ONLINE APPENDIX: ROBOTS OR WORKERS? A MACRO ANALYSIS OF AUTOMATION AND LABOR MARKETS

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Abstract. This appendix provides some derivation details and further results in Leduc and Liu (2019).

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Supplemental Appendices: For Online Publication

Appendix A. Derivations of household's optimizing conditions

Denote by $V_t(B_{t-1}, N_{t-1})$ the value function for the representative household. The household's optimizing problem can be written in the recursive form

$$V_t(B_{t-1}, N_{t-1}) \equiv \max \ \ln C_t - \chi N_t + \beta \mathbb{E}_t \theta_{t+1} V_{t+1}(B_t, N_t), \tag{A.1}$$

subject to the budget constraint

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) + d_t - T_t, \tag{A.2}$$

and the law of motion for employment

$$N_t = (1 - \delta_t) N_{t-1} + q_t^u u_t, \tag{A.3}$$

where the measure of job seekers is given by

$$u_t = 1 - (1 - \delta_t) N_{t-1}. \tag{A.4}$$

The household chooses C_t, B_t, and N_t, taking prices and the average job finding rate as given.

Denote by Λ_t the Lagrangian multiplier for the budget constraint (A.2). The first-order condition with respect to consumption implies that

$$\Lambda_t = \frac{1}{C_t}. \tag{A.5}$$

The optimizing decision for B_t implies that

$$\frac{\Lambda_t}{r_t} = \beta \mathrm{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial B_t}. \tag{A.6}$$

Using the envelope condition with respect to B_{t-1}, we obtain the intertemporal Euler equation

$$1 = \mathrm{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} r_t. \tag{A.7}$$

Denote by μ_{nt} the Lagrangian multiplier associated with the employment law of motion (A.3). The first-order condition with respect to N_t implies that

$$\mu_{nt} = \Lambda_t \left(w_t - \phi - \frac{\chi}{\Lambda_t} \right) + \beta \mathrm{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t}. \tag{A.8}$$

After substituting out u_t in Eq. (A.3) using Eq. (A.4), we obtain the envelope condition with respect to N_{t-1}

$$\frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} = \mu_{nt} (1 - \delta_t)(1 - q_t^u). \tag{A.9}$$

Define the employment surplus as $S_t^H \equiv \frac{\mu_{nt}}{\Lambda_t}$. The first-order condition (A.8), together with the envelope condition (A.9), implies the Bellman equation

$$S_t^H = w_t - \phi - \frac{\chi}{\Lambda_t} + \mathrm{E}_t \frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t}(1-\delta_{t+1})(1-q_{t+1}^u)S_{t+1}^H. \tag{A.10}$$

Appendix B. Summary of equilibrium conditions: Benchmark model

A search equilibrium is a system of 22 equations for 22 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_t, Y_{nt}, Y_{at}, p_{nt}, p_{at}].$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \beta\theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \tag{B.1}$$

(2) Matching function

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \tag{B.2}$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \tag{B.3}$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \tag{B.4}$$

(5) Employment dynamics

$$N_t = (1-\delta_t)N_{t-1} + m_t, \tag{B.5}$$

(6) Number of searching workers

$$u_t = 1 - (1-\delta_t)N_{t-1}, \tag{B.6}$$

(7) Unemployment

$$U_t = 1 - N_t, \tag{B.7}$$

(8) Vacancy dynamics

$$v_t = (1-q_{t-1}^v)(1-q_t^a)v_{t-1} + \delta_t N_{t-1} + \eta_t, \tag{B.8}$$

(9) Automation dynamics

$$A_t = (1-\rho^o)A_{t-1} + q_t^a(1-q_{t-1}^v)v_{t-1}, \tag{B.9}$$

(10) Employment value

$$J_t^e = p_{nt}Z_t - w_t + \mathbb{E}_t \beta\theta_{t+1} \frac{C_t}{C_{t+1}} \left[\delta_{t+1}J_{t+1}^v + (1-\delta_{t+1})J_{t+1}^e\right], \tag{B.10}$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1-q_t^v)\mathbb{E}_t \beta\theta_{t+1}\frac{C_t}{C_{t+1}}\left\{(1-q_{t+1}^a)J_{t+1}^v + q_{t+1}^a J_{t+1}^a - \int_0^{x_{t+1}^*} x dG(x)\right\}. \quad \text{(B.11)}$$

(12) Automation value

$$J_t^a = p_{at} Z_t \zeta_t - \kappa_a + (1-\rho^o)\mathbb{E}_t \beta\theta_{t+1}\frac{C_t}{C_{t+1}} J_{t+1}^a, \quad \text{(B.12)}$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^v, \quad \text{(B.13)}$$

(14) Robot adoption

$$q_t^a = \left(\frac{x_t^*}{\bar{x}}\right)^{\eta_a}, \quad \text{(B.14)}$$

(15) Vacancy creation

$$\eta_t = \left(\frac{J_t^v}{\bar{e}}\right)^{\eta_e}, \quad \text{(B.15)}$$

(16) Aggregate output

$$Y_t = \left[\alpha_n Y_{nt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_n) Y_{at}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \text{(B.16)}$$

(17) Intermediate goods produced by workers

$$Y_{nt} = Z_t N_t, \quad \text{(B.17)}$$

(18) Intermediate goods produced by robots

$$Y_{at} = Z_t \zeta_t A_t, \quad \text{(B.18)}$$

(19) The relative price of intermediate goods produced by workers

$$p_{nt} = \alpha_n \left(\frac{Y_t}{Y_{nt}}\right)^{\frac{1}{\sigma}} \quad \text{(B.19)}$$

