

# Approximating Multisector New Keynesian Models — Extended Version\*

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## Abstract

A calibrated three-sector model with a suitably chosen distribution of price stickiness can closely approximate the dynamic properties of New Keynesian models with a much larger number of sectors. The parameters of the approximate three-sector distribution are such that both the approximate and the original distributions share the same (i) average frequency of price changes, (ii) cross-sectional average of durations of price spells, (iii) cross-sectional standard deviation of durations of price spells, (iv) cross-sectional skewness of durations of price spells, and (v) cross-sectional kurtosis of durations of price spells. This result should prove useful to the literature that takes into account heterogeneity in price stickiness in DSGE models. In particular, it should allow for the estimation of such models at a much reduced computational cost.

*JEL classification codes:* E22, J6, E12

*Keywords:* multisector approximation, heterogeneity, factor specificity

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\*For comments and suggestions we thank an anonymous referee. In this supplement to our main paper we provide several extensions of the model and results. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of San Francisco, the Federal Reserve System or the Central Bank of Brazil. E-mails: [cvianac@econ.puc-rio.br](mailto:cvianac@econ.puc-rio.br) and [fernanda.nechio@sf.frb.org](mailto:fernanda.nechio@sf.frb.org).

# 1 Introduction

The macroeconomics literature has provided substantial evidence of significant heterogeneity in the average frequencies with which firms in different sectors of the economy change their prices, both in the U.S. and in other countries (*e.g.*, [Blinder et al., 1998](#), [Bils and Klenow, 2004](#), [Nakamura and Steinsson, 2008](#), and [Klenow and Malin, 2011](#) for the U.S. economy; [Dhyne et al., 2006](#), and references cited therein for the euro area).

The literature has also shown that accounting for this heterogeneity matters for the dynamic properties of New Keynesian economies (*e.g.*, [Carvalho, 2006](#), [Nakamura and Steinsson, 2010](#), [Carvalho and Nechio, 2011](#) and [Pasten, Schoenle and Weber, 2016](#)). Models that allow for heterogeneity in frequency of price adjustments yield substantially different dynamics for aggregate variables than models that assume that firms adjust prices at a common (average) frequency. In response to monetary shocks, for example, heterogeneous multisector models feature much larger and more persistent real effects than one-sector economies with similar average degrees of nominal rigidities. Heterogeneity in price stickiness also has implications for optimal monetary policy (*e.g.*, [Aoki, 2001](#) and [Eusepi, Hobbijn and Tambalotti, 2011](#)).<sup>1</sup>

Working with large-scale multisector models, however, is computationally costly. As a consequence, while one can find several calibrated multisector models in the literature, the set of papers that entertain estimating such large-scale models is much more limited (exceptions include, for example, [Carvalho and Lee, 2011](#) and [Bouakez, Cardia and Ruge-Murcia, 2009](#)).

In this paper we show how a large-dimension multisector economy can be approximated by a three-sector model that, aside from the number of sectors, features the same model ingredients as the original economy. In particular, we show that for a suitably chosen distribution of price stickiness, aggregate variables in the three-sector model feature similar dynamics than the corresponding variables in the original model. A comparison of the impulse response functions (IRFs) of the main aggregate variables in response to monetary and technology shocks shows that the three-sector economy follows the dynamic properties of the original (larger-scale) model extremely closely.

The empirical evidence shows that the distribution of price stickiness is highly asymmetric, and with a long right tail. Therefore, there is a larger mass of sectors adjusting more frequently and a smaller mass adjusting very infrequently. These features are well summarized by five moments of the distribution, namely; (*i*) average frequency of price changes, (*ii*) cross-sectional

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<sup>1</sup>These papers also study the effects of sector-specific shocks. Because our approximation strategy entails constructing an artificial three-sector economy with price stickiness distribution that matches moments of the larger (original) distribution, there is no direct mapping between sectors in the original and approximated economies. Hence our focus on aggregate shocks.

average of durations of price spells, *(iii)* cross-sectional standard deviation of durations of price spells, *(iv)* the cross-sectional skewness of durations of price spells, and *(v)* cross-sectional kurtosis of durations of price spells.

Our approximation strategy entails constructing an artificial small-scale distribution of price rigidity that matches the same shape features of the original one. More specifically, given these five moments of the original distribution of price stickiness, we build a three-sector distribution by choosing sectoral weights and price adjustment frequencies such that both the approximate and the original distributions share the same aforementioned five moments.

We apply our approximation to the U.S. distribution of price stickiness derived from [Nakamura and Steinsson \(2008\)](#), as well as to artificially-generated arbitrary distributions of price rigidity. We test the performance of our approximation using the dynamic stochastic general equilibrium (DSGE) model presented in [Carvalho and Nechio \(2016\)](#). The model features a multisector economy where heterogeneity in the frequency of price adjustment is the key source of heterogeneity.

The results show that our approximation performs extremely well. In particular, three-sector economies parameterized with distributions of price stickiness that match the five aforementioned moments yield IRFs to monetary and technology shocks that are very close to the ones obtained in the original economies. These results hold for approximations based on both the U.S. distribution and on the artificially-generated arbitrary distributions of price rigidity.

## 2 A three-sector approximation

These features of the distributions of price stickiness are important in determining the dynamics of aggregate variables in New Keynesian models (*e.g.*, [Carvalho, 2006](#), [Nakamura and Steinsson, 2010](#), and [Carvalho and Nechio, 2011](#)). In addition, [Carvalho and Schwartzman \(2015\)](#) show that, under some conditions, moments of the distribution of price stickiness are sufficient statistics for the real effects of nominal shocks.

Therefore, in order to approximate larger-scale economies with smaller-scale ones, one should construct an artificial distribution of price stickiness that mimics the properties of the original distribution. Given the empirical evidence on the skewness and the long tail of the original distribution, this implies that both the approximate and the original distributions should exhibit similar high-order moments.

To that end, we calculate the following moments of the original distribution of price stickiness:

- (i) Average frequency of price changes:

$$\bar{\alpha} = \sum_{s=1}^S f_s \alpha_s,$$

(ii) Cross-sectional average of the expected durations of price spells:

$$\bar{d} \equiv \sum_{s=1}^S f_s \alpha_s^{-1},$$

(iii) Cross-sectional standard deviation of the expected durations of price spells:

$$\sigma_d = \sqrt{\sum_{s=1}^S f_s (\alpha_s^{-1} - \bar{d})^2},$$

(iv) Skewness of the cross-sectional distribution of expected durations of price spells:

$$\mathcal{S}_d = \frac{1}{\sigma_d^3} \sum_{s=1}^S f_s (\alpha_s^{-1} - \bar{d})^3,$$

(v) Kurtosis of the cross-sectional distribution of expected durations of price spells:

$$\mathcal{K}_d = \frac{1}{\sigma_d^4} \sum_{s=1}^S f_s (\alpha_s^{-1} - \bar{d})^4,$$

where, for each sector  $s \in \{1, \dots, S\}$ ,  $f_s$  stands for sector- $s$  expenditure share and  $\alpha_s$  stands for sector- $s$  frequency of price adjustment.

To obtain an artificial distribution that mimics the properties of the original one, we construct a three-sector approximate distribution that matches the above five moments of the original distribution. More specifically, we choose two sectoral weights and three frequencies of price changes for the three-sector distribution such that both the approximate and the original distributions share the same five moments above.<sup>2</sup>

In what follows, we construct approximate distributions based on the U.S. empirical distribution and on artificially-generated arbitrary distributions of price stickiness. In particular, in Section 4 we rely on the U.S. empirical evidence and build a three-sector approximate price stickiness distribution that matches the empirical moments of the price distribution reported in Nakamura and Steinsson (2008). In Section 5 we construct arbitrary (large-scale) distributions and obtain their corresponding approximate three-sector distributions. To assess our approximation based on this moment-matching approach, we use the dynamic stochastic general equilibrium (DSGE) multisector models of Carvalho and Nechio (2016), which we summarize in Section 3, and compare the dynamics of aggregate variables in the original (large-scale) multisector economy to the dynamics of the corresponding variables in the three-sector economies which feature the approximate distribution of price stickiness.

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<sup>2</sup>That is, we choose a set of  $f_1, f_2, \alpha_1, \alpha_2$  and  $\alpha_3$  such that the resulting moments of this three-sector distribution match the moments of the original one. Since we have a three-sector distribution, once  $f_1$  and  $f_2$  are set, by construction,  $f_3 = 1 - (f_1 + f_2)$ .

### 3 Three New Keynesian models of factor specificity

Carvalho and Nechio (2016) present three versions of multisector DSGE models. In all three models, identical infinitely-lived consumers supply labor and capital to intermediate firms that they own, invest in a complete set of state-contingent financial claims, and consume a final good. The latter is produced by competitive firms that bundle varieties of intermediate goods. The monopolistically competitive intermediate firms that produce these varieties are divided into sectors that only differ in their frequency of price changes. Labor and capital are the only inputs in the production of intermediate goods.

The models differ, however, on their assumptions about the mobility of factor inputs across sectors. In their benchmark model, they assume that capital and labor can be reallocated freely across firms in the same sector but cannot move across sectors, i.e., factors are sector-specific. The authors also entertain versions of their model in which factors are firm-specific, i.e., cannot move across sectors nor firms within sectors; and factors are economy-wide and can move freely across firms and sectors.

In the following, we provide some key ingredients of all three versions of Carvalho and Nechio (2016) models, and refer the reader to the paper for detailed descriptions.

#### 3.1 Sector-specific factors

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^S \omega_s \frac{N_{s,t}^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + P_t I_t + E_t [\Theta_{t,t+1} B_{t+1}] \leq \sum_{s=1}^S W_{s,t} N_{s,t} + B_t + T_t + \sum_{s=1}^S Z_{s,t} K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$\begin{aligned} K_{s,t+1} &= (1 - \delta) K_{s,t} + \Phi(I_{s,t}, K_{s,t}) I_{s,t}, \forall s \\ \Phi(I_{s,t}, K_{s,t}) &= \Phi\left(\frac{I_{s,t}}{K_{s,t}}\right) = 1 - \frac{1}{2} \kappa \frac{\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2}{\frac{I_{s,t}}{K_{s,t}}}, \\ I_{s,t} &\geq 0, \forall s \end{aligned}$$

and a standard “no-Ponzi” condition.  $E_t$  denotes the time- $t$  expectations operator,  $C_t$  is consumption of the final good,  $N_{s,t}$  denotes total labor supplied to firms in sector  $s$ ,  $W_{s,t}$  is the associated nominal wage rate, and  $\omega_s$  is the relative disutility of supplying labor to sector  $s$ .  $I_{s,t}$  denotes investment in sector- $s$  capital,  $I_t \equiv \sum_{s=1}^S I_{s,t}$ ,  $K_{s,t}$  is capital supplied to firms in sector  $s$ , and  $Z_{s,t}$  is the associated nominal return on capital.

The final good can be used for either investment or consumption, and sells at the nominal price  $P_t$ .  $B_{t+1}$  accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of  $t + 1$ . Under complete financial markets, agents can choose the value of  $B_{t+1}$  for each possible state of the world at all times, subject to the no-Ponzi condition and the budget constraint.  $T_t$  stands for profits received from intermediate firms. The absence of arbitrage implies the existence of a nominal stochastic discount factor  $\Theta_{t,t+1}$  that prices in period  $t$  any financial asset portfolio with state-contingent payoff  $B_{t+1}$  at the beginning of period  $t+1$ .<sup>3</sup>  $\beta$  is the time-discount factor,  $\sigma^{-1}$  denotes the intertemporal elasticity of substitution,  $\gamma^{-1}$  is the Frisch elasticity of labor supply,  $\delta$  is the rate of depreciation. Finally, the adjustment-cost function,  $\Phi(\cdot)$ , is specified as in [Chari, Kehoe and Mcgrattan \(2000\)](#), where  $\Phi$  is convex and satisfies  $\Phi(\delta) = 1$ ,  $\Phi'(\delta) = 0$ , and  $\Phi''(\delta) = -\frac{\kappa}{\delta}$ .

A representative competitive firm produces the final good, which is a composite of varieties of intermediate goods. Monopolistically competitive firms produce such varieties. These firms are divided into sectors indexed by  $s \in \{1, \dots, S\}$ , each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by their sector  $s$ , and are further indexed by  $j \in [0, 1]$ . The distribution of firms across sectors is given by sectoral weights  $f_s > 0$ , with  $\sum_{s=1}^S f_s = 1$ .

The final good is used for both consumption and investment by combining the intermediate varieties. More specifically, the representative final-good-producing firm solves:

$$\begin{aligned} \max \quad & P_t Y_t - \sum_{s=1}^S f_s \int_0^1 P_{s,j,t} Y_{s,j,t} dj \\ \text{s.t.} \quad & Y_t = \left( \sum_{s=1}^S f_s^{\frac{1}{\eta}} Y_{s,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \\ & Y_{s,t} = \left( f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{s,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \end{aligned}$$

where  $Y_t$  is the final good,  $Y_{s,t}$  is the aggregation of sector- $s$  intermediate goods, and  $Y_{s,j,t}$  is the variety produced by firm  $j$  in sector  $s$ . The parameters  $\eta \geq 0$ , and  $\theta > 1$  are, respectively, the elasticity of substitution across sectors and the elasticity of substitution within sectors.  $P_t$

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<sup>3</sup> To avoid cluttering the notation, we omit explicit reference to the different states of nature.

is the price of the final good,  $P_{s,t}$  is the price index of sector- $s$  intermediate goods, and  $P_{s,j,t}$  is the price charged by firm  $j$  from sector  $s$ .

Monopolistically competitive firms produce varieties of the intermediate good by employing capital and labor. For analytical tractability, we assume that intermediate firms set prices as in [Calvo \(1983\)](#). The frequency of price changes varies across sectors and it is the only source of (ex-ante) heterogeneity.

In each period, each firm  $j$  in sector  $s$  changes its price independently with probability  $\alpha_s$ , and when it does, it chooses  $X_{s,j,t}$  to solve:

$$\begin{aligned} \max \quad & E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \left[ X_{s,j,t} Y_{s,j,t+l} - W_{s,t+l} N_{s,j,t+l} - Z_{s,t+l} K_{s,j,t+l} \right] \\ \text{s.t.} \quad & Y_{s,j,t} = \left( \frac{P_{s,j,t}}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t \\ & Y_{s,j,t} = A_t (K_{s,j,t})^{1-\chi} (N_{s,j,t})^\chi, \\ & \log A_t = \rho_A \log A_{t-1} + \sigma_{\varepsilon_a} \varepsilon_{a,t}, \end{aligned}$$

where  $\chi$  is the elasticity of output with respect to labor,  $A_t$  is a technology shock,  $\rho_A \in [0, 1]$  is the shock persistence, and  $\varepsilon_{a,t}$  is a zero-mean, unit-variance *i.i.d.* shock.

For brevity, we refrain from presenting all equilibrium conditions and refer the reader to [Carvalho and Nechio \(2016\)](#) for details. We note that as a direct implication of the assumption of sector specificity in capital and labor markets, firms' real marginal costs are equalized at the sectoral level.

## 3.2 Firm-specific and economy-wide factors

To consider the firm-specific and the economy-wide multisector economy, one needs to reformulate the consumers' and intermediate firms' problems. In the firm-specific case, in equilibrium, returns to capital and labor are specific at the firm level. In contrast, in the economy-wide case, capital and labor returns are equalized across firms and sectors. Therefore, differently from the model of [Section 3.1](#), in the firm-specific model, firms' real marginal costs are firm-specific, while in the economy-wide model, firms' real marginal costs are equalized across firms and sectors. For additional details about all three models, see [Carvalho and Nechio \(2016\)](#).

### 3.3 Monetary policy

To close the model, as in the benchmark version of the models presented in [Carvalho and Nechio \(2016\)](#), we assume that the growth rate of nominal aggregate demand follows a first-order autoregressive,  $AR(1)$ , process, thus, leaving monetary policy implicit.<sup>4</sup> Denoting nominal aggregate demand by  $M_t \equiv P_t Y_t$ :

$$\Delta m_t = \rho_m \Delta m_{t-1} + \sigma_{\varepsilon_m} \varepsilon_{m,t},$$

where  $m_t \equiv \log(M_t)$ ,  $\rho_m$  determines the autocorrelation in nominal aggregate demand growth, and  $\varepsilon_{m,t}$  is a purely monetary, uncorrelated, zero-mean, unit-variance *i.i.d.* shock. In [Section 6](#) we consider an alternative specification for monetary policy.

All model versions are solved and analyzed using a loglinear approximation around a zero-inflation steady state.

### 3.4 Parameterization

Aside from the distribution of price stickiness, which we detail in the next two sections, we parameterize the three models as in [Carvalho and Nechio \(2016\)](#) and, unless otherwise stated, keep those parameters unchanged in the different variants of the models entertained below.

