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# Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach\*

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## Abstract

We develop a new approach to estimating DSGE models with occasionally binding borrowing constraints and apply it to Mexico's business cycle and financial crisis history. We propose a new endogenous regime-switching specification of the borrowing constraint, develop a general perturbation method to solve the model, and estimate it using Bayesian methods. The estimated model fits the data with well-behaved shocks, identifying three crisis episodes of varying duration and intensity: the early-1980s Debt Crisis, the mid-1990s Tequila Crisis, and the late-2000s Global Financial Crisis. The estimated crisis episodes are much more persistent and in line with the data than traditional models.

**Keywords:** Business Cycles, Bayesian Estimation, Endogenous Regime-Switching, Financial Crises, Mexico, Occasionally Binding Constraints, Persistence, Sudden Stops.

**JEL Codes:** G01, E3, F41, C11.

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# 1 Introduction

The Global Financial Crisis triggered strong renewed interest in understanding the causes, consequences, and remedies of financial crises. In this context, dynamic stochastic general equilibrium (DSGE) models with occasionally binding financial frictions proved successful as laboratories to study the anatomy of both business cycles and crises, and to explore optimal policy responses to these dynamics. This success is because occasionally binding financial frictions are mechanisms that create amplification of regular business cycle dynamics and can capture the fat tail and skewness of the data. Structural estimation of these models is important for inference on key parameters governing the financial frictions, counterfactual analysis, and structural real-time forecasts, yet very challenging.

In this paper, we structurally estimate a model with an occasionally binding borrowing constraint. The paper's contribution is threefold. First, we propose a new specification of the occasionally binding collateral constraint that delivers crisis persistence and dynamics more in line with the data. Second, we develop a perturbation solution method suitable for solving models like ours in a way that permits likelihood-based estimation. Third, we focus on one particular type of crisis, the so-called sudden stop in international capital flows, and apply the proposed framework to the estimation of a medium-scale DSGE model of such crises, investigating sources of business cycles and crises in Mexico since 1981.

As a first step, we propose a new formulation of occasionally binding constraint models. As in the traditional specification of such models, our set up has two states or regimes: in the first, limited leverage amplifies regular shocks and gives rise to fire sales and non-linear crisis dynamics; in the second, access to financing is unconstrained and the economy displays regular business cycles. In our specification, however, the transitions between the two regimes depend on a *range* rather than a *unique* level of leverage, with endogenous switching probabilities that are a function of the borrowing capacity of the economy and the multiplier associated with the leverage constraint. This formulation maps the model with an occasionally binding leverage constraint written as inequality into an endogenous regime-switching model. The paper focuses on a particular constraint and type of crisis, the so called sudden stop in capital flows, but the proposed specification has broader applicability to other types of occasionally binding constraints and frictions.

Next, we develop a perturbation-based solution method for solving the endogenous regime-switching model. The perturbation method is fast enough to permit likelihood-based estimation, is scalable to models larger than the one we estimate in this paper, and displays typical levels of accuracy. We also show analytically that to capture the effects of the endogenous transition probabilities on the policy functions characterizing optimal behavior,

including precautionary behavior, it is necessary to approximate the model solution at least to the second-order, and that these effects would be missed by linear approximations or exogenous regime switching models. Again, the solution method that we develop can be applied to a wide range of models with endogenous regime-switching.

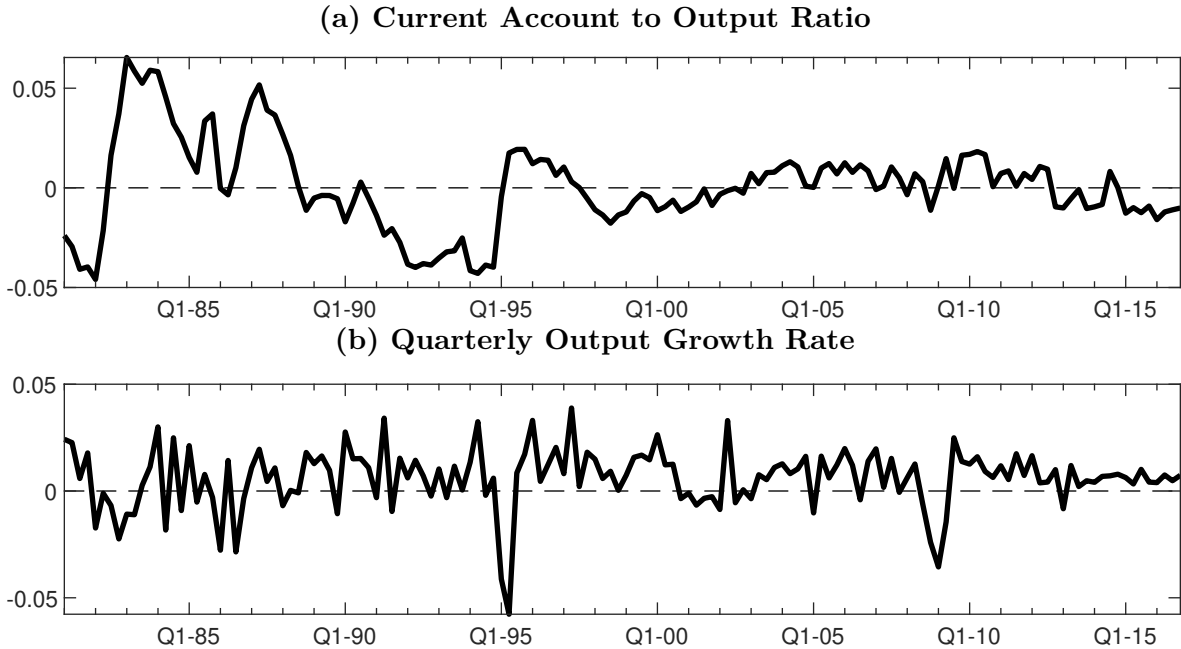
Finally, we apply our borrowing constraint specification and solution method to the Bayesian estimation of a small open-economy model that can characterize both financial crises and business cycles in Mexico. While our application focuses on an emerging market economy, our framework is general and can be applied to the formulation and estimation of other model settings with occasionally binding constraints. For example, the approach that we propose could be applied to the formulation and estimation of models of occasionally binding credit frictions, housing constraints, banking with asymmetric information, downward wage rigidity, or the zero lower bound.

Figure 1 plots two critical variables in our application to Mexico: the current account balance as a share of GDP and the quarterly real GDP growth in deviation from the sample mean. The figure illustrates the regular fluctuations in the data as well as multiple episodes of large current account reversals and persistent output growth declines. Large current account reversals and output drops of heterogeneous size and persistence are the two main empirical features commonly associated with sudden stops in capital flows, not only in Mexico but also in many other emerging markets exposed to volatile capital flows. In this paper, we focus on the challenge of fitting a structural model to Mexico’s business cycle and sudden stop history without imposing ad hoc restrictions on the magnitude or the persistence of the episodes that we identify in estimation.

Despite the econometric challenges in characterizing data like those displayed in Figure 1, our estimated model fits Mexico’s business cycles and sudden stop episodes well, without relying on large shocks to explain crises but instead letting the economic structure of the model explain those events. It produces business cycle statistics that match the second moments of the data and provides evidence that, contrary to the prevailing view, neither productivity nor interest rate shocks are the only, or the most important, drivers of Mexico’s business cycles. Most importantly, our new specification of the collateral constraint identifies crisis episodes and dynamics of duration and intensity that are more in line with the data than those generated from models with traditional inequality borrowing constraints, without any auxiliary empirical restriction imposed on their definition.

In particular, the estimated model identifies three financial crises: the Debt Crisis from 1981:Q3 to 1983:Q2, the Mexican peso crisis commonly referred to as the “Tequila crisis” from 1994:Q1 to 1996:Q1, and the spillover effect from the Global Financial Crisis from 2008:Q4 to 2009:Q3. Unlike the existing quantitative sudden stop literature, the model

**Figure 1: Current Account and Output in Mexico, 1981-2016**



Note: Panel (a) plots Mexico's current account balance as a share of GDP. Panel (b) shows Mexico's quarterly log-change of real GDP. See Appendix F for data sources. Sample period 1981:Q1-2016:Q4.

identifies these without resorting to imposing any ad hoc restrictions on their definition. The identified crisis episodes align well with a purely empirical notion of financial crisis in Mexico (Reinhart and Rogoff, 2009) and display duration about twice as long as the crisis peaks previously identified as sudden stops (Cerra and Saxena, 2008; Mendoza, 2010). The model-simulated dynamics of crisis episodes indicate that they are preceded by slowly unfolding booms and followed by economic stagnation, and are not only driven by a favorable external environment that suddenly reverses, but also domestic factors such as technology, demand, and preference shocks. We also show that different shocks matter for different historical crisis episodes, as well as different phases of a given crisis episode with patterns that fit narrative approaches to Mexico's history of financial crisis.

**Related Literature** Our paper is connected to several strands of literature. The paper relates to the now large literature on the Bayesian estimation of DSGE models (for example, Schorfheide, 2000; Otrok, 2001; Smets and Wouters, 2007; Iacoviello and Neri, 2010; Bianchi, 2013). We extend that successful approach to models with occasionally binding collateral constraints, which have become the benchmark for normative analysis of macro-prudential optimal policy. Welfare-base analysis of optimal macroprudential policies with occasionally binding constraints (Bianchi, 2011; Jeanne and Korinek, 2010; Benigno et al., 2013, 2016; Bianchi and Mendoza, 2018; Devereux et al., 2019; Schmitt-Grohe and Uribe, 2020) critically

depends on calibration assumptions and the collateral constraint formulation. Structural estimation of these parameters and likelihood based model validation can discipline model formulation, which in turn is critical for normative policy recommendations.

In this respect, our paper is closely related to the estimation exercise in [Bocola \(2016\)](#) that is accomplished while solving the model with global methods. However, this estimation is made possible by first estimating the model outside the crisis, and then appending an estimate of the crisis in a second step. While this procedure does not matter for the specific application in [Bocola \(2016\)](#), it is not necessarily applicable more generally. Our approach permits joint estimation of the model inside and outside the crises and is potentially scalable to larger and more complex models, while maintaining a satisfactory level of accuracy relative to global solution methods.

The paper is also closely related to the literature on likelihood-based estimation of Markov switching DSGE models initiated by the seminal contribution of [Bianchi \(2013\)](#), and applied in [Bianchi and Ilut \(2017\)](#) and [Bianchi et al. \(2018\)](#). The filter we use differs in two key respects. First, our regime-switching transition matrix is endogenous. Second, conditional on the regime, we solve the model to the second order. So we employ the Sigma Point Filter to evaluate the likelihood function in place the modified Kalman filter in [Bianchi \(2013\)](#).

In the literature on Markov-switching DSGE models, our paper builds upon the method developed by [Foerster et al. \(2016\)](#), who developed perturbation methods for the solution of exogenous regime-switching models. The perturbation approach that we propose allows for second- and higher-order approximations that go beyond the linear models studied by [Davig and Leeper \(2007\)](#) and [Farmer et al. \(2011\)](#). In fact, we show that at least a second-order approximation is necessary in order to capture the effects of the endogenous switching.

The paper is naturally related to the literature that on focuses on endogenous regime-switching models. [Davig and Leeper \(2008\)](#), [Davig et al. \(2010\)](#), and [Alpanda and Ueberfeldt \(2016\)](#) all consider endogenous regime-switching but employ global solution methods that hinder likelihood-based estimation. [Lind \(2014\)](#) develops a regime-switching perturbation approach for approximating non-linear models, but it requires repeatedly refining the points of approximation and hence it is not suitable for estimation purposes.

Here, our paper relates closely to [Guerrieri and Iacoviello \(2015\)](#), who develop OccBin, a set of procedures for the solution of models with occasionally binding constraints. OccBin is a certainty equivalent solution method that captures non-linearities but not precautionary effects, which are a critical feature of models with occasionally binding collateral constraints.<sup>1</sup> A key feature of our approach is to preserve precautionary saving effects, as agents in the

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<sup>1</sup>[Cuba-Borda et al. \(2019\)](#) study how the solution method and likelihood misspecification interact and possibly compound each other.

model adjust their behavior due to the presence of the constraint even when the constraint does not bind, and vice versa.

The specification of the constraint that we propose and the accompanying perturbation solution method could be easily applied to models with occasionally binding zero-lower bound on interest rates (for example, [Adam and Billi, 2007](#); [Aruoba et al., 2018](#); [Atkinson et al., 2018](#)). Existing methods for the estimation of such models may limit scalability due computational costs ([Gust et al., 2017](#)). Moreover, the occasionally binding zero lower bound is not comparable to the kind of constraints with endogenous collateral value that we estimate in this paper and is used in the normative literature on macroprudential policies. Indeed, endogenous collateral valuation features different amplification mechanisms and entails additional computational complexities.

The application of the methodology that we propose relates to the literature on emerging market business cycles, which includes [Aguiar and Gopinath \(2007\)](#), [Mendoza \(2010\)](#), [Garcia-Cicco et al. \(2010\)](#), [Fernandez-Villaverde et al. \(2011\)](#), among others. Encompassing most shocks previously considered, we include technology, preference, expenditure, interest rate, and terms of trade shocks in our analysis. Relative to [Mendoza \(2010\)](#), we provide a Bayesian estimation of the model and consider a wider set of structural shocks, finding that some of the estimated values of the parameters that are not easily calibrated to the stylized facts of the data differ substantially. Most importantly, we document that we can identify crisis episodes of persistence and magnitude more consistent with the data without imposing any ad hoc restrictions on their amplitude. Relative to [Garcia-Cicco et al. \(2010\)](#), we empirically evaluate the relative importance of interest rate shocks without resorting to the ad hoc debt elastic premium calibrated to be a financial friction in a fully non-linear framework, with a fully articulated specification of the financial frictions driving amplification.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the proposed formulation of the collateral constraint. Section 3 presents our perturbation solution method for endogenous regime-switching models. Section 4 describes the Bayesian estimation procedure and reports results on parameters and model fit. Section 5 presents the main empirical results on business cycle properties and financial crises. Section 6 concludes. The Appendices include technical details and additional empirical results.

## 2 The Model

The framework is a medium scale model for the analysis of business cycles and sudden stop crises in emerging market economies. The model is a small, open, production economy with endogenous labor supply, investment, and an occasionally binding collateral constraint.

The economy is subject to temporary productivity, intertemporal preference, expenditure, interest rate, and terms of trade shocks. We consider a large set of shocks as in [Garcia-Cicco et al. \(2010\)](#).<sup>2</sup> Preferences and production are as in [Mendoza \(2010\)](#). Due to the occasionally binding nature of the constraint, this framework can account not only for normal business cycles, but also for non-linear dynamics and amplifications that are critical aspects of financial crises in both emerging markets and advanced economies in a way that models with always binding financial frictions cannot.

While our application focuses on one particular type of crisis, the so called sudden stop in capital flows, our framework is generally applicable to other macroeconomic models with occasionally binding frictions and crises (e.g. [Kiyotaki and Moore, 1997](#); [Iacoviello, 2005](#); [Gertler and Karadi, 2011](#); [Jermann and Quadrini, 2012](#); [Liu et al., 2013](#); [Gertler and Kiyotaki, 2015](#); [Bocola, 2016](#); [Schmitt-Grohe and Uribe, 2016](#); [Boissay et al., 2016](#); [Eichenbaum et al., 2020](#)). The framework that we propose is particularly suitable to deal with the zero lower bound on interest rates because it takes into account the fact that, in practice, the interest rates are not bound mechanically at zero but effectively so.<sup>3</sup> In the rest of this section, we discuss the representative household-firm and the borrowing constraint specification that is the novel feature of our model. The formal definition of the equilibrium and the full set of equilibrium conditions are reported in [Appendix A](#).

## 2.1 Preferences, Constraints, and Shock Processes

There is a representative household-firm that maximizes the following utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left( C_t - \frac{H_t^\omega}{\omega} \right)^{1-\rho} \right\}, \quad (1)$$

where  $C_t$  denotes consumption,  $H_t$  the supply of labor, and  $d_t$  an exogenous and stochastic preference shock specified below. Households choose consumption, labor, capital  $K_t$ , imported intermediate inputs  $V_t$  given an exogenous stochastic relative price  $P_t$  also specified below, and holdings of real one-period international bonds,  $B_t$ . Negative values of  $B_t$  indicate borrowing from abroad.

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<sup>2</sup>We do not consider permanent technology shocks of the type analyzed by [Aguiar and Gopinath \(2007\)](#) because these long-run components cannot be estimated precisely over samples periods of length comparable to ours. Moreover, [Garcia-Cicco et al. \(2010\)](#) and [Miyamoto and Nguyen \(2017\)](#) find that the permanent technology shock is not quantitatively important in frameworks with financial frictions like ours in the case of Mexico.

<sup>3</sup>The zero lower bound can be implemented in our framework via a shadow rate model where the nominal rate either equals the shadow rate or zero, and the switches between regimes occur when the shadow rate approaches the effective lower bound.



The household-firm faces the following budget constraint:

$$C_t + I_t + E_t = Y_t - \phi r_t (W_t H_t + P_t V_t) - \frac{1}{(1 + r_t)} B_t + B_{t-1}, \quad (2)$$

where  $Y_t$  is gross domestic product given by

$$Y_t = A_t K_{t-1}^\eta H_t^\alpha V_t^{1-\alpha-\eta} - P_t V_t. \quad (3)$$

Here,  $A_t$  denotes the exogenous and stochastic level of technology.  $E_t$  is an exogenous and stochastic expenditure process possibly interpreted as a fiscal or net export shock as in [Garcia-Cicco et al. \(2010\)](#). The term  $\phi r_t (W_t H_t + P_t V_t)$  describes a working capital constraint, stating that a fraction of the wage and intermediate good bill must be paid in advance of production with borrowed funds. The relative price of labor and capital are given by  $W_t$  and  $q_t$ , respectively, both of which are endogenous, but taken as given by the individual household-firm. Gross investment,  $I_t$ , is subject to adjustment costs as a function of net investment:

$$I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left( 1 + \frac{\iota}{2} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right) \right). \quad (4)$$

Household-firms can borrow in international markets issuing one-period bonds that pay a market or country net interest rate  $r_t$ . The country interest rate between period  $t$  and  $t + 1$ ,  $r_t$ , has an exogenous persistent component given by

$$r_t^* = (1 - \rho_{r^*}) \bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t}, \quad (5)$$

and an exogenous transitory component ( $\sigma_r \varepsilon_{r,t}$ ), with  $\varepsilon_{r^*,t}$  and  $\varepsilon_{r,t}$  i.i.d.  $N(0, 1)$ , and  $\sigma_{r^*}$  and  $\sigma_r$  denoting parameters that control the variance of the two components.<sup>4</sup>

As in [Mendoza \(2010\)](#), in our model, the household-firm also faces a endogenous external financing premium on debt (EFPD), measured by the difference between the effective real interest rate, which corresponds to the intertemporal marginal rate of substitution in consumption,  $r_t^h = \mu_t / E_t[\mu_{t+1}]$ , and the market interest rate,  $r_t$ . In fact, the Euler equation for  $b_t$ ,  $\mu_t = \lambda_t + \beta (1 + r_t) \mathbb{E}_t \mu_{t+1}$ , can be rearranged to show that  $EFPD = \mathbb{E}_t[r_t^h - r_t] = \lambda_t / \beta E_t[\mu_{t+1}]$ , where  $\mu_t$  is the Lagrange multiplier on the budget constraint and  $\lambda_t$  is the multiplier on the collateral constraint to be introduced shortly. In

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<sup>4</sup>To aide the computations,  $r_t$  also includes a small endogenous component that depends on the level of debt. As discussed in Section 3, our solution method relies on an approximation around a point between the steady state associated with the constraint always binding and the steady state for the constraint never binding. The elastic debt component aids us in determining the latter, but we have verified it does not play any role in affecting our results.

addition, we do not impose any correlation between the innovations to the interest rate process and the productivity process specified below unlike what typically assumed modelling emerging market business cycles (e.g., [Neumeyer and Perri, 2005](#)).

The remaining exogenous processes for the preference shock  $d_t$ , the temporary technology shock  $A_t$ , the shock to the relative price of intermediate goods  $P_t$ , and the domestic expenditure shock  $E_t$ , are specified as follows:

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, \quad (6)$$

$$\log A_t = (1 - \rho_A)A^* + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}, \quad (7)$$

$$\log P_t = (1 - \rho_P)P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t}, \quad (8)$$

$$\log E_t = (1 - \rho_E)E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t}, \quad (9)$$

where the starred variables and the  $\rho$  coefficients denote the unconditional mean values and the persistence parameters of the processes,  $\varepsilon$  are assumed i.i.d.  $N(0, 1)$  innovations, and the  $\sigma$  parameters control the size of the process variances.<sup>5</sup>

## 2.2 The Occasionally Binding Borrowing Constraint: An Endogenous Regime-Switching Specification

As in typical models with occasionally binding inequality constraints, the economy fluctuates between two states or regimes. In one state, denoted  $s_t = 1$  and called the binding or constrained regime, the following constraint on total borrowing binds strictly:

$$\frac{1}{(1 + r_t)} B_t - \phi (1 + r_t) (W_t H_t + P_t V_t) = -\kappa q_t K_t, \quad (10)$$

with  $\lambda_t$  denoting the corresponding multiplier. Total debt includes borrowing for consumption smoothing plus working capital for the purchase of intermediate inputs and labor for production. Constraint working capital limits the supply response of the economy to shocks in the binding regime. In the other state, denoted  $s_t = 0$  and called the non-binding or unconstrained regime, the total borrowing limit is slack and  $\lambda_t = 0$ . Thus lenders finance all desired borrowing, and the only constraint is the natural debt limit.

Given these two regimes, which represent the occasionally binding nature of the borrowing constraint, we characterize the transition between them *stochastically* in the sense that,

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<sup>5</sup>The stochastic volatility features of emerging economies data as in [Fernandez-Villaverde et al. \(2011\)](#) could be allowed for by introducing regime-switching in the the volatilities of the shocks processes. While possible, we do not allow for regime-switching either in the intercepts or the volatilities of the shocks processes, as we want the collateral constraint to drive regime-switching, rather than shifts in the stochastic processes.

for given values of capital, bond holding, and exogenous processes, there is an *endogenous* probability of switching between the regimes. This formulation contrasts with the *deterministic* relation between leverage and the regime for given values of endogenous and exogenous state variables in typical occasionally binding specification. Specifically, we assume that the probabilities of switching from one regime to the other follow a logistic distribution as in [Bocola \(2016\)](#), where the economy’s transition between a default and non-default state is also a logistic function of a restricted subset of the endogenous state variables of the model.<sup>6</sup>

Define the “borrowing cushion,”  $B_t^*$ , as the distance of total actual borrowing from the debt limit:

$$B_t^* = \frac{1}{(1+r_t)}B_t - \phi(1+r_t)(W_tH_t + P_tV_t) + \kappa q_t K_t, \quad (11)$$

so that when the borrowing cushion is small, total borrowing is high relative to the value of collateral, and leverage also is high. We then assume that the transition from the non-binding to the binding regime depends on  $B_t^*$  according to:

$$\Pr(s_{t+1} = 1 | s_t = 0, B_t^*) = \frac{\exp(-\gamma_0 B_t^*)}{1 + \exp(-\gamma_0 B_t^*)}. \quad (12)$$

Thus, the likelihood that the constraint binds in the *following* period depends on the size of the borrowing cushion in the *current* period.<sup>7</sup> The parameter  $\gamma_0$  controls the steepness of the logistic function, determining the sensitivity of the probability of switching regime to the size of the borrowing cushion. When  $\gamma_0$  is positive, as the cushion declines the probability of switching to the binding regime increases. Note however that, for certain draws from the logistic function, the borrowing cushion could be negative but the economy could still be in the non-binding regime.

Similarly, when the constraint binds, the transition probability to the non-binding regime is a logistic function of the Lagrange multiplier,  $\lambda_t$ :

$$\Pr(s_{t+1} = 0 | s_t = 1, \lambda_t) = \frac{\exp(-\gamma_1 \lambda_t)}{1 + \exp(-\gamma_1 \lambda_t)}. \quad (13)$$

The probability of switching back from the constrained to the unconstrained regime, therefore, depends on the shadow value of the economy’s desired borrowing relative to the limit set by the collateral constraint. As in the case of a switch from the constrained to con-

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<sup>6</sup>[Kumhof et al. \(2015\)](#) also use a logistic function to model the transition to the default regime in the context of a positive analysis of the relation between financial crises and inequality. [Davig, Leeper, and Walker \(2010\)](#) and [Bi and Traum \(2014\)](#) use a similar logistic assumption to study the macroeconomic consequences of fiscal limits.