(20) The relative price of intermediate goods produced by robots

$$p_{at} = (1-\alpha_n) \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\sigma}} \quad \text{(B.20)}$$

(21) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1+\eta_a} q_t^a x_t^* (1-q_{t-1}^v) v_{t-1} + \frac{\eta_e}{1+\eta_e}\eta_t J_t^v = Y_t, \quad \text{(B.21)}$$

(22) Nash bargaining wage

$$\frac{b}{1-b}(J_t^e - J_t^v) = w_t - \phi - \chi C_t + \mathbb{E}_t \frac{\beta\theta_{t+1} C_t}{C_{t+1}}(1-q_{t+1}^u)(1-\delta_{t+1})\frac{b}{1-b}(J_{t+1}^e - J_{t+1}^v), \quad \text{(B.22)}$$

Appendix C. Additional impulse responses

We report some additional impulse responses in this section.

Impulse responses to a job separation shock in the benchmark model. A job separation shock raises both unemployment and vacancies and mechanically boosts hiring through the matching function, as shown in Figure C.1. This finding is consistent with Shimer (2005), who argues that the counterfactual implication of the job separation shock for the correlation between unemployment and vacancies renders the shock unimportant for explaining observed labor market dynamics. The shock reduces the automation probability. Labor productivity increases slightly, since the decline in employment outpaces the decline in aggregate output. The shock also leads to small declines in real wages and the labor income share.

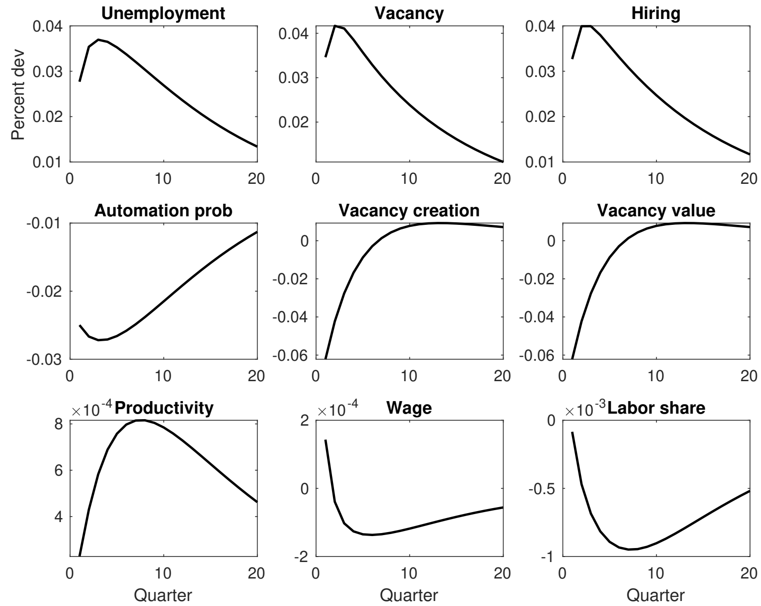

FIGURE C.1. Impulse responses to a job separation shock in the benchmark model.

Impulse responses to a neutral technology shock: benchmark vs. no-automation counterfactuals. We compare the impulse responses to a positive neutral technology shock in the benchmark model with those in two counterfactuals: (1) reducing worker bargaining weight and (2) raising the worker's value of non-market activity (i.e., the unemployment insurance benefits). In both counterfactuals, we turn off the automation threat channel by keeping the automation probability constant at the steady-state level.

Figure C.2 compares the impulse responses to a positive neutral technology shock in the benchmark model (the black solid lines) to the no-automation counterfactual (the blue dashed lines) and the low worker bargaining weight case (the red dashed lines).

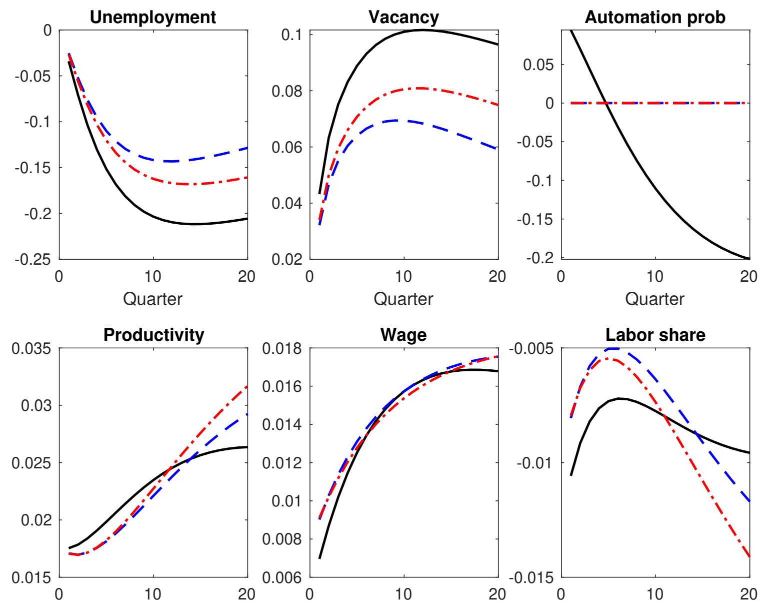

FIGURE C.2. Impulse responses to a positive neutral technology shock in the benchmark model (black solid lines), the counterfactual with no automation (blue dashed lines), and the counterfactual with no automation and low worker bargaining weight (red dot-dash lines).

Figure C.3 compares the impulse responses to a positive neutral technology shock in the benchmark model (the black solid lines) to the no-automation counterfactual (the blue dashed lines) and the high UI benefit counterfactual (the red dashed lines).

