimonious, credit-based justification for the growth in investable funds that is exogenous in previous work.

4.1 Strategies

To find equilibria, we follow a guess-and-verify procedure. We conjecture a strategy profile for investors, and lending caps for lenders, and then verify that the conjectured plans are optimal given the beliefs and prices that they imply.

We begin by conjecturing that lenders set the following LTV bounds. For all $t < t_{pay}$, except for $t = t_c$, they offer the highest allowed $\overline{\phi}_t = \overline{\phi}$. At time t_c, after seeing the price p_{t_c}, it becomes common knowledge that $p_{t_c+1} = \overline{F} R^{t_c+1}$, where $\overline{F} = p_{t_0}/R^{t_0}$ is also common knowledge at this point known. If $\overline{\phi} p_{t_c} > \overline{F} R^{t_c}$, lending $\overline{\phi} p_{t_c}$ against the value of a share worth $\overline{F} R^{t_c}$ would result in losses. In that case, they adjust the maximum LTV ratio $\overline{\phi}_{t_c}$ downward to less than $\overline{\phi}$. Once the price crashes at $t_c + 1$, $\overline{\phi}_{t_c+1}$ increases to $\overline{\phi}$, although loans will be smaller than at time t_c because of lower collateral values. Finally, $\overline{\phi}_{t_{pay}} = 0$ since the price p_t is zero for all $t > t_{pay}$. In sum,

$$\overline{\phi}_t = \begin{cases} \overline{\phi} & \text{if } t \neq t_c \text{ and } t < t_{pay} \\ \min\{\overline{\phi}, \overline{F}R^{t_c}/p_{t_c}\} & \text{if } t = t_c \\ 0 & \text{if } t \geq t_{pay}. \end{cases} \tag{19}$$

Let us next turn to investors. Impatient investors liquidate all their assets to consume $c_{i,t} = b_{i,t} + p_t h_{i,t} - l_{i,t}$ and leave the asset market with $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, \widetilde{l}_{i,t}) = (0,0,0)$. At the debt refinancing stage, they simply collect endowments. Patient investors, as in AB and DM, follow trigger strategies. Type-$\nu(i)$ investors plan to ride the bubble for τ^* periods and sell at time $\nu(i) + \tau^*$, unless the bubble bursts before then. We also assume that patient investors do not consume, and that pre-boom and post-crash, they simply bid the necessary amount to keep price equal to fundamental value. More precisely, the strategy of investor i $\{a_{i,t}, \widetilde{a}_{i,t}\}$ is as follows:

(i) If impatient, $\delta_{i,t} = 0$:

$$a_{i,t} = (m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t}) = \begin{cases} (0, h_{i,t}, 0, 1) \text{ if } t \leq t_{pay}. \\ (0,0,0,1) \text{ if } t > t_{pay}. \end{cases} \tag{20a}$$

$$\widetilde{a}_{i,t} = (r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1}) = (1, y_t R, 0, 0). \tag{20b}$$

(ii) If patient, $\delta_{i,t} = 1/R$, asset market choices are given by $\widetilde{a}_{i,t} = (r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})$

$$a_{i,t} = (m_{i,t}, s_{i,t}, \widetilde{l}_{i,t}, \xi_{i,t}) = \begin{cases} \left(\frac{\theta}{1-\theta}\frac{R^t}{B_t} b_{it}, 0, l_{i,t}, 0\right) & \text{if } t < 0 \\ (b_{i,t} + y_t, 0, l_{i,t}, 0) & \text{if } 0 \leq t < \min\{\nu(i) + \tau^*, t_c + 1\} \\ (0, h_{i,t}, 0, 0) & \text{if } t = \nu(i) + \tau^* < t_c + 1 \\ \left(\frac{\theta}{1-\theta}\frac{\overline{F}R^t}{B_t} b_{it}, 0, l_{i,t}, 0\right) & \text{if } t_c + 1 \leq t \leq t_{pay} \\ (0,0,0,0) & \text{if } t > t_{pay} \end{cases} \tag{20c}$$

And debt refinancing choices $\widetilde{a}_{i,t} = (r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})$ are given by

$h_{i,t+1}$ and $l_{i,t+1}$ are respectively given by $\widetilde{h}_{i,t} - (1 - r_{i,t})(h_{i,t} - s_{i,t})$ and $\overline{\phi}_t p_t h_{i,t+1}$, for all t, where the repayment fraction $r_{i,t}$ is given by

$$r_{i,t} = \begin{cases} 1 & \text{if } t \neq t_c \\ \eta \overline{r}_{i,t_c} & \text{if } t = t_c, \end{cases} \tag{4}$$

and $\eta \in (0,1)$ is small enough to ensure that agents have enough funds to bid prices up to expected dividend after t_c. Safe savings $b_{i,t+1}$ are given by

$$b_{i,t+1} = \begin{cases} R\left[\widetilde{b}_{i,t} + y_t + (1-\overline{r}_{i,t})(h_{i,t} - s_{i,t})e_{i,t} - \overline{r}_{i,t}\widetilde{l}_{i,t}\right] + l_{i,t+1} & \text{if } t < t_{pay} \\ R(\widetilde{b}_{i,t} + y_t + d_t\widetilde{h}_{i,t} - l_{i,t}) & \text{if } t = t_{pay} \\ R(\widetilde{b}_{i,t} + y_t) & \text{if } t > t_{pay}. \end{cases} \tag{5}$$

And for all t,

$$h_{i,t+1} = \widetilde{h}_{i,t} - (1 - r_{i,t})(h_{i,t} - s_{i,t}) \tag{6}$$

and

$$l_{i,t+1} = \overline{\phi}_t p_t h_{i,t+1}. \tag{7}$$

Given strategies, lender and investor beliefs evolve as follows. Lenders—regardless of whether they observe signals—know that t_0 cannot be negative, and that the crash will happen τ^* periods after t_0. If they only observe prices, they can infer that the support of t_0 is infinite, with no upper bound, and a lower bound given by $\max\{0, t - \tau^* + 1\}$. Once they observe the crash, they learn that $t_0 = t_c - \tau^*$. In sum,

$$\widetilde{supp}_t^L(t_0|p^t) = \begin{cases} \{\tau_0 \in \mathbb{Z} | \tau_0 \geq \max\{0, t - \tau^* + 1\}\} & \text{if } 0 \leq t < t_c \\ \{t_c - \tau^*\} & \text{if } t \geq t_c. \end{cases} \tag{21}$$

The actual support of lenders may be more refined than this, if they observe signals. However, in equilibria without risk shifting, this will suffice.

Investors, in addition to observing whether the bubble has burst, also observe signals. Before the crash, investors know that t_0 must be nonnegative and, since the crash has not happened as of time t, greater than $t - \tau^*$. The signal $v(i)$ allows investor i to narrow the support of t_0 between a lower bound $\nu(i) - (N-1)$ and an upper bound $\nu(i)$. In sum, given strategies, the support of t_0 for investor i at the debt refinancing stage of period t is given by

$$\widetilde{supp}_{i,t}(t_0) = \begin{cases} \tau_0 \in \{\max\{0, t-\tau^*+1, \nu(i)-(N-1)\}, ..., \nu(i)\} & \text{if } 0 \le t < t_c \\ \{t_c - \tau^*\} & \text{if } t \ge t_c. \end{cases} \tag{22}$$

To understand this expression, it is most illustrative to consider an investor i with $\nu(i) \ge N-1$. Once she observes the price $p_{\nu(i)-(N-1)+\tau^*}$, she learns that $t_0 = \nu(i)-(N-1)$ if the bubble bursts, or that $t_0 > \nu(i) - (N-1)$ otherwise. Every period after that, the bubble either bursts, revealing the value of t_0, or continues, in which case she drops the lowest value from the support of t_0. As the bubble continues and values are discarded, the probability that the crash happens in the next period increases. In fact, if t_0 happens to be $\nu(i)$, investor i learns the true value of t_0 when she sees that the bubble does not burst at $t = \nu(i) - 1 + \tau^*$, which allows her to rule out the possibility that $t_0 = \nu(i) - 1$ and be certain that $t_0 = \nu(i)$.

