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FISCAL STIMULUS UNDER AVERAGE INFLATION TARGETING

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Abstract. The stimulus effects of expansionary fiscal policy under average inflation targeting (AIT) depends on both monetary and fiscal policy regimes. AIT features an inflation makeup under the monetary regime, but not under the fiscal regime. In normal times, AIT amplifies the short-run fiscal multipliers under both regimes while mitigating the cumulative multiplies due to intertemporal substitution. In a zero-lower-bound (ZLB) period, AIT reduces fiscal multipliers under a monetary regime by shortening the duration of the ZLB through expected inflation makeup. Under the fiscal regime, AIT has a nonlinear effect on fiscal multipliers because of the absence of inflation makeup and the presence of a nominal wealth effect.

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I. Introduction

In response to the recent COVID-19 pandemic, many countries have implemented aggressive monetary and fiscal policy measures to cushion the economic fallout from the pandemic-induced recession. In March 2020, the Federal Reserve cut the federal funds rate to zero and implemented large-scale asset purchasing programs to support the functioning of the financial markets and the economy. Since the onset of the pandemic, the US Congress has also passed several rounds of large-scale fiscal stimulus programs, including the \$2.2 trillion Coronavirus Aid, Relief, and Economic Security Act of March 2020, the \$900 billion coronavirus relief and government funding bill of December 2020, and the \$1.9 trillion American Rescue Plan of March 2021. The scale and the scope of these policy interventions are unprecedented in the post-World War II periods.

Evaluating the effectiveness of these policy interventions requires a theoretical framework that takes into account interactions between monetary policy and fiscal policy. For example, when monetary policy is constrained by the zero lower bound (ZLB), government spending would have a much larger stimulus effect than when monetary policy is unconstrained. In an economy with forward-looking agents, what monetary policy does in normal times when it is unconstrained can also affect the effectiveness of fiscal policy during the ZLB periods. Thus, changes in monetary policy framework can have important implications for fiscal stimulus.

One important recent change in the Federal Reserve's monetary policy framework is the switch from the standard inflation targeting policy to average inflation targeting (AIT) in August 2020. Under AIT, monetary policy allows inflation to overshoot the target level for some periods if inflation has fallen below the target in the past, such that the inflation rate is, on average, close to the targeted level.

Recent studies focus on the implications of AIT for the effectiveness of monetary policy in macroeconomic stabilization. It has been shown that AIT is more effective than the standard inflation targeting policy for attenuating shortfalls in the output gap in an economy where the ZLB occasionally constrains the ability of monetary policy to offset negative demand shocks (Mertens and Williams, 2019, 2020). In the standard New Keynesian models, AIT with sufficiently long history-dependence approximates price-level targeting and improves welfare relative to standard inflation targeting (Budianto et al., 2020; Amano et al., 2020).

Less is known about the implications of AIT for the effectiveness of fiscal policy in stabilizing macroeconomic fluctuations. The goal of this paper is to fill this gap. We study the effectiveness of fiscal stimulus under AIT in a New Keynesian model featuring interactions between fiscal policy and monetary policy, both in normal times when monetary policy is unconstrained and in a liquidity trap with binding ZLB constraints.

Our model builds on the standard New Keynesian framework (Woodford, 2003), featuring a monetary policy rule and a fiscal policy rule in the spirit of Leeper (1991) and Davig and Leeper (2011), with occasionally binding ZLB. We introduce AIT in the monetary policy rule, under which the short-term nominal interest rate reacts to an inflation target that is a moving average of past inflation rates (Budianto et al., 2020). Using this simple theoretical framework, we examine how the history-dependence of the inflation target affects the fiscal multipliers under alternative policy regimes, both in normal times and in ZLB periods.

We first analyze equilibrium determinacy in normal times under AIT and find that two traditional policy regimes, a monetary regime (regime M) and a fiscal regime (regime F) can deliver a unique bounded equilibrium. Under regime M, the central bank pursues an active monetary policy by following the Taylor principle; at the same time, the fiscal authority raises lump-sum taxes sufficiently to finance increases in government spending, such that the Ricardian equivalence holds (Galí et al., 2007; Galí, 2020). Regime F, in contrast, features passive monetary policy and active fiscal policy. Under this regime, fiscal policy responds weakly (or not at all) to the state of government indebtedness, and monetary policy does not raise the nominal interest rate aggressively to stabilize inflation (Woodford, 1998; Kim, 2003; Davig and Leeper, 2011). Through both analytical results and numerical simulations, we show that the stimulus effects of fiscal policy expansions crucially depend on the extent of history-dependence of the AIT (denoted by ρ).

In regime M, a temporary increase in government spending raises aggregate output and inflation. Under the standard inflation-targeting policy, monetary policy responds to the increase in inflation aggressively, resulting in an increase in the real interest rate that crowds out private consumption spending through intertemporal substitution. The increase in government spending will be financed by equivalent increases in future lump-sum taxes, creating a negative wealth effect that further crowds out private consumption. The decline in private consumption dampens the stimulus effect of government spending on aggregate output, leading to a fiscal multiplier that is less than one (Galí et al., 2007).

Under AIT, monetary policy responds to average inflation in the current and past periods. Since average inflation rises slowly following the government spending shock, the nominal interest rate also adjusts slowly, dampening the rise in the real interest rate. The stronger the history dependence of the AIT rule, the slower the adjustments in the nominal interest rate and the smaller the increase in the real interest rate, mitigating the crowding-out effects on private consumption. As a consequence, the impact multiplier of government spending (i.e., the percent increases in aggregate output in the impact period of the government spending shock) increases with ρ.

Over time, however, the initial inflation needs to be compensated by subsequent disinflation, such that average inflation remains at the target level (Mertens and Williams, 2019). This "makeup" feature of AIT implies that the real interest rate would stay above steady state for longer periods than under the standard Taylor rule, resulting in more persistent crowding out of private consumption following the government spending shock. As a consequence, the cumulative multiplier of government spending (i.e., the cumulative percent increases in aggregate output for one-period cumulative increases in government spending) decreases with ρ.

In regime F, an increase in government spending is not associated with sufficient increases in future lump-sum taxes to repay the public debt. With smaller tax hikes expected, the household perceives the newly-issued government debt as an increase in nominal wealth, boosting consumption demand, reinforcing the positive effects of the government spending shock on aggregate demand. Higher government spending also creates higher expected inflation. Under the passive monetary policy, however, the central bank does not respond to inflation by raising the nominal interest rate aggressively, resulting in a lower real interest rate, further boosting consumption demand through intertemporal substitution. Thus, the government spending multiplier under the fiscal regime is greater than that under the monetary regime. This result can be obtained under the standard Taylor rule without consideration of AIT (Beck-Friis and Willems, 2017).

AIT has different implications for fiscal stimulus under the fiscal regime than under the monetary regime. Unlike the monetary regime, AIT under the fiscal regime does not feature an inflation makeup following an expansionary fiscal policy because passive monetary policy allows inflation to stay persistently above its steady-state level. The stronger the history-dependence of the targeted average inflation, the smaller the increases in the nominal interest rate relative to expected inflation, and the greater the declines in the real interest rate. Thus, the impact multiplier of government spending increases with ρ. Similar to the case with the monetary regime, a stronger history-dependence of the targeted inflation implies a flatter time-profile of the nominal interest rate responses to the shock. Thus, the real interest rate also stays persistently above the steady state, depressing aggregate demand in subsequent periods. As a consequence, the cumulative multiplier of government spending decreases with ρ.

We also study the implications of AIT for fiscal stimulus when monetary policy is occasionally constrained by the ZLB. We consider a sharp and persistent contraction in aggregate demand (in particular, a shock that lowers the natural real rate) that pushes the nominal rate to the ZLB, with the liftoff date from the ZLB endogenously determined. We consider the monetary regime and the fiscal regime separately.

Under the monetary regime at the ZLB, a temporary government spending shock has larger stimulus effects than in normal times, because the increase in expected inflation leads to a larger reduction in the real interest rate when the nominal rate stays at zero. This is true under the standard Taylor rule (Christiano et al., 2011; Eggertsson, 2011); and also true under the AIT rule.[1]

An increase in history dependence under AIT, however, reduces the government spending multipliers at the ZLB because of the makeup feature of AIT. During the ZLB periods, inflation increases following a government spending shock. When the economy eventually exits from the ZLB, monetary policy tightening is required to reduce inflation to keep average inflation at the target level. The greater the history dependence of the inflation target, the stronger the makeup effects. A decline in expected inflation reduces current inflation while the economy is still in the ZLB, weakening the stimulus effects of government spending. With sufficiently strong history dependence, however, the economy would never enter the ZLB, such that the fiscal multipliers coincide with those in the normal times with unconstrained monetary policy.

Under the fiscal regime at the ZLB, an increase in government spending acts like a debt-financed tax cut, increasing the household's perceived wealth and boosting private consumption. The increase in aggregate demand raises inflation, lowering the real interest rate and further boosting private consumption through intertemporal substitution. With the nominal rate remaining at zero, the increase in inflation reduces the real rate in the ZLB periods more than it does in normal times, implying larger government spending multipliers at the ZLB than in normal times. Furthermore, under the fiscal regime, the wealth effect and the intertemporal substitution effect both raise private consumption, resulting in a government spending multiplier larger than that under the monetary regime.

Under the fiscal regime, AIT has important implications for fiscal stimulus at the ZLB. Agents expect that, after liftoff from the ZLB, passive monetary policy would allow inflation to stay above the steady state, despite that the government spending shock raises inflation at the ZLB. Stronger history dependence of AIT (i.e., a larger ρ) implies that the nominal interest rate will stay lower for longer after liftoff from the ZLB, allowing inflation to stay above steady state for longer. Although the increase in current inflation at the ZLB boosts aggregate demand through intertemporal substitution, the increases in future inflation outside of ZLB erode the value of nominal wealth associated with the debt-financed tax cut, lowering the impact multiplier of government spending at the ZLB. These two opposing

[1]For empirical evidence that the government spending multiplier is larger in the ZLB periods than in normal times, see, for example, Miyamoto et al. (2018) and Ramey and Zubairy (2018). For a survey of the empirical literature, see Ramey (2019).

forces associated with AIT render the relation between government spending multiplier and the history dependence of AIT nonlinear. With small values of ρ, the impact multiplier increases with ρ; with large values of ρ, the impact multiplier decreases with ρ. If ρ is sufficiently large, then the economy would never enter the ZLB, such that the government spending multiplier coincides with that in normal times.

Our paper is closely related to two strands of the literature. First, our paper is related to the literature on the effects of the AIT rule on welfare and business cycles as cited earlier. This literature does not study its effects on fiscal stimulus and the interactions between fiscal and monetary policies.

Second, our paper is related to the large literature on the effects of government spending in the New Keynesian framework. This literature typically studies the size of the government spending multiplier under the inflation targeting rule in the monetary regime (see Woodford (2011) and Farhi and Werning (2017) for surveys). The multiplier is below or close to one in standard New Keynesian models, but it can rise substantially above one under a variety of assumptions, including the presence of hand-to-mouth consumers (see, e.g., Galí et al. (2007)), a binding ZLB constraint (e.g., Christiano et al. (2011), Eggertsson (2011)), non-separable utility in consumption and hours (e.g. Bilbiie (2011)), and a fiscal regime (e.g. Davig and Leeper (2011)). Galí (2020) shows that, when the ZLB is not binding, a money-financed fiscal stimulus has much larger multipliers than a debt-financed fiscal stimulus. The difference in effectiveness persists, but is much smaller at the ZLB. Billi and Walsh (2021) show that debt-financed fiscal stimulus at the ZLB, unbacked by any promise of future tax increases or spending cuts, not only improves economic stability, but also reduces government debt accumulation. Unlike this literature, we focus on the debt-financed fiscal stimulus under the AIT rule.

The remainder of the paper proceeds as follows. Section II introduces the model. Section III studies fiscal multipliers in normal times, while Section IV studies the impact of the ZLB. Section V concludes. All technical details and proofs are relegated to appendices.

II. Model

We consider a basic cashless new Keynesian model augmented with a government sector. The government levies lump-sum taxes and issues one-period nominal riskless bonds to finance exogenous government spending. The model is fairly standard and can be found in the textbooks of Woodford (2003). The only new element is that we replace the usual inflation targeting rule with the AIT rule. In Section II.1 we present the log-linearized equilibrium system directly and discuss its microfoundation in Appendix A. In Section II.2

we provide a baseline calibration for all numerical results in the paper. In Section II.3 we define fiscal multipliers.

II.1. **Log-linearized Equilibrium System.** The log-linearized equilibrium system is summarized by the following seven equations in seven variables $\left\{\pi_t, i_t, \pi_t^*, \widehat{Y}_t, \widehat{b}_t, \widehat{T}_t, \widehat{G}_t\right\}$:

(1) New Keynesian Philips curve (NKPC)

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_0 \left(\eta_u + \eta_v\right) \left(\widehat{Y}_t - \Gamma \widehat{G}_t\right), \tag{1}$$

where

$$\kappa_0 = \frac{\left(1-\beta\theta\right)\left(1-\theta\right)}{\theta} \cdot \frac{1-\alpha}{1-\alpha+\alpha\epsilon},$$

$$\Gamma = \frac{\eta_u}{\eta_u+\eta_v} \in (0,1), \ \eta_u = \frac{Y\gamma}{Y-G}, \quad \eta_v = \frac{\alpha+\nu}{1-\alpha}.$$

(2) Intertemporal IS curve

$$\widehat{Y}_t - \widehat{G}_t = \mathbb{E}_t \left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \tag{2}$$

where

$$\sigma = \frac{C}{\gamma Y}, \quad r_t^n = -\ln\beta - \Delta_t.$$

(3) Monetary policy rule

$$i_t = \max\left\{-\ln\beta + \phi_\pi \pi_t^*, \ 0\right\}. \tag{3}$$

(4) AIT rule

$$\pi_t^* = \rho \pi_{t-1}^* + (1-\rho)\pi_t. \tag{4}$$

(5) Fiscal policy rule

$$\widehat{T}_t = \phi_b \widehat{b}_{t-1} + \varepsilon_t^\tau. \tag{5}$$

(6) The government budget constraint

$$\beta \widehat{b}_t - \frac{\beta b}{Y}\left(i_t + \ln\beta\right) + \widehat{T}_t = \widehat{b}_{t-1} + \widehat{G}_t - \frac{b}{Y}\pi_t. \tag{6}$$

(7) Government spending

$$\widehat{G}_t = \rho_g \widehat{G}_{t-1} + \varepsilon_t^g. \tag{7}$$

In the system above, π_t, π_t^*, i_t, r_t^n, Y_t, G_t, b_t, and T_t represent respectively the inflation rate, the inflation rate target, the (continuously compounded) nominal interest rate, the real natural interest rate, real output, real government spending, real government debt (principal plus interest), and real lump-sum taxes. Assume that there is zero inflation in a deterministic steady state. In such a steady state the nominal interest rate is equal to the real interest rate $-\ln\beta$. Let a variable without a time-subscript denote its steady-state value.

Define

$$\widehat{G}_t = \frac{G_t - G}{Y}, \ \widehat{T}_t = \frac{T_t - T}{Y}, \ \widehat{b}_t = \frac{b_t - b}{Y},$$

and any other variable with a hat denotes the log deviation from its steady state value. The variable Δ_t represents an exogenous shock to the natural rate, which can be micro-founded by introducing a preference shock. In addition, there are two independent white noise innovations ε_t^τ and ε_t^g in equations (5) and (7), respectively.

The parameters α, β, γ, ϵ, ν, and θ are the capital share, the household's subjective discount factor, the coefficient of relative risk aversion, the elasticity of substitution of differentiated goods, the inverse Frisch elasticity of labor supply, and the probability of keeping prices fixed in any period, respectively.

Equation (1) is the NKPC, accounting for the impact of government spending. The term κ_0 represents the slope of the NKPC curve in absence of government spending and reflects the standard Calvo measure of price stickiness (with $\kappa_0 \to \infty$ as prices become fully flexible). Let $\kappa \equiv \kappa_0 (\eta_u + \eta_v)$. Under flexible prices, we have $\partial\widehat{Y}_t/\partial\widehat{G}_t = \Gamma$ from Equation (1) and thus the government spending multiplier on output is equal to $\Gamma \in (0, 1)$. In this case, the stimulus effects of government spending are driven by the labor wealth effect on labor supply. More specifically, an increase in government spending lowers household wealth with a higher tax burden. This negative wealth effect reduces household consumption but also encourages households to work harder, generating a fiscal multiplier being positive but smaller than 1. With price stickiness, increases in output gap $\widehat{Y}_t - \Gamma\widehat{G}_t$ may generate inflation. The responses of output therefore depend on how monetary authority reacts to inflation by managing the demand block of the economy.