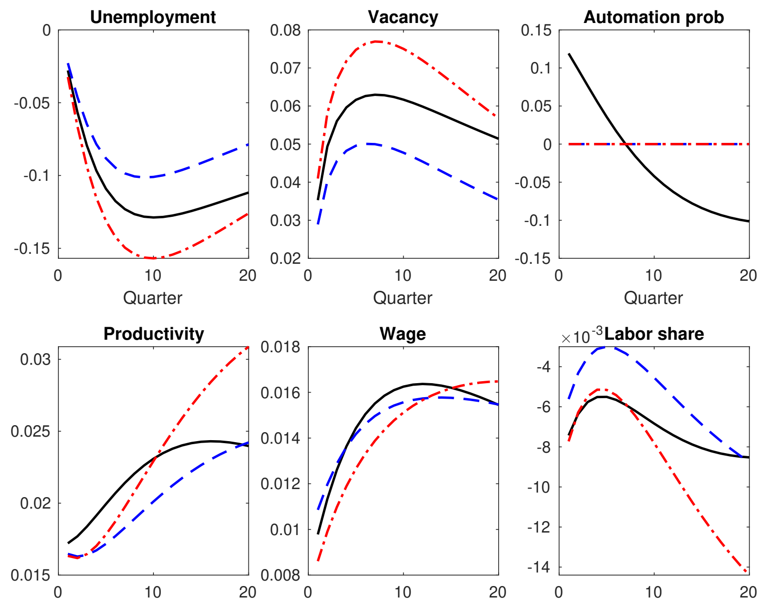

FIGURE C.3. Impulse responses to a positive Fan08Yue shock in the benchmark model (black solid lines), the counterfactual with no automation (blue dashed lines), and the counterfactual with no automation and high unemployment insurance (UI) benefits (red dot-dash lines).

APPENDIX D. MODEL WITH PRODUCTION LAGS

Summary of equilibrium conditions. A search equilibrium is a system of 22 equations for 22 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_t, Y_{nt}, Y_{at}, p_{nt}, p_{at}].$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \tag{D.1}$$

(2) Matching function

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha}, \tag{D.2}$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \tag{D.3}$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \tag{D.4}$$

(5) Employment dynamics

$$N_t = (1 - \delta_t) N_{t-1} + m_t, \tag{D.5}$$

(6) Number of searching workers

$$u_t = 1 - (1-\delta_t)N_{t-1}, \tag{D.6}$$

(7) Unemployment

$$U_t = 1 - N_t, \tag{D.7}$$

(8) Vacancy dynamics

$$v_t = (1 - q^v_{t-1})(1 - q^a_t)v_{t-1} + \delta_t N_{t-1} + \eta_t, \tag{D.8}$$

(9) Automation dynamics

$$A_t = (1-\rho^o)A_{t-1} + q^a_t(1 - q^v_{t-1})v_{t-1}, \tag{D.9}$$

(10) Employment value

$$J^e_t = p_{nt}Z_t - w_t + \mathbb{E}_t\beta\theta_{t+1}\frac{C_t}{C_{t+1}}\left[\delta_{t+1}J^v_{t+1} + (1-\delta_{t+1})J^e_{t+1}\right], \tag{D.10}$$

(11) Vacancy value

$$J^v_t = -\kappa + q^v_t J^e_t + (1-q^v_t)\mathbb{E}_t\beta\theta_{t+1}\frac{C_t}{C_{t+1}}\left\{(1-q^a_{t+1})J^v_{t+1} + q^a_{t+1}J^a_{t+1} - \int_0^{x^*_{t+1}} x dG(x)\right\}. \tag{D.11}$$

(12) Automation value

$$J^a_t = (1-\rho^o)\mathbb{E}_t D_{t,t+1}\left[p_{a,t+1}Z_{t+1}\zeta_{t+1} - \kappa_a + J^a_{t+1}\right] \tag{D.12}$$

(13) Automation threshold

$$x^*_t = J^a_t - J^v_t, \tag{D.13}$$

(14) Robot adoption

$$q^a_t = \left(\frac{x^*_t}{\bar{x}}\right)^{\eta_a}, \tag{D.14}$$

(15) Vacancy creation

$$\eta_t = \left(\frac{J^v_t}{\bar{e}}\right)^{\eta_e}, \tag{D.15}$$

(16) Aggregate output

$$Y_t = \left[\alpha_n Y_{nt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_n)Y_{at}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{D.16}$$

(17) Intermediate goods produced by workers

$$Y_{nt} = Z_t N_t, \tag{D.17}$$

(18) Intermediate goods produced by robots

$$Y_{at} = Z_t\zeta_t A_{t-1}, \tag{D.18}$$

(19) The relative price of intermediate goods produced by workers

$$p_{nt} = \alpha_n \left(\frac{Y_t}{Y_{nt}} \right)^{\frac{1}{\sigma}} \tag{D.19}$$

(20) The relative price of intermediate goods produced by robots

$$p_{at} = (1 - \alpha_n) \left(\frac{Y_t}{Y_{at}} \right)^{\frac{1}{\sigma}} \tag{D.20}$$

(21) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_{t-1} + \frac{\eta_a}{1 + \eta_a} q_t^a x_t^* (1 - q_{t-1}^v) v_{t-1} + \frac{\eta_e}{1 + \eta_e} \eta_t J_t^v = Y_t, \tag{D.21}$$

(22) Nash bargaining wage

$$\frac{b}{1 - b}(J_t^e - J_t^v) = w_t - \phi - \chi C_t + \mathbb{E}_t \frac{\beta \theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u)(1 - \delta_{t+1}) \frac{b}{1 - b}(J_{t+1}^e - J_{t+1}^v), \tag{D.22}$$

Impulse responses. We calibrate the parameters in the model with production lags to the same values as those in the benchmark model.

