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Andres Rodrıguez-Clare
University of California, Berkeley
NBER

Mauricio Ulate
Federal Reserve Bank of San Francisco

Jose P. Vasquez
LSE and CEPR

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Trade with Nominal Rigidities: Understanding the Unemployment and Welfare Effects of the China Shock[*]

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We present a dynamic quantitative trade and migration model that incorporates downward nominal wage rigidities and show how this framework can generate changes in unemployment and labor participation that match those uncovered by the empirical literature studying the "China shock." We find that the China shock leads to average welfare increases in most U.S. states, including many that experience unemployment during the transition. However, nominal rigidities reduce the overall U.S. gains by around one fourth. In addition, there are seven states that experience welfare losses in the presence of downward nominal wage rigidity that would have experienced gains without it.

1   Introduction

A concern about international trade often raised by the public and the popular press is that it may destroy jobs and lead to unemployment. Trade economists have increasingly taken this concern seriously, but the focus has been on the long-run.[1] Thus, we still lack a framework to understand the possibly adverse short-run employment effects of trade shocks. The need for such a framework becomes particularly salient in light of the findings by Autor, Dorn, and Hanson (2013, henceforth ADH) and others indicating that U.S. local labor markets more exposed to the "China shock" experienced significant increases in unemployment and decreases in labor force participation relative to less exposed regions (see Autor et al., 2016; Redding, 2020, for reviews). If trade shocks can lead to temporary increases in unemployment, how does this change the way we evaluate their welfare effects?

In this paper, we propose a dynamic quantitative trade and migration model in which shocks can trigger increases in unemployment and decreases in labor force participation during a transition period, while allowing for the computation of the implied aggregate and distributional welfare effects. The key feature of the model is downward nominal wage rigidity (DNWR) as in Schmitt-Grohe and Uribe (2016), constraining the nominal wage in any period to be no less than a factor δ times the nominal wage in the previous period.[2] We embed this feature into a dynamic model in the spirit of Caliendo, Dvorkin, and Parro (2019, henceforth CDP), which we extend to allow for a difference between the elasticity governing workers' mobility across sectors ($1/\nu$ in our model) and the elasticity governing mobility across local labor markets ($1/\kappa$ in our model).

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[1] Davidson et al. (1999), Helpman et al. (2010), Kim and Vogel (2020b), and Galle et al. (2020) are papers that focus on the long-run impacts of unemployment. Important exceptions looking at short-run employment effects are Dutt et al. (2009) and Dix-Carneiro et al. (2020), which we discuss below.
[2] Several recent papers have found support for the presence of DNWR (e.g., Dickens et al., 2007; Daly and Hobijn, 2014; Grigsby et al., 2019; Hazell and Taska, 2019). We acknowledge that labor-market frictions in the real world go significantly beyond DNWR. However, we aim to show that the DNWR in our model is a powerful yet parsimonious way to capture such frictions.

We calibrate the key model parameters δ, ν, and κ to results from ADH on how labor force participation, unemployment, and population across U.S. labor markets are affected by the China shock. Using dynamic exact hat algebra, we simulate the effects of the China shock for the 2000-2007 period. The results indicate that although the China shock improves the terms of trade for almost all states (i.e., only one state would experience a welfare loss in the absence of DNWR), employment actually falls in most states during the transition, both through an increase in unemployment and a decline in labor force participation. These employment effects have significant welfare implications, as they lead to a one-fourth reduction in the U.S. welfare gains from the China shock, and to absolute welfare losses in eight states.

The intuition behind our results is as follows. With flexible wages, the increase in China's relative productivity would require a downward adjustment in the U.S. relative wage. DNWR prevents this adjustment from taking place through a large decline in the U.S. nominal wage, and a nominal anchor (described below) prevents it from occurring through a large increase in the Chinese dollar wage. The result is temporary unemployment in the U.S. In turn, with home production available to workers, this triggers further declines in labor participation, as more workers prefer to engage in home production rather than face the possibility of unemployment.

Section 2 presents our model. There are multiple sectors linked by an input-output structure, sector-level trade satisfies the gravity equation, and a home-production sector leads to an upward sloping labor supply curve. Trade takes place between regions, and workers can move across regions belonging to the same country. Each period, workers draw idiosyncratic shocks to the utility of working in each sector-region from a nested Gumbel distribution. Based on these draws, the costs of moving, and expected future real income adjusted for unemployment, workers choose which sector-region to participate in. Wages are subject to a DNWR, but are otherwise determined by supply and demand for labor.

Given the presence of the DNWR, we need to close the model with a nominal anchor that prevents nominal wages from rising enough to make the DNWR always

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non-binding.[3] We assume that world nominal GDP in dollars grows at a constant and exogenous rate (which we set to zero without loss of generality). While this nominal anchor is a simplification, it allows us to solve our otherwise-unwieldy dynamic trade and migration model.[4] Qualitatively, we would obtain similar results if we assumed instead that China uses a combination of monetary and exchange rate policies to prevent both an appreciation of its currency and large inflationary pressures – thereby preventing the Chinese wage in dollars from increasing – while the U.S. does not fully offset this with its own policies.

Section 3 describes our data construction. We combine multiple data sources, a set of proportionality assumptions, and implications from a gravity model to construct sector-level trade flows across all region pairs in our sample. We also construct migration flows between all sector-states in the U.S. The resulting dataset contains 87 regions (50 U.S. states, 36 additional countries, and an aggregate rest of the world region), and 15 sectors (home production, 12 manufacturing sectors, services, and agriculture), between 2000 and 2007. Incorporating a service sector is necessary in any analysis of the China shock (due to its size and importance in the U.S. economy), while the agricultural sector is potentially relevant for understanding the effects of the China shock in certain small or rural states.

Section 4 describes our calibration procedure for parameters δ, ν, and κ as well as the China shock, which we operationalize as productivity changes in China that can vary across sectors and years. For any set of parameter values and productivity changes, we use dynamic hat algebra to compute implied annual changes in trade flows as well as the changes in labor-force participation, unemployment, and population over the 2000-2007 period. We then iterate over the parameter values and productivity changes until the sector-level annual changes in U.S. imports from

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[3]Our baseline analysis also assumes that third countries have flexible exchange rates vis-á-vis the dollar, but we explore the alternative of fixed exchange rates in Section 6.

[4]Assuming other types of nominal anchors prevents our model from being solved with an efficient Alvarez-and-Lucas type algorithm that we develop to deal with the DNWR, thereby increasing the time required to solve the model by several orders of magnitude.

China match those predicted in the data and the ADH-style regression coefficients in the model match those obtained by ADH in the data. Thus, we follow the approach suggested by Nakamura and Steinsson (2018) and Redding (2020), namely using causally-identified empirical moments to calibrate our general equilibrium model. The calibration leads to a value of $\delta = 0.98$, implying that – with constant world nominal GDP – wages can fall up to 2% annually without the DNWR becoming binding. This value is similar to the one in Schmitt-Grohe and Uribe (2016).

Section 5 presents the results of the baseline quantitative analysis. In the short run, unemployment increases in the regions most exposed to the China shock, but this is reversed over time as the nominal wage adjusts downward. In the long run, since the real wage governs labor supply and there is no unemployment, employment eventually increases after the economy fully adjusts to the positive terms of trade shock. Overall, the China shock leads to increases in the real wage for almost all regions, including most of the ones for which full employment would require a decline in the nominal wage.

One benefit of our approach is that we can study the effect of the China shock on welfare, and in particular explore how this is affected by DNWR. We compute welfare as the present discounted value of utility flow, with a discount rate of 0.95. We find that welfare increases in most U.S. states, including many that experience unemployment during the transition. For the U.S. as a whole, although the China shock remains beneficial, DNWR reduces the aggregate welfare gains by one fourth (from 31 to 23 basis points).

The spatial heterogeneity in the employment and income effects of the China shock implied by our model is similar to that implied by the empirical results in ADH. This stands in contrast to previous quantitative trade models, such as CDP and Galle et al. (2020), which deliver too little dispersion, as shown in Adao et al. (2020) and Autor et al. (2021). The main reason we obtain a higher dispersion is that, because of the DNWR, our model leads to much larger declines in employment in the most exposed regions, both directly through higher unemployment, and indi-

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rectly through discouraging labor participation.

Section 6 studies how varying some of the key assumptions in the baseline specification affects the conclusions. We consider alternative specifications where we: get rid of migration across U.S. states, impose the same elasticity of mobility across sectors and regions (i.e., $\nu = \kappa$ as in CDP), introduce DNWR only in manufacturing, consider some of the increases in trade surpluses that occurred in China as part of the China shock, and use a different exchange rate regime for third countries. We highlight three important messages arising from these specifications. First, eliminating migration makes little difference for the model's fit with ADH results, while imposing $\nu = \kappa$ significantly worsens the model's fit. Second, assuming that DNWR holds only in manufacturing improves the model's ability to match non-targeted moments. Finally, taking a broader perspective of the China shock so that it also includes an increase in the U.S. trade deficit can make the DNWR more binding and lead to slightly smaller welfare gains from the shock.

Section 7 discusses two additional implications of our analysis. First, we argue that, controlling for terms-of-trade shocks and assuming that labor supply is a function of the real wage, ADH's exposure measure to the China shock becomes a relevant statistic only thanks to DNWR. Second, we discuss how our results for aggregate job losses can be seen as arising from the results in ADH plus a correction for the "missing intercept problem," which is positive in our analysis due to the positive terms-of-trade effect implied by the China shock.

Our paper follows in the footsteps of a large literature that analyzes the impacts of trade shocks on different regions or countries. Quantitative papers such as CDP, Galle et al. (2020), and Adao et al. (2020) focus on the effects of the China shock on regions of the U.S. Our model incorporates nominal rigidities as a mechanism to deliver involuntary unemployment, which is an uncommon feature in this literature despite its prominence in the empirical papers studying the China shock.

Another literature explores the effect of trade on unemployment using search and matching models (e.g. Davidson and Matusz, 2004; Dutt et al., 2009; Helpman

et al., 2010; Hasan et al., 2012; Heid and Larch, 2016; Kim and Vogel, 2020a,b; Galle et al., 2020; Dix-Carneiro et al., 2020). In principle, one could calibrate a trade model with search frictions to make it compatible with ADH moments, but the aggregate implications would be quite different from those in our model. The reason is that models with search and matching frictions essentially lead to forces that amplify the effects of terms-of-trade shocks. Thus, a positive terms-of-trade shock not only leads to an increase in the real wage and labor participation, but also to a decline in unemployment. Since the China shock is a positive terms-of-trade shock for most states, then the search and matching approach would imply that unemployment actually falls in most states (see Galle et al., 2020).[5] In contrast, DNWR can lead to temporary increases in unemployment even when terms of trade improve.

Also related to our paper is Eaton et al. (2013), which studies the extent to which unmodeled cross-country relative wage rigidities can explain the increases in unemployment and decreases in GDP observed in countries undergoing sudden stops. Relative to this paper, our contribution is to show how DNWR can lead to such relative wage rigidities, to extend the analysis to terms-of-trade shocks in a multi-sector model with migration, and to quantify the effect of the China shock on unemployment and nonemployment across U.S. states between 2000 and 2007.

On the side of open-economy macroeconomics, classic contributions such as Clarida et al. (2002) or Gali and Monacelli (2005, 2008, 2016) have introduced nominal rigidities in models with a simplified trade structure. Schmitt-Grohe and Uribe (2016) studies optimal policies in the presence of DNWR in a small open economy. Nakamura and Steinsson (2014), Beraja et al. (2016), and Chodorow-Reich and Wieland (2017) deal with multiple heterogeneous regions in models with nominal rigidities. None of these papers connect to actual sector-level trade flows and hence cannot be used for a detailed quantitative analysis of an event like the China shock.

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[5]An exception here is Dix-Carneiro et al. (2020), which develops a dynamic multi-sector model to study the role of trade imbalances on labor market dynamics. By allowing the China shock to go beyond a trade shock and also lead to an increase in the U.S. trade deficit, their analysis reveals another way in which the China shock could lead to a temporary increase in unemployment.

2 A Dynamic Spatial Trade and Migration Model with Nominal Wage Rigidities

Building on Artuc, Chaudhuri, and McLaren (2010) and more directly on CDP, we consider a dynamic multi-sector quantitative trade model with an input-output structure and forward looking agents that decide in which region and sector to work. Given our goals of matching the results in ADH, we introduce two key extensions to CDP: downward nominal wage rigidity as a mechanism that can generate unemployment, and a nested structure in the households' labor supply decision to allow for different elasticities of moving across regions and sectors. In this section, we present an abridged description of the model, focusing on its non-standard elements and relegating some of the details to Appendix A.

2.1 Basic Assumptions

We assume that the world is composed of multiple economies or "regions" (indexed by i or j). There are M regions inside the U.S. (which will be the 50 U.S. states), plus $I - M$ regions (countries) outside of the U.S. (for a total of I regions). We assume that there is no labor mobility across different countries, but allow for mobility across different states of the U.S. There are $S + 1$ sectors in the economy (indexed by s or k), with sector zero denoting the home production sector and the remaining S sectors being productive market sectors. In each region j and period t, a representative consumer participating in the market economy devotes all income to expenditure $P_{j,t}C_{j,t}$, where $C_{j,t}$ and $P_{j,t}$ are aggregate consumption and the price index respectively. Aggregate consumption is a Cobb-Douglas aggregate of consumption across the S different market sectors with expenditure shares $\alpha_{j,s}$. As in a multi-sector Armington trade model, consumption in each market sector is a CES aggregate of consumption of the good of each of the I regions, with an elasticity of substitution $\sigma_s > 1$ in sector s.

Each region produces the good in sector s with a Cobb-Douglas production function, using labor with share $\phi_{j,s}$ and intermediate inputs with shares $\phi_{j,ks}$, where $\phi_{j,s} + \sum_k \phi_{j,ks} = 1$. Total factor productivity in region j, sector s, and time t is $A_{j,s,t}$. There is perfect competition and iceberg trade costs $\tau_{ij,s,t} \geq 1$ for exports from i to j in sector s. Intermediates from different origins are aggregated in the same way as consumption goods (i.e., CES with elasticity σ_s). Letting $W_{i,s,t}$ denote the wage in region i, sector s, at time t, the price in region j of good s produced by region i at time t is then

$$p_{ij,s,t} = \tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_k P_{i,k,t}^{\phi_{i,ks}}, \tag{1}$$

where $P_{i,k,t}$ is the price index of sector k in region i at time t. Given our Armington assumption, these price indices satisfy

$$P_{j,s,t}^{1-\sigma_s} = \sum_{i=1}^{I} p_{ij,s,t}^{1-\sigma_s}. \tag{2}$$

Let $R_{i,s,t}$ and $L_{i,s,t}$ denote total revenues and employment in sector s of country i, respectively. Noting that the demand of industry k of country j of intermediates from sector s is $\phi_{j,sk} R_{j,k,t}$ and allowing for exogenous deficits as in Dekle et al. (2007), the market clearing condition for sector s in country i can be written as

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t} \left(\alpha_{j,s} \left(\sum_{k=1}^{S} W_{j,k,t} L_{j,k,t} + D_{j,t} \right) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t} \right), \tag{3}$$

where $D_{j,t}$ are transfers received by region j, with $\sum_j D_{j,t} = 0$, and the trade shares $\lambda_{ij,s,t}$ are given by

$$\lambda_{ij,s,t} \equiv \frac{p_{ij,s,t}^{1-\sigma_s}}{\sum_{r=1}^{I} p_{rj,s,t}^{1-\sigma_s}}. \tag{4}$$

In turn, employment must be compatible with labor demand,

$$W_{i,s,t} L_{i,s,t} = \phi_{i,s} R_{i,s,t}. \tag{5}$$

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Before describing our assumptions regarding labor supply and nominal rigidities, it is instructive to consider two standard ways to close the model. First, as in Caliendo and Parro (2015), one could assume that there is perfect labor mobility across sectors within each region and that labor supply is perfectly inelastic. This would imply that wages are equalized across sectors within a region, $W_{i,s,t} = W_{i,t}$ for all s, and that employment must add up to some exogenous level, $\sum_s L_{i,s,t} = \bar{L}_{i,t}$. Alternatively, one could assume that there is a perfectly inelastic labor supply in each sector-region, $L_{i,s,t} = \bar{L}_{i,s,t}$. In this case, one could use equations (1)-(5) to solve for wages (up to a choice of numeraire) at time t given employment levels. Inverting this relationship we get what could be thought of as labor demand, with employment $\{L_{i,s,t}\}$ a function of wages $\{W_{i,s,t}\}$, a concept that we will return to later.

2.2 Labor Supply

Agents can either engage in home production or look for work in the labor market. If they participate in the labor market, they can be employed in any of the S market sectors. We let $\omega_{i,0,t}$ denote consumption associated with home production in region i, and $\omega_{i,s,t}$ denote consumption associated with seeking employment in sector s and region i at time t. We assume that $\omega_{i,0,t}$ is exogenous and does not vary over time, while – as explained further below – $\omega_{i,s,t}$ is endogenous and depends on real wages and unemployment. Additionally, we denote the number of agents that participate in region i, sector s, at time t, by $\ell_{i,s,t}$.

Agents are forward looking and they face a dynamic problem where they discount the future at rate β. As in Artuc et al. (2010), migration decisions are subject to sectoral and spatial mobility costs. There are labor relocation costs $\varphi_{ji,sk}$ of moving from region j, sector s to region i, sector k. These costs are time invariant, additive, and measured in terms of utility. Additionally, agents have additive idiosyncratic shocks for each choice of region and sector, denoted by $\epsilon_{i,s,t}$.

An agent that starts in region j and sector s observes the economic conditions

in all labor markets and the idiosyncratic shocks, then earns real income $\omega_{j,s,t}$ and has the option to reallocate. Denoting with $\nu_{j,s,t}$ the lifetime utility of an agent who is in region j, sector s, at time t, we have

$$\nu_{j,s,t} = U(\omega_{j,s,t}) + \max_{\{i,k\}_{i=1,k=0}^{I,S}} \{\beta\mathbb{E}(\nu_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\},$$

where the expectation is taken over future realizations of the idiosyncratic shocks. In contrast to CDP, we assume that the joint density of the vector ϵ at time t is a nested Gumbel:

$$F(\epsilon) = \exp\left(-\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(-\epsilon_{i,k,t}/\nu\right)\right)^{\nu/\kappa}\right),$$

where $\kappa > \nu$. This allows us to have different elasticities of moving across regions and sectors, which will be essential for the model to match the empirical evidence in ADH. Let $V_{j,s,t} \equiv \mathbb{E}(\nu_{j,s,t})$ be the expected lifetime utility of a representative agent in labor market j, s. As we show in Appendix A.2, we have

$$V_{j,s,t} = U(\omega_{j,s,t}) + \ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa} + \gamma\kappa, \tag{6}$$

where γ is the Euler-Mascheroni constant.

Denote by $\mu_{ji,sk|i,t}$ the number of agents that relocate from market js to ik expressed as a share of the total number of agents that move from js to ik' for any sector k'. Additionally, let $\mu_{ji,s\#,t}$ denote the fraction of agents that relocate from market js to any market in i as a share of all the agents in js. We show in Appendix A.2 that these fractions are given by

$$\mu_{ji,sk|i,t} = \frac{\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}}{\sum_{h=0}^{S}\exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu}} \tag{7}$$

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$$
\mu_{ji,s\#,t} = \frac{\left(\sum_{h=0}^{S} \exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu}\right)^{\nu/\kappa}}{\sum_{m=1}^{I} \left(\sum_{h=0}^{S} \exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa}}. \tag{8}
$$

We know that the total number of agents that move from *js* to *ik* is given by the multiplication of the two previous quantities, $\mu_{ji,sk} = \mu_{ji,sk|i,t} \cdot \mu_{ji,s\#,t}$, and that participation in the different labor markets evolves according to

$$
\ell_{i,k,t+1} = \sum_{j=1}^{I} \sum_{s=0}^{S} \mu_{ji,sk|i,t} \mu_{ji,s\#,t} \ell_{j,s,t} \tag{9}
$$

Without DNWR there would be no unemployment and hence the expected real income of participating in a sector-region would be the associated real wage, $\omega_{i,s,t} = W_{i,s,t}/P_{i,t}$, where $P_{i,t}$ is the aggregate price index in region i at time t,

$$
P_{i,t} = \prod_{s=1}^{S} P_{i,s,t}^{\alpha_{i,s}}. \tag{10}
$$

Equations (1)-(10) combined with $\omega_{i,s,t} = W_{i,s,t}/P_{i,t}$ would characterize the equilibrium of a model that is similar to CDP.[6]

With DNWR agents must take into account the possibility of unemployment when deciding which sector-region to participate in. To simplify the analysis, we assume that the income generated in a sector-region is equally shared between all participants in that sector-region. Since agents get real wage $W_{i,s,t}/P_{i,t}$ with probability $L_{i,s,t}/\ell_{i,s,t}$ if they seek employment in sector s of region i at time t, we have

$$
\omega_{i,k,t} = \frac{W_{i,k,t}}{P_{i,t}} \cdot \frac{L_{i,k,t}}{\ell_{i,k,t}}. \tag{11}
$$

Note that our setup does not allow unemployed workers to engage in home production. As we discuss below, this implies that the threat of unemployment discourages labor force participation, which is a desirable feature that allows the model to match the ADH targets with a reasonable labor supply elasticity.

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[6]The main theoretical differences with CDP would be our nested structure for labor supply and the fact that we do not have a fixed factor as they do.