The intertemporal IS curve, i.e. Equation (2), shows how the aggregate demand is affected by government spending and monetary policy. With price stickiness, the endogenous drops in markups and the resulting increases in labor demand become an additional channel through which government spending can boost output (Woodford, 2011). The changes of markups in turn depend on how aggregate demand is different from what it was expected to be when the prices were set. The forward-looking nature of the intertemporal IS curve therefore suggests that the sizes of fiscal multipliers depend crucially on both the current and the future stance of monetary policy.

Equation (3) describes the monetary policy rule, according to which the central bank reacts to the AIT π_t^* with strength parameter ϕ_π. This rule also incorporates a zero lower bound on the nominal interest rate. The AIT π_t^* satisfies (4), which is equal to an exponential moving average of the current and past actual inflation rates (Woodford, 2003; Budianto et al., 2020). The literature typically assumes that the inflation target is equal to an arithmetic moving average of the current and past actual inflation rates (Nessén and Vestin, 2005; Amano et al., 2020). As discussed by Budianto et al. (2020), using exponential moving average economizes on the number of state variables and thereby facilitates the solution without changing the key insights. The usual inflation targeting is a special case with $\rho = 0$. When $\rho = 1$, we obtain the price-level targeting rule. Without ZLB, we can rewrite Equations (3) and (4) as

$$i_t = -(1-\rho)\ln\beta + \rho i_{t-1} + \phi_\pi(1-\rho)\pi_t,$$

suggesting that the AIT rule creates weaker but more persistent responses of the policy rate to inflation. With the presence of ZLB, the above condition no longer holds. In either case, AIT, a slow-moving state variable, serves as a policy commitment to keep the nominal interest rate persistently low if there is disinflation or deflation today.

Equation (5) describes the fiscal policy rule as in Leeper (1991), according to which the fiscal authority adjusts lump-sum taxes in response to changes of lagged real debt with strength parameter ϕ_b.[2] Equation (6) is the government's intertemporal budget constraint. The government issues nominal debt and collects nominal taxes to finance its nominal spending and interest cost on its existing debt level. Combining (5) and (6), we obtain the debt dynamics

$$\widehat{b}_t = \frac{1}{\beta}(1-\phi_b)\widehat{b}_{t-1} + \frac{1}{\beta}\widehat{G}_t + \frac{b}{Y}(i_t + \ln\beta) - \frac{b}{\beta Y}\pi_t - \frac{1}{\beta}\varepsilon_t^\tau. \tag{8}$$

Equation (7) shows that government spending follows an exogenous AR(1) process. The parameter $\rho_g \in [0,1)$ describes the persistence of government spending.

II.2. **Calibration.** Unless noted otherwise, we adopt the following baseline parameter values. One period in the model corresponds to one quarter. The discount factor is set at $\beta = 0.995$, which implies a steady state (annualized) real interest rate of about 2%. Set the relative risk aversion parameter $\gamma = 1$. Set the capital elasticity parameter $\alpha = 0.33$ as in the business cycle literature. Following Galí (2020), we assume $\nu = 5$, implying that the Frisch elasticity of labor supply is equal to 0.2. As in Galí (2020), we set $\epsilon = 9$, implying a 12.5 percent steady-state price markup. Set $\theta = 0.75$, implying an average price duration of four quarters, a value consistent with much of the empirical micro and macro evidence. According to the US annual data from 1950 to 2019, the average government spending to GDP ratio is about 11% and the average government debt to GDP ratio is about 36%. Thus we set $G/Y = 0.11$ and $b/Y = 1.44$. Similar to Nakamura and Steinsson (2014), we consider different degree of persistence of government spending with $\rho_g = 0$, $\rho_g = 0.5$, and $\rho_g = 0.9$.

To fully solve for the equilibrium dynamics, we need to assign values for the policy parameters ϕ_π and ϕ_b. These values depend on a particular fiscal-monetary policy regime, and thus we will return to this issue after we study equilibrium determinacy.

We suppose that the economy is initially at the deterministic steady state and there is no natural rate shock ($\Delta_t = 0$ for all t) in normal times when the ZLB constraint does not bind.

[2]More precisely, Leeper (1991) assumes that taxes respond to changes of the lagged real principal value of debt. Here we follow Woodford (2003) by assuming debt includes both principal and interest.

To enter a liquidity trap when the ZLB constraint binds, we set $\Delta_t = 0.01$ for $0 \leq t \leq 5$ and $\Delta_t = 0$ for all $t > 5$, so that the annual natural rate stays at -2% for 6 quarters only as in Galí (2020). Assume that all agents have perfect foresight and we focus on perfect foresight numerical solutions.

II.3. **Definition of Fiscal Multipliers.** The main goal of our paper is to study fiscal multipliers. We define the taxation multiplier $TM_t^y(j)$ at horizon $j \geq 0$ on output (y) as the impulse response of output in period $t+j$ to a unit (lump-sum) tax cut in period t and define the government spending multiplier $GSM_t^y(j)$ as the impulse response of output in period $t+j$ to a unit increase in government spending in period t. One unit corresponds to a one percent of steady-steady output. We call the multiplier at $j = 0$ the impact multiplier. For convenience we set $t = 0$ throughout the paper and suppress this subscript for all fiscal multipliers. Similarly, we can define fiscal multipliers on other variables such as inflation and real debt. We define the cumulative fiscal multipliers as the cumulative responses of a unit tax cut or government spending increase.

During normal times, the model admits a log-linear solution and thus we can analytically compute fiscal multipliers for any horizon j (Beck-Friis and Willems, 2017). In particular, the government spending multiplier on output at horizon j is equal to $GSM^y(j) = \partial \widehat{Y}_j / \partial \varepsilon_0^g$ and the cumulative government spending multiplier on output is equal to $(1-\rho_g) \sum_{t=0}^{\infty} \partial \widehat{Y}_t / \partial \varepsilon_0^g$ (e.g., Galí (2020)). We will study this case in Section III.

When the ZLB constraint binds due to negative shocks to natural rates, the economy enters a liquidity trap and the model solution will be nonlinear. The computation of fiscal multipliers will be more complicated and needs numerical methods. Woodford (2011) argues that the duration of fiscal stimulus is important for the size of fiscal multipliers (see Eggertsson (2011), Cogan et al. (2010), and Miao and Ngo (2021)). If the government spending follows a persistent AR(1) process as in (7), then the government spending multiplier will be much smaller because the increased government spending after the economy leaves the liquidity trap has a negative effect on consumption and output in the liquidity trap.

For this reason, we only consider the impact of temporary fiscal stimulus with $\rho_g = 0$. More specifically, let $\{\widehat{Y}_t^a\}$ ($\{\widehat{Y}_t^b\}$) denote the path of output after (before) the temporary government spending increase. We then define the impact government spending multiplier on output as $\left(\widehat{Y}_0^a - \widehat{Y}_0^b\right) / \varepsilon_0^g$ and the cumulative government spending multiplier on output as $\sum_{t=0}^{\infty} \left(\widehat{Y}_t^a - \widehat{Y}_t^b\right) / \sum_{t=0}^{\infty} \varepsilon_t^g$. We will study this case in Section IV.

III. Fiscal Multipliers in Normal Times

In this section we first analyze equilibrium determinacy in normal times when the ZLB constraint does not bind. Then we study the effects of a tax cut and an increase in government purchases on the economy in normal times.

III.1. **Equilibrium Determinacy.** To study equilibrium determinacy, it suffices to consider the perfect foresight case by assuming $\Delta_t = \varepsilon_t^\tau = \varepsilon_t^g = 0$ for all t. We can then write the equilibrium system in a matrix form:

$$\begin{bmatrix} 1 & \sigma & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & \rho-1 & 1 & 0 \\ 0 & \frac{b}{Y} & -\beta\frac{b}{Y}\phi_\pi & \beta \end{bmatrix} \begin{bmatrix} \widehat{Y}_{t+1} \\ \pi_{t+1} \\ \pi^*_{t+1} \\ \widehat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \phi_\pi\sigma & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 1-\phi_b \end{bmatrix} \begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ \pi^*_t \\ \widehat{b}_t \end{bmatrix} + \begin{bmatrix} \widehat{G}_{t+1} - \widehat{G}_t \\ \kappa\Gamma\widehat{G}_t \\ 0 \\ \widehat{G}_{t+1} \end{bmatrix}.$$

Because $\widehat{G}_t$ is an exogenously given stationary process, we can ignore the last forcing vector when analyzing determinacy. Then the predetermined variables are π^*_t and $\widehat{b}_t$ and the non-predetermined variables are $\widehat{Y}_t$ and π_t. Write the above system as $X_{t+1} = \Omega X_t$, where $X_t = \left[\widehat{Y}_t, \pi_t, \pi^*_t, \widehat{b}_t\right]'$ for some matrix Ω. To have a unique bounded equilibrium, we need Ω to have two eigenvalues inside the unit circle and two eigenvalues outside the unit circle.

Following Woodford (2003), we define a fiscal rule (tax rule) as locally Ricardian if when substituted into the government budget constraint (6) it implies that $\left\{\widehat{b}_t\right\}$ remains bounded for all bounded paths of endogenous variables $i_t, \pi_t, \widehat{Y}_t$ and exogenous variable $\widehat{G}_t$. By (8), we deduce that the tax rule in (5) is locally Ricardian if and only if $|(1-\phi_b)/\beta| < 1$. In Appendix B we show that the matrix Ω has an eigenvalue $(1-\phi_b)/\beta$. The remaining three eigenvalues are the roots of the following characteristic equation:

$$f(\lambda) \equiv \lambda^3 - \left(\frac{1+\kappa\sigma}{\beta} + 1 + \rho\right)\lambda^2 + \frac{1}{\beta}\left[1+\rho+\beta\rho+(\phi_\pi+\rho(1-\phi_\pi))\,\kappa\sigma\right]\lambda - \frac{\rho}{\beta} = 0. \quad (9)$$

Let λ_1, λ_2, and λ_3 denote the three roots with $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$. Studying the stability of these eigenvalues leads to the equilibrium determinacy conditions summarized in Proposition 1 below.

Proposition 1. Suppose that $\phi_\pi \geq 0$, $0 \leq \rho < 1$. (i) If the fiscal policy is locally Ricardian (i.e., $|(1-\phi_b)/\beta| < 1$), then the necessary and sufficient condition for determinacy is given by $\phi_\pi > 1$. (ii) If the fiscal policy is locally non-Ricardian ($|(1-\phi_b)/\beta| > 1$), then the necessary and sufficient condition for determinacy is given by $\phi_\pi < 1$.[3]

Proof. See Appendix B. □

[3] We can also easily show that the equilibrium is indeterminate if $|(1-\phi_b)/\beta| < 1$ and $\phi_\pi < 1$ and there is no bounded equilibrium if $|(1-\phi_b)/\beta| > 1$ and $\phi_\pi > 1$.

We call the first case in Proposition 1 the monetary regime and the second case the fiscal regime as in Leeper (1991). According to Leeper (1991), fiscal policy is passive (active) and monetary policy is active (passive) in the monetary (fiscal) regime. Proposition 1 shows that if the fiscal policy is locally (non-) Ricardian, then determinacy requires monetary policy to satisfy (violate) the Taylor principle ($\phi_\pi > 1$). The intuition is that at least in the long run, the nominal interest rates should rise by more than the increase in the inflation rate for inflation dynamics to be stabilized. For the AIT rule (4), the interest rate target rises by the same amount as the actual inflation rate in the long run. Thus, according to the monetary rule (3) in normal times, we must have $\phi_\pi > 1$.

Proposition 1 generalizes the determinacy conditions under the standard Taylor rule (Woodford, 2003) to an economy under the AIT rule. In particular, it shows that the policy parameter space for determinacy is independent of ρ, and thus the determinacy conditions under AIT are the same as those under the standard Taylor rule.

III.2. **Monetary Regime.** To characterize the relation between the fiscal multiplier and the history dependence of AIT under the monetary regime, we include $\widehat{G}_t$ in the state vector. Let $X_t = \left[\widehat{Y}_t, \pi_t, \pi_t^*, \widehat{b}_t, \widehat{G}_t\right]'$. The state-space solution takes the form

$$X_t = H X_{t-1}^p + H_\tau \varepsilon_t^\tau + H_g \varepsilon_t^g, \tag{10}$$

where H, H_τ, and H_g are conformable matrices and $X_{t-1}^p = (\pi_{t-1}^*, \widehat{b}_{t-1}, \widehat{G}_{t-1})$ denote the vector of the predetermined state variables.

The fiscal multipliers can be read from the matrices H_τ and H_g. Under the monetary regime (regime M), the analytical expressions of the fiscal multipliers are summarized in Proposition 2 below.

Proposition 2. Suppose that $\phi_\pi > 1$ and $|(1-\phi_b)/\beta| < 1$ so that the economy is in the monetary regime. Then we have

(1) the eigenvalues of Eq (9) satisfy

$$|\lambda_1| \geq |\lambda_2| > 1, \quad 0 < \lambda_3 < \rho,$$

(2) the taxation multipliers are equal to zero

$$TM_M^y(j) = 0, \; TM_M^\pi(j) = 0,$$

(3) the government spending multipliers are given by

$$GSM_M^y(j) = \lambda_3^j \frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\rho_g)(1-\Gamma)}{J_2} + \rho_g^j \frac{J_1}{J_2}, \tag{11}$$

$$GSM_M^\pi(j) = \frac{(1-\rho_g)(1-\Gamma)\kappa}{J_2}\left[\lambda_3^j(\rho-\lambda_3) + \rho_g^j(\rho_g-\rho)\right], \tag{12}$$

$$GSM_M^{\pi^*}(j) = \frac{(1-\rho)(1-\rho_g)(1-\Gamma)\kappa}{J_2}\left(\rho_g^{j+1} - \lambda_3^{j+1}\right), \tag{13}$$

where J_1 and J_2 are given by

$$J_1 = (\rho_g-\rho)(1-\rho_g)(1-\beta\rho_g) + \rho_g\left[\rho+\phi_\pi(1-\rho)-\rho_g\right]\Gamma\kappa\sigma,$$
$$J_2 = (\rho_g-\rho)(1-\rho_g)(1-\beta\rho_g) + \rho_g\left[\rho+\phi_\pi(1-\rho)-\rho_g\right]\kappa\sigma.$$

Proof. See Appendix C. 1 □

In the monetary regime, the Ricardian equivalence holds so that changes in lump-sum taxes do not affect output and inflation. But government spending affects the economy through two effects: First, an increase in government spending raises aggregate demand. With sticky prices, firms cannot fully raise prices in response to the higher demand. Instead, they lower their markup and increase production, thereby raising labor demand. Second, an increase in government spending raises inflation and, under the monetary regime, the real interest rate rises, crowding out private consumption through intertemporal substitution. The decline in private consumption also increases labor supply through the wealth effect. The net effect is that equilibrium labor hours increase and thus output rises. These channels do not depend on the history dependence of AIT.[4]

However, AIT does affect the time profiles of the adjustments in output and inflation following a government spending increase, and thereby affecting the size of the cumulative fiscal multiplier relative to the impact multiplier. To better understand the size of the fiscal multipliers, we consider the special case with an i.i.d. government spending shock (i.e., $\rho_g = 0$). In this special case, the government spending multipliers under AIT are given by the following corollary.

[4]As $\rho \to 0$, Proposition 2 is reduced to the result for the standard inflation targeting rule in Woodford (2003) and Beck-Friis and Willems (2017).

Corollary III.1. *Suppose the conditions in Proposition 2 hold. Let $\rho_g = 0$. Then we have*

$$GSM_M^y(j) = \begin{cases} -\frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\Gamma)}{\rho} + 1 \in (\Gamma, 1)\,, & \text{if } j = 0, \\ -\lambda_3^j \frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\Gamma)}{\rho} < 0, & \text{if } j \geq 1, \end{cases} \tag{14}$$

$$GSM_M^\pi(j) = \begin{cases} \frac{(1-\Gamma)\kappa\lambda_3}{\rho} > 0, & \text{if } j = 0, \\ -\lambda_3^j \frac{(1-\Gamma)\kappa(\rho-\lambda_3)}{\rho} < 0, & \text{if } j \geq 1, \end{cases} \tag{15}$$

$$GSM_M^{\pi^*}(j) = \lambda_3^j \frac{(1-\rho)(1-\Gamma)\kappa\lambda_3}{\rho} > 0, \quad j \geq 0. \tag{16}$$

Thus, a positive transitory government spending shock boosts output and inflation in the impact period, but reduces output and inflation in subsequent periods before reverting to the steady state.