In particular, we set the intertemporal elasticity of substitution  $\sigma^{-1}$  to  $1/2$ , the (inverse) labor supply elasticity  $\gamma$  to  $0.5$ , the elasticity of output with respect to labor to  $\chi = 2/3$ , and the consumer discount factor  $\beta$  such that it implies a time-discount rate of  $4\%$  per year.

We set the elasticity of substitution between varieties of the same sector to  $\theta = 7$ . Relying on the estimates provided by [Hobijn and Nechio \(2017\)](#), we set the elasticity of substitution across sectors to unity,  $\eta = 1$  (*i.e.* the aggregator that converts sectoral into final output is Cobb-Douglas).

To specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of  $\rho_m$  typically fall in the range of  $0.4$  to  $0.7$ , which maps into a range of roughly  $0.75 - 0.90$  at a monthly frequency.<sup>5</sup> We set  $\rho_m = 0.8$ , and the standard deviation of the shocks  $\sigma_{\varepsilon_m} = 0.6\%$  (roughly  $1\%$  at a quarterly frequency), in line with the same estimation results.<sup>6</sup> For technology shocks, we set  $\rho_a = 0.965$  and the standard deviation of the shocks  $\sigma_{\varepsilon_m} = 0.6\%$  (roughly  $1\%$  at a quarterly frequency).

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<sup>4</sup>This is a common assumption in the literature (e.g. [Mankiw and Reis, 2002](#)).

<sup>5</sup>See, for instance, [Mankiw and Reis \(2002\)](#).

<sup>6</sup>All results for volatilities scale-up proportionately with  $\sigma_{\varepsilon_m}$ .

Finally, we calibrate investment adjustment costs,  $\kappa$ , such that each model roughly matches the standard deviation of investment relative to the standard deviation of GDP in the data. For that, we consider monetary and technology shocks one at a time. When analyzing the response to monetary shocks, we set  $\kappa$  to 40, while when analyzing the response to technology shocks, we set  $\kappa$  to 15. The remaining parameter values are unchanged from the baseline parameterization when considering one or the other shock.

To be able to assess our approximation approach, it remains to specify the original distributions of price stickiness to be approximated. In what follows, we construct approximate distributions based on the U.S. empirical distribution in Sections 4, and based on artificially-generated arbitrary distributions of price stickiness in Section 5.

## 4 Approximating the U.S. empirical distribution

In this section we resort to the available microeconomic evidence on price rigidity in the U.S. to pin down the distribution of price rigidity across sectors.

In particular, we use the statistics on the frequency of regular price changes — those that are not due to sales or product substitutions — reported by [Nakamura and Steinsson \(2008\)](#). More specifically, the U.S. distribution of price stickiness we consider consists of weights and frequencies of price changes for 67 expenditure classes of goods and services.<sup>7</sup> In the model, each class is identified with a sector.

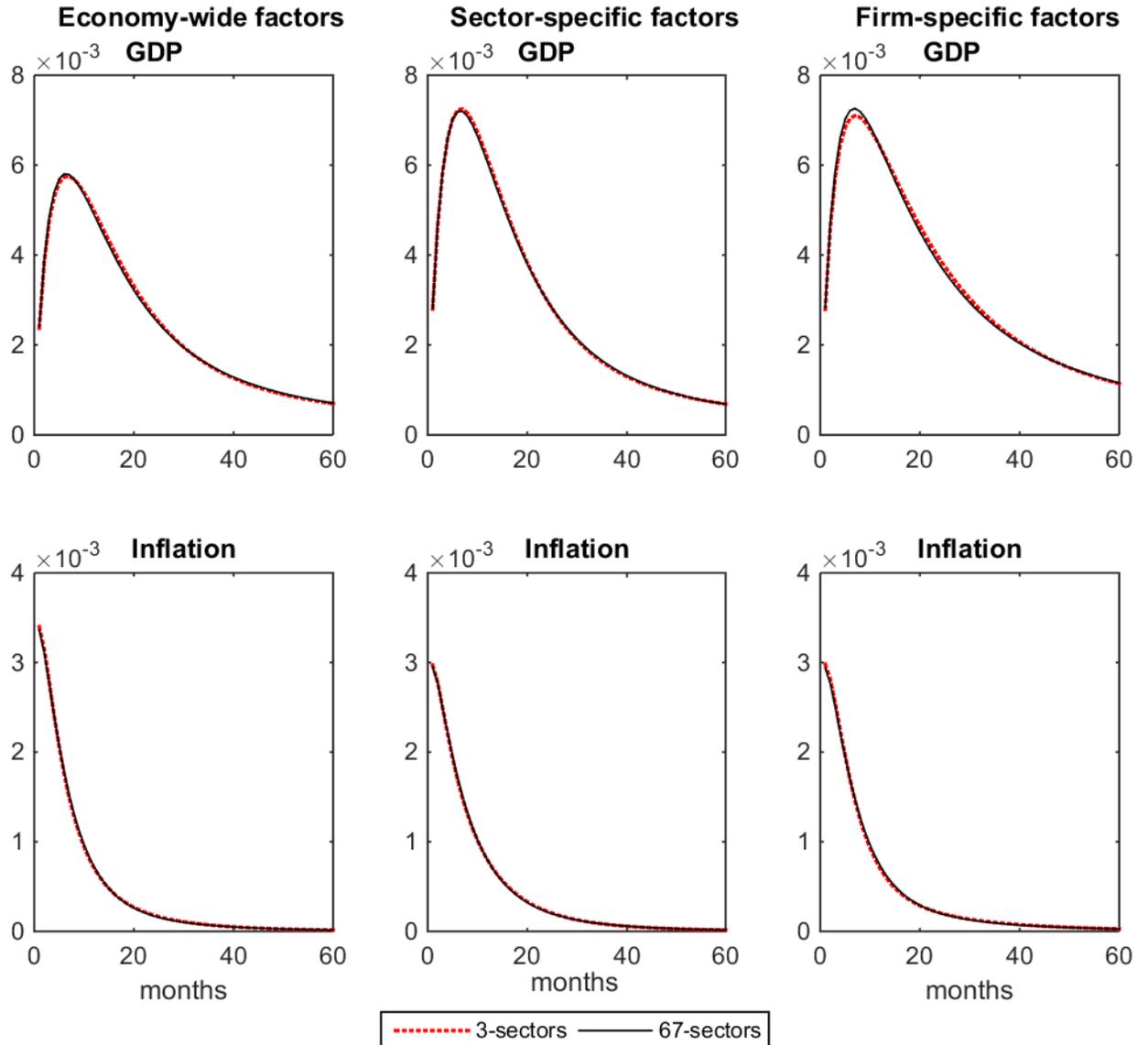
Following the approach described in Section 2, we construct a three-sector approximate distribution. The original and the artificial distributions are such that the average frequency of price changes equals 0.2 (which implies prices changing, on average, once every 4.7 months), the cross-sectional average of durations of price spells equals 11.9 months, the cross-sectional standard deviation of durations of price spells equals 9.3 months, the cross-sectional skewness of durations of price spells equals 0.7 months, and the cross-sectional kurtosis of durations of price spells equals 2.5 months

Figure 1 shows the impulse response functions (IRFs) to a monetary shock for the three

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<sup>7</sup>Note that we aggregate the original 272 categories from [Nakamura and Steinsson \(2008\)](#) up into their corresponding 67 expenditure classes of goods and services. To do so, we discard the category “Girls’ Outerwear,” for which the reported frequency of regular price changes is zero, aggregate up based on the 67 expenditure classes, and renormalize the expenditure weights to sum to unity. The frequency of price changes for each expenditure class is obtained as the weighted average of the frequencies for the underlying categories, using the expenditure weights provided by [Nakamura and Steinsson \(2008\)](#). Expenditure-class weights are given by the sum of the expenditure weights for those categories. As an example of what this aggregation entails, the resulting “New and Used Motor Vehicles” class consists of the categories “Subcompact Cars”, “New Motorcycles”, “Used Cars”, “Vehicle Leasing” and “Automobile Rental”; the “Fresh Fruits” class comprises four categories: “Apples”, “Bananas”, “Oranges, Mandarins etc.” and “Other Fresh Fruits.”

models; economy-wide, sector-, and firm-specific factor markets. For brevity, the figure only reports the IRFs for real GDP and inflation. The approximation for other variables are provided in the Appendix Figures A1 to A3.<sup>8</sup>



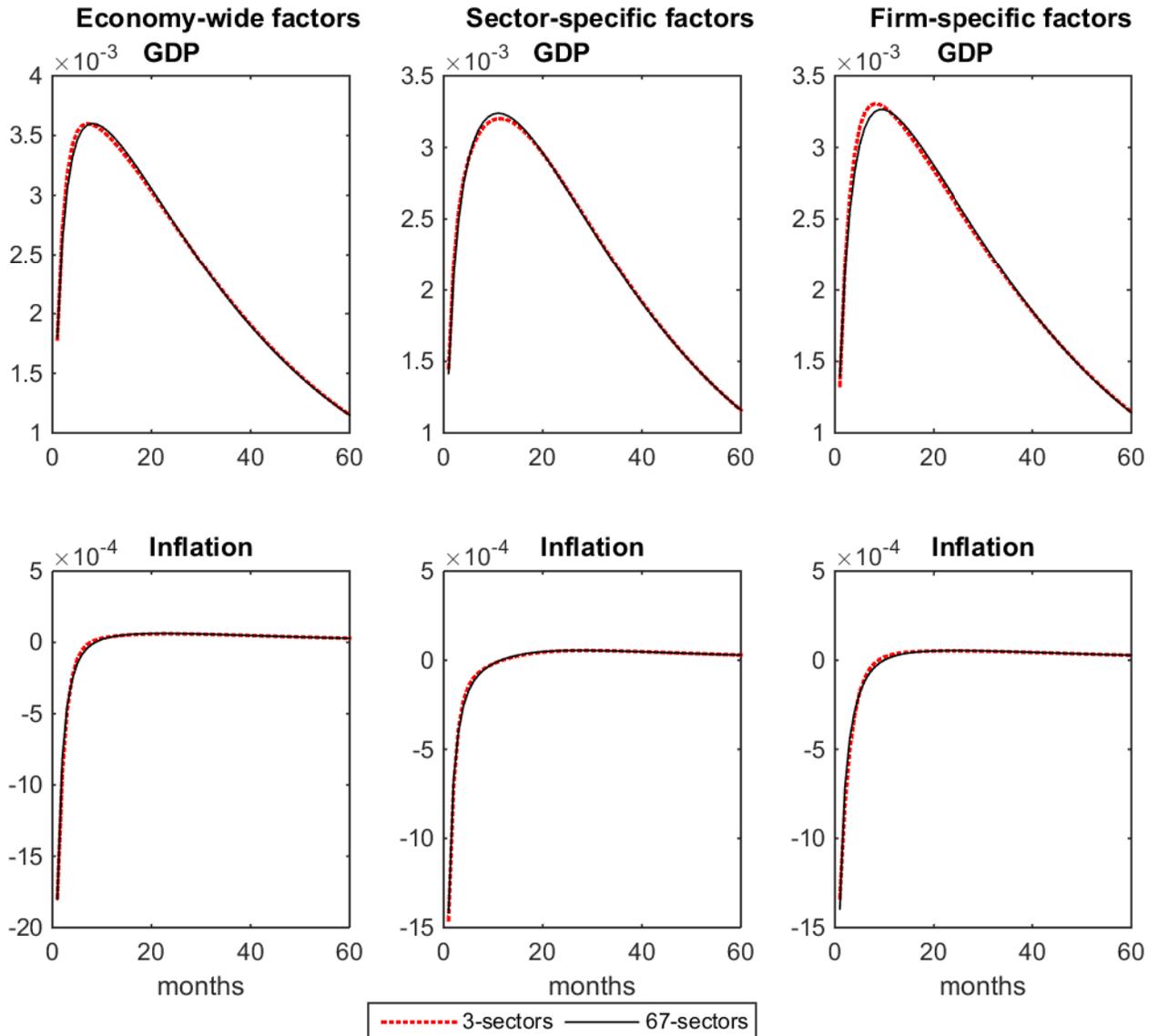
**Figure 1:** Impulse response functions in the 67-sector and the approximate three-sector economies to a monetary shock.

The panels show that for all three models, the three-sector model calibrated with the approximate distribution does a great job in replicating the impulse response functions of output

<sup>8</sup>Note that we aggregate the original 272 categories from Nakamura and Steinsson (2008) up into their corresponding 67 expenditure classes of goods and services. Approximating directly from the 271-sector distribution from Nakamura and Steinsson (2008) yields similar results, as reported in the Appendix Figures A7 and A8.

and inflation (and similarly for the other aggregate variables in the model). The largest approximation error is attained at the firm-specific model and equals 0.0006.

Figure 2 shows the impulse response functions of GDP and inflation to a technology shock for the same three models. Results are qualitatively similar. For all model versions, the three-sector model calibrated with the approximate distribution tracks very closely the dynamics of the impulse response functions of output and inflation in the original 67-sector models. The approximation for other variables are provided in the Appendix Figures A4 to A6.



**Figure 2:** Impulse response functions in the 67-sector and the approximate three-sector economies to a technology shock.

## 5 Approximating arbitrary distributions

The results above are based on an approximate three-sector distribution obtained from the U.S. empirical evidence. In this section we consider our approximation from artificially-generated distributions of price stickiness. To that end, we construct 67-sector distributions in which both the frequency of price adjustment and the sectoral weight are chosen arbitrarily. In particular, we start by constructing a vector of frequencies of price changes that ranges from 0.999 to 0.008, declining in equal steps. Therefore, in this artificial 67-sector distribution, price changes range from, on average, once a month, in the most flexible sector, to once every 120 months, in the stickiest sector. Next, we associate a set of arbitrary sectoral weights to this vector of frequencies. More specifically, we generate sectoral weights from a *Beta* distribution in which both shape parameters range from 0.5 to 10, in incremental steps of 0.25. This combination of shape parameters yields a set of 1521 arbitrary vectors of sectoral weights. Finally, each vector of weights is paired with the arbitrary vector of frequencies of price changes. This pairing yields 1521 sets of 67-sector price stickiness distributions.<sup>9</sup>

We, then, turn to the approximation for each of these arbitrary distributions. For that, we follow the same steps as in Section 4. More specifically, for each arbitrary (67-sector) distribution, we calculate the five empirical moments described in Section 2 and obtain its corresponding approximate three-sector distribution. Next, we parameterize the models of Section 3 under the original and its approximate arbitrary distribution, and compare the responses to monetary and technology shocks under the two parameterizations.

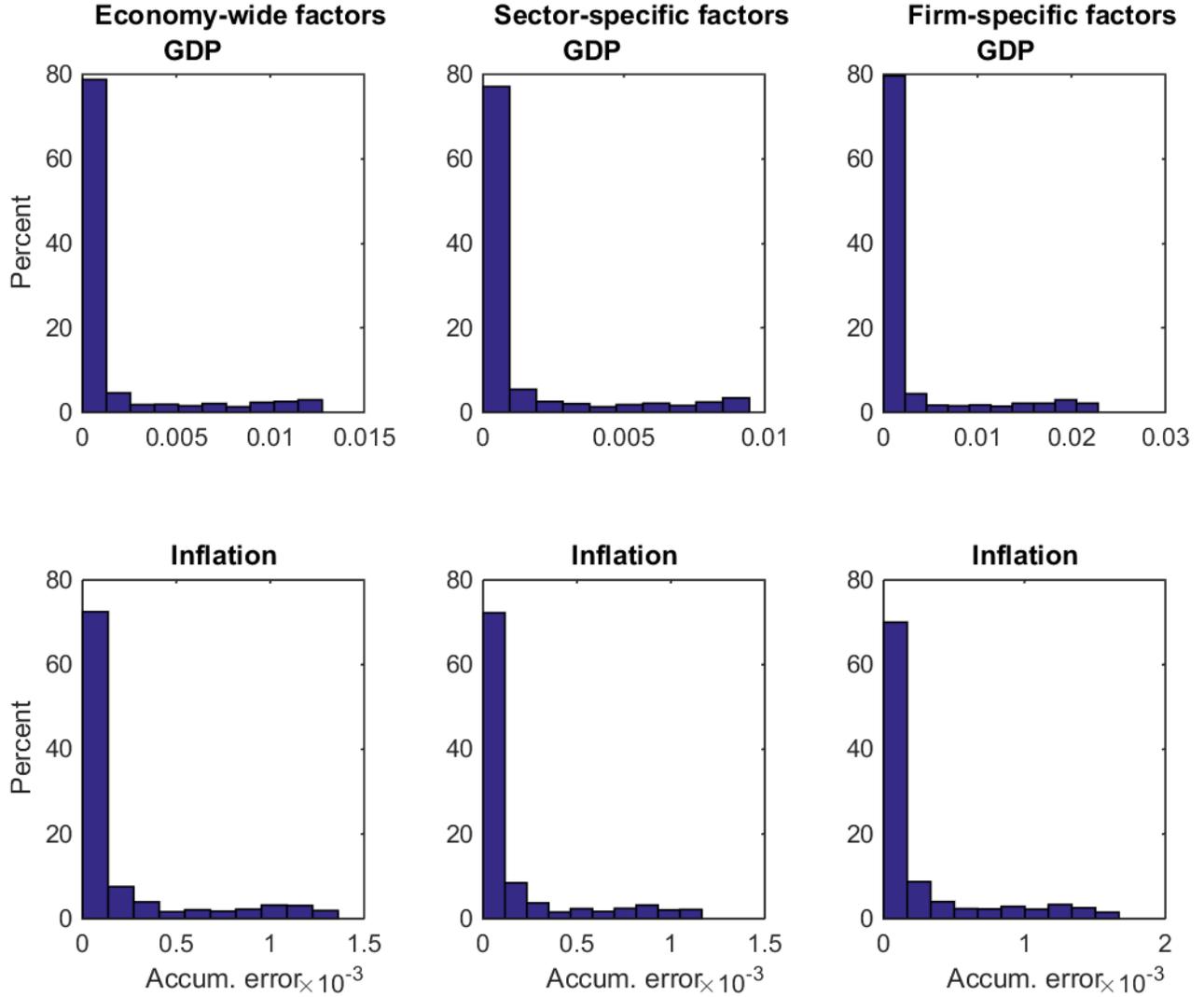
The exercise described above yields 1521 original and approximate parameterizations of the three DSGE economies of Section 3. While we could, in principle, compare how each approximation performs for each economic variable over time (in analogy to Figures 1 and 2), we, instead, report a summary of our results. In particular, for each model, variable and approximation, we calculate the sum over time of the difference between the response of the 67-sector and the approximate three-sector models. This yields a distribution of the cumulative approximation errors for each aggregate variable in the different models.