Finally, since no new prices or signals are observed between the debt refinancing stage of period t and the asset market stage of period $t+1$, the support of t_0 at the asset market stage is $supp_{i,0}(t_0) = \{0, 1, 2, ...\}$ for $t = 0$, and for $t \ge 1$,

$$supp_{i,t+1}(t_0) = \widetilde{supp}_{i,t}(t_0). \tag{23}$$

4.2 Equilibrium boom and bust

4.2.1 Pre-boom

Before the boom, p_t simply equals R^t. As noted above, dividends are expected to arrive after time 0, and thus $d_t = 0$ while $t < 0$. Every period $t < 0$, a mass θ of impatient investors sell $S_t = \theta$ shares to a mass $1-\theta$ of patient investors, who on aggregate bid $M_t = \theta R^t$. It must be the case that patient investors wield enough funds to bid this amount, and thus

$$\theta R^t \le (1-\theta) B_t \tag{24}$$

must hold for all $t \le 0$. At $t = 0$, the inequality must be strict, so that investors can afford to bid more than R^t when the time-0 shock hits.

Since shocks are i.i.d., the average seller starts the period with the same portfolio as the average buyer. Sellers consume a fraction θ of the sum of aggregate initial safe assets B_t plus revenue from selling shares net of debt repayment $\theta[R^t(1-\overline{\phi}/R)]$. Since impatient sellers are the only consumers, aggregate consumption is given by

$$C_t = \theta\left[B_t + R^t(1-\overline{\phi}/R)\right].$$

At the debt refinancing stage, each seller collects her endowment y_t, which she saves. Buyers, in addition to collecting endowments, repay their share of last period's debt $(1-\theta)L_t$ and pledge the full supply $H_t = 1$ of the risky asset to borrow L_{t+1}. Aggregating refinancing constraints across investors yields

$$\frac{B_{t+1}}{R} + (1-\theta)L_t = y_t + [(1-\theta)\,B_t - M_t] + \frac{L_{t+1}}{R}.$$

Substituting $M_t = \theta R^t$ and $L_t = \overline{\phi}R^{t-1}$, and rearranging terms, we find that B_{t+1} is given by

$$B_{t+1} = R[y_t + (1-\theta)B_t] - \theta R^t(R-\overline{\phi}). \tag{25}$$

If, for instance, endowments grow at the risk-free rate $y_t = \overline{y}R^t$ while $t<0$, (25) becomes

$$B_{t+1} = R(1-\theta)B_t + R^{t+1}\left[\overline{y} - \theta(1-\overline{\phi}/R)\right].$$

If $\lim_{s\to\infty} B_{-s}$ is finite and $(1-\theta)R < 1 = 0$, this expression simplifies to $B_t = R^t\left[\overline{y} - \theta(1-\overline{\phi}/R)\right]$. Then, (24) holds with strict inequality as long as $\overline{y} > \theta\left[(1-\theta)^{-1} + 1 - \overline{\phi}/R\right]$.

4.2.2 Innovation and boom

At time 0, following equilibrium strategies, patient investors begin to invest as much as they can into the risky asset. They bid $M_0 = (1-\theta)B_0$ for the $S_0 = \theta$ shares sold by impatient investors, giving rise to a price

$$p_0 = \tfrac{1-\theta}{\theta}B_0.$$

Since inequality (24) is assumed to hold strictly at time 0, p_0 must be greater than 1. Sellers pay back their debt and consume $C_0 = \theta[B_0 + p_0 - \overline{\phi}/R]$. Investors carry no safe assets at the end of the asset market, and thus, $\widetilde{B}_0 = 0$. At the debt refinancing stage, buyers and sellers collect y_t. Moreover, buyers repay their debt $(1-\theta)L_0 = (1-\theta)\overline{\phi}/R$, and—pledging the full $H_t = 1$—take loans $L_1 = \overline{\phi}p_0$. Overall, aggregate safe balances starting $t = 1$ are given by $B_1 = Ry_0 + \overline{\phi}[p_0 - (1-\theta)]$. This pattern where impatient investors sell θ shares to patient investors who bid as much as they can continues while $t \in \{1, ..., t_c - 1\}$, giving rise to prices

$$p_{t+1} = \frac{1-\theta}{\theta}\left[L_{t+1} + R(y_t - (1-\theta)\, L_t)\right]. \tag{26}$$

Substituting $L_t = \overline{\phi}p_{t-1}$ into (26) and dividing by p_t, we obtain

$$\frac{p_{t+1}}{p_t} = \frac{1-\theta}{\theta}\left[\overline{\phi} + R\frac{y_t}{p_t} - (1-\theta)R\overline{\phi}\frac{p_{t-1}}{p_t}\right].$$

We next define $G_t \equiv p_t/p_{t-1}$, and conjecture that this process generates price growth faster than the risk-free rate, i.e., that $G_t > R$. Under this conjecture, and the assumption that endowments do not grow faster than R, y_t/p_t approaches 0 as t grows, and the above converges to

$$G_{t+1} = \frac{1-\theta}{\theta}\overline{\phi}\left[1 - \frac{(1-\theta)R}{G_t}\right]. \tag{27}$$

As illustrated in Figure 3, depending on parameters, this function may have zero, one, or two points of intersection with the 45 degree line. Algebraically, if there is a constant growth rate $G_{t+1} = G_t = G$, it must solve the quadratic equation

$$G^2 - \tfrac{1-\theta}{\theta}\overline{\phi}G + \tfrac{(1-\theta)^2}{\theta}\overline{\phi}R = 0, \tag{28}$$

which has roots given by

$$G = \tfrac{1-\theta}{2\theta}\overline{\phi}\left[1 \pm \sqrt{1 - 4\theta R/\overline{\phi}}\right]. \tag{29}$$

If $\overline{\phi} > 4\theta R$, there are two roots, of which only $G = \overline{\phi}\,(1-\theta)\,/(2\theta)\left[1 + \sqrt{1 - 4\theta R/\overline{\phi}}\right]$ is stable.

If $\overline{\phi} = 4\theta R$, there is only one root $G = \overline{\phi}(1-\theta)/(2\theta)$. Given the dynamics shown in Figure 3, if $\overline{\phi} \geq 4\theta R$ and initial growth is high enough, the price growth rate p_{t+1}/p_t over the course of the boom converges to this constant G, which is increasing in $\overline{\phi}$ and decreasing in θ and R. Whenever G exists and $\overline{\phi} < 1$—which as we will see must be the case in equilibria without risk-shifting—the conjectured inequality $G > R$ will be satisfied.[11] In the case where $4\theta R > \overline{\phi}$, G does not exist. Simulation results depicted in Figure 4 show rapidly growing prices and debt for a number of periods, followed by declining and eventually negative growth rates. With trigger strategies, the fact that price growth will reverse at a date that is common knowledge is problematic and leads to unraveling of the equilibrium.[12] For this reason, in this paper we focus on the case where $4\theta R \leq \overline{\phi}$, so that a constant rate G does exist. In this case, our price-credit feedback loop endogenously generates growing availability of investable funds, much like the growing endowments environment in DM.

In sum, if $\overline{\phi} \geq 4\theta R$, for high-enough realizations of t_0, our model converges to the growing-endowments DM case. We summarize these results in the following Lemma.