2.3 Downward Nominal Wage Rigidity

We denote the number of agents that are actually employed in region i and sector k at time t with $L_{i,k,t}$. In the standard trade model, labor market clearing requires that the labor used in a sector and region be equal to labor supplied to that sector, i.e. $L_{i,k,t} = \ell_{i,k,t}$. We depart from this assumption and instead follow Schmitt-Grohe and Uribe (2016) by allowing for downward nominal wage rigidity, which might lead to an employment level that is strictly below labor supply,

$$L_{i,k,t} \leq \ell_{i,k,t}. \tag{12}$$

All prices and wages up to now have been expressed in U.S. dollars. In contrast, a given region faces DNWR in terms of its local currency unit. Letting $W_{i,k,t}^{LCU}$ denote nominal wages in local currency units, the DNWR takes the following form:

$$W_{i,k,t}^{LCU} \geq \delta_k W_{i,k,t-1}^{LCU}, \qquad \delta_k \geq 0.$$

Letting $E_{i,t}$ denote the exchange rate between the local currency unit of region i and the local currency unit of region 1 (which is the U.S. dollar) in period t (in units of dollars per local currency of region i), then $W_{i,k,t} = W_{i,k,t}^{LCU} E_{i,t}$ and so the DNWR for wages in dollars entails

$$W_{i,k,t} \geq \frac{E_{i,t}}{E_{i,t-1}} \delta_k W_{i,k,t-1}.$$

Since all regions within the U.S. share the dollar as their local currency unit, then $E_{i,t} = 1$ and $W_{i,k,t}^{LCU} = W_{i,k,t} \ \forall \ i \leq M$. This means that the DNWR in states of the U.S. takes the familiar form $W_{i,k,t} \geq \delta_k W_{i,k,t-1}$. For the $I - M$ regions outside of the U.S., the LCU is not the dollar and so the behavior of the exchange rate impacts how the DNWR affects the real economy. The DNWR in dollars can then be captured using a country-specific parameter $\delta_{i,k}$ for each sector, i.e.:

$$W_{i,k,t} \geq \delta_{i,k} W_{i,k,t-1}, \qquad \delta_{i,k} \geq 0. \tag{13}$$

In our baseline specification we assume that all regions outside of the U.S. have a flexible exchange rate and so the DNWR never binds. We capture these assumptions by setting $\delta_{i,k} = \delta_k \ \forall \ i \leq M$ and $\delta_{i,k} = 0 \ \forall \ i > M$. In an extension described in Section 6.4, we consider an alternative scenario in which other countries have fixed exchange rates to the U.S. so that $\delta_{i,k} = \delta_k \ \forall \ i$. Finally, equations (12) and (13) are satisfied with complementary slackness,

$$(\ell_{i,k,t} - L_{i,k,t})(W_{i,k,t} - \delta_{i,k}W_{i,k,t-1}) = 0. \tag{14}$$

2.4 Nominal Anchor

So far, we have introduced nominal elements to the model (i.e., the DNWR), but we have not introduced a nominal anchor that prevents nominal wages from rising so much in each period as to make the DNWR always non-binding. We now want to capture the general idea that central banks are unwilling to allow inflation to be too high because of its related costs (as described in, e.g., Woodford, 2003). In traditional macro models, this is usually implemented via a Taylor rule, where the policy rate reacts to inflation in order to keep price growth in check. Instead, we use a nominal anchor that captures a similar idea in a way that naturally lends itself to quantitative implementation in our trade model.

Specifically, we assume that world nominal GDP in dollars grows at a constant gross rate of γ across years,

$$\sum_{i=1}^{I} \sum_{s=1}^{S} W_{i,s,t} L_{i,s,t} = \gamma \sum_{i=1}^{I} \sum_{s=1}^{S} W_{i,s,t-1} L_{i,s,t-1}. \tag{15}$$

A similar nominal anchor is used in Guerrieri et al. (2021), albeit in the context of a static, closed-economy model. This nominal anchor has some desirable properties. First, it allows us to solve our otherwise-unwieldy model using a fast contraction-mapping algorithm in the spirit of Alvarez and Lucas (2007) that we develop to deal with equations (12)-(14) implied by the DNWR. We describe this special algorithm

in Section A.7. Second, if $\gamma = 1$, it can be seen as capturing a given level of world aggregate nominal demand in the context of a global savings glut (or zero lower bound), in the spirit of papers such as Caballero et al. (2015) and Jeanne (2021). Third, it is flexible enough to allow for unemployment even in the context of two countries that have a single region each.

Intuitively, we would obtain similar results if we removed (15) and assumed instead that something prevents the Chinese wage in dollars from rising. This could occur, for example, if China wants to preserve its competitiveness and uses a combination of monetary and exchange rate policies to prevent the Chinese wage in dollars from increasing, while the U.S. does not counteract this strongly enough with its own policies. Additionally, the nominal anchor only matters in determining how unemployment is generated in the model, but this will be one of our targets in the calibration exercise. Hence, it will be the empirical evidence, and not our assumption for the nominal anchor, that determines the unemployment response.

Consider a shock that requires the relative wage of some sector k in region i to fall in order to maintain full employment in that sector-region. The cause could be a negative productivity shock, an increase in productivity in that sector abroad, or a decline in transfers to the region. If δ_k is low enough, or the exchange rate can depreciate (e.g., $\delta_{i,k}$ is low), then nominal wages can adjust downwards in the required magnitude to avoid unemployment. Alternatively, if γ is high enough then again there would be no unemployment, since no downward adjustment is needed in the nominal wage. However, there are combinations of $\delta_{i,k}$ and γ that can lead to unemployment after the shock, although there would then be a decline in unemployment as the DNWR and the anchor allow for adjustment year after year.

2.5 Equilibrium

Following CDP, we can think of the full equilibrium of our model in terms of a temporary equilibrium and a sequential equilibrium. In our environment with

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DNWR, given last period's nominal world GDP ($\sum_{i=1}^{I} \sum_{s=1}^{S} W_{i,s,t-1} L_{i,s,t-1}$), wages $\{W_{i,s,t-1}\}$, and the current period's labor supply $\{\ell_{i,s,t}\}$, a temporary equilibrium at time t is a set of nominal wages $\{W_{i,s,t}\}$ and employment levels $\{L_{i,s,t}\}$ such that equations (1)-(5) and (12)-(15) hold. Without DNWR then $L_{i,s,t} = \ell_{i,s,t}$ for all i, s, and (relative) wages would be determined by equations (1)-(5), as discussed at the end of Section 2.1, with equations (12)-(15) just serving to pin down nominal wages. DNWR implies that labor demand and supply may not be equalized and so we need the full set of equations in (1)-(5) and (12)-(15) to pin down all variables.

In turn, given starting world nominal GDP ($\sum_{i=1}^{I} \sum_{s=1}^{S} W_{i,s,0} L_{i,s,0}$), labor supply $\{\ell_{i,s,0}\}$, and wages $\{W_{i,s,0}\}$, a sequential equilibrium is a sequence $\{\omega_{i,s,t}, V_{i,s,t}, \mu_{ji,sk|i,t}, \mu_{ji,s\#,t}, \ell_{i,s,t}, W_{i,s,t}, L_{i,s,t},\}_{t=1}^{\infty}$ such that: (i) at every period t $\{W_{i,s,t}, L_{i,s,t}\}$ constitute a temporary equilibrium given $\sum_{i=1}^{I} \sum_{s=1}^{S} W_{i,s,t-1} L_{i,s,t-1}$, $\{W_{i,s,t-1}\}$, and $\{\ell_{i,s,t}\}$, and (ii) $\{\omega_{i,s,t}, V_{i,s,t}, \mu_{ji,sk|i,t}, \mu_{ji,s\#,t}, \ell_{i,s,t}\}_{t=1}^{\infty}$ satisfy equations (6)-(11).

2.6 Dynamic Hat Algebra

Our goal is to use a calibrated version of the model to compute the employment and welfare effects of a trade shock. We do this using data for U.S. states as well as other countries, but without needing to calibrate technology levels and iceberg trade costs along the transition and without requiring data on nominal wages per efficiency unit of labor. We follow the exact hat algebra methodology of Dekle et al. (2007) and its extension to dynamic settings proposed by CDP. Consequently, our counterfactual exercises only require data on revenues $R_{i,s,t}$, value added $Y_{i,b,t} \equiv W_{i,b,t} L_{i,b,t}$, trade deficits $D_{i,t}$, mobility matrices $\mu_{ji,sk|i,t}$ and $\mu_{ji,s\#,t}$, labor supply levels $\ell_{j,s,t}$, and trade shares $\lambda_{ij,s,t}$ in period zero ($t = t_0$), whatever shocks we are interested in, and the model's parameters, namely $\delta_{i,k}, \gamma, \kappa, \nu, \{\sigma_s\}, \{\alpha_{j,s}\}, \{\phi_{i,s}\}$, and $\{\phi_{i,sk}\}$.

We use $\dot{x}_t$ to denote x_t / x_{t-1} for any variable x. In Appendix A.3 we describe how to express the equilibrium system in dots and only leave it in terms of observable data in period zero. We assume that the economy starts from a point where

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every region had full employment.[7] In Appendix A.4 we describe the algorithm that we use to solve the equilibrium system in dots.

We are interested in obtaining the effects of the China shock as it is introduced in an economy that did not previously expect this shock. In order to do this, we use $\hat{x}_t$ to denote the ratio between a relative time difference in the counterfactual economy ($\dot{x}'_t$) and a relative time difference in the baseline economy ($\dot{x}_t$), i.e. $\hat{x}_t = \dot{x}'_t / \dot{x}_t$ for any variable x. Then we compare a counterfactual economy where the knowledge of the China shock is unexpectedly introduced in the year 2001 (and agents have perfect foresight about the path of the shock from then on), with a baseline economy where the China shock does not occur. In Appendix A.5 we describe how to express the equilibrium system in hats and only leave it in terms of observable data in period zero or data coming from the baseline economy. Additionally, Appendix A.6 describes the algorithm used to solve the equilibrium system in hats.

Our general equilibrium model also allows us to compute the welfare effects of the shock. Using the utility framework described in Section 2.2, we can express the welfare change in sector s in region j due to the China shock as

$$\ln\left(\zeta_{j,s}\right) = \sum_{t=1}^{\infty} \beta^t \ln\left(\frac{\hat{\omega}_{j,s,t}}{\left(\hat{\mu}_{jj,ss|j,t}\right)^{\nu}\left(\hat{\mu}_{jj,s\#,t}\right)^{\kappa}}\right).$$

This expression corresponds to the permanent equivalent variation in real income for workers originally employed in region j in sector s, so that $V'_{j,s,0} = V_{j,s,0} + \frac{1}{1-\beta}\ln(\zeta_{j,s})$.[8] For intuition, consider a shock that decreases the expected real wage

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[7] Assuming that the U.S. had full employment in the year 2000 is not problematic, since that year was the peak of a business cycle, with an unemployment rate of just 4%. This is the lowest unemployment rate observed in the U.S. in the last 40 years (except for the period from 2018 onward). The existence of 4% unemployment is consistent with our assumption of "full employment" because the concept of unemployment in our model is that of "cyclical" unemployment, i.e., the unemployment in excess of the natural rate of unemployment.

[8] See Appendix A.8 for details of this derivation. Trade imbalances supported by international transfers imply that consumption (or real expenditure) may differ from real income. We follow Costinot and Rodriguez-Clare (2014) and measure welfare by real income rather than consumption to avoid attributing a positive direct gain to the foreign transfer. Taking into consideration the direct gain

in sector j, s, $\hat{\omega}_{j,s,t} < 1$. Without mobility we would simply have

$$\ln \left(\zeta_{j,s} \right) = \sum_{t=1}^{\infty} \beta^t \ln \left(\hat{\omega}_{j,s,t} \right),$$

which is the present discounted value of the changes in real wage. Mobility allows workers in the sector to move to other sectors and regions, as captured by $\hat{\mu}_{jj,ss|j,t} < 1$ and $\hat{\mu}_{jj,s\#,t} < 1$. Finally, given those mobility measures, higher variability parameters ν and κ imply larger gains from moving out of the affected sector.

The welfare expression above is given at the sector-region level. However, in some parts of the paper we will refer to welfare measures at the region level. Such regional welfare measures are computed as weighted averages of the corresponding sector-region welfare levels, with weights given by the shares of population that participate in a given sector in that region in the initial year.

3 Data

We provide a brief description of our data construction procedure here and relegate additional details to Appendix B. We use trade, production, and employment data for 50 U.S. states, 36 additional countries, and an aggregate rest of the world region, for a total of 87 regions from 2000 to 2007. We consider 14 market sectors: 12 manufacturing sectors, one service sector, and one agricultural sector. All sectors are classified according to the North American Industry Classification System (NAICS).

Labor, consumption, and input shares. For each region j and each sector k, our model requires data to compute the share of labor in production $\phi_{j,k}$, the share of intermediates from other sectors $\phi_{j,sk}$ $\forall s$, and the aggregate consumption shares $\alpha_{j,k}$. We use data from the BEA (for U.S. states) and from WIOD to compute the share of value-added in gross output of region j, which in our model is equivalent to $\phi_{j,k}$. We also scale the relative importance of each U.S. state in the total value added of the

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would risk treating deficits as a gift and assuming away their future costs.

U.S. so that the sum of value added across states matches the aggregate value-added of the U.S. according to WIOD. We compute $\phi_{j,sk}$ as the share of purchases of sector k coming from sector s (the input-output coefficient) using WIOD data.[9]

Bilateral trade flows. Our model also requires data on bilateral trade flows between all region pairs in our sample for each sector in order to compute deficits, revenue, and trade shares for the year 2000. We also require the bilateral trade flows (combined with input-output coefficients) to infer the $\alpha_{j,k}$'s. We construct the bilateral trade flow dataset in four steps, which we summarize here while referring the reader to Appendix B.2 for additional details.

In the first step, we take sector-level bilateral trade between countries directly from WIOD. In the second step, we follow CDP to calculate the bilateral trade flows in manufacturing among U.S. states by combining WIOD and the Commodity Flow Survey (CFS). We first compute the bilateral expenditure shares across regions and sectors from the CFS, and then use a proportionality rule to assign the total U.S. domestic sales from WIOD according to those bilateral shares. This step ensures that the trade flows from the bilateral trade matrix for the 50 U.S. states match the total U.S. internal sales from WIOD in each sector.

In the third step, we use the Import and Export Merchandise Trade Statistics, a dataset compiled by the U.S. Census Bureau, to compute – for manufacturing and agriculture – the sector-level bilateral trade flows between each U.S. state and each of the other countries in our sample. The U.S. Census data on exports at the sector-state-country level starts in 2002, and the data on imports starts in 2008. We use these starting years to project our bilateral trade matrix for previous years until 2000 by assuming that the importance of each state in the total exports (imports) to (from) other countries in each sector remains constant at the 2002 (2008) levels. We use a proportionality rule for the bilateral trade flows between the U.S. states and other countries to match the aggregate trade flows between the U.S. and other countries from WIOD in each sector.

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[9]We assume a common input-output matrix for all U.S. states due to data limitations.

In the fourth and last step, we combine data for region-level production and expenditure in services from the Regional Economic Accounts of BEA, WIOD data, and data on bilateral distances to construct the trade flows in services among all regions consistent with a gravity structure. We follow a similar gravity approach for the case of trade flows in agriculture using data from the Agricultural Census, the National Marine Fisheries Service Census, and WIOD. By construction, the bilateral trade flows in services and agriculture match the aggregates of trade in services and agriculture between all countries (including the U.S.) and the total production of U.S. services and agriculture consumed by the U.S.

Labor flows across sectors and regions. For the U.S. states, we construct the matrix of migration flows $\mu_{ji,sk,t}$ for $t = 2000$ combining data for intersectoral mobility from the Current Population Survey (CPS) with data for interstate mobility from the American Community Survey (ACS). We follow CDP by assuming that interstate movements (j to i) across sectors follow the same pattern as the intrastate moves in the destination state i across sectors. We then apply a proportionality rule to the flows from the CPS so that the total movements between states across sectors add up to the total movements in the ACS. An important limitation of measuring worker mobility across region-sectors using the self-reported information from the CPS and ACS is the well-known problem of artificially large amounts of mobility due to the prevalence of misclassification errors (Murphy and Topel, 1987; Kambourov and Manovskii, 2013; Dvorkin, 2021). To avoid this issue, we smoothed the mobility flows in shares such that the set of migration flows in our first period implies a steady state in the U.S. in that period.[10]

Finally, we assume away migration flows between countries. Thus, there is no need to compute labor flows for that case. In addition, for countries outside of the U.S. we assume that there are no costs of moving across sectors in the single region

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[10]The change in the flows implied by this procedure is extremely small. In particular, the correlation between the original flows and the smoothed ones is 99.69%. We provide more details about our smoothing algorithm at the end of Appendix B.3.

of each country (due to data limitations). Given this assumption, one can infer the matrix of migration flows from the labor distribution in 1999 and 2000. We provide more details in Appendix B.3.

4   Calibration

In this section we describe how we calibrate our main parameters (δ, ν, κ), as well as the China shock. We focus on the effect of the China shock as captured by a set of productivity shocks in China given by $\{\hat{A}_{China,s,t}\}$ that apply only to the 12 manufacturing sectors. Inspired by ADH, and following CDP and Galle et al. (2020), we calibrate these shocks to match the changes in U.S. imports from China predicted from the changes in imports from China to other high-income countries.[11]

We decompose the total productivity shock in sector s and time t into a component coming from a sector-level productivity increase that is constant from 2000 to 2007 and a component coming from a productivity increase over time that is constant across sectors, i.e. $\hat{A}_{China,s,t} = \hat{A}^1_{China,t}\hat{A}^2_{China,s}$. This means we have to estimate 19 parameters. We choose $\{\hat{A}^1_{China,t}\}$ and $\{\hat{A}^2_{China,s}\}$ to match two targets. The first target is the vector of annual predicted changes in U.S. imports from China in all manufacturing sectors combined, obtained from the following regression:

$$\Delta X_{C,US,t} = a + b_1 \Delta X_{C,OC,t} + \varepsilon_t,$$

where $\Delta X_{C,US,t}$ is the change in U.S. imports from China between year $t-1$ and year t in all manufacturing sectors, $\Delta X_{C,OC,t}$ is the change in imports from China by the other high-income countries between year $t-1$ and year t in all manufacturing sectors, and b_1 is the coefficient of interest. We denote the predicted values from this regression by $\{\widehat{\Delta X_{C,US,t}}\}$.

The second target is the vector of predicted changes in U.S. imports from China

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[11]We use the subset of ADH countries that are also present in the 2013 version of the WIOD, namely Australia, Germany, Denmark, Spain, Finland, and Japan. New Zealand and Switzerland are included in the "other high-income countries" category of ADH but are not included in WIOD.

between 2000 and 2007 across sectors, obtained from the following regression

$$\Delta X^{2007-2000}_{C,US,s} \;=\; b_2 \Delta X^{2007-2000}_{C,OC,s} + \varepsilon_s,$$

where $\Delta X^{2007-2000}_{C,US,s}$ is the change in U.S. imports from China between 2000 and 2007 in sector s, $\Delta X^{2007-2000}_{C,OC,s}$ is the change in imports from China by the other high-income countries between 2000 and 2007 in sector s, and b_2 is the coefficient of interest. The predicted values from this regression are denoted $\{\widehat{\Delta X^{2007-2000}_{C,US,s}}\}$.[12] We choose $\{\hat{A}^1_{China,t}\}$ and $\{\hat{A}^2_{China,s}\}$ such that the total productivity changes in China $\{\hat{A}_{China,s,t}\}$ deliver changes in imports in our model that simultaneously match the 7 values of $\{\widehat{\Delta X_{C,US,t}}\}$ and the 12 values of $\{\widehat{\Delta X^{2007-2000}_{C,US,s}}\}$.[13]

The calibration of the key model parameters (described below) is based on matching moments that capture the relative effect of the China shock on labor force participation, unemployment, and population. These moments come from regressions of changes in these variables across regions differentially exposed to the China shock, as captured by an exposure measure that follows the one proposed by ADH:

$$\text{Exposure}_i \equiv \sum_{s=1}^{S} \frac{L_{i,s,2000}}{L_{i,2000}} \frac{\widehat{\Delta X^{2007-2000}_{C,US,s}}}{R_{US,s,2000}}, \tag{16}$$

where $R_{US,s,2000}$ is total U.S. production in sector s in the year 2000, $L_{i,s,2000}$ is the employment of region i in sector s in year 2000, $L_{i,2000} \equiv \sum_s L_{i,s,2000}$, and $\widehat{\Delta X^{2007-2000}_{C,US,s}}$ is the predicted 2000-2007 change in U.S. imports in sector s from China as in ADH and explained above. Besides the calibration, we will also use this exposure measure to present the results of the model for non-targeted variables such as manufacturing and non-manufacturing employment, as well as for welfare, so that we can see how these predictions vary across states differentially exposed to the China shock.

For our baseline specification, we assume that all countries outside the U.S.

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[12]We exclude the constant in this regression because it can lead to negative predicted imports from China, which is impossible. While the regression only has 12 observations, it has an R^2 of 0.99.

[13]The multiplicative nature of $\hat{A}_{China,s,t} = \hat{A}^1_{China,t}\hat{A}^2_{China,s}$, implies that their level is not identified. We use the normalization $\sum_{s=1}^{S} \hat{A}^2_{China,s} = 1$. For more details see Appendix A.9.

have a flexible exchange rate that adjusts in such a way that they retain full employment, implying that $\delta_i = 0$ for all $i > M$. We do not calibrate γ and δ separately – since only their relative value matters – and instead assume that γ is 1, so that the burden of adjustment falls entirely on δ, as in Schmitt-Grohe and Uribe (2016).

We choose δ, ν, and κ simultaneously to match three empirical estimates obtained by ADH. The first one is that a \$1,000 per worker increase in import exposure to China increases the unemployment to population rate by 0.22 percentage points. The second one is that the same rise in import exposure increases the not-in-labor-force (NILF) to population rate by 0.55 percentage points. The third one is that the same rise in import exposure leads to a 0.05 percentage points decrease in population.[14] In broad terms, one could say that δ mostly governs the amount of unemployment generated by exposure to China for given ν and κ, ν mostly governs the amount of change in labor force participation generated by exposure to China for given δ and κ, and κ mostly governs the change in population generated by exposure to China for given δ and ν.

The calibration results in values of $\delta = 0.98$, $\nu = 0.55$, and $\kappa = 12.3$. The value of $\delta = 0.98$ implies that nominal wages can fall up to 2% annually, and falls in the range advocated by Schmitt-Grohe and Uribe (2016) who obtain an annual δ of 0.984 (after "normalizing" γ to one as we do).[15] The quantitative implications of this calibration would be the same if we had instead set $\gamma = 1.02$ and $\delta = 1$, which is consistent with the U.S. typically having 2% annual inflation and nominal wage cuts being relatively infrequent.