Figures 3 and 4 report the main results. They show the histograms of the cumulative approximation errors for real GDP and inflation for the three models in response to a monetary and technology shock, respectively. Both Figures show that, for all three models of factor specificity, the three-sector approximation performs extremely well. In all cases, the cumulative approximation error across all arbitrary price stickiness distributions is in the order of  $10^{-3}$  to  $10^{-2}$ . The Appendix Figures A9 to A14 confirm that these results also hold for the other main

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<sup>9</sup>Note that all distributions have the same vector of price stickiness but vary in the set of sectoral weights.

aggregate variables in the models.

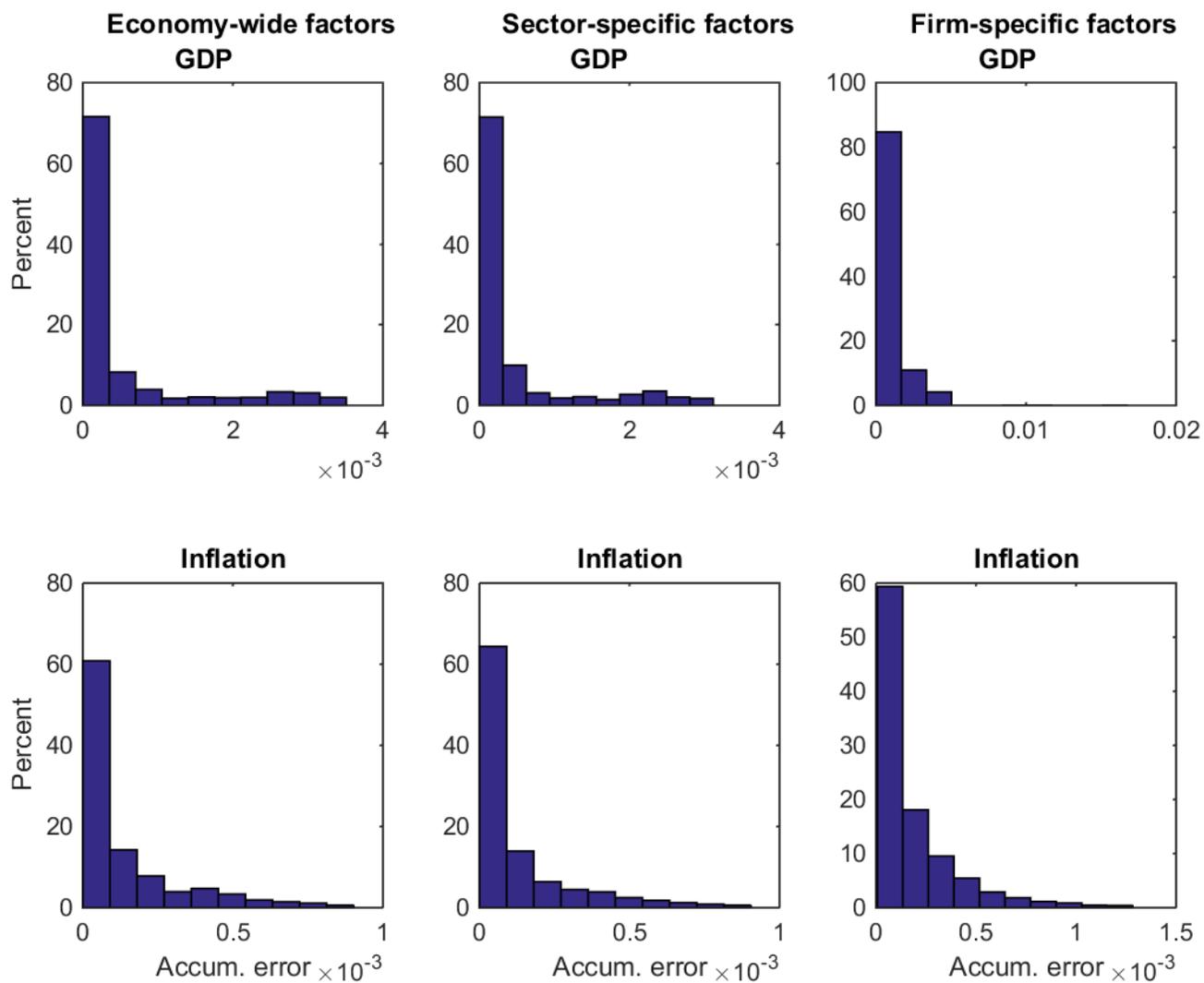


**Figure 3:** Distribution of the cumulative approximation errors in response to a monetary shock.

## 6 Robustness

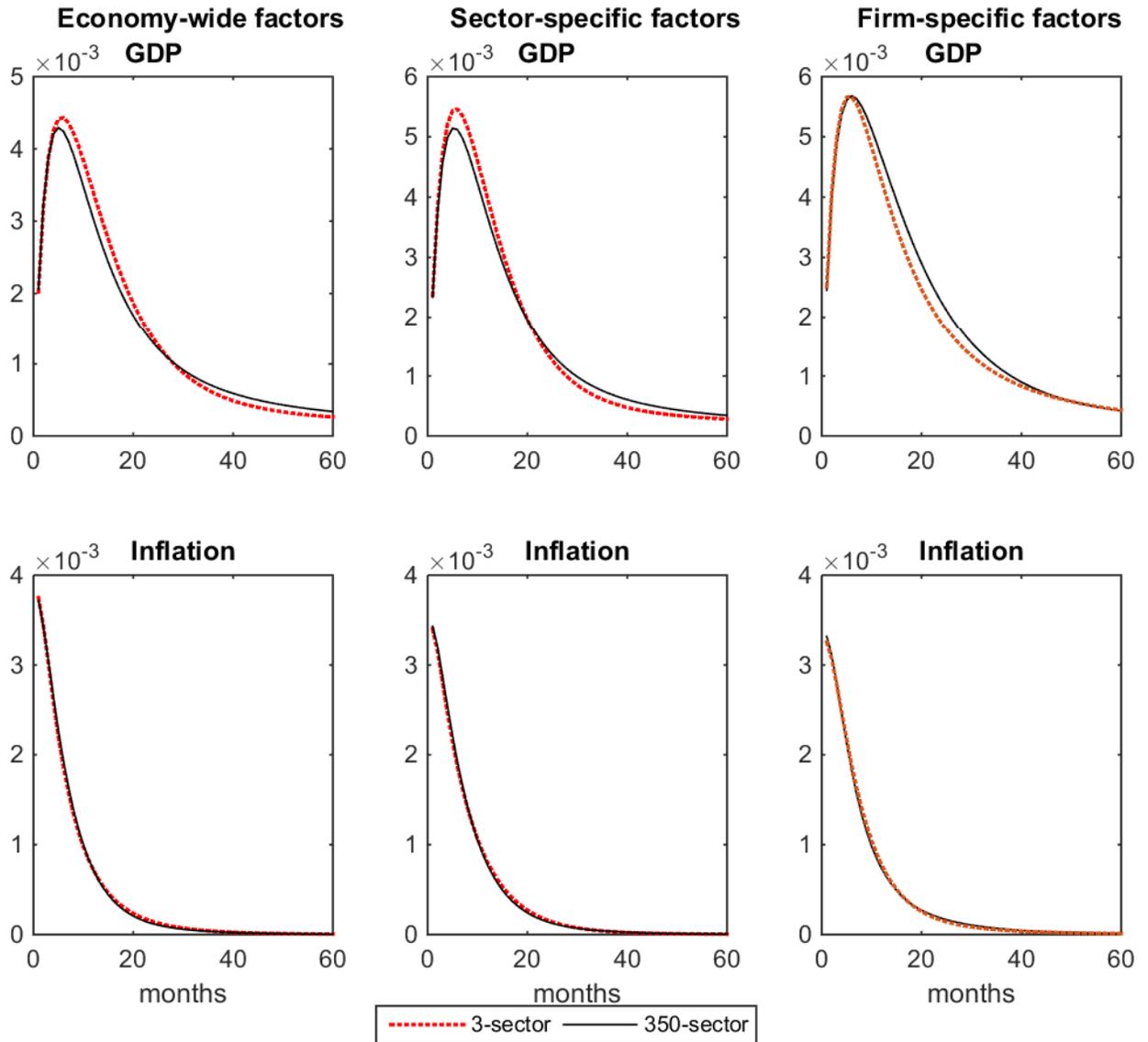
To test for the robustness of our results, in this section we consider an alternative distribution of price stickiness and variations to the models' specifications.

In particular, we start by performing our approximation approach on the price stickiness distribution from [Bils and Klenow \(2004\)](#). We follow the same steps as described in Sections 2 and 4 and construct a three-sector distribution of price stickiness that features the same five moments (described in Section 2) as the original distribution of 350-categories from [Bils](#)



**Figure 4:** Distribution of the cumulative approximation errors in response to a technology shock.

and Klenow (2004). In particular, the original and the artificial distributions are such that the average frequency of price changes equals 0.3 (which implies prices changing, on average, once every 3.8 months), the cross-sectional average of durations of price spells equals 7 months, the cross-sectional standard deviation of durations of price spells equals 7.8 months, the cross-sectional skewness of durations of price spells equals 4.1 months, and the cross-sectional kurtosis of durations of price spells equals 36.9 months. Results, reported in Figure 5, confirm that our approximation approach performs really well.



**Figure 5:** Impulse response functions in the 350-sector and the approximate three-sector economies: monetary shock.

Next, we consider alternative model specifications. More specifically, we assume that mon-

etary policy is conducted according to an interest rate rule subject to persistent shocks:

$$IR_t = \beta^{-1} \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left( \frac{GDP_t}{GDP_t^n} \right)^{\phi_y} \epsilon_{v,t},$$

where  $IR_t$  is the short-term nominal interest rate,  $GDP_t$  is gross domestic product,  $GDP_t^n$  denotes gross domestic product when all prices are flexible,  $\phi_\pi$  and  $\phi_y$  are the parameters associated with Taylor-type interest-rate rules.  $\epsilon_{v,t}$  is a persistent shock with process  $\epsilon_{v,t} = \rho_v \epsilon_{v,t-1} + \sigma_{\epsilon_v} \varepsilon_{v,t}$ , where  $\varepsilon_{v,t}$  is a zero-mean, unit-variance *i.i.d.* shock, and  $\rho_v \in [0, 1)$ .

To calibrate the version of the model with the above interest rate rule, we set  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5/12$ , and  $\rho_v = 0.965$ , and set the investment cost parameter  $\kappa$  to 45, which roughly implies that the model matches the standard deviation of investment in the data relative to the standard deviation of GDP. The remaining parameter values are unchanged from the baseline specification.

We, then, follow the same steps as described in Sections 4 and 5, and assess our approximation using the U.S. empirical distribution of price stickiness and the set of arbitrary distributions, respectively.

Figure 6 shows the impulse response functions of GDP and inflation to an expansionary monetary shock for the three models of Section 3. Figure 7 reports the distribution of the cumulative approximation errors obtained when the three models are calibrated using the arbitrary distributions described in Section 5.<sup>10</sup> The approximation in the DSGE models assuming an interest rule is quite accurate, with the three-sector approximate economy following closely the dynamics of the original one. Figure 7 reinforces our conclusions.

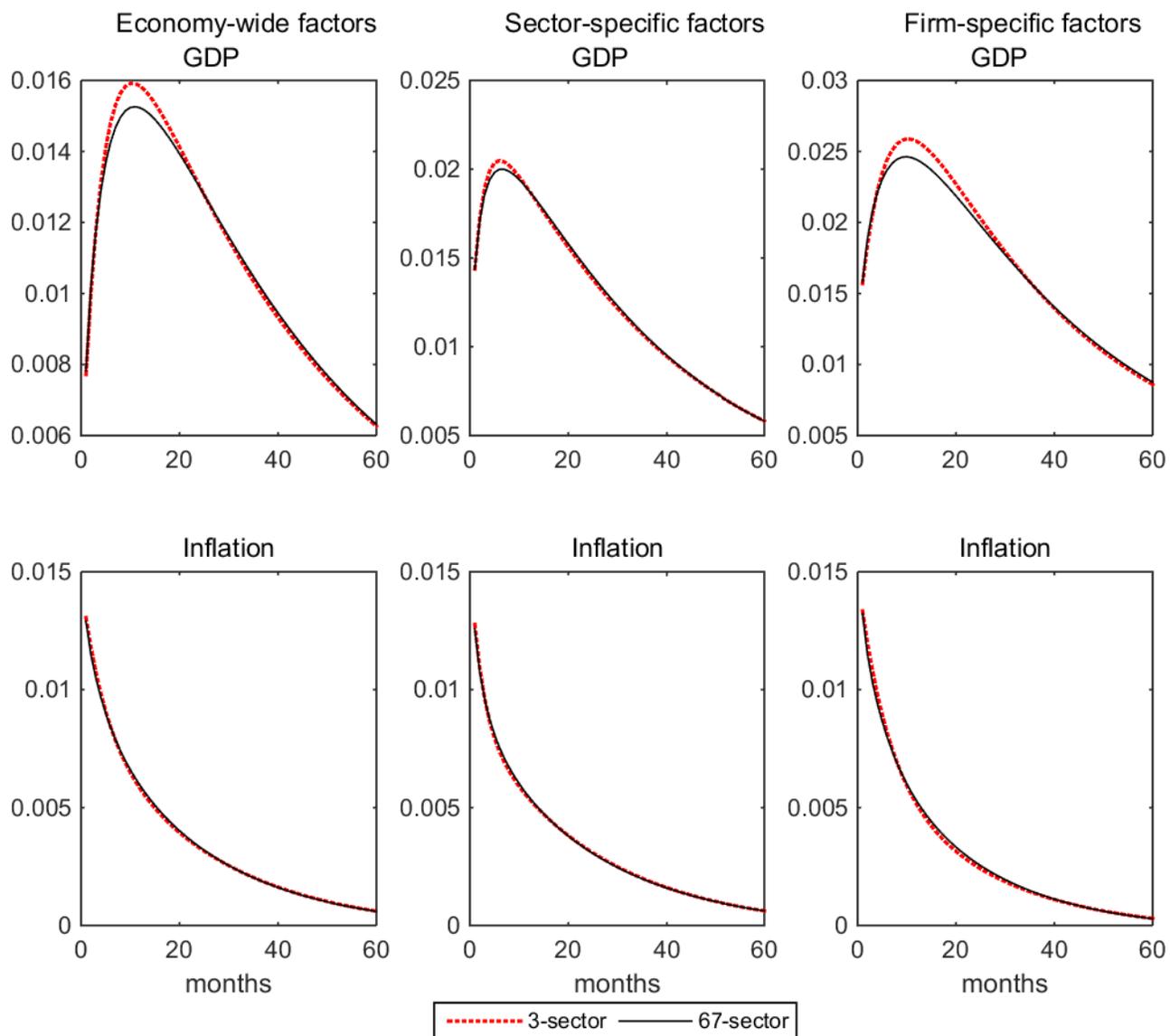
Finally, we also entertain a version of the three models in which we assume a labor-only economy by excluding capital accumulation, and a version in which we modify the Taylor rule to allow for policy inertia. In all cases, our results are qualitatively unchanged and are available upon request.