Lemma 1 *Assume that $\overline{\phi} \geq 4\theta R$ and that $2B_0 > \overline{\phi}(1+\sqrt{1-4\theta R/\overline{\phi}})$. Then, during the boom phase the price growth rate p_{t+1}/p_t converges to a constant G given by $G = (1-\theta)\overline{\phi}\left[1+\sqrt{1-4\theta R/\overline{\phi}}\right]/2\theta$. Moreover, if $2B_0 > \overline{\phi}(1+\sqrt{1+4\theta R/\overline{\phi}})$, the price growth rate is decreasing over time.*

Proof. *A constant growth rate G is a fixed point of equation (27). As we can see in (29), fixed points exist if and only if $\overline{\phi} \geq 4\theta R$. When a fixed point exists, the price growth rate converges to that constant level if the initial growth rate $G_0 = p_0/p_{-1}$ is sufficiently high. A sufficient condition for this—regardless of endowments—is $2B_0 > \overline{\phi}(1+\sqrt{1-4\theta R/\overline{\phi}})$. If $2B_0 > \overline{\phi}(1+\sqrt{1+4\theta R/\overline{\phi}})$, (27) implies that the price growth rate not only converges to a constant, but decreases over time.* ■

4.2.3 Crash and aftermath

At time t_c impatient sellers are joined by patient sellers of type $\nu(i) = t_0$, who sell in (correct) anticipation of the crash. Relative to earlier boom periods, the addition of these sellers raises

[11] To see this, note that if $\overline{\phi} = 4\theta R$, $G = 2(1-\theta)R$. In this case, $G > R$ is equivalent to $\theta < 1/2$, which must be the case if $4\theta R = \overline{\phi} \leq 1$. If $\overline{\phi} > 4\theta R$, the difference between G and R is even greater.

[12] Doblas-Madrid (2014) shows how trigger strategy perfect Bayesian equilibria unravel in this case and proposes a solution, which is to make τ^* a decreasing function of $\nu(i)$. In fact, using this trick, one can consider a finite-horizon model.

S_{t_c}, the number of shares for sale, from θ to $\theta + (1-\theta)/N$, and lowers the number of buyers from $1-\theta$ to $(1-\theta)(1-1/N)$. The price at time t_c is given by

$$p_{t_c} = \frac{(1-\theta)(1-\frac{1}{N})}{\theta + \frac{1-\theta}{N}} \left[\overline{\phi} p_{t_c-1} - (1-\theta) R \overline{\phi} p_{t_c-2} + y_{t_c} \right].$$

Dividing by p_{t_c-1}, approximating $y_{t_c}/p_{t_c-1} \approx 0$, and rearranging terms we obtain

$$\begin{aligned} \frac{p_{t_c}}{p_{t_c-1}} &= \frac{\frac{(1-\theta)(1-\frac{1}{N})}{\theta+\frac{1-\theta}{N}}}{(1-\theta)/\theta} \left[\frac{\overline{\phi} p_{t_c-1} - (1-\theta) R \overline{\phi} p_{t_c-2}}{\overline{\phi} p_{t_c-2} - (1-\theta) R \overline{\phi} p_{t_c-3} + y_{t_c-1}} + \frac{y_{t_c}}{p_{t_c-1}} \right] \\ &\approx \frac{\theta(N-1)}{1+\theta(N-1)} \left[\frac{\overline{\phi} p_{t_c-1} - (1-\theta) R \overline{\phi} p_{t_c-2}}{\overline{\phi} p_{t_c-2} - (1-\theta) R \overline{\phi} p_{t_c-3} + y_{t_c-1}} \right]. \end{aligned}$$

Further rearranging and approximating $y_{t_c}/p_{t_c-3} \approx 0$, $p_{t_c-1} \approx p_{t_c-2} G \approx p_{t_c-3} G^2$, we can further simplify the above to obtain

$$\begin{aligned} \frac{p_{t_c}}{p_{t_c-1}} &\approx \frac{\theta(N-1)}{1+\theta(N-1)} \left[\frac{\overline{\phi} \frac{p_{t_c-1}}{p_{t_c-3}} - (1-\theta) R \overline{\phi} \frac{p_{t_c-2}}{p_{t_c-3}}}{\overline{\phi} \frac{p_{t_c-2}}{p_{t_c-3}} - (1-\theta) R \overline{\phi} + \frac{y_{t_c-1}}{p_{t_c-3}}} \right] \\ &\approx \frac{\theta(N-1)}{1+\theta(N-1)} G. \end{aligned} \tag{30}$$

In addition to revealing information, the reduction in the price growth rate from G at time $t_c - 1$ to $G\{\theta(N-1)/[1+\theta(N-1)]\}$ at time t_c has an effect on sellers' revenue. This loss of revenue is decreasing in θ and N, tending to zero as N becomes large.[13]

Let us keep track of investors' portfolios over the course of period t_c. Individual portfolios vary depending on individual histories, but these histories are independent of signal $\nu(i)$ or current discount factor δ_{i,t_c}. Thus, patient and impatient investors of all types start period t_c holding on average, safe balances $B_{t_c} = R y_{t_c-1} + \overline{\phi} p_{t_{c-1}} - (1-\theta)\overline{\phi} p_{t_c-2}$, risky shares $H_{t_c} = 1$ and liability $L_{t_c} = \overline{\phi} p_{t_{c-1}}$. Impatient investors sell $S_{t_c} = \theta$ shares, repay their share of the debt $\theta \overline{\phi} p_{t_{c-1}}$ and consume $C_{t_c} = \theta[B_{t_c} + p_{t_c} - \overline{\phi} p_{t_c-1}]$, which we can rewrite as $\theta[R y_{t_c-1} + p_{t_c} - (1-\theta)\overline{\phi} p_{t_c-2}]$. The lucky mass $(1-\theta)/N$ of patient type-t_0 investors who sell do not consume. Instead, they save their initial safe assets plus sales proceeds net of debt. They are the only investors holding safe assets after the asset market, carrying the full amount $\widetilde{B}_{t_c} = (1-\theta)/N[R y_{t_c-1} + p_{t_c} - (1-\theta)\overline{\phi} p_{t_c-2}]$.

[13] Such a large-N assumption is in place for most of the analysis in D-M.

Buyers spend all of their safe assets—amounting to $(1-1/N)(1-\theta)B_{t_c}$ in total—buying $\theta+(1-\theta)/N$ shares and leave the asset market with the full supply $H_t=1$.

Since p_{t_c} pricks the bubble and reveals $t_0 = t_c - \tau^*$, by the time agents refinance debt in period t_c, the uncertainty created by private signals has been resolved. Lenders and investors now expect the risky asset to trade at fundamental value and appreciate at the risk-free rate. Specifically,

$$p_t = p_{t_0}R^{t-t_0} = \overline{F}R^t \text{ for all } t = t_{c+1}, ..., t_{pay}. \tag{31}$$

Anticipating the price drop that will occur between periods t_c and t_c+1, lenders tighten credit by setting $\overline{\phi}_{t_c} = p_{t_{c+1}}/p_{t_c} < \overline{\phi}$. This implies that $\overline{r}_{i,t_c}$, as given by (12), may fall below 1, forcing some investors to default. Investors who sold in the asset market are not affected by this this credit crunch, since they already liquidated their shares. All they do at the debt refinancing stage is collect endowments. It is buyers, and in particular buyers who have been patient for a long time, who are most affected by the crunch. Since lenders immediately rebate any remaining equity, and since both assets are perfect substitutes after the crash, investors are indifferent between repaying a debt or surrendering the collateral and receiving the remaining equity. In other words, investor i is indifferent between any repayment fraction r_{i,t_c} between 0 and $\overline{r}_{i,t_c}$. We therefore focus on the case in which they set a repayment fraction $r_{i,t_c} = \eta\overline{r}_{i,t_c}$ that is positive, but small enough to ensure that aggregate safe funds after the crash suffice to buy shares at $\overline{F}R^t$. Lenders seize $(1-r_{i,t_c})(h_{i,t_c}-s_{i,t_c})$ risky shares from investor i, and rebate $\overline{F}R^{t_c+1} - \overline{\phi}p_{t_{c-1}}$, a nonnegative amount since we are considering equilibria without risk-shifting. Investor i retains $h_{i,t_c+1} = m_{i,t_c}/p_{t_c} + r_{i,t_c}(h_{i,t_c}-s_{i,t_c})$ shares in her possession, which she pledges as collateral for new loans $l_{i,t_{c+1}} = \overline{\phi}_{t_c}p_{t_c}h_{i,t_c+1} = p_{t_{c+1}}h_{i,t_c+1}$.