Our estimates for ν and κ compare to a value of $\nu = \kappa = 2.02$ in CDP.[16] Im-

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[14]These results correspond to the ones in Panel B of Table 5 and Panel C of Table 4 in ADH. Following ADH, we also take the 2006-2008 averages of unemployment and labor force participation in our estimation. Some recent papers such as Borusyak et al. (2021) have revised some of the results in ADH. We focus on ADH's results as targets since ADH is the most influential paper in this literature. That said, our quantitative analysis can accommodate any other target.

[15]Using a set of countries that excludes the U.S., Schmitt-Grohe and Uribe (2016) obtain a quarterly value of $\delta = 0.996$. This value corresponds to an annual δ of 0.984. However, they end up using a δ of 0.96 in their paper as a conservative estimate.

[16]Our model has an annual frequency, so we compare our elasticity estimates with the appropriately

posing $\nu = \kappa = 2.02$ as in CDP would lead to effects on labor force participation that are too small and effects on population that are too high relative to those estimated by ADH. Alternatively, we could constrain our model to satisfy $\nu = \kappa$, but without setting this single elasticity to the CDP value of 2.02. Calibrating ν (or κ) and δ to match the unemployment and participation targets from ADH would lead to $\nu = \kappa = 0.56$, similar to our baseline estimate of $\nu = 0.55$, but very far from our other baseline estimate $\kappa = 12.3$ (we discuss this in more detail in Section 6.1). This would lead to a population response to Chinese exposure that is a full order of magnitude greater than the population response in ADH (i.e., -0.52 instead of -0.05). This shows the importance of our nested structure with different elasticities of moving across regions and sectors.

Finally, we assume that the trade elasticity σ_s is constant across sectors and takes the value of 6, consistent with the trade literature (e.g. Costinot and Rodriguez-Clare, 2014). We also use a discount factor β of 0.95 and explore the sensitivity with respect to this parameter in Section 6.5.

5   Effects of the China Shock in the Baseline Model

5.1   Comparison of Cross-Sectional results with ADH

We now use the calibrated model to study the effects of the China shock across U.S. states. We first obtain the changes in real wages, employment, unemployment, labor force participation, and population for all the 87 regions included in our model. Then we run OLS regressions across U.S. states of the changes in the variables of interest on the exposure measure in equation (16). We present the resulting coefficients in Table 1, along with the analogous coefficients from ADH.

Column (1) of Table 1 reports the results of ADH.[17] Rows one, two and five

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annualized version of CDP's single elasticity.

[17]Specifically, we use the ADH estimates presented in their panel B of Table 5, panel B of Table 7, and panel C of Table 4.

Table 1: Employment, population, wage, and welfare effects of exposure to China across U.S. regions and associated parameters generating them

	ADH (1)	Baseline (2)	NM (3)	$\nu = \kappa$ (4)	DNWRM (5)
Change in Population Shares					
Unemployment (targeted)	0.221**	0.221	0.221	0.221	0.221
NILF (targeted)	0.553**	0.553	0.553	0.553	0.553
Mfg Employment	-0.596**	-0.331	-0.337	-0.340	-0.543
Non-mfg Employment	-0.178	-0.442	-0.437	-0.434	-0.230
Percentage Changes					
Population (targeted)	-0.050	-0.050	-0.000	-0.521	-0.050
Mfg Wage	0.150	-0.214	-0.182	-0.049	0.152
Non-mfg Wage	-0.761**	-0.689	-0.717	-0.623	-1.065
Welfare					
Welfare vs exposure		-0.053	-0.079	-0.044	-0.047
Mean welfare change		0.229	0.235	0.225	0.197
Mean welf. change no DNWR		0.310	0.313	0.311	0.298
Parameters					
ν		0.551	0.594	0.562	0.496
κ		12.30		0.562	11.21
δ		0.980	0.980	0.981	0.987

Notes: The changes for the first four coefficients are measured from 2000 to an average of 2006-2008, multiplied by 10/7 to turn into decadal changes. Wages are simply measured in percentage change (between 2000 and 2006-2008), still turned into decadal changes. Welfare is obtained as described at the end of Section 2.6. ν is the parameter that governs substitution between sectors, κ is the one that governs substitution between regions, and δ governs the DNWR. Column 1 reproduces the ADH results from their Tables 4 (panel C, first column), 5 (panel B, first row) and 7 (Panel B, columns 1 and 4), stars denote significance, one star for 5%, and two for 1%. Column 2 gives the results in our baseline specification. Column 3 eliminates migration across U.S. states. Column 4 imposes $\nu = \kappa$. Column 5 imposes the DNWR only in manufacturing. In column (5) κ is not reported, because, without migration, this parameter is irrelevant.

correspond to our targeted regression coefficients. Column (2) of Table 1 presents the results of our baseline model. We focus on the results related to employment and wages in this section, and discuss the welfare effects in Section 5.3.[18] Columns (3) and (4) present results for versions of the model that eliminate migration across U.S. states and that impose the DNWR only in manufacturing. The discussion of these columns will be postponed to Section 6.

Our results in column (2) show that exposure to China measured as in ADH

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[18]We focus on a state-level analysis because this is the level at which one can construct bilateral trade matrices and mobility flows without further assumptions on how the state-level flows are split between different commuting zones. This simplification is a consequence of data availability and not of our model. Moreover, running simple ADH state-level regressions without controls yields similar response-to-exposure coefficients that do not change our conclusions substantively.

leads to a fall in manufacturing and non-manufacturing employment of 0.33 percentage points and 0.44 percentage points, respectively. These are moments that we did not target in our calibration.[19] Our results from the baseline model understate the fall in manufacturing employment and overstate the fall in non-manufacturing employment, but are in a reasonable ballpark.[20]

Regarding the effect of exposure to China on wages, our baseline model indicates that manufacturing wages fall by 21 basis points while the non-manufacturing wage falls by 69 basis points. These results preserve the ordering of the empirical evidence in ADH, with the response of the manufacturing wage to exposure to China being small and the non-manufacturing wage falling more than the manufacturing wage in response to more exposure to the China shock. Since the wage moments are completely untargeted in our exercise, these results are reassuring.[21]

Additionally, our results imply a dispersion in the impacts of the China shock on the employment rate and income per capita across U.S. states that is comparable to the one predicted by the ADH specification in the 2000-2007 data. To assess this, we first compute the predicted variation in the employment to population ratio and income per capita by running ADH's main regression specification on their data at the commuting zone level.[22] We then compute the population-weighted average of these predicted values across all commuting zones within the same state. Finally,

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[19]The only restriction is that the coefficients have to add up to 0.77 since this is the sum of the targeted unemployment and NILF coefficients in ADH.

[20]As we discuss in Section 5, the model can match these two ADH moments better if we assume that the DNWR only applies to the manufacturing sector.

[21]Autor et al. (2014) use worker-level data to show that the wage effects were more prominent for individuals with low initial wages, low initial tenure, and low attachment to the labor force. Therefore, selection is at least part of the explanation for the small relative declines in local average wages documented by ADH. It is important to note that we do not obtain our value of δ from changes in wages. Instead, our DNWR parameter is identified jointly with the mobility elasticities across sectors and regions from changes in unemployment, nonemployment, and migration flows. This alleviates the concern of selection driving wage rigidity and the welfare effects in our calibrated model.

[22]For the variation in employment rate we focus on the change in the ratio of total employment to working age population using data from ADH. For the variation in income per capita we follow the left hand side of equation 8 in Autor et al. (2021) to compute the deviation in changes in income per capita of each commuting zone relative to the national weighted average. We use the total salary income per adult from column 2 of Table 9 in ADH as the measure of income per capita.

we compare these empirical predictions to their model-implied counterparts.

The standard deviation (s.d.) of the changes in the state-level employment to population ratio predicted by the model is 1.35, which is similar to the s.d. of 1.18 implied by the empirical estimates. In turn, the s.d. of the changes in income per capita predicted by the model is 2.5, while the one associated with the empirical estimates is 1.9. These results stand in contrast to previous quantitative models such as CDP and Galle et al. (2020), which imply too little spatial heterogeneity in the employment and income effects relative to ADH (as shown by Adao et al., 2020; Autor et al., 2021). There are two reasons why our model generates more dispersion in employment and income effects. First, because of DNWR, our model leads to much larger declines in employment in the most exposed regions, both directly through higher unemployment, and indirectly through discouraging labor participation. Second, by allowing for a difference between the elasticity of moving across sectors and that of moving across regions and calibrating these elasticities to ADH moments for the effect of the China shock on employment and population, we arrive at lower mobility across states and a higher labor supply elasticity than CDP.

5.2 Aggregate Employment Effects

We now use our general equilibrium model to go beyond cross-sectional implications and obtain the implied aggregate effects of the China shock on unemployment and other variables. In Figure 1, we plot the aggregate U.S. unemployment generated by the China shock according to our model. This variable increases gradually, reaching 1.4 percent in 2007 and falling back to zero by 2010. Notice that in our model, once shocks are no longer hitting the economy, all excess unemployment generated by the DNWR eventually disappears. This occurs because, since the nominal wage can fall up to 2% per year, the nominal wage (and hence the real wage) eventually reaches the level required to make all unemployment disappear. This is a desirable feature of the model since it is hard to square a permanent ag-
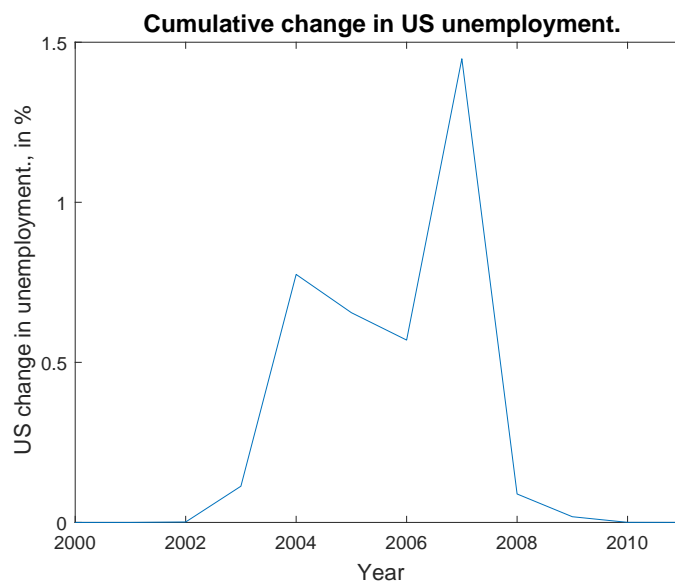
Figure 1: Path of aggregate U.S. unemployment over time between 2000 and 2011 in the baseline specification.

gregate unemployment effect of the China shock with the historically low levels of unemployment observed in the U.S. between 2016 and 2019.

Regarding labor force force participation, there is a sign reversal throughout the transition. On impact, the China shock leads to a temporary decline in participation, stemming from the fact that unemployment discourages participation due to the risk of participating in the labor market but not being able to obtain a job. Aggregate labor force participation falls by up to 25 basis points in 2004. However, when the China shock stops hitting the economy and the nominal wage has room to fully adjust, labor force participation ends up increasing relative to its original level. This increase happens because the China shock is a positive terms-of-trade shock for the U.S., which translates to a higher real wage and an increase in labor supply. By 2010, aggregate labor force participation in the U.S. has already reversed sign and increased roughly 1% relative to its pre-shock value.

The results imply that most states experience both a long-run increase in the real wage and a temporary increase in unemployment. This may seem paradoxical, but it is a natural consequence of a shock that implies both an improvement in the terms

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of trade and a decline in the export price index in a setting with DNWR. To see this more clearly, consider a small open economy and imagine that the price index of its exports falls while the price index of its imports falls even more. Since the terms of trade have improved, the real wage and employment would increase in the absence of nominal frictions. However, the fact that the price index of its exports has fallen requires the nominal wage to decline, and if this decline is higher than $1 - \delta$, there would be temporary unemployment.

We illustrate this mechanism via a simple supply and demand analysis in Figure 2. Both panels in the figure have the nominal wage in the vertical axis and employment in the horizontal axis. The China shock leads to a fall in producer prices, shifting the labor demand down. At the same time, the China shock also leads to a decline in consumer prices, shifting the labor supply to the right. The final result is a fall in the nominal wage from W_0 to W^*, a fall in prices from P_0 to P^* (not illustrated), an increase in the real wage from W_0/P_0 to W^*/P^* (prices fall more than nominal wages), and an increase in the amount of labor supplied from L_0 to L^*.

Panel (b) of Figure 2 shows the adjustment in the presence of DNWR assuming that $\delta^3 W_0 < W^* < \delta^2 W_0$. In the first year, the nominal wage only falls from W_0 to $W_1 \equiv \delta W_0$ and employment falls from L_0 to L_1, as determined by the demand curve. Since the nominal wage does not fully adjust in the first year, the fall in prices is also smaller than in the frictionless case, and hence the labor supply curve only moves from L^S to L_1^S. The gap between the labor supplied at point A and labor demanded L_1 is the level of unemployment. In the second year, nominal wages adjust further down (to $W_2 \equiv \delta W_1 = \delta^2 W_0$), the labor supply curve moves to L_2^S, employment increases from L_1 to L_2, labor supplied moves from point A to point B, and unemployment decreases. In the third year, nominal wages finally adjust fully and there is no more unemployment. Notice that the final equilibrium of the economy is the same with and without DNWR, and it involves higher labor supply, a higher real wage, and no unemployment.
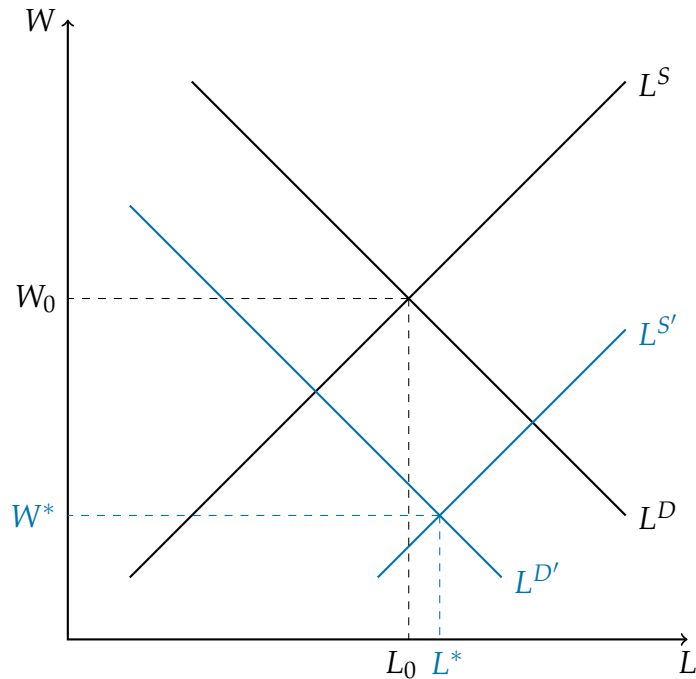
(a) Case without DNWR
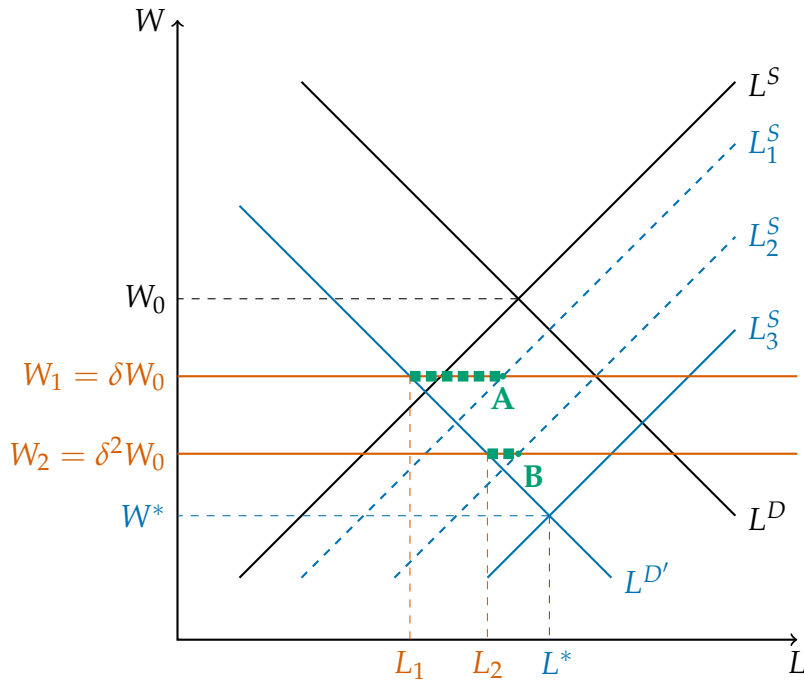
(b) Case with DNWR

Figure 2: Illustration of wage and employment effects, with and without DNWR. The nominal wage is in the vertical axis, hence price movements results in shifts in the labor supply curve. Employment is in the horizontal axis.
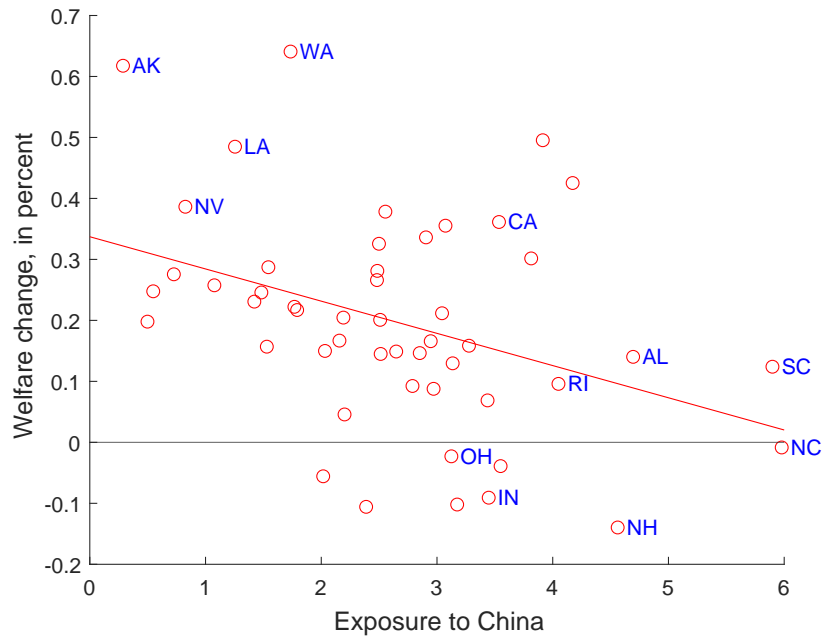
Figure 3: Welfare change vs exposure to China across U.S. states in the baseline specification.

5.3   Welfare Effects

We find that U.S. states more exposed to the China shock experience lower model-implied welfare gains: a \$1,000 per worker increase in exposure to China decreases welfare by around 5.3 basis points (this is the coefficient displayed in Table 1, column 2, row 8). Figure 3 presents a scatter plot of the percentage change in welfare across states against exposure to China, while Figure 4 displays a welfare map across the 50 U.S. states. There are 42 states that gain from the China shock while 8 states experience welfare losses. Of these 8 states, only 1 experiences a worsening of its terms of trade, which implies a lower steady state real wage. The other 7 states that suffer losses actually experience improvements of their terms of trade, but these are dominated by temporary increases in unemployment due to the DNWR.

When we consider the U.S. as a whole, and measure welfare by the population-weighted average across U.S. states, we see that the China shock leads to an increase in welfare of roughly 23 basis points. This is true even though we match the un-
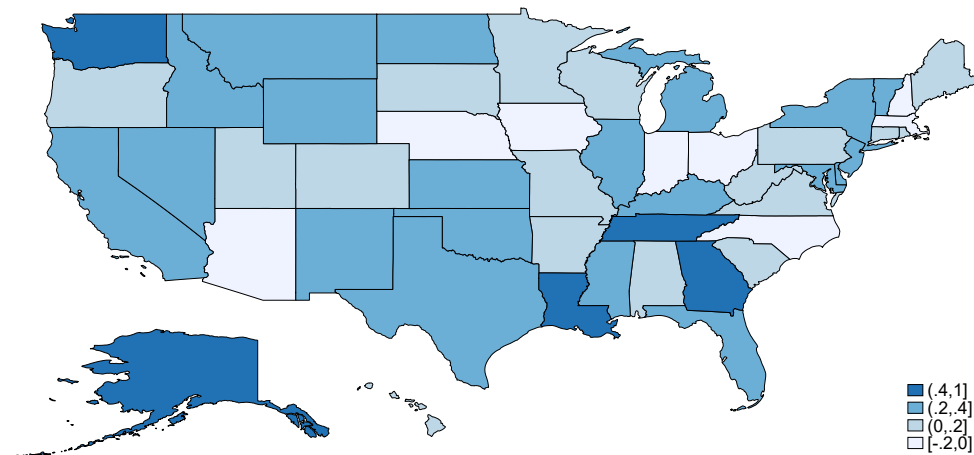
Figure 4: Welfare change across U.S. states in the baseline specification.

employment effects captured by ADH, which have sometimes been interpreted as implying that the China shock had adverse overall welfare effects. We can also compare the results of our baseline model against those from a model without nominal rigidity (i.e., with $\delta = 0$). In this alternative version of the model without DNWR and without recalibrating other parameters (such as ν or κ), all but 1 state experience
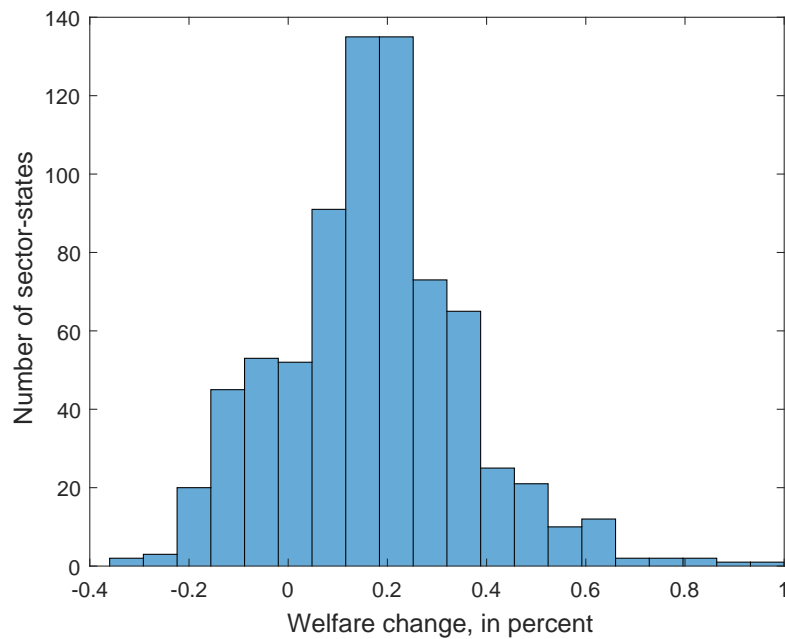
Figure 5: Histogram of welfare changes across different sector-states of the U.S. in the baseline specification.