## 7 Conclusion

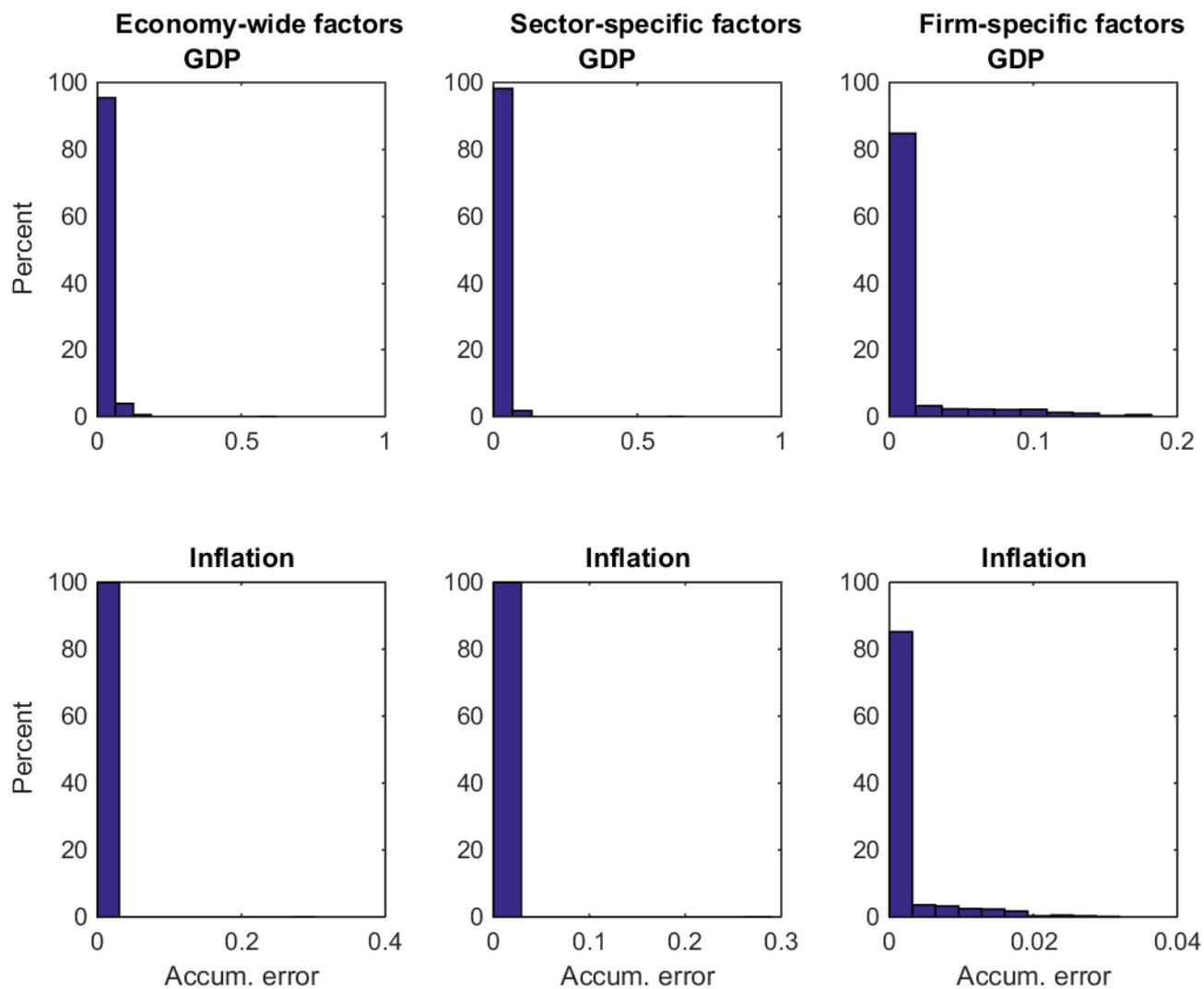
Our results show that a calibrated model with a suitably chosen three-sector distribution of price stickiness can closely approximate the dynamic properties of New Keynesian models calibrated with a much larger distribution. In particular, the set of sectoral weights and frequencies of price adjustment in the approximate distribution are such that the latter and the original distributions share the same (i) average frequency of price changes, (ii) cross-sectional average

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<sup>10</sup>The approximation for all other variables are provided in the Appendix Figures A15 to A20.



**Figure 6:** Impulse response functions in the 67-sector and the approximate three-sector economies: interest rate rule.



**Figure 7:** Distribution of the cumulative approximation errors in response to a monetary shock: interest rate rule.

of durations of price spells, (iii) cross-sectional standard deviation of durations of price spells, (iv) cross-sectional skewness of durations of price spells, and (v) cross-sectional kurtosis of durations of price spells of the original distribution of price stickiness. Under those conditions, we show that, for alternative multisector models with different assumptions for factor inputs mobility, type of shocks and monetary policy specifications, the original and the approximate distributions yield impulse response functions that are remarkably close. This result should prove useful to the literature that takes into account heterogeneity of price stickiness in DSGE models. In particular, it should allow for the estimation of such models at a much reduced computational cost.

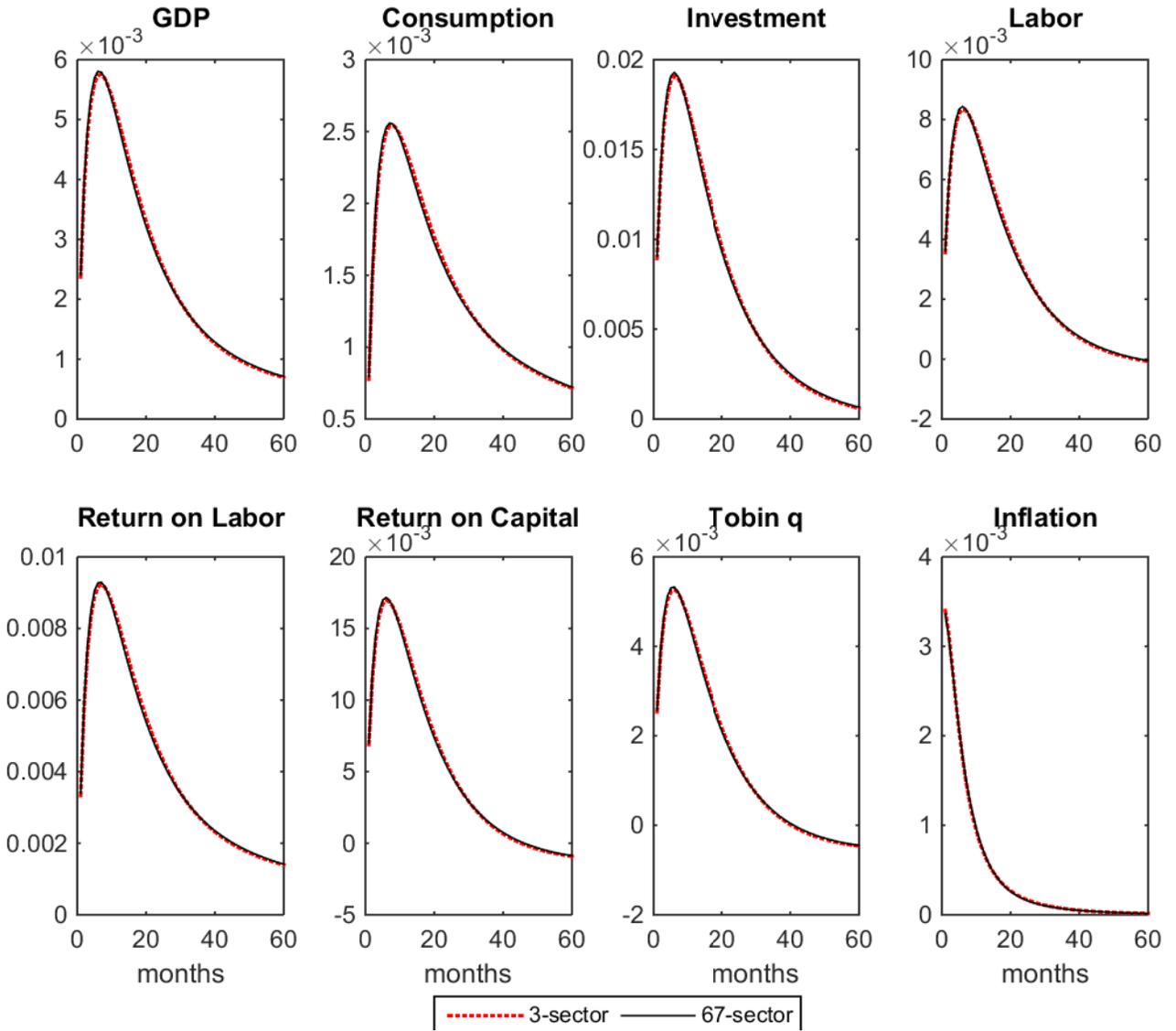
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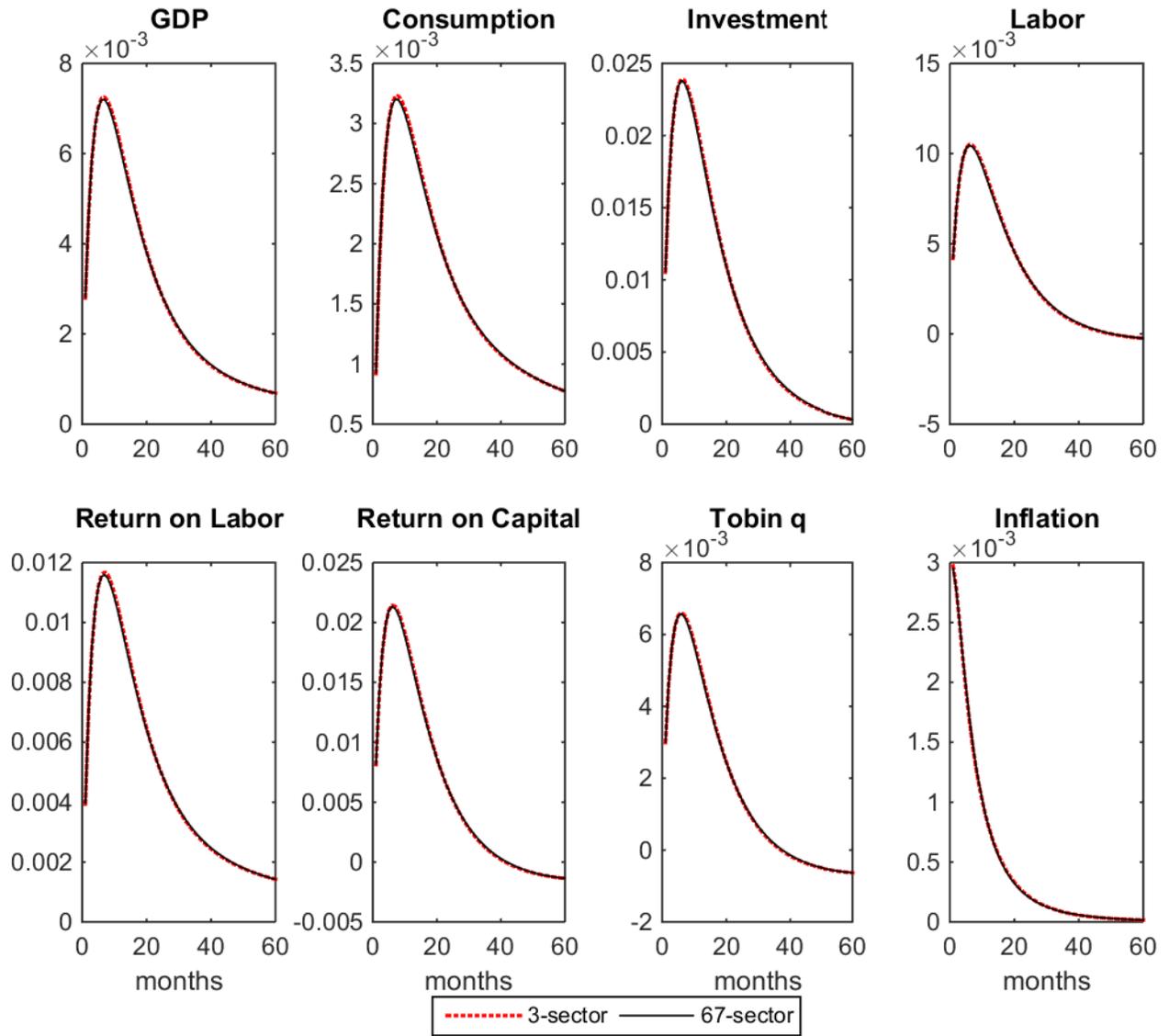
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# A Appendix

