Online Appendix for
Missing Growth from Creative Destruction

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A.1 Heterogeneous elasticities and varying markups

In this section of the Online Appendix, we discuss how our analysis of missing growth can be extended: (i) to the case of non-CES production technologies; and (ii) to accommodate varying markups.

A.1.1 Non-CES production elasticities

Let us first recall that the main equation used in the market share approach in our core analysis makes use of the CES production technology for the final good (i.e., of the assumption of a uniform elasticity of substitution σ across intermediate inputs). There we related the market share of product j to its quality adjusted price relative to the price index, according to the equilibrium expression:

$$s_t(j) \equiv \frac{p_t(j)x_t(j)}{M_t} = \left(\frac{P_t}{p_t(j)/q_t(j)}\right)^{\sigma-1}, \tag{A.1}$$

where P_t is the "true" price index, M_t are nominal expenditure, $p_t(j)/q_t(j)$ is the quality-adjusted price, and σ is the constant elasticity of substitution. From this it is clear that the choice of the value of σ is quantitatively important and so is also the assumption that this elasticity is constant.

Now consider the case where the technology for producing the final good is general constant return to scale production function, with real output Y_t given by

$$Y_t = \frac{M_t}{P\left(p_t(1), ..., p_t(N_t)\right)}, \tag{A.2}$$

where $P\left(p_t(1),...,p_t(N_t)\right)$ is the true price index.

Roy's identity yields the Marshallian demand

$$x_t(j) = \frac{P_j\left(p_t(1),...,p_t(N_t)\right)}{P\left(p_t(1),...,p_t(N_t)\right)} M_t, \tag{A.3}$$

where $P_j\left(p_t(1),...,p_t(N_t)\right) \equiv \frac{\partial P(p_t(1),...,p_t(N_t))}{\partial p_t(j)}$.

In this case the share spent on product j is given by

$$s_j(t) \equiv \frac{p_t(j)x_t(j)}{M_t} = \frac{P_j\left(p_t(1),...,p_t(N_t)\right)p_t(j)}{P\left(p_t(1),...,p_t(N_t)\right)}, \tag{A.4}$$

and the elasticity of that share with respect to the firm's own price is given by

$$\frac{\partial s_t(j)}{\partial p_t(j)}\frac{p_t(j)}{s_t(j)} = \frac{\partial \frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}}{\partial p_t(j)} \frac{p_t(j)}{\frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}} + 1. \tag{A.5}$$

Thus, if we denote the (local) price elasticity of demand, $\frac{\partial \frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}}{\partial p_t(j)} \frac{p_t(j)}{\frac{P_j(p_t(1),...,p_t(N_t))}{P(p_t(1),...,p_t(N_t))}}$, by $-\sigma_j\left(p_t(1),...,p_t(N_t)\right)$, the market share of intermediate producer j is approximated by a similar expression to (A.1), namely:

$$s_j(t) = \left(\frac{P_t}{p_t(j)}\right)^{\sigma_j(\cdot)-1}, \tag{A.6}$$

where $\sigma_j(\cdot)$ is the *local* elasticity.

Hence, as long as we know the *local* elasticity $\sigma_j(\cdot)$ the "market share approach" can still be used to quantify missing growth.

Suppose the elasticity of substitution differs between different type of inputs. Which elasticity of substitution should then be used in the market share approach? More specifically, suppose we have the following production technology for the final good

$$Y = \left(\left[\int_I \left[q(j)y(j)\right]^{\frac{\sigma_I-1}{\sigma_I}} dj\right]^{\frac{\sigma_I}{\sigma_I-1}\frac{\sigma_B-1}{\sigma_B}} + \left[\int_{N\setminus I} \left[q(j)y(j)\right]^{\frac{\sigma_N-1}{\sigma_N}} dj\right]^{\frac{\sigma_N}{\sigma_N-1}\frac{\sigma_B-1}{\sigma_B}}\right)^{\frac{\sigma_B}{\sigma_B-1}},$$

where I is the set of survivors, N is the set of existing plants, σ_I is the elasticity of substitution among surviving products, σ_N is the elasticity of substitution among new products, and σ_B is the elasticity of substitution between all the surviving and all the new products. In this case σ_B is the elasticity that should be used in our market share approach. With $\sigma_I = \sigma_N = \sigma_B$ we are back to the CES case in our core analysis. This we see as the most realistic case to the extent that there is no obvious reason to believe that surviving and new products should differ (surviving products are products that have been new at some point in the past too).

