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WORKING PAPER SERIES

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February 2023

Working Paper 2023-04

https://www.frbsf.org/economic-research/publications/working-papers/2023/04/

Suggested citation:

Cardozo, Cristhian Hernando Ruiz, Jens Henrik Eggert Christensen. 2023 “The Benefit of Inflation-Indexed Debt: Evidence from an Emerging Bond Market,” Federal Reserve Bank of San Francisco Working Paper 2023-04. https://doi.org/10.24148/wp2023-04

The Benefit of Inflation-Indexed Debt: Evidence from an Emerging Bond Market

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&

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Abstract

Portfolio diversification is as important to debt management as it is to asset management. In this paper, we focus on diversification of sovereign debt issuance through greater reliance on inflation-indexed bonds for a representative emerging economy, Colombia. Using an arbitrage-free dynamic term structure model of fixed-coupon and inflation-indexed bond prices, we account for inflation and liquidity risk premia and calculate the net benefit of issuing inflation-indexed bonds over nominal bonds. Our results suggest that the Colombian government could lower its funding costs by as much as 0.69 percent by increasing its issuance of inflation-indexed debt, in particular at long maturities.

JEL Classification: D84, E31, E43, E44, E47, E52, E58, G12

Keywords: term structure modeling, liquidity risk, financial market frictions, central bank credibility, debt management

We thank Gus Kmetz for excellent research assistance. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Bank of the Republic of Colombia, the Federal Reserve Bank of San Francisco, or the Federal Reserve System.

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This version: February 13, 2023.

1 Introduction

Portfolio diversification is as important to debt management as it is to asset management. With respect to sovereign debt issuance, several studies have examined issues of time-varying maturity structure of debt as well as default resolution through maturity extension.[1] In light of the significant fiscal consequences arising from the COVID-19 pandemic and its aftermath, questions regarding the magnitudes of sovereign bond issuance and the optimization of its issuance terms are worthy of further review. This is particularly true for emerging economies whose debt capacity is widely perceived to be lower than the debt capacity of more advanced economies that have a long tradition of deep and well-established financial markets. Furthermore, as global financial conditions have tightened while government deficits remain large, practical questions arise regarding how to finance new and maturing debt in a manner sustainable for both governments and their domestic financial markets.

In this paper, we examine a potentially relevant aspect of this sovereign debt management challenge; namely, whether the government might benefit from switching its debt issuance away from well-established nominal fixed-coupon bonds and towards inflation-indexed bonds.[2,3]

The choice between issuing conventional debt with a fixed notional amount and inflation-indexed debt, which maintains its real value, involves a tradeoff. When the government issues nominal debt, investors assume the inflation risk. Provided investors as a group can be considered rational and forward-looking, they will demand a premium for taking on this risk, referred to as the inflation risk premium. If, instead, the government chooses to issue inflation-indexed debt, investors are protected against inflation risk as the cash flow of such debt (coupons and notional amount) is adjusted with the change in the economy-wide price index. As a consequence, the government can avoid paying the inflation risk premium and lower its debt servicing costs, assuming the government is less averse to inflation than investors.

One reason why the government may be less averse to inflation risk is that it controls the central bank and hence can directly affect inflation outcomes, unlike investors. Another reason is that taxes and many public benefits have structures with nominal rigidities such as fixed income brackets that are only adjusted with a lag, again with the government deciding when and what adjustments are made. This means that unexpected positive shocks to inflation, which are very costly to holders of nominal debt, tend to have a positive effect on the government's fiscal position, at least initially.[4] Moreover, positive inflation shocks in and of themselves erode the real value of the government's nominal debt and transfers wealth from bond holders to the government and taxpayers. These are the standard arguments put

[1] See Arellano and Ramanarayanan (2012) and Michalache (2020), respectively.

[2] Ermolov(2021) studies this question for a large group of advanced economies.

[3] Christensen, Lopez, and Mussche (2021) examine whether the U.S. government could benefit from extending the maximum maturity of its debt issuance, which represents another way to diversify a government's debt portfolio. In our analysis, we stay within the prevailing maturity structure.

[4] See Neely (2022) for a recent discussion in the case of U.S. government debt.

forward in favor of greater government reliance on inflation-indexed debt; see Price (1997) for an overview.

Against this backdrop, one may ask why most government debt around the world remains nominal.[5] What can account for the low issuance of inflation-indexed debt despite this clear economic argument in its favor? The theory of revealed preferences suggests that some costs must be preventing governments from following this seemingly pareto-improving strategy.

One key factor is likely to be frictions in financial markets, which may impede the trading and liquidity of inflation-indexed debt in secondary markets. These frictions could materialize for several reasons. In the following, we identify one key friction that flows from the economic reasoning above and relates to the natural demand for inflation-indexed bonds.

If inflation shocks and changes to investors' outlook for inflation represent a major reason for investors to trade standard fixed-coupon securities, a security that is insulated from this risk should be traded less, all else being equal.[6] This means that the trading environment for inflation-indexed bonds may be rather different and less active than the market for nominal securities. In turn, this may have a determining effect on the type of investors who choose to hold this debt. First, the classic economic argument outlined earlier clearly has *domestic* holders in mind in that they are the ones exposed to the inflation risk protected by the inflation-indexed bonds. Hence, foreign investors may have little appetite for this type of debt.[7] In addition, the lower level of trading may imply that the investor segment holding inflation-indexed bonds could be dominated by buy-and-hold investors. As a consequence, the liquidity risk profile of inflation-indexed debt may be very different from that of nominal debt, and the premium investors demand for assuming the liquidity risk of these harder-to-sell securities could be quite sizable.[8] As a final corollary, provided the trading in inflation-indexed debt securities is indeed concentrated among domestic investors, their liquidity premia should be driven primarily by domestic factors and economic developments.

In contrast, the inflation risk premium, which is compensation demanded by the holders of nominal bonds, is more likely to be determined by global factors and international developments. By the same logic, the liquidity premia of nominal debt securities are also likely to be heavily influenced by foreign factors.[9]

[5]In the United States, for example, Treasury Inflation-Protected Securities (TIPS) only account for about 8 percent of marketable government debt.

[6]Swanson and Williams (2014) find that positive surprises in inflation cause U.S. Treasury yields of all maturities to rise, while Beechey and Wright (2009) report no significant response of U.S. TIPS yields to such surprises.

[7]Beauregard et al. (2022, henceforth BCFZ) document for Mexico that only a very small fraction of its government's inflation-indexed debt is held by foreigners, while more than half of the nominal debt is foreign held.

[8]A large literature has documented sizable liquidity premia in the market for U.S. TIPS; see Andreasen et al. (2021), D'Amico et al. (2018), and Pflueger and Viceira (2016), among many others.

[9]Christensen, Fischer, and Shultz (2021, henceforth CFS) find that the foreign share is a key determinant of liquidity premia in the Mexican bonos market, while BCFZ find that Mexican inflation risk premia are determined by the foreign share in addition to global factors like oil prices, U.S. interest rates, and the VIX, a measure of risk aversion in the U.S. stock market.

In summary, there is a tradeoff to consider when a government decides whether to issue nominal or inflation-indexed debt. Although classic economic arguments seem to favor inflation-indexed debt, trading conditions and differences in investor preferences may cause the liquidity premia of inflation-indexed debt to be sufficiently large as to offset both the inflation risk premium and any liquidity premia in nominal debt. In that case, it may *not* be beneficial to issue inflation-indexed debt. Ultimately, it is an empirical question which of these factors dominates in any given government bond market.

For evidence of the potential benefits of the strategic switch in debt issuance away from nominal bonds and towards inflation-indexed debt, we choose to focus on Colombia, which is a country in which inflation and associated risk premia are likely to play a first-order role given its long history of high and fairly volatile inflation.[10] Additional motivations underlie this choice. First, Colombia has well-functioning markets for both standard nominal fixed-coupon government bonds, so-called bonos del tesoro, and real inflation-indexed government bonds, known as bonos del tesoro UVR (Unidad de Valor Real), with available data starting in 2005. Thus, the debt diversification question we consider is one of relevance to the Colombian government. Second and equally important, we consider Colombia and its government bond market to be a useful and informative case study for the wider set of emerging economies in which both of these two types of debt securities are issued and being traded.[11]

The starting point for our empirical analysis is breakeven inflation (BEI)—the difference between yields on comparable-maturity nominal and real debt. This is a frequently used indicator of inflation expectations. However, as widely noted by most observers, BEI is a noisy measure of expected inflation because it contains both an inflation risk premium and differential liquidity premia. Thus, for our research question and to fully understand the variation in BEI, we need estimates of both the differential liquidity premia in nominal and real bond prices and investors' underlying inflation expectations in order to get an estimate of the inflation risk premium, which is the main theoretical benefit of inflation-indexed debt.

The challenge in accounting for the differential liquidity premia in nominal and real bond prices is to distinguish them from more fundamental factors such as inflation risk premia that would affect asset prices even in a world without any frictions to trading. To achieve this separation, we follow BCFZ, who augment a flexible dynamic term structure model of nominal and real bond prices studied in Carriero et al. (2018) with separate liquidity risk factors for nominal and real bonds using the approach described in Andreasen et al. (2021, henceforth ACR). For each class of bonds, the identification of the liquidity risk factor comes from its unique loading, which mimics the idea that, over time, an increasing amount of the outstanding notional value of individual securities gets locked up in buy-and-hold investors'

[10]For the 2005-2020 period, year-over-year inflation in Colombia as measured by the consumer price index averaged 4.13 percent with a standard deviation of 1.68 percent. For comparison, the respective corresponding statistics were 1.70 percent and 0.83 percent for Canada, 4.07 percent and 1.01 percent for Mexico, and 2.00 percent and 1.33 percent for the United States.

[11]This list includes Brazil, Chile, India, Israel, Korea, Mexico, Peru, and South Africa, among others.

portfolios. This increases their sensitivity to variation in the marketwide liquidity risk captured by the corresponding liquidity risk factor. By observing prices for balanced panels of nominal and real bonds, their respective liquidity risk factors can be separately identified. This separation is particularly salient in emerging bond markets as they tend to be much less deep and liquid than the well-established major international bond markets in the United States and other advanced economies.

In terms of our empirical findings, we make a number of observations. First, we find that the model delivers estimates of investors' inflation expectations that are robust to a range of different model implementations.

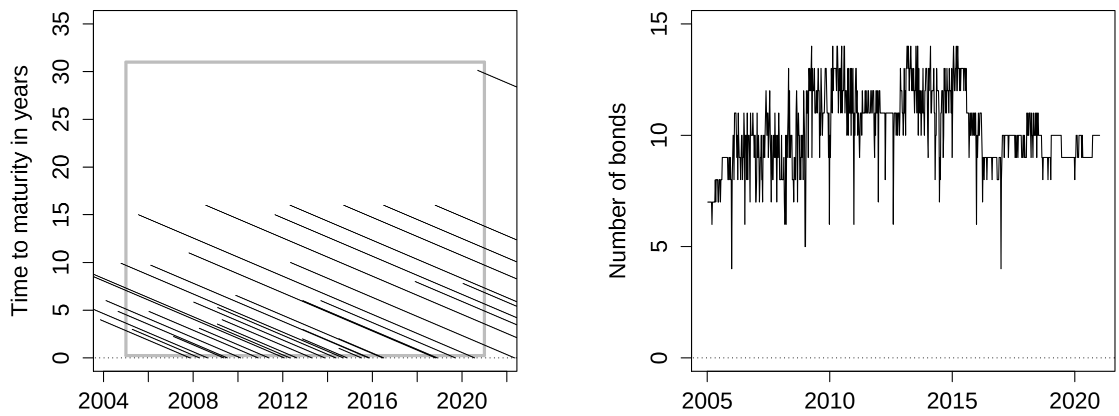
Second, our results indicate that the average liquidity premia embedded in both nominal and real Colombian bond yields exhibit notable time variation. For nominal yields, the estimated liquidity premia average 40 basis points with a standard deviation of 27 basis points. For real yields, the estimated liquidity premia average 225 basis points with a standard deviation of 32 basis points. Thus, as conjectured earlier, the liquidity premia of Colombian inflation-indexed government bonds are significantly larger and more volatile than those of standard Colombian nominal government bonds. Furthermore, these results are consistent with the findings of ACR, who report an average liquidity premium of U.S. TIPS estimated at 34 basis points for the 1997-2013 period, which is well above measures of liquidity premia in regular U.S. Treasury bonds. The difference in liquidity premium levels across the U.S. TIPS and the Colombian bonos UVR markets is likely to be due to the much greater relative liquidity of U.S. Treasury securities. Importantly, the nominal and real bond liquidity risk premia we estimate are practically uncorrelated in levels, and their weekly changes are only mildly negatively correlated. These results suggest that inflation-indexed Colombian bonds indeed are less liquid and less desirable than nominal Colombian bonds. Given the sizable difference in liquidity risk premia favoring nominal bonds, these results leave open the possibility that the trading frictions of inflation-indexed debt in Colombia may more than offset the benefit of avoiding having to pay the inflation risk premium.

Third, the model's decomposition of the liquidity-adjusted or frictionless BEI rates indicates that investors' long-term inflation expectations in Colombia have been stable at a level close to the inflation target set by the Bank of the Republic with some mild fluctuations. This finding implies that most of the variation in the liquidity-adjusted BEI rates is driven by fluctuations in the inflation risk premium, which has trended lower since 2005 and fallen on net slightly more than 300 basis points during our sample period, which ends in December 2020. Furthermore, we compare our estimated inflation risk premium series to estimates from Canada, Mexico, and the United States and find them to be weakly positively correlated, but larger and more volatile, as anticipated. Still, for extended periods, inflation risk in Colombia only commands a premium slightly above the one estimated for Mexico, while it is markedly above those observed for Canada and the United States. These findings underscore that inflation risk is a significant source of risk for investors in Colombian nominal bonds. Furthermore, the sizable inflation risk premia have the potential to tilt the calculus of benefits back in favor of inflation-indexed debt.