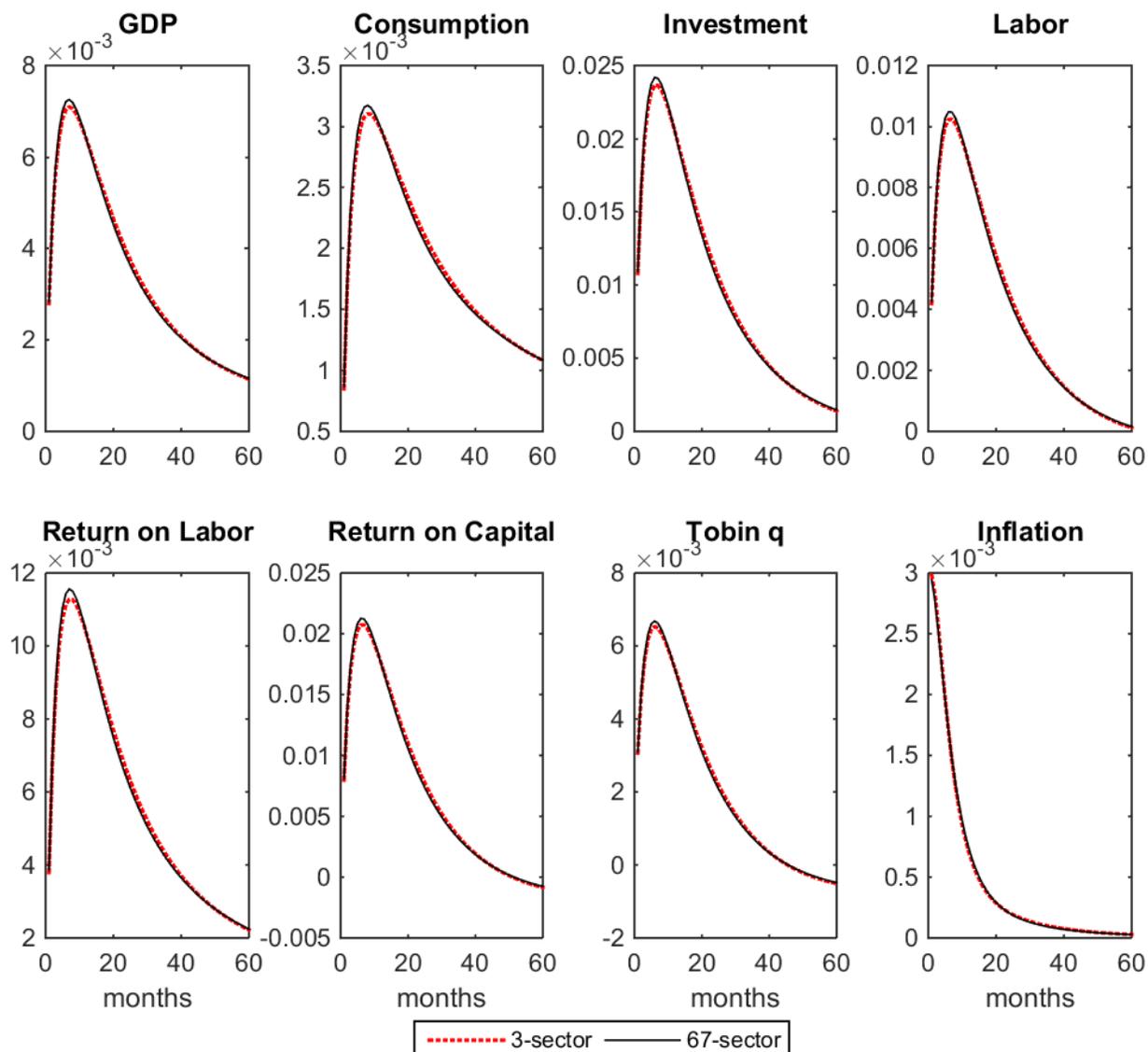
## A.1 Additional evidence



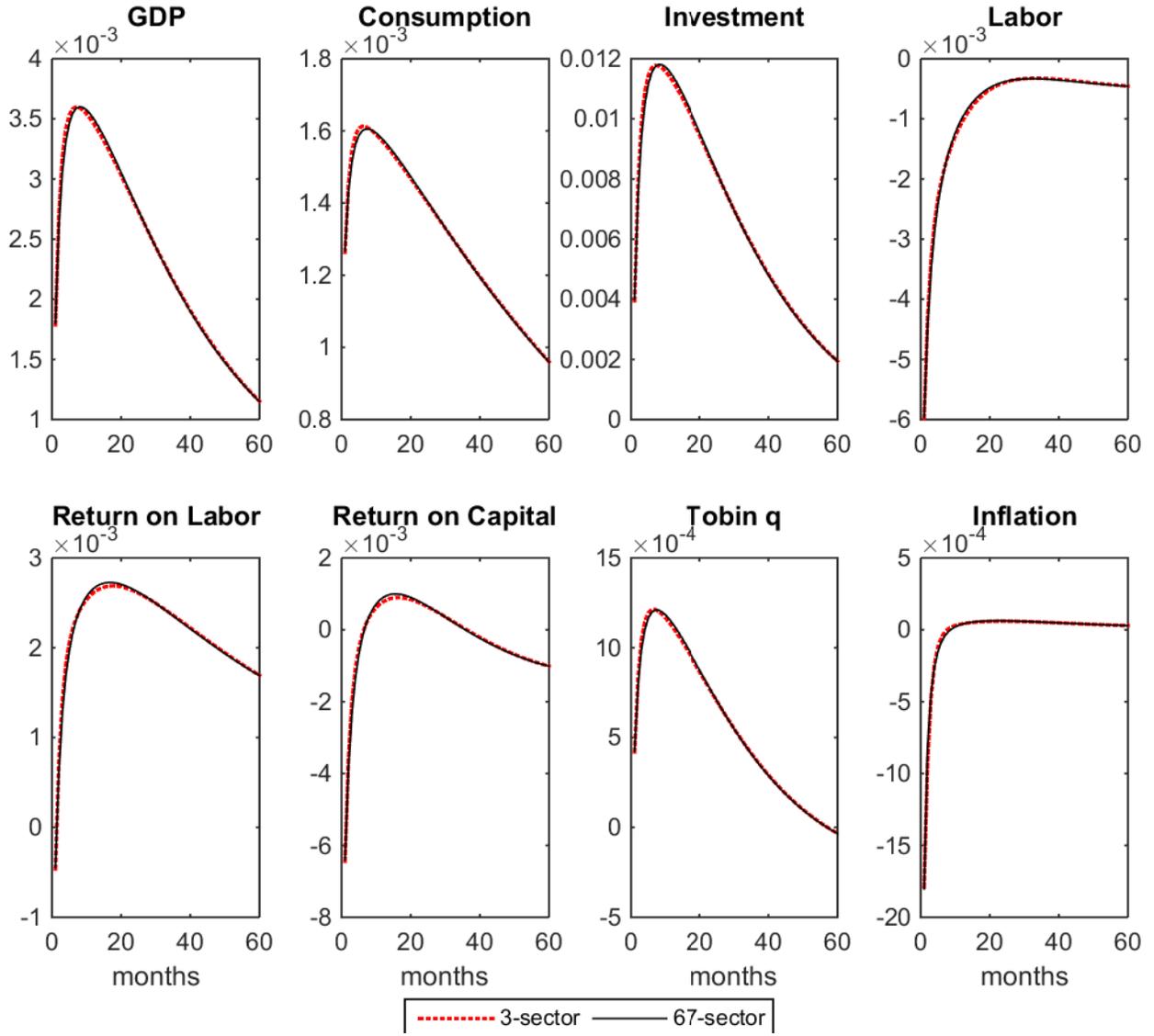
**Figure A1:** Impulse response functions in the 67-sector and the approximate three-sector economies – economy-wide factor markets: monetary shock.



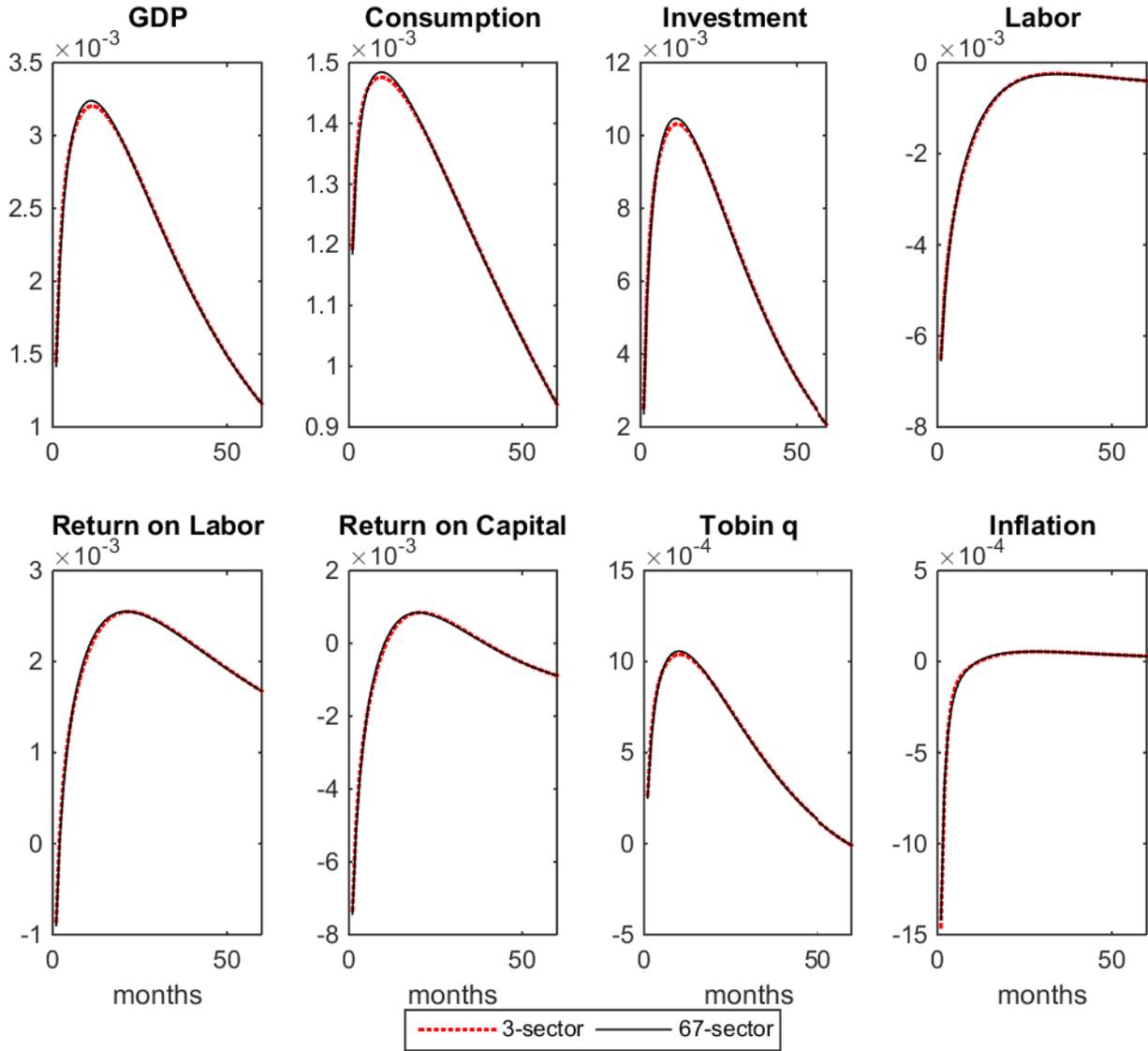
**Figure A2:** Impulse response functions in the 67-sector and the approximate three-sector economies – sector-specific factor markets: monetary shock.



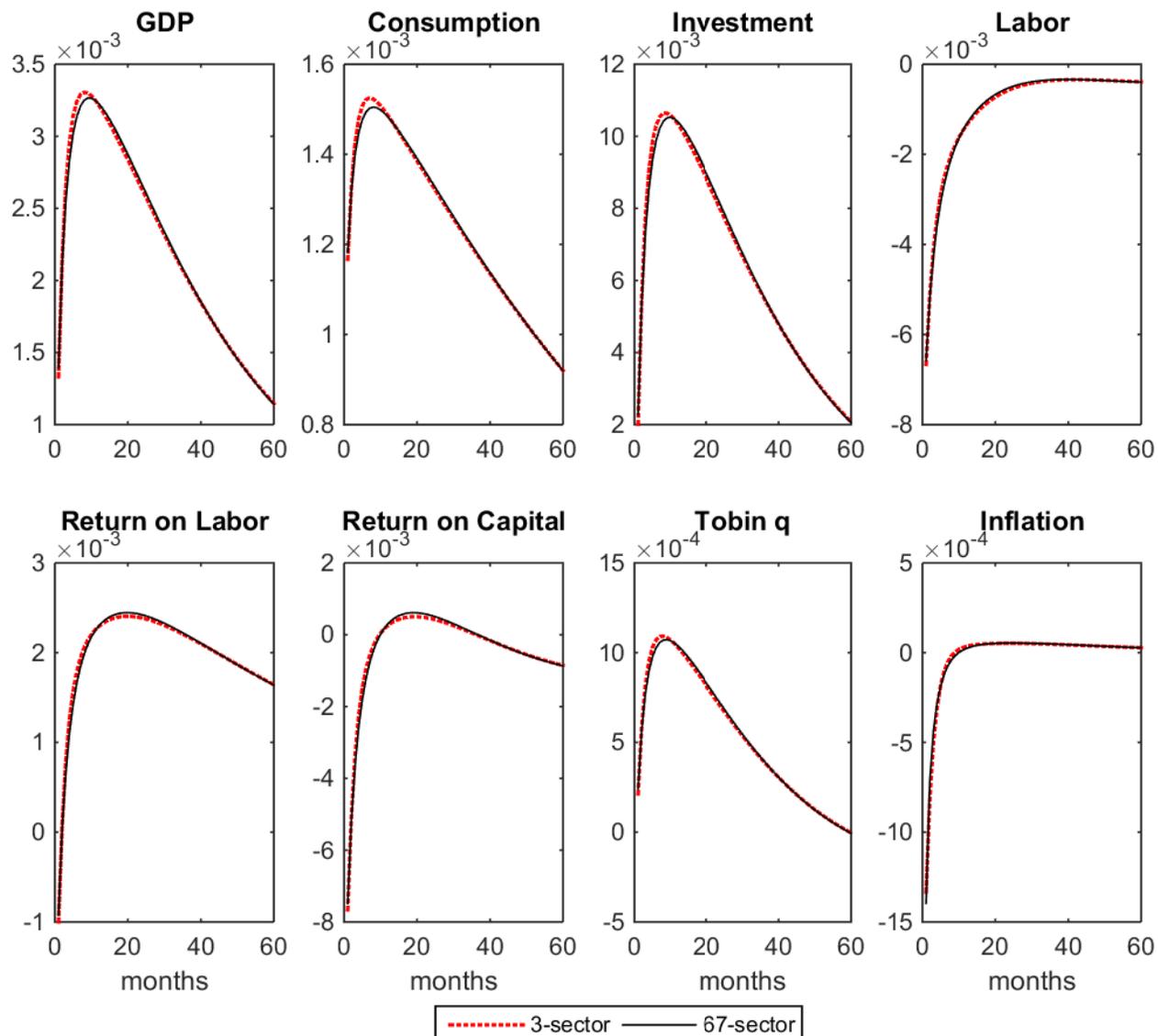
**Figure A3:** Impulse response functions in the 67-sector and the approximate three-sector economies – firm-specific factor markets: monetary shock.



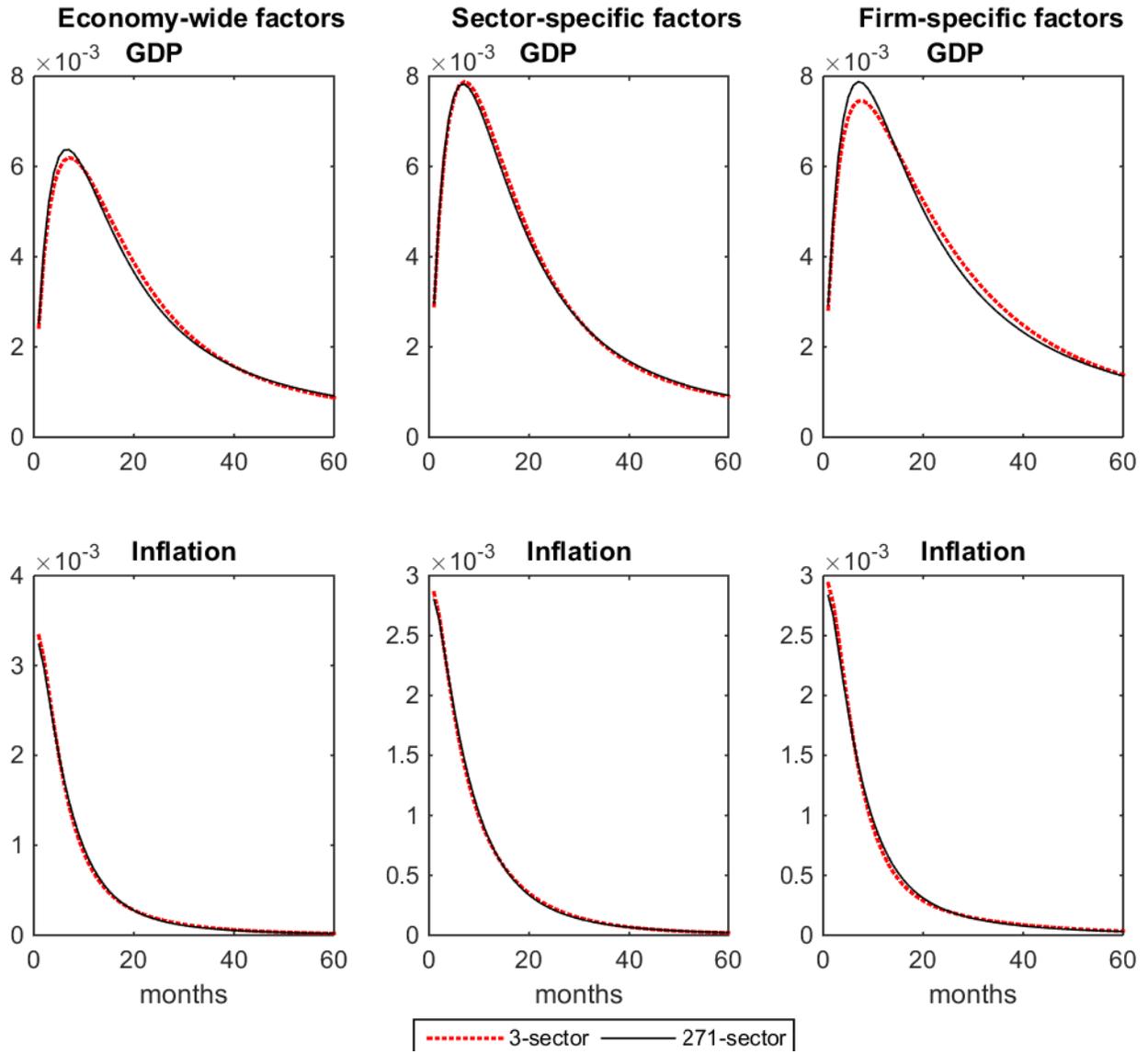
**Figure A4:** Impulse response functions in the 67-sector and the approximate three-sector economies – economy-wide factor markets: technology shock.