This is different from the result in the standard model with the inflation target rule (Beck-Friis and Willems, 2017), which is the special case of Corollary 1 with $\rho \to 0$. Absent history dependence of the inflation target, the macro effects of a transitory government spending shock are also transitory: the shock boosts output and inflation in the impact period, but has no effects thereafter. Specifically, we have

$$GSM_M^y(j) = \begin{cases} \frac{1+\phi_\pi\kappa\sigma\Gamma}{1+\phi_\pi\kappa\sigma}, & \text{if } j = 0, \\ 0, & \text{if } j \geq 1, \end{cases} \quad GSM_M^\pi(j) = \begin{cases} \frac{(1-\Gamma)\kappa}{1+\phi_\pi\kappa\sigma}, & \text{if } j = 0, \\ 0, & \text{if } j \geq 1. \end{cases} \tag{17}$$

To further illustrate the intuition for these analytical results, we plot the impulse response functions follow a positive and transitory government spending shock (with $\rho_g = 0$) under calibrated parameters in Figure 1.[5]

Under the AIT rule, the nominal interest rate reacts to the average inflation rate in the current and past periods. Since the average inflation target π_t^* adjusts slowly, the nominal interest rate i_t also adjusts slowly, even if the government spending shock is purely i.i.d. Thus the real interest rate $r_t = i_t - \mathbb{E}_t\pi_{t+1}$ declines gradually to its steady state level. With the real interest rate staying above steady state, intertemporal substitution implies that private consumption stays below its steady state level persistently. With private consumption crowded out, aggregate output rises on impact, but with a smaller magnitude than the increase in government spending. The increase in aggregate demand in the impact period also raises inflation. However, in the subsequent periods when the government spending shock vanishes and private consumption goes below its steady-state level, aggregate demand

[5] We calibrate $\phi_\pi = 1.5$ and $\phi_b = 0.0177$ so that economy stays in the monetary regime. The choice of $\phi_b = 0.0177$ implies that 5% of the deviation from the target debt ratio is corrected over four quarters, i.e., $[(1-\phi_b)/\beta]^4 = 0.95$ by (8). This choice only affects debt dynamics, but not other variables due to the Ricardian equivalence.

declines, lowering both output and inflation before they revert to their steady-state levels. These dynamic effects on inflation and output reflect the makeup property of the AIT: the initial increase in inflation following the government spending shock triggers persistent monetary policy tightening such that inflation falls below target for some periods in order to maintain the average target level. In the case with the standard inflation targeting rule (i.e., with $\rho = 0$), in contrast, output and inflation would revert to its steady state values immediately after the initial period, and there would be no inflation makeup, as shown in Eq. (17).

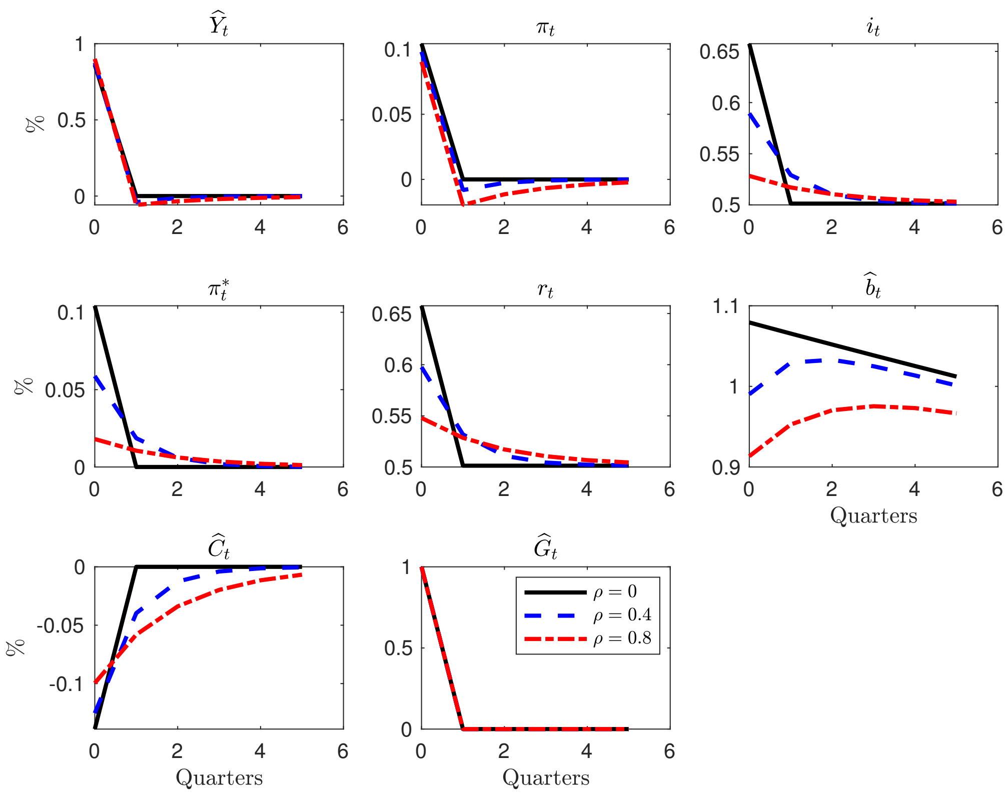

FIGURE 1. Dynamic Effects of Government Spending in the Monetary Regime. Note: we set $\phi_\pi = 1.5$ and $\phi_b = 0.0177$ so that the economy is in the monetary regime and 5% of the deviation from target in the debt ratio is corrected over four quarters.

Monetary policy reactions under the AIT rule have implications for the size of the government spending multiplier. Figure 2 presents the fiscal multipliers against $\rho \in [0, 1)$, the parameter that measures the history-dependence of the AIT rule for several different values of ρ_g, the persistence of the government spending shock. The figure shows that, for any given values of ρ and ρ_g, the cumulative multiplier is smaller than the impact multiplier. This result reflects the makeup feature of AIT. The initial expansion in aggregate demand raises inflation, triggering monetary policy tightening in subsequent periods to bring average inflation rate back to the target level. Monetary policy tightening implies a persistently higher real interest rate, crowding out private consumption persistently, and reducing aggregate output when the government spending returns to steady state. The initial expansion and subsequent declines in aggregate output imply a smaller cumulative multiplier than the impact multiplier.

Figure 2 shows that, for a fixed ρ, both the impact multiplier and the cumulative multiplier on output decreases with ρ_g. The intuition is that a more persistent government spending shock leads to more persistent increases in future inflation and thus more persistent monetary policy tightening. As a consequence, the real interest rate stays above steady state for longer, crowding out private consumption persistently and reducing the expansionary effects of the government spending shock on aggregate output.

The figure also shows that, for a given value of ρ_g, the impact multiplier increases with ρ, while the cumulative multiplier can be non-monotonic with ρ. A higher ρ implies a slower adjustment in the average inflation target π_t^* and thus a flatter time profile of the nominal interest rate. With a smaller increase in the nominal and real interest rates in the impact period, aggregate demand rises more, leading to a larger impact multiplier. However, with a larger ρ, the real interest rates stay above steady state for longer, leading to more persistent crowding-out effects on private consumption, reducing the expansionary effects on output in subsequent periods. The non-monotonic dynamics of the output responses to the government spending shock imply a potentially non-monotonic cumulative multiplier.

III.3. **Fiscal Regime.** The fiscal regime features a passive monetary policy (i.e., $\phi_\pi < 1$) and an active fiscal policy (i.e., $|(1-\phi_b)/\beta| > 1$).[6] In this regime, the Ricardian equivalence fails to hold, such that an increase in government spending today will not be financed by an equivalent increase in future taxes, but it will be partially financed by inflation taxes. Monetary policy is passive, such that the nominal interest rate adjusts by less than one-for-one to an increase in the inflation target. Proposition 3 below characterizes the tax multipliers and government spending multipliers under the fiscal regime.

Proposition 3. Suppose that $0 \leq \phi_\pi < 1$ and $\phi_b < 1-\beta$ so that the economy is in the fiscal regime. We have the following results:

(1) $|\lambda_1| > 1 > \lambda_2 > \rho > \lambda_3 > 0$.

[6] If $\phi_b > 1+\beta$ and $\phi_\pi < 1$, the economy is also in the fiscal regime. But the parameterizations would imply an unrealistic overreaction of tax revenues to debt dynamics. We do not study this case here following the literature on fiscal theory of the price level (Leeper and Leith, 2016; Canzoneri et al., 2010).

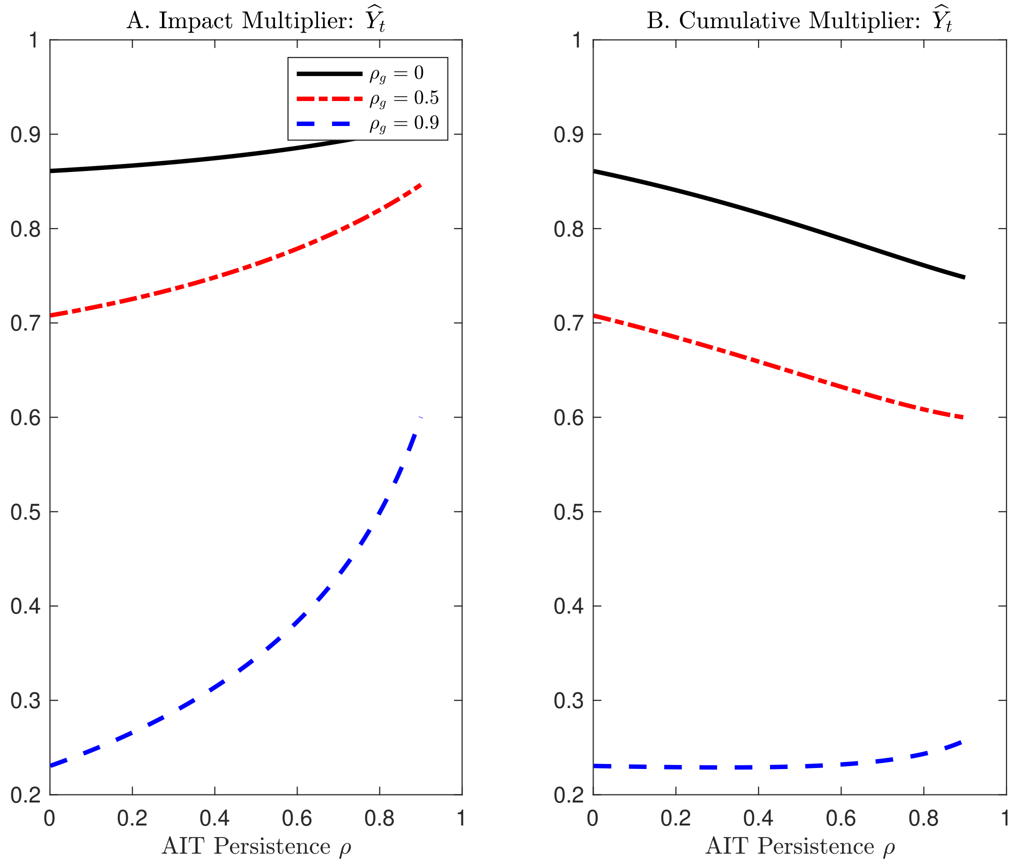

Figure 2. Government Spending Multipliers in the Monetary Regime

(2) the taxation multipliers are given by

$$TM_F^y(j) = \frac{1}{(1-\phi_b)b/Y}\frac{1}{\kappa}\left[\frac{\lambda_2^j(1-\beta\lambda_2)(\lambda_2-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}}+\frac{\lambda_3^j(1-\beta\lambda_3)(\lambda_3-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}}\right] > 0, \tag{18}$$

$$TM_F^\pi(j) = \frac{1}{(1-\phi_b)b/Y}\left[\frac{\lambda_2^j(\lambda_2-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}}+\frac{\lambda_3^j(\lambda_3-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}}\right] > 0. \tag{19}$$

(3) the government spending multipliers are given by

$$GSM_F^y(j) = GSM_M^y(j) + \sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k\left[\rho_g^k - \frac{b}{Y}\cdot GSM_M^\pi(k) + \beta\frac{b}{Y}\phi_\pi\cdot GSM_M^{\pi^*}(k)\right]\times TM_F^y(j), \tag{20}$$

$$GSM_F^\pi(j) = GSM_M^\pi(j) + \sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k\left[\rho_g^k - \frac{b}{Y}\cdot GSM_M^\pi(k) + \beta\frac{b}{Y}\phi_\pi\cdot GSM_M^{\pi^*}(k)\right]\times TM_F^\pi(j). \tag{21}$$

Proof. See Appendix C. 2. □

In the fiscal regime, the Ricardian equivalence does not hold. A deficit-financed tax cut or an increase in government spending is not associated with sufficiently high future taxes to repay the debt. The household perceives the newly-issued government bonds as an increase in nominal wealth, raising consumption through the wealth effect. The increase in consumption demand raises inflation. Since monetary policy is passive in this regime, the nominal interest rises by less than one-for-one in response to the increase in inflation, leading to a decline in the real interest rate. This further stimulates consumption through intertemporal substitution. The fiscal regime thus implies positive multiplier effects on output and inflation for both tax cuts and government spending increases, as shown in Proposition 3.

To better understand the intuition for the results in Proposition 3, consider the special case with $\rho_g = 0$. The expressions for the fiscal multipliers can be simplified significantly, as summarized in the following Corollary.

Corollary III.2. *Suppose that the conditions in Proposition 3 hold. If $\rho_g = 0$, then the government spending multipliers are given by*

$$GSM_F^y(j) = GSM_M^y(j) + \left[1 + \sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k \left[-\frac{b}{Y}\cdot GSM_M^{\pi}(k) + \frac{b}{Y}\beta\phi_\pi \cdot GSM_M^{\pi^*}(k)\right]\right] \times TM_F^y(j), \tag{22}$$

$$GSM_F^{\pi}(j) = GSM_M^{\pi}(j) + \left[1 + \sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k \left[-\frac{b}{Y}\cdot GSM_M^{\pi}(k) + \frac{b}{Y}\beta\phi_\pi \cdot GSM_M^{\pi^*}(k)\right]\right] \times TM_F^{\pi}(j). \tag{23}$$

where $GSM_M^y(j)$, $GSM_M^{\pi}(j)$, $GSM_M^{\pi^*}(j)$, $TM_F^y(j)$, and $TM_F^{\pi}(j)$ are given by (14), (15), (16), (18) and (19), respectively.

These expressions show that the government spending multipliers under the fiscal regime are in general greater than those under the monetary regime. Furthermore, a temporary increase in government spending not financed by equivalent increases in future taxes acts like a debt-financed tax cut in the fiscal regime, which explains the presence of the tax multipliers in the terms within the squared brackets in both equations (22) and (23).

Since the spending increase is not financed by an equivalent increase in future taxes and monetary policy is passive, inflation is expected to rise to reduce the real value of government debts so that the intertemporal government budget constraints can be satisfied; that is, the real value of debts is equal to the present value of future government surpluses. This inflation tax effect is reflected by the term $-\sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k \frac{b}{Y}\cdot GSM_M^{\pi}(k)$ in Eq. (22). An increase in future inflation reduces the real values of nominal wealth, and thus reduces the multiplier effects on output.

Increases in future inflation would raise the average inflation rate, triggering monetary policy tightening. Although monetary policy is passive, the nominal interest rate still rises following an increase in inflation provided that $\phi_\pi > 0$. The increase in the nominal interest rate boosts the return on future nominal wealth, amplifying the positive wealth effect on private consumption and the fiscal multiplier effect on output. This explains the presence of the term $\sum_{k=0}^{\infty}\left(\frac{\beta}{1-\phi_b}\right)^k \frac{b}{Y}\beta\phi_\pi \cdot GSM_M^{\pi^*}(k)$ in Eq. (22).

In the special case with $\rho = 0$, the AIT rule collapses to the standard inflation targeting rule, and we obtain the same results under the fiscal regime as Beck-Friis and Willems (2017).

Figure 3 shows the impulse responses to a positive and transitory government spending shock (with $\rho_g = 0$), for three different values of ρ.[7] The figure shows that a transitory increase in government spending raises consumption, output, inflation, and the nominal interest rate. Since monetary policy is passive, the shock raises inflation and reduces the real interest rate because the nominal interest rate does not adjust aggressively in response to changes in the inflation target. The decline in the real rate further stimulates private consumption, amplifying the multiplier effect on output and inflation.