As for the determinants of Colombian inflation risk premia, we perform regression analysis with a large battery of explanatory variables. The regressions have significant explanatory power with adjusted R^2s of almost 0.80. Furthermore, focusing on our preferred regression specification, we note that increases in global interest rate risk as captured by the MOVE index, increases in global perceptions about credit risk as reflected in the Colombian five-year credit default swap (CDS) rate, and increases in U.S. long-term interest rates all tend to boost long-term inflation risk premia in Colombia. This is consistent with our conjecture that foreigners are likely to favor holding Colombian nominal bonds over inflation-indexed ones, which should make the inflation risk premium more sensitive to global risk factors. Furthermore, the results show that proxies for the liquidity risk in the markets for bonos and bonos UVR matter as well, even though we technically have adjusted for the liquidity risk premia within our model.

With estimates of liquidity and inflation risk premia in hand, we can turn our attention to our main research question about the net benefit to the Colombian government from increasing its issuance of inflation-indexed debt. As already noted, the liquidity risk premia embedded in the prices of bonos UVR average significantly higher than those of bonos. Thus, based on our results, issuance of bonos UVR faces a liquidity disadvantage, as also conjectured earlier. However, this disadvantage is more than overcome by the large inflation risk premia demanded by investors to assume the inflation risk of nominal bonos. Our model allows us to construct synthetic measures of the net benefit of bonos UVR over bonos at constant maturities, where we focus on the 5-year, 10-year, and 15-year horizons to be consistent with the maturities of the securities actually issued in the bonos and bonos UVR markets. Our results show that the average net benefit of bonos UVR issuance during our sample period from January 2005 to December 2020 was 38 basis points, 66 basis points, and 69 basis points, respectively. Thus, while issuance of 5-year securities can be viewed as fairly competitive, although still favoring inflation-indexed bonds, our results clearly favor increased issuance of such bonds at the longer 10-year and 15-year maturities. Furthermore, a replication of our exercise for the Mexican bonos and udibonos market using the same data as BCFZ produces qualitatively similar results, again favoring inflation-indexed debt, in particular at longer maturities.

The analysis in this paper relates to several strands of literature. First, our results regarding liquidity premia could be relevant for the financial market microstructure literature that aims to understand the factors that determine the size and dynamics of liquidity and financial frictions in government bond markets. Second, it speaks to the large literature focused on understanding the role and impact of foreign participation in emerging bond markets. Finally, it has ties to the large literature on sovereign debt management and the benefits of portfolio



(a) Distribution of bonos

(b) Number of bonos

Figure 1: **Overview of the Colombian Bonos del Tesoro Data**
Panel (a) shows the maturity distribution of the Colombian government fixed-coupon bonos considered in the paper. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on January 7, 2005, and end on December 30, 2020, and limited to bonos prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonos at a given point in time.

diversification.

The remainder of the paper is structured as follows. Section 2 contains the data description. Section 3 details the model, the empirical results, and some sensitivity analysis, while Section 4 describes the BEI decomposition and scrutinizes the estimated inflation risk premia. Finally, Section 5 examines the benefits to the Colombian government of issuing bonos UVR and Section 6 concludes. An online appendix contains additional analysis, estimation results, and robustness exercises.

2 Colombian Government Bond Data

This section first describes the Colombian government bond data we use in the model estimation before we proceed to a discussion of the credit risk and bid-ask spreads in the markets for these bonds.

2.1 Bonos del Tesoro

The available universe of individual Colombian government fixed-coupon bonds, known as bonos del tesoro, is illustrated in Figure 1(a). Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in

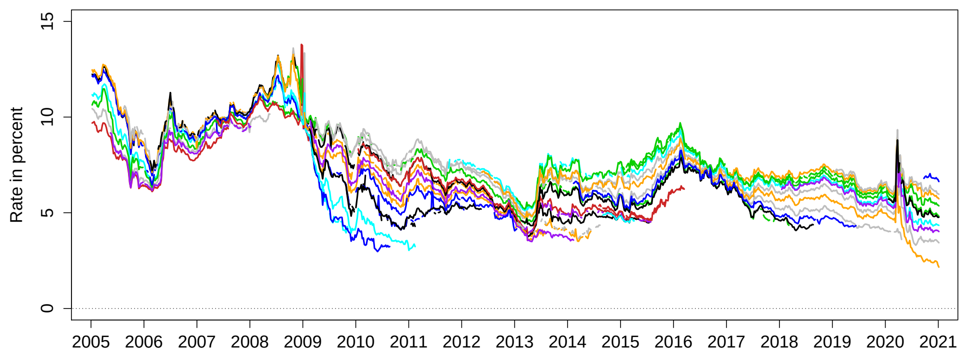


Figure 2: **Yield to Maturity of Colombian Bonos del Tesoro**
Illustration of the yields to maturity implied by the Colombian government fixed-coupon bonos prices. The data are weekly covering the period from January 7, 2005, to December 30, 2020, and censors the last three months for each maturing bond.

Colombian pesos that pay a fixed rate of interest annually. We note that we track the entire universe of bonos issued since January 2005. In addition, we include a few bonds outstanding at the start of our sample period. In general, the Colombian government has been issuing 5-, 10-, and 15-year bonos on a fairly regular basis during this period. In addition, it recently issued its first 30-year bonos, likely to take advantage of the low interest rate levels at the time.[12] As a result, there is a wide variety of bonds with different maturities and coupon rates in the data throughout our sample. This variation provides the foundation for the econometric identification of the factors in the yield curve models we use.

The contractual characteristics of all 35 bonos securities in our sample are reported in Table 1. The number of weekly observations for each bond using three-month censoring before maturity is also reported in the table.

Figure 1(b) shows the distribution across time of the number of bonds included in the sample. We note that the number of bonds has fluctuated around 10 for most of our sample. Furthermore, the occasional sharp drops are days when some bonds have missing observations. Combined with the cross sectional dispersion in the maturity dimension observed in Figure 1(a), this implies that the panel of bond prices is very well-balanced.

Figure 2 shows the time series of the yields to maturity implied by the observed Colombian bonos prices. First, we note that the general yield level in Colombia trended down between 2005 and 2010,[13] but has been fairly stable since then, unlike government bond yields in advanced economies, which have declined significantly during this period; see Holston et

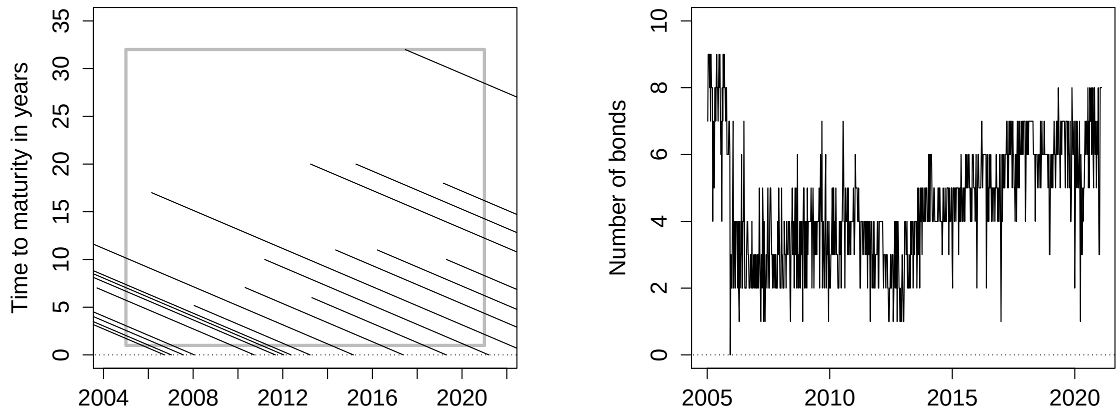
[12]This 30-year bonos with a 7.25% annual coupon is the longest bond the Colombian government has ever issued denominated in pesos, and at auction the bids received were almost twice the amount auctioned.

[13]This downward trend can be explained by various idiosyncratic factors such as better debt management (external debt as a percentage of GDP fell from 40.3% in 2001 to 22.6% in 2010), solid economic growth in the 2004-2007 period, and the deceleration of inflation that resulted in the inflation target being lowered to 3%.

Fixed-coupon bonos	No. obs.	Issuance		Number of auctions	Total notional amount
		Date	Amount		
(1) 15% 8/22/2008	174	8/22/2001	447.4	36	4,516.5
(2) 15% 1/25/2012	154	1/28/2002	289.4	4	1,522.3
(3) 15% 4/26/2012	259	4/24/2002	4.9	32	3,453.8
(4) 12% 11/9/2007	135	11/12/2003	107.2	30	4,904.3
(5) 13% 2/12/2010	220	2/11/2004	103.5	31	4,987.3
(6) 12.5% 7/10/2009	219	8/25/2004	94.0	34	5,440.2
(7) 13.5% 9/12/2014	427	10/12/2004	108.1	36	5,669.0
(8) 10% 4/11/2008	135	4/13/2005	92.9	42	4,461.3
(9) 11% 7/24/2020	748	7/24/2005	71.7	71	6,834.7
(10) 8% 10/28/2015	447	2/8/2006	174.0	53	6,067.4
(11) 7.5% 11/24/2010	215	1/11/2006	55.0	39	4,017.3
(12) 8.75% 5/14/2009	91	2/14/2007	37.4	26	4,399.9
(13) 11% 5/18/2011	141	4/9/2008	164.0	25	4,616.2
(14) 11.25% 10/24/2018	438	10/24/2007	171.6	75	10,385.7
(15) 10.25% 11/14/2013	275	1/9/2008	119.2	35	5,524.4
(16) 9.25% 5/14/2014	260	2/2/2009	96.2	43	8,671.8
(17) 10% 7/24/2024	613	7/24/2008	91.6	112	21,618.0
(18) 9.25% 8/15/2012	169	1/28/2009	191.8	30	6,066.8
(19) 6% 4/17/2013	156	4/17/2009	146.0	17	4,995.4
(20) 7.25% 6/15/2016	319	11/24/2009	142.5	69	11,753.3
(21) 7.5% 8/26/2026	449	8/26/2011	203.0	106	29,191.3
(22) 6% 4/28/2028	411	4/27/2012	470.5	146	32,479.1
(23) 7% 5/4/2022	442	5/4/2012	400.4	97	29,035.0
(24) 5.25% 11/11/2015	84	11/15/2012	237.7	26	3,076.7
(25) 5% 11/12/2014	50	11/15/2012	84.5	21	1,908.8
(26) 5% 11/21/2018	287	11/21/2012	485.1	28	5,289.4
(27) 7% 9/11/2019	279	9/11/2013	470.9	34	9,468.2
(28) 5.5% 7/1/2016	73	7/1/2014	50.2	21	4,924.2
(29) 5% 7/3/2015	33	7/3/2014	250.6	21	4,947.0
(30) 7.75% 9/18/2030	302	9/18/2014	478.0	70	22,399.6
(31) 7% 6/30/2032	207	6/30/2016	673.9	58	27,970.5
(32) 6.25% 11/26/2025	156	11/26/2017	693.5	55	18,089.5
(33) 7.25% 10/18/2034	104	10/18/2018	733.5	101	29,913.8
(34) 5.75% 11/3/2027	50	1/15/2020	711.9	52	20,101.9
(35) 7.25% 10/26/2050	16	9/9/2020	354.5	53	23,499.9

Table 1: **Sample of Colombian Bonos del Tesoro**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of Colombian pesos for the available universe of Colombian government fixed-coupon bonos in the sample. Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2020.

al. (2017) and Christensen and Rudebusch (2019), among others. Second, as in U.S. Treasury yield data, there is notable variation in the shape of the yield curve. At times, like in mid-2007, yields across maturities are relatively compressed. At other times, the yield curve is steep with long-term bonos trading at yields that are 400-500 basis points above those of



(a) Distribution of bonos del tesoro UVR

(b) Number of bonos del tesoro UVR

Figure 3: **Overview of the Colombian Bonos del Tesoro UVR Data**
Panel (a) shows the maturity distribution of the Colombian government inflation-indexed bonos considered in the paper. These securities are also known as Colombian bonos de tesoro UVR. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on January 7, 2005, and limited to bonos prices with more than one year to maturity after issuance. Panel (b) reports the number of outstanding bonos at a given point in time.

shorter-term securities like in 2010.

Finally, regarding the important question of a lower bound, the Bank of the Republic has never been forced to lower its conventional policy rate even close to zero, and the bond yields in the data have remained well above zero throughout the sample period. Thus, there is no need to account for any lower bounds to model these fixed-coupon bond prices, which motivates our focus on Gaussian models.

2.2 Bonos del Tesoro UVR

The Colombian government also issues inflation-indexed bonds, which are bonos del tesoro denominated in consumption units known as Unidades de Valor Real (UVR) and therefore referred to as bonos UVR. Unlike standard fixed-coupon bonds, interest and principal payments on bonos UVR are adjusted for changes in the general price level as reflected in the Colombian consumer price index, abbreviated IPC in Spanish. Hence, UVR is used to convert the real return of bonos UVR into the corresponding value measured in current pesos at any given point in time.