Figures D.1-D.4 show the impulse responses of several macro and labor market variables to a positive, one-standard-deviation shock to the neutral technology, the discount factor, the automation-specific technology, and the job separation rate, respectively.

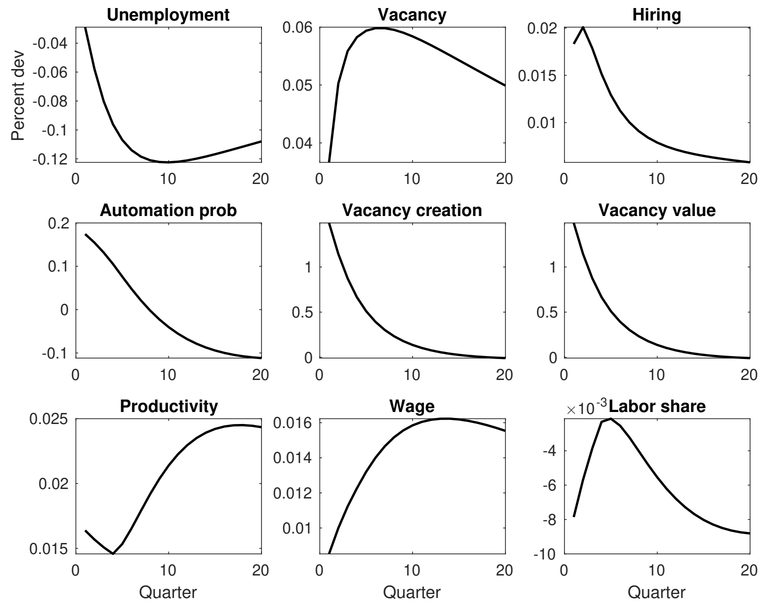

FIGURE D.1. Impulse responses to a positive neutral technology shock in the model with production lags.

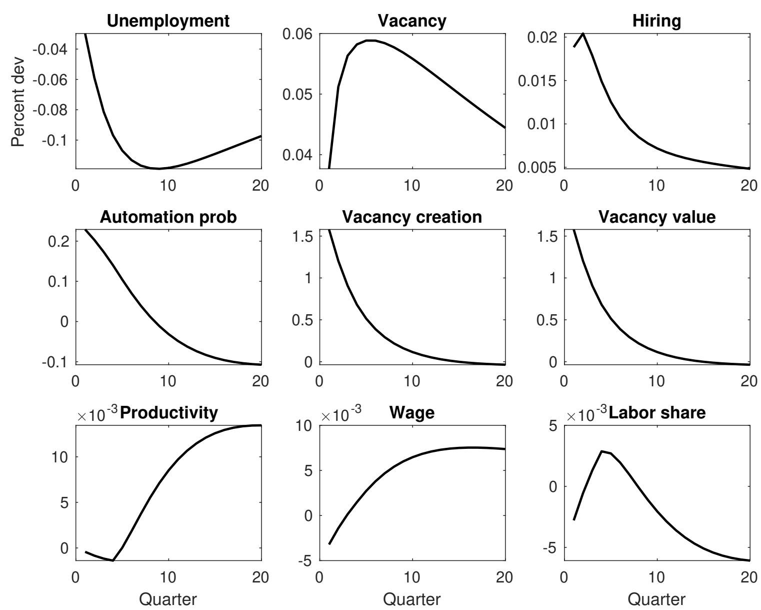

FIGURE D.2. Impulse responses to a positive discount factor shock in the model with with production lags.

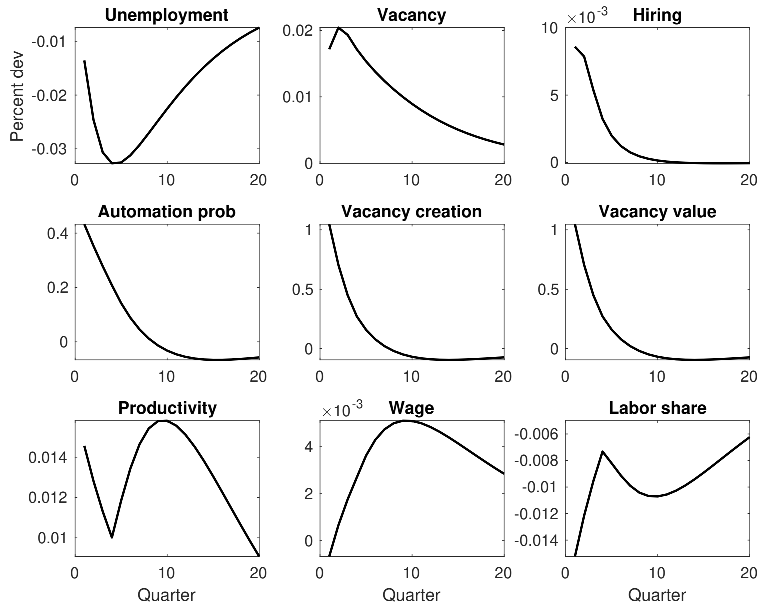

FIGURE D.3. Impulse responses to a positive automation-specific technology shock in the model with production lags.