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welfare gains from the China shock, and the U.S. as a whole experiences gains of 31 basis points.[23] Comparing these two models, we see that the temporary increase in unemployment due to DNWR reduces the aggregate gains from the China shock by around one fourth.

So far, in this section, we have discussed welfare at the state level. However, as mentioned in Section 2.6, our measure of welfare changes is at the sector-region level, which would capture the welfare changes experienced by a worker who starts the period at sector s of region i. Figure 5 presents a histogram of welfare changes for sector-states of the U.S. There is higher variation in this disaggregated measure, with welfare effects ranging from -40 to 100 basis points, compared to the measure at the state level, where the welfare effects range only from -13 to 64 basis points.

6   Alternative Specifications

In this section, we discuss the robustness of the baseline results. First, we discuss how the results change with different assumptions regarding migration across states. At one extreme we shut down migration between U.S. states, while at the other extreme we force the elasticity of moving across states to be the same as the elasticity of moving across sectors as in CDP (implying much higher migration flows). Second, we examine the consequences of introducing the DNWR only in manufacturing (and not in services or agriculture). Third, we introduce some of the increases in trade surpluses that happened in China between 2000 and 2007 as an integral part of the China shock. Fourth, we explore fixed exchange rate regimes for other countries. Finally, we examine how our welfare results change if we assume different discount factors. For each of these specifications we recalibrate parameters $\{\delta, \nu, \kappa\}$ as well as the China shock, $\{\hat{A}_{China,s,t}\}$.

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[23]This is comparable to the gains obtained in recent papers studying the same setting (e.g., CDP, Galle et al., 2020).

6.1 Different Migration Assumptions

Given the potential importance of migration for determining the dispersion of welfare effects across U.S. states, we now study two polar options for migration: no migration and $\nu = \kappa$, which leads to a lot more migration across states in response to the China shock.

In the first case workers only have the option of moving between sectors within a state. We start from a mobility matrix that matches intra-state migration flows from the CPS data, which has good coverage about employment status and industry of each respondent who stayed in the same state between waves of the survey. We then compute the impacts of the China shock in the same way as in the baseline model except for the fact that migration flows across states have been shut down. The results of this extension are described in column (3) of Table 1. Notice that in this case κ is no longer relevant, so we leave this field blank in the table. And evidently, we no longer match the response of population to exposure to China in ADH.

The calibrated ν increases relative to our baseline exercise, but the calibrated δ remains similar. In addition, many (non-targeted) moments, such as the changes in manufacturing and non-manufacturing employment and wages, as well as our inferred welfare changes (with and without DNWR) also stay relatively unchanged. The biggest change occurs in the coefficient measuring the response of welfare to exposure, which decreases from -0.053 to -0.079. The direction of this result is intuitive: if agents cannot leave states that are severely hit by the China shock, the gains and loses from the shock will be more concentrated. The approximately 50% increase in the coefficient measuring the response of welfare to exposure is still surprising, given that we are matching a relatively small effect of exposure on population (i.e. the -0.05 response of population to exposure target that we obtain from ADH). Furthermore, if we look simply at the variance of the welfare change, we get an even bigger discrepancy between the two exercises, as the variance of the welfare change more than doubles when we shut down inter-state migration. This result indicates

that even a seemingly small population response to a trade shock can already decrease the variance in the welfare effects of the shock substantially.

In our second alternative specification, we impose that $\nu = \kappa$, which is necessarily true in the framework of CDP. As in the previous extension, we do not target the population response in ADH, and only target the unemployment and participation responses to exposure. The results are described in column (4) of Table 1. We find that $\nu = \kappa = 0.562$, similar to our baseline estimate of ν, but very different from our baseline estimate of κ. In the restricted model, κ is much lower than in the baseline, leading to a population response to the China shock that is an order of magnitude greater than the one in the baseline model (-0.521 vs -0.050). Other results in column (4), like the calibrated δ and employment changes, are more similar to those from the baseline model. Wage changes are both closer to zero but in a similar ballpark as in the baseline model. In contrast to the case with no migration across states, here the response of welfare to exposure becomes less negative because agents can more easily flow out of the more negatively affected states.

6.2   DNWR Only in Manufacturing

While our baseline specification approximates the non-targeted moments in ADH – the changes in employment and wages in manufacturing and non-manufacturing – relatively well, there is some room for improvement. In particular, one could think of letting the DNWR have a different δ in manufacturing compared to non-manufacturing. In this section, in the interest of parsimony, we investigate the results of introducing the DNWR only in manufacturing (i.e., $\delta_{Mfg} > 0$ and $\delta_{Non-Mfg} = 0$), implying no unemployment in the agriculture and service sectors.[24] We discuss below how this experiment allows us to better match some of the non-targeted moments from ADH and then comment on why this extension could plau-

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[24] An alternative would be to calibrate two different δ's and match some of the currently-non-targeted moments from ADH. The exercise in this section allows us to almost match the non-targeted ADH moments while remaining parsimonious.

sibly capture important real-world features in a reduced-form way.

Column (5) of Table 1 presents the results. Parameters ν and κ decrease a little relative to the baseline calibration, while the required δ increases from 0.98 to 0.987. A higher value of δ is intuitive. Since the DNWR only applies to the manufacturing sector, it needs to bind more strongly in order to match the required response of unemployment to exposure.

The responses of the employment shares in manufacturing and non-manufacturing are very close to the ones in ADH. The wage responses are also similar to ADH, with the manufacturing wage exhibiting a coefficient that is very close to zero and the non-manufacturing wage responding strongly to exposure. Overall, the results in column (5) of Table 1 are very close to the empirical results obtained by ADH and hence this model provides a good benchmark to understand the effects of trade shocks on unemployment, labor force participation, and wages. Using this version of the model, we find that the U.S. experiences an average welfare gain of 20 basis points (weighing each state by its population), which is around two-thirds of what the model without DNWR finds.

Given the improved performance of the version of the model in which there is DNWR only in the manufacturing sector, it is worth discussing whether this is a realistic assumption. There are a few papers documenting a substantial degree of heterogeneity in wage rigidity across sectors and occupations (Radowski and Bonin, 2010; Du Caju et al., 2012). More recently and for the U.S., Hazell and Taska (2019) explore this heterogeneity using a dataset containing wages for new vacancies with specific job descriptions for each establishment. Their paper finds that production workers face a higher degree of DNWR than workers in non-production occupations. If production workers are a higher share of total labor in manufacturing compared to non-manufacturing, this could explain why the DNWR could bind more strongly in manufacturing. Another explanatory element could be the presence of stronger unionization in manufacturing relative to services.[25]

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[25] Apart from the aforementioned evidence that the DNWR could be more binding in manufacturing,

6.3 Changing Deficits as Part of the China Shock

In the previous versions of our model, we kept the deficits for all regions constant in terms of world GDP. However, it is plausible that part of the increase in Chinese surpluses that occurred between 2000 and 2007 could be part of the China shock. In this section, we explore what happens if, besides a Chinese productivity increase, the China shock also includes an increase in the Chinese surplus, which has to be offset by a rise in the deficits of other regions.

We perform two separate exercises where we explore different assumptions regarding how we treat the increase in the Chinese surplus.[26] In the first exercise, we only incorporate the increase in Chinese surpluses that occurred because China's GDP increased relative to world GDP (i.e., we keep the surplus to GDP ratio constant in China), while in the second exercise we incorporate the changes in the Chinese surplus observed in the data. An increase in the surplus of China implies an equal change in the combined deficits of the other countries, since total deficits must always sum to zero. We keep the deficits of all other countries besides the U.S. unchanged in terms of world GDP in both exercises, thus having the U.S. deficit offset the whole increase in China's surplus. We distribute this increased deficit across U.S. states according to their shares in U.S. GDP.

The results for these two exercises are shown in Table 2, which has the same structure as Table 1 (the first column repeats column (2) in Table 1, to facilitate com-

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we suggest a broader reading of this experiment. We interpret the model with DNWR only in manufacturing as a simplified way to capture forces pushing for reallocation of labor away from manufacturing before the impact of the China shock. Even with the same DNWR in manufacturing and non-manufacturing, these forces would make the DNWR in manufacturing more likely to be binding once the China shock hits the economy.

[26]These two exercises keep the assumption of exogenous deficits and focus on how changes in deficits affect our baseline results. Dix-Carneiro et al. (2020) present a model with endogenous deficits and analyze the counterfactual where shocks to China between 2000 and 2014 are equal to the average of shocks across all other economies. They find that the behavior of the U.S. trade deficit is barely affected, implying that little of the change in the U.S. deficit is explained by the exceptional productivity shocks that China experienced over this period. Note that in Dix-Carneiro et al. (2020) there is a single region in the U.S., which makes dealing with endogenous deficits more computationally plausible than in our model with 50 different U.S. states.

Table 2: Employment, wage, and welfare effects of exposure to China across U.S. regions and associated parameters generating them (continued)

	Baseline (2)	Def. Low (6)	Def. High (7)	Fixed ER (8)
Change in Population Shares				
Unemployment (targeted)	0.221	0.221	0.221	0.221
NILF (targeted)	0.553	0.553	0.553	0.553
Mfg Employment	-0.331	-0.340	-0.400	-0.299
Non-mfg Employment	-0.442	-0.434	-0.374	-0.475
Percentage Changes				
Population (targeted)	-0.050	-0.050	-0.050	-0.050
Mfg Wage	-0.214	-0.180	0.015	-0.165
Non-mfg Wage	-0.689	-0.661	-0.541	-0.574
Welfare				
Welfare vs exposure	-0.053	-0.052	-0.051	-0.048
Mean welfare change	0.229	0.232	0.221	0.185
Mean welf. change no DNWR	0.310	0.323	0.386	0.284
Parameters				
ν	0.551	0.548	0.571	0.521
κ	12.30	11.87	10.38	10.37
δ	0.980	0.981	0.986	0.987

Notes: All definitions are the same as the ones in Table 1. Column 2, which contains the results from the baseline specification, repeats Column 2 from Table 1 to facilitate comparison. Column 6 gives the results from our model when we introduce a modest increase in Chinese surplus as part of the China shock, while in column 7 this increase is larger. Column 8 gives the results when other countries have fixed exchange rates relative to the U.S.

parisons of the new results with those in the baseline). Column (6) presents the results of the first exercise, while column (7) shows the results from the second exercise. Column (6) is labeled "Def. Low", because the U.S. deficit increases moderately (i.e., around 10% in total over the 2000-2007 period), while column (7) is labeled "Def. High", because the U.S. deficit rises substantially (i.e., around 57% in total over the 2000-2007 period).

Most of the results in column (6) are similar to those of our baseline calibration, although manufacturing employment falls slightly more, wages fall marginally less, ν and κ both decrease slightly and δ increases a bit. These changes from the baseline model to column (6) are amplified in column (7), because the U.S. deficit grows more in response to the China shock, but the overall pattern is the same.

The most surprising result in our deficit exercises regards the mean welfare

change, which increases as we move from column (2) to (6) but then decreases from column (6) to (7).[27] In fact, the mean welfare change is smaller in column (7) than in our baseline model (column 2), which could seem counter-intuitive, since the U.S. is given a positive transfer. To understand this result, notice that increasing U.S. deficits has two opposing results. On one hand, it increases U.S. wages, which leads to an increase in the mean welfare change. The higher wages are due to the fact that, in the presence of trade costs, a transfer from abroad leads to a worldwide shift in demand towards domestic goods and this in turn improves the recipient's terms of trade, as shown in Dornbusch et al. (1977). On the other hand, it leads to a transition from manufacturing production to non-manufacturing production. This can exacerbate the binding DNWR in manufacturing in certain regions, leading to higher temporary unemployment and lower welfare gains. When going from column (2) to (6) the first effect dominates, leading to an increase in the mean welfare change, whereas when going from column (6) to (7) the second effect dominates, leading to a fall in the mean welfare change. In the absence of DNWR only the first effect would be present. Indeed, notice that the mean welfare change in the absence of DNWR increases monotonically from column (2) to column (6) and then to column (7). In contrast, the fraction of the mean welfare change in the model with DNWR as a share of the mean welfare change in the model without it decreases monotonically from 74% in column (2) to 72% in column (6) and 57% in column (7).

The fact that incorporating a large enough increase in Chinese surplus as part of the China shock can eventually lead to a fall in the U.S. welfare due to the DNWR is very interesting. In particular, it speaks to the flexibility of our dynamic trade and migration model with nominal rigidities as capturing some potentially "real-world" elements that are hard to capture in previous frameworks.

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[27]Recall that we measure welfare as real income rather than consumption – see footnote 8.

6.4 Fixed vs. Flexible Exchange Rates

So far, we have assumed that all countries outside of the U.S. have a fully flexible exchange rate with respect to the U.S. dollar. Adjustments in their exchange rate can then ensure that the DNWR never binds, and hence countries outside of the U.S. never experience unemployment. In this section, we consider the consequences of assuming that all countries have a fixed exchange rate with respect to the U.S. dollar, implying a potentially binding DNWR, $W_{i,s,t} \geq \delta_s W_{i,s,t-1} \; \forall i$. Compared to the baseline, we will no longer have countries devaluing their currencies relative to the U.S. dollar, implying less of a need for the nominal wage to fall in the U.S., and hence lower unemployment in U.S. labor markets.

Column (8) of Table 2 presents the results of this exercise. Overall, our results are remarkably robust to the new assumption regarding the exchange rate regime. The only notable change is that δ increases from 0.98 to 0.987. This increase in δ occurs because the other countries "absorb" part of the China shock, so a higher δ is needed in the U.S. to match the target response of unemployment to exposure.

6.5 Different Discount Factors

We now study how changing the discount factor affects our results. The discount factor matters directly for computing welfare as well as indirectly through its impact on the relocation decisions of forward-looking agents. Since it affects the equilibrium, changing the discount factor requires a full recalibration of our model.

Reassuringly, we find that our calibrated parameter values for δ, κ, and ν do not change much when we vary β. Additionally, the non-targeted moments also do not vary much. The welfare calculations do change substantially, especially because a higher discount factor implies that agents are more patient and do not suffer that much from a temporary period of unemployment.

To fully convey the results, Table 3 shows the different welfare changes in the model with and without DNWR, for several values of the discount factor β. As men-

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Table 3: Welfare gains from the China shock across different discount factors

β	$\delta = 0$ (1)	calibrated δ (2)	% decrease (3)
0.99	0.382	0.362	5.33
0.97	0.344	0.289	15.85
0.95	0.310	0.228	26.22
0.93	0.278	0.177	36.41
0.91	0.250	0.134	46.42

Notes: This table displays the average welfare gains from the China shock, for the U.S. as a whole, across different values of the discount factor β. Column (1) displays the gains in percent when the DNWR is inactive ($\delta = 0$). Column (2) displays the gains in percent for the calibrated δ, ν, and κ that match the ADH targets (these parameters differ in each row of the table because the whole model has to be re-calibrated whenever β changes). Finally, column (3) displays the percentage decrease in the welfare gain when going from $\delta = 0$ to the calibrated δ.

tioned above, for $\beta = 0.95$, our baseline value, the model without DNWR overestimates the welfare gains by approximately 26% relative to our baseline model. For higher discount factors, such as $\beta = 0.99$, this overestimation is just 5%, whereas for lower discount factors, such as $\beta = 0.91$ it is 46%.

7 Discussion

7.1 Different Exposure Measures

The measure of exposure to China that we have been using throughout the paper (defined in equation 16) follows the one in ADH. This measure is a Bartik instrument where the "shift component" is given by the predicted change in imports from China to the U.S. in a sector and the "share component" is given by the share of employment in that sector-region. However, this exposure measure cannot fully capture the welfare effects of the China shock, because it misses the impact through consumer prices.[28]

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[28]Consider, for example, a region that did not produce a good at all (and hence would have a zero employment share) but consumed it in a positive amount. This region would benefit from an increase in Chinese productivity in that sector, even though the ADH measure would imply a zero exposure of that sector to the shock.

As we show in Appendix C, in a simple neoclassical environment with an upward sloping labor supply curve but without nominal rigidities, a sufficient statistic for the first-order changes in employment resulting from the China shock would use net exports as the "share" component, as in

$$\text{Exposure}_i^{NX} \equiv \sum_{s=1}^{S} \frac{TX_{i,s,2000} - TM_{i,s,2000}}{R_{i,2000}} \frac{\Delta \widehat{X_{C,US,s}^{2007-2000}}}{R_{US,s,2000}}, \tag{17}$$

where $TX_{i,s,2000}$ are the total sales of region i in sector s in year 2000, and $TM_{i,s,2000}$ is total expenditure of region i on sector s in year 2000. This captures the effect of the shock on the economy's terms of trade, which in turn affects the equilibrium real wage and employment according to labor demand and supply elasticities. In contrast, when the wage does not adjust because of the DNWR, the employment shares become directly relevant, since the change in employment is determined entirely by the shift in the demand curve. Of course, in a more realistic situation where wages are sometimes sticky in the short run due to the DNWR but can eventually adjust to their frictionless level, then both measures of exposure are expected to be relevant.

To illustrate this point, we regress the state-level changes in welfare and employment generated by the model on both exposure measures (and a constant), with and without the DNWR. The results are reported in Table 4. Columns (1) and (3) reveal that, without DNWR, only the net export exposure measure is significant for employment and welfare, while ADH exposure is not significant. In contrast, columns (2) and (4) show that in the model with DNWR both the ADH exposure measure and the net export exposure measure are significant. These results indicate that a mechanism similar to DNWR is likely to be active in the U.S. economy, and this is what leads to the ADH exposure measure being relevant.[29]

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[29]In our framework, nominal rigidities lead to a separate effect of labor demand on employment over and above those that would come through terms-of-trade effects. Adao et al. (2020) obtain similar effects through a more general reduced-form specification of the labor market where labor supply is a function of the nominal wage and the consumption price entering separately rather than through the real wage.

Table 4: "Horse race" between different exposure measures in the baseline model with and without DNWR

	(1) Welf. Flex.	(2) Welf. DNWR	(3) Empl. Flex.	(4) Empl. DNWR
Constant	0.513**	0.522**	3.204**	4.732**
	(0.043)	(0.048)	(0.338)	(0.861)
ADH Exposure	−0.016	−0.031*	−0.168	−0.944**
	(0.012)	(0.014)	(0.097)	(0.248)
NX Exposure	−0.076**	−0.092**	−0.537**	−1.168**
	(0.012)	(0.014)	(0.097)	(0.248)
N	50	50	50	50
R squared	0.491	0.554	0.460	0.503
Mean dep. var.	0.269	0.198	1.351	−0.821

Notes: This table shows the results of regressing several variables of interest on a constant, ADH exposure, and net export exposure. The exposure variables are described in the text. The dependent variables are: welfare change from the China shock in the baseline model without DNWR (column 1), welfare change from the China shock in the baseline model with DNWR (column 2), percentage change in total employment between 2000 and 2007 in the baseline model without DNWR (column 3), and percentage change in total employment between 2000 and 2007 in the baseline model with DNWR (column 4). Stars denote significance, one star for 5%, and two for 1%.

7.2 Quantifying Job Losses

In this section, we quantify the aggregate employment changes due to the China Shock. Recall that ADH find a cross-sectional estimate indicating that a $1,000 per worker increase in import exposure to China leads to a decrease in the employment to population ratio of 77 basis points (22 basis points from increased unemployment and 55 basis points from reduced labor force participation). We start by using the ADH estimate in a naive calculation that assumes that a U.S. state with zero ADH exposure would have no employment changes (meaning that the cross-sectional regression has an absolute intercept of zero). This calculation implies that the China shock generated employment losses of $0.77 \cdot 2.63 \cdot 220$ million $= 4.4$ million jobs (where 2.63 is the mean exposure, and 220 million is approximately the U.S. population over 16 years between 2000 and 2007).

However, our quantitative results imply that U.S. states with zero ADH exposure to the China shock increased their employment because they experienced a

positive terms-of-trade shock. This means that the cross-sectional regression of employment on exposure to China has a negative intercept.[30] In particular, this intercept is approximately -1.75. A back-of-the-envelope calculation using this intercept would add $1.75 \cdot 220 = 3.85$ million jobs. Combining the intercept number of 3.85 million jobs gained with the cross-sectional estimate of 4.4 million jobs lost would result in a net loss of 550 thousand jobs.

The previous discussion is based on a simple regression to highlight the importance of the "missing intercept". While this approximation using a simple regression is possible because the model allows us to compute aggregate employment effects, we can also compute the actual general equilibrium effect of the shock using our full model. The quantification using the full model implies that 467 thousand jobs were lost by 2007 due to the China shock. This number is similar to the aforementioned back-of-the-envelope calculations incorporating a non-zero intercept.

It is important to point out that in all of these estimates we stop the accounting of job losses in 2007. If we continue the analysis into further years, we would obtain that the China shock actually led to a net job gain in the U.S., since by 2010 labor force participation in our model has already recovered and is approximately 1% higher than its original value. These findings align with those of Bloom et al. (2019), who find sizable net negative employment effects due to the China shock between 2000 and 2007 but weakened impacts after 2007. In Bloom et al. (2019), these findings are consistent with firm and labor market adaptation. In our model, the labor market adaptation comes from the temporary nature of the DNWR and the fact that the China shock becomes less strong over time. At the same time, it is important to note that our model abstracts from structural forces that could lead the adverse impacts of the China shock to endure in the long run, which recent evidence has documented for less-educated commuting zones (Autor et al., 2021).

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[30] As shown in Adao et al. (2020), this intercept not only captures the effects coming directly from the China shock, but also includes indirect effects of the shock on the state's trading partners.