The Colombian government launched its inflation-indexed bond program in the 1990s. However, the data provided to us by staff at the Bank of the Republic only start in January 2005 and end in December 2020, which determines our sample period. The available universe of bonos UVR and their maturity distribution across time is shown in Figure 3(a). It includes

Indexed bonos	No. obs.	Issuance Date	Issuance amount	Number of auctions	Total notional amount in UVR
(1) 8% 9/21/2006	34	9/23/1999	1.1	1	6.4
(2) 6% 1/12/2007	40	1/12/2000	1.2	3	9.5
(3) 8% 7/26/2007	56	9/21/2000	1.3	2	10.5
(4) 8% 1/22/2008	40	2/22/2001	0.4	1	6.0
(5) 8% 9/2/2011	50	8/30/2001	1.3	3	7.0
(6) 7% 1/17/2012	110	1/17/2002	1.3	2	14.8
(7) 7% 5/15/2012	89	5/3/2002	3.0	1	11.3
(8) 7% 2/25/2015	420	2/20/2003	7.0	57	51.1
(9) 7% 9/22/2010	234	9/16/2003	2.9	37	32.8
(10) 4.75% 2/23/2023	521	2/23/2006	0.5	132	100.4
(11) 5.25% 3/20/2013	182	1/16/2008	1.0	36	32.0
(12) 4.25% 5/17/2017	266	4/26/2010	0.7	54	63.4
(13) 3.5% 3/10/2021	364	3/10/2011	2.0	104	76.8
(14) 3% 3/25/2033	323	3/25/2013	1.4	90	44.4
(15) 3.5% 4/17/2019	216	4/17/2013	0.9	48	40.7
(16) 3.5% 5/7/2025	237	5/7/2014	1.5	147	61.6
(17) 4.75% 4/4/2035	224	4/4/2015	1.0	125	92.2
(18) 3.3% 3/17/2027	193	3/17/2016	1.4	176	80.1
(19) 3.75% 6/16/2049	43	6/16/2017	3.9	107	55.7
(20) 3.75% 2/25/2037	93	2/25/2019	1.4	127	83.9
(21) 2.25% 4/18/2029	33	4/18/2019	0.7	100	42.6

Table 2: **Sample of Colombian Bonos del Tesoro UVR**

The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of UVR for the available sample of Colombian government inflation-indexed bonos, also known as bonos del tesoro UVR. Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2020.

the entire universe of bonos UVR issued since 2005 combined with the outstanding stock of bonos UVR at the start of our sample. We note that the issuance is concentrated in the 10- and 20-year segment, while a single 30-year bonos UVR was issued in 2017.

The contractual details of each bonos del tesoro UVR in our sample are reported in Table 2. It also contains the number of weekly observations for each bond in our sample, with the last year before maturity censored to avoid erratic variation in their prices arising from seasonality in the inflation adjustment of their payoffs.

The total number of bonos UVR in our sample across time is shown in Figure 3(b). As with the regular bonos, we stress that the sample of bonos UVR we use is very well-balanced across maturities at all times, which underpins the econometric identification of the state variables in the term structure model we use.

Figure 4 shows the yields to maturity implied by the bonos UVR prices. Similar to what we observe for the nominal bonos yields, the yields of bonos UVR have fluctuated around a fairly stable level since 2010, but with some variation in the steepness of the bonos UVR yield curve. Our model is intended to exploit this variation to deliver estimates of their liquidity

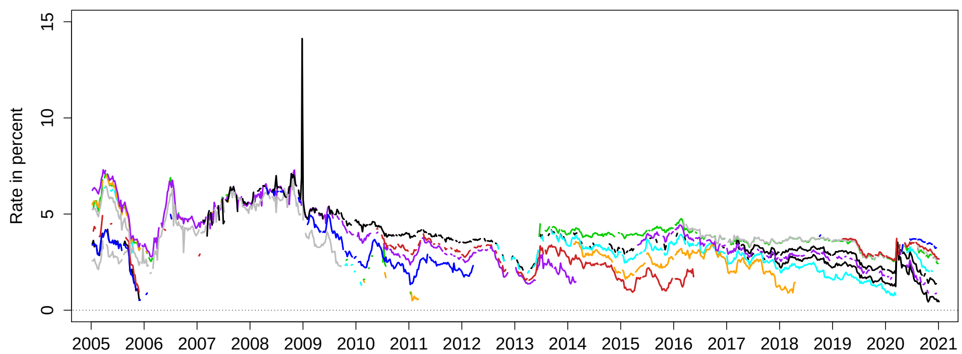


Figure 4: **Yield to Maturity of Colombian Bonos del Tesoro UVR**
Illustration of the yield to maturity implied by the Colombian government inflation-indexed bonos prices considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 7, 2005, to December 30, 2020; (2) censoring of a bond's price when it has less than one year to maturity.

premia, as explained in Section 3.

2.3 The Credit Risk of Colombian Government Bonds

To gauge whether there are any material credit risk issues to consider in modeling Colombian government bond prices, we use rates on CDS contracts. They reflect the annual rate investors are willing to pay to buy protection against default-related losses on these bonds over a fixed period of time stipulated in the contract. Such contracts have been used to price the credit risk of many countries, including Colombia, since the early 2000s.

In Figure 5, we plot the series for the one- and five-year Colombian CDS rate since 2005 with solid gray and black lines, respectively. Also shown with a solid red line is the spread between these two CDS rates. We note that the five-year CDS rate has fluctuated in a fairly narrow range between 100 and 200 basis points, except for a few brief episodes including the Global Financial Crisis in 2008-2009, when Colombian CDS rates temporarily spiked above 300 basis points, and the early stages of the COVID-19 pandemic. This is a level of credit risk on par with most investment-grade firms in the United States, and its variation is mostly very gradual. This suggests that credit risk-related components are unlikely to be the driver of the results we present later on. To further support this view, we note that our measure of the Colombian government debt relative to GDP never goes above 50 percent, which is not a high value by international standards. Furthermore, the slope of the CDS rate curve, measured as the difference between the five-year and one-year CDS rates, is always positive and fairly stable and fluctuates in a narrow range, and it is almost uncorrelated (13 percent) with the one-year CDS rate. Thus, the steepness of the CDS rate curve for

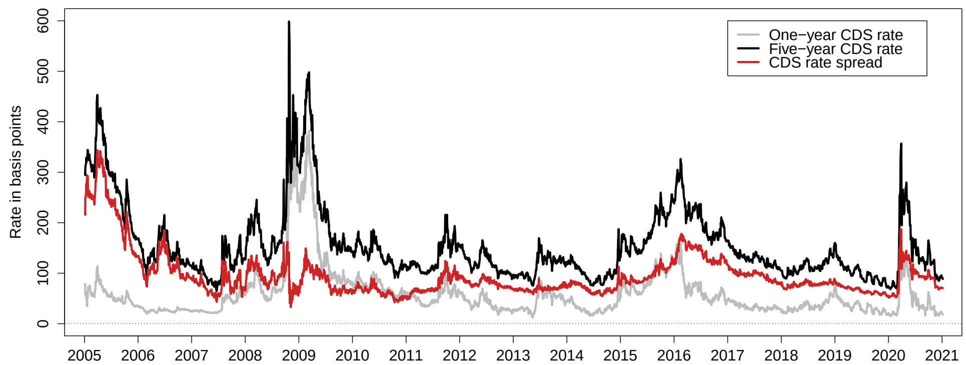


Figure 5: **Colombian CDS Rates**

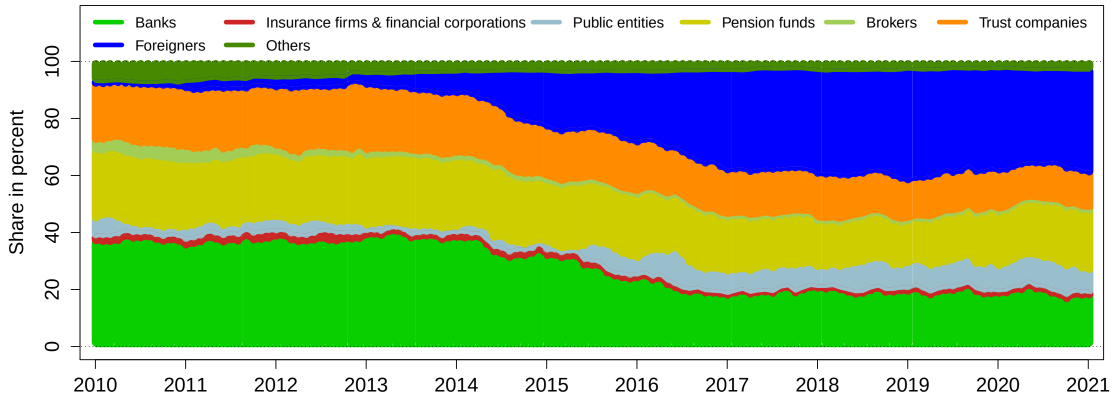
Colombian government debt has little connection to the near-term level of the priced credit risk of the Colombian government. We take this as a sign that the bulk of the variation in Colombian CDS rates reflect investor sentiment and risk aversion rather than actual credit risk.[14] This view finds further support in Gamboa-Estrada and Romero (2022), who analyze CDS rates across Latin-American countries, including Colombia, and find that their levels are mainly driven by a common component and global financial conditions leaving little room for country-specific factors in their determination. Overall, we take this evidence to imply that credit risk is not likely to materially affect our results, and we are therefore comfortable not accounting for credit risk premia in our analysis.

More importantly, on a practical note, there are no differences in the credit risk of nominal and real bonds in the sense that they will receive the same treatment in case the Colombian government stops servicing its debt. Thus, using arguments similar to those made by Fleckenstein et al. (2014) for U.S. Treasuries and TIPS, there is no reason to believe that there are any differentials in the pricing of bonos and bonos UVR tied to credit risk. By implication, our measures and decompositions of Colombian BEI are unaffected by variation in the credit risk premia of Colombian government debt.

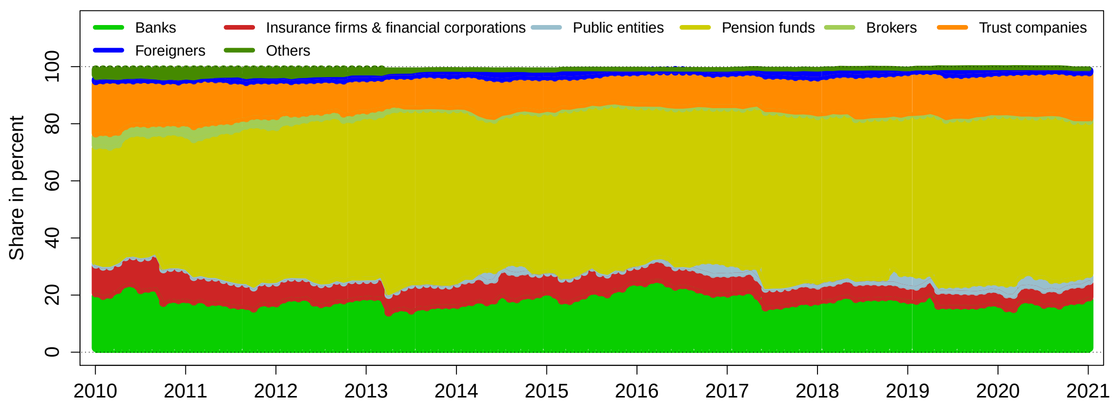
2.4 Colombian Government Bond Holdings

In this section, we provide details on the investor groups holding Colombian government bonds. The data we use have been collected by the Bank of the Republic since 2010 to track market activity in the Colombian sovereign bond markets. Importantly, the data break down investor holdings into multiple groups, which include banks, insurance companies, pension funds, and foreigners, among others.

[14]This is a phenomenon also seen in the pricing of corporate bonds and frequently referred to as the credit spread puzzle; see Christensen (2008) and references therein.



(a) Bonos del tesoro



(b) Bonos del tesoro UVR

Figure 6: **Holdings of Colombian Government Bonds**

Figure 6(a) shows the relative share held by each of these groups of investors in the regular bonos tesoro market. Note that there has been a significant increase in the foreign-held share since 2010, which reflects that foreigners have become the largest investor group with about one-third of the market by the end of our sample. This expansion of the foreign role has come at the expense of the participation of domestic banks, while the holdings of the other local investor groups have changed little on net.

Figure 6(b) shows the corresponding breakdown of holdings in the tesoro UVR market, where the shares are distributed very differently across groups. Most importantly, foreigners are nearly absent in this market. Instead, the holdings are concentrated among domestic investors, such as pension funds, insurance companies, and banks, that all tend to pursue a more stable buy-and-hold strategy for their bond investments.

These observations are consistent with the holding statistics reported by BCFZ for the

Mexican government markets, where foreigners own more than half of the regular fixed-coupon bond market and less than 10 percent of the inflation-indexed bond market.

Overall, this represents tangible evidence in favor of our conjectures laid out in the introduction whereby inflation-indexed bonds mainly meet the needs of domestic buy-and-hold investors exposed to the fluctuations in the Colombian consumer price index (CPI). By implication, foreigners will tend to gravitate towards the nominal bond market.

In the public debt market in Colombia, the main regulatory changes that could have had an impact on the behavior of portfolio investment flows occurred between 2010 and 2013. During that period the country's credit rating reached investment-grade level.[15] After those changes and with global financial markets characterized by benign liquidity conditions driven in part by unconventional monetary policies pursued by major central banks in advanced economies, JP Morgan announced that it would increase Colombia's weight in three of its main emerging market bond indexes in 2014. As a result, the share of foreign investors in the local public debt market, i.e. the tesoro market, rose from approximately 5 percent in mid-2013 to 19 percent at the end of 2014. The foreign participation further increased to levels around 35 percent by 2017, where it has remained since.

In contrast, domestic pension funds and insurance companies continue to participate heavily in the UVR market to hedge their liabilities. This is particularly true for pension funds as their liabilities tend to have a large share of claims indexed to inflation.

Ultimately, these differences in investor concentrations have consequences for the size and dynamics of inflation and liquidity risk premia in each of the two bond markets that we aim to quantify in our empirical analysis.

2.5 Bid-Ask Spreads of Colombian Government Bonds

In this section, to shed light on the trading frictions in the markets for bonos tesoro and bonos tesoro UVR, we compare the median bid-ask spread of the bonos in our sample to the median of the bid-ask spread of the bonos UVR in our sample. These series are available starting in 2013 and shown in Figure 7.[16]

Note that, outside of sharp, short-lived spikes in 2013 and again in 2020, these series have fluctuated near a fairly stable and similar level. The main takeaway is that, with a level of bid-ask spreads around 5 basis points, the trading of these securities is indeed associated with some amount of liquidity risk. Motivated by this evidence, our model assumes that both nominal and real bond prices contain liquidity premia that investors demand to assume their liquidity risk.