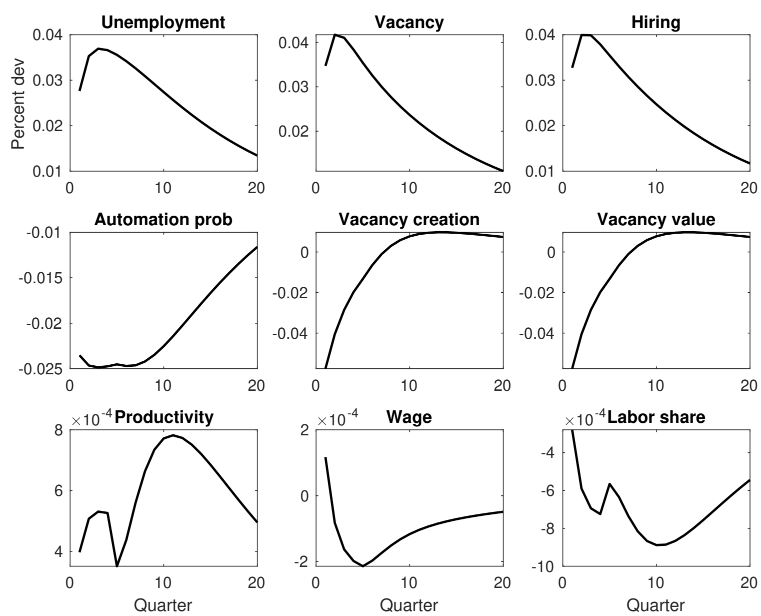

FIGURE D.4. Impulse responses to a positive job separation shock in the model with production lags.

Appendix E. Model with automated jobs

Summary of equilibrium conditions. A search equilibrium is a system of 22 equations for 22 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_t, Y_{nt}, Y_{at}, p_{nt}, p_{at}] .$$

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \tag{E.1}$$

(2) Matching function

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha}, \tag{E.2}$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \tag{E.3}$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \tag{E.4}$$

(5) Employment dynamics

$$N_t = (1-\delta_t)(1-q_t^a)N_{t-1} + m_t, \tag{E.5}$$

(6) Number of searching workers

$$u_t = 1 - (1-\delta_t)(1-q_t^a)N_{t-1}, \tag{E.6}$$

(7) Unemployment

$$U_t = 1 - N_t, \tag{E.7}$$

(8) Vacancy dynamics

$$v_t = (1-q_{t-1}^v)v_{t-1} + \delta_t N_{t-1} + \eta_t, \tag{E.8}$$

(9) Automation dynamics

$$A_t = (1-\rho^o)A_{t-1} + q_t^a(1-\delta_t)N_{t-1}, \tag{E.9}$$

(10) Employment value

$$J_t^e = p_{nt} Z_t - w_t + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} \left\{ \delta_{t+1} J_{t+1}^v + (1-\delta_{t+1}) \left[q_{t+1}^a J_{t+1}^a - \int_0^{x_{t+1}^*} x dG(x) + (1-q_{t+1}^a) J_{t+1}^e \right] \right\}, \tag{E.10}$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1-q_t^v)\mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^v, \tag{E.11}$$

(12) Automation value

$$J_t^a = p_{at} Z_t \zeta_t - \kappa_a + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} (1 - \rho^o) J_{t+1}^a, \tag{E.12}$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^e, \tag{E.13}$$

(14) Robot adoption

$$q_t^a = \left(\frac{x_t^*}{\bar{x}} \right)^{\eta_a}, \tag{E.14}$$

(15) Vacancy creation

$$\eta_t = \left(\frac{J_t^v}{\bar{e}} \right)^{\eta_e}, \tag{E.15}$$

(16) Aggregate output

$$Y_t = \left[\alpha_n Y_{nt}^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_n) Y_{at}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{E.16}$$

(17) Intermediate goods produced by workers

$$Y_{nt} = Z_t N_t, \tag{E.17}$$

(18) Intermediate goods produced by robots

$$Y_{at} = Z_t \zeta_t A_t, \tag{E.18}$$

(19) The relative price of intermediate goods produced by workers

$$p_{nt} = \alpha_n \left(\frac{Y_t}{Y_{nt}} \right)^{\frac{1}{\sigma}} \tag{E.19}$$

(20) The relative price of intermediate goods produced by robots

$$p_{at} = (1 - \alpha_n) \left(\frac{Y_t}{Y_{at}} \right)^{\frac{1}{\sigma}} \tag{E.20}$$

(21) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1 + \eta_a} q_t^a x_t^* (1 - \delta_t) N_{t-1} + \frac{\eta_e}{1 + \eta_e} \eta_t J_t^v = Y_t, \tag{E.21}$$

(22) Nash bargaining wage

$$\frac{b}{1-b}(J_t^e - J_t^v) = w_t - \phi - \chi C_t + \mathbb{E}_t \frac{\beta \theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u)(1 - \delta_{t+1}) \frac{b}{1-b} (J_{t+1}^e - J_{t+1}^v), \tag{E.22}$$

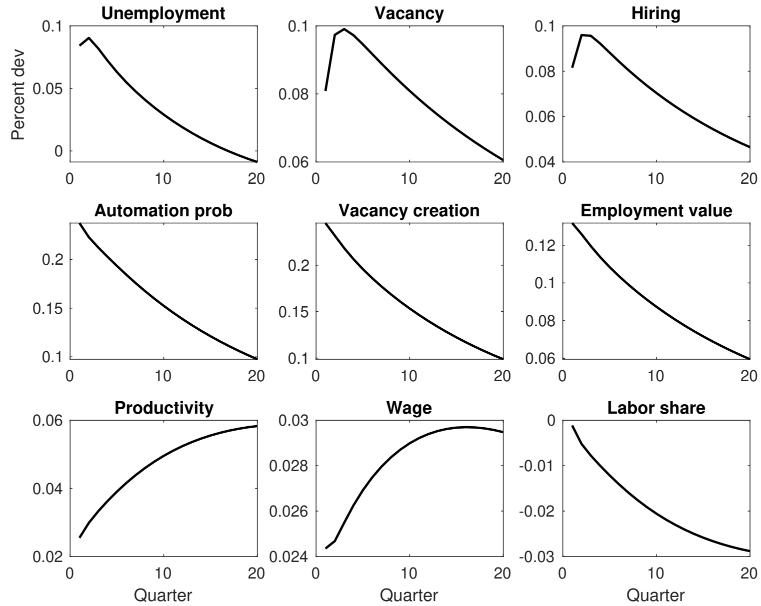

FIGURE E.1. Impulse responses to a positive neutral technology shock in the model with automating jobs.