8 Conclusion

In this paper, we propose a dynamic quantitative trade and migration model with downward nominal wage rigidity and use it to study the path of adjustment in unemployment, labor force participation, population, and welfare after a trade shock. We show that even a shock that improves an economy's terms of trade can lead to unemployment if it requires a fall in the nominal wage that is larger than the one permitted by nominal frictions.

We calibrate the model to match the reduced-form evidence in Autor et al. (2013), and find that the China shock is responsible for up to 1.4 percentage points of the increase in unemployment in the U.S. over the period 2000-2007. This increase can go as high as 3 percentage points for the states affected the most. Although the U.S. as a whole still gains from the China shock, such gains are approximately one fourth lower than the gains without nominal rigidities, and there are seven states that experience welfare losses despite the improvement in their terms of trade.

We acknowledge that the way we have captured nominal forces and trade imbalances in our model is simplistic relative to traditional models in open-economy macroeconomics. A more satisfactory approach from a macroeconomic perspective would model monetary policy by adding a Taylor Rule with a zero lower bound, allow agents to make savings and investment decisions, and incorporate international financial flows affecting exchange rates, among other features. We have instead chosen to capture these forces via simple rules so that we can have a rich trade structure with many countries and sectors, intermediate inputs, and forward looking migration decisions while still being able to conduct the quantitative analysis in a transparent way (i.e., using dynamic exact hat algebra). Our aim is that this exercise serves to identify the key elements that future models need to incorporate.

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Appendices for Online Publication

A Model Details

A.1 Production

Technology to produce the differentiated good of industry s in region i at time t is

$$Y_{i,s,t} = \left(\phi_{i,s}^{-\phi_{i,s}} \prod_{k=1}^{S} \phi_{i,ks}^{-\phi_{i,ks}} \right) A_{i,s,t} L_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} M_{i,ks,t}^{\phi_{i,ks}},$$

where $M_{i,ks,t}$ is the quantity of the composite good of industry k used in region i to produce in sector s at time t, $\phi_{i,s}$ is the labor share in region i, sector s, $\phi_{i,ks}$ is the share of inputs that sector s uses from sector k in region i, and $1 - \phi_{i,s} = \sum_{k=1}^{S} \phi_{i,ks}$. The resource constraint for the composite good produced in region j, sector k, at time t is

$$M_{j,k,t} = C_{j,k,t} + \sum_{s=1}^{S} M_{j,ks,t}.$$

In turn, the resource constraint for good s produced by region i is $Y_{i,s,t} = \sum_{j=1}^{I} \tau_{ij,s,t} Y_{ij,s,t}$. The composite in sector k is produced according to

$$M_{j,k,t} = \left(\sum_{i=1}^{I} Y_{ij,k,t}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}.$$

Let $P_{i,s,t}$ be the price of $M_{i,s,t}$, $p_{ij,s,t}$ be the price of $Y_{i,s,t}$ in j at time t, and $W_{i,s,t}$ be the nominal wage in region i, sector s, at time t. We know that $p_{ii,s,t} = A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} P_{i,k,t}^{\phi_{i,ks}}$, $p_{ij,s,t} = \tau_{ij,s,t} p_{ii,s,t}$, and $P_{j,s,t} = \left(\sum_{i=1}^{I} p_{ij,s,t}^{1-\sigma_s} \right)^{1/(1-\sigma_s)}$. Combining these we obtain:

$$P_{j,s,t}^{1-\sigma_s} = \sum_{i=1}^{I} \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} P_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s},$$

The price of final output in region j at time t is given by $P_{j,t} = \prod_{s=1}^{S} P_{j,s,t}^{\alpha_{j,s}}$. Multiplying the resource constraint for $M_{j,k,t}$ by $P_{j,k,t}$ we get

$$Z_{j,k,t} = P_{j,k,t} C_{j,k,t} + \sum_{s=1}^{S} P_{j,k,t} M_{j,ks,t},$$

where $Z_{j,k,t} \equiv P_{j,k,t} M_{j,k,t}$ denotes the total expenditure of region j in industry k. Let the share of that expenditure spent on imports from i be $\lambda_{ij,k,t} \equiv \frac{p_{ij,k,t} Y_{ij,k,t}}{Z_{j,k,t}}$. We know that

$$\lambda_{ij,k,t} = \frac{p_{ij,k,t}^{1-\sigma_k}}{\sum_l p_{lj,k,t}^{1-\sigma_k}} = \frac{p_{ij,k,t}^{1-\sigma_k}}{P_{j,k,t}^{1-\sigma_k}} = \frac{\left(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,k,t}^{\phi_{i,k}} \prod_{s=1}^{S} P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}}{\sum_{r=1}^{I} \left(\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,k,t}^{\phi_{r,k}} \prod_{s=1}^{S} P_{r,s,t}^{\phi_{r,sk}} \right)^{1-\sigma_k}}.$$

Let $R_{i,k,t} = p_{ii,k,t} Y_{i,k,t}$ represent the sales of good k by region i. Multiplying the resource constraint for $Y_{i,k,t}$ above by $p_{ii,k,t}$ we get $p_{ii,k,t} Y_{i,k,t} = \sum_{j=1}^{I} \tau_{ij,k,t} p_{ii,k,t} Y_{ij,k,t}$, and hence $R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t} Z_{j,k,t}$. Plugging in from the resource constraint for $Z_{j,k,t}$ we have

$$R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t} \left(P_{j,k,t} C_{j,k,t} + \sum_s P_{j,k,t} M_{j,ks,t} \right).$$

Note that $P_{j,k,t} M_{j,ks,t} = \phi_{j,ks} R_{j,s,t}$. Additionally, the total amount available for consumption in region j at time t is the sum of total labor income (denoted $I_{j,t}$, notice $I_{j,t} \equiv \sum_{k=1}^{S} W_{j,k,t} L_{j,k,t}$) and the deficit (denoted $D_{j,t}$). So we get $P_{j,k,t} C_{j,k,t} = \alpha_{j,k} \left(I_{j,t} + D_{j,t} \right)$, hence

$$R_{i,k,t} = \sum_{j=1}^{I} \lambda_{ij,k,t} \left(\alpha_{j,k} \left(I_{j,t} + D_{j,t} \right) + \sum_s \phi_{j,ks} R_{j,s,t} \right).$$

We know that a fraction $\phi_{i,k}$ of $R_{i,k,t}$ is payed to labor, hence $W_{i,k,t} L_{i,k,t} = \phi_{i,k} R_{i,k,t}$.

A.2   Labor Supply

As mentioned in the text, an agent's utility in region j, sector s, at time t is given by

$$v_{j,s,t} = U(\omega_{j,s,t}) + \max_{\{i,k\}_{i=1,k=0}^{I,S}} \{ \beta \mathbb{E}(v_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t} \},$$

with the joint density of vector ϵ being i.i.d over time and nested Gumbel,

$$F(\epsilon) = \exp\left(-\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(-\epsilon_{i,k}/\nu\right)\right)^{\nu/\kappa}\right)$$

with $\nu \leq \kappa$. If there is an strict inequality such that $\nu < \kappa$, that means that the elasticity across sectors $(1/\nu)$ is greater than the elasticity across locations $(1/\kappa)$. Denote $V_{i,k,t+1} \equiv \mathbb{E}[\nu_{i,k,t+1}]$. In this appendix, we will prove two main results. First, the probability that an agent in js will choose to move to ik conditional on moving to region i is

$$\mu_{ji,sk|i,t} = \frac{\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}}{\sum_{h=0}^{S}\exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu}},$$

while the probability that an agent in js will move to any sector in region i is

$$\mu_{ji,s\#,t} = \frac{\left(\sum_{h=0}^{S}\exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\left(\sum_{h=0}^{S}\exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa}}.$$

Second,

$$\mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{I,S}}\left\{\beta\mathbb{E}(\nu_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\right\}\right] = \ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa} + \gamma\kappa,$$

where γ is the Euler-Mascheroni constant. The previous expression implies

$$V_{j,s,t} = U(\omega_{j,s,t}) + \ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa} + \gamma\kappa.$$

To show the first result, note that an agent that is in market js at time t will choose to switch to ik if and only if the following expression holds for all mh:

$$\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \geq \beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t},$$

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which is equivalent to $\epsilon_{m,h,t} \le \nu x_{im,kh} + \epsilon_{i,k,t}$, where

$$x_{im,kh} \equiv \frac{\beta\left(V_{i,k,t+1} - V_{m,h,t+1}\right) - \left(\varphi_{ji,sk} - \varphi_{jm,sh}\right)}{\nu}.$$

Denoting

$$\Phi_{j,s,t} \equiv \mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{I,S}} \left\{\beta\mathbb{E}(\nu_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\right\}\right]$$

We know that

$$\Phi_{j,s,t} = \sum_{i=1}^{I} \sum_{k=0}^{S} \int_{-\infty}^{+\infty} \left(\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t}\right) G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t}$$

where $G_{ik}(\epsilon_{i,k,t}, x_{i,k,t})$ is the partial derivative of $F(\cdot)$ w.r.t. to the ik element of the vector ϵ, with the ik element of the vector evaluated at $\epsilon_{i,k,t}$ and the generic element in position mh of the vector evaluated at $\nu x_{im,kh,t} + \epsilon_{i,k,t}$. Given our function $F(\epsilon)$ above, the partial derivative w.r.t the element in position ik is

$$\frac{\partial F(\epsilon)}{\partial \epsilon_{i,k}} = \frac{1}{\kappa}\left(\sum_h \exp\left(-\epsilon_{i,h}/\nu\right)\right)^{\nu/\kappa-1} \exp\left(-\epsilon_{i,k}/\nu\right) \exp\left[-\sum_m \left(\sum_h \exp\left(-\epsilon_{m,h}/\nu\right)\right)^{\nu/\kappa}\right]$$

We then have

$$G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) = \frac{1}{\kappa}\left(\sum_h \exp\left(-x_{ii,kh,t}\right)\right)^{\nu/\kappa-1} \exp\left(-\epsilon_{i,k,t}/\kappa\right)$$
$$\cdot \exp\left[-\exp\left(-\epsilon_{i,k,t}/\kappa\right)\sum_m \left(\sum_h \exp\left(-x_{im,kh,t}\right)\right)^{\nu/\kappa}\right]$$

where we have used the fact that $x_{ii,kk,t} = 0$. Integrating this over $\epsilon_{i,k,t}$ yields

$$\int_{-\infty}^{+\infty} G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t} = \frac{\left(\sum_h \exp\left(-x_{ii,kh,t}\right)\right)^{\nu/\kappa-1}}{\sum_m \left(\sum_h \exp\left(-x_{im,kh,t}\right)\right)^{\nu/\kappa}} \int_{-\infty}^{+\infty} \frac{1}{\kappa}\exp\left(-\epsilon_{i,k,t}/\kappa\right)$$
$$\cdot T\exp\left[-\exp\left(-\epsilon_{i,k,t}/\kappa\right)T\right] d\epsilon_{i,k,t},$$

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where

$$T \equiv \sum_m \left(\sum_h \exp\left(-x_{im,kh,t}\right) \right)^{\nu/\kappa}.$$

But note that

$$\int_{-\infty}^{+\infty} \frac{1}{\kappa} \exp\left(-\epsilon_{i,k,t}/\kappa\right) T \exp\left[-\exp\left(-\epsilon_{i,k,t}/\kappa\right) T \right] d\epsilon_{i,k,t} = 1,$$

because the integrand is the density associated with $\exp\left(-\exp\left(-\epsilon_{i,k,t}/\kappa\right) T\right)$, a univariate Gumbel. Hence, the previous expression simplifies to

$$\int_{-\infty}^{\infty} G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t} = \frac{\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}}{\sum_h \exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu}} \frac{\left(\sum_h \exp\left(\beta V_{i,h,t+1} - \varphi_{ji,sh}\right)^{1/\nu} \right)^{\nu/\kappa}}{\sum_m \left(\sum_h \exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu} \right)^{\nu/\kappa}}$$

It is easy to see that the first fraction is $\mu_{ji,sk|i,t}$, while the second one is $\mu_{ji,s\#,t}$.

Now we want to solve for

$$\mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{I,S}} \left\{ \beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \right\} \right]$$

Let's compute

$$\mathbb{E}\left[\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \,\middle|\, \arg\max_{mh} \left\{ \beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t} \right\} = ik \right]$$

To do this, first note that the joint probability that $\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \leq a$ while at the same time $\arg\max_{mh} \left\{ \beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t} \right\} = ik$, is

$$\int_{-\infty}^{a - \left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)} G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t}$$

$$= \frac{\left(\sum_h \exp\left(-x_{ii,kh}\right)\right)^{\nu/\kappa - 1}}{\sum_m \left(\sum_h \exp\left(-x_{im,kh}\right)\right)^{\nu/\kappa}} \int_{-\infty}^{a - \left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)} \frac{1}{\kappa} T \exp\left(-z/\kappa\right) \exp\left(-T \exp\left(-z/\kappa\right)\right) dz$$

A change of variables with $y = \exp(z)$ implies that $dy/y = dz$ and

$$\int_{-\infty}^{a - \left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)} \frac{1}{\kappa} T \exp\left(-z/\kappa\right) \exp\left(-T \exp\left(-z/\kappa\right)\right) dz$$

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$$= \exp\left(-T\exp\left[\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)/\kappa\right]\exp\left(-a/\kappa\right)\right)$$

Thus, the **joint** probability we are interested in is

$$\frac{\left(\sum_h \exp\left(-x_{ii,kh}\right)\right)^{\nu/\kappa-1}}{\sum_m \left(\sum_h \exp\left(-x_{im,kh}\right)\right)^{\nu/\kappa}} \exp\left(-T\exp\left[\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)/\kappa\right]\exp\left(-a/\kappa\right)\right)$$

and hence the probability of $(\beta V_{i,k,t+1} - \varphi^{ji,sk} + \epsilon_{i,k,t} \leq a)$ **conditional** on

$$\arg\max_{mh}\left\{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\right\} = ik,$$

is

$$\exp\left(-\tilde{T}\exp\left(-a/\kappa\right)\right),$$

where now

$$\tilde{T} \equiv T\exp\left[\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)/\kappa\right].$$

In turn, this implies that

$$\mathbb{E}\left[\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \,\middle|\, \arg\max_{mh}\left\{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\right\} = ik\right]$$
$$= \int_{-\infty}^{+\infty} a\, d\exp\left(-\exp\left(-\frac{\left(a - \ln \tilde{T}^{\kappa}\right)}{\kappa}\right)\right),$$

where we have used

$$\tilde{T}\exp\left(-a/\kappa\right) = \exp\left(-\frac{\left(a - \ln \tilde{T}^{\kappa}\right)}{\kappa}\right).$$

This is the expectation of a variable distributed Gumbel with location parameter $\mu = \ln \tilde{T}^{\kappa}$ and scale parameter $\beta = \kappa$. But we know that the expectation of a variable distributed Gumbel with μ and β is $\mu + \beta\gamma$, where γ is the Euler-Mascheroni constant, hence we have

$$\int_{-\infty}^{\infty} a\, d\exp\left(-\tilde{T}\left(\exp a\right)^{-\kappa}\right) = \ln \tilde{T}^{\kappa} + \gamma\kappa.$$

This implies that

$$\mathbb{E}\left[\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \,\middle|\, \arg\max_{mh}\left\{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\right\} = ik\right]$$

$$= \ln\left(\sum_m\left(\sum_h \exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa} + \gamma\kappa$$

Since this does not depend on ik, then we have

$$\mathbb{E}\left[\max_{\{i,k\}_{i=1,k=0}^{I,S}}\left\{\beta\mathbb{E}(v_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\right\}\right] = \ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S} \exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa} + \gamma\kappa,$$

as we wanted to show.

A.3 Equilibrium in Relative Time Changes (Dots)

Now we will describe the equilibrium equations in relative changes from one period to the next. We use the notation $\dot{x}_t = x_t/x_{t-1}$. We start by deriving the dot equations for the labor market block of the economy. We will denote $u_{j,s,t} \equiv \exp(V_{j,s,t})$ and assume that the utility function takes log form: $U(\omega_t) = \log(\omega_t)$. We have,

$$\frac{\mu_{ji,sk|i,t+1}}{\mu_{ji,sk|i,t}} = \frac{\exp\left(\beta V_{i,k,t+2} - \varphi_{ji,sk}\right)^{1/\nu} / \exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}}{\sum_{h=0}^{S}\exp\left(\beta V_{i,h,t+2} - \varphi_{ji,sh}\right)^{1/\nu} / \sum_{h'=0}^{S}\exp\left(\beta V_{i,h',t+1} - \varphi_{ji,sh'}\right)^{1/\nu}}$$

$$= \frac{\exp\left(V_{i,k,t+2} - V_{i,k,t+1}\right)^{\beta/\nu}}{\sum_{h=0}^{S}\mu_{ji,sh|i,t}\exp\left(V_{i,h,t+2} - V_{i,h,t+1}\right)^{\beta/\nu}},$$

while

$$\frac{\mu_{ji,s\#,t+1}}{\mu_{ji,s\#,t}} = \frac{\left(\sum_{h=0}^{S}\exp\left(V_{i,h,t+2} - V_{i,h,t+1}\right)^{\beta/\nu}\mu_{ji,sh|i,t}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\mu_{jm,s\#,t}\left(\sum_{h=0}^{S}\exp\left(V_{m,h,t+2} - V_{m,h,t+1}\right)^{\beta/\nu}\mu_{jm,sh|m,t}\right)^{\nu/\kappa}}.$$

Since $u_{j,s,t} \equiv \exp(V_{j,s,t})$ then

$$\dot{u}_{t+2}^{nj} \equiv u_{j,s,t+2}/u_{j,s,t+1} = \frac{\exp(V_{j,s,t+2})}{\exp(V_{j,s,t+1})} = \exp(V_{j,s,t+2} - V_{j,s,t+1})$$

$$(\dot{u}_{j,s,t+2})^{\frac{\beta}{\nu}} = \exp(V_{j,s,t+2} - V_{j,s,t+1})^{\frac{\beta}{\nu}}.$$

Introducing this in the previous results and writing the equations for period t instead of $t+1$, we obtain

$$\mu_{ji,sk|i,t} = \frac{\mu_{ji,sk|i,t-1}\dot{u}_{i,k,t+1}^{\frac{\beta}{\nu}}}{\sum_{h=0}^{S}\mu_{ji,sh|i,t-1}\dot{u}_{i,h,t+1}^{\frac{\beta}{\nu}}} \tag{A1}$$

$$\mu_{ji,s\#,t} = \frac{\mu_{ji,s\#,t-1}\left(\sum_{h=0}^{S}\mu_{ji,sh|i,t-1}\dot{u}_{i,h,t+1}^{\frac{\beta}{\nu}}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\mu_{jm,s\#,t-1}\left(\sum_{h=0}^{S}\mu_{jm,sh|m,t-1}\dot{u}_{m,h,t+1}^{\frac{\beta}{\nu}}\right)^{\nu/\kappa}}. \tag{A2}$$

Take the difference between $V_{j,s,t+1}$ and $V_{j,s,t}$ using equation (6) to get

$$
\begin{aligned}
V_{j,s,t+1} - V_{j,s,t} &= U(\omega_{j,s,t+1}) - U(\omega_{j,s,t}) \\
&+ \ln\left(\frac{\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(\beta V_{i,k,t+2} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa}}{\left(\sum_{m=1}^{I}\left(\sum_{h=0}^{S}\exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa}\right)^{\kappa}}\right) \\
e^{V_{j,s,t+1} - V_{j,s,t}} &= \exp(\ln(\omega_{j,s,t+1}/\omega_{j,s,t}))\exp\left(\ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\dot{u}_{i,k,t+2}^{\frac{\beta}{\nu}}\mu_{ji,sk|i,t}\right)^{\nu/\kappa}\mu_{ji,s\#,t}\right)\right).
\end{aligned}
$$

Thus, we finally obtain

$$\dot{u}_{j,s,t+1} = \dot{\omega}_{j,s,t+1}\left(\sum_{i=1}^{I}\mu_{ji,s\#,t}\left(\sum_{k=0}^{S}\mu_{ji,sk|i,t}\dot{u}_{i,k,t+2}^{\frac{\beta}{\nu}}\right)^{\nu/\kappa}\right)^{\kappa}. \tag{A3}$$

The equilibrium in changes includes equations (A1), (A2), (A3), together with the dot versions of the remaining equations in (1) - (15).

A.4   Algorithm to Solve the Dot System

Group the equations of the dot equilibrium system into 3 categories:

1. The ones that are needed to obtain new migration and new labor supply from a guess

of utilities (block 1):

$$\mu_{ji,sk|i,t} = \frac{\mu_{ji,sk|i,t-1}\dot{u}_{i,k,t+1}^{\frac{\beta}{\nu}}}{\sum_{h=0}^{S}\mu_{ji,sh|i,t-1}\dot{u}_{i,h,t+1}^{\frac{\beta}{\nu}}}$$

$$\mu_{ji,s\#,t} = \frac{\mu_{ji,s\#,t-1}\left(\sum_{h=0}^{S}\mu_{ji,sh|i,t-1}\dot{u}_{i,h,t+1}^{\frac{\beta}{\nu}}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\mu_{jm,s\#,t-1}\left(\sum_{h=0}^{S}\mu_{jm,sh|m,t-1}\dot{u}_{m,h,t+1}^{\frac{\beta}{\nu}}\right)^{\nu/\kappa}}$$

$$\ell_{i,s,t} = \sum_{j=1}^{I}\sum_{k=0}^{S}\mu_{ji,ks|i,t-1}\mu_{ji,k\#,t-1}\ell_{j,k,t-1}$$

With these equations, if one has an initial distribution of labor supply ($\ell_{i,s,0}$), initial mobility matrices ($\mu_{ji,sk|i,0}$ and $\mu_{ji,s\#}$) and an initial guess for the utility dots ($\dot{u}_{i,s,t}^{(0)} \ \forall \ t$), one can obtain the entire path of labor supplies ($\ell_{i,s,t} \ \forall \ t > 0$), and the entire path of mobility matrices ($\mu_{ji,sk|i,t}$ and $\mu_{ji,s\#,t} \ \forall \ t > 0$) without needing to use the other equations at all.