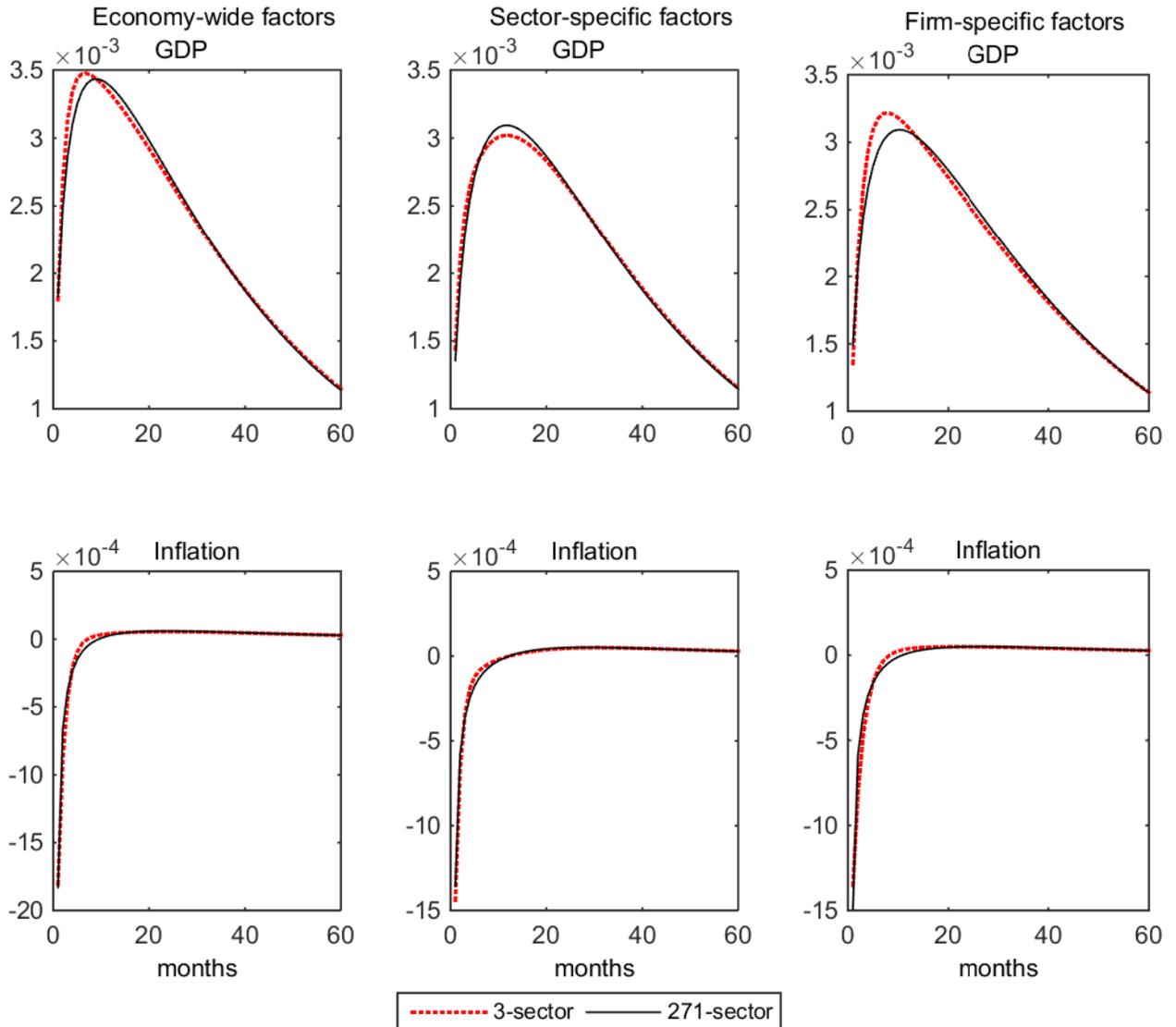
**Figure A5:** Impulse response functions in the 67-sector and the approximate three-sector economies – sector-specific factor markets: technology shock.



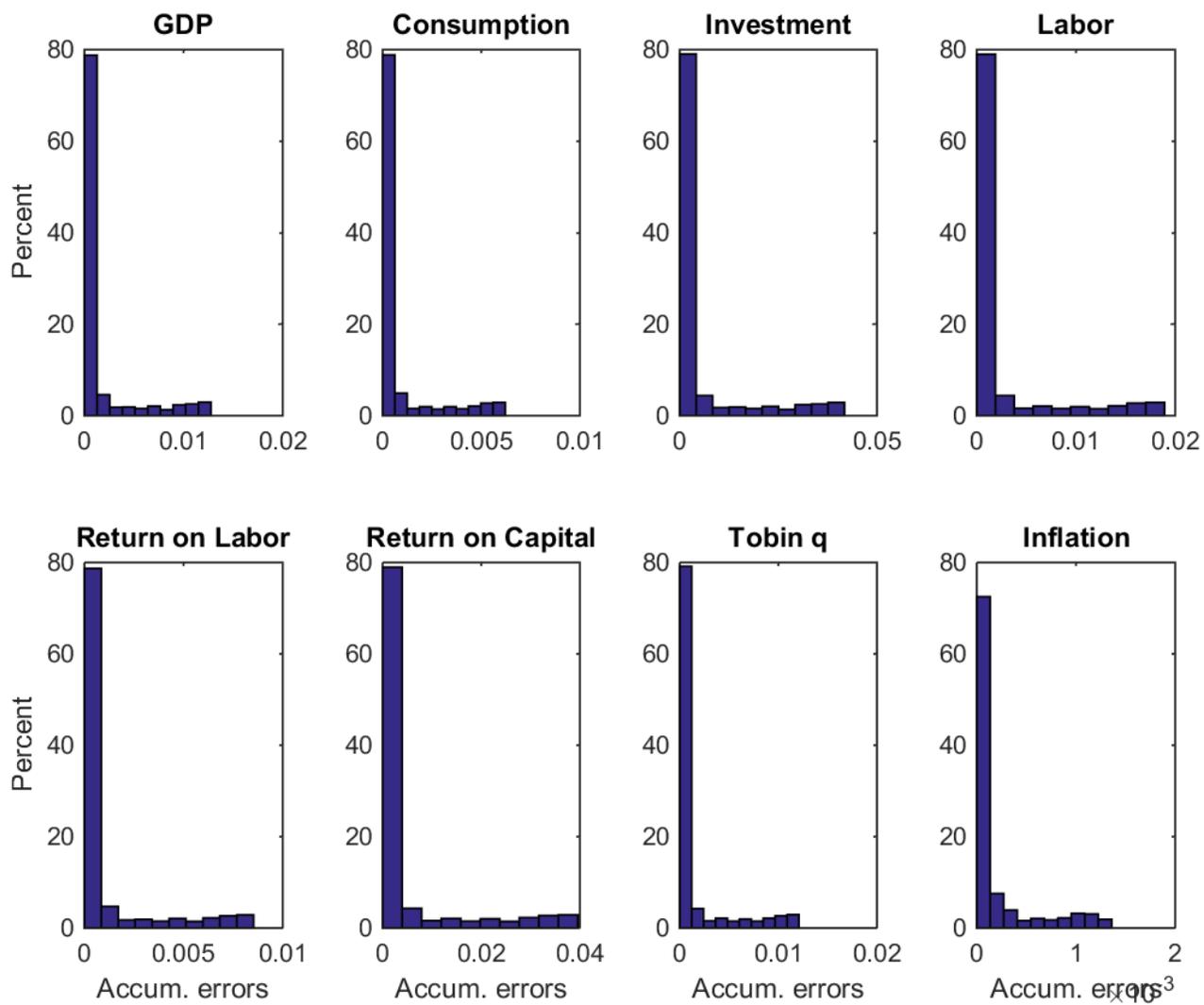
**Figure A6:** Impulse response functions in the 67-sector and the approximate three-sector economies – firm-specific factor markets: technology shock.



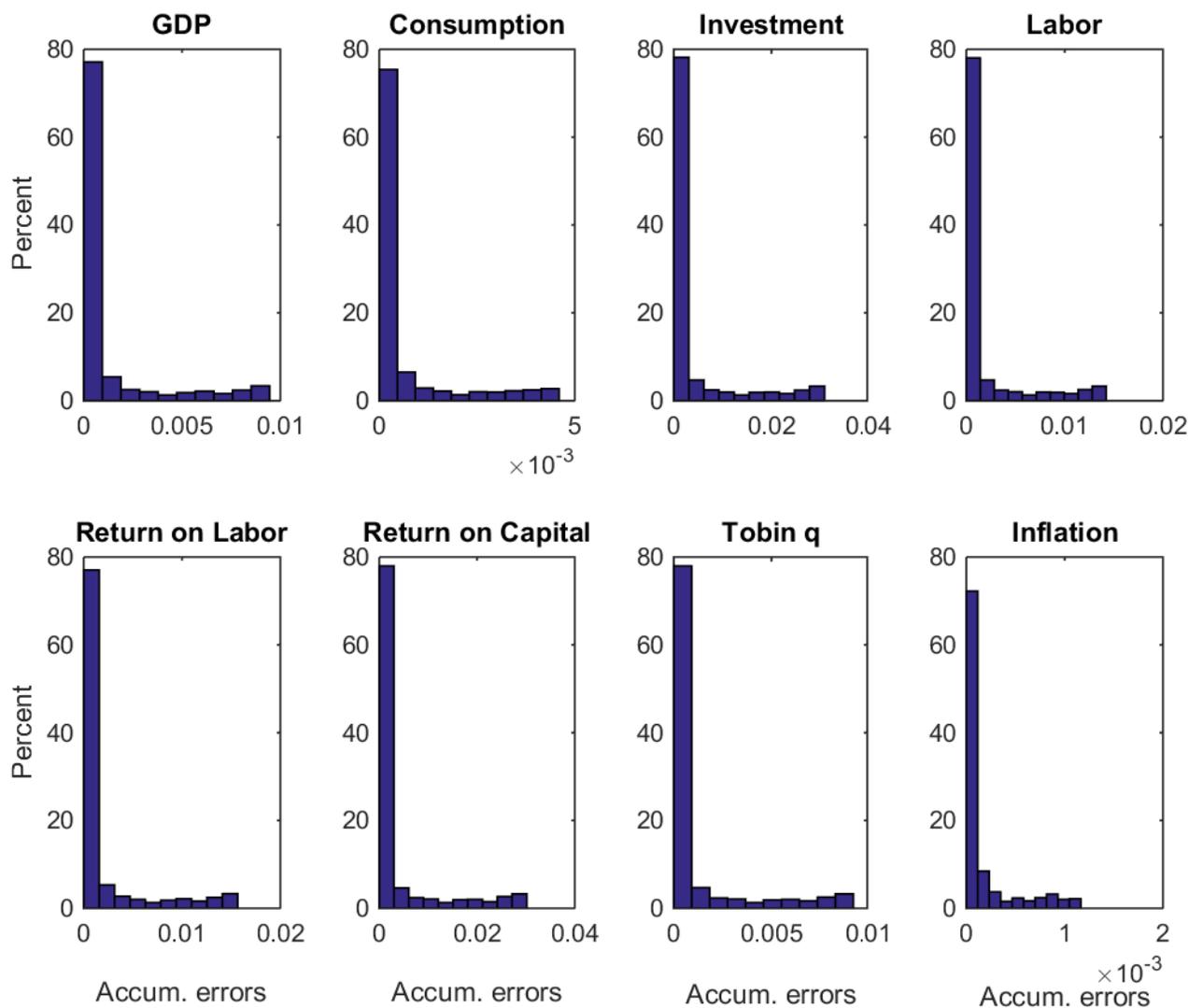
**Figure A7:** Impulse response functions in the 271-sector and the approximate three-sector economies: monetary shock.



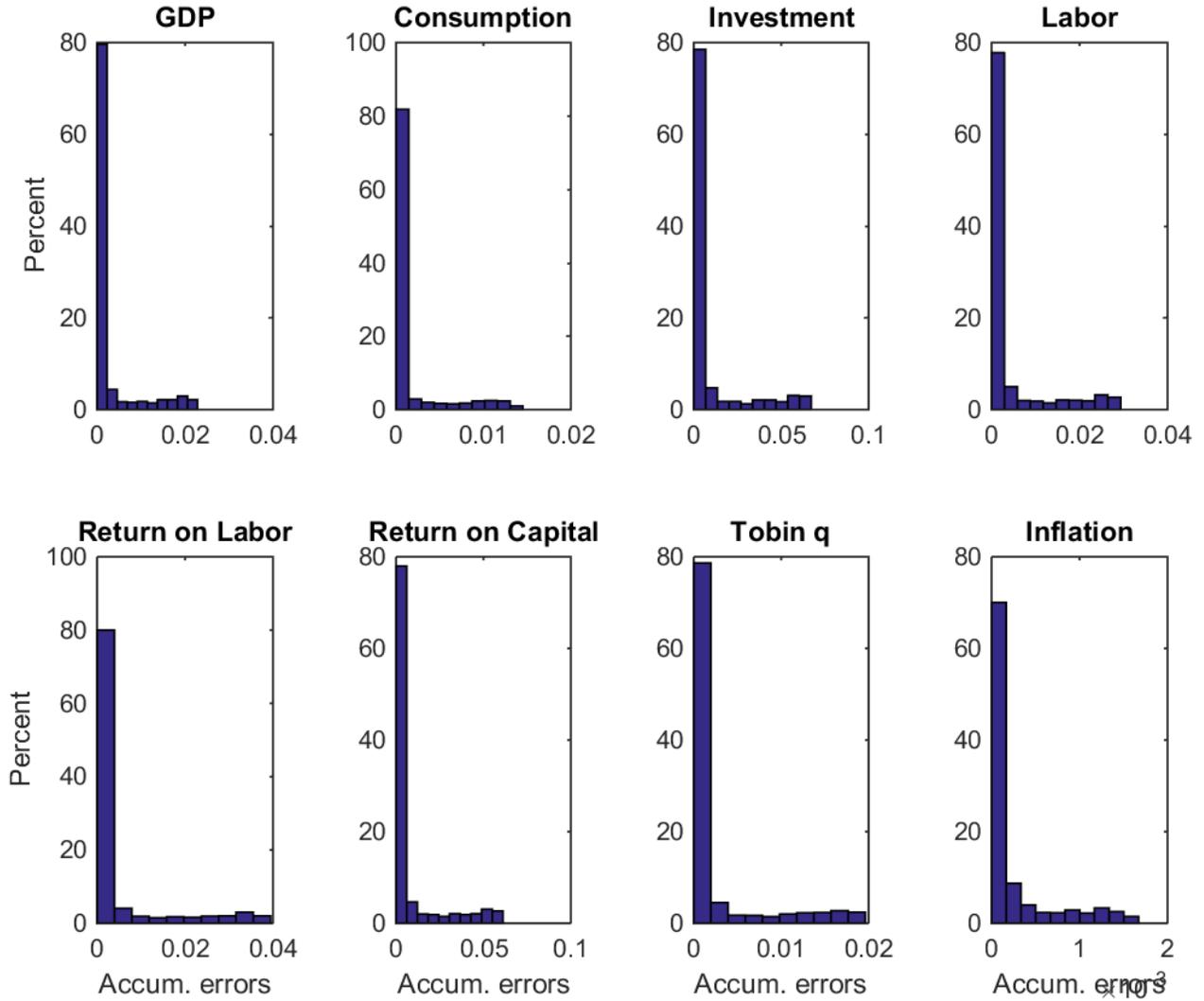
**Figure A8:** Impulse response functions in the 271-sector and the approximate three-sector economies: technology shock.