Impulse responses. For ease of comparison, we calibrate the parameters in the model with automated jobs to the same values as those in the benchmark model.

Figures E.1-E.4 show the impulse responses of several macro and labor market variables to a positive, one-standard-deviation shock to the neutral technology, the discount factor, the automation-specific technology, and the job separation rate, respectively.

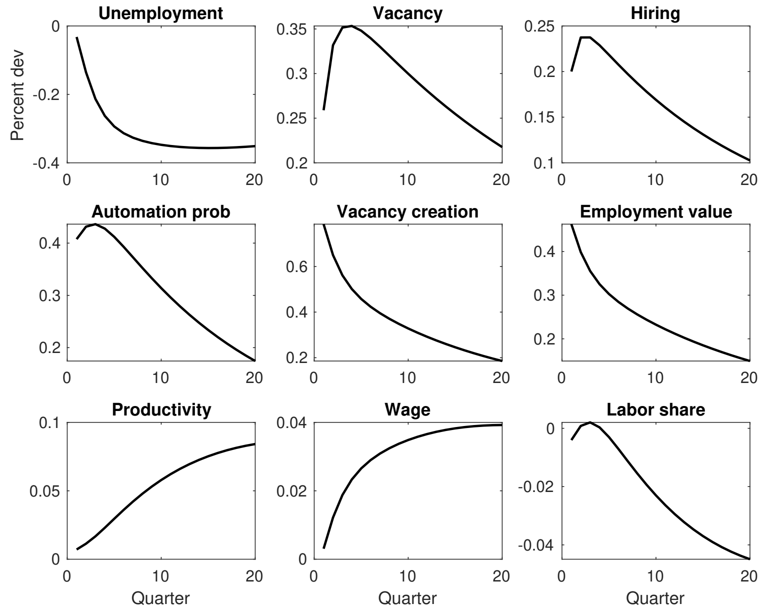

Figure E.2. Impulse responses to a positive discount factor shock in the model with automating jobs.

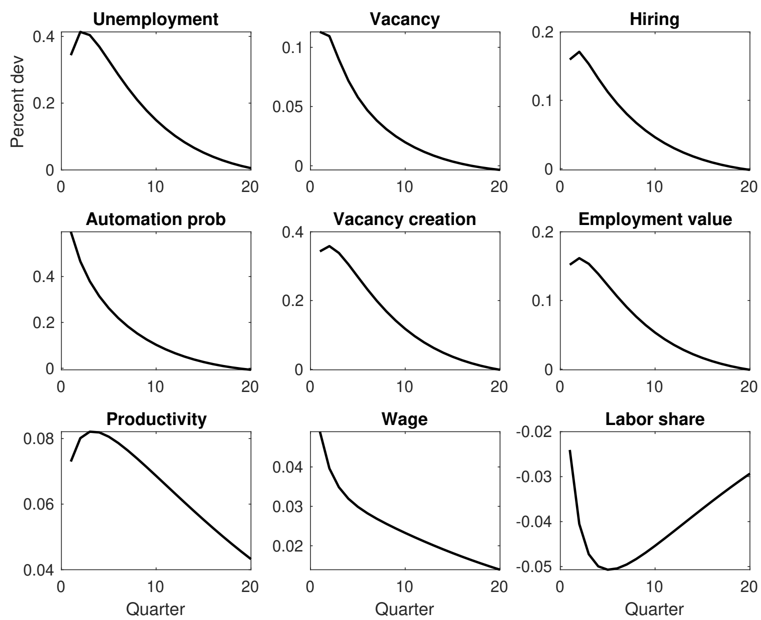

Figure E.3. Impulse responses to a positive automation-specific technology shock in the model with automating jobs.

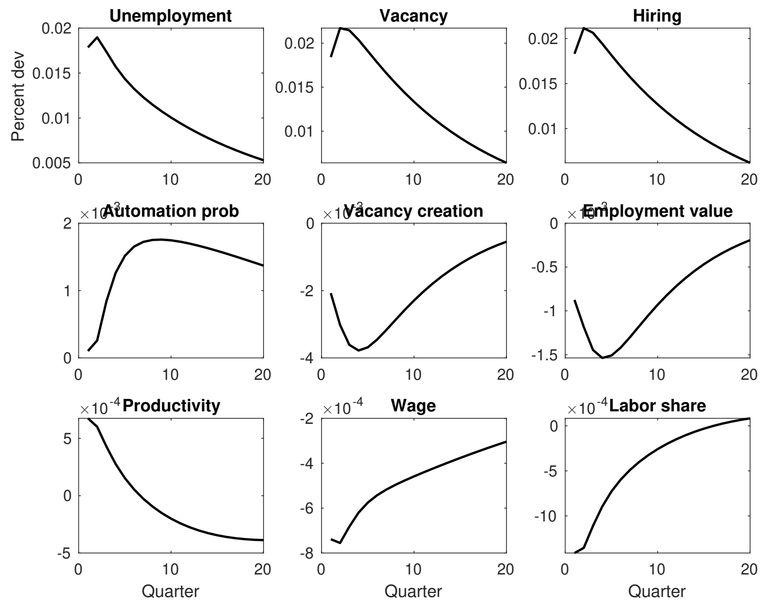

Figure E.4. Impulse responses to a positive job separation shock in the model with automating jobs.