2. The ones that are needed to obtain the temporary equilibrium (wages, actual labor, sectoral prices, trade shares, revenue levels) from a given set of shocks and labor supply (block 2):

$$\dot{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I}\lambda_{ji,s,t-1}\left(\dot{\tau}_{ji,s,t}\dot{A}_{j,s,t}^{-1}\dot{W}_{j,s,t}^{\phi_{j,s}}\prod_{k=1}^{S}\dot{P}_{j,k,t}^{\phi_{j,ks}}\right)^{1-\sigma_s}$$

$$\lambda_{ij,s,t} = \frac{\lambda_{ij,s,t-1}(\dot{\tau}_{ij,s,t}\dot{A}_{i,s,t}^{-1}\dot{W}_{i,s,t}^{\phi_{i,s}}\prod_{k=1}^{S}\dot{P}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,t-1}(\dot{\tau}_{rj,s,t}\dot{A}_{r,s,t}^{-1}\dot{W}_{r,s,t}^{\phi_{r,s}}\prod_{k=1}^{S}\dot{P}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}}$$

$$R_{i,s,t} = \sum_{j=1}^{I}\lambda_{ij,s,t}\left(\alpha_{j,s}\left(\sum_{s}\dot{W}_{j,s,t}\dot{L}_{j,s,t}Y_{j,s,t-1}+D_{j,t}\right)+\sum_{k=1}^{S}\phi_{j,sk}R_{j,k,t}\right)$$

$$\dot{W}_{i,s,t}\dot{L}_{i,s,t}Y_{i,s,t-1} = \phi_{i,s}R_{i,s,t}$$

$$\prod_{q=1}^{t}\dot{L}_{i,s,q} \leq \prod_{q=1}^{t}\dot{\ell}_{i,s,q} \ , \ \dot{W}_{i,s,t} \geq \delta_{i,s} \ , \ \text{Complementary Slackness}$$

$$\gamma\sum_{i=1}^{I}\sum_{s=1}^{S}Y_{i,s,t-1} = \sum_{i=1}^{I}\sum_{s=1}^{S}\dot{W}_{i,s,t}\dot{L}_{i,s,t}Y_{i,s,t-1}$$

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3. The ones that are needed to update the guess for the path of utilities (block 3):

$$\dot{P}_{i,t} = \prod_{s=1}^{S} \dot{P}_{i,s,t}^{\alpha_{i,s}}$$

$$\dot{\omega}_{i,s,t} = \frac{\dot{W}_{i,s,t}\dot{L}_{i,s,t}}{\dot{P}_{i,t}\dot{\ell}_{i,s,t}} \text{ (but with } \dot{\omega}_{i,s,t} = 1 \text{ if } s = 0 \text{)}$$

$$\dot{u}_{j,s,t+1} = \dot{\omega}_{j,s,t+1} \left(\sum_{i=1}^{I} \mu_{ji,s\#,t} \left(\sum_{k=0}^{S} \mu_{ji,sk|i,t}\dot{u}_{i,k,t+2}^{\frac{\beta}{\nu}} \right)^{\nu/\kappa} \right)^{\kappa}$$

The algorithm would work as follows:

1. Guess a path for the utility dots (which can be all of them being equal to one).

2. Use block one to obtain paths for the μ's and ℓ's using the guessed path for utility.

3. Use block two to solve the temporary equilibrium using the path for the ℓ's.

4. Use block three to obtain a new guess for the utility dots. This uses the fact that in a far enough point in the future (called T) even the new guess of utility dots should have $\dot{u}_{i,s,T}^{(1)} = 1$. With $\dot{u}_{i,s,T}^{(1)} = 1$, the path for μ's and the sectoral compensations one can obtain $\dot{u}_{i,s,T-1}^{(1)}$. And from those obtain $\dot{u}_{i,s,T-2}^{(1)}$, and so on until $\dot{u}_{i,s,1}^{(1)}$.

5. If the two guessed paths of utility dots $\dot{u}^{(0)}$ and $\dot{u}^{(1)}$ are close enough, stop the algorithm, otherwise return to item one with the new guess and iterate again.

A.5 Equilibrium in Counterfactual Relative to Baseline (Hats)

Now we want to describe the equilibrium equations in ratios of changes in a counterfactual economy relative to the same changes in the baseline economy. We will use the notation $\hat{x}_t = \dot{x}'_t/\dot{x}_t$, where $\dot{x}'_t$ is the relative change from period $t-1$ to t in the counterfactual economy and $\dot{x}_t$ is the same thing but for the baseline economy. First, we want to get the evolution of $\mu'_{ji,sk|i,t}$. Start from equation (A1) for the case of the counterfactual economy,

$$\mu'_{ji,sk|i,t} = \frac{\mu'_{ji,sk|i,t-1}(\dot{u}'_{i,k,t+1})^{\frac{\beta}{\nu}}}{\sum_{h=0}^{S} \mu'_{ji,sh|i,t-1}(\dot{u}'_{i,h,t+1})^{\frac{\beta}{\nu}}}.$$

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Divide this by the same expression in the case of the baseline economy and rearrange to get:

$$
\mu'_{ji,sk|i,t} = \frac{\mu'_{ji,sk|i,t-1} \dot{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{\nu}}}{\sum\limits_{h=0}^{S} \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{\nu}}}.
\tag{A4}
$$

To obtain the evolution of $\mu'_{ji,s\#,t}$, start from equation (A2) for the counterfactual economy,

$$
\mu'_{ji,s\#,t} = \frac{\mu'_{ji,s\#,t-1} \left(\sum_{h=0}^{S} \mu'_{ji,sh|i,t-1} (\dot{u}'_{i,h,t+1})^{\frac{\beta}{\nu}} \right)^{\nu/\kappa}}{\sum_{m=1}^{I} \mu'_{jm,s\#,t-1} \left(\sum_{h=0}^{S} \mu'_{jm,sh|m,t-1} (\dot{u}'_{m,h,t+1})^{\frac{\beta}{\nu}} \right)^{\nu/\kappa}}.
$$

Divide this by the same expression in the case of the baseline economy and rearrange to get:

$$
\mu'_{ji,s\#,t} = \frac{\mu'_{ji,s\#,t-1} \dot{\mu}_{ji,s\#,t} \left(\sum\limits_{h=0}^{S} \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{\nu}} \right)^{\nu/\kappa}}{\sum\limits_{m=1}^{I} \mu'_{jm,s\#,t-1} \dot{\mu}_{jm,s\#,t} \left(\sum\limits_{h=0}^{S} \mu'_{jm,sh|m,t-1} \dot{\mu}_{jm,sh|m,t} \hat{u}_{m,h,t+1}^{\frac{\beta}{\nu}} \right)^{\nu/\kappa}}.
\tag{A5}
$$

Now we want to derive an expression for utility in hats. Start from equation (A3) for the counterfactual economy (but for period t instead of $t+1$):

$$
\dot{u}'_{j,s,t} = \dot{\omega}'_{j,s,t} \left(\sum_{i=1}^{I} \mu'_{ji,s\#,t-1} \left(\sum_{k=0}^{S} \mu'_{ji,sk|i,t-1} (\dot{u}'_{i,k,t+1})^{\frac{\beta}{\nu}} \right)^{\nu/\kappa} \right)^{\kappa}.
$$

Dividing by this equation in the baseline economy and rearranging yields

$$
\hat{u}_{j,s,t} = \hat{\omega}_{j,s,t} \left(\sum_{i=1}^{I} \mu'_{ji,s\#,t-1} \dot{\mu}_{ji,s\#,t} \left(\sum_{k=0}^{S} \mu'_{ji,sk|i,t-1} \dot{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{\nu}} \right)^{\nu/\kappa} \right)^{\kappa}.
\tag{A6}
$$

However, at $t = 1$ the equilibrium conditions are slightly different. This is the result of the timing assumption in CDP (which we adopt in this paper too), that the counterfactual fundamentals are unknown before $t = 1$. This means that at $t = 0$, $\hat{u}_{j,s,0} = 1$, $\mu'_{ji,sk|i,0} = \mu_{ji,sk|i,0}$, $\mu'_{ji,s\#,0} = \mu_{ji,s\#,0}$, and $\ell'_{i,k,1} = \ell_{i,k,1} = \sum_{j=1}^{I} \sum_{s=0}^{S} \mu_{ji,sk|i,0} \mu_{ji,s\#,0} \ell_{j,s,0}$. To account for the

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unexpected change in fundamentals at $t = 1$, the right equations are

$$\mu'_{ji,sk|i,1} = \frac{\theta_{ji,sk|i,0}\hat{u}^{\beta/\nu}_{i,k,2}}{\sum_{h=0}^{S} \theta_{ji,sh|i,0}\hat{u}^{\beta/\nu}_{i,h,2}} \tag{A7}$$

$$\mu'_{ji,s\#,1} = \frac{\mu_{ji,s\#,1}\left(\sum_{h=0}^{S} \theta_{ji,sh|i,0}\hat{u}^{\beta/\nu}_{i,h,2}\right)^{\nu/\kappa}}{\sum_{m=1}^{I} \mu_{jm,s\#,1}\left(\sum_{h=0}^{S} \theta_{jm,sh|m,0}\hat{u}^{\beta/\nu}_{m,h,2}\right)^{\nu/\kappa}} \tag{A8}$$

$$\hat{u}_{j,s,1} = \hat{\omega}_{j,s,1}\left(\sum_{i=1}^{I} \mu_{ji,s\#,1}\left(\sum_{k=0}^{S} \theta_{ji,sk|i,0}\hat{u}^{\beta/\nu}_{i,k,2}\right)^{\nu/\kappa}\right)^{\kappa}, \tag{A9}$$

where

$$\theta_{ji,sk|i,0} \equiv \mu_{ji,sk|i,1}\hat{u}^{\beta/\nu}_{i,k,1}.$$

The equilibrium in hats includes equations (A4), (A5), (A6), together with the hat versions of the remaining equations in (1) - (15).

A.6 Algorithm to Solve the Hat System

As in the previous algorithm, group the equations into 3 categories:

1. The ones that are needed to obtain new mobility shares and new labor supply from a guess of utilities (block 1):

$$\mu'_{ji,sk|i,t} = \frac{\mu'_{ji,sk|i,t-1}\dot{\mu}_{ji,sk|i,t}\hat{u}^{\frac{\beta}{\nu}}_{i,k,t+1}}{\sum\limits_{h=0}^{S} \mu'_{ji,sh|i,t-1}\dot{\mu}_{ji,sh|i,t}\hat{u}^{\frac{\beta}{\nu}}_{i,h,t+1}}$$

$$\mu'_{ji,s\#,t} = \frac{\mu'_{ji,s\#,t-1}\dot{\mu}_{ji,s\#,t}\left(\sum\limits_{h=0}^{S} \mu'_{ji,sh|i,t-1}\dot{\mu}_{ji,sh|i,t}\hat{u}^{\frac{\beta}{\nu}}_{i,h,t+1}\right)^{\nu/\kappa}}{\sum\limits_{m=1}^{I} \mu'_{jm,s\#,t-1}\dot{\mu}_{jm,s\#,t}\left(\sum\limits_{h=0}^{S} \mu'_{jm,sh|m,t-1}\dot{\mu}_{jm,sh|m,t}\hat{u}^{\frac{\beta}{\nu}}_{m,h,t+1}\right)^{\nu/\kappa}}$$

$$\ell'_{i,s,t} = \sum_{j=1}^{I}\sum_{k=0}^{S} \mu'_{ji,ks|i,t-1}\mu'_{ji,k\#,t-1}\ell'_{j,k,t-1}$$

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But period one works differently:

$$\mu'_{ji,sk|i,1} = \frac{\theta_{ji,sk|i,0}\hat{u}^{\beta/\nu}_{i,k,2}}{\sum_{h=0}^{S}\theta_{ji,sh|i,0}\hat{u}^{\beta/\nu}_{i,h,2}}$$

$$\mu'_{ji,s\#,1} = \frac{\mu_{ji,s\#,1}\left(\sum_{h=0}^{S}\theta_{ji,sh|i,0}\hat{u}^{\beta/\nu}_{i,h,2}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\mu_{jm,s\#,1}\left(\sum_{h=0}^{S}\theta_{jm,sh|m,0}\hat{u}^{\beta/\nu}_{m,h,2}\right)^{\nu/\kappa}}$$

$$\theta_{ji,sk|i,0} \equiv \mu_{ji,sk|i,1}\hat{u}^{\beta/\nu}_{i,k,1}$$

With these equations, if one has an initial distribution of labor supply ($\ell'_{i,s,0}$, which should be the same as $\ell_{i,s,0}$), the mobility matrices in the baseline economy and an initial guess for the utility hats ($\hat{u}^{(0)}_{i,s,t} \ \forall \ t$), one can obtain the entire path of labor supplies ($\ell'_{i,s,t} \ \forall \ t > 0$), and the entire path of mobility matrices without needing to use the other equations at all.

2. The ones that are needed to obtain the temporary equilibrium (wages, actual labor, sectoral prices, trade shares, revenue levels) from a given set of shocks and labor supply (block 2):

$$\hat{P}^{1-\sigma_s}_{i,s,t} = \sum_{j=1}^{I}\lambda'_{ji,s,t-1}\dot{\lambda}_{ji,s,t}\left(\hat{\tau}_{ji,s,t}\hat{A}^{-1}_{j,s,t}\hat{W}^{\phi_{j,s}}_{j,s,t}\prod_{k=1}^{S}\hat{P}^{\phi_{j,ks}}_{j,k,t}\right)^{1-\sigma_s}$$

$$\lambda'_{ij,s,t} = \frac{\lambda'_{ij,s,t-1}\dot{\lambda}_{ij,s,t}(\hat{\tau}_{ij,s,t}\hat{A}^{-1}_{i,s,t}\hat{W}^{\phi_{i,s}}_{i,s,t}\prod_{k=1}^{S}\hat{P}^{\phi_{i,ks}}_{i,k,t})^{1-\sigma_s}}{\hat{P}^{1-\sigma_s}_{j,s,t}}$$

$$R'_{i,s,t} = \sum_{j=1}^{I}\lambda'_{ij,s,t}\left(\alpha_{j,s}\left(\sum_{s}\hat{W}_{j,s,t}\hat{L}_{j,s,t}Y'_{j,s,t-1}\dot{W}_{j,s,t}\dot{L}_{j,s,t} + D'_{j,t}\right) + \sum_{k=1}^{S}\phi_{j,sk}R'_{j,k,t}\right)$$

$$\phi_{i,s}R'_{i,s,t} = \hat{W}_{i,s,t}\hat{L}_{i,s,t}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}$$

$$\prod_{q=1}^{t}\hat{L}_{i,s,q}\dot{L}_{i,s,q} \leq \prod_{q=1}^{t}\ell'_{i,s,q} \ , \ \hat{W}_{i,s,t}\dot{W}_{i,s,t} \geq \delta_{i,s} \ , \text{ Complementary Slackness}$$

$$\sum_{i=1}^{I}\sum_{s=1}^{S}Y'_{i,s,t-1} = \frac{1}{\gamma}\sum_{i=1}^{I}\sum_{s=1}^{S}\hat{W}_{i,s,t}\hat{L}_{i,s,t}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}$$

With these equations, if we have a set of shocks in hats ($\hat{\tau}$ and $\hat{A}$, as well as deficits in the counterfactual economy D'), together with initial values for the counterfactual economy (like trade shares and nominal incomes) and the solution for the baseline economy (including trade shares dot, wages dot and labor dot), we can solve for hat

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prices, new trade shares in levels, new revenues in levels, actual labor hats and wages hat.

3. The ones that are needed to update the guess for the path of utilities (block 3):

$$
\hat{P}_{i,t} = \prod_{s=1}^{S} \hat{P}_{i,s,t}^{\alpha_{i,s}}
$$

$$
\hat{\omega}_{i,s,t} = \frac{\hat{W}_{i,s,t}\hat{L}_{i,s,t}}{\hat{P}_{i,t}\hat{\ell}_{i,s,t}} \quad \text{(but with } \hat{\omega}_{i,s,t} = 1 \text{ if } s = 0\text{)}
$$

$$
\hat{u}_{j,s,t} = \hat{\omega}_{j,s,t} \left(\sum_{i=1}^{I} \mu'_{ji,s\#,t-1} \dot{\mu}_{ji,s\#,t} \left(\sum_{k=0}^{S} \mu'_{ji,sk|i,t-1} \dot{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{\nu}} \right)^{\nu/\kappa} \right)^{\kappa}
$$

But period one works differently:

$$
\hat{u}_{j,s,1} = \hat{\omega}_{j,s,1} \left(\sum_{i=1}^{I} \mu_{ji,s\#,1} \left(\sum_{k=0}^{S} \theta_{ji,sk|i,0} \hat{u}_{i,k,2}^{\beta/\nu} \right)^{\nu/\kappa} \right)^{\kappa}
$$

$$
\theta_{ji,sk|i,0} \equiv \mu_{ji,sk|i,1} \hat{u}_{i,k,1}^{\beta/\nu}.
$$

The algorithm would work as follows:

1. Guess a path for the utility hats (which can be all of them being equal to one).

2. Use block one to obtain paths for the μ' and ℓ' using the guessed path for the utility hat and the solution for the baseline economy.

3. Use block two to solve the temporary equilibrium using the path for ℓ', the hat shocks and the solution for the baseline economy.

4. Use block three to obtain a new guess for the utility hats. This uses the sectoral compensations obtained in the previous step and the fact that in a far enough point in the future (called T) the change in utility in the baseline economy should be the same as the change in utility in the counterfactual, so we should have $\hat{u}_{i,s,T}^{(1)} = 1$. With $\hat{u}_{i,s,T}^{(1)} = 1$, the path for the μ' and the sectoral compensations one can obtain $\hat{u}_{i,s,T-1}^{(1)}$. And from those obtain $\hat{u}_{i,s,T-2}^{(1)}$, and so on until $\hat{u}_{i,s,2}^{(1)}$. $\hat{u}_{i,s,1}^{(1)}$ needs to be obtained with a special equation.

5. If the two guessed paths of utility hats $\hat{u}^{(0)}$ and $\hat{u}^{(1)}$ are close enough, stop the algorithm, otherwise return to item one with the new guess and iterate again.

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A.7 Algorithm to Solve the Temporary Equilibrium

Block two of the previously described outer algorithms (which solve the equilibrium system in dots or hats) solves for the temporary equilibrium of the baseline or counterfactual economy. Given the presence of an inequality constraint due to the DNWR, solving this temporary equilibrium is an unwieldy process that would be infeasible with any traditional solver. To overcome this limitation, we develop an augmented version of Alvarez and Lucas (2007) to be able to handle the existence of DNWR. This inner algorithm is very efficient and allows us to solve the temporary equilibrium of the full model with DNWR extremely fast (provided we use the nominal anchor described in equation 15). In this appendix, we describe this inner algorithm in the case of the hat system. The inner algorithm for the dot system is analogous.

Notice first that, if one knows a given period's wages in hats (as well as the solution for the baseline economy, the previous period's trade shares, and the shocks to trade costs and technology), it is possible to obtain the corresponding prices in hats from the equation:

$$
\hat{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \lambda'_{ji,s,t-1} \dot{\lambda}_{ji,s,t} \left(\hat{\tau}_{ji,s,t} \hat{A}_{j,s,t}^{-1} \hat{W}_{j,s,t}^{\phi_{j,s}} \prod_{k=1}^{S} \hat{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s},
$$

using traditional contraction mapping algorithms. The new trade shares can then easily be obtained from the following equation,

$$
\lambda'_{ij,s,t} = \frac{\lambda'_{ij,s,t-1} \dot{\lambda}_{ij,s,t} (\hat{\tau}_{ij,s,t} \hat{A}_{i,s,t}^{-1} \hat{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} \hat{P}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\hat{P}_{j,s,t}^{1-\sigma_s}}.
$$

Knowing the previous elements, employment in hats, the previous periods output levels, and the shock to deficits, allows one to solve for revenues using the linear (albeit massive) system described by the following set of equations

$$
R'_{i,s,t} = \sum_{j=1}^{I} \lambda'_{ij,s,t} \left(\alpha_{j,s} \left(\sum_{s} \hat{W}_{j,s,t} \hat{L}_{j,s,t} Y'_{j,s,t-1} \dot{W}_{j,s,t} \dot{L}_{j,s,t} + D'_{j,t} \right) + \sum_{k=1}^{S} \phi_{j,sk} R'_{j,k,t} \right).
$$

The previous argument implies that we can write revenues in the counterfactual economy in a given period as a function of that same period's wages and employment hats, i.e.

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$R'_{i,s,t}(\hat{\mathbf{W}}, \hat{\mathbf{L}})$ (where the bold W and L stand for the vector of wages and employment hats in all the regions and sectors).