A.1.2 Varying markups

Our baseline analysis carries over to the case where markups are heterogeneous but uncorrelated with the age of the firm or with whether or not there was a successful innovation (own incumbent or new entrant innovation) in the firm's sector.

Now, suppose that: (i) the markups of unchanged products grow at gross rate g; (ii) the markups of new varieties are equal to g_n times the "average markup" in the economy in the last period; (iii) markups grow at gross rate g_i if there is an incumbent own innovation; (iv) markups after a successful creative destruction innovation is g_d times the markup of the eclipsed product. This amounts to replacing Assumption 1 in the main text by:[1]

$$\frac{q_{t+1}(j)}{\mu_{t+1}(j)} = \frac{\gamma_n}{g_n}\left(\frac{1}{N_t}\int_0^{N_t}\left(\frac{q_t(i)}{\mu_t(i)}\right)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}, \ \forall j \in (N_t, N_{t+1}].$$

Under the above assumptions the market share approach can still provide a precise estimate of missing growth, as long as: (a) we still make the assumption that the statistical office is measuring changes in markups of surviving product properly since changes in nominal prices are observed; (b) the market share relates to the quality-adjusted price in the same way for young and old firms, but recall that we are focusing our market share analysis on plants that have appeared in the data set for at least five years.

However, allowing for changing markups affects the expression for missing growth, which now becomes

$$MG = \frac{1}{\sigma-1}\log\left(1+\frac{\lambda_d\left[\left(\frac{\gamma_d}{g_d}\right)^{\sigma-1} - g^{1-\sigma} - \lambda_i\left(\left(\frac{\gamma_i}{g_i}\right)^{\sigma-1} - g^{1-\sigma}\right)\right] + \lambda_n\left(\frac{\gamma_n}{g_n}\right)^{\sigma-1}}{g^{1-\sigma} + \lambda_i\left(\left(\frac{\gamma_i}{g_i}\right)^{\sigma-1} - g^{1-\sigma}\right)}\right).$$

In particular, allowing for changing markups introduces an additional source of missing growth having to do with the fact that the subsample of (surviving) products are not representative of all firms in their markup dynamics.

A.2 Missing growth with capital

The purpose of this section of the Online Appendix is to extend our "missing growth" framework to a production technology with capital as an input, and to see how this affects estimated missing growth as a fraction of "true" growth.

[1] Note that this covers several possible theories governing the dynamics of markups. In particular it covers the case where firms face a competitive fringe from the producer at the next lower quality rung, in which $g_i > 1$ and $g < 1$. It also covers the case where newly born plants start with a low markup and markups just grow over the live-cycle of a product, in which $g_d < 1$, $g_n < 1$ and $g > 1$.

A.2.1 A simple Cobb-Douglas technology with capital

Instead of the linear technology in the main text, we assume the following Cobb-Douglas production technology for intermediate inputs

$$y(j) = (k(j)/\alpha)^{\alpha} \left(l(j)/(1-\alpha)\right)^{1-\alpha}.$$

It is straightforward to see how this generalization affects the main equations in the paper. If R denotes the rental rate of capital, then the true aggregate price index becomes

$$P = p\left(\int_0^N q(j)^{\sigma-1}dj\right)^{\frac{1}{1-\sigma}},$$

with just $p = p(j) = \mu R^{\alpha} W^{1-\alpha}$.

Again we assume that the statistical office perfectly observes the nominal price growth $\frac{p_{t+1}(j)}{p_t(j)}$ of the surviving incumbent products. Since the Cobb-Douglas production technologies are identical across all intermediate inputs the capital-labor ratio equalizes across all firms and we have in equilibrium

$$y(j) = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(\frac{K}{L}\right)^{\alpha} l(j),$$

where K and L denote the aggregate capital and labor stocks in the economy.