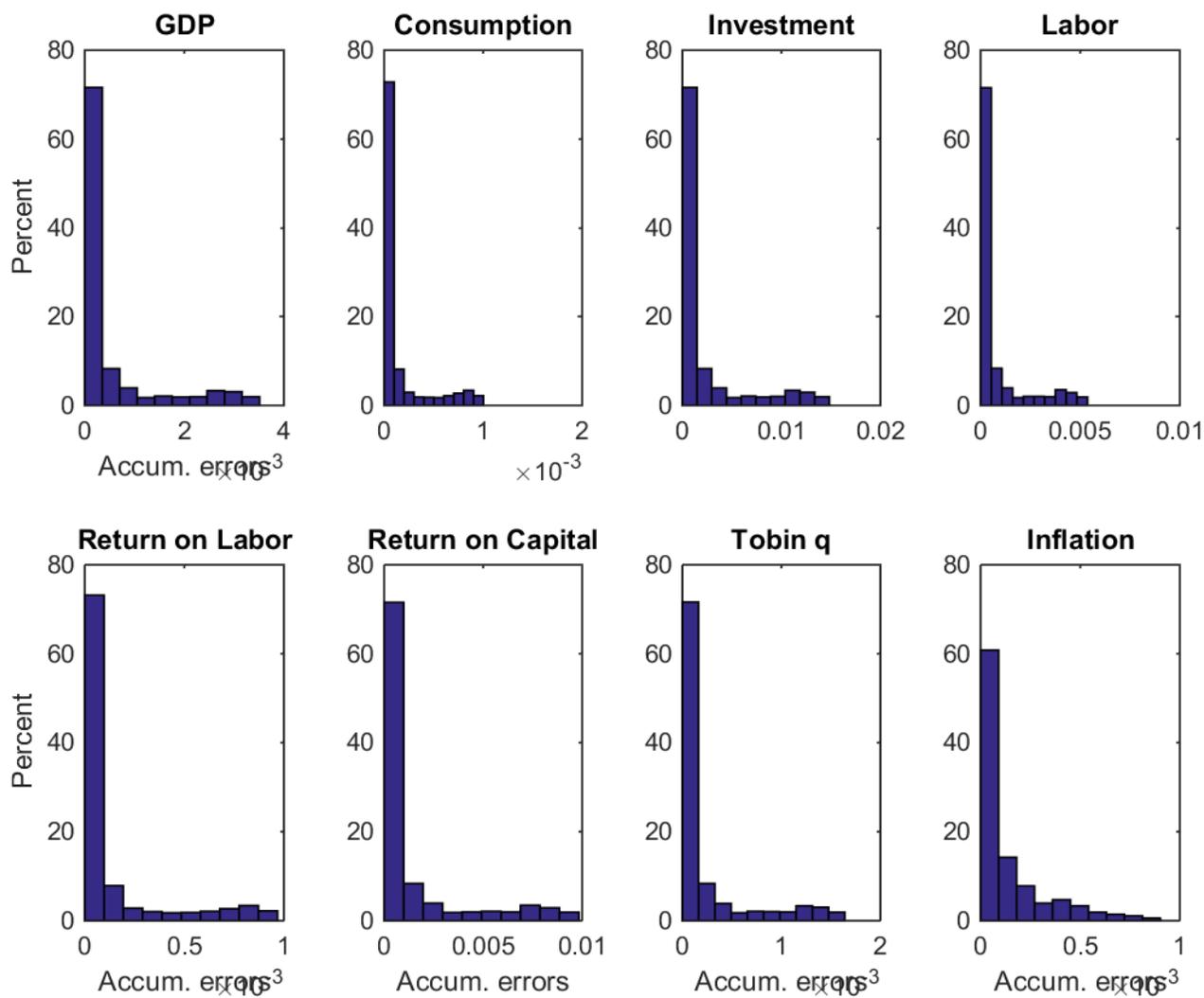
**Figure A9:** Distribution of the cumulative approximation errors – economy-wide factor markets: monetary shock.



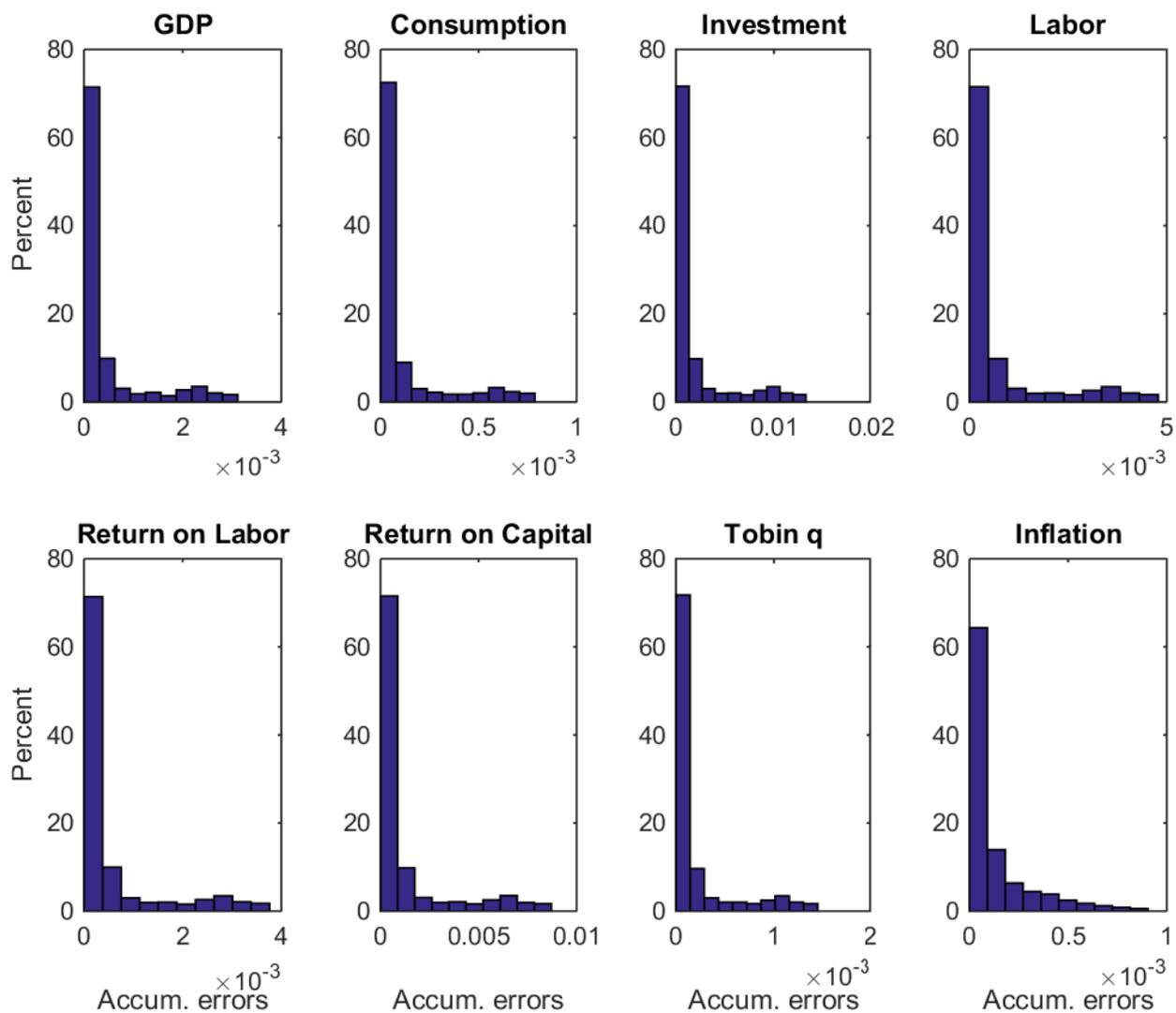
**Figure A10:** Distribution of the cumulative approximation errors – sector-specific factor markets: monetary shock.



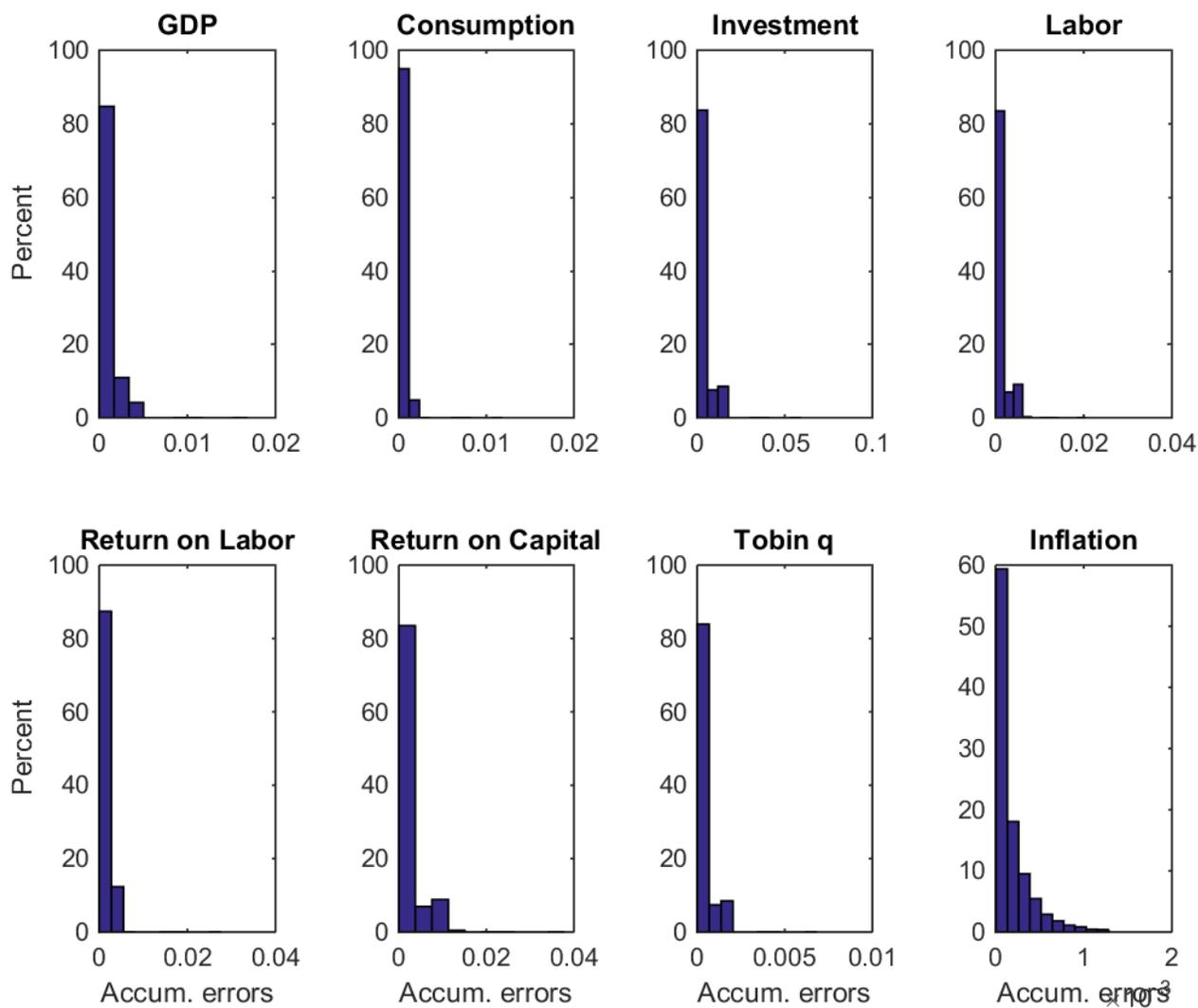
**Figure A11:** Distribution of the cumulative approximation errors – firm-specific factor markets: monetary shock.



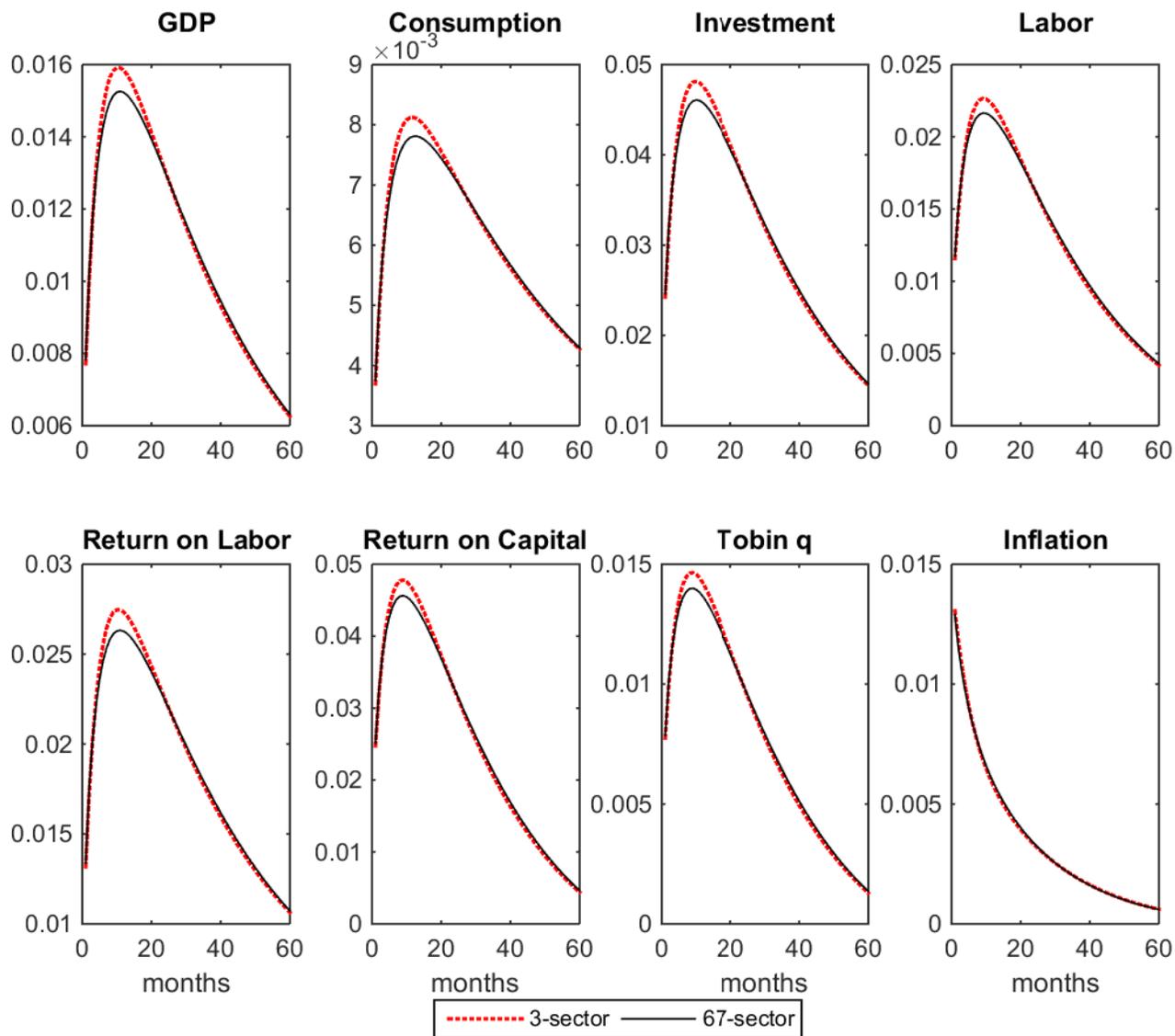
**Figure A12:** Distribution of the cumulative approximation errors – economy-wide factor markets: technology shock.



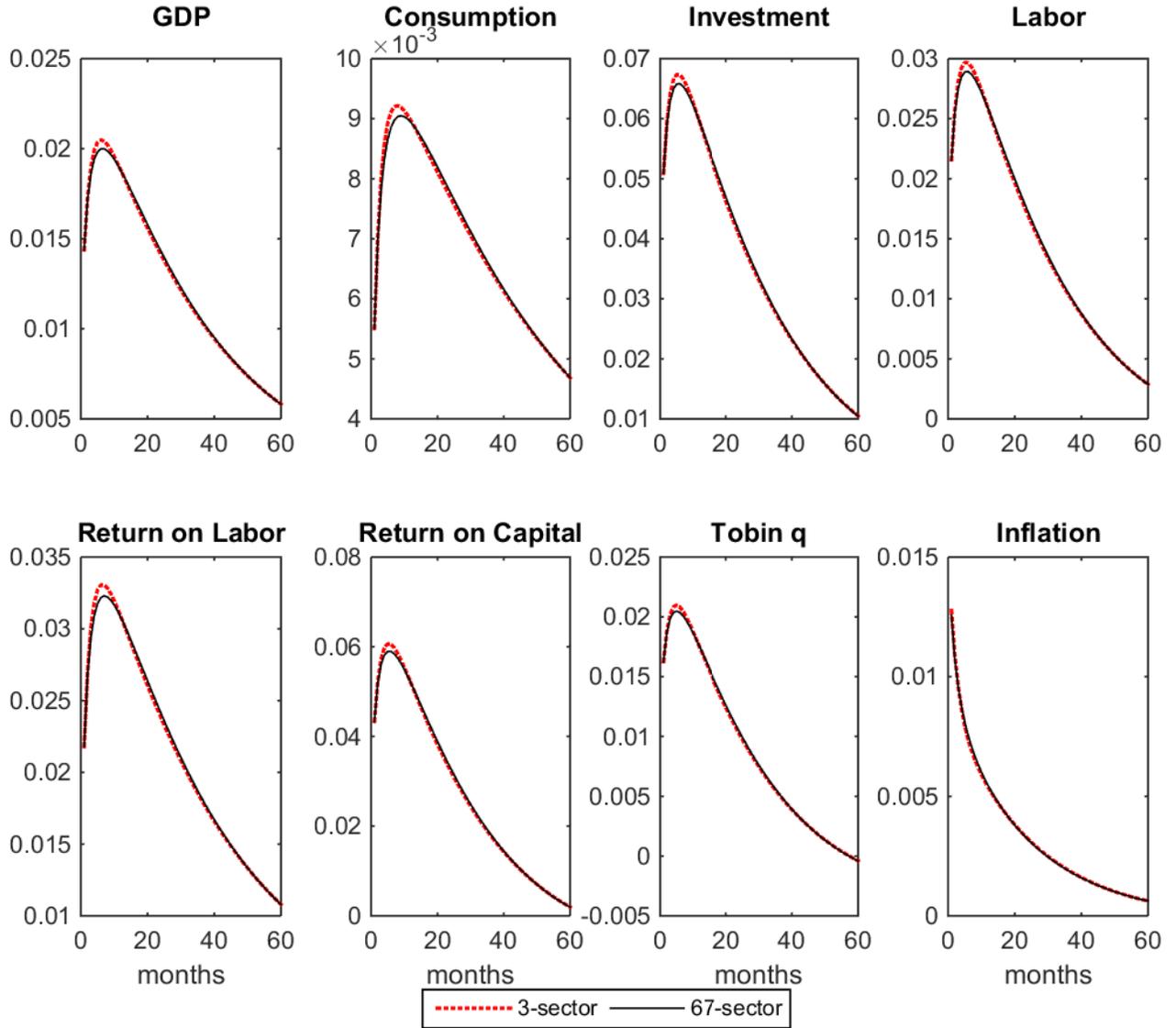
**Figure A13:** Distribution of the cumulative approximation errors – sector-specific factor markets: technology shock.