Appendix F. Model with heterogeneous worker skills

Summary of equilibrium conditions. A search equilibrium in the model with heterogeneous workers is a system of 23 equations for 23 variables summarized in the vector

$$[C_t, r_t, Y_t, m_t, u_t, v_t, q_t^u, q_t^v, q_t^a, N_t, U_t, \eta_t, J_t^e, J_t^v, J_t^a, A_t, x_t^*, w_{nt}, w_{st}, Y_{nt}, Y_{at}, p_{nt}, p_{at}] \, .$$

We write the equations in the same order as in the dynare code.

(1) Household's bond Euler equation:

$$1 = \mathrm{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} r_t, \tag{F.1}$$

(2) Matching function

$$m_t = \mu u_t^{\alpha} v_t^{1-\alpha}, \tag{F.2}$$

(3) Job finding rate

$$q_t^u = \frac{m_t}{u_t}, \tag{F.3}$$

(4) Vacancy filling rate

$$q_t^v = \frac{m_t}{v_t}, \tag{F.4}$$

(5) Employment dynamics

$$N_t = (1-\delta_t) N_{t-1} + m_t, \tag{F.5}$$

(6) Number of searching workers

$$u_t = 1 - (1-\delta_t) N_{t-1}, \tag{F.6}$$

(7) Unemployment

$$U_t = 1 - N_t, \tag{F.7}$$

(8) Vacancy dynamics

$$v_t = (1 - q_{t-1}^v)(1 - q_t^a) v_{t-1} + \delta_t N_{t-1} + \eta_t, \tag{F.8}$$

(9) Automation dynamics

$$A_t = (1 - \rho^o) A_{t-1} + q_t^a (1 - q_{t-1}^v) v_{t-1}, \tag{F.9}$$

(10) Employment value

$$J_t^e = p_{nt} Z_t - w_{nt} + \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} \left[\delta_{t+1} J_{t+1}^v + (1 - \delta_{t+1}) J_{t+1}^e \right], \tag{F.10}$$

(11) Vacancy value

$$J_t^v = -\kappa + q_t^v J_t^e + (1 - q_t^v) \mathbb{E}_t \beta \theta_{t+1} \frac{C_t}{C_{t+1}} \left[(1 - q_{t+1}^a) J_{t+1}^v + q_{t+1}^a J_{t+1}^a - \int_0^{x_{t+1}^*} x dG(x) \right]. \tag{F.11}$$

(12) Automation value

$$J_t^a = \gamma_a p_{at} Z_t \zeta_t^{\gamma_a} \left(\frac{\bar{s}}{A_t}\right)^{1-\gamma_a} - \kappa_a + (1-\rho^o)\mathbb{E}_t \beta\theta_{t+1} \frac{C_t}{C_{t+1}} J_{t+1}^a, \tag{F.12}$$

(13) Automation threshold

$$x_t^* = J_t^a - J_t^v, \tag{F.13}$$

(14) Robot adoption

$$q_t^a = \left(\frac{x_t^*}{\bar{x}}\right)^{\eta_a}, \tag{F.14}$$

(15) Vacancy creation

$$\eta_t = \left(\frac{J_t^v}{\bar{e}}\right)^{\eta_e}, \tag{F.15}$$

(16) Aggregate output

$$Y_t = \left[\alpha_n Y_{nt}^{\frac{\sigma-1}{\sigma}} + (1-\alpha_n) Y_{at}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{F.16}$$

(17) Intermediate goods produced by workers

$$Y_{nt} = Z_t N_t, \tag{F.17}$$

(18) Intermediate goods produced by robots

$$Y_{at} = Z_t (\zeta_t A_t)^{\gamma_a} \bar{s}^{1-\gamma_a}, \tag{F.18}$$

(19) The relative price of intermediate goods produced by workers

$$p_{nt} = \alpha_n \left(\frac{Y_t}{Y_{nt}}\right)^{\frac{1}{\sigma}} \tag{F.19}$$

(20) The relative price of intermediate goods produced by robots

$$p_{at} = (1-\alpha_n) \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\sigma}} \tag{F.20}$$

(21) Resource constraint

$$C_t + \kappa v_t + \kappa_a A_t + \frac{\eta_a}{1+\eta_a} q_t^a x_t^* (1 - q_{t-1}^v) v_{t-1} + \frac{\eta_e}{1+\eta_e} \eta_t J_t^v = Y_t, \tag{F.21}$$

(22) Nash bargaining wage

$$\frac{b}{1-b}(J_t^e - J_t^v) = w_{nt} - \phi - \chi C_t + \mathbb{E}_t \frac{\beta\theta_{t+1} C_t}{C_{t+1}} (1 - q_{t+1}^u)(1 - \delta_{t+1}) \frac{b}{1-b}(J_{t+1}^e - J_{t+1}^v), \tag{F.22}$$

(23) Skilled wage

$$w_{st} = (1-\gamma_a) Z_t \left(\frac{\zeta_t A_t}{\bar{s}}\right)^{\gamma_a} \tag{F.23}$$