<sup>7</sup>As [Appendix C](#) shows, this timing difference with respect to models with traditional specifications of the occasionally borrowing constraint does not affect its positive properties.

strained regime, the parameter  $\gamma_1$  affects the sensitivity of this probability to the value of the multiplier. When  $\gamma_1$  is positive, as the multiplier declines, the probability of exiting the binding regime increases. As in the non-binding regime, for certain draws from the logistic, it is possible that the desired level of borrowing is less than the level forced upon the economy by the binding constraint and this will manifest itself with a negative collateral constraint multiplier.<sup>8</sup>

Putting equations (12) and (13) together, the regime-switching model has the following endogenous transition matrix

$$\mathbb{P}_t = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B_t^*)}{1 + \exp(-\gamma_0 B_t^*)} & \frac{\exp(-\gamma_0 B_t^*)}{1 + \exp(-\gamma_0 B_t^*)} \\ \frac{\exp(-\gamma_1 \lambda_t)}{1 + \exp(-\gamma_1 \lambda_t)} & 1 - \frac{\exp(-\gamma_1 \lambda_t)}{1 + \exp(-\gamma_1 \lambda_t)} \end{bmatrix}. \quad (14)$$

Agents have full information with rational expectations about these transition probabilities that, in general equilibrium, are affected by all endogenous variables in the model.

### 2.3 Remarks on the Endogenous Regime-Switching Specification

A few remarks are useful on how our stochastic formulation of the borrowing constraint works and differs relative to the typical inequality formulation used in the literature.

First, in our setup, agents in the non-binding regime know that higher leverage increases the probability of switching to the binding regime, and vice-versa. Critically, this knowledge preserves the interaction in agents' behavior between regimes and gives rise to precautionary behavior, distinguishing this class of models from those in which financial frictions are always binding (Gertler and Karadi, 2011) or are approximated with solution methods that eliminate the interactions across regimes as in Guerrieri and Iacoviello (2015). One could certainly consider a wider range of variables governing the regime transition. Our choice, as in Bocola (2016), reflects both the need to adopt a parsimonious representation and to choose variables motivated by the sudden stop literature. As we shall see, the specification proposed allows us to identify all parameters of interest and fit the data well with small, well-behaved shocks.

Second, as we already noted, for certain draws of the regime state from the logistic distribution, the borrowing cushion and multiplier can take negative values. This feature has the important implication that the build up to, or the duration of, a financial crisis can be more persistent than in traditional models. As a consequence, as we will illustrate below, our model can account for crisis episodes with varying dynamics and durations and provides

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<sup>8</sup>By construction, the transition probabilities equal 0.5 when their arguments are zero. This assumption can be easily relaxed by introducing a constant into the arguments of equations (12-13). However, preliminary estimates that allow for such degrees of freedom suggested that these parameters were effectively zero. For ease of presentation, we omitted them from the beginning of the analysis.

a more data-consistent account of crisis dynamics without imposing any additional ad-hoc restrictions on the sudden stop definition. For example, negative values of the borrowing cushion  $B_t^*$  in the non-binding regime are possible if the probability of a switch to the binding regime is elevated, but the switch has not been drawn yet, prolonging the boom phase and postponing the beginning of the crisis episode. Conversely, the economy can remain in the binding regime even if the multiplier is no longer positive, but the non-binding regime has not been drawn yet, prolonging the crisis duration and contributing to a sluggish recovery.<sup>9</sup>

This important model property is consistent with a growing body of microeconomic evidence suggesting that a deterministic specification of the switch between regimes in models with occasionally binding collateral constraints may not accurately capture lending and borrowing behaviors at the household and firm or bank level. For example, [Chodorow-Reich and Falato \(2017\)](#) and [Greenwald \(2019\)](#) show that loan covenants are frequently violated, triggering renegotiation rather than suddenly cutting off borrowers from funding once activated. [Campello et al. \(2010\)](#) also provide survey information on the behavior of financially constrained firms, and [Ivashina and Scharfstein \(2010\)](#) examine loan level data, showing that firms draw down pre-existing credit lines in order to satisfy their liquidity needs to avoid hitting borrowing limits. In emerging market economies, official reserves play a similar liquidity role. Thus, in practice, collateral constraints bind for a *range* of leverage ratios rather than at any *one level* as in model with inequality constraints. The aggregate dynamics in our model are also consistent with the macroeconomic empirical evidence of [Jorda et al. \(2013\)](#), who show that the exact level of leverage at which a crisis occurs varies considerably across crisis episodes.<sup>10</sup>

Third, our stochastic specification nests a deterministic case in which regime switching is triggered with probability 1 by a unique level of leverage. Moreover, we show in [Appendix C](#) that the numerical solution of our model is practically not distinguishable from its inequality counterpart, while at the same time offering a dramatically improved trade off between accuracy in terms of Euler equation errors and speed in terms of solution time.

Fourth and finally, as in other specifications of occasionally binding collateral constraints in the literature (e.g., [Bianchi, 2011](#); [Schmitt-Grohe and Uribe, 2020](#); [Farhi and Werning, 2020](#)), our formulation is not derived as an equilibrium outcome of a fully specified con-

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<sup>9</sup>In other words, as the transition probability are endogenous, they are time-varying. In contrast, the exogenous Markov-switching setup ([Davig and Leeper, 2007](#); [Farmer et al., 2011](#); [Bianchi, 2013](#); [Foerster et al., 2016](#)) has a constant probability of transitioning between regimes that is independent of the structural shock realizations and the agent decisions. For this reason, our endogenous-switching framework is capable of generating long- or short-lived binding-regime episodes depending on the realization of shocks and agents' decisions.

<sup>10</sup>The notion of “debt intolerance” discussed by [Reinhart and Rogoff \(2009\)](#) and credit surface of [Fostel and Geanakoplos \(2015\)](#) also are consistent with our stochastic specification.

tracting environment. Our specification however could be derived by generalizing existing contract environments. The traditional inequality formulation of our borrowing constraint,

$$\frac{1}{(1+r_t)}B_t - \phi(1+r_t)(W_tH_t + P_tV_t) \geq -\kappa q_t K_t. \quad (15)$$

can be derived as an implication of incentive compatibility constraints on borrowers if limited enforcement prevent lenders from collecting more than a fraction  $\kappa$  of the value of the assets owned by a defaulting borrower as for instance discussed in [Bianchi and Mendoza \(2018\)](#). An endogenous transition between the two regimes associated with this constraint could be derived in a standard costly state verification framework ([Bernanke and Gertler, 1989](#)). For example, [Matsuyama \(2007\)](#) shows how borrower net worth causes endogenous switching between investment projects with different productivity levels, which feeds back into borrower net worth. [Martin \(2008\)](#) shows that endogenous regime switches in financial contracts from pooling to separating and vice-versa leads to switching in lending standards between a “lax” and a “tight” regime.<sup>11</sup>

### 3 Solving the Endogenous Switching Model

This Section describes our solution method for endogenous regime-switching models. The model proposed can in principle be solved using global methods, as for example in [Davig et al. \(2010\)](#). However, as we discuss in Appendix C, in the case of our application, with two endogenous and five exogenous state variables, the regime switching indicator, plus six exogenous shocks, using a global solution method is prohibitive precluding likelihood-based estimation. Instead, we solve the model using a perturbation approach, which allows for an accurate approximation fast enough to permit estimation and that is potentially applicable to larger models.

We first describe how to implement occasionally binding constraints represented as endogenous regime switching, the approximation point, how to define a steady state in this setup, and the Taylor-series expansions. We then discuss the importance of approximating at least to a second-order in our framework. The competitive equilibrium of the endogenous regime-switching model is defined formally in Appendix A. The derivation of the Taylor-series expansions and other details of the solution method are reported in Appendix B.

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<sup>11</sup>Heterogeneous agent models with financial frictions imposed at the individual level could also be consistent with a stochastic specification of the switching between regimes. For example, [Fernandez-Villaverde et al. \(2019\)](#) study financial frictions and the wealth distribution, where endogenous aggregate risk generates an endogenous regime-switching process.

### 3.1 The Regime-Switching Slackness Condition

A critical step in applying our perturbation methods to the model above, is to ensure that the two variables  $B_t^*$  and  $\lambda_t$  are zero if the economy is in the relevant regime:  $\lambda_t = 0$  in the non-binding regime, and  $B_t^* = 0$  in the binding regime. This step ensures that the slackness condition  $B_t^* \lambda_t = 0$  always holds. To implement this condition and be consistent with regime-switching DSGE models in which the *parameters* are the model objects that change state, we define two auxiliary regime-dependent parameters,  $\varphi(s_t)$  and  $\nu(s_t)$ , such that  $\varphi(0) = \nu(0) = 0$ , and  $\varphi(1) = \nu(1) = 1$ .<sup>12</sup> Next, we introduce the following regime-switching slackness condition:

$$\varphi(s_t) B_{ss}^* + \nu(s_t) (B_t^* - B_{ss}^*) = (1 - \varphi(s_t)) \lambda_{ss} + (1 - \nu(s_t)) (\lambda_t - \lambda_{ss}), \quad (16)$$

where  $B_{ss}^*$  and  $\lambda_{ss}$  are the steady state borrowing cushion and collateral constraint multiplier, respectively, defined more precisely below. It is now easy to see from equation (16) that, as desired,  $\lambda_t = 0$  when  $s_t = 0$ , and  $B_t^* = 0$  when  $s_t = 1$ , always satisfying the slackness condition  $B_t^* \lambda_t = 0$  that characterizes the representative household-firm’s optimization problem. Yet, given a regime  $s_t$ , equation (16) remains continuously differentiable for any value of  $B_t^*$  or  $\lambda_t$ , as no inequality constraint is imposed.

Technically, equation (16) “preserves” information in the perturbation approximation that we introduce in Section 3.3, since, at first order, both variables  $B_t^*$  and  $\lambda_t$  are constant at zero in the respective regimes. The use of the regime-dependent switching parameters,  $\varphi(s_t)$  and  $\nu(s_t)$ , follows from the Partition Principle of Foerster et al. (2016), which separates parameters based upon whether they affect the steady state or not. Intuitively,  $\varphi(s_t)$  captures the *level* of the economy changing across regimes (e.g., the level of capital is lower when the constraint binds), while  $\nu(s_t)$  captures the dynamic responses differing across regimes (e.g., the response of investment to shocks changes when the constraint binds).

### 3.2 Defining the Steady State

Given the regime-switching slackness condition (16), we define a steady state as a state in which all shocks have ceased and the regime-switching variables that affect the level of the economy ( $\varphi(s_t)$ ) take the *ergodic mean* values associated with the steady state transition

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<sup>12</sup>In our model, these parameters coincide with the regime-switching indicator variable  $s_t$ , but in more general settings they may not. The notation provides a general formulation of the modified slackness condition that is applicable to other setups. See, for example, the discussion of our stochastic specification in the context of other model settings in Binning and Maih (2017).

matrix:

$$\mathbb{P}_{ss} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B_{ss}^*)}{1 + \exp(-\gamma_0 B_{ss}^*)} & \frac{\exp(-\gamma_0 B_{ss}^*)}{1 + \exp(-\gamma_0 B_{ss}^*)} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}. \quad (17)$$

Since this matrix depends on the steady state level of the borrowing cushion and the multiplier,  $B_{ss}^*$  and  $\lambda_{ss}$ , which in turn depend upon the ergodic mean of the regime-switching parameter  $\varphi(s_t)$ , such a steady state is the solution of a fixed point problem that is described in more detail in Appendix B.

More specifically, consider the model regime-specific parameters defined above and distinguish between  $\varphi(s_t)$ , which affect the level behavior of the economy, and  $\nu(s_t)$ , which affect only its dynamics with no effects on the steady state. Then denote with  $\xi = [\xi_0, \xi_1]$  the ergodic vector of  $\mathbb{P}_{ss}$ . Next, apply the Partition Principle of Foerster et al. (2016) to focus only on parameters that affect the level of the economy, and write the ergodic mean of  $\varphi(s_t)$ , denoted  $\bar{\varphi}$ , as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1). \quad (18)$$

Defining the steady state as the state in which the auxiliary parameter  $\varphi(s_t)$  is at its ergodic mean value  $\bar{\varphi}$  implies that the approximation point constructed is a weighted average of the steady states of two separate models: one in which only the non-binding regime occurs, and one in which only the binding regime occurs. How close our approximation point is to each of these two other steady states, therefore, depends on the frequency of being in each of the two regimes.<sup>13</sup>

### 3.3 The Solution and Its Properties

Equipped with the steady state of the endogenous regime-switching economy, we construct a second-order approximation to the policy functions by taking derivatives of the equilibrium conditions. We relegate details of these derivations to the Appendix B, but here we provide a summary.

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<sup>13</sup>The ergodic mean is a natural candidate as the perturbation point in models with endogenous state variables and where at least one regime has limited expected duration. These two features imply that the ergodic mean is in the area of the state space in which the economy operates most frequently. In the specific case of the model in this paper, debt and capital are slow-moving state variables, and the binding regime tends to be self-limiting—that is, being in the binding regime causes the economy to reduce leverage and hence switch back to the non-binding regime. Thus, the economy will rarely reach the area around the steady state of the “binding regime only.” Alternative methods for finding solutions to endogenous regime-switching models, such as Maih (2015) and Barthélemy and Marx (2017), propose using regime-dependent steady states as multiple approximation points. Such a strategy would not be suitable for our purposes because the binding regime steady state is a poor approximation point given that the regime is infrequent and usually of shorter duration than normal cycles of expansions and contractions.



For each regime  $s_t$ , the policy functions of our model take the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \quad \mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \quad (19)$$

where  $\mathbf{x}_t$  denotes predetermined variables,  $\mathbf{y}_t$  non-predetermined variables,  $\varepsilon_t$  the set of shocks, and  $\chi$  a perturbation parameter such that when  $\chi = 1$  the fully stochastic model results and when  $\chi = 0$  the model reduces to the non-stochastic steady state defined above. Using these functional forms, we can express the equilibrium conditions conditional on regime  $s_t$  as

$$\mathbb{F}_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = 0. \quad (20)$$

We then stack the regime-dependent conditions for  $s_t = 0$  and  $s_t = 1$ , denoting the resulting system of equations with  $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ , and successively differentiate with respect to  $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ , evaluating them at the steady state. The systems

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \quad \mathbb{F}_{\varepsilon}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \quad \mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \quad (21)$$

can then be solved for the unknown coefficients of the first-order Taylor expansion of the policy functions in equation (19). Note importantly here that since we are only approximating the decision rules and not the probabilities in equation (14), the probabilities remain bounded between zero and one.

A second-order approximation of the policy functions in equation (19) can be found by taking the second derivatives of  $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ . In the end, we have two sets of matrices:  $H_{s_t}^{(1)}$  and  $G_{s_t}^{(1)}$ , characterizing the first-order coefficients; and  $H_{s_t}^{(2)}$  and  $G_{s_t}^{(2)}$  characterizing the second-order coefficients. Therefore, the approximated policy functions are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \quad (22)$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \quad (23)$$

where  $S_t = \left[ (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' \quad \varepsilon_t' \quad 1 \right]'$ .

Our perturbation method produces stable approximated policy functions, but does not guarantee the existence or uniqueness of the solution. As we discuss in Appendix B, this limitation is shared with global solution methods of models of occasionally binding constraints that check convergence of the numerical algorithm without guaranteeing existence or uniqueness.<sup>14</sup>

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<sup>14</sup>While in some simpler models with collateral constraints it is possible to impose parametric restrictions that rule out multiple equilibria (Schmitt-Grohe and Uribe, 2020; Benigno et al., 2016), in the case of our

Our solution method is fast, and can readily be scaled to handle larger models. In all, we have 23 equations that characterize the equilibrium, two endogenous and five exogenous state variables, one regime indicator, and six shocks. Our computational approach is similar to that in [Fernandez-Villaverde et al. \(2015\)](#). We use Mathematica to take symbolic derivatives and export these derivatives so that we can use Matlab to solve the model repeatedly for different parameterizations. The model solves in about a second on a standard laptop.<sup>15</sup>

As [Appendix C](#) discusses in more detail, the proposed solution method is also accurate. We investigate its accuracy by applying it to the calibrated model in [Mendoza and Villalvazo \(2020\)](#) and comparing our endogenous regime switching specification solved by perturbation with the traditional inequality constraint specification solved with global methods. We compare first and second moments for all model variables and show that our solution method yields results practically not distinguishable from those obtained from a traditional inequality specification of the borrowing constraint, except for persistence and volatility of the bond holding and the trade balance. As we argue above and we show in [Appendix C](#), however, these differences illustrates the strengths of the specification that we propose. Specifically, we find Euler equation errors in line with the accuracy of perturbation methods applied to exogenous regime-switching models ([Foerster et al., 2016](#)) and models without regime-switching ([Aruoba et al., 2006](#)). We also document a solution speed more than 800 times faster than the global method. Indeed, the solution speed gain is what makes our objective of likelihood-based estimation feasible.

### 3.4 Approximation Order and Endogenous Switching

Our endogenous regime-switching framework must be solved at least to the second order to capture the effects of the endogenous probabilities on the policy rules, which include precautionary effects that vary with the state of the economy. If we were to use only a first-order approximation, we would not capture the precautionary behavior stemming from rational expectations about the dependency of the probability of a regime change on the borrowing cushion and the multiplier. The following Proposition states this result formally.

**Proposition 1 (Irrelevance of Endogenous Switching in a First-Order Approximation).** *The first-order approximation to the endogenous regime-switching model is identical to the first-order approximate solution of an exogenous regime-switching model in which the*

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model, as in [Mendoza \(2010\)](#) and [Bianchi and Mendoza \(2018\)](#), there are no such restrictions and uniqueness must be verified heuristically. We check the mean-squared stability of the first-order approximation paired with the steady state transition matrix  $\mathbb{P}_{ss}$  ([Farmer et al., 2011](#); [Foerster et al., 2016](#)), and additionally check for explosive simulations.

<sup>15</sup>A general code that applies the solution algorithm is available on the authors' webpages.

*transition probabilities are given by the steady-state value of the time-varying transition matrix.*

**Proof.** See Appendix B.

The Proposition illustrates that using a second-order approximation to the solution is necessary to characterize the model properties associated with the endogenous nature of the regime-switching, including particularly precautionary behavior. This result is analogous to the one stating that, in models with only one regime, first-order solutions are invariant to the size of the shocks, second-order solutions captures precautionary behavior, and third-order solutions are needed to capture the effects of stochastic volatility (Fernandez-Villaverde et al., 2015). Unfortunately, however, the need to use a second-order approximation along with regime-switching creates additional challenges for estimation purposes. We now turn to our strategy to address these issues.

## 4 Estimating the Endogenous Switching Model

We estimate the model with a full information Bayesian procedure. The posterior distribution has no analytical solution and we use Markov-Chain Monte Carlo (MCMC) methods to sample from it. Since the Metropolis-Hastings algorithm that we use for sampling is a standard tool, we omit a discussion of this step in our procedure. The details of the construction of the state space representation and the filtering steps for the evaluation of the likelihood are reported in Appendix D.

A critical challenge in sampling from the posterior is the evaluation of the likelihood function. We face three difficulties here relative to linear DSGE models (for example, Smets and Wouters, 2007). The first is the non-linearity due to the presence of multiple regimes. The second is the need to approximate to the second-order the model solution that governs the decision rules in each regime. The third is the fact that the transition probabilities are endogenous. Bianchi (2013) develops an algorithm to address the first difficulty. Here we must deal with the second-order approximation and endogenous probabilities in a tractable manner. One alternative is the Particle Filter (Fernandez-Villaverde and Rubio-Ramirez, 2007). However, the regime switching leads to discarding a large number of simulated particles, lowering accuracy for a given number of particles and greatly increasing the computational cost of obtaining a given level of accuracy (Doucet et al., 2001).

To address these challenges, we use the Unscented Kalman Filter (UKF) with Sigma Points (Julier and Uhlmann, 1999). The Sigma Point filter has been shown to be an efficient way to estimate regime switching models (Binning and Maih, 2015). As we show in Section

4.3, the estimated shocks are well-behaved, providing direct evidence that the model and associated filter is effective at capturing the nonlinear dynamics of sudden stops while not relying on exogenous shocks with large outliers.

Given the complexity of our non-linear model, our estimation approach takes four steps. First, we estimated a version of the model without working capital and the occasionally binding constraint, which yields an initial estimate of the exogenous processes and the non-financial parameters. Second, conditional on these initial estimates, we performed a grid search over the remaining parameters ( $\kappa$ ,  $\phi$ ,  $\gamma_0$ , and  $\gamma_1$ ) to find high posterior regions. Third, from the high posterior regions of the grid search, we used a mode-finding routine to identify the posterior mode, which forms the basis for our empirical results. Lastly, we sampled 500,000 times from the posterior with a random-walk Metropolis-Hastings algorithm to explore the parameter space around the mode and characterize credible sets for the parameter estimates.<sup>16</sup>

## 4.1 Observables, Data, and Measurement Errors

We estimate the model with quarterly data for GDP growth (gross output less intermediate input payments), consumption growth, investment growth, and intermediate import price growth, as well as the current account-to-output ratio, and a measure of the country real interest rate.<sup>17</sup>

As there are six shocks with six observables, we do not need measurement errors. However, measurement errors in the observation equation improve performance of the non-linear filter and accounts for any actual measurement error in the data. We follow [Garcia-Cicco et al. \(2010\)](#) and limit their variance to 5% of the variance of the observable variables. As a consequence, our model will fit the data relatively closely on average; thus, how it performs across cycles and crises and whether it relies on large shocks to fit the data will be important in assessing model performance.

## 4.2 Calibrated Parameters and Prior Distributions

Our objective is to provide likelihood-based estimates of parameters governing the model's dynamics in both the binding and non-binding regime, on which we have some prior information, and those that govern the transitions between regimes, on which we do not have

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<sup>16</sup>For the last MCMC step, we adjusted the scale of the proposal density until we achieved an acceptance rate of 0.25.

<sup>17</sup>See Appendix F for details on variable definitions and data sources. The country interest rate is constructed following [Uribe and Yue \(2006\)](#) and it is the US 3-Month Treasury Bill minus ex post US CPI inflation rate plus Mexico's EMBI Spread.

**Table 1: Calibrated Parameters**

Parameter	Description	Value
$\beta$	Discount Factor	0.9798
$\rho$	Risk Aversion	2.0000
$\omega$	Labor Supply	1.8460
$\eta$	Capital Share	0.3053
$\alpha$	Labor Share	0.5927
$\delta$	Depreciation Rate	0.0228
$P^*$	Mean Import Price	1.0280
$E^*$	Mean Expenditure	0.2002

any prior information. To improve estimation of these parameters of interest, we calibrate a set of parameters listed in Table 1 on which we have strong prior information. In particular, we set these parameters largely following [Mendoza \(2010\)](#), who calibrated them based on the stylized facts from Mexico’s National Accounts, adapting the calibration to our model specification and the quarterly data frequency.<sup>18</sup>

We set two types of priors on the parameters to be estimated. The first type of prior is set on the parameters. These priors are shown in Table 2. We use relatively diffuse priors to reflect our lack of *a priori* information on their values, using them to impose sign restrictions and place lower prior probability on parameter values that generate implausible moments in model simulations. The second type of prior is on a model-implied object: the steady state transition probability of switching from the binding to to the non-binding regime, given by the steady state value of equation (12),  $\Pr(s_{t+1} = 1 | s_t = 0, B_{ss}^*)$ . This prior is a Beta distribution with mean 0.25 and variance of 0.25. It puts lower probability mass on combinations of parameters that either generate extremely infrequent transitions to the binding regime, or that imply the economy exits the binding regime almost immediately.<sup>19</sup>

### 4.3 Estimated Parameters and Model’s Fit

We now discuss the estimated parameters and the model’s fit to the data. Table 2 reports the mode, the median, the 5th, and the 95th percentile of the posterior distribution of the estimated structural parameters of interest. The estimated mean interest rate, slightly below 1.75% per quarter, is close to the value estimated by [Mendoza \(2010\)](#). The posterior coverage interval for this variable is fairly diffuse, indicating some uncertainty about its true value.

The remaining parameters have tightly estimated posteriors, so we will focus the discus-

<sup>18</sup>See Appendix E for more details on the calibration and the targeted data moments.

<sup>19</sup>Priors on model-implied objects have been used by, for example, [Otrok \(2001\)](#) and [Del Negro and Schorfheide \(2008\)](#).