At t_c+1, given our assumption that lenders hold repossessed shares until maturity, the pattern of activity in the asset market reverts to the one before the boom. The number of shares for sale is $S_{t_c+1} = \theta H_{t_c+1}$, a fraction θ of shares in the hands of investors H_{t_c+1}. Buyers bid a fraction $\theta\overline{F}R^tH_{t_c+1}/[(1-\theta)B_t]$ of $b_{i,t}$ every period. As in the pre-boom phase, it must be that investors can obtain enough resources to bid $\overline{F}R^t$ per share. That is, it must be that

$$(1-\theta)B_{t_c+1} \geq p_{t_c+1}S_{t_c+1}.$$

This inequality can always be guaranteed by choosing a low enough η and thus restricting the number of shares for sale. In fact, choosing a low enough η, we can ensure that this holds for all periods after t_c until t_{pay}.

4.3 Characterizing bubble duration

To characterize τ^* in equilibrium, we must verify that investors are willing to play equilibrium strategies. For expositional convenience, we focus on the case where the realization of t_0 is high enough that we can approximate $p_{t+1} = p_t G$ for all $t \in \{t_0, ..., t_c\}$. (Our reasoning—as we will argue shortly—will also apply to lower realizations of t_0.) Since impatient investors have zero discount factors, it is straightforward that they are willing to sell and consume. What is not trivial is the trade-off faced by patient investors during the boom. They must be willing to fully invest into the risky asset, planning to leave the market at time $\nu(i) + \tau^*$, unless the bubble bursts before. The size and duration of equilibrium bubbles depends crucially on the willingness of patient investors to continue investing despite the crash risk. To be precise, in equilibrium patient investors must be willing to: *(i)* Sell when the strategy dictates that they should sell and *(ii)* Buy when the strategy dictates that they should buy. Part *(i)* holds for any $\tau^* \geq 1$, because an individual investor will choose to sell if she knows that other investors of her same type are selling. That is, if investor i knows that others of her same type $\nu(i)$ are selling at t, she knows that the price will certainly reveal the sales, and the bubble will burst in the next period.[14] Part *(ii)*, however, is not obvious, since it is not trivial that an investor will always buy as dictated by equilibrium strategies, especially as crash risk grows.

To understand buyers' choices, consider a type-$\nu(i)$ investor. Her beliefs about possible values of t_0 given by (22), which combines information from signal and prices. The signal $\nu(i)$ implies that the support of t_0 is the set $\{\max\{0, \nu(i) - (N-1)\}, \ldots, \nu(i)\}$. The investor further eliminates values from her support of t_0 as she observes that the bubble does not burst from period $\max\{0, \nu(i) - (N-1)\} + \tau^*$ onward. (If t_0 had been equal to $\max\{0, \nu(i) - (N-1)\}$, the bubble would have burst at $\max\{0, \nu(i) - (N-1)\} + \tau^*$.) If she happens to be "first in line", i.e., if $\nu(i) = t_0$ she unequivocally learns that $t_0 = \nu(i)$ when she observes that the price at time $\nu(i) - 1 + \tau^*$ continues to grow at the rate G. In equilibrium, she must be willing to buy until

[14] See D-M for a discussion of multiplicity and the role played by discrete types and noise.

the scheduled selling date $\nu(i) + \tau^*$, despite the fact that the probability of a crash (conditional on the bubble not having burst yet) increases as earlier realizations of t_0 are discarded. Crash risk is highest at the start of period $\nu(i) - 1 + \tau^*$, just one period before the investor is supposed to sell. It is at this point, with the support of t_0 containing only two values $\{\nu(i) - 1, \nu(i)\}$, that preemptive selling is most tempting. If investor i is willing to buy at this time, she is also willing to buy at all earlier times.

At time $\nu(i)-1+\tau^*$, the buy-versus-sell tradeoff works as follows. If investor i sells, she avoids the crash and sells at a price which varies slightly depending on t_0. If $t_0 = \nu(i) - 1$ impatient investors and type-$\nu(i) - 1$ investors will sell and the price growth rate $p_{\nu(i)-1+\tau^*}/p_{\nu(i)-2+\tau^*}$ will be lower than G by a factor $\theta(N-1)/[1+\theta(N-1)]$. If $\nu(i)-1$ only impatient investors will sell and the price growth rate will be G. If investor i buys, she gets caught in the crash if $t_0 = \nu(i) - 1$, and otherwise rides the bubble for one more period, selling at the price $\theta(N-1)/[1+\theta(N-1)]p_{t_0}G^{\tau^*}$. With the support of t_0 given by $\{\nu(i)-1, \nu(i)\}$, equilibrium beliefs (18) assign probabilities $1/(1+\lambda)$ and $\lambda/(1+\lambda)$ to $\nu(i) - 1$ and $\nu(i)$, respectively. Therefore, buying is optimal if

$$\left(\frac{\theta(N-1)}{1+\theta(N-1)} + \lambda\right) p_{t_0} G^{\tau^*-1} \leq \frac{p_{t_0}}{G} R^{\tau^*} + \lambda \frac{\theta(N-1)}{1+\theta(N-1)} p_{t_0} \frac{G^{\tau^*}}{R}$$

Dividing through by $p_{t_0}G^{\tau^*-1}$ and rearranging terms, we can rewrite the above inequality as

$$\frac{\theta(N-1)}{1+\theta(N-1)} + \lambda \leq \left(\frac{G}{R}\right)^{-\tau^*} + \lambda \frac{\theta(N-1)}{1+\theta(N-1)} \frac{G}{R} \tag{32}$$

This expression is easiest to interpret when N is large enough that $\theta(N-1)/[1+\theta(N-1)]$ is close to one, in which case it simplifies to

$$1 \leq \frac{1}{1+\lambda} \left(\frac{G}{R}\right)^{-\tau^*} + \frac{\lambda}{1+\lambda} \frac{G}{R}. \tag{33}$$

Whether it is better to buy as dictated by the strategy or to deviate by selling preemptively can be reduced to a simple calculation. Relative to selling, buying yields crash losses with probability $1/(1+\lambda)$ and one more period of appreciation with probability $\lambda/(1+\lambda)$. Thus, the higher λ and G/R, the greater the bubble duration τ^* that can be supported in equilibrium. The interpretation of the more general inequality (32) is in essence the same. As payoffs change continuously as functions of $\theta(N-1)/[1+\theta(N-1)]$, the inequality changes quantitatively, but the qualitative interpretation remains the same. In fact, we use (32) to characterize the set of values of τ^* that can be supported in equilibrium and summarize our findings in Proposition **2**.

Proposition 2 *Suppose that* $G = \overline{\phi}\,(1-\theta)\left[1+\sqrt{1-4\theta R/\overline{\phi}}\right]/(2\theta)$*,* $\overline{\phi} \geq 4\theta R$*, and* $2B_0 > \overline{\phi}(1+\sqrt{1-4\theta R/\overline{\phi}})$*. Moreover assume that there is no risk-shifting in equilibrium. Then: (a) If* $G/R < [1+\theta(N-1)]/[\theta(N-1)]+1/\lambda$*, equilibrium can be supported for any* τ^* *between* 0 *and* $-\ln(\{\theta(N-1)/[1+\theta(N-1)]\} + (1-\lambda\{\theta(N-1)/[1+\theta(N-1)]\}G/R))/\ln(G/R)$*.(b) If* $G/R \geq [1+\theta(N-1)]/[\theta(N-1)]+1/\lambda$*, any integer* $\tau^* \geq 0$*, can be supported in equilibrium.*