The purpose of the remainder of the paper is to quantify the relative importance of these

[15] Granted in March 2011 by Standard & Poor's, in May 2011 by Moody's, and in June 2011 by Fitch Ratings.

[16] The shown bid-ask spread series are calculated using the daily median of the difference between the best bid and the best ask for each government bond. This is done for bonos tesoro and bonos tesoro UVR separately.

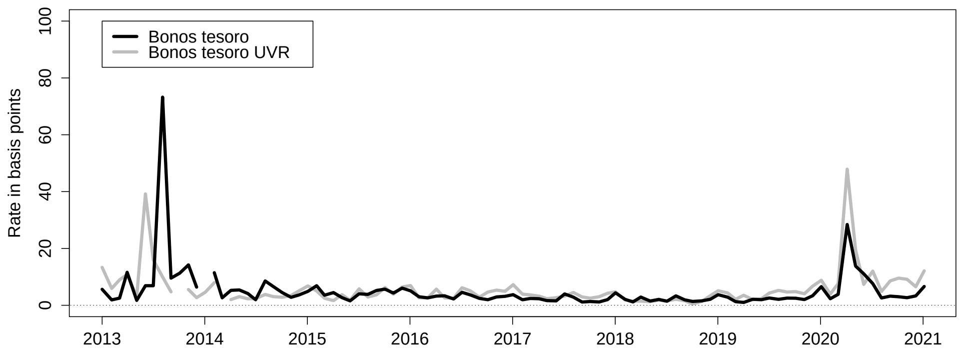


Figure 7: **Bid-Ask Spreads of Colombian Government Bonds**

bond risk premia in the pricing of bonos and bonos UVR and what adjustments for them may imply about bond investors' underlying inflation expectations and associated inflation risk premia.

3 Model Estimation and Results

In this section, we first describe the dynamic term structure model of nominal and real yields that we use to account for the liquidity bias in their pricing. We then detail how BEI is decomposed within the model. We end the section with a brief overview of the main estimation results, including the estimated nominal and real bond liquidity premia.

3.1 An Arbitrage-Free Model of Nominal and Real Yields with Liquidity Risk

To precisely measure nominal and real liquidity premia, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R. With that goal in mind we choose to focus on the tractable affine dynamic term structure model of nominal and real yields briefly summarized below. We emphasize that, even though the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the corresponding canonical Gaussian model.

To begin, let $X_t = (L_t^N, S_t^N, C_t^N, X_t^N, L_t^R, S_t^R, X_t^R)$ denote the state vector of our seven-factor model, which we refer to as the $G^{X^N,X^R}(7)$ model using the terminology of BCFZ. Here, (L_t^N, S_t^N, C_t^N) represent level, slope, and curvature factors in the nominal yield curve, while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve.[17] Finally, (X_t^N, X_t^R) represent the added nominal and real liquidity risk factors. Our joint model of

[17] Chernov and Mueller (2012) provide evidence of a hidden factor in the nominal yield curve that is observable from real yields and inflation expectations. Our model accommodates this stylized fact via the (L_t^R, S_t^R) factors.

nominal and real yields is a liquidity-augmented extension of the five-factor model used by Carriero et al. (2018) to analyze nominal and real U.K. gilt yields.

The instantaneous nominal and real risk-free rates are defined as

$$r_t^N = L_t^N + S_t^N, \quad (1)$$
$$r_t^R = L_t^R + S_t^R. \quad (2)$$

The risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dC_t^N \\ dX_t^N \\ dL_t^R \\ dS_t^R \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & -\lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_N^{\mathbb{Q}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_R^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_N^{\mathbb{Q}} \\ 0 \\ 0 \\ \theta_R^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t^N \\ C_t^N \\ X_t^N \\ L_t^R \\ S_t^R \\ X_t^R \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{C^N,\mathbb{Q}} \\ dW_t^{X^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \\ dW_t^{X^R,\mathbb{Q}} \end{pmatrix},$$

where Σ is assumed to be a diagonal matrix as per Christensen et al. (2011).

Due to the liquidity risk in the markets for nominal and real bonds, their yields are sensitive to liquidity pressures. As a consequence, the pricing of nominal and real bonds is not performed with the frictionless short rates in equations (1) and (2), but rather with discount functions that account for the liquidity risk as in ACR:

$$\overline{r}_t^{N,i} = r_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N = L_t^N + S_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N, \quad (3)$$
$$\overline{r}_t^{R,j} = r_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R = L_t^R + S_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R, \quad (4)$$

where t_0^i and t_0^j denote the dates of issuance of the specific nominal and real bonds, respectively, and $\beta^{N,i}$ and $\beta^{R,j}$ are their sensitivities to the variation in their respective liquidity risk factors. Furthermore, the decay parameters $\delta^{N,i}$ and $\delta^{R,j}$ are assumed to vary across securities.

Christensen and Rudebusch (2019) show that the net present value of one unit of currency paid by nominal bond i at time $t + \tau^i$ has the following exponential-affine form[18]

$$\begin{aligned} P_t^N(t_0^i, \tau^i) &= E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau^i} \overline{r}^{N,i}(s,t_0^i)ds}\right] \\ &= \exp\Big(B_1^N(\tau^i)L_t^N + B_2^N(\tau^i)S_t^N + B_3^N(\tau^i)C_t^N + B_4^N(t, t_0^i, \tau^i)X_t^N + A(t, t_0^i, \tau^i)\Big). \end{aligned}$$

By similar arguments, the net present value of one unit of the consumption basket paid

[18]The calculations leading to our bond pricing results can be found in the online supplementary appendix.

by real bond j at time $t+\tau^j$ has the following exponential-affine form

$$\begin{aligned} P_t^R(t_0^j,\tau^j) &= E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^j}\overline{r}^{R,j}(s,t_0^j)ds}\Big] \\ &= \exp\Big(B_1^R(\tau^j)L_t^R + B_2^R(\tau^j)S_t^R + B_3^R(t,t_0^j,\tau^j)X_t^R + A(t,t_0^j,\tau^j)\Big). \end{aligned}$$

Now, consider the whole value of the nominal bond i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C^i. Its price is given by[19]

$$\begin{aligned} \overline{P}_t^{N,i}(t_0^i,\tau^i,C^i) &= C^i(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{N,i}(s,t_0^i)ds}\Big] + \sum_{k=2}^{n} C^i E^{\mathbb{Q}}\Big[e^{-\int_t^{t_k}\overline{r}^{N,i}(s,t_0^i)ds}\Big] \\ &\quad + E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\overline{r}^{N,i}(s,t_0^i)ds}\Big]. \end{aligned}$$

Next, consider the whole value of the real bond j issued at time t_0^j with maturity at $t+\tau^j$ that pays an annual coupon C^j. Its clean price is given by[20]

$$\begin{aligned} \overline{P}_t^{R,j}(t_0^j,\tau^j,C^j) &= C^j(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{R,j}(s,t_0^j)ds}\Big] + \sum_{k=2}^{n} C^j E^{\mathbb{Q}}\Big[e^{-\int_t^{t_k}\overline{r}^{R,j}(s,t_0^j)ds}\Big] \\ &\quad + E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^j}\overline{r}^{R,j}(s,t_0^j)ds}\Big]. \end{aligned}$$

The only minor omission in the real bond price formula above is that we do not account for the lag in the inflation indexation of the real bond payoff, but the potential error should be modest in most cases; see Grishchenko and Huang (2013) and D'Amico et al. (2018) for evidence in the case of the U.S. TIPS market.

To complete the model description, we need to specify the risk premia that connect the factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the objective $\mathbb{P}$-measure, where we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^7$ and $\gamma^1 \in \mathbf{R}^{7\times 7}$ contain unrestricted parameters. Thus, the resulting unrestricted $G^{X^N,X^R}(7)$ model has $\mathbb{P}$-dynamics given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 7×7 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 7×1 vector of mean levels, and Σ is a 7×7 lower triangular volatility matrix. This is the transition equation in the

[19]This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

[20]Unlike U.S. TIPS, Colombian bonos del tesoro UVR have no embedded deflation protection option, which makes their pricing straightforward.

extended Kalman filter estimation of this model.

3.2 Decomposing BEI

Christensen et al. (2010) show that the price of a nominal zero-coupon bond with maturity in τ years can be written as

$$P_t^N(\tau) = P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right),$$

where $P_t^R(\tau)$ is the price of a real zero-coupon bond that pays one consumption unit in τ years, M_t^R is the real stochastic discount factor, and Π_t is the price level.[21]

By taking logarithms, this can be converted into

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real *frictionless* zero-coupon yields as described in the previous section, while the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau} \ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau} \ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right] \tag{5}$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau} \ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0. \tag{6}$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2012).

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

[21] The full details of the decomposition can be found in the online supplementary appendix.

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

3.3 Model Estimation and Econometric Identification

Due to the nonlinearity of the bond pricing formulas, the models cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012); see Christensen and Rudebusch (2019) for details. To make the fitted errors comparable across bonds of various maturities, we follow ACR and scale each bond price by its duration. Thus, the measurement equation for the nominal bond prices takes the following form:

$$\frac{\overline{P}_t^j(\tau^i)}{D_t^j(\tau^i)} = \frac{\widehat{P}_t^j(\tau^i)}{D_t^j(\tau^i)} + \varepsilon_t^{j,i}, \quad j \in \{N, R\},$$

where $\widehat{P}_t^j(\tau^i)$ is the model-implied price of bond i and $D_t^j(\tau^i)$ is its duration, which is fixed and calculated before estimation. See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equations.

Since the liquidity factors are latent factors that we do not observe, their levels are not identified without additional restrictions. As a consequence, when we include the nominal liquidity factor X_t^N, we let the first 15-year bonos issued after the start of our sample window have a unit loading on the liquidity factor, that is, bonos del tesoro number (9) in our sample issued on July 24, 2005, with maturity on July 24, 2020, and a coupon rate of 11 percent has $\beta^{N,i} = 1$. As for the real liquidity factor X_t^R, we let the bonos del tesoro UVR number (10) issued on February 23, 2006, with maturity on February 23, 2023, and a coupon rate of 4.75 percent have $\beta^{R,j} = 1$.

Furthermore, we note that the liquidity decay parameters, $\delta^{N,i}$ and $\delta^{R,j}$, can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences based on the evidence presented in CFS. Also, for numerical stability during the model optimization, we impose the restrictions that the liquidity sensitivity parameters, $\beta^{N,i}$ and $\beta^{R,j}$, fall within the range from 0 to 250, which turns out not to be a binding constraint at the optimum.

In addition, we assume that all nominal bond price measurement equations have *i.i.d.* fitted errors with zero mean and standard deviation σ_ε^N. Similarly, all real bond price measurement equations have fitted errors that are assumed to be *i.i.d.* with zero mean and standard deviation σ_ε^R.

We also incorporate long-term forecasts of inflation from the Consensus Forecasts survey for Latin America in our model estimation. These include monthly data on inflation forecasts for the following full calendar year and semiannual data on 5-year, 10-year, and so-called 5yr5yr inflation forecasts, which represent long-term inflation forecasts covering a 5-year period starting 5 years ahead.[22] As demonstrated by Kim and Orphanides (2012), the inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics, which can otherwise suffer from significant finite-sample bias.[23]

The measurement equation for the survey expectations takes the form

$$\pi_t^{CF}(\tau) = \pi_t^e(\tau) + \varepsilon_t^{CF},$$

where τ is the forecast horizon. The required expected inflation is calculated using equation (5), which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\left(0, (\sigma_\varepsilon^{CF})^2\right)$. As for the value of σ_ε^{CF}, we follow D'Amico et al. (2018) and fix it at 75 basis points in order to not overly influence the estimation results by including the survey forecasts. Alternatively, this approach can be interpreted as treating the survey forecasts as relatively noisy and infrequent measures of bond investors' inflation expectations.

Finally, we assume that the state variables are stationary, which is standard in the literature. As a consequence, we start the Kalman filter at the unconditional mean and covariance matrix.

3.4 Results

In this section, we briefly summarize the main estimation results, while additional details are provided in the online appendix.

To examine the model fit, pricing errors are computed based on the implied yield on each coupon bond to make these errors comparable across securities. That is, for the price on the ith coupon bond $P_t^i(\tau, C^i)$, we find the value of $y_t^{i,c}$ that solves

$$P_t^i(\tau^i, C^i) = C^i(t_1 - t)\exp\left\{-y_t^{i,c}(t_1 - t)\right\} + \sum_{j=2}^{N} C^i \exp\left\{-y_t^{i,c}(t_j - t)\right\} + \exp\left\{-y_t^{i,c}(t_N - t)\right\}. \quad (7)$$

For the model-implied estimate of this bond price, denoted $\hat{P}_t^i(\tau, C^i)$, we find the corresponding implied yield $\hat{y}_t^{i,c}$ and report the pricing error as $y_t^{i,c} - \hat{y}_t^{i,c}$.

For the nominal bonos del tesoro, the root mean-squared error (RMSE) for all bonds combined is 10.91 basis points, while the corresponding statistics for the real bonos del tesoro UVR is 16.35 basis points. Thus, the $G^{X^N,X^R}(7)$ model provides a good fit to both sets of bond prices.

As for the monthly data on inflation forecasts for the following full calendar year and semiannual data on 5-year, 10-year, and so-called 5yr5yr inflation forecasts, the RMSEs are

[22] Similar to Christensen et al. (2010) and Abrahams et al. (2016), we do not include inflation data in the model estimation. This omission is expected to, at most, have a small impact on our results due to the relatively long maturities of most of our real yield observations; see D'Amico et al. (2018) for evidence.

[23] Also, see Bauer et al. (2012).