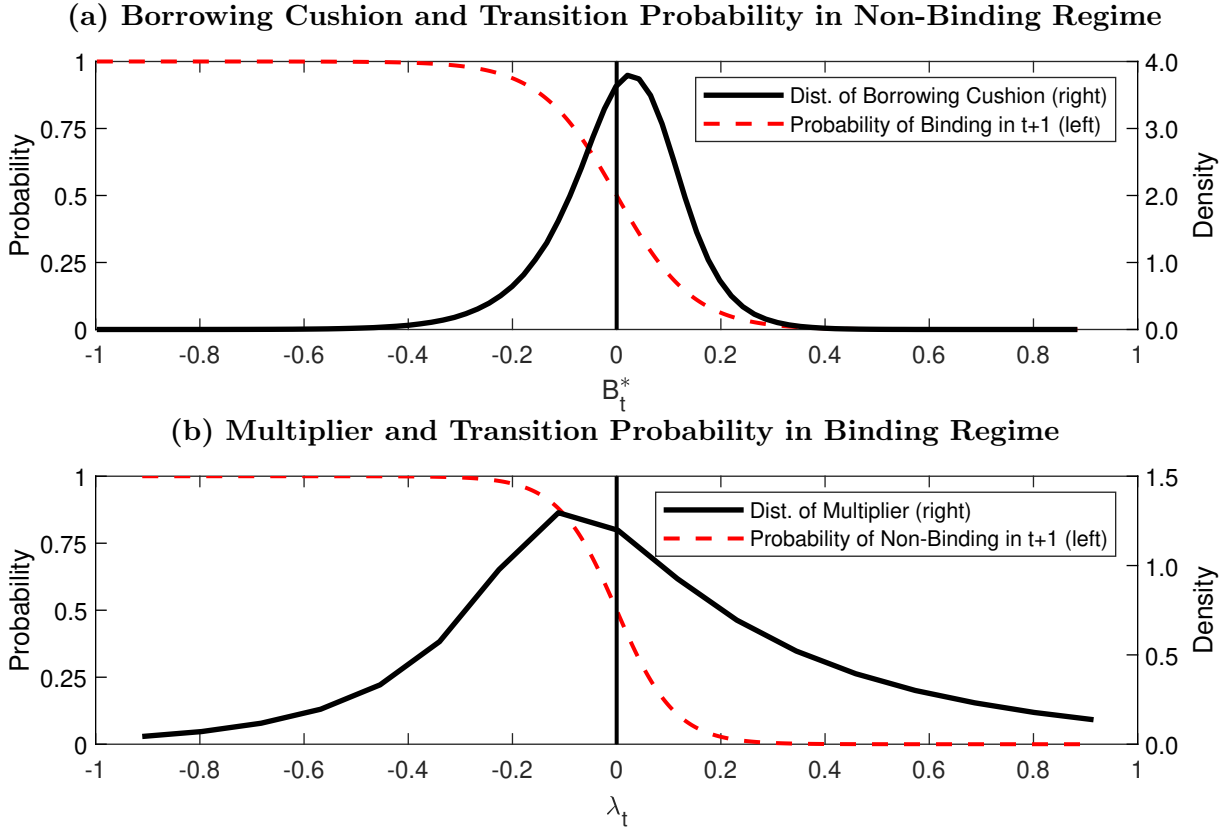
**Table 2: Estimated Parameters**

Par.	Description	Prior	Posterior			
			Mode	5%	50%	95%
$\iota$	Capital Adj.	N(10,5)	12.703	12.649	12.701	12.724
$\phi$	Working Cap.	U(0,1)	0.7113	0.7102	0.7153	0.7207
$r^*$	Mean Int. Rate	N(0.0177,0.01)	0.0172	0.0115	0.0165	0.0216
$\kappa$	Leverage	U(0,1)	0.1727	0.1592	0.1756	0.1989
$\rho_a$	Autocor. TFP	B(0.6,0.2)	0.9796	0.9653	0.9793	0.9881
$\rho_e$	Autocor. Exp	B(0.6,0.2)	0.9111	0.9066	0.9132	0.9237
$\rho_p$	Autocor. Imp Price	B(0.6,0.2)	0.9711	0.9609	0.9754	0.9549
$\rho_d$	Autocor. Pref.	B(0.6,0.2)	0.9810	0.9753	0.9810	0.9843
$\rho_{r^*}$	Autocor. Persist. Int. Rate	B(0.6,0.2)	0.8929	0.8782	0.8896	0.8995
$\sigma_a$	SD TFP	IG(0.01,0.01)	0.0083	0.0066	0.0081	0.0098
$\sigma_e$	SD Exp.	IG(0.1,0.1)	0.1806	0.1672	0.1816	0.1892
$\sigma_p$	SD Imp. Price	IG(0.1,0.1)	0.0471	0.0382	0.0452	0.0524
$\sigma_d$	SD Pref.	IG(0.1,0.1)	0.1123	0.0998	0.1123	0.1194
$\sigma_r$	SD Trans. Int. Rate	IG(0.01,0.01)	0.0028	0.0013	0.0025	0.0044
$\sigma_{r^*}$	SD, Persist Int. Rate	IG(0.01,0.01)	0.0047	0.0037	0.0047	0.0059
$\gamma_0$	Logistic, Enter Binding	U(0,150)	13.552	10.903	13.712	18.014
$\gamma_1$	Logistic, Exit Binding	U(0,150)	17.798	15.784	17.800	19.806

Notes: Estimated parameters, with prior distribution and posterior moments. Priors are Normal, Uniform, Beta, or Inverse Gamma; prior distributions show mean and variance, except for uniform where lower and upper bounds are shown. Posterior distribution shows mode, along with 5-th, 50-th, and 95-th percentiles from the MCMC posterior draws.

sion on posterior modes. The model provides precise estimates of the investment adjustment cost, working capital, and leverage parameters, and the parameters of the logistic function. These parameters cannot be directly calibrated from the stylized facts of the data—unlike, for example, capital or labor shares—but are nonetheless critical for the model dynamics and properties. In particular, the estimated investment adjustment cost parameter ( $\iota$ ) is 12.7. This parameter is model dependent and its magnitude has no interpretation outside of a particular model. The estimate for the working capital constraint parameter indicates that 71% of the wage and intermediate good bill needs to be paid in advance with borrowed funds. This estimate is substantially higher than the 25.79% value set by [Mendoza \(2010\)](#) and much lower than the 100% used by [Neumeier and Perri \(2005\)](#) or the 125% used by [Uribe and Yue \(2006\)](#). The estimate is close to the 60% estimated by [Ates and Saffie \(2016\)](#) who use interest payments and production costs from Chilean microeconomic data. The estimated value of the leverage parameter in the borrowing constraint ( $\kappa$ ) is 0.17, indicating less than a fifth of the value of capital serves as collateral. The estimate is slightly tighter than the benchmark value of 0.20 chosen by [Mendoza \(2010\)](#), and on the low end of the 0.15

**Figure 2: Logistic Functions and Distributions of Their Arguments**



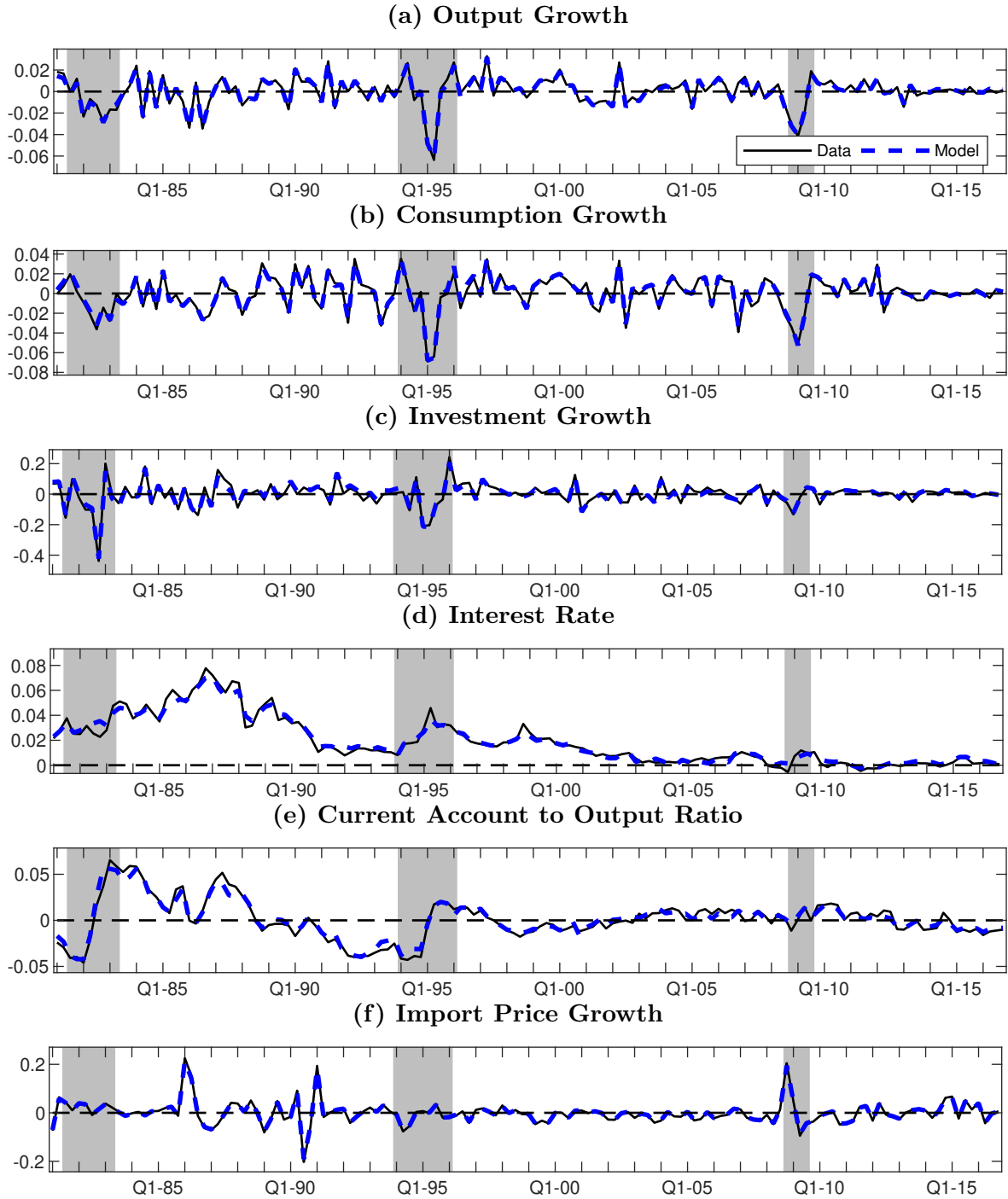
Note: The top panel shows the model-implied distribution of the borrowing cushion  $B^*$  in the non-binding regime, and the logistic transition function to the binding regime as in equation (12) implied by our estimates. The bottom panel shows the model-implied distribution of the multiplier  $\lambda$  in the binding regime, and the transition function to the non-binding regime as in equation (13) implied by our estimates.

to 0.30 range of alternative values considered in that calibration.

The posterior modes of the logistic parameters in equations (12) and (13) are 13.6 and 17.8, respectively, estimated in a tight range relative to the very loose prior. These estimates are significantly different from zero, thus providing strong evidence that the data reject a model specification in which the transition probabilities are exogenous, which is allowed for under the prior distribution.

Figure 2 plots the implied probabilities from equation (12) and (13), evaluated at the posterior mode value of  $\gamma_0$  and  $\gamma_1$ , together with the estimated ergodic distributions of their arguments, the borrowing cushion,  $B^*$  and the the constraint multiplier,  $\lambda$ . The figure shows that the ergodic distribution of the borrowing cushion is centered on a positive value, as the economy spends most of its time in the non-binding regime, above the borrowing limit. As the borrowing cushion falls, the probability of switching to the binding regime increases, and gradually reaches 1 for small negative values, with very little probability mass on large

Figure 3: Data and Model Estimates



Note: The figure plots observable variables used in estimation (solid black lines) and fitted values (i.e., model implied smoothed estimated series based upon the full sample, dashed blue lines). Gray areas indicate model-identified periods of crisis defined in Section 5.2.

negative realizations of the borrowing cushion.

On the other hand, once the economy is in the binding regime, the ergodic distribution



of the multiplier is centered on small negative values, with more probability mass on the right tail than the left tail. As  $\lambda$  approaches 0 from the positive side of the support, the probability of switching to the non-binding regime increases and quickly reaches 1, with a mode on a small negative value. Nonetheless, there is a significant probability mass for larger negative values. As we explained earlier, negative values of  $\lambda$  reflect instances in which had the economy been in the non-binding regime, the borrowing cushion would be positive, but a switch to the non-binding regime at has not been drawn yet.<sup>20</sup>

The observables used in estimation are shown in Figure 3, along with the model estimates. The Figure also includes the model-identified crises (gray regions), which we define and discuss in Section 5.2. As we have already noted (Figure 1), during the sample period there are both regular cycles and crisis episodes, with rather substantial declines in output, consumption, and investment growth, and reversals in the current account to output ratio. In between crisis episodes, their fluctuations are more limited. Since we have assumed measurement error with a variance of 5% of the observables, the model tracks the data closely. The tracking is consistent throughout the sample, during both regular business cycle and crisis periods.<sup>21</sup> For example, around the 1995 “Tequila Crisis,” the data show large drops and rebounds in output, consumption, and investment growth, and a very sharp reversal in the current account to output ratio. If, by contrast, one were to observe a loss of fit during crisis episodes, it would suggest that our estimated model finds it difficult to match the data dynamics during these episodes of critical interest in the empirical analysis.

A key feature of the likelihood based estimation that we perform is that we can recover the historical shock series that drive our observables. This is not possible in calibrated models and in turn allows us to analyze, in Section 4.3, the relative importance of different shocks in different phases of the different crises historically realized. In other words, we can empirically—rather than theoretically—assess which shocks drove crises and cycles in Mexico’s history.

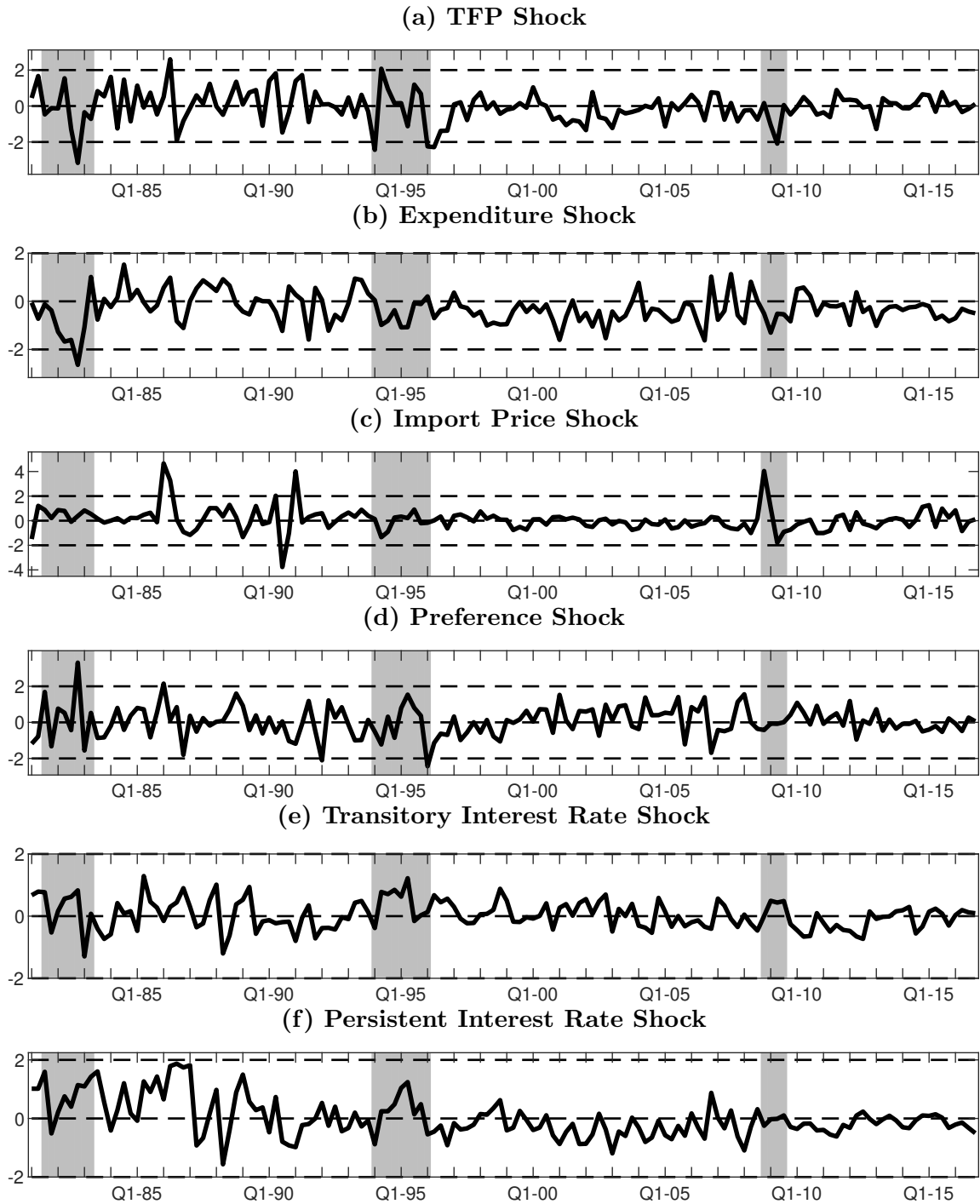
Figure 4 plots the estimated model implied shocks in standard deviation units, together with a two-standard deviations band. Since the model tracks the observed series closely and evenly over time, the recovered structural shocks are informative on the model’s mechanisms ability to drive the outsize movements in the data seen during crises. If only normal shocks are needed, then the model’s internal propagation must be accounting for the abnormal fluctuations in the data; alternatively, reliance on large, low probability shocks would cast doubt on the model’s ability at capturing the crisis dynamics. Well-behaved realized shocks

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<sup>20</sup>Sufficiently negative values of  $\lambda$ , approximately below  $-0.2$ , produce a nearly deterministic switch back to the binding regime. The ergodic distribution of  $\lambda$  in the binding regime (Figure 2b) implies that the probability of exiting that regime exceeds 99% about a quarter of the time.

<sup>21</sup>See Appendix G for additional details.

Figure 4: Model Estimated Shocks



Notes: The figure plots the estimated model implied shocks, in standard deviation units, together with a two-standard deviations band (black dashed lines). Gray areas indicate model-identified periods of crisis defined in in Section 5.2.

also provide evidence supporting our choice of using the Sigma Point filter. As we can see in Figure 4, the estimated model fits the data without relying on large or skewed shocks, including particularly during crisis times. Instead, it explains crisis dynamics using the model’s internal propagation mechanisms that amplify the effects of normally sized shocks. Shocks slightly outside the two-standard deviations band are estimated only right before the 1982 debt crisis, possibly do to the limited number of observations before the peak of that crisis episode. TFP, expenditure, and preference shocks, however, are all well within the band during the that event. Moreover, the few shock realizations outside the two-standard deviation band are both positive and negative. The largest shocks occur for the import price shock, which is directly implied from the observable data on import prices. The shock processes thus provide evidence that the skewness that is evident in the raw data is picked up by the regime switching and the propagation mechanism of the model rather than the estimated shocks. Indeed, Appendix G shows that the realized errors are approximately normal distributed, thus validating the assumption needed to apply the UKF.

## 5 The Anatomy of Mexico’s Business Cycles and Financial Crises

In this Section, we study Mexico’s history of business cycles and sudden stop crises through the lens of the estimated model. We first compare moments in the model and the data and assess the relative importance of different shocks by means of a variance decomposition. We then focus on sudden stop crises: how the model allows us to identify them, the historical drivers of the episodes identified, and the model’s dynamics before, during, and after these episodes.

### 5.1 Business Cycles

Table 3 compares data and simulated model second moments, reporting results for three variables used in estimation (output growth, consumption growth, investment growth, and the country interest rate), and one critical variable, the trade balance ratio, not used in estimation.<sup>22</sup> The model matches the business cycle data moments well, fitting both the relative volatilities and the correlations with output. Consumption is significantly more volatile than

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<sup>22</sup> All business cycle and crisis statistics rely on simulated data based on the posterior mode estimates. We generate 100,000 samples of 144 quarters length (the same as our data sample), after a burn-in period of 1,000 quarters. We then compute median values across these 10,000 runs. We use a pruning method (Andreasen et al., 2018) to avoid explosive simulation paths. All reported simulated moments are unconditional, rather than conditional on a particular regime.

**Table 3: Simulated Second Moments: Data and Model**

Data Series	Relative Std. Dev.		Correlations	
	Data	Model	Data	Model
Output Growth	1.00	1.00	1.00	1.00
Consumption Growth	1.25	1.92	0.73	0.98
Investment Growth	5.37	5.75	0.53	0.90
Trade Balance to Output Ratio	1.24	0.80	-0.20	-0.21
Country Interest Rate	1.36	0.15	-0.11	-0.03

Notes: The table compares second moments of the data, relative to the same moments simulated from the model.

output, which is a robust stylized fact of emerging market business cycles. The model underestimates the relative volatility or the trade balance ratio and, particularly, the country interest rate. The model implied comovements of all variables match the data counterparts remarkably well, again with the exception of the country interest rate, whose correlation is not estimated precisely in the model. The trade balance, in particular, is counter-cyclical as in the data, with a model-implied autocorrelation coefficient (not reported) well below one without resorting to using the debt elasticity parameters as in [Garcia-Cicco et al. \(2010\)](#).

Table 4 reports variance decompositions. The table illustrates that all shocks play an important role, even though different shocks matter more for different variables. Output and consumption are predominantly driven by productivity, preference, expenditure, and terms of trade shocks, respectively. Investment is significantly affected by expenditure, preference, productivity, terms of trade, and persistent interest rate shocks. Expenditure and persistent interest rate shocks are the most important drivers of the trade balance, while the country interest rate is mostly driven by persistent interest rate shocks, and to a lesser extent by the temporary interest rate shock. Expenditure and preference and both interest rate shocks play a more important role than productivity and terms of trade shocks for financial variables and the collateral multiplier.

The estimated variance shares are not directly comparable with those in the literature. The results suggest that both real and financial shocks matter for Mexico business cycles. We find a lower share for productivity and interest rate shocks than [Fernandez and Gulan \(2015\)](#), although we also consider terms of trade and demand shocks. We also find a variance share explained by terms of trade shocks that is very close to the structural VAR model estimated by [Schmitt-Grohe and Uribe \(2018\)](#). The variance share explained by interest rate shocks is smaller than in [Garcia-Cicco et al. \(2010\)](#), who use an ad-hoc specification of the financial friction as a debt elastic country premium, without amplification from the financial accelerator ([Fernandez and Gulan, 2015](#)), or the working capital ([Neumeier and](#)

**Table 4: Estimated Unconditional Variance Decomposition**

Variables / Shocks			Import		Temp.	Pers.
	TFP	Expend.	Prices	Pref.	Int. Rate	Int. Rate
Output	33.2	17.2	15.7	25.4	2.5	6.0
Consumption	30.3	23.4	14.3	20.6	3.8	7.6
Investment	19.2	29.8	10.3	25.6	4.6	10.5
Trade Bal/Output	9.5	35.2	8.8	17.2	9.2	20.1
Interest Rate	0.0	0.0	0.0	0.0	21.1	78.9
Borrowing Cush.	10.6	32.3	9.9	21.3	9.9	16.0
Debt/Output	15.2	25.5	7.6	40.9	1.4	9.5
Multiplier	9.5	40.5	9.5	18.1	9.6	12.8

Note: The variance decomposition is normalized to sums to 100 by row, but estimates may not add up to 100 exactly due to rounding. The decomposition is computed by setting each shock to zero to compute its marginal impact on each variable.

[Perri, 2005](#); [Mendoza, 2010](#); [Ates and Saffie, 2016](#)).

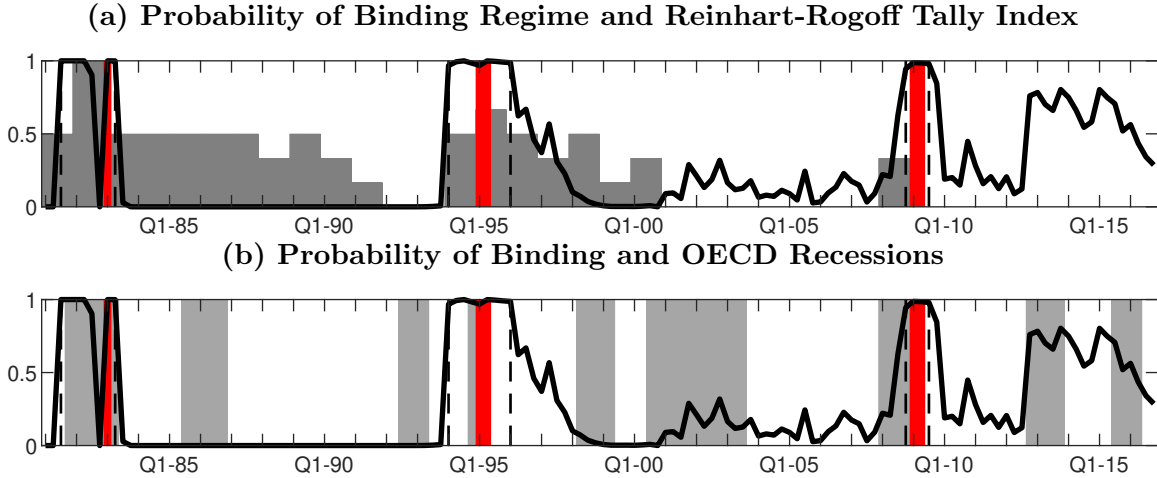
## 5.2 Financial Crises

The defining feature of our model is its ability to identify shocks and characterize dynamics not only over regular business cycles, but also during periods of a particular type of crisis, the so-called sudden stop in capital flows. So we now turn to the model-implied reading of Mexico’s history of financial crises. We start by defining financial crises episodes in a model consistent manner. Next, we investigate the drivers of the three historical episodes of sudden stop that the estimated model identifies in the data: the Debt Crisis of the 1980s, the 1995 ‘Tequila’ Crisis, and the spillover of the Global Financial Crisis (GFC) in 2008-2009. Finally, we investigate the model’s dynamics before, during and after crisis episodes.