What remains is to show how to solve the following system in wages and employment hats for all regions and sectors:

$$\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}, \hat{\mathbf{L}}) = \hat{W}_{i,s,t} \hat{L}_{i,s,t} Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t}$$

$$\hat{L}_{i,s,t} \leq L^U_{i,s,t}, \; \hat{W}_{i,s,t} \geq W^L_{i,s,t}, \; \text{Complementary Slackness (C.S.)}$$

$$\sum_{i=1}^{I}\sum_{s=1}^{S} Y'_{i,s,t-1} = \sum_{i=1}^{I}\sum_{s=1}^{S} \hat{W}_{i,s,t} \hat{L}_{i,s,t} Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t}.$$

This is where we will use an augmented version of the Alvarez and Lucas (2007) algorithm that accounts for the presence of DNWR. Imagine that one has an initial guess for wages and employment in hats, denoted $\hat{W}^{(0)}_{i,s,t}$ and $\hat{L}^{(0)}_{i,s,t}$. We use an algorithm that updates this guess as follows:

$$\hat{W}^{(1)}_{i,s,t} = \max\left\{ \frac{(1-\lambda)\hat{W}^{(0)}_{i,s,t}\hat{L}^{(0)}_{i,s,t} + \lambda \frac{\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L^U_{i,s,t}}, W^L_{i,s,t}\right\}$$

$$\hat{L}^{(1)}_{i,s,t} = \min\left\{ L^U_{i,s,t}, \frac{(1-\lambda)\hat{W}^{(0)}_{i,s,t}\hat{L}^{(0)}_{i,s,t} + \lambda \frac{\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W^L_{i,s,t}}\right\}.$$

These new guesses obviously satisfy $\hat{L}^{(1)}_{i,s,t} \leq L^U_{i,s,t}$ and $\hat{W}^{(1)}_{i,s,t} \geq W^L_{i,s,t}$. The new guesses also satisfy the C.S. condition. To see this, notice that it cannot happen that:

$$\hat{W}^{(1)}_{i,s,t} = \frac{(1-\lambda)\hat{W}^{(0)}_{i,s,t}\hat{L}^{(0)}_{i,s,t} + \lambda \frac{\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L^U_{i,s,t}}$$

$$\hat{L}^{(1)}_{i,s,t} = \frac{(1-\lambda)\hat{W}^{(0)}_{i,s,t}\hat{L}^{(0)}_{i,s,t} + \lambda \frac{\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W^L_{i,s,t}},$$

since that would require:

$$\frac{(1-\lambda)\hat{W}^{(0)}_{i,s,t}\hat{L}^{(0)}_{i,s,t} + \lambda \frac{\phi_{i,s} R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L^U_{i,s,t}} \geq W^L_{i,s,t}$$

$$L_{i,s,t}^U \geq \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'(\hat{\mathbf{W}}^{(0)},\hat{\mathbf{L}}^{(0)})}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L}.$$

Putting the last two inequalities together we get:

$$(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'(\hat{\mathbf{W}}^{(0)},\hat{\mathbf{L}}^{(0)})}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}} \geq W_{i,s,t}^L L_{i,s,t}^U \geq (1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'(\hat{\mathbf{W}}^{(0)},\hat{\mathbf{L}}^{(0)})}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}},$$

which is impossible unless both inequalities hold with equality (in which case all the relevant conditions are satisfied anyway). This means that unless we are in a knife edge case (where everything works fine) we are going to be either in the point:

$$\left(\hat{L}_{i,s,t}^{(1)}, \hat{W}_{i,s,t}^{(1)}\right) = \left(L_{i,s,t}^U, \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L_{i,s,t}^U}\right),$$

or in the point:

$$\left(\hat{L}_{i,s,t}^{(1)}, \hat{W}_{i,s,t}^{(1)}\right) = \left(\frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L}, W_{i,s,t}^L\right),$$

which means that the C.S. condition is satisfied. It is also true that the new guess satisfies the nominal anchor if the previous guess did. To see this, notice that (from the observation that we are always in either of those special points) the following always holds:

$$\hat{W}_{i,s,t}^{(1)}\hat{L}_{i,s,t}^{(1)} = (1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R_{i,s,t}'}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}}.$$

Multiplying this by $Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}$ and summing it over i and s we get:

$$\sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(1)}\hat{L}_{i,s,t}^{(1)}Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t} = (1-\lambda)\sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)}Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}$$
$$+ \lambda \sum_{i=1}^I \sum_{s=1}^S \frac{\phi_{i,s}R_{i,s,t}'}{Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}}Y_{i,s,t-1}'\dot{W}_{i,s,t}\dot{L}_{i,s,t}.$$

Focusing on the last term, it is possible to show that:

$$\sum_{i=1}^{I}\sum_{s=1}^{S}\phi_{i,s}R'_{i,s,t} = \sum_{j=1}^{I}\sum_{r=1}^{S}\hat{W}_{j,r,t}^{(0)}\hat{L}_{j,r,t}^{(0)}Y'_{j,r,t-1}\dot{W}_{j,r,t}\dot{L}_{j,r,t}.$$

This makes it clear that:

$$\sum_{i=1}^{I}\sum_{s=1}^{S}\hat{W}_{i,s,t}^{(1)}\hat{L}_{i,s,t}^{(1)}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t} = \sum_{i=1}^{I}\sum_{s=1}^{S}\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}.$$

Therefore, if the initial guess satisfies the nominal anchor the new guess will do so as well. Finally, when the algorithm converges, for example at iteration N, the following holds:

$$\hat{W}_{i,s,t}^{(N)}\hat{L}_{i,s,t}^{(N)} = (1-\lambda)\hat{W}_{i,s,t}^{(N)}\hat{L}_{i,s,t}^{(N)} + \lambda\frac{\phi_{i,s}R'_{i,s,t}}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}},$$

which implies $\hat{W}_{i,s,t}^{(N)}\hat{L}_{i,s,t}^{(N)}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t} = \phi_{i,s}R'_{i,s,t}$, indicating that the final guess solves our desired system. We use the following initial guess which satisfies the nominal anchor,

$$\hat{W}_{i,s,t}^{(0)} = \frac{1}{\dot{W}_{i,s,t}}, \qquad \hat{L}_{i,s,t}^{(0)} = \frac{1}{\dot{L}_{i,s,t}}.$$

A.8 Welfare

We start from our previous result that

$$V_{j,s,t} = \ln(\omega_{j,s,t}) + \kappa\ln\left(\sum_{i=1}^{I}\left(\sum_{k=0}^{S}\exp\left(\beta V_{i,k,t+1} - \varphi_{ji,sk}\right)^{1/\nu}\right)^{\nu/\kappa}\right) + \gamma\kappa.$$

Using

$$\mu_{jj,s\#,t} = \frac{\left(\sum_{h=0}^{S}\exp\left(\beta V_{j,h,t+1} - \varphi_{jj,sh}\right)^{1/\nu}\right)^{\nu/\kappa}}{\sum_{m=1}^{I}\left(\sum_{h=0}^{S}\exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa}},$$

we then have

$$\sum_{m=1}^{I}\left(\sum_{h=0}^{S}\exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu}\right)^{\nu/\kappa} = \mu_{jj,s\#,t}^{-1}\left(\sum_{h=0}^{S}\exp\left(\beta V_{j,h,t+1} - \varphi_{jj,sh}\right)^{1/\nu}\right)^{\nu/\kappa}.$$

Next, using

$$\mu_{jj,ss|j,t} = \frac{\exp\left(\beta V_{j,s,t+1}\right)^{1/\nu}}{\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1} - \varphi_{jj,sh}\right)^{1/\nu}},$$

we have

$$\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1} - \varphi_{jj,sh}\right)^{1/\nu} = \mu_{jj,ss|j,t}^{-1} \exp\left(\beta V_{j,s,t+1}\right)^{1/\nu},$$

and hence

$$\mu_{jj,s\#,t}^{-1} \left(\sum_{h=0}^{S} \exp\left(\beta V_{j,h,t+1} - \varphi_{jj,sh}\right)^{1/\nu} \right)^{\nu/\kappa} = \mu_{jj,s\#,t}^{-1} \mu_{jj,ss|j,t}^{-\nu/\kappa} \exp\left(\beta V_{j,s,t+1}\right)^{1/\kappa}.$$

This implies that

$$\kappa \ln \left(\sum_{m=1}^{I} \left(\sum_{h=0}^{S} \exp\left(\beta V_{m,h,t+1} - \varphi_{jm,sh}\right)^{1/\nu} \right)^{\nu/\kappa} \right) = \beta V_{j,s,t+1} - \kappa \ln\left(\mu_{jj,s\#,t}\right) - \nu \ln\left(\mu_{jj,ss|j,t}\right).$$

We then write

$$V_{j,s,t} = \ln(\omega_{j,s,t}) - \kappa \ln\left(\mu_{jj,s\#,t}\right) - \nu \ln\left(\mu_{jj,ss|j,t}\right) + \gamma\kappa + \beta V_{j,s,t+1}.$$

Iterating this equation forward, we obtain

$$V_{j,s,t} = \sum_{r=t}^{\infty} \beta^{r-t} \left(\ln(\omega_{j,s,t}) - \kappa \ln\left(\mu_{jj,s\#,t}\right) - \nu \ln\left(\mu_{jj,ss|j,t}\right) + \gamma\kappa \right).$$

We define the CV in consumption for market js at time $t = 0$ to be the scalar $\zeta_{j,s}$ such that

$$V'_{j,s,0} = V_{j,s,0} + \sum_{r=0}^{\infty} \beta^r \ln(\zeta_{j,s}) = \sum_{r=0}^{\infty} \beta^r \left(\ln\left(\frac{\omega_{j,s,r} \zeta_{j,s}}{\left(\mu_{jj,ss|j,r}\right)^{\nu} \left(\mu_{jj,s\#,r}\right)^{\kappa}} \right) + \gamma\kappa \right).$$

Rearranging the definition, we can write:

$$
\begin{aligned}
(V'_{j,s,0} - V_{j,s,0}) &= \ln(\zeta_{j,s}) \sum_{r=0}^{\infty} \beta^r \\
\ln(\zeta_{j,s}) &= (1-\beta)(V'_{j,s,0} - V_{j,s,0})
\end{aligned}
$$

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$$= \sum_{r=1}^{\infty} \beta^r \ln \left(\frac{\hat{\omega}_{j,s,r}}{\left(\hat{\mu}_{jj,ss|j,r} \right)^{\nu} \left(\hat{\mu}_{jj,s\#,r} \right)^{\kappa}} \right),$$

which is the expression that we will use for the "welfare change" stemming from the China shock, formally the compensating variation in consumption due to the shock.

A.9   More on Calibration

As we discussed in the main text, the multiplicative nature of our productivity decomposition, $\hat{A}_{China,s,t} = \hat{A}^1_{China,t} \hat{A}^2_{China,s}$, implies that their level is not identified. For example, if we multiply all the $\hat{A}^2_{China,s}$ by a constant c and we divide all the $\hat{A}^1_{China,t}$ by c, then we would have the same $\hat{A}_{China,s,t}$. Thus, we use the normalization $\sum_{s=1}^{S} \hat{A}^2_{China,s} = 1$. Correspondingly, the model is only able to produce changes in imports that satisfy $\sum_{t=2001}^{2007} \Delta X^{model}_{C,US,t} = \sum_{s=1}^{S} \Delta X^{2007-2000,model}_{C,US,s}$. This condition is automatically satisfied by the actual changes, i.e. $\sum_{t=2001}^{2007} \Delta X_{C,US,t} = \sum_{s=1}^{S} \Delta X^{2007-2000}_{C,US,s}$, but not necessarily by the predicted changes, due to the lack of a constant in the second regression. We adjust the predicted changes in manufacturing so that they satisfy: $\sum_{t=2001}^{2007} \widehat{\Delta X_{C,US,t}} = \sum_{s=1}^{S} \widehat{\Delta X^{2007-2000}_{C,US,s}}$, this adjustment is very small. In all of our applications we match our targets with an accuracy greater than 99.9%.

B   Data Construction

In this appendix section, we provide details on the construction of the data we briefly described in Section 3. We divide this appendix into three parts. Appendix B.1 describes all data sources. Appendix B.2 discusses how we combine the different data sources to compute an internally consistent bilateral trade-flow matrix for all sectors for the years when all the data is available. It also discusses how we use the previous step to project a bilateral trade-flows between states and countries for the years before full data availability. Finally, Appendix B.3 discusses the construction of the initial employment allocations for all regions and the bilateral migration flows between sectors and U.S. states.

B.1 Data Description and Sources

List of sectors. We use a total of 14 sectors. The list includes 12 manufacturing sectors, one catch-all services sector, and one agriculture sector. We follow CDP in the selection of the 12 manufacturing sectors. These are: **1)** Food, beverage, and tobacco products (NAICS 311-312, WIOD sector 3); **2)** Textile, textile product mills, apparel, leather, and allied products (NAICS 313-316, WIOD sectors 4-5); **3)** Wood products, paper, printing, and related support activities (NAICS 321-323, WIOD sectors 6-7); **4)** Mining, petroleum and coal products (NAICS 211-213, 324, WIOD sectors 2, 8); **5)** Chemical (NAICS 325, WIOD sector 9); **6)** Plastics and rubber products (NAICS 326, WIOD sector 10); **7)** Nonmetallic mineral products (NAICS 327, WIOD sector 11); **8)** Primary metal and fabricated metal products (NAICS 331-332, WIOD sector 12); **9)** Machinery (NAICS 333, WIOD sector 13); **10)** Computer and electronic products, and electrical equipment and appliance (NAICS 334-335, WIOD sector 14); **11)** Transportation equipment (NAICS 336, WIOD sector 15); **12)** Furniture and related products, and miscellaneous manufacturing (NAICS 337- 339, WIOD sector 16). There is a **13)** Services sector which includes Construction (NAICS 23, WIOD sector 18); Wholesale and retail trade sectors (NAICS 42-45, WIOD sectors 19-21); Accommodation and Food Services (NAICS 721-722, WIOD sector 22); transport services (NAICS 481-488, WIOD sectors 23-26); Information Services (NAICS 511-518, WIOD sector 27); Finance and Insurance (NAICS 521-525, WIOD sector 28); Real Estate (NAICS 531-533, WIOD sectors 29-30); Education (NAICS 61, WIOD sector 32); Health Care (NAICS 621-624, WIOD sector 33); and Other Services (NAICS 493, 541, 55, 561, 562, 711-713, 811-814, WIOD sector 34).[31]

List of countries: We use data for 50 U.S. states, 37 other countries including a constructed rest of the world. The list of countries is: Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, the Slovak Republic, Slovenia, S. Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.

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[31]The only difference with respect to CDP in the definition of manufacturing sectors is that we include Mining (NAICS 211-213) together with Petroleum and Coal Products (NAICS 324) in our sector 4.

Data on bilateral trade between countries. World Input-Output Database (WIOD). Release of 2013. We use data for 2000-2007. We map the sectors in the WIOD database to our 14 sectors in the following way: **1)** Food Products, Beverage, and Tobacco Products (c3); **2)** Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4-c5); **3)** Wood Products, Paper, Printing, and Related Support Activities (c6-c7); **4)** Petroleum and Coal Products (c8); **5)** Chemical (c9); **6)** Plastics and Rubber Products (c10); **7)** Nonmetallic Mineral Products (c11); **8)** Primary Metal and Fabricated Metal Products (c12); **9)** Machinery (c13); **10)** Computer and Electronic Products, and Electrical Equipment and Appliances (c14); **11)** Transportation Equipment (c15); **12)** Furniture and Related Products, and Miscellaneous Manufacturing (c16); **13)** Construction (c18), Wholesale and Retail Trade (c19-c21), Transport Services (c23-c26), Information Services (c27), Finance and Insurance (c28), Real Estate (c29- c30); Education (c32); Health Care (c33), Accommodation and Food Services (c22), and Other Services (c34); **14)** Agriculture and Mining (c1-c2). We follow Costinot and Rodriguez-Clare (2014) to remove the negative values in the trade data from WIOD.

Data on bilateral trade in manufacturing between U.S states. We combine the 2002 and 2007 Commodity Flow Survey (CFS) with the WIOD database. The CFS records shipments between U.S states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). We follow CDP and use CFS 2007 tables that cross-tabulate establishments by their assigned NAICS codes against commodities (SCTG) shipped by establishments within each of the NAICS codes. These tables allow for mapping of SCTG to NAICS.

Data on bilateral trade in manufacturing and agriculture between U.S states and the rest of the countries. We obtain sector-level imports and exports between the 50 U.S. states and the list of other countries from the Import and Export Merchandise Trade Statistics, which is compiled by the U.S. Census Bureau. This dataset reports imports and exports in each NAICS sector between each U.S. state and each other country in the world. Data for exports at the state×sector level starts in 2002. Data for imports at the state×sector level starts in 2008.

Data on sectoral and regional value added share in gross output. Value added for each of the 50 U.S. states and 14 sectors can be obtained from the Bureau of Economic Analysis

(BEA) by subtracting taxes and subsidies from GDP data. In the cases when gross output was smaller than value added we constrain value added to be equal to gross output. For the list of other countries we obtain the share of value added in gross output using data on value added and gross output data from WIOD.

Data on services expenditure and production. We compute bilateral trade in services using a gravity approach explained in Appendix B.2. As part of this calculations we require data on production and expenditure in services by region. We obtain U.S. state-level services GDP from the Regional Economic Accounts of the Bureau of Economic Analysis (BEA). We obtain U.S. state-level services expenditure from the Personal Consumption Expenditures (PCE) database of BEA. Finally, for the list of other countries we compute total production and expenditure in services from WIOD.

Data on agriculture expenditure and production. We also compute bilateral trade in agriculture using a gravity approach explained in Appendix B.2. To get production in agriculture for the U.S. states we combine the 2002 and 2007 Agriculture Census with the National Marine Fisheries Service Census to get state-level production data on crops and livestock and seafood. We infer state-level expenditure in agriculture from our gravity approach explained in Appendix B.2. Finally, for the list of other countries we compute total production and expenditure in agriculture from WIOD.

Data on population and geographic coordinates. As part of the gravity approach to compute bilateral trade in services, we also need to compute bilateral distances between regions. We follow the procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. We thus require data on the most populated cities in each country, the cities' coordinates and population, and each country's population. We obtain this information from the United Nations' Population Division website. In particular, we use the population of urban agglomerations with 300,000 inhabitants or more in 2018, by country, for 2000-2007. For Austria, Cyprus, Denmark, Estonia, Hungary, Ireland, Lithuania, Slovakia and Slovenia we use the two most populated cities.[32] For the case of U.S. states, we use population and coordinates data for each U.S county within each U.S state. The data for the U.S. counties comes from the U.S. CENSUS.

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[32]For the specific case of Cyprus, the cities' information comes from the country's Statistical Service.

Data on employment and migration flows. For the case of countries, we take data on employment by country and sector from the WIOD Socio Economic Accounts (WIOD-SEA). For the case of U.S. states, we take sector-level employment (including unemployment and non-participation) from the 5% sample PUMS files of the 2000 Census. We only keep observations with age between 25 and 65, who are either employed, unemployed, or out of the labor force. We construct a matrix of migration flows between sectors and U.S. states by combining data from the American Community Survey (ACS) and the Current Population Survey (CPS). Finally, we abstract from international migration.

B.2 Construction of the Bilateral Trade Flows Between Regions

We follow the notation from Costinot and Rodriguez-Clare (2014) and omit the time subscripts t that are relevant in our quantitative model. Define $X_{ij,ks}$ as sales of intermediate goods from sector k in region i to sector s in region j, and $X_{ij,kF}$ as the sales of sector k in region i to the final consumer of region j. Our final objective is to construct a bilateral trade flows matrix between all regions in our sample with elements equal to $X_{ij,k} = \sum_s X_{ij,ks} + X_{ij,kF}$. This matrix allows us to compute the trade shares $\lambda_{ij,k}$, and the sector-level revenues $R_{j,k} = \sum_l X_{jl,k}$ for each region, which are crucial elements in our hat algebra described in Section 2.6.

As additional definitions, take $E_{j,k} = \sum_i X_{ij,k}$ as the total expenditure of region j in sector k, $F_{j,k} = \sum_i X_{ij,kF}$ as the final consumption in region j of sector k, $F_j = \sum_k F_{j,k}$ as the total final consumption of region j, and $X_{j,ks} = \sum_i X_{ij,ks}$ as the total purchases that sector s in region j makes from sector k. We construct the matrix of $X_{ij,k}$ in four parts explained below. With some abuse of notation, we refer to a region i as a U.S. state (country) by $i \in US$ ($i \notin US$).

Step 1: Bilateral trade between countries. In the first step we focus on the case where both i and j are countries. Thus, we simply take $X_{ij,k} = X_{ij,k}^{WIOD}$, where $X_{ij,k}^{WIOD}$ are the bilateral trade flows that come directly from the WIOD database.

Step 2: Manufacturing trade among U.S. states. In the second step we focus on manufacturing bilateral trade between U.S. States. For this, we combine the closest Commodity Flow Survey (CFS) for each year with WIOD Data for the total trade of the U.S. with itself.

We first compute the shares that each state i exports to state j in sector k represent in the total trade of sector k according to CFS. Then, we calculate the total exports of state i to state j in sector k as WIOD's U.S. trade with itself in sector k multiplied by the share computed in the previous step to ensure that bilateral trade between states adds up to the WIOD total.

Step 3: Manufacturing trade between U.S. states and countries. For the third step, we combine Census and WIOD data to calculate the trade flows between each of the 50 U.S. states and the other 37 country regions. There is limited availability for the state$\times$sector-level trade data coming from the CENSUS. Data for exports at the state$\times$sector-level starts in 2002 and data for imports starts in 2008. We scale state-level imports and exports data from the Import and Export Merchandise Trade Statistics to match the U.S. totals in WIOD. More precisely, the exports (imports) of state i to (from) country j in manufacturing sector k are computed as a proportion of WIOD's U.S. export (imports) to (from) country j in sector k. This proportion is equal to the exports (imports) of state i to (from) country j in sector k relative to the total U.S. exports (imports) to (from) country j in sector k.

Since the Import and Export Merchandise Trade Statistics data for exports starts in 2002 and for imports starts in 2008, the bilateral trade flows between regions for the years before the data starts cannot be computed directly from the data. We adapt our computation method to take into account this issue. All previous procedures remain the same. Denote $X_{ij,k}^{base}$ as the matrix $X_{ij,k}$ for the first year where the exports or imports data is available (the base year). Define the share of exports of U.S. State i in sector k, going to country j in the base year as $y_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{h \in US} X_{hj,k}^{base}}$ $\forall i \in US$, $j \notin US$. Similarly, define the share of imports of U.S. state j in sector k, coming from country i in the base year as $e_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{l \in US} X_{il,k}^{base}}$ $\forall i \notin US$, $j \in US$. Finally for each sector k in manufacturing or agriculture; and any year before the base year define $X_{ij,k} = e_{ij,k}^{base} X_{i\,US,k}^{WIOD}$ $\forall i \notin US$, $\forall j \in US$ and $X_{ij,k} = y_{ij,k}^{base} X_{US\,j,k}^{WIOD}$ $\forall i \in US$, $\forall j \notin US$.