Unlike the monetary regime, AIT under the fiscal regime does not feature an inflation makeup following an expansionary fiscal policy because passive monetary policy allows inflation to stay persistently above its steady-state level. As shown in Figure 1, the stronger the history-dependence of the targeted average inflation, the smaller the increases in the nominal interest rate relative to expected inflation, and the greater the declines in the real interest rate.

Figure 4 presents the impact multipliers and the cumulative multipliers against ρ in the fiscal regime for several values of ρ_g. Compared to the multipliers in the monetary regime, three features stand out. First, the impact multiplier in the fiscal regime is larger and larger than 1. As we have discussed, the larger fiscal multiplier stems from two channels: (i) the spending increase is not financed by equivalent tax increases in the future, creating a positive nominal wealth effect on private consumption; and (ii) the real interest rate falls because of the passive monetary policy, creating an intertemporal substitution effect that further stimulates private consumption. Of course, the increase in inflation acts like a tax, partially offsetting the stimulus effect.

Second, the cumulative multiplier is larger than the impact multiplier if the persistence in government spending is not too high. This result arises because the real interest rate

[7] In calculating the impulse responses, we set $\phi_\pi = 0.8$ such that monetary policy is passive and $\phi_b = 0.0025$ such that fiscal policy is active. The impulse responses to a tax cut are similar.

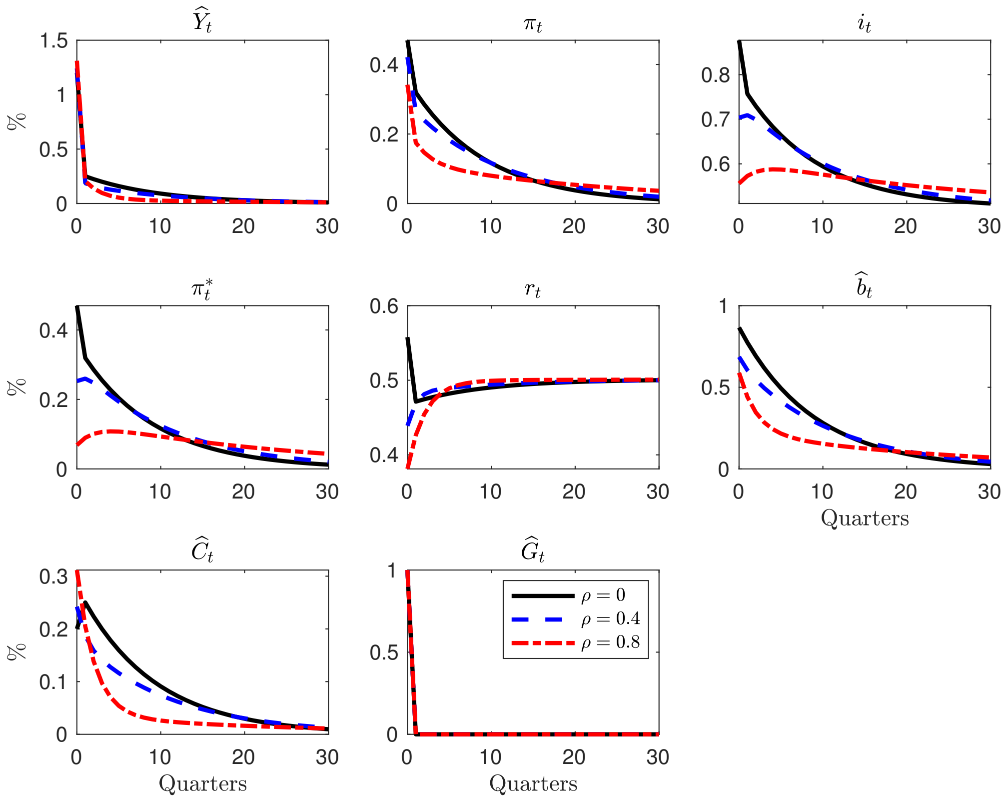

Figure 3. Dynamic Effects of Government Spending in the Fiscal Regime. Note: we set $\phi_\pi = 0.8$ and $\phi_b = 0.0025$ so that the economy is in the fiscal regime.

declines persistently under the passive AIT rule, such that output rises persistently following a transitory fiscal policy shock. If the government spending is very persistent, the negative nominal wealth effect of inflation may dominate the intertemporal substitution channel, resulting a cumulative multiplier smaller than the impact multiplier.

Third, similar to the monetary regime, the cumulative multiplier decreases with the persistence of government spending shocks. The underlying reason is also similar: a more persistent government spending shock implies that the real interest rate will stay high for longer, leading to more persistent crowding out of private consumption. Different from the monetary regime, however, the impact multiplier in the fiscal regime increases with the persistence of the shock. This is because a more persistent government spending shock leads to a larger nominal wealth effect in the fiscal regime, whereas the nominal wealth effect is absent in the monetary regime.

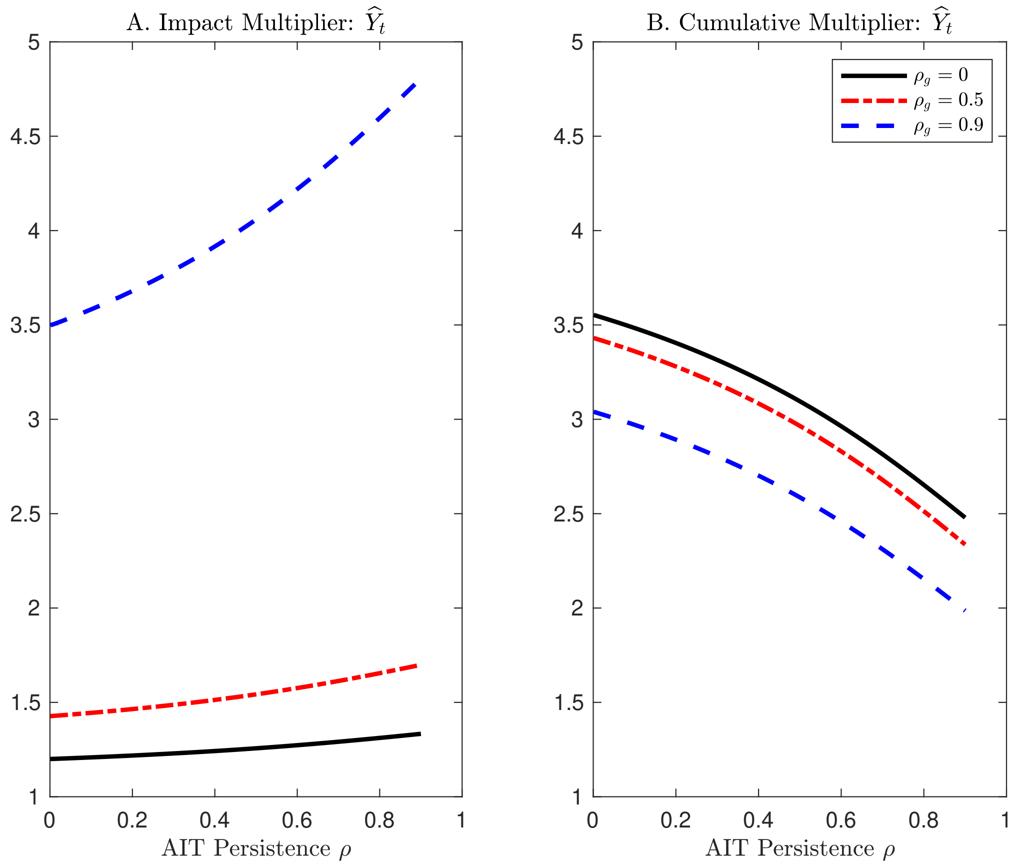

FIGURE 4. Government Spending Multipliers in the Fiscal Regime

IV. Fiscal Multipliers in a Liquidity Trap

We now examine the implications of AIT rules for fiscal stimulus when monetary policy is constrained by the ZLB. We consider a sharp and persistent contraction in aggregate demand that pushes the nominal rate to the ZLB, with the liftoff date from the ZLB endogenously determined. In particular, we consider an unexpected drop in the natural real rate to -2% for 6 quarters and then returns to its steady state of 2% (in annualized terms). All else being equal, the initial drop in the natural real rate pushes the nominal interest rate to the ZLB. We show that the effectiveness of fiscal stimulus conditional on the path of the aggregate demand shock depends on whether the economy is under the monetary regime or the fiscal regime, and in each regime, it also depends on the history-dependence of the AIT rule.

IV.1. **Monetary Regime.** We first consider the monetary regime featuring an active monetary policy(with $\phi_\pi = 1.5$) and a passive fiscal policy (with $\phi_b = 0.0177 > 1 - \beta$).

Figure 5 shows the impulse responses to the negative aggregate demand shock for several different values of ρ without any fiscal policy responses. Under the standard inflation targeting rule with $\rho = 0$, the shock reduces aggregate output and inflation, pushing the nominal

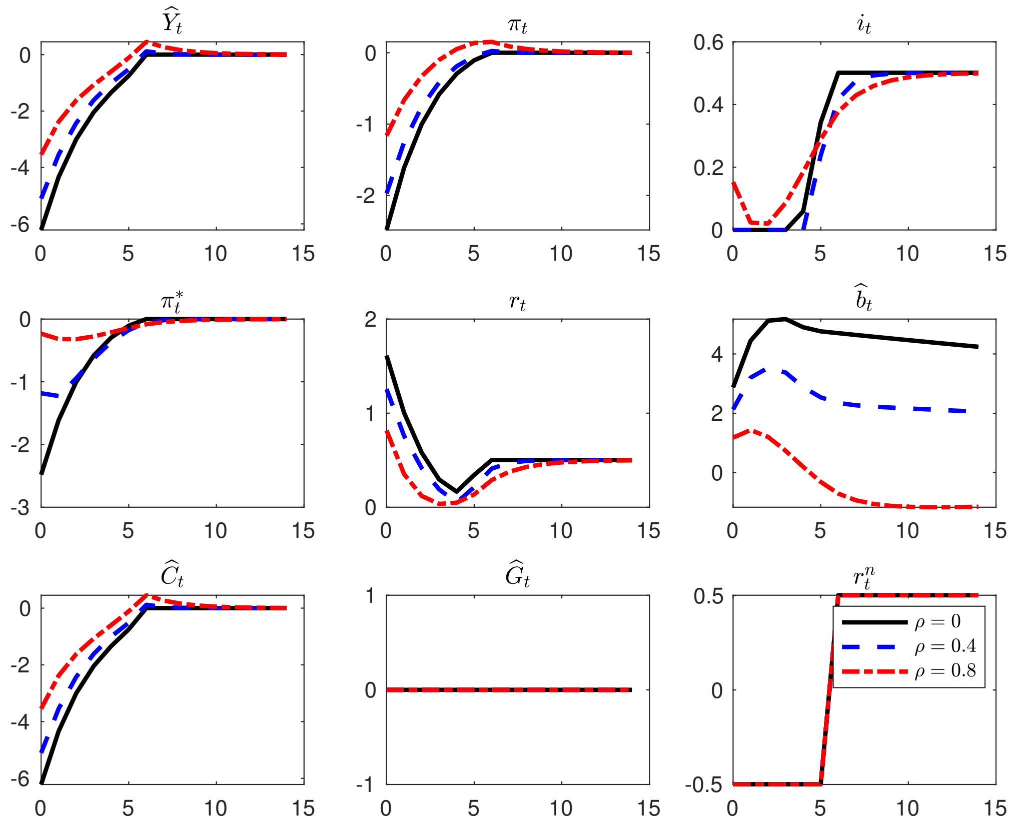

FIGURE 5. Impulse responses to a negative aggregate demand shock under the monetary regime, with no fiscal policy responses.

interest rate to the ZLB for the first 4 quarters. When the ZLB constraint is binding, declines in the inflation rate raise the real interest rate, further exacerbating the recession. Over time, as the negative demand shock unwinds, inflation rises back to its steady state, reducing the real interest rate while the ZLB constraint is still binding. Eventually, the nominal interest rate lifts off from the ZLB and returns to its steady state level in 6 quarters after the shock. These results are the same as in the standard New Keynesian models.

Under the AIT rule, the nominal interest rate responds to average inflation π_t^* instead of actual inflation. Stronger history-dependence of the AIT (i.e., higher ρ) implies more gradual changes in the inflation target and thus more gradual adjustments in the nominal interest rate as well. With a sufficiently large value of ρ (e.g., $\rho = 0.8$), the inflation target adjusts very slowly, and the nominal rate does not hit the ZLB constraint at all.

AIT also implies that, after liftoff from the ZLB, the nominal and real interest rates would stay lower for longer. Forward-looking households would thus respond by consuming more and saving less. In this sense, the AIT rule acts like forward guidance. This explains why AIT helps dampen the recessionary effects of the negative demand shock, leading to smaller drops in consumption, output, and inflation.

Conditional on the negative aggregate demand shock that pushes the economy into the ZLB, we now consider the impact of a temporary fiscal policy shock. In particular, we consider a one-time increase in government spending of 1% of steady-state real GDP in the impact period of the demand shock, which returns to steady-state thereafter (i.e., $\varepsilon_0^g = 1$ and $\rho_g = 0$).

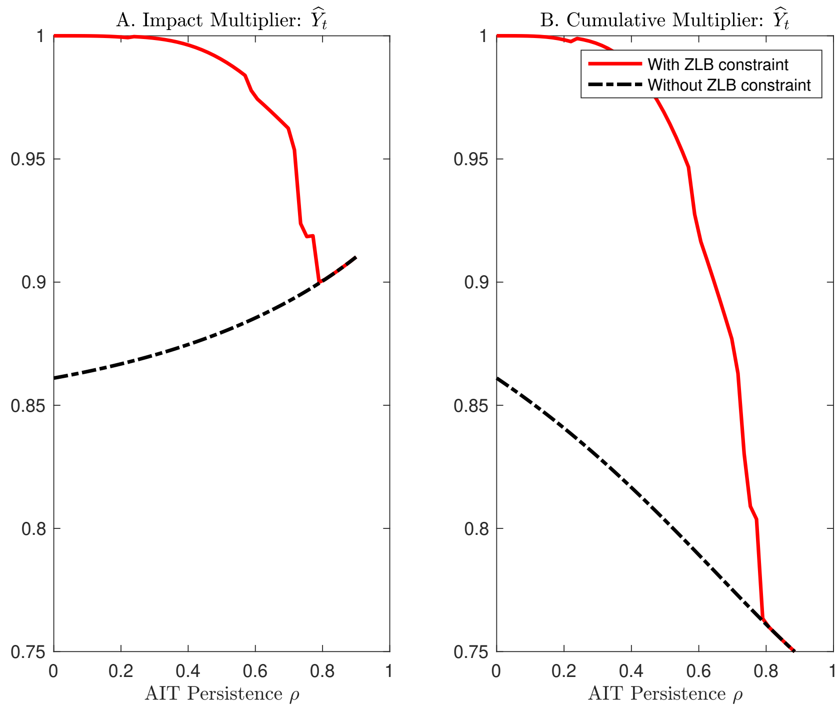

FIGURE 6. Government Spending Multipliers in the Monetary Regime.
Note: The figure plots the fiscal multipliers under the monetary regime as a function of history-dependence of the AIT rule (measured by ρ). It shows the impact effects (left panel) and the cumulative effects (the right panel) of a one-time increase in government spending conditional on a persistent aggregate demand shock that pushes the economy into the ZLB. The dashed lines show the fiscal multipliers in normal times. The solid lines show the multipliers when the ZLB constraint is binding.

Figure 6 plots the fiscal multipliers as a function of the history-dependence of the AIT rule measured by the parameter ρ. In particular, for each given value of ρ, we plot the impact effects (the left panel) and the cumulative effects of a temporary government spending increase conditional on the negative aggregate demand shock that pushed the economy into the ZLB. We plot the fiscal multipliers both in the normal times (the dashed lines) and during the ZLB periods (the solid lines).

The figure shows that, for a small value of ρ, the impact and cumulative fiscal multipliers are both larger during the ZLB periods than in normal times. This result obtains because the increase in inflation and inflation expectations following the government spending shock reduces the real interest rate during the ZLB period, stimulating private consumption through intertemporal substitution. In contrast, in normal times, nominal interest rate would have to rise in response to increases in inflation and the inflation target. This result generalizes that in the literature that studies the fiscal multipliers under the ZLB with the standard Taylor rule (Christiano et al., 2011; Eggertsson, 2011), with is nested as a special case of our AIT rule with $\rho = 0$. The result is also consistent with the empirical evidence of Miyamoto et al. (2018) and Ramey and Zubairy (2018).