### 5.2.1 Defining and Identifying Sudden Stop Episodes

The estimated model allows us to *make inference* on whether the economy is in the binding regime, and hence identify periods of sudden stop crisis in a model-consistent manner. In the model, the household-firm knows the regime, but the estimation procedure does not and the regime must be inferred based on the information in the data. The estimation results, therefore, provide a time-varying estimate of the smoothed probability of being in each regime that is based upon the full sample information. The estimation procedure also provides an estimate of the time-varying transition probability in equations (12)-(13), showing that an exogenous regime-switching specification would be strongly rejected by the data. These probabilities are reported in Appendix G.

Figure 5: Mexico’s Model-identified Crisis Episodes



Notes: Black line shows the model implied smoothed probability of being in the binding regime. The dark gray regions in panel (a) indicates [Reinhart and Rogoff \(2009\)](#) tally index of financial crisis, normalized so that it takes values between 0 (no crisis) and 6 (most severe). Light gray regions in panel (b) indicate OECD recession dates for Mexico. Red bars indicate crisis periods identified as in calibrated models with traditional inequality constraints, vertical blacked dash lines indicate the beginning and the end of the estimated crisis episodes; see text for details.

Figure 5 plots this model-based estimate of the probability of being in each regime (solid black line). Using this estimate, we define a *crisis quarter* as one in which the economy is in the binding regime with probability higher than 90% probability.<sup>23</sup> A *crisis episode* can then be defined as a sequence of such periods. Figure 5 shows that our estimated model identifies three crisis episodes whose start and end quarters are marked by the vertical dashed lines. The first is the Debt Crisis, occurring from 1981:Q3 to 1983:Q2, with its peak in 1983:Q1. The second episode is the Tequila Crisis, during 1994:Q1-1996:Q1, with its peak in 1995:Q1-Q2. The last one is the the Global Financial Crisis that in Mexico led to sudden stop during 2008:Q4-2009:Q3, peaking in 2009:Q1-Q2.

Figure 5 also reports a purely empirical definition of financial crisis (dark grey shaded areas in Panel a) and the OECD dating of the business cycle of Mexico (light grey shaded areas in Panel b). The empirical notion of financial crisis reported is a normalized version of the *crisis tally* index of [Reinhart and Rogoff \(2009\)](#) (RR).<sup>24</sup> Figure 5 clearly illustrates that our estimated probability of being in a binding regime, which is our model-consistent

<sup>23</sup>Such a threshold is intuitive and justifiable but somewhat arbitrary. The duration of the identified sudden stop episode, however, is robust to using a wider range of values.

<sup>24</sup>The RR tally index ranges from 0 to 6, depending on whether a country-year observation is deemed to be in one or more of the following 6 varieties of crisis, assigning the value of one if a variety is present: Currency, Inflation, Stock Market, Sovereign Domestic or External Debt, and Banking Crisis. See Chapter 1 of [Reinhart and Rogoff \(2009\)](#) for more details. We follow their methodology to extend the index to cover our full sample. In Figure 5, the index is normalized to range between 0 and 1.

definition of a crisis, aligns quite well with the RR tally index. The crisis episodes that our model identifies track the RR tally index very well in the case of the Tequila and GFC episodes, and are much more persistent than the crisis typically identified in the sudden stop literature in calibrated models with traditional inequality constraints (red bars). Our model’s crisis signal is less persistent than the tally index in the aftermath of the Debt crisis. This result is to be expected, however, as our model economy is not designed to capture debt overhang or financial intermediation disruptions that drive the classification of RR between 1983 and 1989 and in the mid-1990s.

Importantly, our model estimates of these three sudden stop episodes do not mistake ordinary recessions, not associated with spikes in the tally index, for crisis periods. Mexico’s OECD recessions are illustrated by the light dark shaded areas in Figure 5 Panel (b). The estimated probability of a binding regime is close to 0 during the OECD recessions before the Tequila crisis, during the US recession in 2001, and the Argentine crisis in 2000-2001. The estimated probability of a binding regime also does not register stress during the 1998 Russian default and US Long-Term Capital Management debacle that affected only the currency and stock market, without triggering a sudden stop in Mexico.

For comparison, Figure 5 also reports crisis episodes as typically defined based on calibrated models with occasionally binding constraints (red bars). In this view, crises are sequences of periods in which (i) the borrowing constraint multiplier is positive and (ii) the model generated output growth is negative by more than a certain threshold, while the current account balance reversal meet another empirical condition (for example, [Mendoza, 2010](#); [Benigno et al., 2013](#), among many others).<sup>25</sup> This second set of auxiliary conditions is necessary in that setting because fully model-consistent definitions of a sudden stop as instances in which the borrowing constraint multiplier is positive are counterfactual and cannot match the frequency and the severity of actual sudden stops episodes as seen in the data. Thus, calibrated models with traditional inequality constraints need additional empirical assumptions to identify a sudden stop in the model simulations. In sharp contrast, our estimated model provides a fully model-consistent definition of a sudden stop.

Overall, Figure 5 shows that our model provides an accurate data-driven signal of when the economy is likely to have experienced a crisis, without mistaking regular recessions or large currency and stock market movements for financial crisis episodes. Most importantly,

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<sup>25</sup>For comparison purposes, the red bars in Figure 5 are defined as sequences of quarters in which, in addition to the criterion met by the back solid line, the model estimate of output growth (reported in Panel a of Figure 3) is negative by more than one standard deviation, and the model estimate of the current account ratio (reported in Panel e of Figure 3) increases by more than one standard deviation. Thus, here, we replaced the first criterion with our model-consistent definition of crisis period, namely that the smoothed estimate of the probability of being in the binding regime is at least 90%.

these crisis episodes are identified by relying on a purely model-consistent definition of sudden stop, which is the smoothed probability on being in the binding regime.

### 5.2.2 Drivers of Mexico’s Sudden Stop Episodes

We now turn to which shocks drove the three identified episodes in Mexico’s history. As we saw earlier, our estimated model fits Mexican data well (Figure 3), including during the crisis episodes identified in Figure 5, without relying on large shocks. We now examine the sudden stop episodes identified, evaluating the relative importance of different shocks driving the economy before, during and after the Debt Crisis of the early 1980s, the Tequila Crisis of 1994-1995, and the spillover on Mexico from the GFC that originated in the United States in 2008-09. This exercise is possible because we have a likelihood-based estimation of the model that produces sequences of shocks, shown in Figure 4, allowing for the construction of such historical counterfactuals.

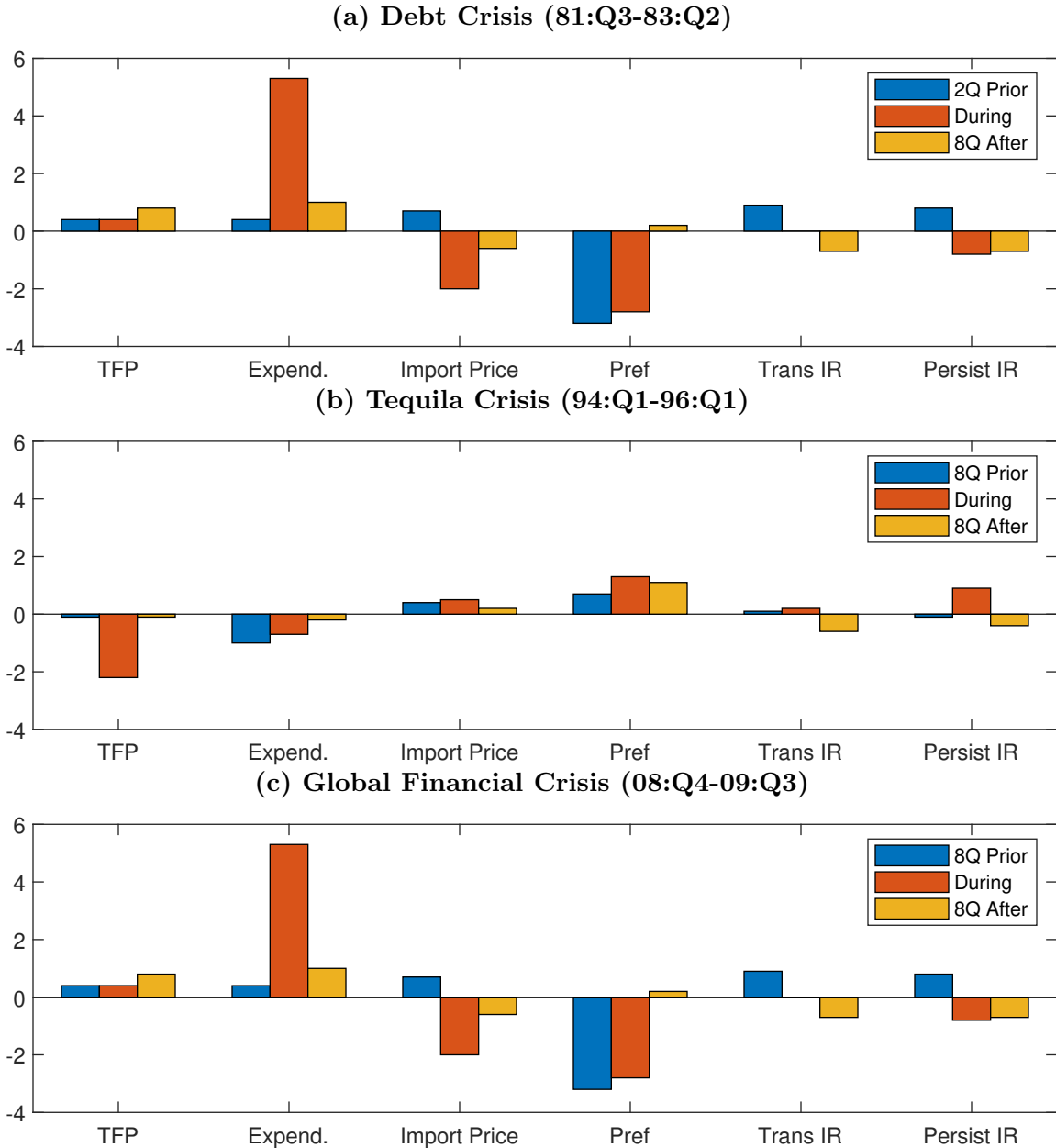
While many counterfactuals are possible, there is no standard benchmark for this class of models, and here we present results from one possible approach. The multiple sources of non-linearity in the model, the endogenous regime-switching and the second-order approximation pose a challenge for computing historical counterfactuals. The task is complicated by the fact that shocks not only have non-linear effects on the endogenous variables, but also on the realization of the regimes in subsequent periods. To address these issues, rather than decomposing the change in individual endogenous variables, we compute a summary measure, namely the importance of each shock in terms of contribution to the likelihood. Specifically, we counterfactually recalculate the model likelihood, evaluated at the posterior mode, turning one shock off at the time, while leaving all other shocks at their estimated values, over a particular sub-sample and repeat the calculation for all six shocks. The details of these calculations are in Appendix G, but in summary, the measure is the average importance of a shock in a given period relative to all shocks in the model, relative to their average importance over the full sample.

Figure 6 reports this likelihood-based measure of shock relative importance. For each of the three crisis episodes, we consider the crisis episodes themselves as defined above, as well as two years before and two years after the episode. Positive (negative) numbers denote relatively more (less) important shocks in a given sub period. For example, a value of +1 means that the shock is relatively more important in that period relative to its average importance over the full sample. By definition, the percentages in Figure 6 sum to zero for each bar of the same color in each panel.

Each shock plays some role in explaining different crises or different phases of a particular



**Figure 6: Estimated Relative Importance of Shocks in Mexico's Crises**



Notes: The figure plots a likelihood-based measure of the importance of each shock relative to all shocks in the model, during different sub-sample periods, compared to their average relative importance over the full sample, in percentage point differences. For example, a value of +1 indicates that the shock has an average 1 percentage point greater relative importance in the sub-sample relative to its average importance over the full sample. See Appendix G for details. The prior period before the 1983 Debt Crisis is limited by the data sample length.

crisis, highlighted by the fact that no individual shock has a disproportionate importance in any particular period. Consider first the Debt Crisis. Since this episode starts right at the beginning of the sample period, we can only look at the two quarters before its start.

The counterfactual suggests that, in the immediate run up, the most important shocks were imported intermediate input prices and both temporary and persistent interest rate shocks, consistent with the drop in oil prices starting in 1981 and the Volcker disinflation in the United States. The crisis episode itself and its aftermath appears driven by the expenditure and technology shocks, possibly reflecting the import and fiscal contraction typically associated with a sudden stop and its aftermath, and the loss of efficiency associated with sudden adjustment of expenditure plans due to tighter financial frictions.

Next, consider the Tequila Crisis. According to our counterfactual, in the run up to this crisis episode, the most important drivers were the preference shock and the imported input price shock. The preference shock possibly captures the political developments (the uprising in Chiapas and the assassination of a leading presidential candidate) that are well known to have contributed to crisis's precipitation. The terms of trade shock likely captures the persistent decline in the oil price that fell persistently during the three years prior to the crisis, after a sustained recovery after the 1986 crash.<sup>26</sup>

The importance of these two shocks increases during the crisis episode, even though the shock to the persistent component of the interest rate also becomes much more important, reflecting the sudden increase in US interest rates during this period. The likelihood weight of the preference and interest rate shocks declines after the crisis, while the weight in the likelihood of the technology shocks increases during this phase in relative terms. In the post-crisis period, the preference shock continues to play a role, while the importance of shocks to both components of the interest rate decline markedly.

Overall, therefore, according to our estimation results, the main drivers of the Tequila crisis are the preference, the persistent interest rate, and the terms of trade shocks. This is notwithstanding the fact that the model also allows for temporary exogenous changes in the country specific component of the interest rate and a amplification mechanism from the occasionally binding borrowing constraint. In contrast, in calibrated sudden stop models with occasionally binding constraints, or estimated models with a debt elastic interest rate premium, sudden stops are driven by technology and interest rate shocks. Our estimation results, therefore, suggest that these models, which rely on auxiliary empirical assumptions for the identification of sudden stops, or ad-hoc specifications of the risk premium shock appended to interest rates, might be misspecified from an empirical standpoint.

Lastly, consider the GFC episode. The counterfactual likelihood analysis suggests that, before the crisis, expenditure and to a lesser extent a shock to the persistent component of the interest rate were the most important drivers. This is consistent with the lax interna-

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<sup>26</sup>Oil is the most important driver of Mexico's fiscal and current account during this period as clearly documented in the IMF adjustment programs of that period.

tional financial conditions, strong external demand, and loose Mexican fiscal policy stance prevailing before the GFC. However, all other shocks become more important during the crisis episode itself, consistent with the collapse of the oil price in during the fall of 2008 and spike in emerging market spreads. In the aftermath of the episode, the likelihood weight of the import price shock and temporary interest rate shock diminishes, while that of the preference and persistent interest shocks increases, confirming the importance of international financial conditions during this episode.

### 5.2.3 Simulated Sudden Stops: Duration, Frequency, and Dynamics

We saw earlier that our model-consistent definition of a sudden stop crisis aligns well with the RR tally index, which is based on a narrative approach to identification of crisis episodes. This important result stems from the stochastic specification of the borrowing constraint that is also a more realistic characterization of how lending limits applies to individual households and firms in the micro data. In contrast, as we also argued before, sudden stops identified using calibrated models with traditional inequality occasionally binding constraints have a much shorter and counterfactual duration, even after imposing empirically motivated additional ad hoc restrictions. In this section, we provide more details on the properties of our model generated crises, studying their frequency, duration, and severity.<sup>27</sup>

Figure 7 reports duration and frequency statistics of simulated sudden stops as defined by the economy being in the binding regime. Note in fact that in the simulations in this section, the household-firm always knows in which regime the economy is. Therefore, there is no sampling uncertainty about being in the binding regime, in contrast to the econometrician’s perspective reported in Figure 5. The figure shows that the estimated model can generate substantial heterogeneity in crisis episode duration and frequency. This highlights the importance of combining prior information on the structural parameters of the model with the likelihood of the data to identify data-consistent historical sudden stop episodes as proposed in the paper.

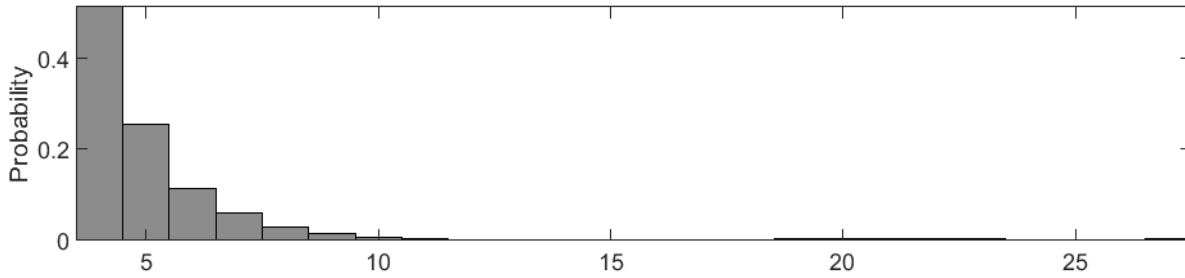
Panel (a) is a histogram of simulated crisis episodes. The shortest of the three crisis episode identified in our data (the GFC) lasted four quarters, and hence here we consider episodes in which the economy is in the binding regime for at least four consecutive quarters. The average conditional duration is 4.95 consecutive quarters. Some of the simulated episodes last up to 22 consecutive quarters though, but they are very rare events, as they make up less than a half-percent of all simulated episodes. Our model, therefore, can account for a wide range of crisis durations when estimated with data for other countries.

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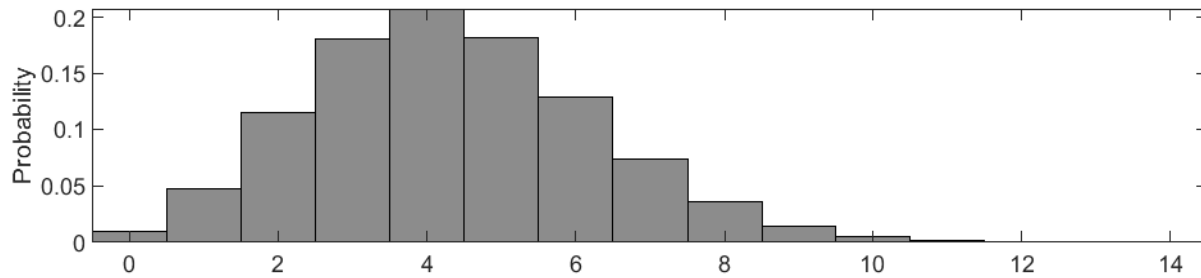
<sup>27</sup>See footnote 22 for simulation details.

Figure 7: Model-Simulated Crisis Duration and Frequency

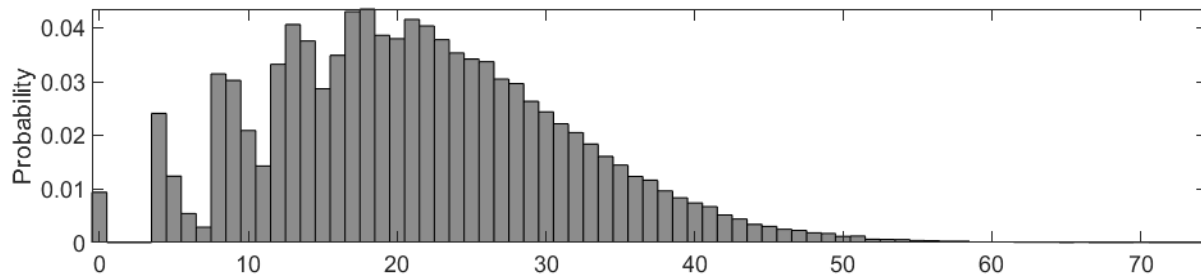
(a) Crisis Episodes of at least Four Consecutive Quarters



(b) Frequency of Crisis Episodes of Any Duration per Sample



(c) Number of Quarters in Binding Regime per Sample



Note: Histograms of the model-implied (a) conditional crisis duration, (b) frequency of distinct episodes of any duration in 144 quarter-long samples, and (c) number of quarters in the binding regime in 144 quarter-long samples. Crisis period and crisis episodes defined as in Section 5.2.1.

Panel (b) plots a histogram of crisis episodes of at least four quarters *per sample period* of 144 quarters length. The most common occurrence is four distinct crisis episodes, which is close to our estimation results with three distinct events in Mexico's history, two of which last about eight quarters. Nonetheless, again, there is significant heterogeneity, as some samples contain no crisis episodes at all, while others experience as many as eight-ten shorter-lived crisis events per sample.

Building on Panels (a) and (b), Panel (c) counts the *total number* of quarters in crisis episodes out of 144. The model-implied mean is 21.5 quarters, again consistent with our estimates in Figure 5 that implies that Mexico spent 21 quarters in a crisis state over the sample period. The standard deviation is about 10 quarters, with a long right-tail and a

maximum of 62 quarters in the constrained regime.

Figure 8 turns to model-simulated crisis dynamics. As the more severe historical episodes of sudden stop identified in Figure 5 in the early 1980s and mid-1990s lasted about 8 quarters, the Figure plots the model dynamics during crisis episodes of eight consecutive quarters (starting at  $t = 0$  and ending at  $t = 7$ , vertical dashed lines), as well as 5 years (20 quarters) before the beginning of the episode and 5 years (20 quarters) after the end of the event. Variables are in log-levels, normalized to zero at the beginning of the pre-crisis period, which is time  $t = -20$ . The lower part of the figure also reports the average value of the observable variables used in estimation across the three identified crisis episodes (red lines).

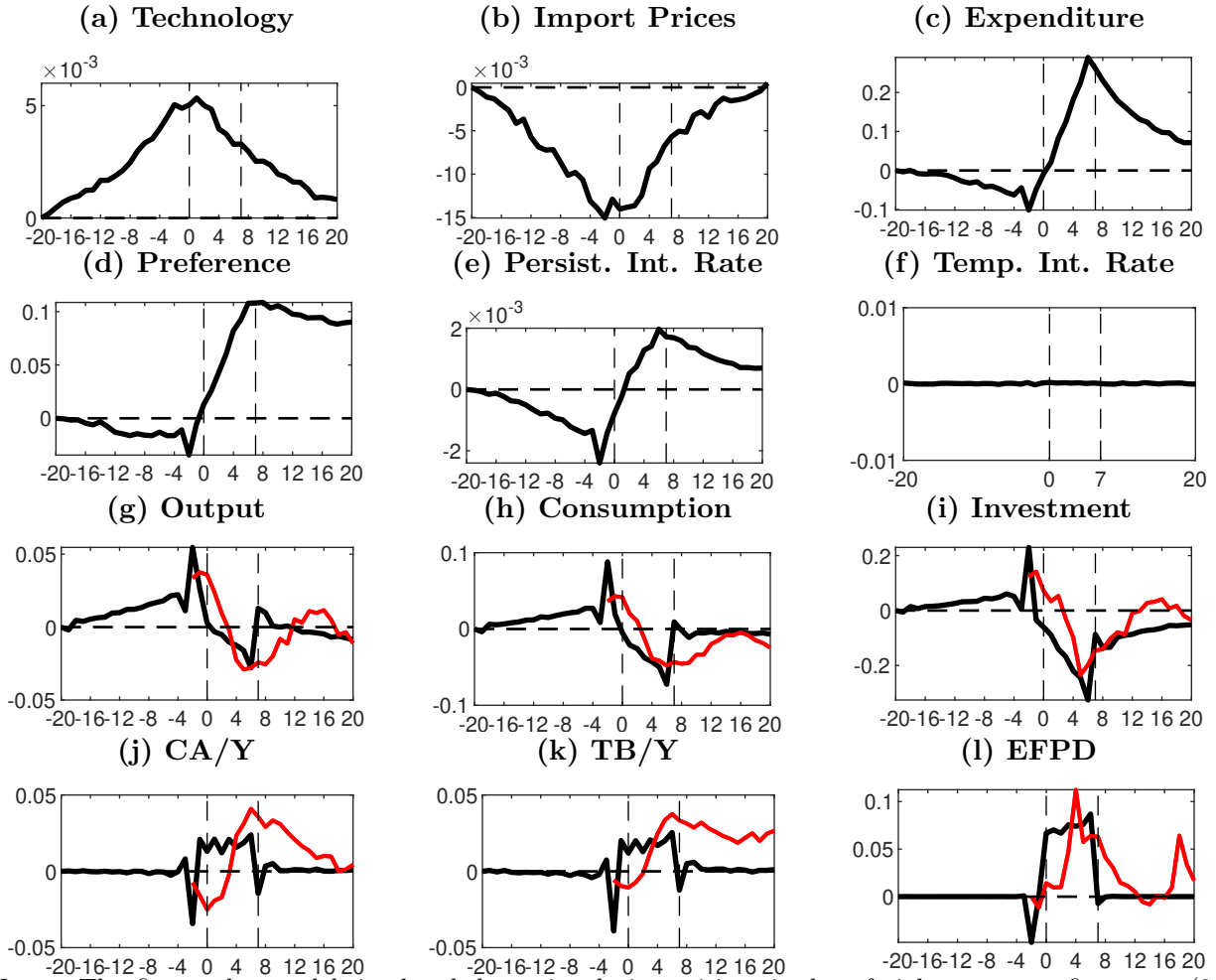
The top part of Figure 8 (panels a to f) shows the distinctive combinations of shocks that drive the economy before, during and after the crisis episode. Crisis episodes are preceded by a long-lasting “boom” phase, driven by improving technology and a favorable international environment, with improving terms of trade and a decline in the persistent component of the interest rate. These three forces drive the expansion gradually, with increasing output, consumption and investment, in a manner consistent with the empirical characterizations of the boom phases of financial crises (Boissay et al., 2016).