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$K^{\mathbb{P}}_{\cdot,6}$	$K^{\mathbb{P}}_{\cdot,7}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	10.2994	11.4429	5.4497	14.7588	1.9074	-18.7883	1.7623	0.1741	σ_{11}	0.0090
	(0.3737)	(0.3232)	(0.2693)	(0.4718)	(0.5270)	(0.4841)	(0.1563)	(0.0600)		(0.0006)
$K^{\mathbb{P}}_{2,\cdot}$	-17.8121	-18.5402	-11.4100	-21.9464	-8.8420	37.1534	-3.7883	-0.1080	σ_{22}	0.0001
	(0.4804)	(0.4702)	(0.4397)	(0.5405)	(0.5820)	(0.6488)	(0.3081)	(0.0313)		(0.1010)
$K^{\mathbb{P}}_{3,\cdot}$	18.8673	16.8532	13.2323	18.8663	12.7932	-38.5258	4.7732	-0.0242	σ_{33}	0.0411
	(0.5523)	(0.5392)	(0.5283)	(0.6317)	(0.6740)	(0.5983)	(0.3814)	(0.0379)		(0.0017)
$K^{\mathbb{P}}_{4,\cdot}$	11.4370	10.9803	6.7873	13.9177	1.8157	-21.5846	2.0855	0.0183	σ_{44}	0.0150
	(0.3813)	(0.3874)	(0.2945)	(0.5319)	(0.5498)	(0.5127)	(0.1912)	(0.0076)		(0.0011)
$K^{\mathbb{P}}_{5,\cdot}$	4.1746	6.5610	2.8134	9.1910	5.6750	-9.5865	1.0950	0.0811	σ_{55}	0.0084
	(0.3585)	(0.3434)	(0.2064)	(0.4551)	(0.5268)	(0.4734)	(0.1101)	(0.0257)		(0.0006)
$K^{\mathbb{P}}_{6,\cdot}$	-6.1001	-7.9724	-3.2790	-7.9983	4.0558	12.4306	-0.1722	-0.0606	σ_{66}	0.0190
	(0.4242)	(0.3834)	(0.2510)	(0.5377)	(0.5941)	(0.4992)	(0.1446)	(0.0163)		(0.0008)
$K^{\mathbb{P}}_{7,\cdot}$	-2.8107	-3.8636	-5.4988	-7.5267	-0.2925	8.3293	-0.1452	-0.2862	σ_{77}	0.1519
	(0.6522)	(0.5783)	(0.6489)	(0.6512)	(0.5958)	(0.6956)	(0.3562)	(0.1900)		(0.0126)

Table 3: **Estimated Dynamic Parameters of the $G^{X^N,X^R}(7)$ Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the $G^{X^N,X^R}(7)$ model. The estimated value of λ^N is 0.5031 (0.0097), while $\lambda^R = 0.2082$ (0.0222), $\kappa_N^{\mathbb{Q}} = 3.1280$ (0.2029), $\theta_N^{\mathbb{Q}} = 0.0033$ (0.0003), $\kappa_R^{\mathbb{Q}} = 9.9984$ (0.5728), and $\theta_R^{\mathbb{Q}} = 0.0217$ (0.0023). The maximum log likelihood value is 61,117.10. The numbers in parentheses are the estimated parameter standard deviations.

34.87 basis points, 31.78 basis points, 25.76 basis points, and 27.91 basis points, respectively, which are all well below the 75 basis points assumed in the model estimation. Thus, the model is also able to simultaneously deliver a reasonably accurate fit to the full term structure of available survey inflation forecasts.

The estimated dynamic parameters in the $G^{X^N,X^R}(7)$ model are reported in Table 3. We note that the estimated mean and volatility parameters for the seven state variables are surprisingly similar to those reported by BCFZ using Mexican data. Hence, the Colombian and Mexican government bond markets seem to share some fundamental characteristics.

3.5 The Estimated Bonos and Bonos UVR Liquidity Premia

We now use the estimated $G^{X^N,X^R}(7)$ model to extract the liquidity premium in the bonos and bonos UVR prices. To compute these premia, we first use the estimated parameters and the filtered states $\{X_{t|t}\}_{t=1}^T$ to calculate the fitted bond prices $\{\hat{P}_t^i\}_{t=1}^T$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\{\hat{y}_t^{c,i}\}_{t=1}^T$ by solving the fixed-point problem

$$\begin{aligned}\hat{P}_t^i &= C(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n} C\exp\left\{-(t_k-t)\hat{y}_t^{c,i}\right\} \\ &\quad + \exp\left\{-(T-t)\hat{y}_t^{c,i}\right\},\end{aligned} \tag{8}$$

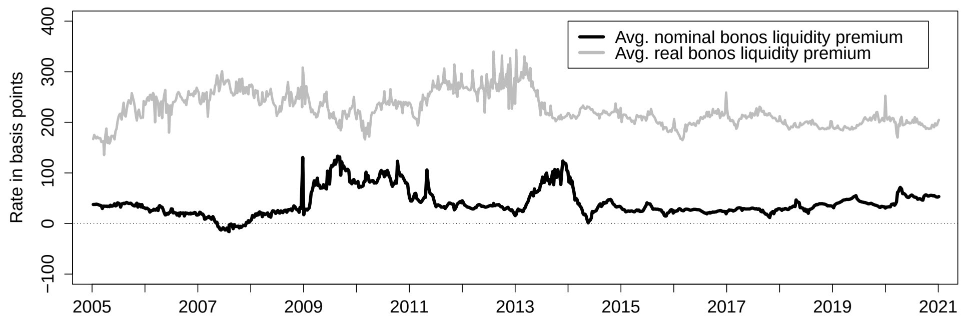


Figure 8: **Average Estimated Liquidity Premia of Colombian Bonos and Bonos UVR**
Illustration of the average estimated liquidity premium of Colombian bonos and bonos UVR for each observation date implied by the $G^{X^N,X^R}(7)$ model. The liquidity premia are measured as the estimated yield difference between the fitted yield to maturity of individual bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from January 7, 2005, to December 30, 2020.

for $i = 1, 2, ..., n$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^T$ is approximately the rate of return on the ith bond if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields without correcting for liquidity risk, we compute a new set of model-implied bond prices from the estimated $G^{X^N,X^R}(7)$ model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^N = 0$ for all t as well as $\sigma_{44} = 0$ and $\theta_N^Q = 0$ for the nominal bonos, and $X_{t|t}^R = 0$ for all t as well as $\sigma_{77} = 0$ and $\theta_R^Q = 0$ for the real bonos UVR. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^T$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using (8). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the ith bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{9}$$

This can be calculated for bonos and bonos UVR separately.

Figure 8 shows the average bonos and bonos UVR liquidity premium series, denoted $\bar{\Psi}_t^N$ and $\bar{\Psi}_t^R$, across the outstanding set of each type of bond at each point in time. The bonos average liquidity premium series has a mean equal to 40.46 basis points with a standard deviation of 26.60 basis points, while the average bonos UVR liquidity premium has a significantly higher mean equal to 225.47 basis points with a standard deviation of 32.42 basis points. Hence, according to our model, the liquidity risk in the bonos UVR market is an order of magnitude above that in the standard bonos market. Furthermore, their correlation in levels is -21 percent, while it is -3 percent in first differences. Thus, the liquidity risk in the two markets is hardly correlated, which is similar to what BCFZ report for the Mexican government bond market.

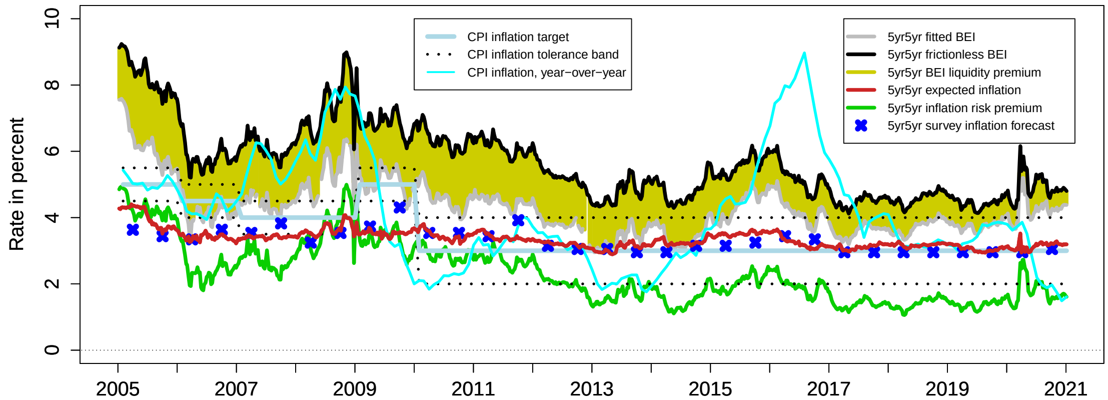


Figure 9: **Decomposition of 5yr5yr BEI**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Colombian bonos and bonos UVR prices separately and its decomposition based on the $G^{X^N,X^R}(7)$ model estimated with an unrestricted specification of $K^{\mathbb{P}}$ and a diagonal specification of Σ into: (1) the estimated frictionless BEI, (2) expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from January 7, 2005, to December 30, 2020.

As discussed in the introduction, the trading of nominal and inflation-indexed debt could be concentrated among two very different investor groups. We take the very low correlation in the estimated liquidity premium series from each market to be a sign that this is indeed the case.

4 Empirical BEI Decomposition

In this section, we explore the properties of the BEI decomposition implied by the $G^{X^N,X^R}(7)$ model with a particular emphasis on both the model-implied expected inflation and the associated inflation risk premium that investors in nominal bonos demand to assume their inflation risk. First, we examine the BEI decomposition and the realism of the model-implied inflation expectations. We then analyze the inflation risk premia and their determinants, including an international comparison.

4.1 BEI Decomposition

In this section, we examine the BEI decompostion implied by the estimated $G^{X^N,X^R}(7)$ model. To be consistent with the existing literature, we focus on a horizon long enough into the future that most transitory shocks to the economy can be expected to have vanished. At the same time, the horizon must be practically relevant and covered by the available maturities in the underlying bond data. Balancing these considerations, we limit our analysis to the five-year forward BEI rate that starts five years ahead, denoted 5yr5yr BEI.

The result of decomposing 5yr5yr BEI as described in Section 3.2 is shown in Figure 9. The solid gray line shows the fitted 5yr5yr BEI obtained by estimating a standard three-factor arbitrage-free Nelson-Siegel (AFNS) model to nominal bonos and real bonos UVR prices separately. This can be compared to the estimated 5yr5yr frictionless BEI implied by the $G^{X^N,X^R}(7)$ model and is shown with a solid black line in the figure. The difference between these two measures of 5yr5yr BEI represents the net liquidity premium or distortion of the observed BEI series due to bond-specific liquidity risk premia in both bonos and bonos UVR prices. The fact that the 5yr5yr frictionless BEI is entirely above the 5yr5yr fitted BEI implies that the distortions due to liquidity risk are systematically larger in the real yields compared to those in the nominal yields at the 5yr5yr horizon, consistent with the evidence in Figure 8.

Due to its theoretical consistency, the $G^{X^N,X^R}(7)$ model allows us to break down the 5yr5yr frictionless BEI into an expected inflation component, shown with a solid red line in Figure 9, and the residual inflation risk premium, shown with a solid green line. Also shown in the figure with a solid light blue line is the inflation target of the Bank of the Republic, which has varied over time but has been stable at 3 percent since 2010. Also shown with dotted black lines is the $\pm$ 1 percentage point tolerance band around the target rate. For comparison, the figure also shows the 5yr5yr expected CPI inflation reported semiannually in the Consensus Forecasts surveys. Although these survey inflation forecasts are included in the model estimation, the model-implied expected inflation rate does deviate from them at times thanks to the assumed standard deviation of 75 basis points for the associated measurement errors. Still, the closeness of the model's expected inflation to all the considered survey forecasts reported earlier underscores its ability to appropriately capture the term structure of inflation expectations among investors in the Colombian bonos and bonos UVR markets. Finally, Figure 9 also shows the year-over-year change in the Colombian CPI with a solid cyan line to provide a measure of the actual inflation outcomes during this 16-year period. Since 2010, CPI inflation has mostly remained within the tolerance band except for a brief period in 2015-2016. We take this evidence to imply that both the survey inflation forecasts and the model-implied inflation expectations can be viewed as anchored at a level consistent with the central bank's inflation target.

4.1.1 Comparison with Another BEI Decomposition

To further validate the BEI decomposition implied by the $G^{X^N,X^R}(7)$ model, we compare it to the BEI decomposition from an existing model of Colombian nominal and real government bond yields described in Espinosa-Torres et al. (2017, henceforth EMM). They use nominal and real yields with maturities ranging from one to eight years and estimate a six-factor

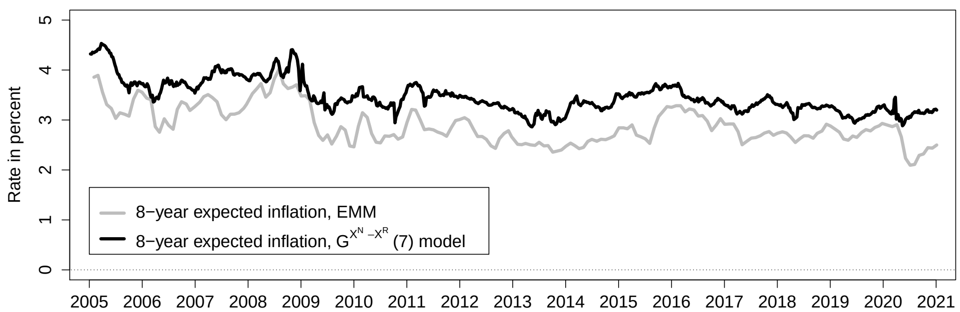


Figure 10: **Comparison of Market-Based 8-year Expected Inflation**

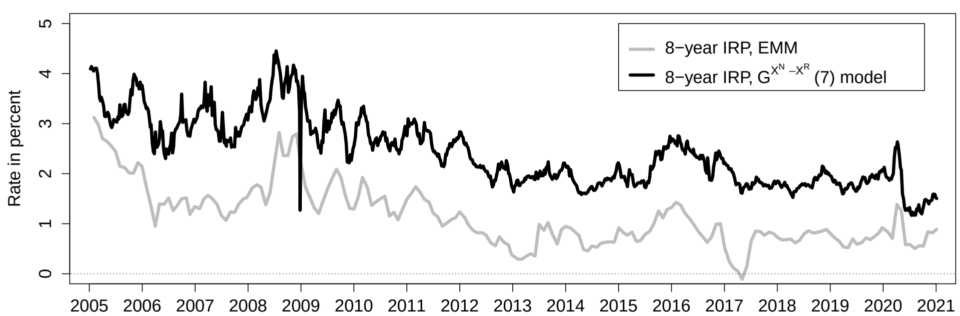


Figure 11: **Comparison of 8-Year Inflation Risk Premium**

dynamic term structure model using the approach of Adrian et al. (2013). Furthermore, their model makes adjustments for liquidity premia in both markets using an approach similar to Abrahams et al. (2016). Finally, no CPI information is used in the model estimation.