An increase in history dependence under AIT, however, reduces the government spending multipliers at the ZLB because of the makeup feature of AIT. During the ZLB periods, inflation increases following a government spending shock. When the economy eventually exits from the ZLB, monetary policy tightening is required to reduce inflation to keep average inflation at the target level. The greater the history dependence of the inflation target, the stronger the makeup effects. A decline in expected inflation reduces current inflation while the economy is still in the ZLB, weakening the stimulus effects of government spending. With sufficiently strong history dependence, however, the economy would never enter the ZLB (as shown in Figure 5), such that the fiscal multipliers coincide with those in the normal times with unconstrained monetary policy (as shown in Figure 6).

IV.2. **Fiscal Regime.** We next consider the fiscal regime featuring a passive monetary policy(with $\phi_\pi = 0.8 < 1$) and an active fiscal policy (with $\phi_b = 0.0025 < 1 - \beta$).

Figure 7 shows the impulse responses to a negative demand shock under the fiscal regime that pushes the economy into the ZLB, with no contemporaneous fiscal policy responses. The shock is the same as that we have considered in Section IV.1 for the monetary regime. Similar to the case with the monetary regime, the negative demand shock leads to a short-run recession, reducing aggregate consumption, output, and inflation, and pushing the nominal interest rate to the ZLB for small values of ρ. The recession reduces the present value of fiscal surprise, raising the real value of government debt. Since future taxes do not increase sufficiently to pay off the debt, future inflation is expected to increase. Thus, inflation overshoots its steady state in the short run. The inflation overshooting shortens the duration of the ZLB periods. As shown in the figure, with sufficiently large values of ρ, the average inflation target does not decline sufficiently for the economy to enter the ZLB.

Now consider a one-time increase in government spending in the impact period of the demand shock that is equivalent to 1% of steady-state real GDP. Figure 8 plots the impact and cumulative fiscal multipliers as a function of ρ. For each given value of ρ, we plot the

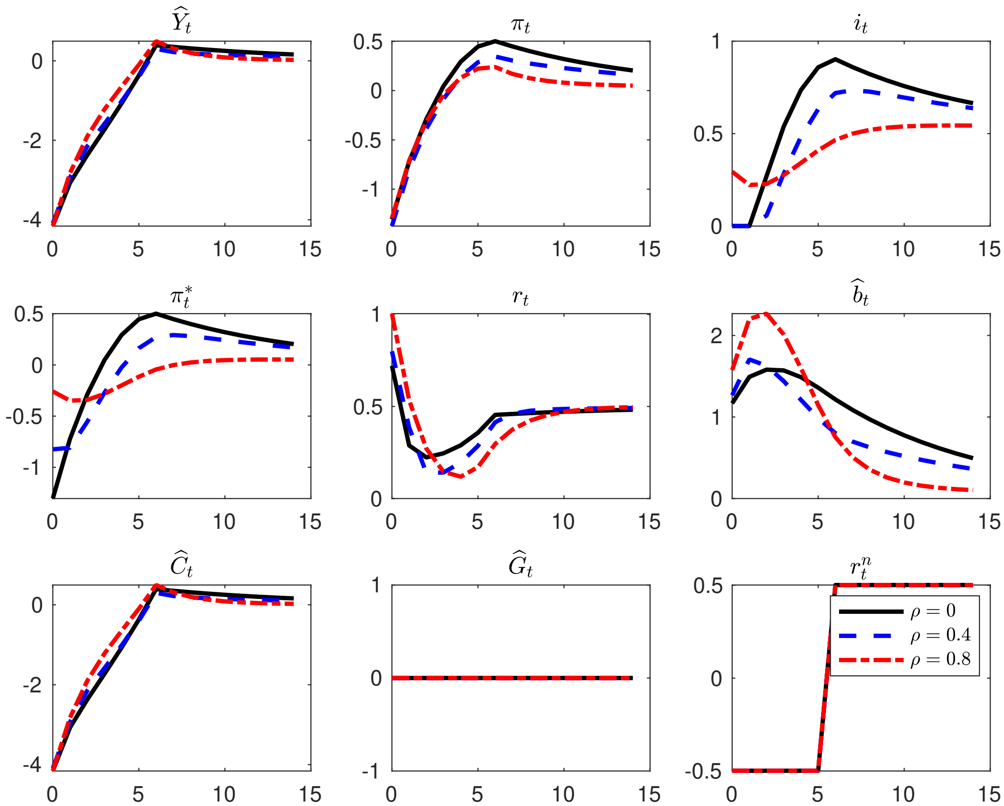

Figure 7. Impulse responses to a negative aggregate demand shock under the fiscal regime, with no fiscal policy responses.

impact effects (the left panel) and the cumulative effects (the right panel) of the government spending shock conditional on the realizations of the negative demand shock, for both the normal times (the dashed lines) and the ZLB periods (the solid lines).

Under the fiscal regime at the ZLB, an increase in government spending acts like a debt-financed tax cut, increasing the household's perceived nominal wealth and boosting private consumption. The increase in aggregate demand raises inflation, lowering the real interest rate and further boosting private consumption through intertemporal substitution. When the nominal rate reaches the ZLB, the increase in inflation reduces the real rate more than it does in normal times, implying larger impact multipliers at the ZLB than in normal times. Furthermore, under the fiscal regime, the wealth effect and the intertemporal substitution effect both raise private consumption, resulting in an impact multiplier larger than one and larger than that under the monetary regime.

The cumulative multiplier in the ZLB period, however, is smaller than in normal times, because the spending increase is not financed by equivalent future tax increases, but it is financed partially by increases in future inflation. Agents expect that, after liftoff from the ZLB, passive monetary policy would allow inflation to stay above steady state, despite that

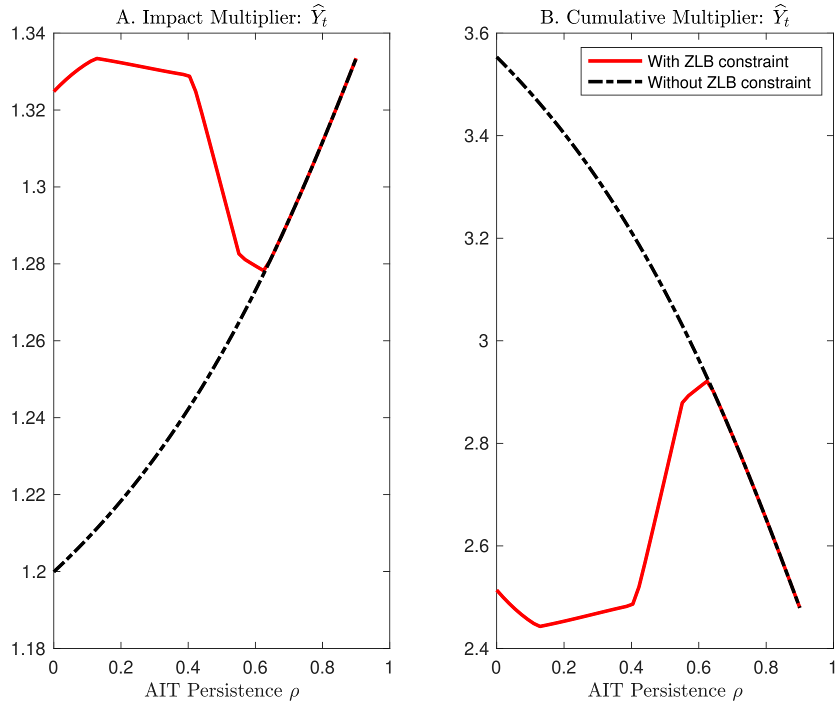

FIGURE 8. Government Spending Multipliers in the Fiscal Regime.
Note: The figure plots the fiscal multipliers under the fiscal regime as a function of history-dependence of the AIT rule (measured by ρ). It shows the impact effects (left panel) and the cumulative effects (the right panel) of a one-time increase in government spending conditional on a persistent aggregate demand shock that pushes the economy into the ZLB. The dashed lines show the fiscal multipliers in normal times. The solid lines show the multipliers when the ZLB constraint is binding.

the government spending shock raises inflation at the ZLB. Inflation acts as a tax on the household's perceived nominal wealth, reducing the cumulative multiplier. Furthermore, in normal times, the nominal interest rate would increase with the inflation target, raising the return on nominal wealth. During the ZLB periods, however, this channel is absent such that the cumulative multiplier is smaller than in normal times.

Under the fiscal regime, AIT has important implications for fiscal stimulus at the ZLB. Stronger history dependence of AIT (i.e., a larger ρ) implies that the nominal interest rate will stay lower for longer after liftoff from the ZLB, allowing inflation to stay above steady state for longer. Although the increase in current inflation at the ZLB boosts aggregate demand through intertemporal substitution, the increases in future inflation outside of ZLB erode the value of nominal wealth associated with the debt-financed tax cut, lowering the impact multiplier of government spending at the ZLB. These two opposing forces associated with AIT render the relation between the government spending multipliers and the history dependence of AIT nonlinear. With small values of ρ, the impact multiplier increases with ρ; with large values of ρ, the impact multiplier decreases with ρ. If ρ is sufficiently large, then the economy would never enter the ZLB, such that the government spending multiplier coincides with that in normal times.

V. Conclusion

We have studied the implications of AIT, a feature of the new monetary policy framework of the Federal Reserve, for the effectiveness of fiscal stimulus. Our study is based on the standard New Keynesian framework with interactions between monetary policy and fiscal policy, generalized to incorporate AIT targeting and allowing for occasionally binding ZLB constraints. We show that the fiscal multiplier effects of a government spending shock depend on (1) the prevailing policy regime (monetary vs. fiscal regime), (2) the prevailing economic conditions (ZLB or normal times), and (3) the history-dependence of the AIT rule.

In normal times under a monetary regime with active monetary policy and passive fiscal policy, government spending raises aggregate demand and inflation on impact. The history-dependence of AIT dampens the adjustments in the nominal interest rate, mitigating the crowd-out effects on private consumption. Over time, monetary policy tightening is required to reduce inflation and keep average inflation at target, resulting in a smaller cumulative fiscal multiplier than the impact multiplier. Under a fiscal regime with an active fiscal policy and passive monetary policy, a government spending shock pushes up inflation persistently, with no inflation makeup because of weak reactions of monetary policy to average inflation. Thus, the real interest rate stays below steady state persistently, resulting in a larger cumulative multiplier than the impact multiplier.

In a liquidity trap where a negative demand shock pushes the nominal interest rate to the ZLB, increasing history-dependence of the AIT rule shortens the duration of the liquidity trap. In this situation, a government spending shock raises inflation and thus reduces the real interest rate, resulting in a fiscal multiplier that is larger than that in normal times. Under the monetary regime, however, greater history-dependence of the AIT rule leads to a smaller fiscal multiplier because higher inflation now needs to be compensated by lower inflation later. Under a fiscal regime, there is no inflation makeup and the history-dependence of the AIT rule has a nonlinear effect on the size of the government spending multipliers in a liquidity trap.

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For Online Publication Appendix

Appendix A. Details of the model

A. 1. **Model environment.**

A. 1.1. *Households.* There exists a continuum of households. The representative household chooses consumption C_t, labor N_t, and holdings of real government bond b_t to maximize the life-time utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_t^p \left[u(C_t) - v(N_t) \right],$$

where

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \text{ and } v(N_t) = \chi \frac{N_t^{1+\nu}}{1+\nu},$$

subject to the following budget constraint:

$$C_t + T_t + \frac{b_t}{R_t} = W_t N_t + \frac{b_{t-1}}{\Pi_t} + \Omega_t,$$

where T_t is the lump-sum tax, R_t is the gross nominal return rate on government bond, $b_t = B_t/P_t$ is the real government bond, W_t is the real wage, and Ω_t is the real dividend payout from firms. Notice that Z_t^p denotes preference shock, which is exogenously given.

The optimal choices for consumption and labor supply are characterized by corresponding first-order conditions:

$$1 = \mathbb{E}_t \beta \frac{Z_{t+1}^p u'(C_{t+1})}{Z_t^p u'(C_t)} \frac{R_t}{\Pi_{t+1}}, \tag{A.1}$$

$$u'(C_t) W_t = v'(N_t). \tag{A.2}$$

A. 1.2. *Firms.* There is a continuum of retailers $j \in [0, 1]$ who produce differentiated intermediate goods according to

$$Y_{jt} = f(N_{jt}) = A N_{jt}^{1-\alpha}.$$

A final goods producer bundles the differentiated intermediate goods into the final goods with the CES production function:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1.$$

It follows that each intermediate goods producer faces a downward-slopping demand curve:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon} Y_t,$$

where the price index is given by

$$P_t \equiv \left[\int_0^1 P_{jt}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}.$$

Assume that the intermediate goods producers are subject to Calvo pricing frictions such that in each period they have a constant probability of $1-\theta$ to be able to reset their price P_{jt}. The optimal pricing condition implies that

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k Z_{t+k}^p u'(C_{t+k}) \left[P_t^* - \frac{\epsilon}{\epsilon-1} MC_{t+k|t}^j P_{t+k}\right] Y_{t+k} P_{t+k}^{\epsilon-1} = 0, \tag{A.3}$$

where $MC_{t+k|t}^j$ is the firm-specific marginal cost that satisfies

$$MC_t^j = W_t / f'(N_t^j). \tag{A.4}$$

Define the average marginal cost as

$$MC_t = W_t / f'(N_t). \tag{A.5}$$

Combining labor supply condition (A.2) with (A.5) yields

$$MC_t \cdot f'(N_t) = W_t = \frac{v'(N_t)}{u'(C_t)}. \tag{A.6}$$

Reorganizing yields

$$MC_t \cdot u'(Y_t - G_t) = \frac{v'(N_t)}{f'(N_t)} = \widetilde{v}'(Y_t), \tag{A.7}$$

where

$$\tilde{v}(Y_t) \equiv v(f^{-1}(Y_t)).$$

A. 1.3. *Monetary and Fiscal Policies.* In each period, the government issues nominal bond B_t, which is sold at discounted value B_t/R_t. The government budget constraint is given by

$$G_t + \frac{b_{t-1}}{\Pi_t} = T_t + \frac{b_t}{R_t}, \quad t \geq 0,$$

where $b_t \equiv B_t/P_t$ is the real government bond issued at date t, G_t is the government spending, and T_t is the lump-sum tax.

Following Leeper (1991), suppose that the government adjusts real lump-sum tax in response to changes in outstanding real public debt:

$$\frac{T_t - T}{Y} = \phi_b \frac{b_{t-1} - b}{Y} + \varepsilon_t^\tau,$$

where ε_t^τ is the fiscal transfer shock.

Assume that the monetary authority sets nominal interest rate $\ln R_t$ in response to the average inflation target $\ln \Pi_t^*$:

$$\ln R_t = \ln R^r + \varphi_\pi \ln \Pi_t^*,$$

where the average inflation target $\ln \Pi_t^*$ is a moving average of past inflation, namely

$$\ln \Pi_t^* = \rho \ln \Pi_{t-1}^* + (1-\rho) \ln \Pi_t.$$

A. 1.4. *Market Clearing Conditions.* From the labor market clearing condition, we have

$$N_t = \int_0^1 N_{jt} dj.$$

The resource constraint is given by

$$Y_t = C_t + G_t,$$

where G_t follows the process:

$$\frac{G_t - G}{Y} = \rho_g \frac{G_{t-1} - G}{Y} + \varepsilon_t^g.$$

A. 2. **Steady State.** Let variables without time subscripts denote their steady state values. Assume that the government spending to output ratio G/Y and the public debt to output ratio b/Y are exogenously given. We will focus on the steady state with zero net inflation ($\Pi = 1$) so that all intermediate goods producers set the same prices.

From the Euler equation (A.1), we have the real interest rate given by $R^r = 1/\beta$. With $\Pi = 1$, we have $R = R^r \Pi = 1/\beta$. The optimal pricing condition (A.3) suggests that the real marginal cost in steady state is given by

$$MC = MC^j = \frac{\epsilon - 1}{\epsilon}. \tag{A.8}$$

It follows that we can rewrite (A.7) as

$$MC \cdot u' \left[Y(1 - \frac{G}{Y}) \right] = \tilde{v}'(Y). \tag{A.9}$$

Combining (A.8), (A.9), and the exogenous government spending-output ratio G/Y, we can solve for the steady state output Y. According to the production technology $Y = Y_j = f(N) = AN^{1-\alpha}$, we can pin down N. And consumption can be derived from the resource constraint $C = Y(1 - G/Y)$. Using the government budget constraint, we have

$$\frac{G}{Y} + \left(1 - \frac{1}{R^r}\right) \frac{b}{Y} = \frac{T}{Y}.$$

This suggests that for given values of G/Y and b/Y, we can derive the relative size of the lump-sum tax T/Y. By using these ratios and the value of Y derived above, we can pin down the steady state value of G, b, and T. Lastly, we have the real wage given by $W = MC \cdot f'(N)$.