The economy enters the crisis episode at  $t = 0$ , after a final acceleration, driven by an increase in expenditures and a fall in patience. The crisis episode is precipitated by a small negative technology shock and a sudden reversal of the favorable external environment that drove the boom phase. During the crisis episode, the terms of trade deteriorate and the cost of funding increases; the effective cost of borrowing spikes, driven by the external finance premium on debt (EFPD). Technology stagnates and patience increases sharply during the crisis. The constraint on borrowing curtails consumption smoothing and limits the supply side of the economy through the working capital constraint, causing output, consumption and investment to drop sharply. The autonomous component of expenditure continues to increase during the crisis period, which is consistent with the import compression typically associated with sudden stops.

Looking at the magnitude and persistence of the dislocation caused by the crisis episode, we see again that the model does a remarkably good job at tracking the data without imposing additional ad hoc restrictions on the amplitude of the episodes identified. The output drop from peak to trough is eight percentage points, only slightly more than the average of the three crisis episodes identified in our sample period and with a speed of decline that is comparable to the data. Similarly, both simulated consumption and investment dynamics track the data very closely.

When the crisis occurs, the current account and the trade balance suddenly revert, after a sharp deterioration right before the beginning of the crisis event, by about six percentage

Figure 8: Dynamics of Crisis Episodes



Notes: The figure plot model-simulated dynamics during crisis episodes of eight quarters, five years (20 quarters) before the crisis, and five years after the crisis. The economy is in the binding regime from period  $t = 0$  to period  $t = 7$  (vertical dashed lines). The plotted dynamics in all panels are medians across all crisis episodes identified, in log-levels, setting  $t - 20 = 0$ . The red lines indicate the means of the observable variables used in estimation across the three identified crisis episodes. The red lines start at  $t = -2$  due to proximity of 1982 debt crisis to the beginning of our sample. The crisis period and crisis episodes, denoted by the vertical dashed lines denote the beginning and the end of the episode of eight consecutive quarters in the binding regime.

points as a share of output from trough to peak. This swing is only slightly less than in the data, but occurs significantly faster. The EFPD premium, which is driven by the borrowing constraint multiplier in the binding regime, displays pattern similar to the external balances, although it declines more in line with the persistence in the data after the crisis event.

Note here that the adjustment would be even faster, typically completed within one year or 4 quarters, in calibrated models with occasionally binding constraints (see, for example, [Mendoza, 2010](#)). Thus, as we also see in our estimates in [Figure 5](#) and illustrate in more

details in Figure C.3 in Appendix C, the current account and trade balance reversal in our model is twice as persistent that in the traditional models.<sup>28</sup> We also note that in traditional models with occasionally binding constraints, data consistency in terms of magnitude of the reversal is achieved by imposing additional auxiliary restriction on the sudden stop definition. Without those ad hoc restrictions, those models cannot generate sudden stops of magnitude comparable to the data. In sharp contrast, the dynamics reported in Figure 8 are based on a purely model-based definition of sudden stops.

The economy rebounds quickly from these crisis episodes, but only partially—about half of the ground lost during the crisis, or about 4 percentage points at the end of the crisis episode. After the initial rebound, a combination of persistently adverse external and internal shocks coalesces to produce a protracted output decline. The international cost of borrowing as captured by the persistent interest rate shock remains elevated. The terms of trade are depressed for an extended period of time after the crisis episode, and the productivity decline is very long lasting. Expenditure and patience also do not recover to pre-crisis levels even five years after the end of the episode.

Consistent with the data, and also in line with empirical evidence on the long-term consequences of financial crises in other emerging markets in (Cerra and Saxena, 2008), during the post-crisis period, investment stagnates below its pre-crisis level. As a result, output resumes to decline, not aided by the persistent contraction in the exogenous component of expenditure, and despite a declining persistent component of the interest rate. Again, while in the data the recovery phase is slower and more anemic than in our model simulated episodes, traditional models of sudden stops with occasionally binding constraints do not generate sluggish recoveries as at all as in Figure C.3 in Appendix C clearly illustrates.

## 6 Conclusions

In this paper we propose a new approach to specifying and solving DSGE models with occasionally binding constraints that is suitable for structural estimation using full information methods. We can then obtain estimates of critical model parameters and conduct likelihood-based inference and counterfactual experiments. The critical step in our approach is to specify the occasionally binding nature of the borrowing constraint stochastically, so that the formulation can be mapped into an endogenous regime-switching model.

We apply this new approach to a workhorse medium-scale model of a particular type of financial crises, the so-called sudden stops in capital flows, and estimate it with Bayesian

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<sup>28</sup>See Appendix C for a more the comparison between our endogenous-switching and a traditional occasionally binding sudden stop model dynamics at annual frequency.

methods on quarterly data for Mexico since 1981. We find that the estimated model fits Mexico's business cycle and crisis episodes well without relying on large or skewed shocks, that critical parameter estimates differ from values previously used in the literature, and that different shocks matter for different variables and phases of financial crisis dynamics in a way that was hitherto not possible to document. In particular, we show that the model can generate heterogeneous crisis episodes of varying duration, frequency, and intensity. Specific combinations of shocks typically drive the economy before, during, and after crisis episodes. Finally, we document that our estimated model identifies sudden stops that are significantly longer lasting and more in line with narratives of Mexico's history of financial crises than those typically simulated with traditional inequality specifications of the collateral constraint, without imposing ad hoc restrictions on their amplitude.

We regard the estimation of larger models—including those with nominal or labor market frictions, those with permanent and temporary productivity shocks over longer sample periods, or those with financial intermediation or equilibrium default, and models with a zero lower bound constraint—as important areas of future research. Such models would be useful lenses to understand episodes like the Great Depression and the Global Financial Crisis. Another important area of future research is also to study contracting environments in which the economy switches endogenously but stochastically between lending regimes, consistent with growing microeconomic evidence of covenant contract violations and financing cut off periods of varying duration as we assumed.

## References

- Adam, K. and R. M. Billi (2007). Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates. *Journal of Monetary Economics* 54(3), 728–752.
- Aguiar, M. and G. Gopinath (2007). Emerging Market Business Cycles: The Cycle Is the Trend. *Journal of Political Economy* 115, 69–102.
- Alpanda, S. and A. Ueberfeldt (2016). Should Monetary Policy Lean Against Housing Market Booms? Staff Working Papers 16-19, Bank of Canada.
- Andreasen, M. M., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2018). The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications. *Review of Economic Studies* 85(1), 1–49.
- Aportela Rodriguez, F., J. A. A. Ituarte, and Y. C. Aguayo (2001). Comportamiento Histórico de Las Tasas de Interés Reales en México: 1951-2001. Documento de Investigación (Working Paper) No. 2001-0, Dirección General de Investigación Económica, Banco de Mexico.



- Aruoba, B., P. Cuba-Borda, and F. Schorfheide (2018). Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries. *Review of Economic Studies* 85(1), 87–118.
- Aruoba, S. B., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2006). Comparing Solution Methods for Dynamic Equilibrium Economies. *Journal of Economic Dynamics and Control* 30(12), 2477–2508.
- Ates, S. and F. Saffie (2016). Fewer but Better: Sudden Stops, Firm Entry, and Financial Selection. International Finance Discussion Papers 1187, Board of Governors of the Federal Reserve System (U.S.).
- Atkinson, T., A. W. Richter, and N. Throckmorton (2018). The Zero Lower Bound and Estimation Accuracy. Working Papers 1804, Federal Reserve Bank of Dallas.
- Barthélemy, J. and M. Marx (2017). Solving Endogenous Regime Switching Models. *Journal of Economic Dynamics and Control* 77(C), 1–25.
- Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. Young (2013). Financial Crises and Macro-Prudential Policies. *Journal of International Economics* 89(2), 453–470.
- Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2016). Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective. *Journal of Monetary Economics* 84(C), 147–165.
- Bernanke, B. and M. Gertler (1989). Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review* 79(1), 14–31.
- Bi, H. and N. Traum (2014). Estimating Fiscal Limits: The Case Of Greece. *Journal of Applied Econometrics* 29(7), 1053–1072.
- Bianchi, F. (2013). Regime Switches, Agents’ Beliefs, and Post-World War II U.S. Macroeconomic Dynamics. *Review of Economic Studies* 80(2), 463–490.
- Bianchi, F. and C. Ilut (2017). Monetary/Fiscal Policy Mix and Agents’ Beliefs. *Review of Economic Dynamics* 26, 113–139.
- Bianchi, F., C. Ilut, and M. Schneider (2018). Uncertainty Shocks, Asset Supply and Pricing over the Business Cycle. *Review of Economic Studies* 85(2), 810–854.
- Bianchi, J. (2011). Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review* 101(7), 3400–3426.
- Bianchi, J. and E. G. Mendoza (2018). Optimal Time-Consistent Macroprudential Policy. *Journal of Political Economy* 126(2), 588–634.
- Binning, A. and J. Maih (2015). Sigma Point Filters for Dynamic Nonlinear Regime Switching Models. Working Paper 2015/10, Norges Bank.
- Binning, A. and J. Maih (2017). Modelling Occasionally Binding Constraints Using Regime-Switching. Working Paper 2017/23, Norges Bank.

- Bocola, L. (2016). The Pass-Through of Sovereign Risk. *Journal of Political Economy* 124(4), 879–926.
- Boissay, F., F. Collard, and F. Smets (2016). Booms and Banking Crises. *Journal of Political Economy* 124(2), 489–538.
- Campello, M., J. R. Graham, and C. R. Harvey (2010). The Real Effects of Financial Constraints: Evidence from a Financial Crisis. *Journal of Financial Economics* 97(3), 470–487.
- Cerra, V. and S. C. Saxena (2008). Growth Dynamics: The Myth of Economic Recovery. *American Economic Review* 98(1), 439–457.
- Chodorow-Reich, G. and A. Falato (2017). The Loan Covenant Channel: How Bank Health Transmits to the Real Economy. Working Papers 23879, NBER.
- Costa, O., M. Fragoso, and R. Marques (2005). *Discrete-Time Markov Jump Linear Systems*. Springer.
- Cuba-Borda, P., L. Guerrieri, M. Iacoviello, and M. Zhong (2019). Likelihood Evaluation of Models with Occasionally Binding Constraints. *Journal of Applied Econometrics* 34(7), 1073–1085.
- Davig, T. and E. Leeper (2007). Generalizing the Taylor Principle. *American Economic Review* 97(3), 607–635.
- Davig, T. and E. M. Leeper (2008). Endogenous Monetary Policy Regime Change. In *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, pp. 345–391. National Bureau of Economic Research, Inc.
- Davig, T., E. M. Leeper, and T. B. Walker (2010). “Unfunded Liabilities” and Uncertain Fiscal Financing. *Journal of Monetary Economics* 57(5), 600–619.
- Del Negro, M. and F. Schorfheide (2008). Forming Priors for DSGE Models (and how it Affects the Assessment of Nominal Rigidities). *Journal of Monetary Economics* 55(7), 1191–1208.
- Devereux, M. B., E. Young, and C. Yu (2019). Capital Controls and Monetary Policy in Sudden-stop Economies. *Journal of Monetary Economics* 103, 52–74.
- Doucet, A., N. J. Gordon, and V. Krishnamurthy (2001). Particle Filters for State Estimation of Jump Markov Linear Systems. *IEEE Transactions on Signal Processing* 49(3), 613–624.
- Eichenbaum, M., S. Rebelo, and M. Trabandt (2020). The Macroeconomics of Epidemics. Working Paper 26882, National Bureau of Economic Research.
- Farhi, E. and I. Werning (2020). Taming a Minsky Cycle. Manuscript.

- Farmer, R., D. Waggoner, and T. Zha (2011). Minimal State Variable Solutions to Markov-Switching Rational Expectations Models. *Journal of Economic Dynamics and Control* 35(12), 2150–2166.
- Fernandez, A. and A. Gulan (2015). Interest Rates, Leverage, and Business Cycles in Emerging Economies: The Role of Financial Frictions. *American Economic Journal: Macroeconomics* 7(3), 153–188.
- Fernandez-Villaverde, J., P. Guerron-Quintana, J. Rubio-Ramirez, and M. Uribe (2011). Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review* 101(6), 2530–61.
- Fernandez-Villaverde, J., P. Guerron-Quintana, and J. F. Rubio-Ramirez (2015). Estimating Dynamic Equilibrium Models with Stochastic Volatility. *Journal of Econometrics* 185(1), 216–229.
- Fernandez-Villaverde, J., S. Hurtado, and G. Nuno (2019). Financial Frictions and the Wealth Distribution. PIER Working Paper 19-015, Penn Institute for Economic Research.
- Fernandez-Villaverde, J. and J. F. Rubio-Ramirez (2007). Estimating Macroeconomic Models: A Likelihood Approach. *Review of Economic Studies* 74(4), 1059–1087.
- Foerster, A. (2015). Financial Crises, Unconventional Monetary Policy Exit Strategies, and Agents Expectations. *Journal of Monetary Economics* 76(C), 191–207.
- Foerster, A., J. F. Rubio-Ramirez, D. F. Waggoner, and T. Zha (2016). Perturbation Methods for Markov-switching Dynamic Stochastic General Equilibrium Models. *Quantitative Economics* 7(2), 637–669.
- Fostel, A. and J. Geanakoplos (2015). Leverage and Default in Binomial Economies: A Complete Characterization. *Econometrica* 83(6), 2191–2229.
- Gabriel, G. (2008). Hechos estilizados del ciclo economico en mexico. Working Paper 2008-14, Banco de Mexico.
- Garcia-Cicco, J., R. Pancrazi, and M. Uribe (2010). Real Business Cycles in Emerging Countries? *American Economic Review* 100(5), 2510–2531.
- Gertler, M. and P. Karadi (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics* 58(1), 17–34.
- Gertler, M. and N. Kiyotaki (2015). Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. *American Economic Review* 105(7), 2011–2043.
- Greenwald, D. (2019). Firm Debt Covenants and the Macroeconomy: The Interest Coverage Channel. Working Paper 5909-19, MIT Sloan.
- Guerrieri, L. and M. Iacoviello (2015). OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily. *Journal of Monetary Economics* 70(C), 22–38.

- Gust, C., E. Herbst, D. Lopez-Salido, and M. Smith (2017). The Empirical Implications of the Interest-Rate Lower Bound. *American Economic Review* 107(7), 1971–2006.
- Iacoviello, M. (2005). House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review* 95(3), 739–764.
- Iacoviello, M. and S. Neri (2010). Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics* 2(2), 125–64.
- Ivashina, V. and D. Scharfstein (2010). Bank Lending During the Financial Crisis of 2008. *Journal of Financial Economics* 97(3), 319–338.
- Jeanne, O. and A. Korinek (2010). Managing Credit Booms and Busts: A Pigouvian Taxation Approach. Working Papers 16377, NBER.
- Jermann, U. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review* 102(1), 238–271.
- Jorda, O., M. Schularick, and A. Taylor (2013). When Credit Bites Back: Leverage, Business Cycles, and Crises. *Journal of Money, Credit and Banking* 45(S2), 3–28.
- Julier, S. J. and J. K. Uhlmann (1999). A New Extension of the Kalman Filter to Nonlinear Systems. In *Proc. SPIE*, Volume 3068, pp. 182–193.
- Kim, C.-J. and C. R. Nelson (1999). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, Volume 1. The MIT Press.
- Kiyotaki, N. and J. Moore (1997). Credit Cycles. *Journal of Political Economy* 105(2), 211–248.
- Kumhof, M., R. Rancière, and P. Winant (2015). Inequality, Leverage, and Crises. *American Economic Review* 105(3), 1217–1245.
- Lind, N. (2014). Regime-Switching Perturbation for Non-Linear Equilibrium Models. Working Paper.
- Liu, Z., P. Wang, and T. Zha (2013). Land-Price Dynamics and Macroeconomic Fluctuations. *Econometrica* 81(3), 1147–1184.
- Maih, J. (2015). Efficient Perturbation Methods for Solving Regime-Switching DSGE Models. Working Paper 2015/01, Norges Bank.
- Martin, A. (2008). Endogenous Credit Cycles. Economics Working Papers 916, Department of Economics and Business, Universitat Pompeu Fabra.
- Matsuyama, K. (2007). Credit traps and credit cycles. *American Economic Review* 97(1), 503–516.
- Mendoza, E. and S. Villalvazo (2020). FiPIt: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints. *Review of Economic Dynamics* 37, 81–102.

- Mendoza, E. G. (2010). Sudden Stops, Financial Crises, and Leverage. *American Economic Review* 100(5), 1941–1966.
- Miyamoto, W. and T. L. Nguyen (2017). Business Cycles in Small Open Economies: Evidence from Panel Data between 1900 and 2013. *International Economic Review* 58(3), 1007–1044.
- Neumeyer, P. A. and F. Perri (2005). Business Cycles in Emerging Economies: the Role of Interest Rates. *Journal of Monetary Economics* 52(2), 345–380.
- Otrok, C. (2001). On Measuring the Welfare Cost of Business Cycles. *Journal of Monetary Economics* 47(1), 61–92.
- Reinhart, C. M. and K. S. Rogoff (2009). *This Time is Different: Eight Centuries of Financial Folly*. Princeton University Press.
- Schmitt-Grohe, S. and M. Uribe (2016). Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment. *Journal of Political Economy* 124, 1466–1514.
- Schmitt-Grohe, S. and M. Uribe (2018). How Important are Terms of Trade Shocks? *International Economic Review* 59(1), 85–111.
- Schmitt-Grohe, S. and M. Uribe (2020). Multiple Equilibria in Open Economies with Collateral Constraints. *Review of Economic Studies* 59(1), 85–111.
- Schorfheide, F. (2000). Loss Function-based Evaluation of DSGE Models. *Journal of Applied Econometrics* 15(6), 645–670.
- Smets, R. and F. Wouters (2007). Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach. *American Economic Review* 97(3), 586–606.
- Uribe, M. and V. Z. Yue (2006). Country Spreads and Emerging Countries: Who Drives Whom? *Journal of International Economics* 69(1), 6–36.

## Appendix A Model and Equilibrium Definition

This Appendix derives the model's equilibrium conditions and defines a competitive equilibrium.

### A.1 Derivation of Equilibrium Conditions

The household-firm maximizes the utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left( C_t - \frac{H_t^\omega}{\omega} \right)^{1-\rho} \right\}, \quad (\text{A.1})$$

subject to

$$C_t + I_t = A_t K_{t-1}^\eta H_t^\alpha V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t (W_t H_t + P_t V_t) - E_t - \frac{1}{(1+r_t)} B_t + B_{t-1} \quad (\text{A.2})$$

where gross investment follows

$$I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left( 1 + \frac{\iota}{2} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right) \right). \quad (\text{A.3})$$

In the binding regime, the collateral constraint is given by

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) = -\kappa q_t K_t \quad (\text{A.4})$$

with the corresponding multiplier denoted  $\lambda_t$ . In the non-binding regime, the collateral constraint equation drops out, and the multiplier is  $\lambda_t = 0$ . The first-order conditions of this problem are the following:

$$d_t \left( C_t - \frac{H_t^\omega}{\omega} \right)^{-\rho} = \mu_t; \quad (\text{A.5})$$

$$(1 - \alpha - \eta) A_t K_{t-1}^\eta H_t^\alpha V_t^{-\alpha-\eta} = P_t \left( 1 + \phi r_t + \frac{\lambda_t}{\mu_t} \phi (1+r_t) \right); \quad (\text{A.6})$$

$$\alpha A_t K_{t-1}^\eta H_t^{\alpha-1} V_t^{1-\alpha-\eta} = \phi W_t \left( r_t + \frac{\lambda_t}{\mu_t} (1+r_t) \right) + H_t^{\omega-1}; \quad (\text{A.7})$$

$$\mu_t = \lambda_t + \beta (1+r_t) \mathbb{E}_t \mu_{t+1}; \quad (\text{A.8})$$

$$\mathbb{E}_t \mu_{t+1} \beta \left( \begin{array}{l} 1 - \delta + \left( \frac{\iota}{2} \left( \frac{K_{t+1}}{K_t} \right)^2 - \frac{\iota}{2} \right) \\ + \eta A_{t+1} K_t^{\eta-1} H_{t+1}^\alpha V_{t+1}^{1-\eta-\alpha}; \end{array} \right) = \mu_t \left( 1 - \iota + \iota \left( \frac{K_t}{K_{t-1}} \right) \right) - \lambda_t \kappa q_t. \quad (\text{A.9})$$

Market prices for capital and labor satisfy the following two conditions:

$$q_t = 1 + \iota \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right); \quad (\text{A.10})$$

$$W_t = H_t^{\omega-1}. \quad (\text{A.11})$$

Defining the borrowing cushion,  $B_t^*$ , as the difference between the amount of borrowing and the debt limit

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) + \kappa q_t K_t, \quad (\text{A.12})$$

the regime-switching slackness condition is given by

$$\varphi(s_t) B_{ss}^* + \nu(s_t) (B_t^* - B_{ss}^*) = (1 - \varphi(s_t)) \lambda_{ss} + (1 - \nu(s_t)) (\lambda_t - \lambda_{ss}) \quad (\text{A.13})$$

where  $\varphi(s_t)$  and  $\nu(s_t)$  are regime-switching parameters controlling the level and the dynamics of the economy, respectively, and  $B_{ss}^*$  and  $\lambda_{ss}$  are the regime-switching steady-state values of  $B_t^*$  and  $\lambda_t$ .

As we discussed in the text, the country interest rate and the exogenous processes are given by

$$r_t = r_t^* + \sigma_r \varepsilon_{r,t} + \psi_r \left( e^{\bar{B}-B_t} - 1 \right) \quad (\text{A.14})$$

where

$$r_t^* = (1 - \rho_{r^*}) \bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t} \quad (\text{A.15})$$

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t} \quad (\text{A.16})$$

$$\log E_t = (1 - \rho_E) \log E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t} \quad (\text{A.17})$$

$$\log P_t = (1 - \rho_P) \log P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t} \quad (\text{A.18})$$

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, \quad (\text{A.19})$$

where the errors terms  $\varepsilon_{.,t}$  are i.i.d  $N(0, 1)$ , and  $\psi_r$  is set to an arbitrarily small value so that it does not affect any of the model properties.