Proof. To verify that type-$\nu(i)$ investors are willing to sell at $\nu(i)+\,\tau^*$, note that for any positive τ^*, if other investors of type $\nu(i)$ are selling at $\nu(i)+\,\tau^*$, the price will reveal the sale and precipitate a crash. Any investor of type $\nu(i)$ will therefore be willing to sell. To verify that type-$\nu(i)$ investors are willing to buy as long as the boom continues and $t < \nu(i)+\,\tau^*$, we first consider the case in which $t_0+\tau^*$ is large. By Lemma 1, the price growth rate p_{t+1}/p_t converges to $G = \overline{\phi}\,(1-\theta)\left[1+\sqrt{1-4\theta R/\overline{\phi}}\right]/(2\theta)$ as t grows, and therefore, for all types $\nu(i) > 0$, willingness to buy is governed by (32). Therefore, the upper bound on τ^* given by $-\ln(\{\theta(N-1)/[1+\theta(N-1)]\} + (1-\lambda\{\theta(N-1)/[1+\theta(N-1)]\}G/R))/\ln(G/R)$ is derived directly by solving (32) for τ^*. In the event that $t_0 = 0$, agents of type $\nu(i) = 0$ are always willing to buy while $t < t_0+\tau^*$ since they know the value of t_0 as soon as they observe signals. Finally, although (32) is derived for the case where $t_0+\tau^*$ is large, willingness to buy is even stronger if $t_0+\,\tau^*$ is not large. By Lemma 1, if $2B_0 > \overline{\phi}(1+\sqrt{1-4\theta R/\overline{\phi}})$, price growth rates during the boom start out higher than G, gradually falling towards this constant over time. Q.E.D. ■

In equilibria with $\tau^* > N$, the model generates *strong bubbles* in the sense of Allen et al. (1993), where—for all $t > t_0+N$, all investors know with certainty that the risky asset is overvalued, and nevertheless continue to buy it (unless they become impatient). Up to period t_0+N, bubbles are weaker in the sense that some agents still believe that the boom may be fundamental.

A straightforward but important implication of Proposition 1 is the positive link between the maximum possible bubble duration τ^* and the LTV cap $\overline{\phi}$ and the negative link between τ^* and the interest rate R. These results, which are in line with macroprudential wisdom, follow directly from the fact that G is increasing in $\overline{\phi}$ and decreasing in R, while τ^* is increasing in G/R.

4.4 Loan-to-value limits and risk-shifting

The equilibrium analysis thus far takes absence of risk shifting as a given. We have maintained the assumption that, either through repayment or through collateral repossession in default, lenders fully recover principal and interest on all of their loans. In such an environment, the competitive lending rate is equal to the risk-free rate R and investors bear the full risk of their speculative activity.

In this Section, we show that, as long as θ and $\overline{\phi}$ are not too large, bubbly equilibria without risk shifting exist. While the derivation of the precise parameter thresholds is somewhat involved, the idea behind it is quite simple. At $t_c - 1$, lenders lend $\overline{\phi} p_{t_c-1}$ per unit of collateral, but this collateral is only worth $\overline{F} R^{t_c}$ by next period's refinancing stage. Risk shifting is avoided if (and only if) $\overline{F} R^{t_c}$ suffices to repay $\overline{\phi} p_{t_c-1}$. Equilibria without risk shifting exist if this restriction is compatible with condition (33) which determines bubble duration, and with the relationship between G and $\overline{\phi}$ arising from the interaction between price and debt in the boom. For tractability, we focus on the large-N case. The specific parameter restrictions and the derivation are provided in Proposition 2 and its proof.

Absence of credit losses thus requires that is thus necessary to verify that the defaults happening at time t_c do not generate credit losses. There may also be defaults at time $t_c + 1$, given that $\overline{\phi} p_{t_c+1}$ is less than $\overline{\phi}_{t_c} p_{t_c} = p_{t_c+1}$. However, since future collateral values are known with certainty when loans are made at time t_c, lenders perfectly foresee future prices and—although p_{t_c} allows them to make bigger loans—adjust $\overline{\phi}_{t_c}$ accordingly to avoid credit losses. In sum, to verify that condition (16) holds, we need to verify that

$$\overline{\phi} p_{t_c-1} \leq p_{t_0} R^{\tau^*}, \tag{34}$$

which, approximating $p_{t_c-1} = p_{t_0} G^{t_c-1}$, reduces to

$$\frac{\overline{\phi}}{G} \leq \left(\frac{G}{R}\right)^{-\tau^*}, \tag{35}$$

which related the lending limit $\overline{\phi}$ to the percentage price fall in the crash. If (35) holds with equality, $\overline{\phi}$ is as high as possible while still ruling out credit losses. In turn, the size of the price drop in the crash is a crucial input to the sell-verus-wait condition that limits how large τ^* can be. Combining (35) with the large-N version of the sell-or-wait condition (33) and rearranging terms yields

$$G\left[1-\lambda\left(\frac{G}{R}-1\right)\right] \leq \overline{\phi}. \tag{36}$$

If the conjectured strategies, lending policies and prices do indeed constitute an equilibrium, this no-risk-shifting condition relating G and $\overline{\phi}$ must hold. Moreover, as seen above, G must be derived from $\overline{\phi}$, θ, and R according to expression (28), which we can solve for $\overline{\phi}$ and rewrite as

$$\frac{\theta}{1-\theta}\frac{G^2}{G-(1-\theta)R} = \overline{\phi} \tag{37}$$

In Figure 6, we plot both (36) and (37) jointly in a $(G, \overline{\phi})$ diagram. For small enough values of θ, there are two points of intersection, but only the point with the higher value of G corresponds to the stable root $G = \overline{\phi}\,(1-\theta)\,/(2\theta)\left[1+\sqrt{1-4\theta R/\overline{\phi}}\right]$. If θ surpasses a certain threshold, there are no points of intersection. In that case, equilibria without risk-shifting do not exist. To solve for said points of intersection, we equate the left-hand side of (36) to that of (37), and—after some tedious but straightforward algebra—find that the two curves intersect at

$$G = R\left[1-\frac{\theta}{2}+\frac{1-2\theta\pm\sqrt{[1-\lambda\theta(1-\theta)]^2-4\theta(1-\theta)}}{2\lambda(1-\theta)}\right] \tag{38}$$

Substituting G into (36) or (37) we find the corresponding level of $\overline{\phi}$.

These results are summarized in Proposition 2.

Proposition 3 *Suppose that N is large, p_{t+1}/p_t is close to G for all $t \geq t_0$, and that $[1-\lambda\theta(1-\theta)]^2 \geq 4\theta(1-\theta)$. Then, there is a nonempty region of the parameter space in which bubbly equilibria without risk-shifting exist. Moreover, bubbles in this region can become arbitrarily large.*

Proof. *Of all loans made by lenders, those made at time $t_c - 1$ are most likely to become 'underwater' loans and generate credit losses. At $t_c - 1$, lenders extend loans amounting to $\overline{\phi}p_{t_c-1}$ per unit of collateral. Loans collateralized by units that are sold at t_c are easily repaid, since p_{t_c} is higher than $p_{t_{c-1}}$. However, the lower growth rate of the price reveals that the bubble has burst, lowering* *the value of the bubble to* $\overline{F}R^{t_c}$*. This value is the same, regardless of whether lenders sell the seized shares or hold them to collect the dividend. It is thus necessary to verify that the defaults happening at time* t_c *do not generate credit losses. There may not be credit losses for loans made after the crash, since future collateral values are known with certainty when loans are made and thus lenders adjust loan sizes. Specifically, at time* t_c*, although* p_{t_c} *allows them to make bigger loans, lenders adjust* $\overline{\phi}_{t_c}$ *accordingly to avoid credit losses. In sum, to verify that condition (16) holds, we need to verify that*

$$\overline{\phi} p_{t_c-1} \leq p_{t_0} R^{\tau^*}, \tag{39}$$

which, approximating $p_{t_c-1} = p_{t_0} G^{t_c-1}$*, reduces to*

$$\frac{\overline{\phi}}{G} \leq \left(\frac{G}{R}\right)^{-\tau^*}, \tag{40}$$

which related the lending limit $\overline{\phi}$ *to the percentage price fall in the crash. If (35) holds with equality,* $\overline{\phi}$ *is as high as possible while still ruling out credit losses. In turn, the size of the price drop in the crash is a crucial input to the sell-versus-wait condition that limits how large* τ^* *can be. Combining (35) with the large-*N *version of the sell-or-wait condition (33) and rearranging terms yields*

$$G\left[1 - \lambda\left(\frac{G}{R} - 1\right)\right] \leq \overline{\phi}. \tag{41}$$