A. 3. **Linearized System.** Let $\Delta_t = \ln\left(Z^p_{t+1}/Z^p_t\right)$. Define

$$\widehat{Y}_t = \frac{Y_t - Y}{Y}, \widehat{b}_t = \frac{b_t - b}{Y}, \widehat{G}_t = \frac{G_t - G}{Y}, \pi_t = \frac{\Pi_t - \Pi}{\Pi}, \pi^*_t = \ln\left(\Pi^*_t/\Pi^*\right), i_t = \ln R_t.$$

Linearizing (A.1) yields the linearized dynamic IS curve

$$\widehat{Y}_t - \widehat{G}_t = \mathbb{E}_t\left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) - \sigma\mathbb{E}_t\left(i_t - \pi_{t+1} + \ln\beta + \Delta_t\right), \tag{A.10}$$

where

$$\sigma = -\frac{u'(C)}{Yu''(C)} = \frac{C}{\gamma Y}.$$

Using (A.4) and (A.5), we obtain the firm-level real marginal cost as

$$\widetilde{MC}^j_t - \widetilde{MC}_t = -b\left(\widetilde{P}^j_t - \widetilde{P}_t\right),$$

where $b = \alpha\epsilon/(1-\alpha)$. Combining with the first-order approximation of (A.3) yields

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)}{\theta(1+b)}\widetilde{MC}_t + \beta\mathbb{E}_t\pi_{t+1}.$$

From (A.7), the average real marginal cost can be written as

$$\widetilde{MC}_t = \eta_v\widehat{Y}_t + \eta_u\left(\widehat{Y}_t - \widehat{G}_t\right),$$

where

$$\eta_v \equiv Y\frac{\tilde{v}''(Y)}{\tilde{v}'(Y)} = \frac{\alpha+\nu}{1-\alpha},$$
$$\eta_u \equiv -Y\frac{u''(C)}{u'(C)} = \gamma\frac{Y}{C}.$$

Combining the results above, we obtain the linearized NKPC

$$\pi_t = \kappa\left(\widehat{Y}_t - \Gamma\widehat{G}_t\right) + \beta\mathbb{E}_t\pi_{t+1},$$

where $\kappa = \kappa_0\left(\eta_v + \eta_u\right)$, $\kappa_0 = (1-\beta\theta)(1-\theta)/\left(\theta(1+b)\right)$, and $\Gamma = \eta_u/(\eta_v+\eta_u)$.

For given values of $\{\pi^*_{-1}, \widehat{b}_{-1}\}$ and exogenous processes of $\{\Delta_t, \varepsilon^g_t, \varepsilon^\tau_t\}$, the linearized system can be summarized by 5 variables (π_t, i_t, π^*_t, $\widehat{Y}_t$, $\widehat{b}_t$) with the following 5 equations:

(1) IS curve,

$$\widehat{Y}_t - \widehat{G}_t = \mathbb{E}_t\left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) - \sigma\left[i_t - \mathbb{E}_t\pi_{t+1} + \ln\beta + \Delta_t\right]. \tag{A.11}$$

(2) NKPC,

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\left(\widehat{Y}_t - \Gamma\widehat{G}_t\right). \tag{A.12}$$

(3) Monetary policy rule,

$$i_t = \max\left\{-\ln\beta + \phi_\pi \pi_t^*,\ 0\right\}. \tag{A.13}$$

(4) AIT,

$$\pi_t^* = \rho\pi_{t-1}^* + (1-\rho)\pi_t. \tag{A.14}$$

(5) Debt dynamics,

$$\beta\widehat{b}_t = (1-\phi_b)\widehat{b}_{t-1} + \widehat{G}_t + \beta\frac{b}{Y}(i_t + \ln\beta) - \frac{b}{Y}\pi_t - \varepsilon_t^\tau, \tag{A.15}$$

where

$$\widehat{G}_t = \rho_g \widehat{G}_{t-1} + \varepsilon_t^g.$$

Note that the shock processes $\{\Delta_t, \varepsilon_t^\tau, \varepsilon_t^g\}$ are exogenously given.

Appendix B. Proofs of Proposition 1.

B. 1. **Determinacy conditions under locally Ricardian fiscal policy.** Assume the fiscal policy is locally Ricardian, i.e. $|1-\phi_b|/\beta < 1$, and that the monetary policy rule is given by

$$i_t = -\ln\beta + \phi_\pi\pi^* + \phi_y\widehat{Y}_t.$$

Let $X_t = \left[\widehat{Y}_t, \pi_t, \pi_t^*, \widehat{b}_t\right]'$. With perfect foresight and without shocks, the linearized system can be summarized by $AX_{t+1} = BX_t$, namely

$$\begin{bmatrix} 1 & \sigma & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & \rho-1 & 1 & 0 \\ -\beta\frac{b}{Y}\phi_y & \frac{b}{Y} & -\beta\frac{b}{Y}\phi_\pi & \beta \end{bmatrix} \begin{bmatrix} \widehat{Y}_{t+1} \\ \pi_{t+1} \\ \pi_{t+1}^* \\ \widehat{b}_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\phi_y\sigma & 0 & \phi_\pi\sigma & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 1-\phi_b \end{bmatrix} \begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ \pi_t^* \\ \widehat{b}_t \end{bmatrix}. \tag{B.1}$$

Then we have

$$A^{-1}B = \begin{bmatrix} 1+\phi_y\sigma+\frac{\kappa\sigma}{\beta} & -\frac{\sigma}{\beta} & \phi_\pi\sigma & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\ \frac{\kappa(\rho-1)}{\beta} & \frac{1-\rho}{\beta} & \rho & 0 \\ \frac{b}{Y}\left[\zeta\kappa + \phi_y(1+\phi_y\sigma)\right] & -\frac{b}{Y}\zeta & \frac{b}{Y}\phi_\pi(\rho+\phi_y\sigma) & \frac{1-\phi_b}{\beta} \end{bmatrix}, \tag{B.2}$$

where $\zeta \equiv \left[1-\beta\phi_\pi(1-\rho)+\beta\sigma\phi_y\right]/\beta^2$.

The linearized system has two predetermined variables, namely π_t^* and $\widehat{b}_t$. Therefore, to have a unique bounded solution, we need to have exactly two eigenvalues of $A^{-1}B$ being inside the unit circle. Noting that the matrix $A^{-1}B$ is a block lower triangular matrix,[8] we have one eigenvalue being $(1-\phi_b)/\beta$, with the rest being determined by the matrix:

$$\Omega = \begin{bmatrix} 1+\phi_y\sigma+\frac{\kappa\sigma}{\beta} & -\frac{\sigma}{\beta} & \phi_\pi\sigma \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{\kappa(\rho-1)}{\beta} & \frac{1-\rho}{\beta} & \rho \end{bmatrix}.$$

The corresponding characteristic function of Ω is given by

$$\begin{aligned} f(\lambda) \equiv & \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 \\ = & \lambda^3 - \left(\frac{1+\kappa\sigma}{\beta} + (1+\rho+\phi_y\sigma)\right)\lambda^2 \\ & + \frac{\rho + (1+\beta\rho)(1+\phi_y\sigma) + (\phi_\pi + \rho(1-\phi_\pi))\,\kappa\sigma}{\beta}\lambda - \frac{\rho(1+\phi_y\sigma)}{\beta}. \end{aligned} \tag{B.3}$$

Suppose that the fiscal policy is locally Ricardian, i.e. $|(1-\phi_b)/\beta| < 1$, then we need exactly one additional eigenvalue being inside the unit circle to ensure determinacy. According to Woodford (2003, Proposition C.2 on page 672), for Ω to have exactly one eigenvalue inside the unit circle, the sufficient and necessary conditions are characterized by the following three cases:

- Case I:

$$1 + A_2 + A_1 + A_0 < 0,$$
$$-1 + A_2 - A_1 + A_0 > 0.$$

- Case II:

$$1 + A_2 + A_1 + A_0 > 0,$$
$$-1 + A_2 - A_1 + A_0 < 0,$$

  and

$$A_0^2 - A_0A_2 + A_1 - 1 > 0.$$

- Case III:[9]

$$1 + A_2 + A_1 + A_0 > 0,$$
$$-1 + A_2 - A_1 + A_0 < 0,$$

[8]For a block lower triangular matrix, its eigenvalues are given by the eigenvalues of the diagonal block matrices. This can be proved if we notice that the determinant of a block lower triangular matrix is product of the determinants of diagonal matrices.

[9]As noted by Woodford (2003), we include $A_0^2 - A_0A_2 + A_1 - 1 < 0$ so as to make sure the three cases are mutually disjoint. The condition is actually not needed.

and

$$A_0^2 - A_0 A_2 + A_1 - 1 < 0,$$
$$|A_2| > 3.$$

Using the definitions of A_0, A_1, and A_2 in (B.3), we can show that

$$1 + A_2 + A_1 + A_0 = \frac{\kappa\sigma}{\beta}(1-\rho)\left(\phi_\pi + \frac{1-\beta}{\kappa}\phi_y - 1\right), \tag{B.4}$$

and that

$$\begin{aligned}-1 + A_2 - A_1 + A_0 = &- 1 - \left[\frac{1}{\beta} + \rho + \frac{\kappa\sigma}{\beta} + (1+\phi_y\sigma)\right] \\ &- \left[(\rho + (1-\rho)\phi_\pi)\frac{\kappa\sigma}{\beta} + (1+\phi_y\sigma)(\frac{1}{\beta}+\rho) + \frac{\rho}{\beta}\right] \\ &- \frac{\rho}{\beta}(1+\phi_y\sigma) < 0, \text{ if } \phi_y \geq 0 \text{ and } \phi_\pi \geq 0 \end{aligned} \tag{B.5}$$

Therefore, Case I violates equation (B.5). It follows that the necessary and sufficient conditions for determinacy are completely summarized by Case II and Case III.

Using (B.4), the condition $1 + A_2 + A_1 + A_0 > 0$ in Case II and Case III can be rewritten as

$$\rho \neq 1, \quad \phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1. \tag{B.6}$$

In addition, conditions $A_0^2 - A_0A_2 + A_1 - 1 > 0$ (from Case II) and $|A_2| > 3$ (from Case III) can be rewritten as:

$$\frac{\rho}{\beta}\left(\frac{\rho}{\beta} - 1\right)(\phi_y\sigma + 1 - \beta)\left(\phi_y\sigma + 1 - \frac{1}{\rho} + \frac{\kappa\sigma}{\beta - \rho}\right) + \frac{\kappa\sigma}{\beta}(1-\rho)\phi_\pi > 0, \tag{B.7}$$

$$1 + \frac{1}{\beta} + \rho + \phi_y\sigma + \frac{\kappa\sigma}{\beta} > 3, \tag{B.8}$$

respectively.

To summarize, recalling Woodford (2003), the necessary and sufficient conditions for the determinacy with locally Ricardian fiscal policy are given by (B.6), and one of the two conditions (B.7) and (B.8).

Imposing $\phi_y = 0$, we can further simplify the results as: for $0 \leq \rho < 1$ and $|1 - \phi_b|/\beta < 1$, $\phi_\pi > 1$ is both sufficient and necessary for determinacy. With $\phi_y = 0$, condition (B.6) can be simplified as

$$\rho \neq 1, \quad \phi_\pi > 1. \tag{B.9}$$

This suggests that $\phi_\pi > 1$ is a necessary condition for determinacy.

To show that (B.9) is also sufficient for determinacy, we consider two cases. Firstly, suppose that $\rho > 2 - 1/\beta - \kappa\sigma/\beta$, then we can easily verify that (B.8) is satisfied. Secondly, suppose that $\rho < 2 - 1/\beta - \kappa\sigma/\beta$. Combining with $\phi_\pi > 1$, it can be shown that (B.7) is satisfied. More specifically, we can rewrite (B.7) as

$$\phi_\pi > \frac{1-\beta}{\beta\kappa\sigma}\left[-\beta - \bar{\kappa}\sigma + \rho + \frac{\kappa\sigma}{1-\rho}\right] \equiv H(\rho).$$

It is obvious that $H(\rho)$ is increasing in ρ. We notice that

$$H(2-1/\beta) = 1 + \left(2 - \frac{1}{\beta} - \beta - \kappa\sigma\right)\frac{1-\beta}{\beta\kappa\sigma} < 1.$$

This suggests that for $\rho < 2 - 1/\beta - \kappa\sigma/\beta$, we have

$$H(\rho) < H(2 - 1/\beta - \kappa\sigma/\beta) < H(2 - 1/\beta) < 1.$$

Therefore, if $\phi_\pi > 1$, then condition (B.7) is satisfied. This completes the proof of Part (i) of Proposition 1.

B. 2. **Determinacy under locally non-Ricardian fiscal policy.** Assume the fiscal policy is locally non-Ricardian, i.e. $|1-\phi_b|/\beta > 1$, and that the monetary policy rule is given by

$$i_t = -\ln\beta + \phi_\pi \pi^* + \phi_y \widehat{Y}_t.$$

The linearized system is summarized by (B.1). We denote the matrix $A^{-1}B$ in (B.2) as

$$H \equiv A^{-1}B \equiv \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix},$$

which is a block lower triangular matrix. With two predetermined variables, i.e. $\widehat{b}_t$ and π^*, we need exactly two eigenvalues of H being inside the unit circle to ensure determinacy. Equivalently, we need exactly two eigenvalues of H^{-1} being outside the unit circle.[10] Noting that

$$H^{-1} = \begin{bmatrix} H_{11}^{-1} & 0 \\ -H_{22}^{-1}H_{21}H_{11}^{-1} & H_{22}^{-1} \end{bmatrix},$$

with $H_{22} = (1-\phi_b)/\beta$ and $|(1-\phi_b)/\beta| > 1$. This suggests that one of the eigenvalues of H^{-1} is $\beta/(1-\phi_b)$, which is inside the unit circle. Among the three eigenvalues of H_{11}^{-1}, we therefore need exactly two eigenvalues of H_{11}^{-1} being outside the unit circle, and exactly one eigenvalue being inside the unit circle.

[10]Suppose that the eigenvalues of H are given by λ_i, $i = 1, 2, 3, 4$. Then the eigenvalues of H^{-1} are given by λ_i^{-1}, $i = 1, 2, 3, 4$. In addition, we have $|\lambda_i^{-1}| = 1/|\lambda_i|$.

The characteristic function associating with H_{11}^{-1} is given by

$$
\begin{aligned}
h(\lambda) \equiv & \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 && \text{(B.10)}\\
= & \lambda^3 - \left[\frac{1}{\rho} + \frac{\beta}{1+\phi_y\sigma}\left(1+\phi_y\sigma + \frac{1}{\beta} + \frac{1}{\rho}\frac{\kappa\sigma}{\beta}\left(\rho + (1-\rho)\,\phi_\pi\right)\right)\right]\lambda^2 && \text{(B.11)}\\
& + \frac{\beta}{1+\phi_y\sigma}\left[1+\frac{1}{\rho}\left(1+\phi_y\sigma + \frac{1+\kappa\sigma}{\beta}\right)\right]\lambda - \frac{1}{\rho}\frac{\beta}{1+\phi_y\sigma}, && \text{(B.12)}
\end{aligned}
$$

with $A_0 < 0$, $A_1 > 0$, and $A_2 < 0$.

According to Woodford (2003), to ensure H_{11}^{-1} has exactly one eigenvalue being inside the unit circle, the necessary and sufficient conditions are characterized by the following three cases:

- Case I:

$$
\begin{aligned}
1 + A_2 + A_1 + A_0 &< 0,\\
-1 + A_2 - A_1 + A_0 &> 0.
\end{aligned}
$$

- Case II:

$$
\begin{aligned}
1 + A_2 + A_1 + A_0 &> 0,\\
-1 + A_2 - A_1 + A_0 &< 0,
\end{aligned}
$$

  and

$$
A_0^2 - A_0A_2 + A_1 - 1 > 0.
$$

- Case III:[11]

$$
\begin{aligned}
1 + A_2 + A_1 + A_0 &> 0,\\
-1 + A_2 - A_1 + A_0 &< 0,
\end{aligned}
$$

  and

$$
\begin{aligned}
A_0^2 - A_0A_2 + A_1 - 1 &< 0,\\
|A_2| &> 3.
\end{aligned}
$$

Using (B.12), it can be shown that $-1 + A_2 - A_1 + A_0 < 0$. It follows that the necessary and sufficient conditions are given by Cases II and III.