In the paper, we also use a number of auxiliary variables defined as as

$$\text{GDP: } Y_t = A_t K_{t-1}^\eta H_t^\alpha V_t^{1-\alpha-\eta} - P_t V_t \quad (\text{A.20})$$

$$\text{Debt-to-GDP Ratio: } \Phi_t^b = \frac{B_t}{Y_t} \quad (\text{A.21})$$

$$\text{Current Account-to-GDP Ratio: } \Phi_t^{ca} = \frac{B_t - B_{t-1}}{Y_t} \quad (\text{A.22})$$

$$\text{Trade Balance-to-GDP Ratio: } \Phi_t^{tb} = \frac{Y_t - E_t - C_t - I_t}{Y_t} \quad (\text{A.23})$$

$$\text{External Financing Premium on Debt: } EFPD_t = \frac{\lambda_t}{\beta \mathbb{E}_t \mu_{t+1}}. \quad (\text{A.24})$$

## A.2 Regime-Switching Equilibrium Definition

A competitive equilibrium of our economy is a sequence of quantities  $\{K_t, B_t, C_t, H_t, V_t, I_t, A_t, E_t, B_t^*\}$  and prices  $\{P_t, r_t^*, r_t, q_t, w_t, \mu_t, \lambda_t\}$  that, given the 5 exogenous processes (A.16)-(A.15), satisfy the first-order conditions for the representative household-firm (A.5)-(A.9), the market price equations (A.10)-(A.11), the market clearing conditions (A.2)-(A.3), the debt cushion definition (A.12), regime-switching slackness condition (A.13), and the equation for the interest rate (A.14).

## Appendix B Perturbation Solution Method

This Appendix provides details about two aspects of the solution method: (1) the definition of, and solution for, the steady state of the endogenous regime-switching economy; and (2) the perturbation method that generates second order Taylor expansions to the solution of the economy around the steady state.

### B.1 Regime Switching Equilibrium

Write the 23 equilibrium conditions above as

$$\mathbb{E}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi^{\varepsilon_{t+1}}, \varepsilon_t, \theta_{t+1}, \theta_t) = 0. \quad (\text{B.1})$$



Here  $\mathbf{y}_t$  denotes the non-predetermined variables,  $\mathbf{x}_t$  predetermined variables,  $\varepsilon_t$  the exogenous shocks,  $\theta_t$  the regime-switching parameters, and  $\chi$  the perturbation parameter. There are 7 predetermined variables

$$\mathbf{x}_{t-1} = [K_{t-1}, B_{t-1}, A_{t-1}, P_{t-1}, E_{t-1}, d_{t-1}, r_{t-1}^*] \quad (\text{B.2})$$

and 16 non-predetermined variables

$$\mathbf{y}_t = [C_t, H_t, V_t, I_t, k_t, r_t, q_t, W_t, \mu_t, \lambda_t, B_t^*, Y_t, \Phi_t^b, \Phi_t^{ca}, \Phi_t^{tb}, EFPD_t], \quad (\text{B.3})$$

with 6 exogenous shocks

$$\varepsilon_t = [\varepsilon_{A,t}, \varepsilon_{E,t}, \varepsilon_{P,t}, \varepsilon_{d,t}, \varepsilon_{r,t}, \varepsilon_{r^*,t}], \quad (\text{B.4})$$

and 2 regime-switching parameters

$$\theta_t = [\varphi(s_t), \nu(s_t)]. \quad (\text{B.5})$$

In general, the regime-switching parameters are partitioned into those that affect the steady state,  $\theta_{1,t}$ , and those that do not,  $\theta_{2,t}$ . In the case of our specific application, the partition is

$$\theta_{1,t} = [\varphi(s_t)] \quad \theta_{2,t} = [\nu(s_t)]. \quad (\text{B.6})$$

In order to solve the model, we assume the functional forms

$$\theta_{1,t+1} = \bar{\theta}_1 + \chi \hat{\theta}_1(s_{t+1}), \quad \theta_{1,t} = \bar{\theta}_1 + \chi \hat{\theta}_1(s_t) \quad (\text{B.7})$$

$$\theta_{2,t+1} = \theta_2(s_{t+1}), \quad \theta_{2,t} = \theta_2(s_t) \quad (\text{B.8})$$

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \quad (\text{B.9})$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \quad \mathbf{y}_{t+1} = g_{s_{t+1}}(\mathbf{x}_t, \chi \varepsilon_{t+1}, \chi) \quad (\text{B.10})$$

and

$$\mathbb{P}_{s_t, s_{t+1}, t} = \pi_{s_t, s_{t+1}}(\mathbf{y}_t). \quad (\text{B.11})$$

Now, substituting these functional forms in the 23 equilibrium conditions and being more

explicit about the expectation operator, given  $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$  and  $s_t$ , we have:

$$F_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \int \sum_{s'=0}^1 \pi_{s_t, s'}(g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)) f \begin{pmatrix} g_{s_{t+1}}(h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \chi \varepsilon', \chi), \\ g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \\ \mathbf{x}_{t-1}, \chi \varepsilon', \varepsilon_t, \\ \bar{\theta} + \chi \hat{\theta}(s'), \bar{\theta} + \chi \hat{\theta}(s_t) \end{pmatrix} d\mu \varepsilon'. \quad (\text{B.12})$$

where  $d\mu \varepsilon'$  denotes the joint pdf of the shocks.

Finally, stacking all conditions by regime yields:

$$\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) = \begin{bmatrix} F_{s_t=0}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \\ F_{s_t=1}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \end{bmatrix} = 0. \quad (\text{B.13})$$

## B.2 Steady State Definition and Solution

The model has two features that make defining a steady state challenging. First, as it is common in a regime-switching framework, some structural parameters may be switching. In the case of our application, there is only one switching parameter that affects the steady state,  $\varphi(s_t)$ . Nonetheless, in principle, one could allow for regime switching also in the parameters of the exogenous processes,  $a^*(s_t)$  and  $p^*(s_t)$ , or the structural parameter  $\kappa^*(s_t)$ , which would affect the level of the economy and the steady state calculations.<sup>29</sup> Following [Foerster et al. \(2016\)](#), we define the steady state in terms of the ergodic means of these parameters across regimes. To define the steady state, we set  $\varepsilon_t = 0$  and  $\chi = 0$ , which implies that the steady state is given by

$$f(\mathbf{y}_{ss}, \mathbf{y}_{ss}, \mathbf{x}_{ss}, \mathbf{x}_{ss}, 0, 0, \bar{\theta}_1, \theta_2(s'), \bar{\theta}_1, \theta_2(s)) = 0 \quad (\text{B.14})$$

for all  $s', s$ .

In our case, the transition matrix evaluated at steady state  $\mathbb{P}_{ss}$  is endogenous, since it depends on variables that in turn depend on the steady state value of the transition matrix. To find a solution for the steady state, we proceed in two steps. First, we assume the steady state transition matrix is known and solve for all the steady state prices and quantities. Second, we use the steady state values of the borrowing cushion  $B_{ss}^*$  and multiplier  $\lambda_{ss}$  from

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<sup>29</sup>As it is well known, over finite periods of time, unit root processes and processes with structural break or regime changes are observationally equivalent from a statistical standpoint. Allowing for regime changes in the process for  $A_t$ , therefore, would be a way to accommodate permanent productivity shocks as in [Aguiar and Gopinath \(2007\)](#). Similarly, stochastic volatility could be allowed for by introducing regime switching in the some or all of the shock variances.

Step 1 to update the steady state transition matrix. We then iterate to convergence.

**Step 1: Solve steady state using a given steady state transition matrix.** First, assume that the steady state transition matrix at iteration  $i$ ,  $\mathbb{P}_{ss}^{(i)}$ , is known. Next, let  $\xi = [\xi_0, \xi_1]$  denote the ergodic vector of  $\mathbb{P}_{ss}^{(i)}$ . Then, as noted in the paper, define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1).$$

The steady state of the regime-switching economy depends on these ergodic means, and we can now solve for the steady states of all variables. First, we can partially solve for some of the steady state directly

$$A_{ss} = 1, d_{ss} = 1, E_{ss} = E^*, P_{ss} = P^*, q_{ss} = 1, r_{ss}^* = \bar{r}^* \quad (\text{B.15})$$

Suppose now that we knew  $r_{ss}$ . Then, we can obtain:

$$\Omega_v \equiv \frac{A_{ss} K_{ss}^\eta H_{ss}^\alpha V_{ss}^{1-\alpha-\eta}}{P_{ss} V_{ss}} = \frac{1 + \phi r_{ss} + \phi(1 + r_{ss})(1 - \beta(1 + r_{ss}))}{1 - \alpha - \eta} \quad (\text{B.16})$$

$$\Omega_h \equiv \frac{A_{ss} K_{ss}^\eta H_{ss}^\alpha V_{ss}^{1-\alpha-\eta}}{W_{ss} H_{ss}} = \frac{1 + \phi(r_{ss} + (1 + r_{ss})(1 - \beta(1 + r_{ss})))}{\alpha} \quad (\text{B.17})$$

$$\Omega_k \equiv \frac{A_{ss} K_{ss}^\eta H_{ss}^\alpha V_{ss}^{1-\alpha-\eta}}{K_{ss}} = \frac{1}{\eta} \left( \frac{1 - \kappa(1 - \beta(1 + r_{ss}))}{\beta} - 1 + \delta \right) \quad (\text{B.18})$$

$$H_{ss} = \left( \frac{A_{ss}}{\Omega_k^\eta \Omega_h^\alpha (P_{ss} \Omega_v)^{1-\alpha-\eta}} \right)^{\frac{1}{\alpha(\omega-1)}} \quad (\text{B.19})$$

$$V_{ss} \equiv \frac{\Omega_h}{P_{ss} \Omega_v} H_{ss}^\omega \quad (\text{B.20})$$

$$K_{ss} = \frac{\Omega_h}{\Omega_k} H_{ss}^\omega \quad (\text{B.21})$$

$$Y_{ss} = \Omega_h H_{ss}^\omega - P_{ss} V_{ss} \quad (\text{B.22})$$

$$W_{ss} = H_{ss}^{\omega-1} \quad (\text{B.23})$$

$$I_{ss} = \delta K_{ss} \quad (\text{B.24})$$

$$k_{ss} = K_{ss} \quad (\text{B.25})$$

$$B_{ss} = \bar{B} - \log \left( 1 + \frac{r_{ss} - r^*}{\psi_r} \right) \quad (\text{B.26})$$

$$C_{ss} = Y_{ss} - \phi r_{ss} (W_{ss} H_{ss} + P_{ss} V_{ss}) - E_{ss} + B_{ss} \left( 1 - \frac{1}{(1 + r_{ss})} \right) - I_{ss} \quad (\text{B.27})$$

$$\mu_{ss} = \left( C_{ss} - \frac{H_{ss}^\omega}{\omega} \right)^{-\rho} \quad (\text{B.28})$$

$$\lambda_{ss} = (1 - \beta(1 + r_{ss})) \mu_{ss} \quad (\text{B.29})$$

$$B_{ss}^* = \frac{1}{(1 + r_{ss})} B_{ss} - \phi(1 + r_{ss})(W_{ss} H_{ss} + P_{ss} V_{ss}) + \kappa K_{ss} \quad (\text{B.30})$$

$$\Phi_{ss}^b = \frac{B_{ss}}{Y_{ss}} \quad (\text{B.31})$$

$$\Phi_{ss}^{ca} = 0 \quad (\text{B.32})$$

$$\Phi_{ss}^{tb} = \frac{Y_{ss} - E_{ss} - C_{ss} - I_{ss}}{Y_{ss}} \quad (\text{B.33})$$

$$EFPD_{ss} = \frac{\lambda_{ss}}{\beta \mu_{ss}}. \quad (\text{B.34})$$

The variable  $r_{ss}$  can then be derived as the solution of

$$\bar{\varphi} B_{ss}^* = (1 - \bar{\varphi}) \lambda_{ss}. \quad (\text{B.35})$$

**Step 2: Updating the transition matrix.** Step 1 yields the variables  $B_{ss}^*$  and  $\lambda_{ss}$ , and hence provides a new value of the transition matrix for iteration  $i + 1$ :

$$P_{ss}^{(i+1)} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B_{ss}^*)}{1 + \exp(-\gamma_0 B_{ss}^*)} & \frac{\exp(-\gamma_0 B_{ss}^*)}{1 + \exp(-\gamma_0 B_{ss}^*)} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}, \quad (\text{B.36})$$

which can be checked against the guess in Step 1. We then iterate to convergence until

$$\|P_{ss}^{(i+1)} - P_{ss}^{(i)}\| < \textit{tolerance},$$

where in our application we use a tolerance of  $10^{-10}$ .

### B.3 Generating Approximations

To compute a second order approximation to the endogenous regime-switching model solution, we largely follow Foerster et al. (2016), adapting to the case with endogenous probabilities.

We take the stacked equilibrium conditions  $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ , and differentiate with respect to  $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ . The first-order derivative with respect to  $\mathbf{x}_{t-1}$  produces a polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0. \tag{B.37}$$

In Foerster et al. (2016), when the transition probabilities are exogenous and fixed, this system needs to be solved via Gröbner bases, which finds all possible solutions in order to check them for stability. The relevant stability concept is mean square stability (MSS), which requires the expectation of first and second moments to be finite (see Costa et al., 2005). In our case with endogenous probabilities, the check for MSS is not applicable, so we focus on finding a single solution and ignore the possibility of multiple solutions or indeterminacy, a common simplification in the regime-switching literature with and without endogenous switching (e.g. Farmer et al., 2011; Foerster, 2015; Maih, 2015; Lind, 2014). This simplification is also common to global solution methods of models with occasionally binding constraints, where numerical methods converge to a given solution but do not guarantee uniqueness of that solution. Instead, the focus typically is on checking robustness of the solution to initial conditions. While in some simpler models with collateral constraints it is possible to impose parametric restrictions that rule out multiple equilibria (Schmitt-Grohe and Uribe, 2020; Benigno et al., 2016), in the case of our model, as in Mendoza (2010) and Bianchi and Mendoza (2018), there are no such restrictions and uniqueness must be verified numerically.

To find a model solution, we guess a set of policy functions for regime  $s_t = 1$ , which reduces the equilibrium conditions  $\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0; s_t = 0)$  to a fixed-regime eigenvalue problem, and solve for the policy functions for  $s_t = 0$ . Then, using this initial solution as a guess, we solve for regime  $s_t = 0$  under the fixed-regime eigenvalue problem, and iterate to convergence.

After solving the iterative eigenvalue problem, the remaining systems to solve are

$$\mathbb{F}_\varepsilon(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \quad (\text{B.38})$$

$$\mathbb{F}_\chi(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \quad (\text{B.39})$$

and the second order systems of the form

$$\mathbb{F}_{\mathbf{i}, \mathbf{j}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \quad \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \varepsilon, \chi\}. \quad (\text{B.40})$$

Recalling now that the decision rules have the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \quad (\text{B.41})$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi), \quad (\text{B.42})$$

the second-order approximation are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \quad (\text{B.43})$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \quad (\text{B.44})$$

where  $S_t = \left[ (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' \quad \varepsilon_t' \quad 1 \right]'$ , with  $\mathbf{x}_{ss}$  denoting the value of the steady-state variables.

## B.4 Proof of Proposition 1: Irrelevance of Endogenous Switching in the First-Order Solution

To prove Proposition 1, take the first-order derivatives of (B.13) with respect to its arguments, evaluated at the steady state. This yields:

$$\begin{aligned} \mathbb{F}_{\mathbf{x}, s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t, s', y}(\mathbf{y}_{ss}) g_{\mathbf{x}, s_t} f_{ss}(s', s_t) \\ &+ \sum_{s'} \pi_{s_t, s'}(\mathbf{y}_{ss}) \left[ \begin{array}{l} f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x}, s'} h_{\mathbf{x}, s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\mathbf{x}, s_t} \\ + f_{\mathbf{x}_t}(s', s_t) h_{\mathbf{x}, s_t} + f_{\mathbf{x}_{t-1}}(s', s_t) \end{array} \right] \end{aligned} \quad (\text{B.45})$$

$$\begin{aligned} \mathbb{F}_{\varepsilon, s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t, s', y}(\mathbf{y}_{ss}) g_{\varepsilon, s_t} f_{ss}(s', s_t) \\ &+ \sum_{s'} \pi_{s_t, s'}(\mathbf{y}_{ss}) \left[ \begin{aligned} &f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x}, s'} h_{\varepsilon, s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\varepsilon, s_t} \\ &+ f_{\mathbf{x}_t}(s', s_t) h_{\varepsilon, s_t} + f_{\varepsilon_t}(s', s_t) \end{aligned} \right] \end{aligned} \quad (\text{B.46})$$

and

$$\begin{aligned} \mathbb{F}_{\chi, s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t, s', y}(\mathbf{y}_{ss}) g_{\chi, s_t} f_{ss}(s', s_t) \\ &+ \sum_{s'} \pi_{s_t, s'}(\mathbf{y}_{ss}) \left[ \begin{aligned} &f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x}, s'} h_{\chi, s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\chi, s_t} \\ &+ f_{\mathbf{x}_t}(s', s_t) h_{\chi, s_t} \\ &+ f_{\theta_{t+1}}(s', s_t) \hat{\theta}(s_{t+1}) + f_{\theta_t}(s', s_t) \hat{\theta}(s_t) \end{aligned} \right] \end{aligned} \quad (\text{B.47})$$

Note now that, by definition of a steady state,  $f_{ss}(s', s_t) = 0$ , and so the first term of each of these expressions equals zero. Hence, we are left with the expressions for the exogenous transition probabilities as in [Foerster et al. \(2016\)](#), given by  $\mathbb{P}_{ss} = \pi_{s_t, s'}(\mathbf{y}_{ss})$ . QED.

## Appendix C Solution Accuracy and Comparison with Traditional Inequality Specification

Table C.1: Accuracy and Solution Speed

	FiPIt	Mendoza (2010)	End. Switch.
Euler Equation Errors ( $\log_{10}$ units)			
Bond - Mean	-6.27	na	-2.92
Bond - Max	-1.56	na	-1.61
Capital - Mean	-7.04	na	-3.61
Capital - Max	-6.68	na	-2.41
Computing Time (seconds)	810	na	1.00

In order to gauge the accuracy and speed of our solution method and to evaluate the endogenous regime switching specification of the borrowing constraint relative to the traditional inequality one, we compare a suitably modified and calibrated version of our model to [Mendoza \(2010\)](#) solved with the FiPIt method of [Mendoza and Villalvazo \(2020\)](#).<sup>30</sup> Specifically, we compare our endogenous regime switching model solved with perturbation methods to the original [Mendoza \(2010\)](#) and a slightly different version solved with FiPIt in [Mendoza](#)

<sup>30</sup>See [Binning and Maih \(2017\)](#) for an analysis of the properties of our solution method applied to other structural models, such as the zero lower bound, in which they found a high degree of accuracy.

**Table C.2: Model and Solution Comparison**

	FiPIt	Mendoza (2010)	End. Switch.
Means			
gdp	393.619	388.339	393.471
<b>c</b>	274.123	267.857	268.844
inv	67.481	65.802	67.199
<b>nx/gdp (%)</b>	1.5	2.4	3.1
k	765.171	747.709	764.403
<b>b/gdp (%)</b>	1.3	-10.4	-18.4
q	1	1	1
<b>lev (%)</b>	-10.3	-15.9	-19.6
v	42.617	41.949	42.620
wc	76.658	75.455	76.641
Standard Deviations (%)			
gdp	3.94	3.85	3.81
<b>c</b>	4.03	3.69	3.63
inv	13.33	13.45	10.76
<b>nx/gdp</b>	2.94	2.58	1.60
k	4.49	4.31	4.16
<b>b/gdp</b>	19.62	8.9	1.86
q	3.2	3.23	2.54
<b>lev</b>	9.22	4.07	0.70
v	5.89	5.84	5.87
wc	4.35	4.26	4.21
Correlations with with gdp			
gdp	1	1	1
<b>c</b>	0.842	0.931	0.987
inv	0.641	0.641	0.777
<b>nx/gdp</b>	-0.117	-0.184	-0.378
k	0.761	0.744	0.851
<b>b/gdp</b>	-0.12	-0.298	-0.206
q	0.387	0.406	0.509
<b>lev</b>	-0.111	0.258	-0.116
v	0.832	0.823	0.829
wc	0.994	0.987	0.994
Autocorrelations			
gdp	0.825	0.815	0.81
<b>c</b>	0.83	0.766	0.79
inv	0.501	0.483	0.43
<b>nx/gdp</b>	0.601	0.447	0.20
k	0.962	0.963	0.98
<b>b/gdp</b>	0.99	0.087	0.78
q	0.447	0.428	0.34
<b>lev</b>	0.992	0.04	0.89
v	0.777	0.764	0.76
wc	0.801	0.777	0.77
Sudden Stop Statistics (%)			
Prob. Positive Multiplier	2.6	na	1.38



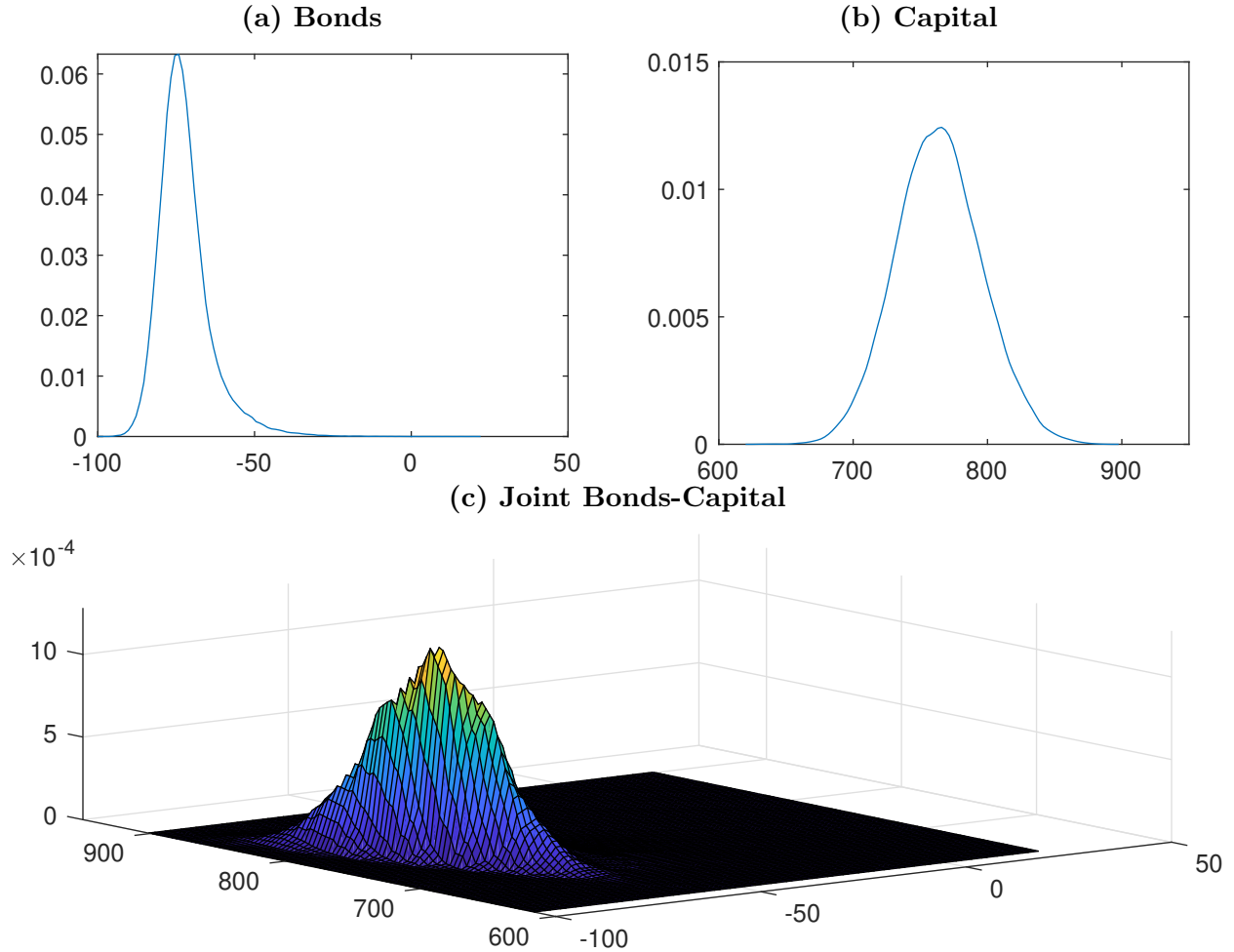
and Villalvazo (2020). To do so, we calibrate our model at the annual frequency, retaining only the three shocks that are in Mendoza (2010) and Mendoza and Villalvazo (2020)—the interest rate, productivity, and intermediate input price shocks—shutting down the expenditure and preference shocks and the temporary interest rate shock. We then compare Euler equation errors and solution speed, simulated model moments, ergodic distributions and decision rules for bond holding and capital, and sudden stop dynamics.

In the endogenous regime switch model, we calibrate all common model parameters as in Mendoza and Villalvazo (2020). The parameters that are not common are the logistic function parameters, which we set to  $\gamma_0 = \gamma_1 = 1$ . In principle, we could introduce a difference between the two logistic parameters but for simplicity we focus on a symmetric calibration that roughly matches the probability of a positive multiplier on the total borrowing constraint. Both Mendoza (2010) and Mendoza and Villalvazo (2020) use finite state Markov processes that restrict the range of possible outcomes, while we use discrete time autoregressive processes over a continuous support. We set our parameters so that the unconditional moments of the two set of processes coincide. However, this difference in shock process specification may introduce some discrepancy between the two methods that we cannot fully control.