If the conjectured strategies, lending policies and prices do indeed constitute an equilibrium, this no-risk-shifting condition relating G *and* $\overline{\phi}$ *must hold. Moreover, as seen above,* G *must be derived from* $\overline{\phi}$*,* θ*, and* R *according to expression (28), which we can solve for* $\overline{\phi}$ *and rewrite as*

$$\frac{\theta}{1-\theta}\frac{G^2}{G-(1-\theta)R} = \overline{\phi} \tag{42}$$

In Figure 6, we plot both (36) and (37) jointly in a $(G, \overline{\phi})$ *diagram. For small enough values of* θ*—specifically if* $[1 - \lambda\theta(1-\theta)]^2 - 4\theta(1-\theta)$*—there are two points of intersection, but only the point with the higher value of* G *corresponds to the stable root* $G = \overline{\phi}\,(1-\theta)\,/(2\theta)\left[1 + \sqrt{1 - 4\theta R/\overline{\phi}}\right]$*. If* θ *surpasses a certain threshold, there are no points of intersection. In that case, equilibria without risk-shifting do not exist. To solve for said points of intersection, we equate the left-hand side of*

(36) to that of (37), and—after some tedious but straightforward algebra—find that the two curves intersect at

$$G = R\left[1 - \frac{\theta}{2} + \frac{1 - 2\theta \pm \sqrt{[1 - \lambda\theta(1-\theta)]^2 - 4\theta(1-\theta)}}{2\lambda(1-\theta)}\right] \tag{43}$$

Substituting this expression for G into (36) or (37) we find the corresponding level of $\overline{\phi}$. Finally, it follows from (43) and (36), that by lowering θ—and $\overline{\phi}$ falling in order to ensure risk-shifting—one can always raise G and τ^*. Q.E.D. ■

4.5 Discussion and Extensions

Binding borrowing constraints are essential because they allow us to circumvent the impossibility results by Tirole (1982). As in DM, there are two reasons for this. First, investors know that if the bubble does not burst, it will grow fuelled by more funds. The prospect of continued growth provides an incentive to ride the bubble that is absent if all the funds that are expected to arrive are already there, in which case trading overvalued assets is a zero-sum game. The second reason why borrowing constraints are essential is informational. Since investors bid as much as they can borrow, their private estimate of the worth of the risky asset is not revealed. In the absence of borrowing constraints, uncertainty about t_0 would be revealed as soon as the boom began.[15]

In the interest of tractability and parsimony, we have sought to make this credit-based extension while staying as close as possible to DM. In particular, the key investor trade-offs that limit bubble duration unchanged are not affected by risk-shifting or other new considerations arising from the credit market. Nevertheless, and although we believe that it is clearest to introduce changes one at a time, it is interesting to discuss how alternative assumptions would influence results. In particular, we discuss liquidation of repossessed assets, loan-to-income requirements and endogenous timing of refinancing.

[15] Another assumption with key informational implications is the trading protocol under which that agents make buy/sell decisions in a first stage and observe prices in a second stage. This allows type-t_0 agents to sell at t_c, since other agents commit to buying in the first stage of the period and cannot withdraw their bids after observing the price. This protocol resembles an exchange with market orders, which are always filled with certainty, but without a price guarantee. Under Walrasian timing—or in a market with limit orders—our results would not obtain, since buyers at t_c would be able to condition their purchase on the price. Only if there was enough noise in the price process, this assumption could be relaxed, since buyers would not be able to distinguish the beginning of the crash from random day-to-day fluctuations. D-M incorporates some of these ideas, although he does not allow enough noise to allow for Walrasian timing.

Our assumption that lenders hold repossessed shares until maturity is analytically convenient, and weakly optimal for lenders. However, the scenario in which lenders liquidate repossessed assets appears to be more realistic. If lenders liquidates collateral, they would only be concerned with the liquidation price to the extent that it allowed them to recover their debt. Thus, in equilibria in which there was positive equity after the crash, a glut of liquidations could depress prices below fundamental value. As long as price remained above the remaining debt, lenders would not be affected by the decrease in price.[16] This 'undershooting' pattern where prices may recover after a period of defaults and liquidations may be reminiscent of recent events in US housing and equity markets. Unfortunately, incorporating this in our model would greatly complicate the notation, exposition and analysis. Characterizing maximum bubble duration τ^* would no longer be analytically feasible, since inequality (32) would have to be modified to account for the greater crash losses, as well as the opportunity of early sellers to profit twice by reentering the market at below-fundamental prices.

The model could also be extended to accommodate risk-shifting. In that case, borrowers in default would be 'under water' and thus the issue of undershooting would not arise, since lenders would be motivated to maximize the sales price. The beliefs of lenders, which do not play a role in our model, would become crucial in the risk-shifting case. They would determine interest rates, credit limits, and thus the growth rate of the bubble and its maximum duration. Our conjecture is that risk-shifting would be conducive to bubbles, since investors would be able to pass losses to lenders, although this effect would be attenuated by higher interest rates.

Another common factor affecting lending standards in practice is the loan-to-income ratio. A cross-country study by Lim, et al. (2011) provides empirical support for this idea. The authors compare the use and performance of loan-to-value constraints versus debt-to-income constraints together with other macroprudential policy tools. Their regression results (p. 53) show that the implementation of a debt-to-income cap is more effective than a loan-to-value cap in reducing the growth rates of real estate prices and credit. In fact, in our model, loan-to-endowment or loan-to-dividend caps would not give rise to booming dynamics, and thus would preclude bubbles. That said, this requirement could also prevent prices from reaching fudamen-

[16] In fact, this is a commonly used argument to explain why foreclosures are sold at discounted prices. If it is costly for lenders to hold a property that is 'above water', they have a strong incentive to lower the price in order to sell quickly.

tal value until the asset begins to pay dividends.

Finally, the frequency with which agents refinance could be another margin of adjustment. In the presence of multi-period loans and a transaction cost of refinancing, price growth and frequent refinancing might emerge in equilibrium.

5 Conclusion

We extend the rational model of speculative bubbles developed by Doblas-Madrid (2012) by adding a credit market as a source of funds that fuels bubbles. We assume that lenders face a constraint on loan-to-value (LTV) ratios, where value is determined by the most recent—instead of expected future—price. Under such constraints, initial asset prices limit how much investors can borrow, which in turn limits the immediate response of asset prices to a shock. Adjustment to the shock—instead of a one-time jump—takes place over a number of periods in which booming prices and credit reinforce each other. Under asymmetric information à la Abreu and Brunnermeier (2003), rational agents continue bidding prices past fundamental values, inflating a bubble. The ensuing crash redistributes wealth from unlucky to lucky investors. We show that, for LTV caps below a certain threshold, our model generates equilibria without risk-shifting. In these equilibria, unlucky investors are forced to default when the bubble bursts, but nevertheless remain 'above water'. That is, lenders are able to avoid credit losses because seized collateral remains valuable enough to recover principal and interest on loans. By focusing on equilibria without risk-shifting, we are able to isolate the speculative motive for riding bubbles from the risk-shifting motive emphasized by Allen and Gale (2000). By reducing the speed at which prices grow, monetary and credit policies can shorten, or even eliminate bubbles. Specifically, such policies include raising interest rates and lowering LTV ratios. While these policies seem intuitively desirable, our model falls short of having implications for optimal policy. Risk neutrality and fixed supply of the risky asset pay huge dividends in terms of tractability, but they also imply that, in our model, there is no misallocation of productive resources and that the zero-sum redistribution caused by bubbles is welfare neutral. In our view, extending the model to address these limitations, in addition to those mentioned in the previous section, is a worthwhile objective for future work.