We assume that labor supply is constant over time and we assume a closed economy where profits, Π, labor earnings and capital income are spent on the final output good such that

$$P \cdot Y = W \cdot L + R \cdot K + \Pi.$$

Then we can derive the equilibrium output of an intermediate input j (the analog of expression (9) in the main text), which yields

$$y_t(j) = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)} K_t^{\alpha} L^{1-\alpha} q_t(j)^{\sigma-1}\left(\int_0^{N_t} q_t(j'^{\sigma-1}dj'\right)^{-1}. \quad \text{(A.7)}$$

The aggregate production function can now be written in reduced form as

$$Y_t = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)} Q_t K_t^{\alpha} L^{1-\alpha},$$

where $Q_t \equiv \left(\int_0^{N_t} q_t(j)^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}}$. The term Q_t summarizes how quality/variety gains affect total productivity for given capital stock K_t.

Allowing for capital does not change anything in the model-based market share approach since we still have

$$\frac{S_{I_t,t+1}}{S_{I_t,t}} = \left(\frac{P_{t+1}}{P_t}\right)^{\sigma-1}\left(\widehat{\frac{P_{t+1}}{P_t}}\right)^{-(\sigma-1)}.$$

This equation can (still) be used to estimate missing growth as in Proposition 6 in the main text.[2] Hence the missing growth figures we obtained in Section 3.1.3 of the main text (e.g., 0.56 percentage points in the baseline specification over the period 1983–2013) are unaffected when we introduce capital as specified above. The only important thing to note here is that this missing growth is "missing growth in the Q term" since under the assumption that nominal price growth is perfectly well observed by the statistical office we have:

$$MG = \left(\frac{P_t}{P_{t+1}}\right)\left(\widehat{\frac{P_{t+1}}{P_t}}\right) = \left(\frac{Q_{t+1}}{Q_t}\right)\left(\widehat{\frac{Q_t}{Q_{t+1}}}\right).$$

What may (potentially) change when introducing capital is how this missing growth should be compared to measured productivity growth. This issue is discussed in the remaining sections of this Online Appendix.

A.2.2 Finding "true" growth

So far we saw that our market share analysis in the main text remains valid when introducing capital, in the sense that it allows us to compute the bias in $\frac{Q_{t+1}}{Q_t}$. We now want to combine this missing growth estimate with information on measured growth to calculate "true" growth. The main question then is: what is the "right" estimate for measured growth $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$? Once we have found this "right" estimate of measured growth we can simply calculate true growth as

$$\left(\frac{Q_{t+1}}{Q_t}\right) = MG \cdot \left(\widehat{\frac{Q_{t+1}}{Q_t}}\right), \tag{A.8}$$

where MG is 1.0056 for the whole period in the baseline specification.

A potentially difficulty here is that the capital stock, K_t, may itself grow over time.[3] Suppose K_t is growing at a constant rate over time, then part of the aggregate output growth $\frac{Y_{t+1}}{Y_t}$ is generated by capital deepening. Relatedly, if the capital stock grows over time the question arises as to whether this capital growth is perfectly measured or not. Finally, the long-run growth path of the capital stock will also matter and consequently we need to specify the saving and investment behaviors which underlie this growth of capital stock, and also need to take a stand as to whether there is investment specific technical change etc. The answer to all these questions have implication for the interpretation of the measured TFP growth and how it relates to $\widehat{\frac{Q_{t+1}}{Q_t}}$.

We first assume that the long-run growth rate of K_t results from a constant (exogenous) saving rate and abstract from investment specific technical change

[2] This also easily generalizes to any constant return to scale production function.

[3] If instead K_t was like "land", i.e., constant over time then the measured $\left(\widehat{\frac{Q_{t+1}}{Q_t}}\right)$ would be equal to the measured Hicks-neutral TFP growth.

(see Section A.2.2.1). Furthermore we assume that all growth due to capital deepening is perfectly well observed and measured by the statistical office (see Section A.2.2.2) . Then, in Section A.2.2.3, we consider two alternative assumptions as to which part of physical capital growth is measured and analyze how these affect true growth estimates.

A.2.2.1 Capital accumulation

We assume that the final output good can be either consumed or invested. Furthermore we assume a constant exogenous saving/investment rate in the economy (we thus abstract from intertemporal optimization), i.e.,

$$K_{t+1} = K_t(1-\delta) + sY_t, \tag{A.9}$$

where s is the constant savings rate and δ is the depreciation rate of capital.