Table C.1 reports the statistics on the Euler equation errors and the computing time. The table shows that there is a clear trade off between speed and accuracy. Our method is about 800 times faster than the FiPIt, with log-10 absolute Euler equation errors that are 1-3 times larger than FiPIt. Moreover, the size of our method’s Euler equation errors are in line with the values typically found solving exogenous regime switching models with perturbation methods (Foerster et al., 2016) and models without regime switching (Aruoba et al., 2006). To put these numbers in perspective, the implied accuracy differences represent only around a dollar error per 1,000 dollars of consumption, which is very small in absolute terms by the standards in the literature.

Table C.2 compare first and second moments and the probability of a positive multiplier across the three models. Unfortunately, this comparison is more problematic. First, as we explained above, the shock processes and structural parameters cannot be equalized exactly across models. Second, as we discuss in the text, the economics of our specification of the borrowing constraint is different than the other two models, as it implies that the collateral constraint binds for a range of leverage values rather than a specific level, given the endogenous and the exogenous states. One should therefore expect some differences in terms of second moments, and especially volatility and persistence of current account or the trade balance. Third and finally, the FiPIt surprisingly departs in important ways from Mendoza (2010) in terms of simulated moments, as bold font highlights in the table clearly

**Figure C.1: Endogenous Switching Model Ergodic Distributions**



Notes: The figure plots the simulated ergodic distribution of the bonds and capital of our endogenous switching model calibrated as discussed above.

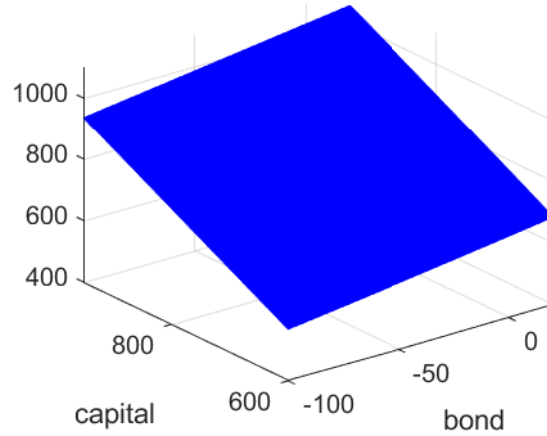
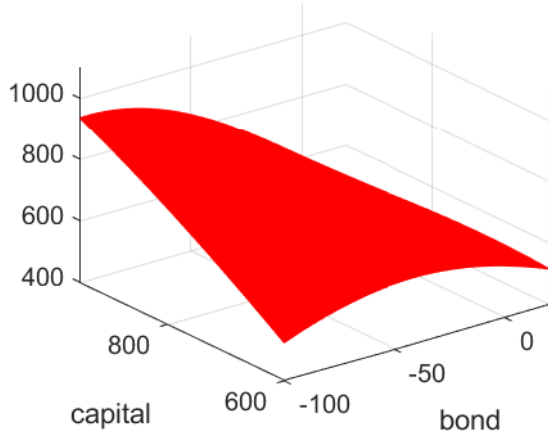
show. In particular, the FiPIt (and all its variations in [Mendoza and Villalvazo \(2020\)](#)) generate a counterfactually positive net foreign asset position and a puzzling positive net export to GDP ratio. Recalling that none of these three different model specifications target these moments explicitly in the calibration, small differences are not surprising. However, the FiPIt also significantly departs from [Mendoza \(2010\)](#) in terms of net foreign assets volatility and persistence. It is therefore not obvious what the benchmark should be here.

Comparing first moments across the three models, we can see that both the FiPIt and our endogenous regime switching model are very close to the original [Mendoza \(2010\)](#), with the exception of the net foreign asset position to GDP ( $b/GDP$ ) and the net export to GDP ratio ( $nx/GDP$ ). Surprisingly, the ergodic mean of  $b/GDP$  in the FiPIt model is *positive*, which is the opposite sign than in [Mendoza \(2010\)](#), while it has the correct sign in our model. Note also that the ergodic level of  $nx/GDP$  in the FiPIt is also positive, while it should be

**Figure C.2: Endogenous Switching Model Ergodic Decision Rules**

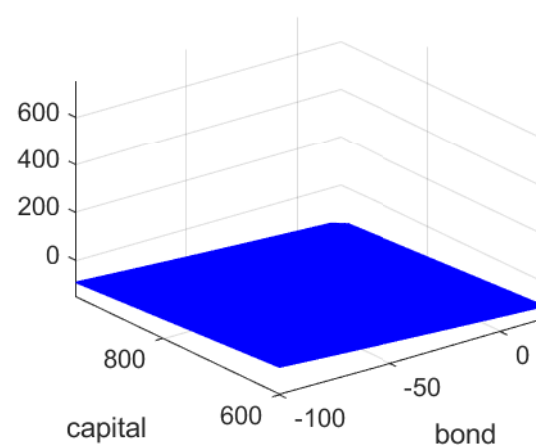
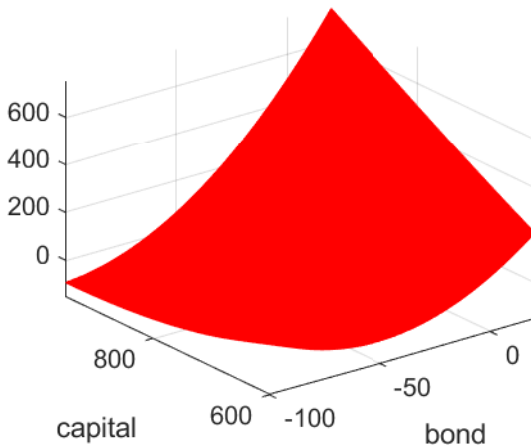
**(a) Capital: Non-binding Regime**

**(b) Capital: Binding Regime**



**(c) Bonds: Non-binding Regime**

**(d) Bonds: Binding Regime**



Notes: The figure plots the bond and capital decision rules in our endogenous switching model calibrated as discussed above, by regime, evaluated at the steady state values of the technology, interest rate, and intermediate input price processes.

negative in the ergodic distribution if the bond position is positive. While explaining these major deviations of the FiPIt from [Mendoza \(2010\)](#) is beyond the scope of this paper, we note that the values generated by our model are significantly closer to the data than both the FiPIt model and the [Mendoza \(2010\)](#) model. Indeed, a large body of literature on emerging market business cycles and sudden stop crises target a negative and sizable negative net foreign asset position, consistent with the data.<sup>31</sup> We also note here that Mexico's NFA position, from 1970 to 2015, averaged -37% of GDP and was never positive. Our model

<sup>31</sup>See, for instance, [Bianchi \(2011\)](#) that targets and NFA position equal to -29% and [Bianchi and Mendoza \(2018\)](#) that targets a -25% level.

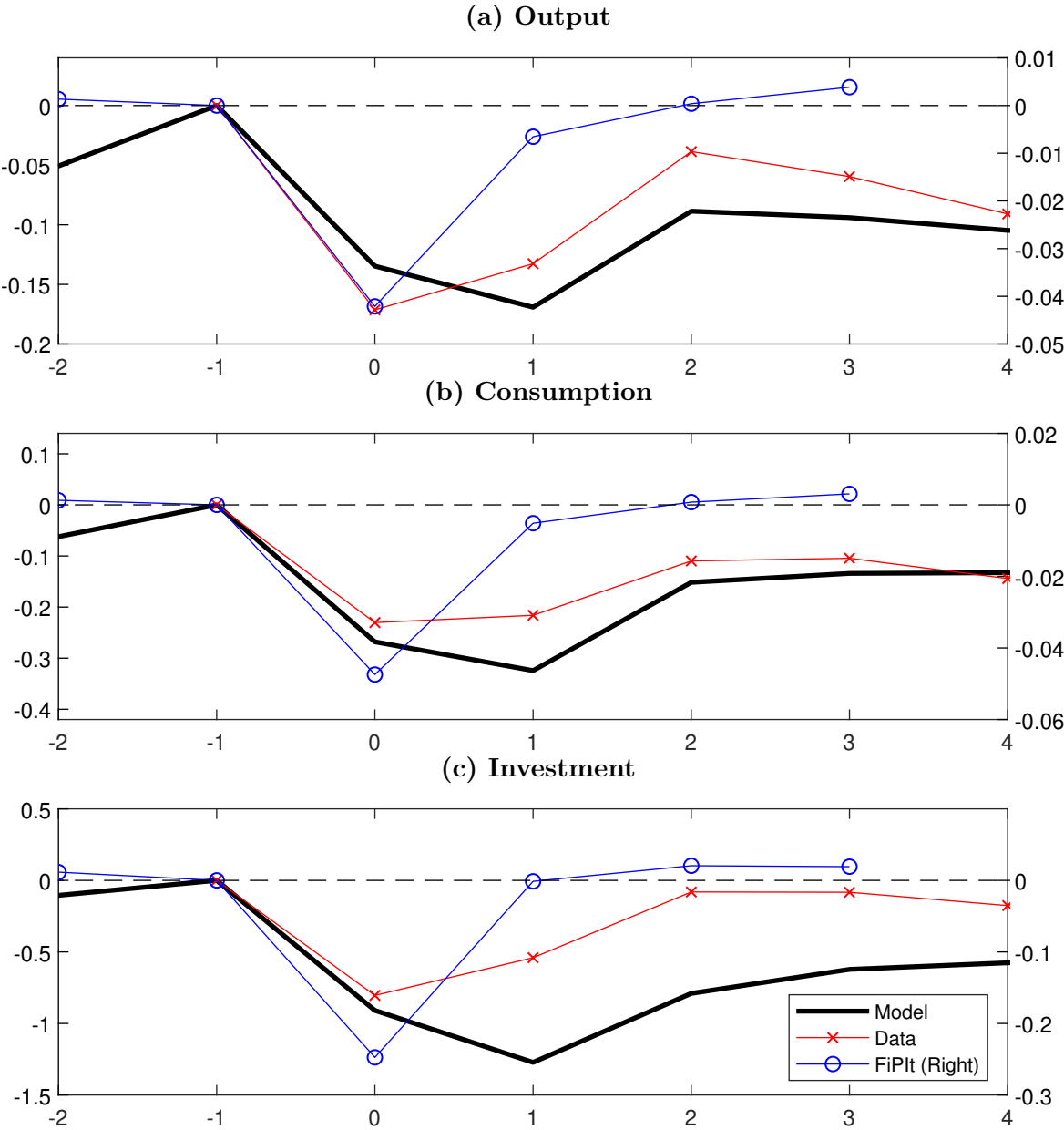
characterization of the data, therefore, is more accurate given a comparable parametrization to the FiPIt.

Consistently with the mechanics of our specification of the borrowing constraint, the standard deviation of these two variables is smaller in our model than in both the FiPIt and the original model. But again, we note that the volatility of  $b/GDP$  is much higher in the FiPIt than in [Mendoza \(2010\)](#). Similar differences between these three models can be detected in terms of persistence and correlations with GDP. Recalling that all models are ultimately wrong, the comparison for all moments along a large number of variables illustrates that our specification of the occasionally binding borrowing constraint provides a very accurate characterization of the economy second moment with differences that are within the margin of error in the literature, as for instance represented by the gaps between the FiPIt and [Mendoza \(2010\)](#). Indeed, as we can see from [Figure C.1](#), the shape of the bond distribution and the sudden stop statistics are very similar to those in [Figure 1](#) in [Mendoza and Villalvazo \(2020\)](#).

[Figure C.2](#) plots the bond and capital decision rules in the binding and the non-binding regimes of our model, evaluated at the steady state values of the technology, interest rate, and intermediate input price processes. In traditional models with occasionally binding constraints, the magnitude, dynamics, and frequency of financial crises depends critically on the behavior of these decision rules near, and at, the constraint. The decision rules of our endogenous switching model display marked non-linear behaviour along two dimensions: the curvature in the non-binding regime, and the difference in slope, for the same value of the state variable, in the binding and non-binding regime. The curvature in the non-binding regime reflects how likely and how severely the credit constraint may be expected to bind at  $t + 1$  when it does not bind at time  $t$ . These policy functions illustrate the ability of our model and its solution to capture the occasionally binding nature of the borrowing constraint consistent with [Proposition 1](#), which is the critical feature of the model.

Finally, [Figure C.3](#) compares crisis dynamics at annual frequency generated from the estimated endogenous regime switching model (solid black lines), the FiPIt model (thin blue line with circles), and selected observable variables that we used in estimation (red, thin lines with stars) aggregated at the annual frequency. Again, the comparison illustrates how a traditional specification of the occasionally binding borrowing constraints lacks the persistence necessary to match the data, despite assuming persistent shock processes and imposing auxiliary ad hoc restrictions on the sudden stop definition to match the depth of the crisis episode.

Figure C.3: Comparing Crisis Dynamics across Models



Notes: The Figure reports crisis dynamics for the estimated endogenous regime switching model (solid black lines) and the observable variable used in estimation (red, thin starred liens), but at an annual frequency, in addition to the annual dynamics in the FiPIt model of [Mendoza and Villalvazo \(2020\)](#). The crisis occurs at  $t = 0$ , and all series are normalized to zero at  $t = -1$  for ease of comparison. Model and data are on the same scale (left), while FiPIt is on a smaller scale (right). See Figure 8 in the text for additional details.

# Appendix D Bayesian Estimation Procedure

## D.1 State Space

For likelihood estimation, the state space representation is

$$\mathcal{X}_t = \mathcal{H}_{s_t}(\mathcal{X}_{t-1}, \varepsilon_t) \quad (\text{D.1})$$

$$\mathcal{Y}_t = \mathcal{G}_{s_t}(\mathcal{X}_t, \mathcal{U}_t), \quad (\text{D.2})$$

where  $\mathcal{X}_t$  denotes the state,  $\mathcal{Y}_t$  denotes the observation,  $\varepsilon_t$  denotes the structural shocks, and  $\mathcal{U}_t$  denotes the observation errors.

Recall the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \quad (\text{D.3})$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t), \quad (\text{D.4})$$

where  $S_t = \left[ (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' \quad \varepsilon_t' \quad 1 \right]'$ . Therefore, we can define the state variables as

$$\mathcal{X}_t = \left[ \mathbf{x}_t' \quad \mathbf{x}_{t-1}' \quad \mathbf{y}_t' \quad \mathbf{y}_{t-1}' \quad \varepsilon_t \right]'. \quad (\text{D.5})$$

The nonlinear transition equations,

$$\mathcal{X}_t = \mathcal{H}_{s_t}(\mathcal{X}_{t-1}, \varepsilon_t) \quad (\text{D.6})$$

can be represented as

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \\ \mathbf{x}_{t-1} \\ \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix}. \quad (\text{D.7})$$

The observation equation

$$\mathcal{Y}_t = \mathcal{G}_{s_t}(\mathcal{X}_t, \mathcal{U}_t) \quad (\text{D.8})$$

is given by

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t/Y_t \\ \Delta P_t \end{bmatrix} = D \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t \quad (\text{D.9})$$

where  $D$  denotes a selection matrix of the form

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t/Y_t \\ \Delta P_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1_{[y_t]} & -1_{[y_t]} & 0 \\ 0 & 0 & 1_{[c_t]} & -1_{[c_t]} & 0 \\ 0 & 0 & 1_{[i_t]} & -1_{[i_t]} & 0 \\ 0 & 0 & 1_{[r_t]} & 0 & 0 \\ 0 & 0 & 1_{[\Phi_t^{ca}]} & 0 & 0 \\ 1_{[P_t]} & -1_{[P_t]} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t. \quad (\text{D.10})$$

## D.2 Filtering

To filter the likelihood, we use the Unscented Kalman Filter (UKF) ([Julier and Uhlmann, 1999](#)). The UKF calculates the state mean and covariance by propagating deterministically chosen sigma-points through the nonlinear functions. The transformed points are then used to calculate the mean and covariance matrix. As [Julier and Uhlmann \(1999\)](#) note, the critical assumption to apply the UKF is that the prediction density and the filtering density are both Gaussian. The filtering and smoothing procedure largely follow [Binning and Maih \(2015\)](#), so here we just outline the procedure.

The filter starts by combining the state vector and exogenous disturbances into a single vector,  $\mathcal{X}_{t-1}^a = [\mathcal{X}_{t-1}, \varepsilon_t]'$ , with the following mean and covariance matrix conditional on  $Y_{1:t-1}$  and regime  $s_{t-1}$ :

$$\mathcal{X}_{t-1}^a(s_{t-1}) = \begin{bmatrix} \mathcal{X}_{t-1|t-1}(s_{t-1}) \\ 0_\varepsilon \end{bmatrix} \quad (\text{D.11})$$

$$P_{t-1}^a(s_{t-1}) = \begin{bmatrix} P_{t-1|t-1}^x(s_{t-1}) & 0 \\ 0 & I \end{bmatrix}. \quad (\text{D.12})$$

The sigma-points  $\mathcal{X}_{i,t-1}^a(s_{t-1})$  that consist of the sigma-points for state variables

$\mathcal{X}_{i,t-1}^x(s_{t-1})$  and the sigma-points for exogenous shocks  $\mathcal{X}_{i,t-1}^\varepsilon(s_{t-1})$  are chosen as follows:

$$\mathcal{X}_{0,t-1}^a(s_{t-1}) = \mathcal{X}_{t-1}^a(s_{t-1}) \quad (\text{D.13})$$

$$\mathcal{X}_{0,t-1}^a(s_{t-1}) = \mathcal{X}_{t-1}^a(s_{t-1}) \quad (\text{D.14})$$

$$\mathcal{X}_{i,t-1}^a(s_{t-1}) = \mathcal{X}_{t-1}^a(s_{t-1}) + (h\sqrt{P_{t-1}^a(s_{t-1})})_i \text{ for } i = 1 \dots L \quad (\text{D.15})$$

$$\mathcal{X}_{i,t-1}^a(s_{t-1}) = \mathcal{X}_{t-1}^a(s_{t-1}) - (h\sqrt{P_{t-1}^a(s_{t-1})})_{i-L} \text{ for } i = L + 1 \dots 2L, \quad (\text{D.16})$$

where  $h = \sqrt{3}$  and  $L$  denotes the number of state variables and exogenous shocks. The weights for the sigma-points are given by:

$$w_0 = \frac{h - L}{2h} \quad (\text{D.17})$$

$$w_i = \frac{1}{2h} \text{ for } i = 1 \dots 2L \quad (\text{D.18})$$

The sigma-points and the assigned weights are then used to calculate the expected mean and covariance by propagating sigma-points through transition equations and taking weighted average:

$$\mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) = H_{s_t}(\mathcal{X}_{i,t-1}^x(s_{t-1}), \mathcal{X}_{i,t-1}^\varepsilon(s_{t-1})) \quad (\text{D.19})$$

$$\mathcal{X}_{t|t-1}(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) \quad (\text{D.20})$$

$$P_{t|t-1}^x(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \tilde{\mathcal{X}}_i \tilde{\mathcal{X}}_i^T \quad (\text{D.21})$$

$$\mathcal{Y}_{t|t-1}(s_{t-1}, s_t) = D\mathcal{X}_{t|t-1}(s_{t-1}, s_t) \quad (\text{D.22})$$

where  $\tilde{\mathcal{X}}_i = \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) - \mathcal{X}_{t|t-1}(s_{t-1}, s_t)$ . From these conditions, we get the Gaussian approximation predictive density  $p(\mathcal{X}_t | \mathcal{Y}_{1:t-1}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t-1}(s_{t-1}, s_t), P_{t|t-1}^x(s_{t-1}, s_t))$ . The predictive density is then updated using the standard Kalman filter rule:

$$P_{t|t-1}^y(s_{t-1}, s_t) = DP_{t|t-1}^x(s_{t-1}, s_t)D^T + R \quad (\text{D.23})$$

$$P_{t|t-1}^{xy}(s_{t-1}, s_t) = P_{t|t-1}^x(s_{t-1}, s_t)D^T \quad (\text{D.24})$$

$$K_t(s_{t-1}, s_t) = P_{t|t-1}^{xy}(s_{t-1}, s_t)(P_{t|t-1}^y(s_{t-1}, s_t))^{-1} \quad (\text{D.25})$$



$$\mathcal{X}_{t|t}(s_{t-1}, s_t) = \mathcal{X}_{t|t-1}(s_{t-1}, s_t) + K_t(s_{t-1}, s_t)(\mathcal{Y}_t - \mathcal{Y}_{t|t-1}(s_{t-1}, s_t)) \quad (\text{D.26})$$

$$P_{t|t}^x(s_{t-1}, s_t) = P_{t|t-1}^x(s_{t-1}, s_t) - K_t(s_{t-1}, s_t)P_{t|t-1}^y(s_{t-1}, s_t)K_t^T(s_{t-1}, s_t) \quad (\text{D.27})$$

This updating step gives  $p(\mathcal{X}_t|\mathcal{Y}_{1:t}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t}(s_{t-1}, s_t), P_{t|t}^x(s_{t-1}, s_t))$ . As a by-product of the filter, we can get the density of  $\mathcal{Y}_t$  conditional on  $\mathcal{Y}_{1:t-1}$ ,  $s_t$ , and  $s_{t-1}$

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1}, s_{t-1}, s_t; \theta) = N(\mathcal{Y}_{t|t-1}(s_{t-1}, s_t), P_{t|t-1}^y(s_{t-1}, s_t)) \quad (\text{D.28})$$

Since the UKF with regime switches creates a large number of nodes at each iteration where the filtered mean and covariance matrix need to be evaluated, we implement the following collapsing procedure suggested by [Kim and Nelson \(1999\)](#):

$$\mathcal{X}_{t|t}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})} \left\{ \sum_{i=1}^M \Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t}) \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j) \right\} \quad (\text{D.29})$$

$$P_{t|t}^x(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})} \left\{ \sum_{i=1}^M \Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t}) [P_{t|t}^x(s_{t-1} = i, s_t = j) \right. \\ \left. + (\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))(\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))^T] \right\} \quad (\text{D.30})$$

where  $\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t})$  and  $\Pr(s_t|\mathcal{Y}_{1:t})$  are obtained from the following Hamilton filter

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}) = \Pr(s_t|s_{t-1}) \Pr(s_{t-1}|\mathcal{Y}_{1:t-1}) \quad (\text{D.31})$$

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}) = \frac{p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1}) \Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})}{\sum_{s_t} \sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1}) \Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})} \quad (\text{D.32})$$

$$\Pr(s_t|\mathcal{Y}_{1:t}) = \sum_{s_{t-1}} \Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}). \quad (\text{D.33})$$

The resulting conditional marginal likelihood is

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1}; \theta) = \sum_{s_t} \sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1}) \Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}). \quad (\text{D.34})$$

### D.3 Smoothing

Once we evaluated the likelihood of the data with the UKF, performed the filtering using the UKF for  $t = 1, \dots, T$ , we can also obtain

$\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T})$ ,  $\Pr(s_t|\mathcal{Y}_{1:T})$ ,  $x_{t|T}(s_t, s_T)$ , and  $P_{t|T}^x(s_t, s_T)$ :

$$\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) = \frac{\Pr(s_{t+1}|\mathcal{Y}_{1:T}) \Pr(s_t|\mathcal{Y}_{1:t}) \Pr(s_{t+1}|s_t)}{\Pr(s_{t+1}|\mathcal{Y}_{1:t})} \quad (\text{D.35})$$

$$\Pr(s_t|\mathcal{Y}_{1:T}) = \sum_{s_{t+1}} \Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) \quad (\text{D.36})$$

$$\mathcal{X}_{t|T}(s_t, s_{t+1}) = \mathcal{X}_{t|t}(s_t) + \tilde{K}_t(s_t, s_{t+1})(\mathcal{X}_{t+1|T}(s_{t+1}) - \mathcal{X}_{t+1|t}(s_t, s_{t+1})) \quad (\text{D.37})$$

$$P_{t|T}^x(s_t, s_{t+1}) = P_{t|t}^x(s_t) - \tilde{K}_t(s_t, s_{t+1})(P_{t+1|T}^x(s_{t+1}) - P_{t+1|t}^x(s_t, s_{t+1}))\tilde{K}_t(s_t, s_{t+1})^T.$$

Given the above smoothing algorithm, we implement another collapsing procedure similar to that in the filtering step:

$$\mathcal{X}_{t|T}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})} \left\{ \sum_{j=1}^M \Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T}) \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j) \right\}, \quad (\text{D.38})$$

$$\begin{aligned} P_{t|T}^x(s_t = j) &= \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})} \left\{ \sum_{j=1}^M \Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T}) [P_{t|T}^x(s_t = i, s_{t+1} = j) \right. \\ &\left. + (\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))(\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))^T \right\}. \end{aligned} \quad (\text{D.39})$$