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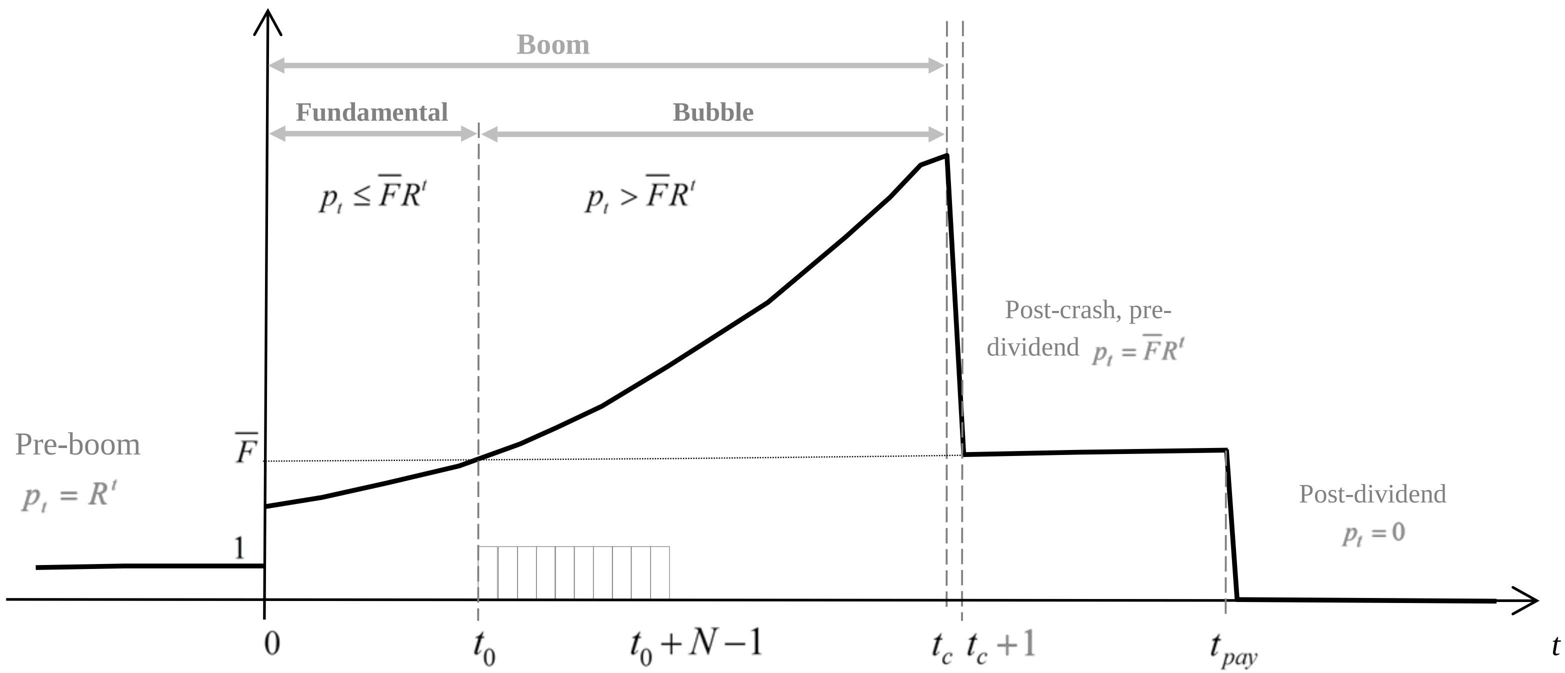

Figure 1 — The boom starting at time 0 is at first fundamental, but turns into a bubble at the imperfectly observed time t_0. Investors receive signals $\nu(i) = t_0, \ldots, t_0 + N - 1$. Bubble duration τ^* will be endogenously determined in equilibrium, and the bubble will burst at $t_c = t_0 + \tau^*$. The risky asset pays off at time t_{pay}, which also depends on the realization of $\overline{F}$ and hence on t_0.

ASSET MARKET - Start with $(b_{i,t}, h_{i,t}, l_{i,t})$, information $\tilde{I}_{i,t} = p^{t-1} \cup \{\delta_{i,t}\}$.

- STAGE 1: Choose $(m_{i,t}, s_{i,t}, \tilde{l}_{i,t}, \xi_{i,t})$
- STAGE 2: $p_t = \frac{M_t}{S_t}$ $\qquad h_{i,t} = h_{i,t} - s_{i,t} + \frac{m_{i,t}}{p_t}$ $\qquad \tilde{l}_{i,t} = l_{i,t} \frac{h_{i,t} - s_{i,t}}{h_{i,t}}$

$$c_{i,t} = \xi_{i,t} \left[b_{i,t} - m_{i,t} + \max \left\{ p_t - \frac{l_{i,t}}{h_{i,t}}, 0 \right\} s_{i,t} \right]$$

$$\tilde{b}_{i,t} = (1 - \xi_{i,t}) \left[b_{i,t} - m_{i,t} + \max \left\{ p_t - \frac{l_{i,t}}{h_{i,t}}, 0 \right\} s_{i,t} \right]$$

DEBT REFINANCING Start with $(\tilde{b}_{i,t}, h_{i,t}, \tilde{l}_{i,t})$ and information $\tilde{I}_{i,t} = I_{i,t} \cup \{p_t\}$.

- Lenders set LTV cap $\bar{\phi}_t$
- Investor chooses $(r_{i,t}, b_{i,t+1}, h_{i,t+1}, l_{i,t+1})$ s.t.

$$\frac{b_{it+1}}{R} + r_{i,t}\tilde{l}_{i,t} = \tilde{b}_{i,t} + y_t + (1 - r_{i,t})(h_{i,t} - s_{i,t}) e_{i,t} + \frac{l_{i,t+1}}{R}$$

$$l_{i,t+1} \leq \bar{\phi}_t p_t h_{i,t+1}, \qquad h_{i,t+1} = h_{i,t} - (1 - r_{i,t})(h_{i,t} - s_{i,t}), \qquad r_{i,t} \in [0, \bar{r}_{i,t}]$$

$$e_{i,t} = \max \left\{ 0, E\left[p_{t+1} \mid \tilde{I}_{i,t} \text{ and } r_{i,t} < 1 \right] - \frac{l_{i,t}}{h_{i,t}} \right\}$$

Figure 2 - Within-period timing of shocks and actions for periods $t < t_{pay}$.