Suppose that $Q_{t+1}/Q_t = g$ is constant over time. This in turn implies that in the long run the capital-output ratio will stabilize at

$$\frac{K}{Y} = \frac{s}{g^{\frac{1}{1-\alpha}} - 1 + \delta}. \tag{A.10}$$

Along this balanced growth path investment, capital, and wages all grow at the same constant gross rate $g^{\frac{1}{1-\alpha}}$.

A.2.2.2 Measured output growth

Under the above assumption for capital accumulation, in the long run, true output growth is given by

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.11}$$

Note that the first term on the right-hand side captures direct quality/variety gains, whereas the second term captures output growth due to capital deepening. In the following we assume that the second term is perfectly well measured whereas the first term is mismeasured as specified in our theory.[4] Under this assumption, measured output growth is equal to

$$\widehat{\frac{Y_{t+1}}{Y_t}} = \widehat{\frac{Q_{t+1}}{Q_t}}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.12}$$

[4] This assumption rests on the view that the part of growth driven by capital deepening materializes—for given quality and variety—in increasing $y(j)$ (see (A.7)) which the statistical office should be able to capture (otherwise we would have still another source of missing growth).

A.2.2.3 Two alternative approaches on measured growth in capital stock

Next, we need to take a stand on how to measure the growth rate of capital stock. For given measured capital growth, the statistical office can compute the rate of Hicks-neutral TFP growth implicitly through the following equation:

$$\frac{\widehat{Q_{t+1}}}{Q_t}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\widehat{K_{t+1}}}{K_t}\right)^{\alpha}\frac{\widehat{TFP_{t+1}}}{TFP_t}. \tag{A.13}$$

First "macro" approach Here we assume that the bias in the measure of capital stock is the same as that for measuring real output.[5] Then the measured growth rate of capital stock in the long run is equal to

$$\frac{\widehat{K_{t+1}}}{K_t} = \frac{\widehat{Y_{t+1}}}{Y_t} = \frac{\widehat{Q_{t+1}}}{Q_t}\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.14}$$

Substituting this expression for measured capital growth in (A.13) in turn yields

$$\frac{\widehat{TFP_{t+1}}}{TFP_t} = \left(\frac{\widehat{Q_{t+1}}}{Q_t}\right)^{1-\alpha}\left(\frac{Q_{t+1}}{Q_t}\right)^{\alpha}. \tag{A.15}$$

Substituting this into (A.8) then leads to:

$$\left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}} = MG\cdot\left(\frac{\widehat{TFP_{t+1}}}{TFP_t}\right)^{\frac{1}{1-\alpha}}. \tag{A.16}$$

In other words, one should add MG to measured growth in TFP (in labor augmenting units) to get total "true" quality/variety growth in labor augmenting units. This is exactly what we are doing in our core analysis in the main text. Thus under the assumptions underlying this first approach the whole analysis and quantification of missing growth in our core analysis carries over to the extended model with capital. Let us repeat what underlies this approach: first, the focus is on the long-run when the capital-output ratio stabilizes at its balanced growth level; second, investment specific technical change is ruled out, so that the bias in measuring the growth in capital stock is the same as that in measuring the growth in real output.[6]

[5] This is a reasonable assumption to the extent that: (i) the same final good serves both as consumption good and as investment good; (ii) if the long-run growth rate of Q_t is constant, i.e., $Q_{t+1}/Q_t = g$, then the bias in measuring capital stock growth (when using a perpetual inventory method) is in the long run identical to the bias in measuring real output growth.

[6] To get some intuition, note that we can also write the production function as

$$Y_t = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}Q_t^{\frac{1}{1-\alpha}}\left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}}L. \tag{A.17}$$

Second "micro" approach Here we assume that the growth in capital stock is perfectly measured by the statistical office,[7] i.e.,

$$\frac{\widehat{K_{t+1}}}{K_t} = \left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{1-\alpha}}. \tag{A.18}$$

Plugging this expression in (A.13) gives:

$$\frac{\widehat{TFP_{t+1}}}{TFP_t} = \frac{\widehat{Q_{t+1}}}{Q_t}, \tag{A.19}$$

so that:

$$\frac{Q_{t+1}}{Q_t} = MG \cdot \frac{\widehat{TFP_{t+1}}}{TFP_t}. \tag{A.20}$$

This in turn implies that our missing growth estimate should be added to measured TFP growth in Hick-neutral terms to obtain Hicks-neutral "true" TFP growth. Assuming $\alpha = 1/3$, this approach would increase missing growth as a fraction of true growth from 22% (=2.49/0.56 see Table 2 in the main text) to 38%.