## Appendix E Calibrated Parameters

To calibrate the parameters that we do not estimate, we largely follow [Mendoza \(2010\)](#), targeting the same moments, but adapting the computations to our model specification. We start by calibrating certain parameters based on the steady state of the model without working capital and the borrowing constraint—i.e., with  $\phi = 0$  and  $\bar{\varphi} = 0$ , which implies  $\lambda_{ss} = 0$ . In addition, we set

$$\beta(1 + r_{ss}) = 1, \quad (\text{E.1})$$

and

$$\Omega_v = \frac{1}{1 - \alpha - \eta}, \Omega_h = \frac{1}{\alpha}, \Omega_k = \frac{1}{\eta} \left( \frac{1}{\beta} - 1 + \delta \right). \quad (\text{E.2})$$

The implied factor payment ratios are

$$\frac{P_{ss}V_{ss}}{Y_{ss} + P_{ss}V_{ss}} = \frac{1}{\Omega_v} = 1 - \alpha - \eta \quad (\text{E.3})$$

$$\frac{W_{ss}H_{ss}}{Y_{ss}} = \frac{1}{\Omega_h \left(1 - \frac{1}{\Omega_v}\right)} = \frac{\alpha}{\alpha + \eta} \quad (\text{E.4})$$

$$\frac{\left(\frac{1}{\beta} - 1 + \delta\right) K_{ss}}{Y_{ss}} = \frac{\frac{1}{\beta} - 1 + \delta}{\Omega_k \left(1 - \frac{1}{\Omega_v}\right)} = \frac{\eta}{\alpha + \eta}. \quad (\text{E.5})$$

Using the shares in [Mendoza \(2010\)](#) that are based on National Accounts data, we obtain

$$\left[ \begin{array}{l} 1 - \alpha - \eta = 0.102 \\ \frac{\alpha}{\alpha + \eta} = 0.66 \end{array} \right] \implies \left[ \begin{array}{l} \alpha = 0.59268 \\ \eta = 0.30532 \end{array} \right]. \quad (\text{E.6})$$

We then set the depreciation rate to an annual value of 8.8 percent, so that

$$(1 - \delta)^4 = 1 - 0.088 \implies \delta = 0.022766. \quad (\text{E.7})$$

The capital-to-(gross annual) output ratio is 1.758, so the capital-to-(gross quarterly) output ratio,  $\Omega_k^{-1}$ , implies

$$\Omega_k^{-1} = \left( \frac{1}{\eta} \left( \frac{1}{\beta} - 1 + \delta \right) \right)^{-1} = 4 * 1.758 \implies \beta = 0.97977. \quad (\text{E.8})$$

In turn, this yields an annualized real interest rate of

$$(1 + r_{ss})^4 = \left( \frac{1}{\beta} \right)^4 = 1.0852, \quad (\text{E.9})$$

which is very close to the value used in [Mendoza \(2010\)](#), but it is obtained under different discounting assumptions. From the resource constraint,

$$\frac{C_{ss}}{Y_{ss}} + \frac{I_{ss}}{Y_{ss}} + \frac{E_{ss}}{Y_{ss}} = 1 + \left( 1 - \frac{1}{1 + r_{ss}} \right) \frac{B_{ss}}{Y_{ss}}, \quad (\text{E.10})$$

we obtain

$$0.65 + 0.172 + 0.11 = 1 + (1 - \beta) \frac{B_{ss}}{Y_{ss}} \implies \frac{B_{ss}}{Y_{ss}} = -3.3605. \quad (\text{E.11})$$

This implies

$$\frac{B_{ss}}{4Y_{ss}} = -0.840127, \quad (\text{E.12})$$

from which we have

$$Y_{ss} = \frac{\alpha + \eta}{\alpha} \left( \frac{1}{P_{ss}^{1-\alpha-\eta} \left( \frac{1}{\eta} \left( \frac{1}{\beta} - 1 + \delta \right) \right)^\eta \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{1}{1-\alpha-\eta} \right)^{1-\alpha-\eta}} \right)^{\frac{\omega}{\alpha(\omega-1)}} = 1.8202, \quad (\text{E.13})$$

and

$$E^* = \frac{E_{ss}}{Y_{ss}} Y_{ss} = 0.11 * 1.8202 = 0.20022, \quad (\text{E.14})$$

as well as

$$B_{ss} = -0.840127 * 4 * 1.8202 = -6.11685. \quad (\text{E.15})$$

Finally, conditional on  $r^*$  and  $\psi_r$ ,  $\bar{B}$  is pinned down via

$$\bar{B} = \log \left( 1 + \frac{r_{ss} - r^*}{\psi_r} \right) + B_{ss}. \quad (\text{E.16})$$

## Appendix F Data Appendix

National accounts are from the National Statistic Office. The data series used in the analysis merge two sets of official statistics by updating the level of the accounts based on 1993 constant prices with the quarterly rate of growth of the accounts based on 2008 constant prices. The merging is necessary as the deflators to splice the accounts in levels were not available at the time of last download of the data (May 2017). The two sets of national accounts overlap from 1993:Q1 to 2006:Q4. Over this period, the difference in annual rate of growth is less than 0.01 percent in absolute value for GDP, less than 0.05 percent for consumption, less than 2 percent for investment, and less than 1 and 3 percent for imports and exports, respectively. The correlations between the series are more than 0.9 for all series except investment that is 0.84, pointing to possibly larger measurement errors in this variable. The differences are smaller the closer to the end of the sample. For this reason, we choose to update the 1993 accounts rather than backdate the 2008 ones.

The specific sources of the data are as follows:

- 1980:Q1-2006:Q4 (Labeled 1993 accounts)—Supply and demand of goods and services. Original Series (not seasonally adjusted). Constant prices, annual 1993 = 100. We obtained these from the Central Bank of Mexico ([Gabriel, 2008](#)).
- 2006:Q1-2016:Q4 (Labeled 2008 accounts)—Supply and demand of goods and services. Original series (not seasonally adjusted). Constant prices, annual 2008 = 100 (Oferta y

demanda de bienes y servicios. Series originales. A precios constantes 2008). Available from <http://www3.inegi.org.mx/sistemas/tabuladosbasicos/tabdirecto.aspx>.

The data are not seasonally adjusted and show a strong seasonal pattern. To seasonally adjust all series (assumed to be  $I(1)$  processes), we adjust the log-difference using the X-12 procedure with the additive option in Eviews. We then use the log of the first observation of the raw series (not seasonally adjusted) and cumulate the seasonally adjusted log-difference. The net exports to GDP series, used to validate the model externally but not as an observable variable in estimation, is calculated as real exports minus real imports divided by real GDP.

The current account as a percentage of GDP is from the balance of payment statistics, obtained from the OECD Economic Outlook Database (Series MEX.CBGDPR.Q, OECD-EO-MEX-CBGDPR-Q).

As a proxy for the relative price of intermediate goods, entered as observable in estimation, we use a measure of Mexico's terms of trade obtained from Banco de México (PPI Producer and International Trade Price Indexes, series SP12753).

Mexico's country interest rate is calculated following [Uribe and Yue \(2006\)](#) as

$$r_t = r_t^* + spread_t \tag{F.1}$$

where  $r^*$  is the US real interest rate, and  $spread$  is a proxy for Mexico's country risk or sovereign spread. We compute  $r^*$  as 3-month Treasury Constant Maturity Rate adjusted for ex post CPI (annualized) quarterly inflation, using period average data. The source of these data is FRED. For the country spread, as customary, we use the Mexico's component of the JP Morgan EMBI.

Unfortunately, the EMBI spread is available only starting from 1993. In order to estimate the country spread before 1993, we rely on empirical modeling of the relation between *domestic* real interest rates and country risk at the Banco de Mexico ([Aportela Rodriguez et al., 2001](#)) that estimates a close and stable relation between a measure of the domestic real interest rate and the EMBI spread over the period over which both these variables are available. The only quarterly interest series available we are aware of going back to 1980 is a three-month nominal short-term rate obtained from Banco de Mexico (Average monthly yield on 90-days Cetes, series SF3338).<sup>32</sup> So we estimate a relationship between this nominal interest rate,  $i_t$ , and the EMBI during the period over which the EMBI is observable, adjusting for inflation,  $\pi_t$ , which was an important source of nominal interest rate variation

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<sup>32</sup>There are three missing monthly observations in this series: August and September 1986 and November 1988. We fill these gaps using July 1986 for 1986Q3 and the average of October and December 1988 for 1988:Q4.

in the 1980s, and then invert it. Specifically, we posit the following simplified version of the model that (Aportela Rodriguez et al., 2001) estimate:

$$i_t = \alpha_0 + \alpha_1\pi_t + \alpha_2EMBI_t. \quad (\text{F.2})$$

We then solve the fitted equation for the country risk component of the domestic real interest rate, which we denote as  $EM\hat{B}I_t$ . The estimated regression is (t-statistics in parentheses and  $R^2 = 0.883$ ):

$$\hat{i}_t = \underset{(-0.42)}{-0.00346} + \underset{(4.46)}{0.397}\pi_t + \underset{(7.37)}{2.770}EM\hat{B}I_t. \quad (\text{F.3})$$

## Appendix G Additional Estimation Results

In this appendix we report additional empirical results.

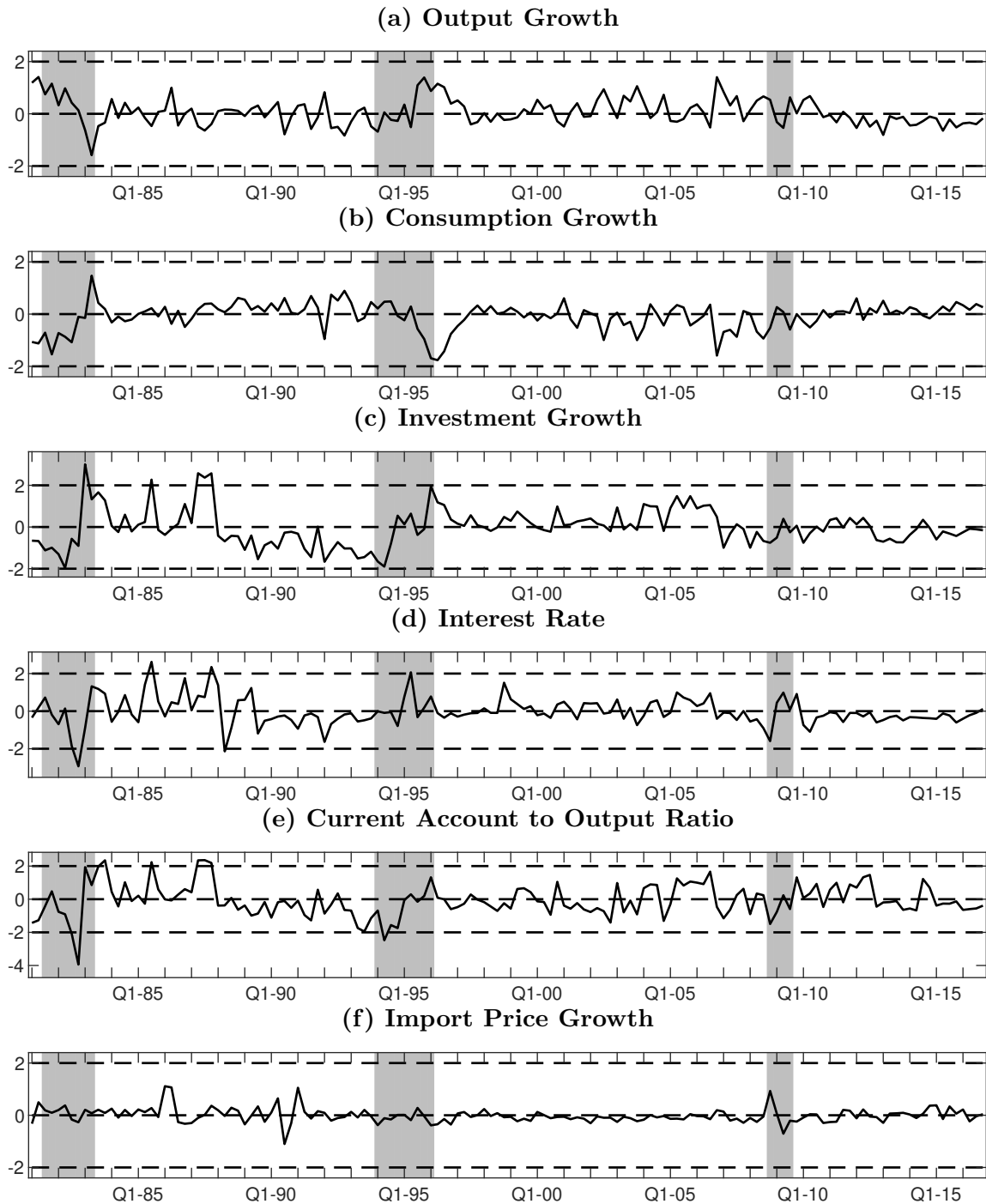
### G.1 Estimated Shocks and Transition Probabilities

Figure F.1 plots the implied measurement error from Figure 3, in standard deviation units. It shows that there are only three realizations outside the two-standard deviation bands during the Debt crisis. All other realizations, for all variables, all periods, including crisis and non crisis periods, are within the bands.

Figure F.2 shows a histogram of the estimated structural shocks plotted in Figure 4, in standard deviation units. The figure shows that the TFP, import price, and transitory interest rate shock are well behaved consistent with the fact that the null hypothesis of normality cannot be rejected by the data. We can also see that the expenditure and the persistent interest rate shocks distribution has some left skewness, picked up by the formal test of normality, even though the rejection is not strong. The preference shock histogram is the only one that shows some sign of fat left tail, as we can also see from Figure 4 in the text. This excess skewness reflects in part a small trend in the exogenous processes required to match the data that in principle could be accounted for by specifying a stochastic process with time-varying mean.

Figure G.1 plots the pseudo-real-time (i.e. filtered) estimated transition probabilities. Panel (a) plots the probability of switching from the non-binding to the binding regime, while Panel (b) plots the probability of switching from the binding to the non-binding regime. In other words, they plot the estimated counterpart of the transition probabilities in equations (11-12) together with the model identified crisis periods. These probabilities provide the probability of switching from one regime to the other as the model travels through the

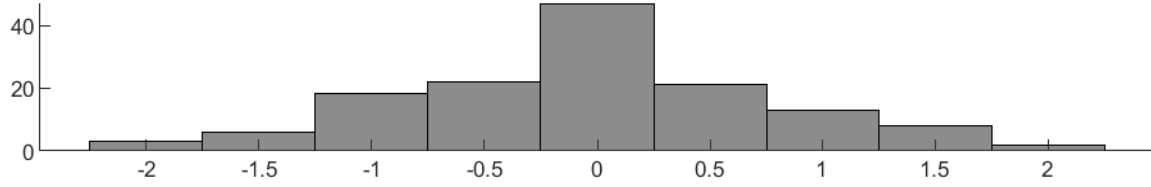
Figure F.1: Estimated Measurement Errors



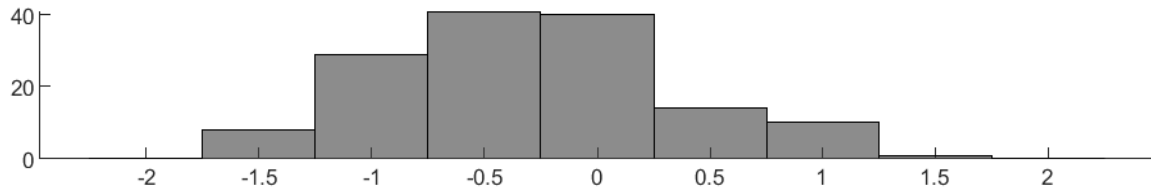
Note: The figure plots realized measurement errors (i.e., observable minus model estimate) for all variables. The gray areas indicate model-identified periods of crisis as defined in Section 5.2.1.

Figure F.2: Histogram of Estimated Shocks

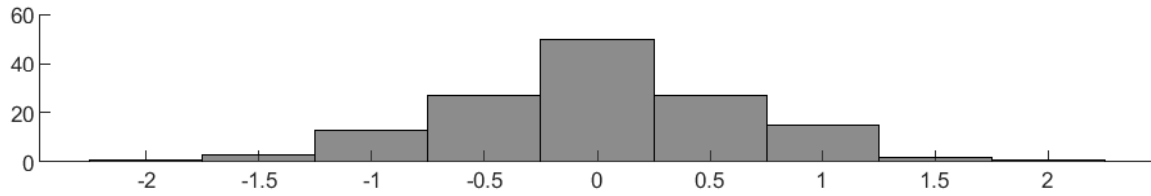
(a) TFP Shock



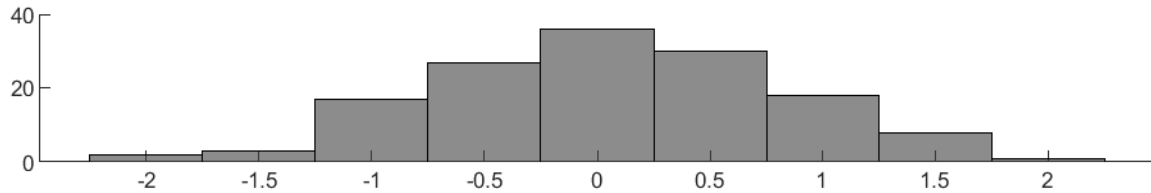
(b) Expenditure Shock



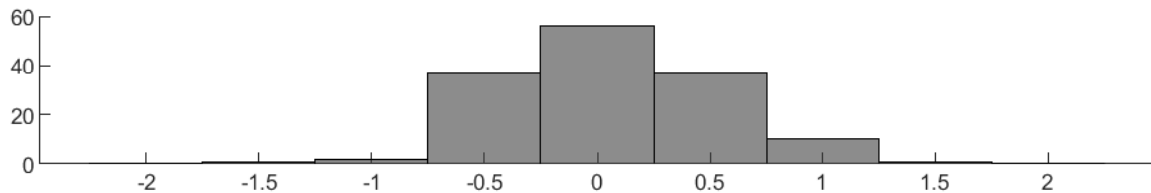
(c) Import Price Shock



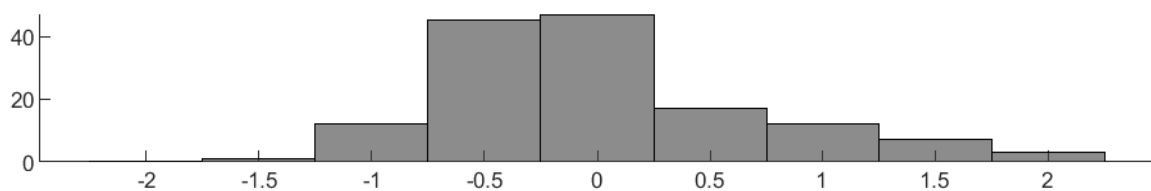
(d) Preference Shock



(e) Transitory Interest Rate Shock



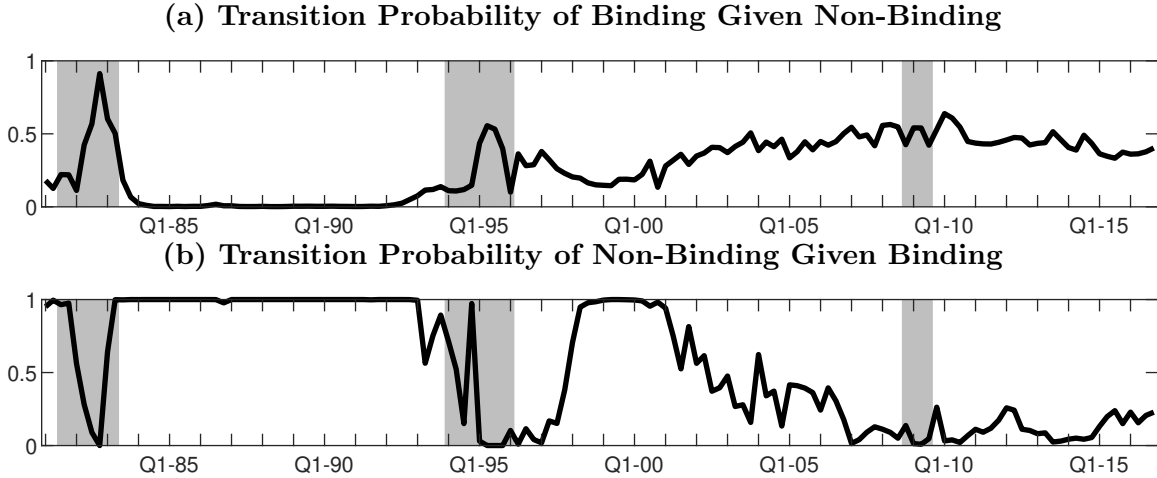
(f) Persistent Interest Rate Shock



Notes: The figure plots histograms of the model implied shocks, in standard deviation units



Figure G.1: Transition Probabilities



Notes: The top panel shows the model-implied filtered probability of transitioning into the binding regime in the subsequent period, conditional on being in the non-binding regime in the current period. The bottom panel shows the filtered probability of transitioning to the non-binding regime, conditional on being in the binding regime. The shaded areas indicate model-identified crisis periods as defined in Section 5.2.1.

sample. Their behavior is driven by the estimated parameters  $\gamma_0$  and  $\gamma_1$  and the estimated values of  $B^*$  and  $\lambda$ . Both probabilities are time-varying and thus clearly indicate that a model with exogenous and constant switching probabilities would be misspecified.

## G.2 Assessing the Relative Importance of Shocks

To study the importance of shocks before, after, and during financial crises episodes we construct a likelihood-based indicator of the relative importance of each shock. Let  $LL$  denote the maximized log-likelihood over the full sample, and let  $CLL_{i,t}$  denote the counterfactual full-sample log likelihood when shock  $i$  is set to zero in quarter  $t$  (i.e.  $\varepsilon_{i,t} = 0$ ). The loss of fit in terms of likelihood points,

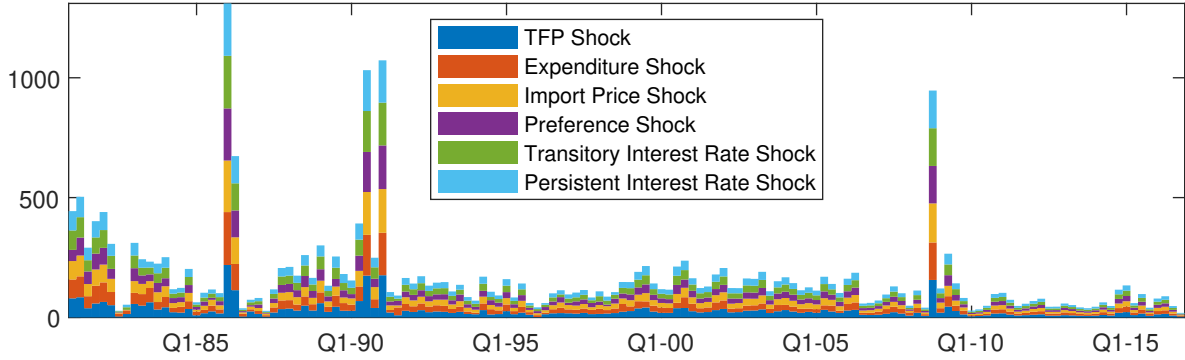
$$\Delta_{i,t} = LL - CLL_{i,t}, \quad (\text{G.1})$$

can be interpreted as an measure of the importance of  $\varepsilon_{i,t}$  in period  $t$ .

Figure G.2 shows how this measure evolves. There is an obvious shortcoming to this approach: since we are computing the difference in the likelihood over the full sample, earlier observations tend to have larger values of  $\Delta_{i,t}$  due to the fact that they impact all subsequent quarters.<sup>33</sup> For this reason, we compute a measure of relative importance. The

<sup>33</sup>The figure, however, shows that period shocks that have unusual weight in the likelihood do not realize during the identified crisis periods.

**Figure G.2: Historical Importance of Shocks, Log Points**



Notes: For each quarter, shows the likelihood contribution (in log points) for each shock, computed by setting the given shock to zero in the given quarter.

importance of  $\varepsilon_{i,t}$  relative to other shocks at time  $t$  can be assessed as

$$\Lambda_{i,t} = \frac{\Delta_{i,t}}{\sum_j \Delta_{j,t}}, \quad (\text{G.2})$$

where the denominator of this expression is the sum of the likelihood losses across all shocks in period  $t$ . In the paper, we report this measure of average relative importance in a given subsample period of  $T$  quarters,  $(t + T) - (t)$ , compared to the full sample averages,  $\bar{\Lambda}_i$ . Thus, the subsample measure reported in the main text can be computed by

$$\sum_{t=t_0}^{t_1} \Lambda_{i,t} - \bar{\Lambda}_i. \quad (\text{G.3})$$