Figure 10 compares the estimated eight-year expected inflation from the two models. Although clearly positively correlated, the EMM estimate is uniformly lower. This is explained by its lower estimate of the net liquidity premium in Colombian BEI rates. Crucially, the EMM estimate implies that long-term inflation expectations in Colombia have frequently been below the target of the central bank. This appears to be somewhat at odds with the fact that realized CPI inflation has mostly come in above the target during our sample period.

Figure 11 compares the estimated eight-year inflation risk premium from the two models. The two series are also highly positively correlated, but with a sizable and persistent difference in magnitude that again can be traced back to differences in the estimated liquidity premia. Furthermore, the low level of the inflation risk premium implied by the EMM model seems to be somewhat counterintuitive given the high level of both inflation and its volatility in Colombia.

Overall, by more accurately accounting for the liquidity premia in both bonos and bonos UVR prices, our model is able to produce what appears to be more realistic estimates of investors' inflation expectations and the premia they demand for being exposed to inflation risk. This is instrumental to our assessment later on of the net benefit to the Colombian government of issuing inflation-indexed debt.

4.2 Analysis of the Model-Implied Inflation Expectations

In this section, we examine the properties of the inflation expectations implied by the $G^{X^N,X^R}(7)$ model in greater detail. First, we evaluate its ability to forecast inflation for the coming calendar year by comparing its performance to that of the Consensus Forecast survey. Second, we assess how anchored inflation expectations appear to be in Colombia using a statistical measure from the literature before we end the section by exploiting the estimated model dynamics to study the outlook for the 5yr5yr expected inflation over a three-year horizon.

4.2.1 Performance Comparison with Consensus Forecasts

In this section, we explore whether the desirable properties of the $G^{X^N,X^R}(7)$ model-implied long-term inflation expectations documented so far allow it to also generate realistic short-term inflation dynamics.

We structure the forecast exercise to match the monthly Consensus Forecasts survey. At the start of each month, the professional forecasters are asked about their expectations for the change in the CPI for both the current and the following calendar years. To have a series of pure forecasts that are not distorted by incoming information on realized inflation outcomes, we focus on the monthly survey forecasts of CPI inflation over the next calendar year. We then use the estimated $G^{X^N,X^R}(7)$ model to generate the matching model-implied CPI inflation forecasts. This has the advantage that the model-implied forecasts reflect information available at the end of each month and therefore lag the official survey dates by between one and two weeks. Thus, this exercise is by design conservative, although we stress the model forecasts are based on the full-sample estimates, unlike the survey forecasts, which are real-time forecasts by construction. Finally, our sample of monthly Consensus Forecasts starts in January 2009. Hence, to align the exercise with the available survey forecasts, we start the sample of corresponding model output in December 2008 and end it in November 2020, a total of 144 forecasts.

As benchmarks, we include two additional forecasting methods. The first is the classic random walk assumption of no change for which the one-year inflation forecast each month equals the past 12-month change in the Colombian CPI. Hence, the fact that the forecast does not start until the beginning of the next calendar year is without importance. The second is constructed from the observed BEI rates and equals the one-year forward BEI rate that starts at the beginning of the next calendar year and hence align exactly with the forecast

Model	Mean	RMSE	MAE
Random walk	9.38	222.99	186.97
BEI	58.43	181.15	141.53
Consensus Forecasts	35.80	166.13	128.11
$G^{X^N,X^R}(7)$ model	38.44	169.43	132.41

Table 4: **Summary Statistics of CPI Inflation Forecast Errors**
This table reports the mean forecasting errors (Mean), the root mean-squared forecasting errors (RMSE), and the mean absolute forecasting errors (MAE). The $G^{X^N,X^R}(7)$ model forecasts are computed from the full sample estimation results. The forecast errors are reported as the true value minus the model-implied prediction, and all numbers are reported in annual basis points.

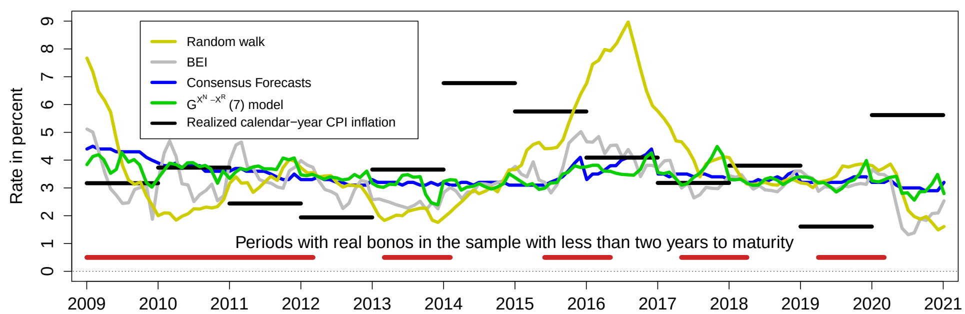


Figure 12: **CPI Inflation Forecasts and Realizations**

horizon in the Consensus Forecasts surveys.[24]

The summary statistics of the monthly forecast errors from the four forecast methods are reported in Table 4. First, we note that observed BEI rates outperform the random walk assumption. This suggests that the bond yield data are informative about inflation dynamics. However, as an inflation forecasting tool, observed BEI rates are inferior to both the survey forecasts and the $G^{X^N,X^R}(7)$ model-implied forecasts because of the noise added by both the inflation risk premium and differential liquidity premia. This explains its higher RMSE forecast errors and mean absolute forecast errors. Importantly, this also underscores the value of adjusting for these risk premia within the $G^{X^N,X^R}(7)$ model. Finally, in terms of the direct comparison to the survey forecasts, we note that the $G^{X^N,X^R}(7)$ model produces slightly higher forecast errors as measured by all three reported statistics. Given the flexible structure of the $G^{X^N,X^R}(7)$ model and its high number of parameters and state variables, we consider this an encouraging outcome.

In comparing the forecast series, Figure 12 shows that the survey forecasts are very stable,

[24]Similar to Figure 9, the BEI rates are obtained by estimating a standard three-factor AFNS model to nominal bonos and real udibonos prices separately.

even at the short calendar-year-ahead horizon examined here, another sign that inflation expectations in Colombia appear to be well anchored. In contrast, BEI rates and the $G^{X^N,X^R}(7)$ model-implied forecasts are slightly more volatile. Furthermore, the figure also includes the subsequent CPI calendar-year inflation realizations shown with solid black lines. Lastly, the random walk forecasts are the most volatile as they span the full swings in realized one-year inflation by construction.

To better understand the periodic deviations between the survey and $G^{X^N,X^R}(7)$ model-implied forecasts, we note that the deviations are positively correlated with the periods during which there are bonos UVR with less than two years to maturity in our sample, highlighted with solid red lines in the figure. Given that the latter are periods when the bonos UVR data may be particularly informative about investors' near-term inflation expectations, it seems reasonable that these would also be times when the model-implied inflation expectations are more likely to differ from those reported in the surveys.

Overall, these results and findings lead us to conclude that the $G^{X^N,X^R}(7)$ model is able to generate realistic inflation dynamics with properties that match those of the actual CPI series, even though we stress that no inflation data is included in the model estimation.

4.2.2 A Statistical Measure of Inflation Anchoring

For an inflation-targeting central bank like the Bank of the Republic, an important policy question is to what extent inflation expectations in Colombia appear to be anchored at a level consistent with the announced inflation target. In this section, to focus more squarely on that question, we consider a statistical measure of inflation anchoring inspired by Grishchenko et al. (2019).

This measure is centered around the conditional probability of our chosen anchoring measure—the 5yr5r expected inflation rate, here denoted $\pi^e_{t+\tau}(5yr5yr)$—being within the (2 percent, 4 percent) tolerance band used by the Bank of the Republic since 2010. That is, we are interested in the following conditional probability:

$$P\Big(\pi^e_{t+\tau}(5yr5yr) \leq 0.04\Big|X_t\Big) - P\Big(\pi^e_{t+\tau}(5yr5yr) \leq 0.02\Big|X_t\Big),$$

where τ is the considered horizon. Hence, this measure emphasizes whether the crucial 5yr5yr expected inflation among bond investors and other financial market participants is likely to remain within the tolerance band.

Since $\pi^e_t(5yr5yr)$ is affine in the state variables,

$$\pi^e_t(5yr5yr) = A^\pi + \Big(B^\pi\Big)' X_t,$$

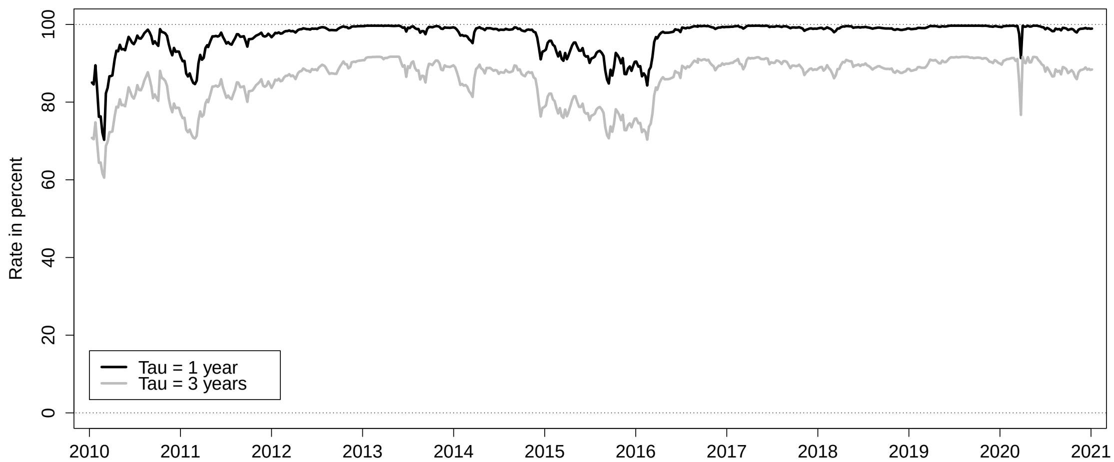


Figure 13: **Probability of 5yr5yr Expected Inflation Remaining Anchored**

it follows from the Gaussian dynamics of our model that

$$\pi^e_{t+\tau}(5yr5yr) \sim N\Big(A^\pi + \Big(B^\pi\Big)' E^{\mathbb{P}}_t[X_{t+\tau}], \Big(B^\pi\Big)' V^{\mathbb{P}}_t[X_{t+\tau}]B^\pi\Big).$$

Thus, the involved probabilities are easily calculated given that the first and second moments of X_t within the $G^{X^N,X^R}(7)$ model follow well-known formulas.

Figure 13 shows these probabilities based on our estimated model at the one- and three-year horizon starting in 2010 when the Bank of the Republic announced its current inflation target of 3 percent. As noted in Figure 9, the estimated value of $\pi^e_{t+\tau}(5yr5yr)$ has tended to be within the tolerance band. As a consequence, it is not surprising that the probability of it remaining within the band one year ahead has fluctuated close to 100 percent since 2010. However, as we increase the considered horizon to three years, the probability declines uniformly to a level between 75 percent and 85 percent. This is thanks to the increase in the uncertainty in the underlying projections as we lengthen the forecast horizon.

Mapping to the results reported for the United States and the euro area in Grishchenko et al. (2019), we note that the probabilities we obtain for Colombia are remarkably close to theirs and much higher than those reported by BCFZ for Mexico. This shows that, although inflation overall is more volatile in an emerging market economy like Colombia, this need not translate into materially more uncertain or less well-anchored long-term inflation expectations.

4.2.3 Outlook for Long-Term Inflation Expectations

To assess the outlook for long-term inflation expectations based on the $G^{X^N,X^R}(7)$ model more fully by taking the entire distribution of potential outcomes into account, we follow the

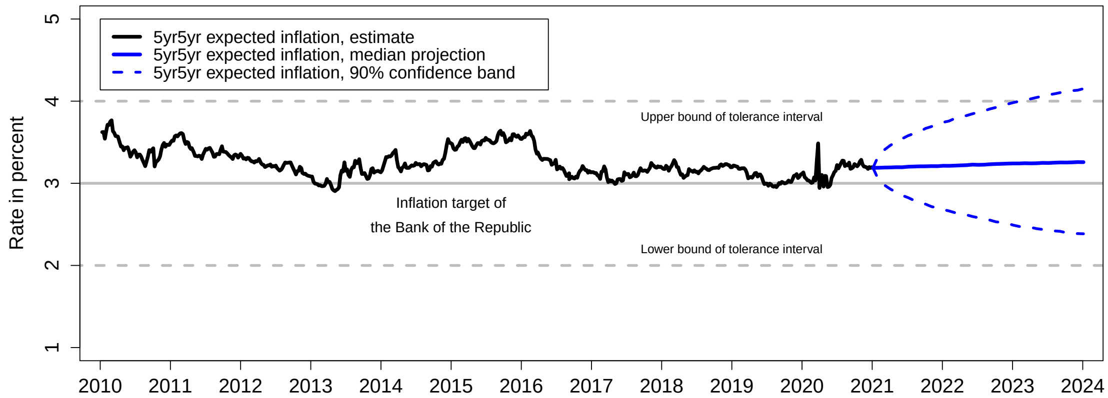


Figure 14: **Three-Year Projections of 5yr5yr Expected Inflation**

approach of Christensen et al. (2015) and simulate 10,000 factor paths over a three-year horizon, conditioned on the shapes of the nominal and real yield curves and investors' embedded forward-looking expectations as of the end of December 2020 (i.e., using the estimated state variables and factor dynamics as of December 30, 2020). The simulated factor paths are then converted into forecasts of 5yr5yr expected inflation. Figure 14 shows the median projection and the 5th and 95th percentile values for the simulated 5yr5yr expected inflation over the three-year forecast horizon.[25]

The model projections indicate that the long-term inflation expectations are likely to very gradually trend marginally higher from their December 2020 estimate of 3.19 percent. Thus, long-term inflation expectations in Colombia appear to be well anchored at a level close to the inflation target of the Bank of the Republic, although it is important to stress the sizable uncertainty surrounding estimates of long-term inflation expectations, as reflected in the wide 90 percent confidence band and consistent with the probabilities reported in the previous section.