Step 4: Trade in services and trade in agriculture. We compute bilateral trade flows for services and agriculture separately using a gravity structure that matches WIOD totals for trade between countries (including the U.S.). We start with the standard gravity equation (for simplicity, we remove the subscript of the sector) $X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j} \right)^{-\varepsilon} E_j$, where $P_j^{-\varepsilon} = \sum_i \left(w_i \tau_{ij} \right)^{-\varepsilon}$. We know that $\sum_j X_{ij} = R_i$ and hence $\sum_j \left(\frac{w_i \tau_{ij}}{P_j} \right)^{-\varepsilon} E_j = R_i$. This implies

$w_i^{-\varepsilon}\Pi_i^{-\varepsilon} = R_i$, where $\Pi_i^{-\varepsilon} = \sum_j \tau_{ij}^{-\varepsilon} P_j^{\varepsilon} E_j$. Let $\tilde{P}_j \equiv P_j^{-\varepsilon}$ and $\tilde{\Pi}_i \equiv \Pi_i^{-\varepsilon}$, and $\tilde{\tau}_{ij} \equiv \tau_{ij}^{-\varepsilon}$. Given $\{E_j\}$, $\{R_i\}$, and $\{\tilde{\tau}_{ij}\}$, one we can get $\{\tilde{P}_j\}$ and $\{\tilde{\Pi}_i\}$ from the following system:

$$\tilde{P}_j = \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i$$

$$\tilde{\Pi}_i = \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \tag{B1}$$

The solution for $\{\tilde{P}_j, \tilde{\Pi}_i\}$ is unique up to a constant (Fally, 2015). This indeterminacy requires a normalization. We thus impose $\tilde{P}_1 = 100$ in each exercise. Then one can compute our outcome of interest $\{X_{ij}\}$ from

$$X_{ij} = \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j. \tag{B2}$$

Computation of the bilateral resistance $\tilde{\tau}_{ij}$. To solve the gravity system, we must first compute $\tilde{\tau}_{ij} \ \forall i, \ j$. We proceed by assuming the following functional form: $\tilde{\tau}_{ij} = \beta_0^{\iota_{ij}} dist_{ij}^{\beta_1} \exp\left(\xi_{ij}\right)$, where ι_{ij} is an indicator variable equal to 1 if $i = j$, and ξ_{ij} is an idiosyncratic error term. β_1 captures the standard distance elasticity and β_0 captures the additional *inverse* resistance of trading with others versus with oneself.

To calculate $dist_{ij}$, we follow the same procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. The idea is to calculate the distance between two countries based on bilateral distances between the largest cities of those two countries, those inter-city distances being weighted by the share of the city in the overall country's population (Head and Mayer, 2002).

We use population for 2010 and coordinates data for all U.S. counties, and all cities around the world with more than 300,000 inhabitants. For those countries with less than two cities of this size, we take the two largest cities. Coordinates are important to calculate the physical bilateral distances in kms between each county r in state i and county s in state j ($d_{rs} \ \forall r \in i$, $s \in j$ and $\forall i, j = 1, ..., 50$), and define $dist \ (ij)$ as:

$$dist \ (ij) = \left(\sum_{r \in i} \sum_{s \in j} \left(\frac{pop_r}{pop_i} \right) \left(\frac{pop_s}{pop_j} \right) d_{rs}^{\theta} \right)^{1/\theta}, \tag{B3}$$

where pop_h is the population of country/state h. We set $\theta = -1$.

Given our definition of $\tilde{\tau}_{ij}$ we can write the gravity equation between countries as $X_{ij} = \beta_0^{\iota_{ij}} dist_{ij}^{\beta_1} \exp\left(\xi_{ij}\right) \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j$. Taking logs we can write the previous equation as:

$$\ln X_{ij} = \delta_i^o + \delta_j^d + \tilde{\beta}_0 \iota_{ij} + \beta_1 \ln dist_{ij} + \xi_{ij}, \tag{B4}$$

where $\tilde{\beta}_0 = \ln \beta_0$ and the δs are fixed effects. We first estimate the equation above separately for services and agriculture using a 2000-2011 panel of bilateral trade flows between countries from WIOD. We present our OLS estimation results in Table B.1. Columns (1) and (2) refer to the estimated coefficients for the case of services and agriculture, respectively. Both regressions include year-by-origin and year-by-destination fixed effects. We take these estimates and compute the bilateral resistance term in each sector as $\hat{\tilde{\tau}}_{ij} = exp(\hat{\tilde{\beta}}_0 \iota_{ij} + \hat{\beta}_1 \ln dist_{ij})$.

Table B.1: Estimation of Own-Country Dummy and Distance Elasticity

	(1)	(2)
Dep. Var.: $\ln X_{ij,t}$	Services	Agriculture
ι_{ij}	7.357 ***	4.143***
	(0.126)	(0.145)
$\ln dist_{ij}$	-0.376***	-1.745***
	(0.037)	(0.020)
Year$\times$Orig.	Yes	Yes
Year$\times$Dest.	Yes	Yes
Observations	17,328	17,328
Adjusted R^2	0.66	0.76

Notes: This table displays the OLS estimates of specifications analogous to the one in equation (B4). The outcome variable $\ln X_{ij,t}$ is the log exports of country i sent to country j. The own-country dummy ι_{ij} is defined as an indicator function equal to one whenever country i is the same as country j. Finally, $\ln dist_{ij}$ is the log distance between country i and country j. This variable is computed according to equation (B3). Robust standard errors are presented in parenthesis. *** denotes statistical significance at the 1%.

Trade in services. As inputs, we need total expenditures in services for each region (E_i), as well as total production in services (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we take these variables from the Regional Economic Accounts of the Bureau of Economic Analysis. We scale the state-level services production

and expenditures so that they aggregate to the U.S. totals in WIOD.

We incorporate the information on bilateral trade in services between countries (including the U.S.) that comes from WIOD to the gravity system of equation (B1) by first writing the system as $\tilde{P}_j = \sum_{i \notin US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i + \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i$ and $\tilde{\Pi}_i = \sum_{j \notin US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j + \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j$. Then, we define $\tilde{\lambda}_j \equiv 1 - \frac{\sum_{i \notin US} X_{ij}}{E_j}$ for $j \notin US$ (the share of imports of region $j \notin US$ coming from the U.S.) and $\tilde{\lambda}_i^* \equiv 1 - \frac{\sum_{j \notin US} X_{ij}}{R_i}$ for $i \notin US$ (total exports of region $i \notin US$ to other regions not in the U.S.). Using these two definitions and substituting $\tilde{\tau}_{ij} = X_{ij} \tilde{\Pi}_i \tilde{P}_j R_i^{-1} E_j^{-1}$ whenever $i, j \notin US$ in the previous system of equations we have the final system we solve for services:

$$
\begin{aligned}
\tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \in US \\
\tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \in US \\
\tilde{\lambda}_j \tilde{P}_j &= \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \notin US \\
\tilde{\lambda}_i^* \tilde{\Pi}_i &= \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \notin US
\end{aligned}
$$

Once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the final bilateral trade matrix according to equation (B2).

Trade in agriculture. As inputs, we need total expenditures in services for each region (E_i), as well as total production in agriculture (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we compute total production (R_i) by combining data from the Agriculture Census and the National Marine Fisheries Service Census. We scale the state-level agriculture production so that it aggregates to the U.S. total in WIOD. However, it is not possible to find state-level agriculture expenditure for U.S. states. To overcome this data unavailability, we combine the U.S. input-output matrix $(\phi_{j,ks})$ together with the shares of value-added in gross production $(\phi_{j,k})$ in order to compute a value of (E_i) that is consistent with the full bilateral trade matrix for all regions and all sectors.

In order to describe our procedure, note that the total expenditure of region j in sector k $(E_{j,k})$ could be written as $E_{j,k} = \sum_s \tilde{\phi}_{j,ks} R_{j,s} + F_{j,k}$, where $\tilde{\phi}_{j,ks} = \phi_{j,ks}(1 - \phi_{j,s})$. We make two assumptions. First, we assume that $\tilde{\phi}_{j,ks} = \tilde{\phi}_{US,ks} \ \forall j \in US$, which means that we assume

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common input-output matrix and value-added shares across U.S. states and equal to the ones of the U.S. as a whole. Second, we assume identical Cobb-Douglas preferences across U.S. states. This means that when $j \in US$ we have that $F_{j,k} = \frac{F_j}{F_{us}} F_{US,k} = F_j \gamma_k$, where $\gamma_k \equiv \frac{F_{US,k}}{F_{us}}$. Using these two assumptions we get $F_j = E_{j,k} - \sum_s \tilde{\phi}_{j,ks} R_{j,s} + \sum_{r \neq k} \left(E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s} \right)$. Substituting the previous equation in the definition of $E_{j,k}$ for the agriculture sector ($k = AG$), and $j \in US$ we find

$$E_{j,AG} = \sum_s \tilde{\phi}_{j,AGs} R_{j,s} + \frac{\gamma_{AG}}{1 - \gamma_{AG}} \sum_{r \neq AG} \left(E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s} \right),$$

which can be computed using state-level production of all sectors and state-level expenditure data of all other sectors (excluding agriculture), combined with the U.S.-level input-output matrix, value-added shares, and sector-level consumption shares.

Once we obtain the state-level expenditure values in agriculture, we can proceed with the gravity system in equation (B1). As in the case of services, we incorporate the information on bilateral trade in agriculture between countries that comes from WIOD. We also incorporate the bilateral trade in agriculture between U.S. states and other countries coming from the Import and Export Merchandise Trade Statistics. Thus, we only need to focus on $\{\tilde{P}_j\}_{j \in US}$ and $\{\tilde{\Pi}_i\}_{i \in US}$. Define $\chi_i^* = 1 - \sum_{j \notin US} \frac{X_{ij}}{R_i}$ for $i \in US$ (the share of sales of state i that stay in the U.S.) and $\chi_j = 1 - \sum_{i \notin US} \frac{X_{ij}}{E_{j,k}}$ for $j \in US$ (the share of purchases of state i that come from the U.S.). The final system we solve for agriculture becomes:

$$\chi_j \tilde{P}_j = \Sigma_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i, \forall j \in US$$
$$\chi_i^* \tilde{\Pi}_i = \Sigma_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j, \forall i \in US$$

As before, once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the bilateral trade in agriculture between U.S. states according to equation (B2).

B.3 Initial Employment Allocations for each Region and Bilateral Migration Flows between Sectors and U.S. States

This subsection explains how to compute the initial labor allocation and the bilateral labor mobility matrix. Most of the steps follow CDP.

Employment allocation in each region and sector. For the case of countries outside of the U.S., we first compute the employment distribution by country-sector from the WIOD-SEA. We treat unemployed and out of labor force as an additional sector. The data for that sector combines WIOD-SEA's worker population and each country's labor force participation rate from World Bank data. Since SEA does not include the RoW directly and since the remaining countries in SEA are too few, we define RoW's employment such that its production to employment ratio equals the respective average ratio of the other 37 countries. This calculation is done separately for each sector.

For the case of U.S. states, we calculate the employment level for each state and sector (including unemployment and non-participation) in the year 2000 from the 5 % sample PUMS files of the 2000 Census. We only keep observations type "P" (persons) aged 25 to 65, who are either employed, unemployed, or out of the labor force. We take unemployment plus non-participation as a different sector. Finally, we apply proportionality so that the aggregate employment at the sector level coincides with the totals for the U.S. in WIOD.

Workers' mobility matrix for U.S. states. Let $L_{ji,sk}$ be the number of workers who move from state j and sector s to state i and sector k between two periods (we ignore the time subscript for simplicity). We want to compute the mobility matrix for the shares $\mu_{ji,sk}$, for each origin state j, origin sector s, destination state i, and destination sector k, with the shares defined as $\mu_{ji,sk} = \frac{L_{ji,sk}}{\sum_{i'} \sum_{k'} L_{ji',sk'}}$. To do this we combine data from the American Community Survey (ACS) and the Current Population Survey (CPS) as explained below.

The ACS provides details of workers' current employment status, sector, and state. It also asks the state in which respondents lived the prior year. However, this survey does not provide information regarding people's employment status and sector in the previous year. This means that we can construct from the ACS data $L_{ji,\#k}^{ACS} \, \forall j, i \in US$ and destination sector k (interstate flows but without knowing the sector of origin). The CPS provides details of

people's employment status and industry each month, but it does not provide information regarding movements across states. This means that we can construct from the CPS data $L_{jj,sk}^{CPS} \forall j \in US$ and any origin or destination sectors s,k (intra-state flows of people between sectors).[33]

To combine both $L_{ji,\#k}^{ACS} \forall j,i \in US$ and $L_{jj,sk}^{CPS} \forall j \in US$ to compute the labor transitions across states and sectors, we follow CDP by assuming that interstates movements (j to i) across sectors follow the same pattern that intrastate moves in the destination state i across sectors. We then apply proportionality to the flows from CPS to sum up to total flows in ACS (which do not require additional assumptions and are available for interstate movements). This means that we define $L_{ji,sk} = \sum_{k'} L_{ji,\#k'}^{ACS} \times \frac{L_{ii,sk}^{CPS}}{\sum_{s'} \sum_{k'} L_{ii,s'k'}^{CPS}}$ $\forall i,j \in US, \ \forall s,k$. Note that $\sum_k \sum_s L_{ji,sk} = \sum_q L_{ji,\#q}^{ACS}$ (so the total movements between states add up to the total movements from ACS.). Also note that $\frac{L_{jj,sk}}{L_{jj,sk'}} = \frac{L_{jj,sk}^{CPS}}{L_{jj,sk'}^{CPS}}$ (so that the relative importance between destination sectors comes from CPS data). Finally, in the few cases when the diagonal value of the matrix (same state and sector in origin and destination) is zero, we change it to the minimum non-zero diagonal value.

Smoothing flows in shares. As discussed in the main text, using self-reported information from the CPS and ACS surveys to measure mobility flows is known to be problematic due to the prevalence of misclassification errors (Murphy and Topel, 1987; Kambourov and Manovskii, 2013; Dvorkin, 2021). For instance, Dvorkin (2021) shows that interindustry mobility rates computed using uncorrected PSID data could be around twice as large than alternative data for which misclassification is likely absent, even when using broad one-digit ISIC sector codes.

To avoid the artificially large mobility flows due to the misclassification issue, we smoothed the mobility flows in shares such that the set of migration flows in our first period implies a steady state in the U.S. in that period. This smoothing means that given a set of

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[33]The CPS surveys households in a 4-8-4 format; that is, it interviews the household for four consecutive months, gives them an 8-month break, and interviews them again for four straight months. We match CPS observations (individuals) across time using the interview number. The first four monthly interviews are 12 months apart from the final four interviews, and the first four and final four are consecutive in months. Since we are interested in recording annual changes, we only keep interview months (1,5) which is equivalent to following each individual for the first twelve months she appears in the survey. To avoid noise in our sample, we use observations for the previous two years and the following two years for the year of interest.

$\mu_{ji,sk}$ coming from the data for 2000, we find a new set of flows $\mu'_{ji,sk}$ that satisfy the following conditions:

1. They are greater than zero $\mu'_{ji,sk} \geq 0$ and they sum to one for each sender market over all receiver markets: $\sum_{i=1}^{I} \sum_{k=0}^{S} \mu'_{ji,sk} = 1$.

2. They imply a steady state with the labor data in 2000, which we will denote with $L_{j,s}$. This means that if the original distribution of labor is described by the $L_{j,s}$'s, this distribution is preserved after the flows occur: $L_{i,k} = \sum_{j=1}^{I} \sum_{s=0}^{S} \mu'_{ji,sk} L_{j,s}$.

3. The probability that someone in any given region-sector is stays in region i is the same across the original and the new mobility matrices $\sum_{k=0}^{S} \mu_{ii,sk} = \sum_{k=0}^{S} \mu'_{ii,sk}$.

4. If the original mobility matrix has a given flow as zero, then this must still be the case in the new mobility matrix: $\mu'_{ji,sk} = 0$ if $\mu_{ji,sk} = 0$.

5. The new $\mu'_{ji,sk}$ minimize the sum of square differences between the new μ's and the original ones, i.e.:

$$\sum_{j=1}^{I} \sum_{i=1}^{I} \sum_{s=0}^{S} \sum_{k=0}^{S} (\mu'_{ji,sk} - \mu_{ji,sk})^2.$$

We solve the previous problem of minimizing the sum of squared differences subject to the constraints in items 1-5. The change in the flows implied by this procedure is very small. In particular, the correlation between the original $\mu_{ji,sk}$ and the $\mu'_{ji,sk}$ is 99.69%.

Mobility matrix for non-U.S. regions. We do not take the mobility matrix for each country outside of the U.S. from the data, which would be cumbersome because we have 37 other countries. However, it can be shown (details provided upon request), that for a country with a single region (such as non-U.S. countries in our context), the fact that there are no mobility costs can be captured by setting a special mobility matrix between 1999 and 2000. Thus, we compute the elements of that mobility matrix between 1999 and 2000. To do this, we take as given the labor distribution in 1999 ($L_{i,s,0}$) and 2000 ($L_{i,s,1}$) and compute the following formula:

$$\mu_{ii,sk,0} = \frac{L_{i,k,1}}{\sum_{r=1}^{S} L_{i,r,0}}$$

Notice that the flows between sector s and sector k do not depend on information of the sender sector (s), which is implicitly encoding the information that in the countries outside

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of the U.S., mobility between sectors is unlimited.

C   Exposure measures

Consider an economy producing a set of homogeneous goods across sectors $s = 1, ..., S$ with prices p_s. Labor is the only factor of production that is mobile across sectors, and there are decreasing returns to labor in each sector so that $q_s = F_s(l_s)$ with $F_s'(\cdot) > 0$ and $F_s''(\cdot) < 0$. Preferences are given by $U(c) - V(l)$, where $l \equiv \sum_s l_s$, $U(c)$ is homogeneous of degree one, and $V'(\cdot) > 0$ and $V''(\cdot) > 0$. We are interested in the effect of a foreign shock on employment in two different cases. In the first case the wage w is fixed and labor is fully determined by labor demand (we assume that labor supply is higher than labor demand at the fixed wage w), while in the second case the wage is fully flexible and clears the labor market. Below we show that further assuming that $\varepsilon(l_s) \equiv -\frac{F_s''(l_s)l_s}{F_s'(l_s)} = \varepsilon$ for all s and $\mu(l) \equiv \frac{V''(l)l}{V'(l)} = \mu$, then in the case of a fixed wage we have

$$d \ln l = \frac{1}{\varepsilon} \sum_s \left(\frac{p_s q_s}{I} \right) d \ln p_s \tag{C1}$$

while in the case of flexible wages we have

$$d \ln l = \frac{1}{\varepsilon + \mu} \sum_s \left(\frac{p_s q_s - p_s c_s}{I} \right) d \ln p_s, \tag{C2}$$

where $I \equiv \sum_s p_s q_s$. Thus, if the wage is fixed and if we know the log changes in prices resulting from the foreign shock then we can interact them with revenue shares, $\frac{p_s q_s}{I}$, to construct a Bartik-style sufficient statistic for the first order effect on employment. In contrast, if the wage fully adjusts to equalize labor supply and demand, then the appropriate weights (share components in the Bartik measure) for the price changes are instead given by net exports as a share of GDP, to capture the implied terms-of-trade effects. If the economy is small, then prices are exogenous and one could further replace $d \ln p_s$ by the underlying Chinese productivity shocks.

Let's start with the case where w is fixed. Fully differentiating the equilibrium condition $p_s F_s'(l_s) = w$ implies $d \ln l_s = \frac{d \ln p_s}{\varepsilon_s(l_s)}$, where $\varepsilon(l_s) \equiv -\frac{F_s''(l_s)l_s}{F_s'(l_s)}$. We then have $d \ln l = \sum_s m_s \frac{d \ln p_s}{\varepsilon_s(l_s)}$,

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where $m_s \equiv \frac{l_s}{\sum_s l_s}$. Assuming that $\varepsilon_s(l_s) = \varepsilon$ we know that $p_s q_s / I = m_s$ and hence we get (C1).

Now let's consider the case with a flexible wage. The equilibrium is given by w, l, λ and $\{l_s, c_s\}_s$ such that the following equations hold

$$p_s F_s'(l_s) = w \tag{C3}$$

$$\frac{\partial U_s}{\partial c_s} = \lambda p_s \tag{C4}$$

$$V'(l) = \lambda w \tag{C5}$$

$$\sum_s l_s = l \tag{C6}$$

$$\sum_s p_s c_s = \sum_s p_s f_s(l_s). \tag{C7}$$

Differentiating equation (C5) yields $\mu(l) d \ln l = d \ln \lambda + d \ln w$, where $\mu(l) \equiv \frac{V''(l)l}{V'(l)}$. Thus

$$d \ln l = \frac{d \ln (w/P)}{\mu(l)}, \tag{C8}$$

with $P \equiv 1/\lambda$. Next, totally differentiating equations (C3) and (C6) yields $d \ln p_s - \varepsilon d \ln l_s = d \ln w$ and $\sum_s m_s d \ln l_s = d \ln l$. Combined, the previous two equations imply $\sum m_s d \ln p_s - \varepsilon d \ln l = d \ln w$, which combined with (C8) implies (after some rearranging):

$$d \ln (w/P) = \frac{\mu}{\mu + \varepsilon} \left(\sum m_s d \ln p_s - d \ln P \right). \tag{C9}$$

But equation (C4) implies that $\sum_s \frac{\partial U_s}{\partial c_s} c_s = \lambda \sum_s p_s c_s$. Since $U(c)$ is homogeneous of degree one this implies $U(c) = \lambda \sum_s p_s c_s$. Totally differentiating this equation yields $\sum_s \frac{\partial U_s}{\partial c_s} dc_s = (\sum_s p_s c_s) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s$. Using equation (C4) we get $\sum_s \lambda p_s dc_s = (\sum_s p_s c_s) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s$, which, after simplifying, implies

$$d \ln P = d \ln (1/\lambda) = \sum_s \theta_s d \ln p_s, \tag{C10}$$

where $\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s}$. Plugging into (C9) and combining with (C8) we get

$$d \ln l = \frac{1}{\mu + \varepsilon} \sum (m_s - \theta_s) d \ln p_s = d \ln l.$$

Finally, note that $m_s \equiv \frac{l_s}{\sum_s l_s} = \frac{w l_s}{\sum_s w l_s} = \frac{p_s F'_s(l_s) l_s}{\sum_s p_s F'_s(l_s) l_s}$. Using $\varepsilon(l_s) \equiv -\frac{F''_s(l_s) l_s}{F'_s(l_s)} = \varepsilon$, we know that $F_s(l_s) \propto l_s^{1-\varepsilon}$ and $F'_s(l_s) \propto (1-\varepsilon) l_s^{-\varepsilon}$, hence $m_s = \frac{p_s F_s(l_s)}{\sum_s p_s F_s(l_s)} = \frac{p_s q_s}{\sum_s p_s q_s} = \frac{p_s q_s}{I}$. On the other hand, using (C7) we have $\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s} = \frac{p_s c_s}{I}$. Combining all of this we obtain (C2).