To simplify the conditions, we notice that

$$
1 + A_2 + A_1 + A_0 = \left(1 - \frac{1}{\rho}\right)\frac{\beta}{1+\phi_y\sigma}\frac{\bar{\kappa}\sigma}{\beta}\left[\frac{1-\beta}{\kappa}\phi_y + \phi_\pi - 1\right].
$$

[11]As noted by Woodford (2003), we include $A_0^2 - A_0A_2 + A_1 - 1 < 0$ so as to make sure the three cases are mutually disjoint. The condition is actually not needed.

For Cases II and III, we would require that $1 + A_2 + A_1 + A_0 > 0$. With $0 < \rho < 1$, the condition is equivalent to

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y < 1. \tag{B.13}$$

Similarly, we can show that the condition $A_0^2 - A_0 A_2 + A_1 - 1 > 0$ is equivalent to

$$\frac{1}{\rho}\frac{\beta}{1+\phi_y\sigma}\left[\left(1+\phi_y\sigma+\frac{1+\kappa\sigma}{\beta}-\frac{1}{\rho}\right)\left(1-\frac{\beta}{1+\phi_y\sigma}\right)+\rho-\frac{\kappa\sigma}{\rho(1+\phi_y\sigma)}\left(1-\rho\right)\phi_\pi\right]-1>0, \tag{B.14}$$

and that the condition $|A_2| > 3$ is equivalent to

$$\frac{1}{\rho}+\beta+\frac{1}{1+\phi_y\sigma}\left(1+\frac{\kappa\sigma}{\rho}\left(\rho+\left(1-\rho\right)\phi_\pi\right)\right)>3. \tag{B.15}$$

To summarize, according to Woodford (2003), the necessary and sufficient conditions for the determinacy with locally non-Ricardian fiscal policy are given by (B.13), and one of the two conditions (B.14) and (B.15).

Now suppose that $\phi_y = 0$. We can further simplify the conditions: we will show that $\phi_\pi < 1$ is both necessary and sufficient for the determinacy of the fiscal regime. Note that condition (B.13) can be rewritten as $\phi_\pi < 1$ if $\phi_y = 0$. This also suggests that $\phi_\pi < 1$ is a necessary condition for determinacy. To show $0 \le \phi_\pi < 1$ is a sufficient condition for determinacy, we notice that (B.14) can be simplified as

$$\phi_\pi < \frac{1-\beta}{\kappa\sigma}\left[-1+\frac{1}{\beta}\left(\rho+\frac{\rho}{1-\rho}\kappa\sigma\right)\right] \equiv M(\rho), \tag{B.16}$$

and (B.15) can be rewritten as

$$\phi_\pi > \frac{2-\beta-\kappa\sigma}{\kappa\sigma}\frac{1}{1-\rho}\left(\rho-\frac{1}{2-\beta-\kappa\sigma}\right). \tag{B.17}$$

We need to show that either (B.16) or (B.17) is satisfied if $0 \le \phi_\pi < 1$. To see this, we consider three cases. Firstly, suppose that $2-\beta-\kappa \le 0$, then (B.17) is trivially satisfied, since the right-hand side of the inequality is negative. Secondly, suppose that $2-\beta-\kappa > 0$ and that $\rho < 1/(2-\beta-\kappa\sigma)$, then (B.17) is still trivially satisfied. Thirdly, suppose that $2-\beta-\kappa > 0$ and that $\rho \ge 1/(2-\beta-\kappa\sigma)$. We notice that $M(\rho)$ is monotonically increasing in ρ, with

$$M(\rho) \ge M(\frac{1}{2-\beta-\kappa\sigma}) > M(\frac{1}{2-\beta}) = \frac{1}{\beta}+\frac{1-\beta}{\kappa\sigma}\left(\frac{1}{\beta(2-\beta)}-1\right) > \frac{1}{\beta}.$$

Therefore, if $\phi_\pi < 1$, then (B.16) is satisfied. This completes the proof of Part (ii) of Proposition 1.

Appendix C. Fiscal Multipliers in Normal Times

Define the multipliers at horizon $j \geq 0$ for output and inflation as follows:

$$TM^y(j) \equiv -\frac{\partial \widehat{Y}_{t+j}}{\partial \varepsilon_t^\tau}, \quad TM^\pi(j) \equiv -\frac{\partial \pi_{t+j}}{\partial \varepsilon_t^\tau}, \quad GSM^y(j) \equiv \frac{\partial \widehat{Y}_{t+j}}{\partial \varepsilon_t^G}, \quad GSM^\pi(j) \equiv \frac{\partial \pi_{t+j}}{\partial \varepsilon_t^G}.$$

Note that the taxation multipliers are defined with a minus-sign so that a positive multiplier implies that a tax cut boosts output and inflation.

Denote the expectational errors as $\delta_t^y = \widehat{Y}_t - \mathbb{E}_{t-1}\widehat{Y}_t$ and $\delta_t^\pi = \pi_t - \mathbb{E}_{t-1}\pi_t$, respectively. Let $X_t = \left[\widehat{Y}_t, \pi_t, \pi_t^*, \widehat{b}_t, \widehat{G}_t\right]'$ and $\delta_t = [\varepsilon_t^\tau, \varepsilon_t^g, \eta_t^y, \eta_t^\pi]'$. The equilibrium system can be summarized by

$$\begin{bmatrix} 1 & \sigma & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & \rho - 1 & 1 & 0 & 0 \\ 0 & \frac{b}{Y} & -\beta\frac{b}{Y}\phi_\pi & \beta & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \widehat{Y}_{t+1} \\ \pi_{t+1} \\ \pi_{t+1}^* \\ \widehat{b}_{t+1} \\ \widehat{G}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sigma\phi_\pi & 0 & \rho_g - 1 \\ -\kappa & 1 & 0 & 0 & \kappa\Gamma \\ 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 1-\phi_b & 0 \\ 0 & 0 & 0 & 0 & \rho_g \end{bmatrix} \begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ \pi_t^* \\ \widehat{b}_t \\ \widehat{G}_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & \sigma \\ 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1}^\tau \\ \varepsilon_{t+1}^g \\ \eta_{t+1}^y \\ \eta_{t+1}^\pi \end{bmatrix}.$$

Premultiplying both sides by the inverse of the matrix on the left yields

$$X_{t+1} = \Gamma_1 X_t + \Psi \delta_{t+1},$$

where

$$\Gamma_1 = \begin{bmatrix} 1 + \frac{\kappa\sigma}{\beta} & -\frac{\sigma}{\beta} & \phi_\pi\sigma & 0 & \rho_g - 1 - \frac{\Gamma\kappa\sigma}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 & \frac{\Gamma\kappa}{\beta} \\ -\frac{\kappa(1-\rho)}{\beta} & \frac{1-\rho}{\beta} & \rho & 0 & \frac{\Gamma\kappa(1-\rho)}{\beta} \\ \frac{b}{Y}\frac{\kappa}{\beta^2}\left[1 - \beta\phi_\pi(1-\rho)\right] & -\frac{b}{Y}\frac{1}{\beta^2}\left[1 - \beta\phi_\pi(1-\rho)\right] & \frac{b}{Y}\phi_\pi\rho & \frac{1-\phi_b}{\beta} & \frac{\rho_g}{\beta} - \frac{b}{Y}\frac{\Gamma\kappa}{\beta^2}(1 - \beta\phi_\pi(1-\rho)) \\ 0 & 0 & 0 & 0 & \rho_g \end{bmatrix},$$

and

$$\Psi = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1-\rho \\ -\frac{1}{\beta} & \frac{1}{\beta} & 0 & \frac{b}{Y}\left[(1-\rho)\phi_\pi - \frac{1}{\beta}\right] \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The eigenvalues $\{\lambda_i\}$ of the matrix Γ_1 are given by

$$(\lambda_1, \lambda_2, \lambda_3, \rho_g, \frac{1-\phi_b}{\beta}),$$

with $(\lambda_1, \lambda_2, \lambda_3)$ being the solutions to $f(\lambda) = 0$, where

$$f(\lambda) = \lambda^3 - \left(\frac{1+\kappa\sigma}{\beta} + (1+\rho)\right)\lambda^2 + \frac{1 + \rho + \beta\rho + (\phi_\pi + \rho(1-\phi_\pi))\kappa\sigma}{\beta}\lambda - \frac{\rho}{\beta}. \qquad \text{(C.1)}$$

Without loss of generality, we assume $|\lambda_1| \geq |\lambda_2| \geq |\lambda_1|$. We consider the case with $\rho > 0$.[12] Denote each of the eigenvectors of matrix Γ_1 by $q_i = [q_{i1}, q_{i2}, q_{i3}, q_{i4}, q_{i5}, q_{i6}]'$. More specifically, for $i = 1, 2, 3$, the eigenvector corresponding to eigenvalue λ_i is given by

$$q_i = \begin{bmatrix} \frac{(\beta\lambda_i - 1)(\phi_b + \beta\lambda_i - 1)(\lambda_i - \rho)}{\frac{b}{Y}\kappa\lambda_i[(1-\beta(1-\rho)\phi_\pi)\lambda_i - \rho]} \\ -\frac{(\phi_b + \beta\lambda_i - 1)(\lambda_i - \rho)}{\frac{b}{Y}\lambda_i[(1-\beta(1-\rho)\phi_\pi)\lambda_i - \rho]} \\ -\frac{(\phi_b + \beta\lambda_i - 1)(1-\rho)}{\frac{b}{Y}[(1-\beta(1-\rho)\phi_\pi)\lambda_i - \rho]} \\ 1 \\ 0 \end{bmatrix}. \tag{C.2}$$

The eigenvector corresponding to eigenvalue $\lambda_4 = \rho_g$ is given by

$$q_4 = \begin{bmatrix} [1 - \phi_b - \beta\rho_g] J_1 \\ -[1 - \phi_b - \beta\rho_g](1-\rho_g)(\rho - \rho_g)(1-\Gamma)\kappa \\ -\rho_g [1 - \phi_b - \beta\rho_g](1-\rho_g)(\rho - 1)(1-\Gamma)\kappa \\ \rho_g \frac{b}{Y}(1-\Gamma)(1-\rho_g)\left(\rho_g(1 + \beta\phi_\pi(\rho - 1)) - \rho\right)\kappa - \rho_g J_2 \\ [1 - \phi_b - \beta\rho_g] J_2 \end{bmatrix}, \tag{C.3}$$

where

$$\begin{aligned} J_1 =& \rho_g^3\beta - \rho_g^2(1 + \beta + \beta\rho + \kappa\sigma\Gamma) + \rho_g\left[1 + \Gamma\kappa\sigma\phi_\pi + \rho(1 + \beta + (1-\phi_\pi)\Gamma\kappa\sigma)\right] - \rho, \\ J_2 =& \rho_g^3\beta - \rho_g^2(1 + \beta + \beta\rho + \kappa\sigma) + \rho_g\left[1 + \kappa\sigma\phi_\pi + \rho(1 + \beta + (1-\phi_\pi)\kappa\sigma)\right] - \rho. \end{aligned}$$

The eigenvector corresponding to eigenvalue $\lambda_5 = (1 - \phi_b)/\beta$ is given by

$$q_6 = [0, 0, 0, 1, 0]'. \tag{C.4}$$

Let Λ be the diagonal matrix of the eigenvalues of Γ_1, and Q be the matrix containing all eigenvectors. Define $Z_t = Q^{-1}X_t$, $v_t = Q^{-1}\Psi\delta_t$, we then can rewrite the system as

$$\begin{aligned} X_t =& \Gamma_1 X_{t-1} + \Psi\delta_t, \\ X_t =& Q\Lambda Q^{-1} X_{t-1} + \Psi\delta_t, \\ Q^{-1}X_t =& \Lambda Q^{-1} X_{t-1} + Q^{-1}\Psi\delta_t, \\ Z_t =& \Lambda Z_{t-1} + v_t, \\ Z_{jt} =& \lambda_j Z_{jt-1} + v_{jt}. \end{aligned} \tag{C.5}$$

Since we have two forward-looking variables, the determinacy of the system requires that there exist exactly two eigenvalues being outside the unit circle. To avoid explosive path for the solution, it requires that $Z_{jt} = 0$ and $v_{jt} = 0$ for any $|\lambda_j| > 1$. In the following, we

[12]If $\rho = 0$, the proof is still valid if the eigenvectors given by (C.2) are normalized properly.

derive the solutions for the fiscal multipliers in the monetary regime and the fiscal regime separately.

C. 1. **Fiscal multiplies under the monetary regime.** For monetary regime, we have $|(1-\phi_b)/\beta| < 1$. To ensure determinacy, we need exactly two eigenvalues being outside the unit circle. By the assumption of $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$, we obtain $|\lambda_1| \geq |\lambda_2| > 1 > |\lambda_3|$. Recalling (9), we have $f(0) = -\rho/\beta < 0$, $f(\rho) = \rho(1-\rho)\phi_\pi\kappa\sigma/\beta > 0$, suggesting $\lambda_3 \in (0, \rho)$. Then we have $v_{1t} = v_{2t} = 0$, and $Z_{1t} = Z_{2t} = 0$ to ensure a unique bounded solution. It follows that

$$\Psi\delta_t = Qv_t,$$

or

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1-\rho \\ -\frac{1}{\beta} & \frac{1}{\beta} & 0 & 0 & -\frac{b}{\beta Y}\left[1-\beta\phi_\pi(1-\rho)\right] \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^\tau \\ \varepsilon_t^g \\ 0 \\ \eta_t^y \\ \eta_t^\pi \end{bmatrix} = \begin{bmatrix} q_{31} & q_{41} & 0 \\ q_{32} & q_{42} & 0 \\ q_{33} & q_{43} & 0 \\ 1 & q_{44} & 1 \\ 0 & q_{45} & 0 \end{bmatrix} \begin{bmatrix} v_{3t} \\ v_{4t} \\ v_{5t} \end{bmatrix},$$

where we reformulate the left-hand side so that the coefficient matrix is invertible. Premultiplying both sides by the inverse of the matrix on the left, and using the resulting first three rows of equations, we can solve for

$$\begin{aligned} v_{3t} =& \frac{q_{42}(1-\rho) - q_{43}}{q_{45}\left[q_{33} - (1-\rho)q_{32}\right]}\varepsilon_t^g, \\ v_{4t} =& \frac{1}{q_{45}}\varepsilon_t^g, \end{aligned}$$

as well as the expression for v_{5t} (irrelevant).

Recalling that $X_t = QZ_t$ and $Z_{1t} = Z_{2t} = 0$, we obtain

$$\begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ \pi_t^* \\ \widehat{b}_t \\ \widehat{G}_t \end{bmatrix} = \begin{bmatrix} q_{31} & q_{41} & 0 \\ q_{32} & q_{42} & 0 \\ q_{33} & q_{43} & 0 \\ 1 & q_{44} & 1 \\ 0 & q_{45} & 0 \end{bmatrix} \begin{bmatrix} Z_{3t} \\ Z_{4t} \\ Z_{5t} \end{bmatrix}.$$

Combining with (C.5) yields

$$\begin{aligned} \frac{\partial \widehat{Y}_{t+j}}{\partial \varepsilon_t^g} =& q_{31}\frac{\partial Z_{3t+j}}{\partial \varepsilon_t^g} + q_{41}\frac{\partial Z_{4t+j}}{\partial \varepsilon_t^g} \\ =& \lambda_3^j q_{31}\frac{q_{42}(1-\rho) - q_{43}}{q_{45}\left[q_{33} - (1-\rho)q_{32}\right]} + \rho_g^j \frac{q_{41}}{q_{45}}. \end{aligned}$$

Using expressions for the eigenvectors given by (C.2) and (C.3), we then obtain (11). Similarly, we can derive other multipliers in Proposition 2.

C. 2. **Fiscal multiplies under the fiscal regime.** For the fiscal regime, we have $|(1-\phi_b)/\beta| > 1$. To ensure determinacy, we need exactly one additional eigenvalue being outside the unit circle. By the assumption of $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$, we obtain $|\lambda_1| > 1 > |\lambda_2| \geq |\lambda_3|$. Recalling (9) that $f(0) = -\rho/\beta < 0$, $f(\rho) = \rho(1-\rho)\phi_\pi\kappa\sigma/\beta > 0$, $f(1) = (1-\rho)(\phi_\pi - 1)\kappa\sigma/\beta < 0$. By intermediate value theorem, there exist two roots of $f(\lambda) = 0$ lying separately in the intervals of $(0, \rho)$ and $(\rho, 1)$. In the fiscal regime, we have $|\lambda_3| \leq |\lambda_2| < 1$ while $|\lambda_3| > 1$. Therefore, we obtain $0 < \lambda_3 < \rho < \lambda_2 < 1$.