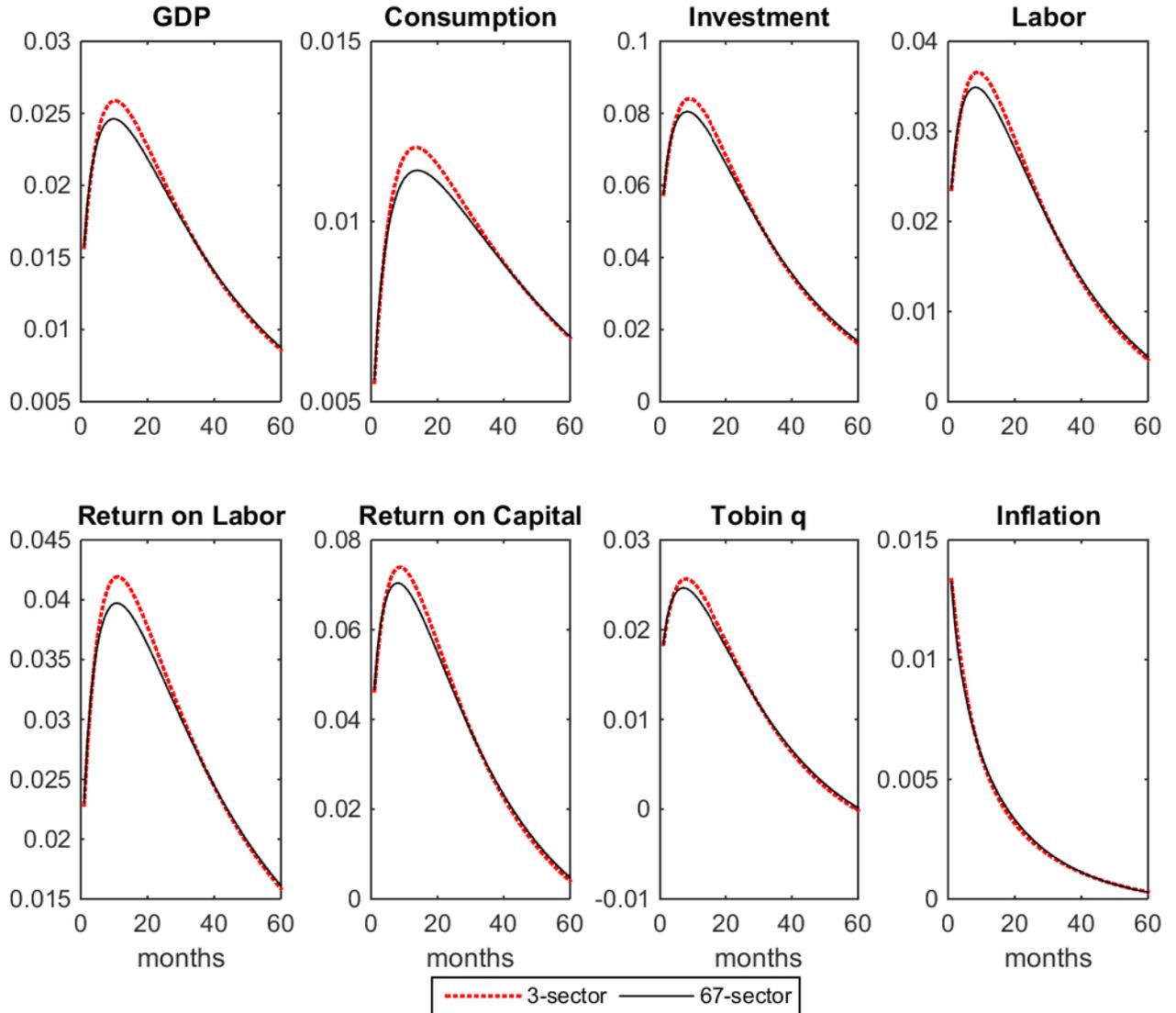
**Figure A14:** Distribution of the cumulative approximation errors – firm-specific factor markets: technology shock.



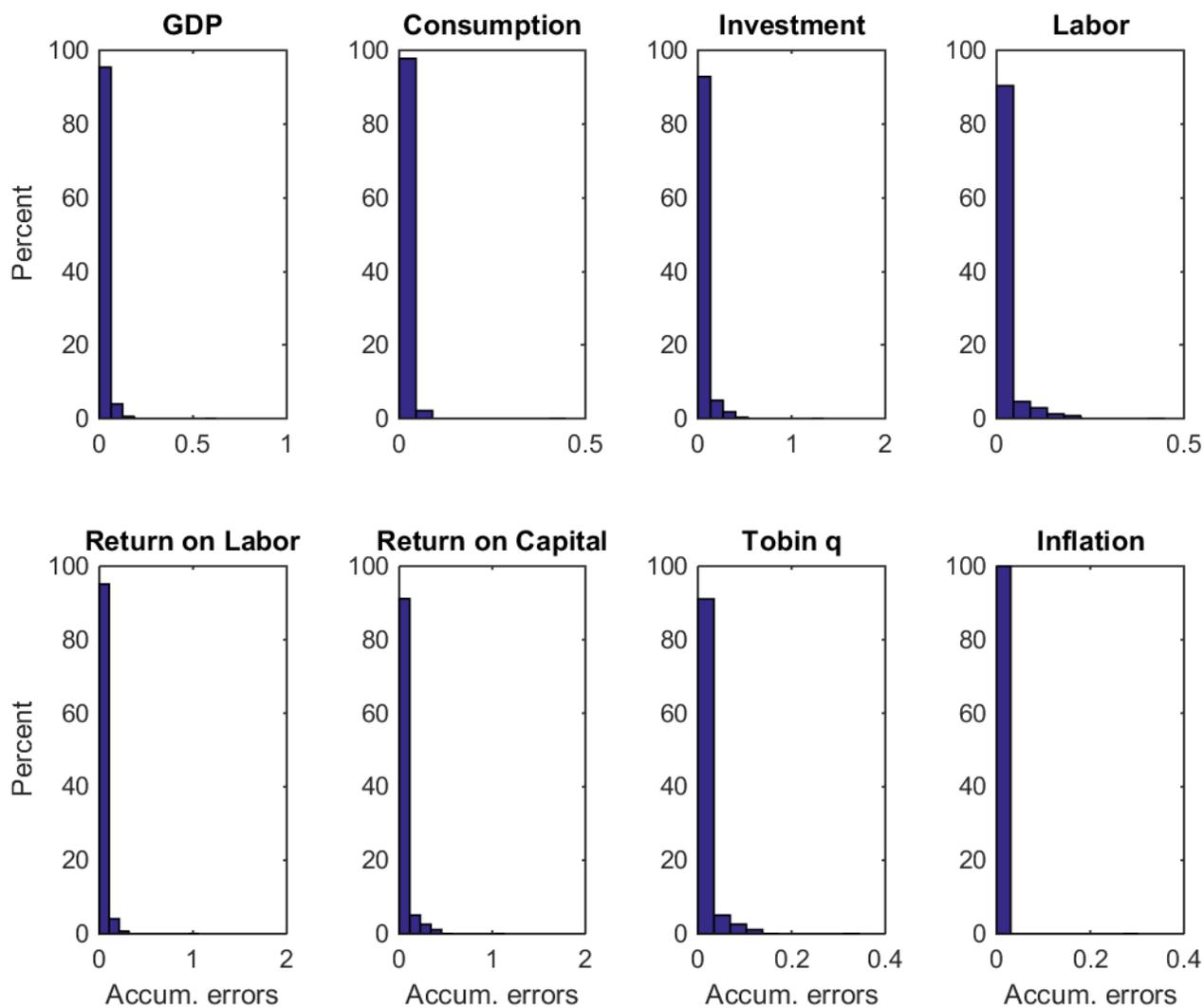
**Figure A15:** Impulse response functions in the 67-sector and the approximate three-sector economies – economy-wide factor markets: interest rate rule.



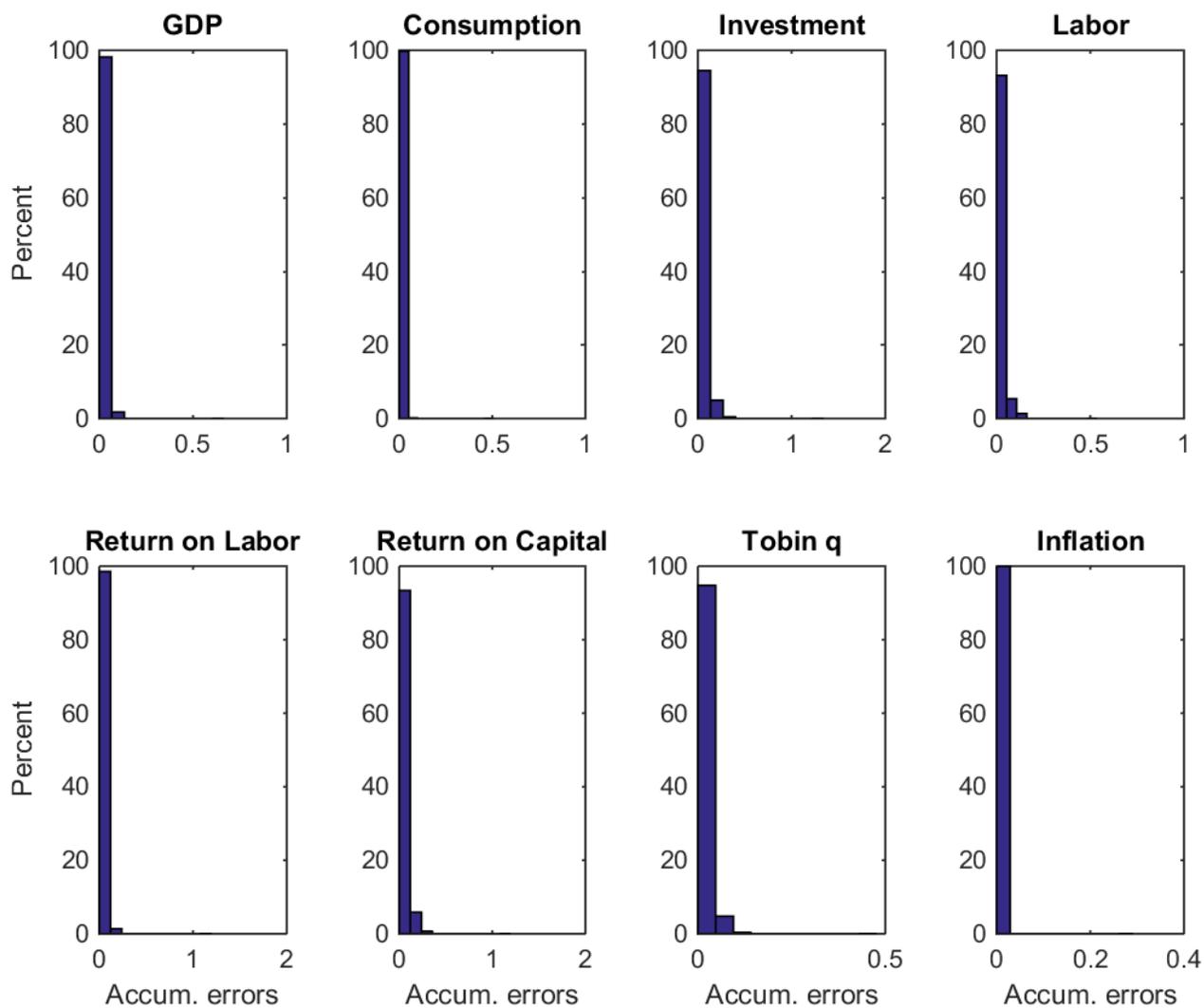
**Figure A16:** Impulse response functions in the 67-sector and the approximate three-sector economies – sector-specific factor markets: interest rate rule.



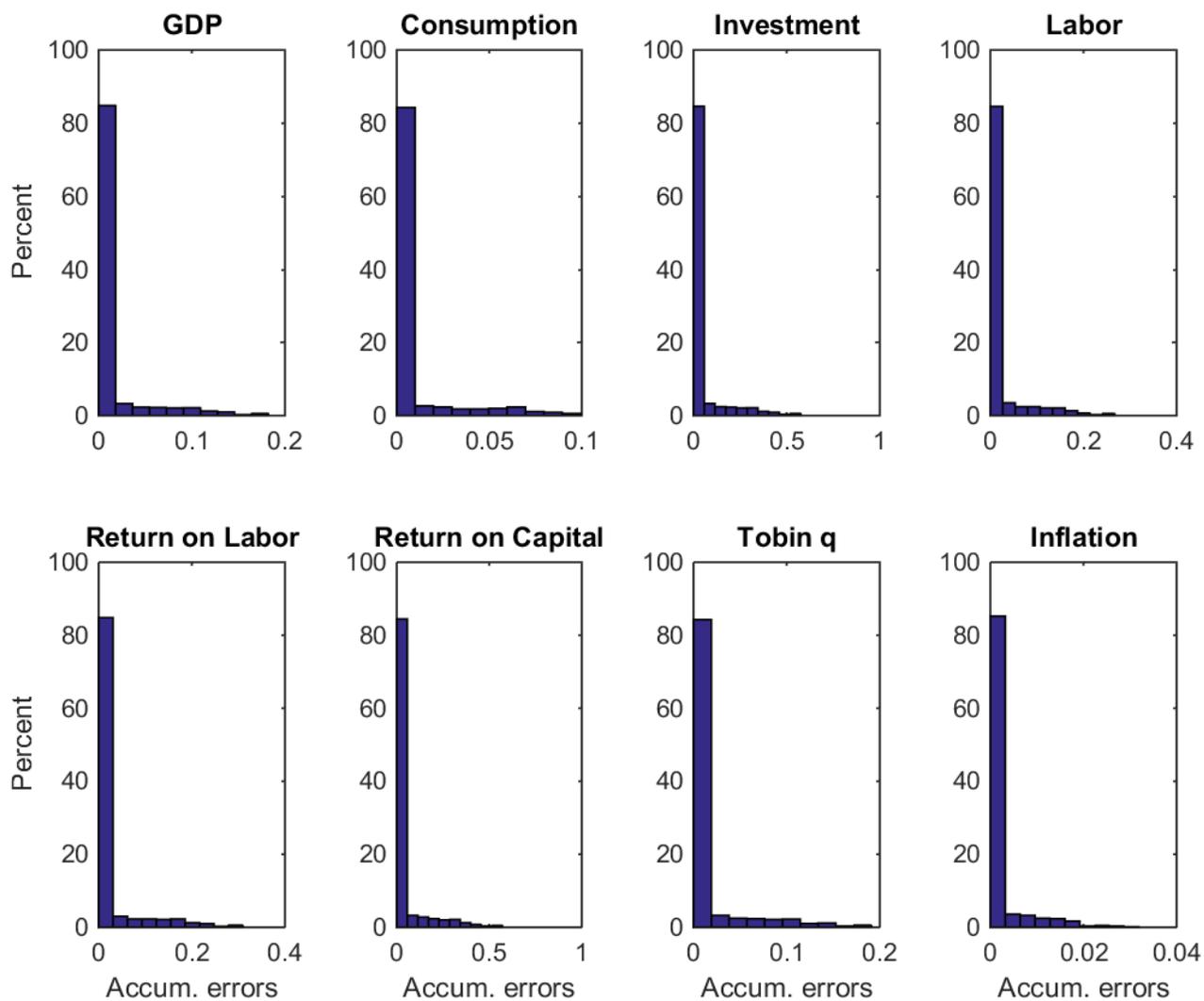
**Figure A17:** Impulse response functions in the 67-sector and the approximate three-sector economies – firm-specific factor markets: interest rate rule.



**Figure A18:** Distribution of the cumulative approximation errors – economy-wide factor markets: interest rate rule.



**Figure A19:** Distribution of the cumulative approximation errors – sector-specific factor markets: interest rate rule.



**Figure A20:** Distribution of the cumulative approximation errors – firm-specific factor markets: interest rate rule.