4.2.4 Summary

In this section, we have performed a careful examination of the $G^{X^N,X^R}(7)$ model-implied inflation dynamics. First and most importantly, we find that long-term inflation expectations in Colombia appear to be stable at a level slightly above 3 percent. This makes us draw the conclusion that inflation expectations in Colombia appear to be well anchored close to the 3 percent inflation target of the Bank of the Republic.[26] Furthermore, based on the high

[25]Note that the lines do not represent paths from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

[26]De Pooter et al. (2014) reached a similar conclusion for Brazil, Chile, and Mexico.

probabilities of statistical measures of inflation anchoring, which are comparable to those reported for advanced economies, we feel extra confident drawing that conclusion. Lastly, the documented reasonableness of the model's estimated inflation dynamics also gives us confidence in its estimated inflation risk premia, which we analyze next.

4.3 Analysis of Inflation Risk Premia

In this section, we first explore what determines the size of and variation in Colombian inflation risk premia using regression analysis. This is followed by an international comparison to Canadian, Mexican, and U.S. inflation risk premia.

4.3.1 Determinants of Inflation Risk Premia

While the long-term inflation expectations in Colombia are largely determined by the inflation target of the Bank of the Republic, it is less clear what factors would matter for the size of Colombian long-term inflation risk premia. To explain the variation of the 5yr5yr Colombian inflation risk premium series, we therefore run a battery of standard regressions with it as the dependent variable and a wide set of explanatory variables that are thought to play a role for inflation risk premia as explained in the following.

To begin, we are interested in the role of factors that are believed to matter for bonos and bonos UVR market liquidity specifically or bond market liquidity more broadly as they could matter for the estimated inflation risk premia, even though we have explicitly accounted for bonos and bonos UVR liquidity premia in the model estimation. First, we include the average bonos age and the one-month realized volatility of the 10-year bonos yield as proxies for bonos liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et al. (2013), we also include a noise measure of bonos prices to control for variation in the amount of arbitrage capital available in this market. Lastly, we include the average bonos UVR age to proxy for liquidity risk in the market for those bonds. Combining these four explanatory variables tied to market liquidity and functioning produces the results reported in regression (1) in Table 5. We note a high adjusted R^2 of 0.62. The average bonos age has a highly significant negative coefficient. This implies that an increase in the liquidity risk of bonos is associated with lower inflation risk premia. The other measures of financial frictions tend to push up the Colombian inflation risk premia.

After having explored the role of liquidity factors, we examine the effects of factors reflecting risk sentiment domestically and globally on the inflation risk premia. This set of variables includes the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The set also contains the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same 10-year maturity. This on-the-run (OTR) premium

Explanatory variables	(1)	(2)	(3)
Avg. bonos age	-42.58***		-30.45***
	(5.183)		(5.191)
One-month bonos yield volatility	0.391		-0.230
	(0.417)		(0.347)
Bonos noise measure	1.974**		0.541
	(0.869)		(0.668)
Avg. bonos UVR age	0.169		7.789***
	(3.509)		(2.996)
VIX		0.950	0.573
		(0.686)	(0.572)
OTR premium (bps)		-0.292	-0.526
		(1.092)	(1.031)
MOVE Index		0.667**	0.591**
		(0.318)	(0.268)
TED spread (bps)		-0.328***	-0.0988
		(0.116)	(0.104)
5yr CDS rate (bps)		0.590***	0.423***
		(0.112)	(0.0925)
10yr US Treasury yield (%)		38.92***	17.25***
		(6.825)	(6.030)
WTI		0.429	-0.219
		(0.271)	(0.321)
Constant	377.7***	-59.47**	170.1***
	(30.07)	(28.21)	(48.74)
N	835	835	835
Adjusted R^2	0.62	0.72	0.78

Table 5: **Regression Results for the 5yr5yr Inflation Risk Premium**
The table reports the results of regressions with the estimated 5yr5yr inflation risk premium as the dependent variable and 11 explanatory variables. Standard errors computed by the Newey-West estimator (with 13 lags) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

is a frequently used measure of financial frictions in the U.S. Treasury market. To control for factors related to the uncertainty about the interest rate environment, we include the MOVE index. The fourth variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate. This spread represents a measure of the perceived general credit risk in global financial markets. As an additional indicator of credit risk and credit risk sentiment, we use the five-year CDS rate for Colombia shown in Figure 5. The next variable in the set is the 10-year U.S. Treasury yield from the Federal Reserve's H.15 database, which is included to control for reach-for-yield effects in advanced economies. This may be particularly relevant for our sample during the period between December 2008 and December 2015 and again in 2020 when U.S. short-term interest rates were constrained by the zero lower bound. Finally, we include the West Texas Intermediate (WTI) Cushing crude oil price to proxy for energy prices, which represent a significant risk to the inflation outlook in many countries around the world, including Colombia. The results of the regression with these seven explanatory variables is reported in regression (2) in Table 5. This produces a notable adjusted R^2 of 0.72. Furthermore, the MOVE index, the five-year CDS rate, and the 10-year U.S. Treasury yield are among the most significant variables and with the expected positive sign. This contrasts with the TED spread, which has a negative coefficient.

To assess the robustness of the results from these two regressions, we include all 15 explanatory variables with the results reported in column (3) in the table. Although this joint regression produces a high adjusted R^2 of 0.78, we do see a few variables with switches in the signs of their estimated coefficients.

Given the mixed results from the large regression models, we use informed priors to identify a simple preferred regression model to explain the variation in the 5yr5yr inflation risk premium series. First, we do want to account for liquidity risk in both bonos and bonos UVR. This makes us include the average bonos and bonos UVR age. Second, the MOVE index is widely used as a measure of interest rate uncertainty that matters for risk premia in both bond and stock markets. Third, perceptions about credit risk as measured by CDS rates appear to matter. As a consequence, we include the five-year CDS rate as well. Finally, as a small open economy, the Colombian government bond market is significantly affected by the interest rate level prevailing in the U.S. Treasury market, which we proxy with the 10-year U.S. Treasury yield. Thus, we run a second set of regressions with these five variables individually and combined. This allows us to identify a final preferred regression model for the Colombian 5yr5yr inflation risk premium series.

The results are reported in Table 6. Regression (6) with all five representative variables combined delivers an adjusted R^2 of 0.77. This is very close to the value obtained when we include all 11 variables. Hence, this supports our selection of this particular set of representative variables.[27] These results also underscore that our five representative variables are responsible for essentially all of the significant explanatory power. Furthermore, all five variables are statistically significant. Most importantly, their regression coefficients have con-

[27] In a robustness exercise, we repeat the regression using the 2011-2020 subsample and get qualitatively similar results; see the online appendix.

Explanatory variables	(1)	(2)	(3)	(4)	(5)	(6)
Avg. bonos age	-48.51***					-29.99***
	(4.860)					(4.402)
Avg. bonos UVR age		-26.99***				8.310***
		(6.252)				(2.491)
MOVE Index			1.751***			0.489***
			(0.132)			(0.174)
5yr CDS rate (bps)				0.771***		0.456***
				(0.107)		(0.0862)
10yr US Treasury yield (%)					49.05***	13.88**
					(7.886)	(5.564)
Constant	432.9***	359.9***	90.93***	113.1***	86.70***	165.8***
	(24.49)	(35.55)	(15.19)	(17.60)	(21.40)	(36.52)
N	835	835	835	835	835	835
Adjusted R^2	0.58	0.12	0.38	0.39	0.39	0.77

Table 6: **Preferred Regression Results for the 5yr5yr Inflation Risk Premium**
The table reports the results of regressions with the estimated 5yr5yr inflation risk premium as the dependent variable and the five representative explanatory variables identified in the initial round of regressions. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks *, **, and *** indicate significance at the 5 percent, 1 percent, and 0.1 percent levels, respectively.

sistent and sensible signs. As a consequence, we consider regression (6) to be our preferred explanatory regression model for the Colombian 5yr5yr inflation risk premium series.

As for the involved magnitudes, we note that a one-year increase in the average bonos age lowers the 5yr5yr inflation risk premium by 30 basis points. Thus, the persistent increase in average bonos age during our sample from 1.67 years to 5.65 years has been a key factor in the persistent decline in the Colombian 5yr5yr inflation risk premium since 2005. On the other hand, increases in the average bonos UVR age, the MOVE index, and the CDS rate tend to put significant upward pressure on Colombian inflation risk premia. Lastly, a one-percentage-point increase in the 10-year U.S. Treasury yield will tend to boost the Colombian 5yr5yr inflation risk premium by about 14 basis points. For comparison, BCFZ report an estimate of 12 basis points for the effect of the 10-year U.S. Treasury yield on Mexican inflation risk premia. Hence, the spillover effects from U.S. interest rates onto the inflation risk premia in Colombia and Mexico appear to be very similar.

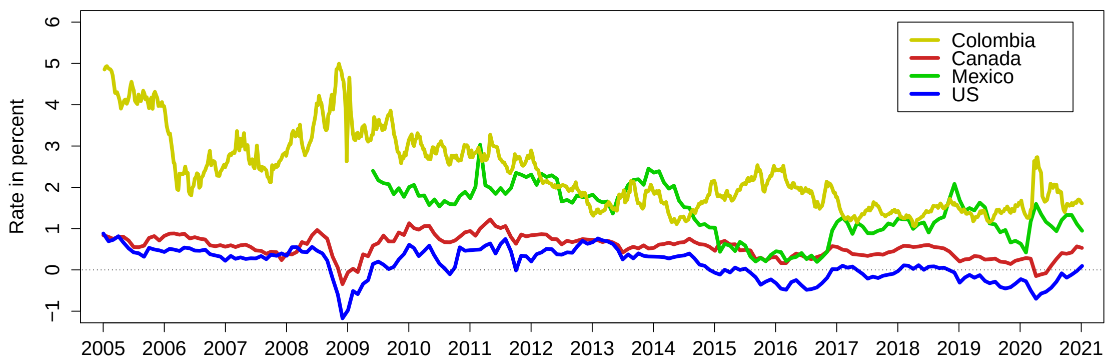


Figure 15: **International Panel of 5yr5yr Inflation Risk Premia**
Illustration of the estimated 5yr5yr inflation risk premium series from Mexican, Canadian, and U.S. nominal and real bond prices as described in the text. The shown data cover the period from May 31, 2009, to December 30, 2019.

4.3.2 International Comparison of Inflation Risk Premia

To go beyond the regression analysis above, we compare the estimated 5yr5yr inflation risk premium for Colombia with matching estimates from Canadian, Mexican, and U.S. nominal and real yields.[28] Figure 15 shows all four series for the available overlapping sample period.

The Canadian and U.S. inflation risk premia are highly positively correlated (84 percent). The Mexican inflation risk premium series is also positively correlated with each for the overlapping period, 60 percent and 69 percent, respectively. Thus, both in terms of size and time variation, Mexican inflation risk premia share similarities with those observed in Canadian and U.S. bond markets.

The estimated 5yr5yr inflation risk premium for Colombia has a much lower correlation with the Canadian, U.S., and Mexican estimates of 25 percent, 23 percent, and 39 percent, respectively. Furthermore, Colombian inflation risk premia are more volatile with a standard deviation of 92 basis points compared with 27 basis points, 37 basis points, and 63 basis points for the Canadian, U.S., and Mexican series, respectively.

The mostly positive and small inflation risk premia in Canada and the United States are consistent with the findings from simple macro-finance representative agent models; see Hördahl and Tristani (2012). For the United States, D'Amico et al. (2018) also report empirical estimates of inflation risk premia, which are mostly positive and relatively small. In turn, to observe larger and more volatile inflation risk premia in an emerging market economy such as Colombia would seem like a reasonable result given the higher and more volatile CPI

[28]The Canadian estimate is taken from Christensen, Rudebusch, and Shultz (2021), the Mexican estimate comes from BCFZ, while the U.S. estimate represents an update of the model described in ACR using all available TIPS.

inflation in Colombia compared with Canada and the United States.

Given that CPI inflation has averaged a notch higher in Colombia compared to Mexico in addition to being notably more volatile, it also seems reasonable that Colombian inflation risk premia are higher and more volatile than those observed in Mexico.

4.3.3 Summary

In this section, we examine the properties of the Colombian 5yr5yr inflation risk premium. First, we explored its determinants. Our preferred regression model showed that inflation risk premia in Colombia are significantly affected by international factors, namely the global interest rate level proxied by the U.S. 10-year yield, the uncertainty surrounding the global interest rate level as captured by the MOVE index, and global perceptions about credit risk as reflected in the Colombian five-year CDS rate. In addition, the liquidity risk of domestic government bond markets also plays a role as proxied by the average age of the bonos and bonos UVR in our sample. Overall, these results largely conform to our conjecture laid out in the introduction that, provided the bonos market is dominated by foreign investors, the inflation risk premium demanded by investors in nominal bonos should indeed be sensitive to global risk factors.