The model determinacy requires that $v_{1t} = v_{5t} = 0$ and $Z_{1t} = Z_{5t} = 0$. Therefore, we can rewrite $\Psi\delta_t = Qv_t$ as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1-\rho \\ -\frac{1}{\beta} & \frac{1}{\beta} & 0 & 0 & -\frac{b}{\beta Y}\left[1-\beta\phi_\pi(1-\rho)\right] \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^\tau \\ \varepsilon_t^g \\ 0 \\ \eta_t^y \\ \eta_t^\pi \end{bmatrix} = \begin{bmatrix} q_{21} & q_{31} & q_{41} \\ q_{22} & q_{32} & q_{42} \\ q_{23} & q_{33} & q_{43} \\ 1 & 1 & q_{44} \\ 0 & 0 & q_{45} \end{bmatrix} \begin{bmatrix} v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix},$$

where we reformulate the left-hand side so that the coefficient matrix is invertible. Premultiplying both sides by the inverse of the matrix on the left, and using the resulting first three rows of equations, we have

$$\begin{aligned} v_{2t} =& \frac{q_{33}-(1-\rho)q_{32}}{\frac{b}{Y}(q_{23}q_{32}-q_{22}q_{33})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{23}-q_{33})+(1-\rho)(q_{32}-q_{22})\right]}\varepsilon_t^\tau \\ &+\frac{(q_{45}-\beta q_{44})\left[(1-\rho)q_{32}-q_{33}\right]+\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{33}q_{42}-q_{32}q_{43})}{\frac{b}{Y}(q_{23}q_{32}-q_{22}q_{33})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{23}-q_{33})+(1-\rho)(q_{32}-q_{22})\right]}\frac{1}{q_{45}}\varepsilon_t^g, \end{aligned} \tag{C.6}$$

$$\begin{aligned} v_{3t} =& \frac{q_{23}-(1-\rho)q_{22}}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}\varepsilon_t^\tau \\ &+\frac{(q_{45}-\beta q_{44})\left[(1-\rho)q_{22}-q_{23}\right]+\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{23}q_{42}-q_{22}q_{43})}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}\frac{1}{q_{45}}\varepsilon_t^g, \end{aligned} \tag{C.7}$$

$$v_{4t} = \frac{1}{q_{45}}\varepsilon_t^g. \tag{C.8}$$

Recalling that $X_t = QZ_t$ and $Z_{1t} = Z_{5t} = 0$, we have

$$\begin{bmatrix} \widehat{Y}_t \\ \pi_t \\ \pi_t^* \\ \widehat{b}_t \\ \widehat{G}_t \end{bmatrix} = \begin{bmatrix} q_{21} & q_{31} & q_{41} \\ q_{22} & q_{32} & q_{42} \\ q_{23} & q_{33} & q_{43} \\ 1 & 1 & q_{44} \\ 0 & 0 & q_{45} \end{bmatrix} \begin{bmatrix} Z_{2t} \\ Z_{3t} \\ Z_{4t} \end{bmatrix}.$$

Therefore, we have

$$\begin{aligned} TM_F^y(j) \equiv & -\frac{\partial \widehat{Y}_{t+j}}{\partial \varepsilon_t^\tau} = -\lambda_2^j q_{21}\frac{\partial Z_{2t}}{\partial\partial\varepsilon_t^\tau} - \lambda_3^j q_{31}\frac{\partial Z_{3t}}{\partial\varepsilon_t^\tau} \\ = & -\lambda_2^j q_{21}\frac{q_{33}-(1-\rho)q_{32}}{\frac{b}{Y}(q_{23}q_{32}-q_{22}q_{33})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{23}-q_{33})+(1-\rho)(q_{32}-q_{22})\right]} \\ & -\lambda_3^j q_{31}\frac{q_{23}-(1-\rho)q_{22}}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}. \end{aligned} \tag{C.9}$$

Combining the condition above with (C.2), (C.6), and (C.7), we obtain

$$TM_F^y(j) = \frac{1}{(1-\phi_b)b/Y}\frac{1}{\kappa}\left[\lambda_2^j\frac{(1-\beta\lambda_2)(\lambda_2-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}} + \lambda_3^j\frac{(1-\beta\lambda_3)(\lambda_3-\rho)}{\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_3-\rho}{1-\phi_b-\beta\lambda_3}-\frac{[1-\beta\phi_\pi(1-\rho)]\lambda_2-\rho}{1-\phi_b-\beta\lambda_2}}\right].$$

Similarly, one can obtain the expression for $TM_F^\pi(j)$ as in Proposition 3.

Using (C.6), (C.7), (C.8), and (11), we have the government spending multipliers for output as

$$\begin{aligned} GSM_F^y(j) = & \frac{\partial \widehat{Y}_{t+j}}{\partial \varepsilon_t^g} = \lambda_2^j q_{21}\frac{\partial Z_{2t}}{\partial\varepsilon_t^g} + \lambda_3^j q_{31}\frac{\partial Z_{3t}}{\partial\varepsilon_t^g} + \rho_g^j q_{41}\frac{\partial Z_{4t}}{\partial\varepsilon_t^g} \\ = & -\lambda_3^j q_{31}\frac{q_{42}(1-\rho)-q_{43}}{q_{45}\left[q_{33}-(1-\rho)q_{32}\right]} + GSM_M^y(j) \\ & + \lambda_2^j q_{21}\frac{(q_{45}-\beta q_{44})\left[(1-\rho)q_{32}-q_{33}\right]+\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{33}q_{42}-q_{32}q_{43})}{\frac{b}{Y}(q_{23}q_{32}-q_{22}q_{33})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{23}-q_{33})+(1-\rho)(q_{32}-q_{22})\right]}\frac{1}{q_{45}} \\ & + \lambda_3^j q_{31}\frac{(q_{45}-\beta q_{44})\left[(1-\rho)q_{22}-q_{23}\right]+\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{23}q_{42}-q_{22}q_{43})}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}\frac{1}{q_{45}}. \end{aligned}$$

Splitting the last two terms and combining with $TM_F^y(j)$ in (C.9) yield

$$\begin{aligned} & GSM_F^y(j) \\ = & TM_F^y(j) \times \frac{q_{45}-\beta q_{44}}{q_{45}} + GSM_M^y(j) \\ & + \lambda_2^j q_{21}\frac{\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{33}q_{42}-q_{32}q_{43})}{\frac{b}{Y}(q_{23}q_{32}-q_{22}q_{33})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{23}-q_{33})+(1-\rho)(q_{32}-q_{22})\right]}\frac{1}{q_{45}} \end{aligned}$$

$$
\begin{aligned}
&+\lambda_3^j q_{31}\frac{\beta\left[(1-\rho)q_{42}-q_{43}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{23}q_{42}-q_{22}q_{43})}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}\frac{1}{q_{45}}\\
&-\lambda_3^j q_{31}\frac{q_{42}(1-\rho)-q_{43}}{q_{45}\left[q_{33}-(1-\rho)q_{32}\right]}.
\end{aligned}
$$

The last two terms can be reorganized as

$$
\begin{aligned}
&\lambda_3^j\frac{q_{31}}{q_{45}}\frac{(1-\rho)q_{22}-q_{23}}{\frac{b}{Y}(q_{22}q_{33}-q_{23}q_{32})\left[1-\beta\phi_\pi(1-\rho)\right]+\beta\left[(q_{33}-q_{23})+(1-\rho)(q_{22}-q_{32})\right]}\\
\times&\frac{\beta\left[q_{43}-(1-\rho)q_{42}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{32}q_{43}-q_{33}q_{42})}{q_{33}-(1-\rho)q_{32}}.
\end{aligned}
$$

Therefore, using the expression for $TM_F^y(j)$ in (C.9) yields

$$
\begin{aligned}
&GSM_F^y(j)\\
=&GSM_M^y(j)+TM_F^y(j)\times\frac{q_{45}-\beta q_{44}}{q_{45}}\\
&+TM_F^y(j)\times\frac{\beta\left[q_{43}-(1-\rho)q_{42}\right]+\frac{b}{Y}\left[1-\beta\phi_\pi(1-\rho)\right](q_{32}q_{43}-q_{33}q_{42})}{q_{33}-(1-\rho)q_{32}}\frac{1}{q_{45}}.
\end{aligned}
$$

Substituting the eigenvectors in (C.2) and (C.3) into the above condition, we obtain

$$
\begin{aligned}
GSM_F^y(j)=&GSM_M^y(j)+TM_F^y(j)\cdot\frac{1-\phi_b}{1-\phi_b-\beta\rho_g}\\
&-TM_F^y(j)\frac{b}{Y}\frac{(1-\rho_g)(1-\Gamma)\kappa}{J_2}\left[\frac{1-\phi_b}{1-\phi_b-\beta\lambda_3}(\rho-\lambda_3)+\frac{1-\phi_b}{1-\phi_b-\beta\rho_g}(\rho_g-\rho)\right]\\
&+TM_F^y(j)\frac{b}{Y}\frac{(1-\rho)(1-\rho_g)(1-\Gamma)\kappa}{J_2}\beta\phi_\pi\left[-\frac{1-\phi_b}{1-\phi_b-\beta\lambda_3}\lambda_3+\frac{1-\phi_b}{1-\phi_b-\beta\rho_g}\rho_g\right].
\end{aligned}
$$

Recalling that

$$
\begin{aligned}
GSM_M^\pi(j)=&\frac{(1-\rho_g)(1-\Gamma)\kappa}{J_2}\left[\lambda_3^j(\rho-\lambda_3)+\rho_g^j(\rho_g-\rho)\right],\\
GSM_M^{\pi^*}(j)=&\frac{(1-\rho)(1-\rho_g)(1-\Gamma)\kappa}{J_2}\left[-\lambda_3^{j+1}+\rho_g^{j+1}\right],
\end{aligned}
$$

we then have

$$
GSM_F^y(j)=GSM_M^y(j)+\sum_{i=0}^{\infty}\left(\frac{\beta}{1-\phi_d}\right)^i\left[\rho_g^i-\frac{b}{Y}\cdot GSM_M^\pi(i)+\frac{b}{Y}\beta\phi_\pi\cdot GSM_M^{\pi^*}(i)\right]\times TM_F^y(j).
$$

Similarly, we can show that

$$
GSM_F^\pi(j)=GSM_M^\pi(j)+\sum_{i=0}^{\infty}\left(\frac{\beta}{1-\phi_d}\right)^i\left[\rho_g^i-\frac{b}{Y}\cdot GSM_M^\pi(i)+\frac{b}{Y}\beta\phi_\pi\cdot GSM_M^{\pi^*}(i)\right]\times TM_F^\pi(j).
$$

C. 3. **Multipliers in normal time with transitory government spending shocks.** Suppose that $\rho_g = 0$ and $\rho > 0$, then we have

$$GSM_M^y(0) = -\frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\Gamma)}{\rho} + 1,$$
$$GSM_M^y(j) = -\lambda_3^j \frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\Gamma)}{\rho}, \forall j \geq 1.$$

Since we know $f(0) = -\rho/\beta \leq 0$ and $f(\rho) = \rho(1-\rho)\phi_\pi\kappa\sigma/\beta \geq 0$. Hence, $\lambda_3 \in [0, \rho]$. This suggests that $GSM_M^y(0) < 1$ and $GSM_M^y(j) < 0$ for $j \geq 1$.

By continuity, we have $\lim_{\rho\to 0}\lambda_3 = 0$. Total differentiating $f(\lambda_3) = 0$ yields

$$\frac{\partial\lambda_3}{\partial\rho} = \frac{\lambda_3^2 - \lambda_3\left[\beta + (1-\phi_\pi)\kappa\sigma\right]/\beta + 1/\beta}{3\lambda_3^2 - 2\left[(1+\kappa\sigma)/\beta + 1 + \rho\right]\lambda_3 + \left[1 + \rho + \beta\rho + (\phi_\pi + \rho(1-\phi_\pi))\kappa\sigma\right]/\beta}.$$

This suggests that $\lim_{\rho\to 0}\lambda_3/\rho = \lim_{\rho\to 0}\partial\lambda_3/\partial\rho = 1/(1+\phi_\pi\kappa\sigma)$ by L'Hopital's rule. Therefore, we have

$$\begin{aligned}\frac{(1-\beta\lambda_3)(\rho-\lambda_3)(1-\rho_g)(1-\Gamma)}{J_2} &= -(1-\Gamma)(1-\beta\lambda_3)(1-\frac{\lambda_3}{\rho}) \\ &\to -(1-\Gamma)(1-\frac{1}{1+\phi_\pi\kappa\sigma}),\end{aligned}$$

as $\rho \to 0$. This implies that when $\rho_g = 0$, and $\rho \to 0$, we have

$$GSM_M^y(0) \to \frac{1+\phi_\pi\kappa\sigma\Gamma}{1+\phi_\pi\kappa\sigma},$$
$$GSM_M^y(j) = 0, \forall j \geq 1.$$

This is the same result as in Woodford (2011) and Beck-Friis and Willems (2017).

C. 4. **Proof of $TM_F^y(j) > 0$ and $TM_F^\pi(j) > 0$.** The taxation multipliers can be rewritten as

$$TM_F^y(j) = \frac{1}{\frac{b}{Y}\kappa}\frac{1}{\left[1-\beta\phi_\pi(1-\rho)\right](1-\phi_b)-\beta\rho}\left[\lambda_2^j\frac{(1-\beta\lambda_2)(\lambda_2-\rho)}{\frac{\lambda_2}{1-\phi_b-\beta\lambda_2}-\frac{\lambda_3}{1-\phi_b-\beta\lambda_3}} + \lambda_3^j\frac{(1-\beta\lambda_3)(\lambda_3-\rho)}{\frac{\lambda_3}{1-\phi_b-\beta\lambda_3}-\frac{\lambda_2}{1-\phi_b-\beta\lambda_2}}\right],$$
$$TM_F^\pi(j) = \frac{1}{\frac{b}{Y}}\frac{1}{\left[1-\beta\phi_\pi(1-\rho)\right](1-\phi_b)-\beta\rho}\left[\lambda_2^j\frac{\lambda_2-\rho}{\frac{\lambda_2}{1-\phi_b-\beta\lambda_2}-\frac{\lambda_3}{1-\phi_b-\beta\lambda_3}} + \lambda_3^j\frac{\lambda_3-\rho}{\frac{\lambda_3}{1-\phi_b-\beta\lambda_3}-\frac{\lambda_2}{1-\phi_b-\beta\lambda_2}}\right].$$

Suppose that $\rho > 0$, we can show that $0 < \lambda_3 < \rho < \lambda_2 < 1$. To obtain this result, we can show by recalling (9) that $f(0) = -\rho/\beta < 0$, $f(\rho) = \rho(1-\rho)\phi_\pi\kappa\sigma/\beta > 0$, $f(1) = (1-\rho)(\phi_\pi-1)\kappa\sigma/\beta < 0$. By intermediate value theorem, there exist two roots of $f(\lambda) = 0$ lying separately in the intervals of $(0,\rho)$ and $(\rho, 1)$. In the fiscal regime, we have $|\lambda_3| \leq |\lambda_2| < 1$ while $|\lambda_3| > 1$. Therefore, we obtain $0 < \lambda_3 < \rho < \lambda_2 < 1$.

By definition of fiscal regime, we have $(1 - \phi_d)/\beta > 1$ in the fiscal regime. These two observations imply that coefficients of λ_2^j and λ_3^j inside the squared bracket are positive. Moreover, using $(1 - \phi_d)/\beta > 1$, we have that

$$
\begin{aligned}
\left[1 - \beta\phi_\pi(1-\rho)\right](1-\phi_b) - \beta\rho =& \beta\left[\left[1 - \beta\phi_\pi(1-\rho)\right](1-\phi_b)/\beta - \rho\right] \\
>& \beta(1-\rho)(1-\beta\phi_\pi) \\
>& 0.
\end{aligned}
$$

Therefore, combining with $|\lambda_3| \leq |\lambda_2| < 1$, we know that $TM_F^y(j) > 0$, $TM_F^\pi(j) > 0$, and both of the multipliers decay monotonically over horizon j.