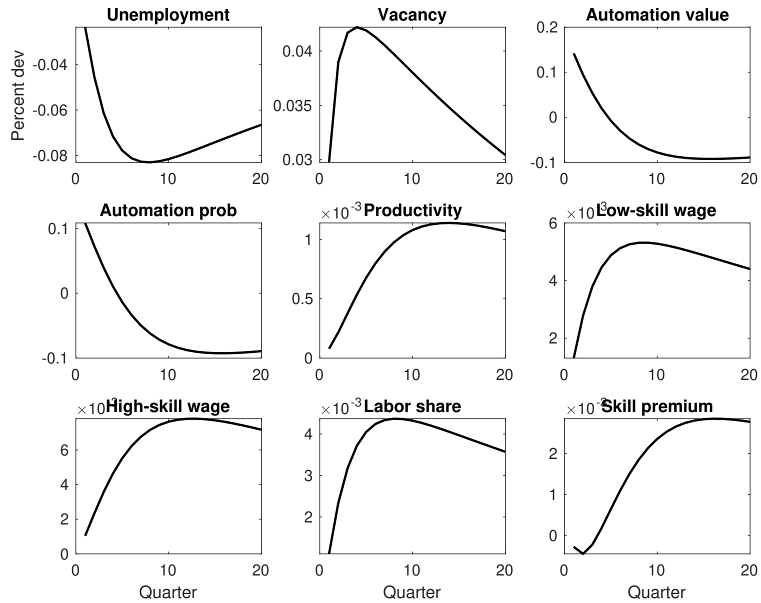

FIGURE F.1. Impulse responses to a positive discount factor shock in the model with heterogeneous skills.

Impulse responses. We use the calibrated and estimated parameter values in the benchmark model where appropriate and calibrate three additional parameters in this generalized model. We set $\gamma_a = 0.32$, such that the skilled labor share is 68% of the revenue generated by the technology using robots and skilled workers as inputs. We normalize the supply of skilled workers and calibrate the average level of the automation-specific productivity (relative to the neutral technology) such that the model implies a steady-state skill premium of 55%, in line with the ratio of median weekly earnings of workers with a bachelor's degree or higher to those of workers with some college or associate degrees.

We have shown the impulse responses to a neutral technology shock in the main text. Here, we present the impulse responses to the other three shocks, shown in Figures F.1-F.3.

REFERENCES

LEDUC, S. AND Z. LIU (2019): "Robots or Workers? A Macro Analysis of Automation and Labor Markets," Federal Reserve Bank of San Francisco Working Paper No. 2019-17.

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25—49.

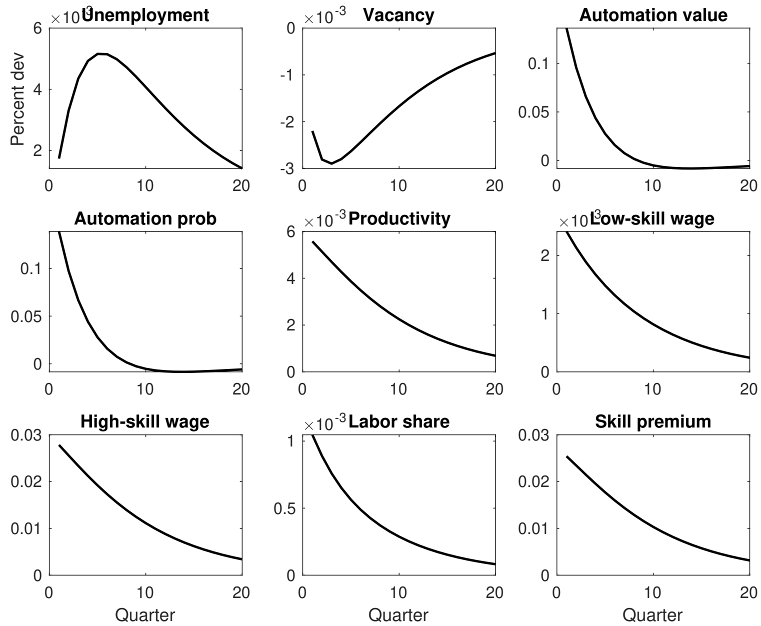

FIGURE F.2. Impulse responses to a positive automation-specific technology shock in the model with heterogeneous skills.

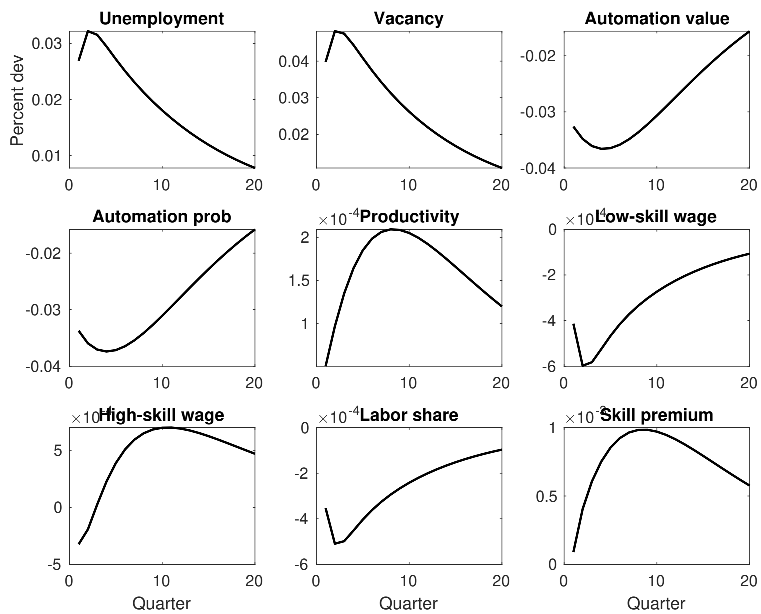

FIGURE F.3. Impulse responses to a positive job separation shock in the model with heterogeneous skills.