This was followed by an international comparison, which revealed that, although positively correlated with estimates from other countries, inflation risk premia in Colombia are unique in addition to being larger and more volatile. These characteristics seem reasonable in light of the fact that CPI inflation in Colombia has been higher and more volatile over the past 15 years than in the countries included in the comparison.

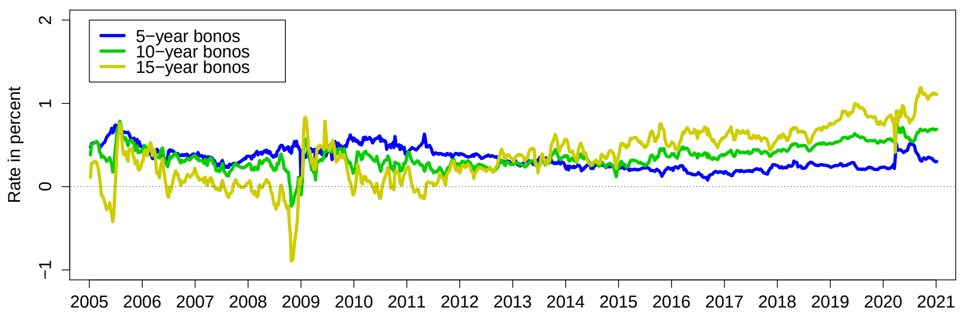
Equipped with an estimate of Colombian inflation risk premia along with estimates of the liquidity risk premia of bonos and bonos UVR, we are ready to tackle our main research question about the benefit to the Colombian government of issuing inflation-indexed debt. This is the focus of the next section.

5 Cost-Benefit Analysis of Inflation-Indexed Debt

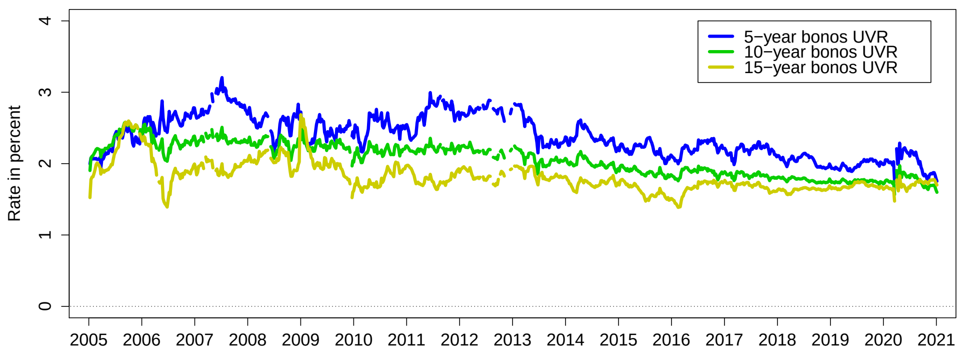
In this section, we assess the relative costs and benefits between issuing bonos and bonos UVR to the Colombian government, which represents an important policy application of our estimation results.

Given that the Colombian government issues bonds with fairly fixed maturities (5-year, 10-year, 15-year), we need to measure both liquidity and inflation risk premia at those fixed constant maturities. To that end, we first calculate fitted nominal and real zero-coupon yields for all relevant maturities.[29] Note that these yields embed the liquidity premia of the nominal and real bonds. We then use the estimated frictionless factor dynamics within the

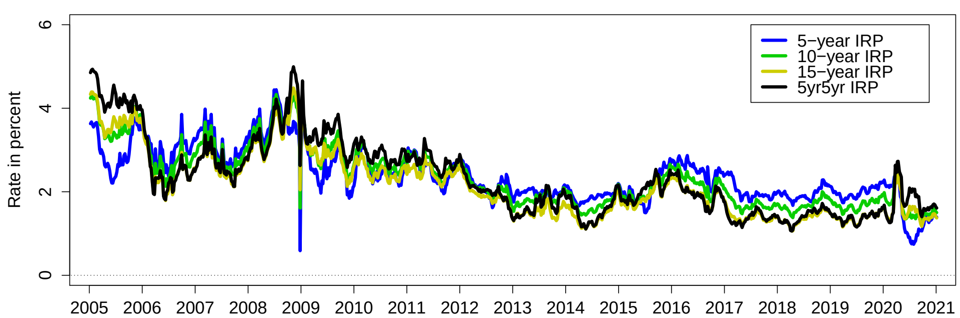
[29] Similar to Figure 9, the required nominal and real zero-coupon yields are obtained by estimating a standard three-factor AFNS model to nominal bonos and real udibonos prices separately.



(a) Bonos liquidity premia



(b) Bonos UVR liquidity premia



(c) Inflation risk premia

Figure 16: **Term Structure of Bond Risk Premia**

$G^{X^N,X^R}(7)$ model to calculate the corresponding frictionless nominal and real zero-coupon yields that do not contain any liquidity risk premia. The difference between the fitted and frictionless nominal zero-coupon yields then becomes an alternative synthetic estimate of the bonos liquidity premia at constant maturities. These are shown in Figure 16(a) for three maturities: 5-year, 10-year, and 15-year. Repeating this for the real yields produces the

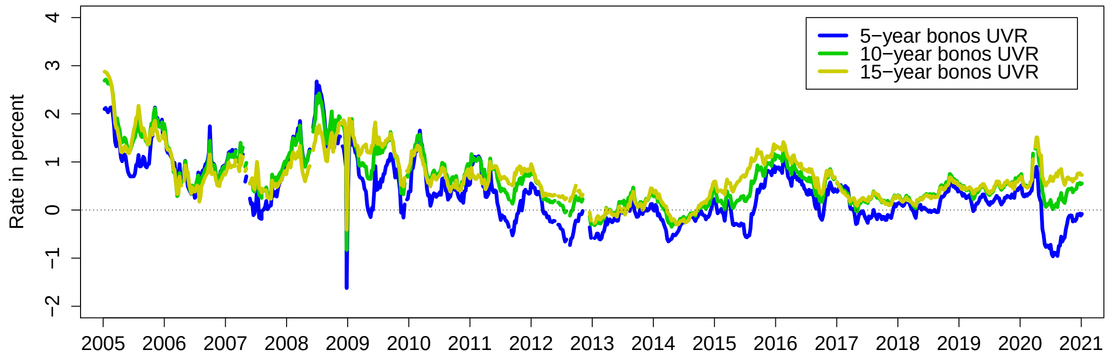


Figure 17: **Term Structure of Net Benefit of Bonos UVR Issuance**

synthetic estimates of bonos UVR liquidity premia at the same three maturities shown in Figure 16(b).

In the final step, we use the frictionless nominal and real zero-coupon yields implied by the $G^{X^N,X^R}(7)$ model to construct the corresponding frictionless BEI at the same three fixed maturities and deduct the associated model-implied expected inflation to obtain the corresponding three inflation risk premium series shown in Figure 16(c). For reference, we also plot the 5yr5yr inflation risk premium used in the regression analysis in the previous section.

Note the diverse shape of the term structure for each of these three types of risk premia. While the term structures of the inflation and bonos liquidity risk premia both vary between flat, upward sloping, and downward sloping, the slope of the bonos UVR liquidity risk premium curve is mostly downward sloping. This diverse pattern underscores the importance of combining the full term structure of bond prices for both bonos and bonos UVR with a full joint term structure model of both nominal and real yields.

Now, for each fixed maturity, adding the inflation risk premium and the bonos liquidity premium before deducting the bonos UVR liquidity premium produces a measure of the net benefit of issuing bonos UVR over bonos to the Colombian government. The resulting three net benefit series at the 5-year, 10-year, and 15-year maturities are shown in Figure 17.

The average net benefit of issuing bonos UVR at the 5-, 10-, and 15-year maturity is estimated at 0.38 percent, 0.66 percent, and 0.69 percent, respectively. Thus, while bonos issuance can be considered to be competitive, at least periodically, at the liquid 5-year maturity, our results are unfavorable regarding issuance of 10-year and 15-year bonos. Based on our estimates, it is recommendable that the Colombian government tilt its issuance of such long-term bonds towards the bonos UVR market. This result shows that the sizable inflation risk premia in Colombia dominate over the relative liquidity disadvantage of bonos

UVR compared to regular bonos, in particular at longer maturities.

5.1 Evidence from Mexico

As a point of reference, we repeat the analysis for Mexico by estimating the $G^{X^N,X^R}(7)$ model using the monthly Mexican data examined in BCFZ, which starts in May 2009. Furthermore, we note that inflation-indexed bonds in Mexico are known as udibonos, which is the term used in the following.

Given that the Mexican government also issues bonds with fairly fixed maturities (5-year, 10-year, 20-year), we calculate the net benefit measure of issuing udibonos for those three maturities with the resulting series shown in Figure 18. In this case, we see a fairly consistent upward sloping pattern with the net benefit of issuing 5-year udibonos being lower than the benefits of issuing longer-term udibonos. Still, all three series are positive on average: 0.15 percent, 0.28 percent, and 0.49 percent, respectively, at the 5-, 10-, and 20-year horizon. Hence, these results lead to a similar conclusion, namely issuance at the 5-year maturity appear to be competitive for both types of debt with a mild advantage towards inflation-indexed issuance, while at the long-term maturities the results point to a more material benefit from increased issuance of inflation-indexed bonds.

To summarize, without other strategic benefits from nominal debt, a strict actuarial cost-benefit analysis would recommend an increased issuance of inflation-indexed debt in both Colombia and Mexico, in particular at longer maturities. Furthermore, the government can introduce regulatory rules for financial institutions and institutional investors such as pension funds and life insurance companies with some form of beneficial treatment of holdings of long-term inflation-indexed debt to provide additional incentives to gain exposure to this particular market.

6 Conclusion

In this paper, we introduce a flexible joint model of nominal and real yields that accounts for liquidity risk premia in both nominal and real bond prices. We estimate the model on a representative sample of nominal and real bond prices from Colombia. This allows us to provide estimates of the liquidity-adjusted frictionless BEI along with its decomposition into investors' underlying inflation expectations and associated inflation risk premia.

Our results indicate that long-term inflation expectations in Colombia appear to be well anchored at a level close to the inflation target set by the Bank of the Republic of Colombia. Still, it is important to note that inflation in Colombia was notably more volatile during our sample period compared to its major neighbors to the North. As a consequence, inflation uncertainty represents a real risk for investors in the Colombian bonos market, which likely explains why we find that Colombian inflation risk premia are larger and more volatile than

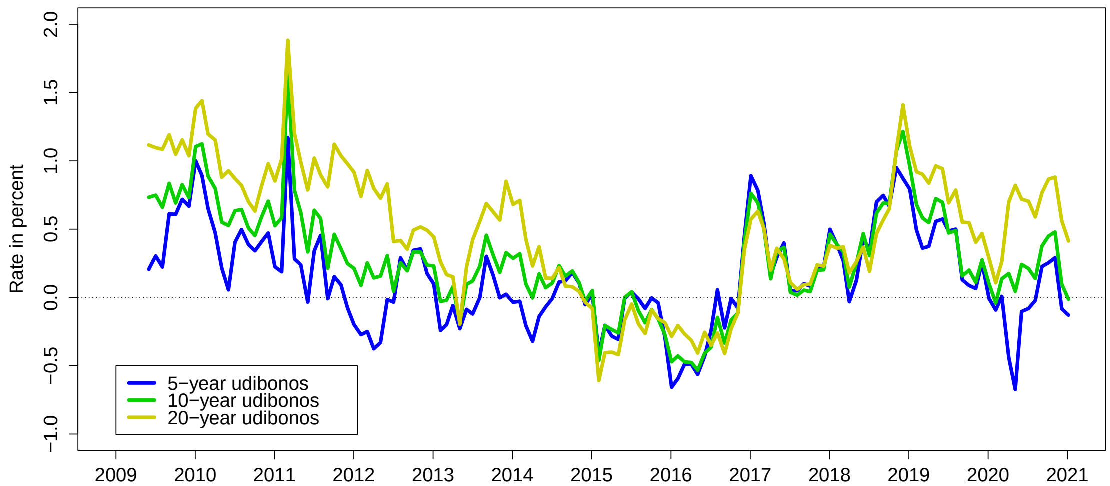


Figure 18: **Term Structure of Net Benefit of Mexican Udibonos Issuance**

corresponding estimates from Canada, Mexico, and the United States.

A comprehensive analysis of the determinants of long-term inflation risk premia in Colombia identifies five variables of particular importance, namely the MOVE index, the five-year CDS rate, and the 10-year U.S. Treasury yield in addition to proxies of the liquidity risk in the markets for bonos and bonos UVR. While the last two variables can be considered domestic, the first three variables are international and have positive coefficients, meaning that increases in global interest rate risk, global perceptions about credit risk, and U.S. long-term interest rates tend to boost long-term inflation risk premia in Colombia. Thus, to maintain the credibility of its monetary policy target, the Bank of the Republic will have to carefully navigate these global influences on its domestic bond markets. We leave it for future research to explore whether this holds for other open emerging market economies with inflation-targeting central banks.

With estimates of bonos and bonos UVR liquidity premia and general inflation risk premia in hand, we are finally able to assess the net benefit of bonos UVR issuance to the Colombian government. Here, our results show a clear advantage of bonos UVR over comparable bonos in the relevant 5-year to 15-year maturity range that is particularly pronounced for long-term bonos UVR. Thus, barring other strategic motives behind issuance of long-term bonos, our results suggest that a tilt towards greater issuance of long-term bonos UVR would be cost effective to the Colombian government.

Finally, we feel compelled to stress that our model framework can be applied to other emerging market economies with established nominal and real bond markets such as Brazil, Chile, and Peru, among many others. However, we leave those applications for future research.

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