A.2.3 Wrapping-up

In this Appendix we argued that our core analysis can easily be extended to production technologies involving physical capital. Under our first (macro) approach the missing growth estimates remain exactly the same as in our core analysis based on the model without capital. And moving to our second (micro) approach only increases our missing growth estimates. In that sense, the macro approach can be viewed as being more conservative.

Since under the assumptions above the growth rate in the capital-output ratio, $\frac{K_t}{Y_t}$, (which is zero in the long run) is properly measured, we see that missing growth automatically obtains a labor-augmenting interpretation and should consequently be compared to TFP growth estimates expressed in labor augmenting terms.

[7] We see this approach as being more "micro" for the following reason. Suppose we only have data about the only one industry. Then we could use our market share approach together with data about the revenue shares of different products to estimate missing output growth in this particular industry. It would then be reasonable to compare this number to the Hicks-neutral TFP growth in this industry, within the implicit assumption that the statistical office perfectly measures the growth in capital stock in the industry when calculating TFP growth. Next, one could sum-up "missing growth" and measured Hicks-neutral TFP growth to compute "true" TFP growth. This true TFP growth would of course itself be mismeasured if there is mismeasurement in the growth of capital stock: this would add yet another source of missing growth.

A.3 Missing growth in manufacturing and non-manufacturing sectors

In the paper, we reported missing growth by the market share method for all sectors in the economy. We also calculated missing growth within manufacturing and non-manufacturing sectors. Table A.1 displays the result. In the first column, we reiterate the baseline results in the market share section of our paper. The second and third columns report missing growth in manufacturing and non-manufacturing, respectively. Missing growth in non-manufacturing is about 0.1 percentage points larger than our baseline results but also appears to be constant over time. Missing growth in manufacturing, however, is only 0.03 percentage points on average between 1983–2013.

Table A.1: Manufacturing and non-manufacturing sector

	All	Mfg	Non-mfg
1983–2013	**0.56**	**0.03**	**0.67**
1983–1995	0.60	0.23	0.71
1996–2005	0.41	-0.13	0.51
2006–2013	0.69	-0.07	0.79

Notes: This table presents missing growth estimates for the whole 1983–2013 period (as well as different sub-periods) by manufacturing and non-manufacturing sectors. The growth numbers are expressed in (average) percentage points per year. The results in column "All" identical to the baseline results in the paper. The elasticity of substitution, σ, is 4 and the lag, k, is 5 throughout the table.

A.4 Our notation vs. GHK code notation

Table A.2: GHK NOTATIONS VS. OUR NOTATION

Parameter	Our model	GHK equivalent
Share of non-obsolete products with OI innovation	$\lambda_i(1-\lambda_d)$	$\frac{\lambda_i}{(1-\delta_o)}$
Share of non-obsolete products having incumbent CD	0	$\frac{\delta_i(1-\lambda_i)}{(1-\delta_o)}$
Share of non-obsolete products having entrant CD	λ_d	$\frac{\delta_e(1-\lambda_i)}{(1-\delta_o)}$
Measure of incumbent or entrant NV in $t+1$	λ_n	$\kappa_i+\kappa_e+\delta_o$
Relative to the number of products in t		
Share of obsolescence	0	δ_o
Net expected step size of CD innovation	$\gamma_d^{\sigma-1}-1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}]-1)$
Net expected step size of OI innovation	$\gamma_i^{\sigma-1}-1$	$\frac{1-\delta_o}{1-\delta_o\psi}(E[s_q^{\sigma-1}]-1)$
Quality of NV innovation rel to average productivity last period	γ_n	$s_\kappa^{\frac{1}{\sigma-1}}$
Average quality of product becoming obsolete in $t+1$ relative to average quality in t	n/a	ψ
Elasticity of substitution	σ	σ