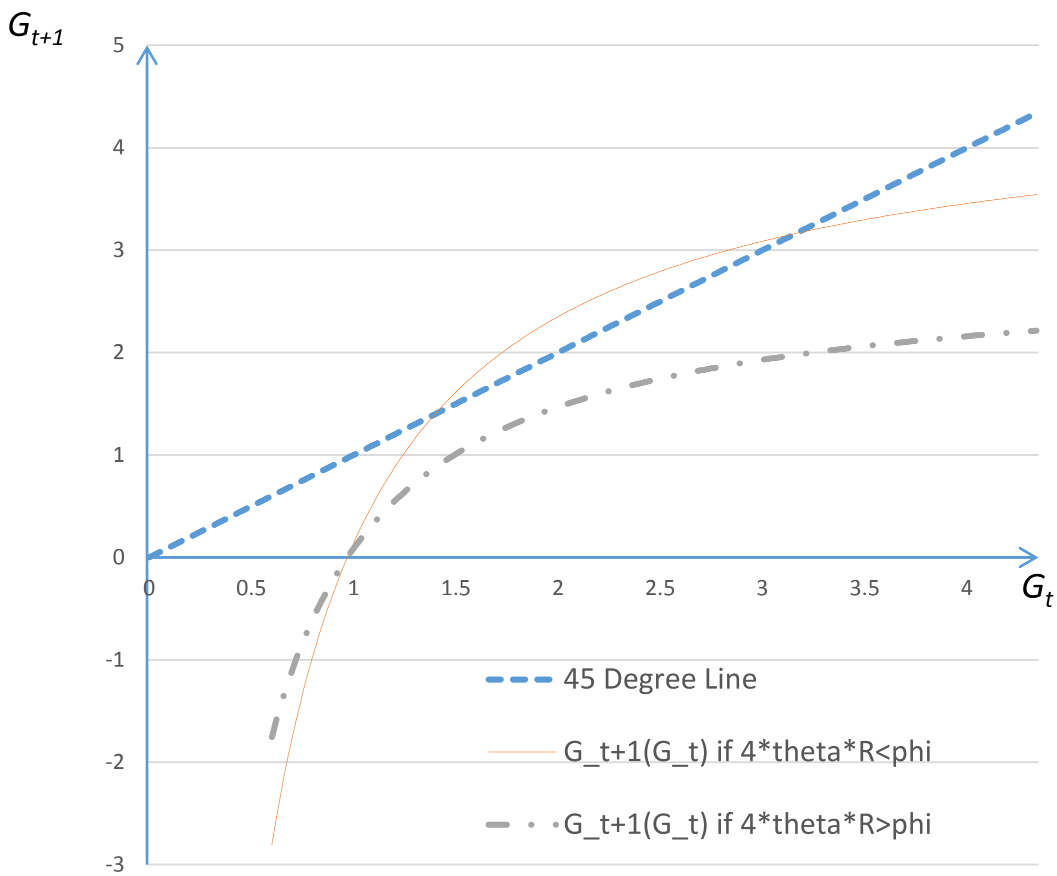

Figure 3 — The endogenous price growth rate G_t converges to a constant G if $4\theta R \leq \bar{\phi}$. Only $G = (1-\theta)\bar{\phi}\left[1+(1-4\theta R/\bar{\phi})^{1/2}\right]/[2\theta]$, the larger of the two roots is stable. Since y_t / p_t falls as the boom progresses, prices growth rates are decreasing over time. Thus, when $4\theta R \leq \bar{\phi}$, convergence happens 'from above'.

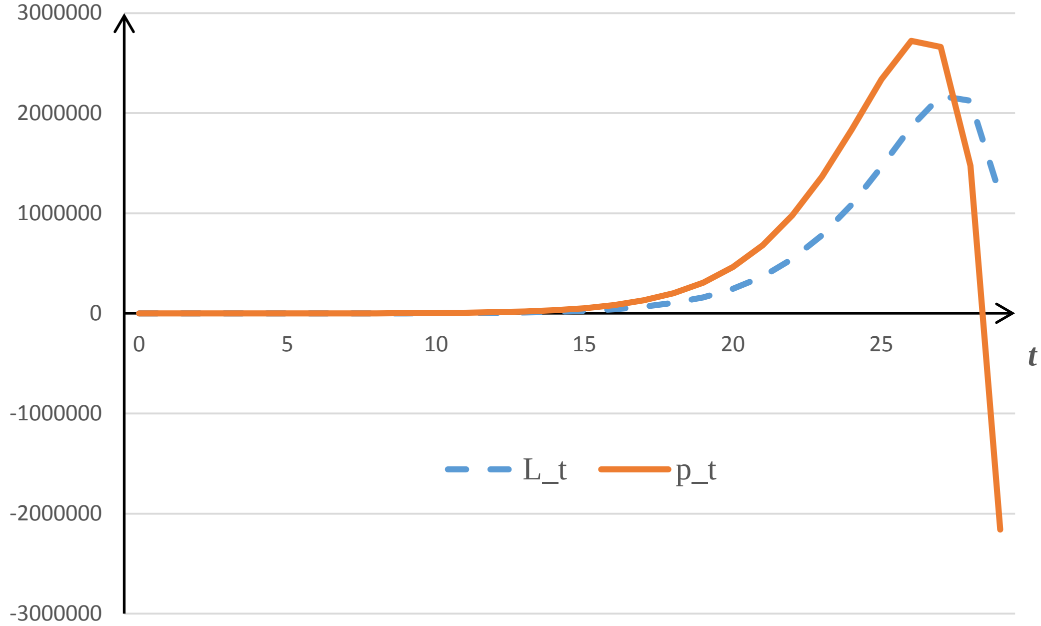

Figure 4 — This figure depicts an example of boom-phase price and debt dynamics if $4\theta R > \overline{\phi}$. In this case, self-reinforcing loop between prices and credit generates only a temporary boom, because in the long run, consumption outflows outweigh credit inflows. In this paper, we focus on the case where $4\theta R \leq \overline{\phi}$ and the boom can be sustained.

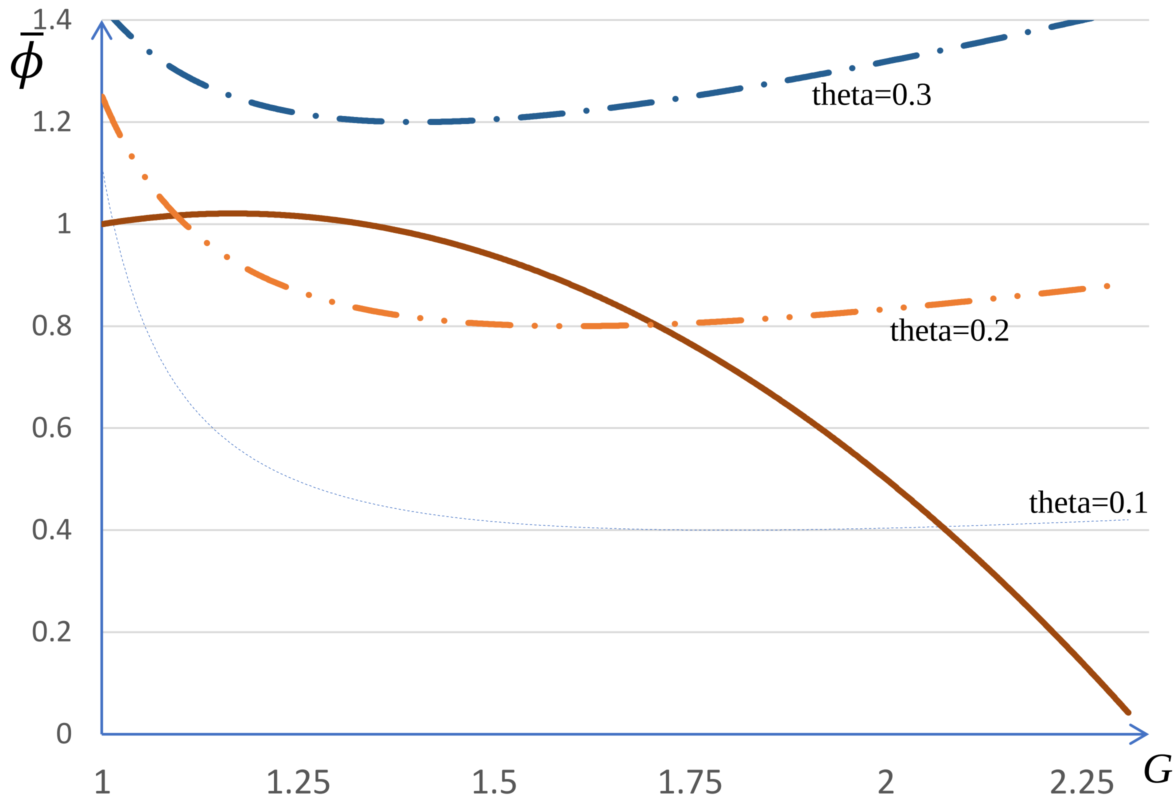

Figure 5 — The no-risk-shifting condition is satisfied by all $(G,\bar{\phi})$ pairs below the solid line. The dotted lines plot, for different values of θ, G as a function of $\bar{\phi}$ arising from the price-credit loop during the boom. As long as $1 \geq \theta(1-\theta)(4+2\lambda)+[\theta(1-\theta)\lambda]^2$ the two curves intersect. When there are two points of intersection, only the one with the larger value of G is stable.