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Estimating Natural Rates of Unemployment: A Primer*

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Abstract

Before the pandemic, the U.S. unemployment rate reached a historic low that was close to estimates of its underlying longer-run value and the short-run level associated with an absence of inflationary pressures. After two turbulent years, unemployment returned to its pre-pandemic low, and the estimated underlying longer-run unemployment rate appeared largely unchanged. However, economic disruptions pushed up the short-run noninflationary rate substantially, as high as 6%. This primer examines these different measures of the natural rate of unemployment and discusses how they can provide useful insights for policymakers.

JEL Classification: E24, E32, J60.
Keywords: unemployment, business cycles, natural rates

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1 Introduction

The U.S. unemployment rate in December 2022 was 3.5%, equal to its pre-pandemic 50-year low of 3.5% recorded in February 2020. Despite these similarly low levels, the economic environment now is very different than before the pandemic. The low unemployment at the end of the expansion following the Great Recession coincided with a period of very low inflation: personal consumption expenditures (PCE) price inflation hovered around 1.5% for much of 2019, below the Federal Reserve's 2% average inflation goal. By contrast, recent low unemployment rates are associated with rates of inflation in excess of 5%.

With this contrast in mind, policymakers often rely on two different unemployment benchmarks, or so-called natural rates of unemployment, to assess appropriate monetary policy (Crump, Nekarda and Petrosky-Nadeau, 2020). A first benchmark, the longer-run unemployment rate, provides a guide for normal economic activity in the longer run, after all the shocks that are thought to cause a current business cycle, either an expansion or a contraction, have dissipated. The second benchmark assesses the degree of economic slack and inflationary pressures in the short run and medium run. This "noninflationary rate of unemployment" associated with price stability provides a guide to how likely current labor market conditions are to be connected with inflationary pressures. In sum, these two concepts of the natural rate of unemployment help policymakers address separate concerns when assessing the current state of the economy.

This paper discusses and provides computer programs to implement some common approaches to estimating the unobserved longer-run and noninflationary benchmarks for the natural rate of unemployment following the discussion in Crump, Nekarda and Petrosky-Nadeau (2020).[1] In particular, we review the widely used Congressional Budget Office's (CBO) noncyclical rate of unemployment, and two alternative ground up approaches to estimating the longer-run natural rate of unemployment. The first seeks to extract longer run trends for dissaggregated demographic groups with statistical filtering methods, while the second seeks to infer the "potential minimum" rates of unemployment for different demographic groups based on recent business cycle peaks adapting a methodology used by DeLong and Summers (1988) to measure the economy's level of potential output.

The second benchmark rate, meant to assess the degree of inflationary pressures in the short and medium run, derived from an assumed relationship between price infla-

[1]The appendix provides additional technical details for the approaches reviewed here and outlines the accompanying programs to obtain a particular estimate of a benchmark rate of unemployment (link to programs here). We are grateful to several authors who have share their estimates reported in this paper.

tion and deviations of actual unemployment from this benchmark – the Phillips curve. We describe and implement the most common approach to estimate this benchmark rate of unemployment, a state space representation in which the noninflationary rate of unemployment is an unobserved variable with an assumed structure to its dynamics. The values of this state variable are then determined by the movements of observed unemployment and inflation rates via the Phillips curve, while simultaneously accounting for other factors that affect inflationary pressures in the economy.

The two benchmarks coincide at times, as they did in late 2019. At other times, there can be a sizable gap, as is the case today, with the noninflationary rate of unemployment well above its longer-run level. This divergence provides useful context for the recent Federal Open Market Committee (FOMC) decision to tighten policy to bring inflation back towards its longer-run goal for price stability.

2 Unemployment rates expected to prevail in the longer run (u^*_{LR})

The structure of the economy and the underlying dynamics of the labor market—factors that change slowly over time—are thought to determine the natural rate of unemployment in the longer run. Although researchers use a wide range of approaches to estimate the longer-run natural rate, there is a commonality in spirit that can be described as follows. Suppose there are J demographic groups, indexed by j, each with an unemployment rate $u(j,t)$ at date t and share of the labor force $\omega^{lf}(j,t)$. The aggregate rate of unemployment $u(t)$ can be expressed as the labor force share weighted sum of group specific unemployment rates:

$$u(t) = \sum_j \omega^{lf}(j,t) \times u(j,t).$$

Denote each group's unemployment rate expected to prevail in the longer run by $u_{LR}(j,t)$. Likewise, denote a labor force share for each demographic group $\omega^{lf}_{LR}(j,t)$ to be used in aggregating these longer run rates of unemployment. An estimate of the aggregate rate of unemployment expected to prevail in the longer run, $u^*_{LR}(t)$, is then obtained from aggregating the group-specific $u_{LR}(j,t)$ weighting each group by their respective labor force share:

$$u^*_{LR}(t) = \sum_j \omega^{lf}_{LR}(j,t) \times u_{LR}(j,t).$$

We focus first on the Congressional Budget Office (CBO) estimate of the "noncyclical rate of unemployment." The CBO follows an approach that mainly relies on changes in the composition of the labor force. According to Shackleton (2018), the longer-run or noncyclical rate of unemployment is based on an assumption that the U.S. labor market was at its longer-run state in 2005, and that this was true for different populations grouped by age, sex, race and ethnicity, and educational attainment. Using 2005 as a long-run benchmark for each demographic group's unemployment rate, the CBO constructs an aggregate longer-run rate of unemployment for the United States in which changes over time reflect the evoluation of each group's actual share of the labor force.[2] As a result, all movements in the CBO's estimate of the longer-run rate of unemployment come from slow-moving changes in the makeup of the workforce.

Figure 1 shows the CBO's noncyclical rate of unemployment (dashed blue line) from 1985 through 2021, along with a range of alternative estimates (shaded area), some of which we describe next. In general, the longer-run estimates change very gradually over time, in contrast to the higher-frequency cyclical fluctuations in the actual unemployment rate (red line).

A second related approach uses statistical methods to estimate the longer-run trends for different population groups' unemployment rates from historical experience before aggregating them into an overall longer-run natural rate of unemployment. This approach, which can be categorized as "longer-run trends," tends to imply higher longer-run rates of unemployment than the CBO estimate. This is especially true around the prolonged period of relatively elevated unemployment in the aftermath of the 2007–08 financial crisis. Our own application of this approach, shown as the blue solid line in Figure 1, draws on monthly microdata from the Current Population Survey and a bandpass filter (Christiano and Fitzgerald, 2003) to extract the changes in each population group's unemployment rates over multiple decades.[3] Our approach yields an estimate for the longer-run rate of unemployment of 6.0% in 2005, compared with the CBO's 5.0% estimate. By the fourth quarter of 2021, the two approaches result in very close estimates of 4.5% for the CBO and 4.4% with our approach.

A third approach seeks to infer the "potential minimum" rates of unemployment for

[2] That is, each group's $u_{LR}(j,t)$ for any date after 2005 is assumed to be equal to the group's average rate of unemployment in 2005. Prior to 2005 the CBO estimates are based on a Phillips curve relationship for married men, as described in Shackelton (2018), although it is well approximated by applying the post-2005 approach backwards. The weights $\omega_{LR}^{lf}(j,t)$ are the actual shares of the labor force for past and present estimates and projected shares for projections of the longer run rate of unemployment at future dates.

[3] Appendix A.1 provides further details. Estimates of longer run rates of unemployment close to this approach include Morris, Rich and Tracy (2019), Tasci (2012), Hornstein and Kudlyak (2019), Tüzemen (2019).

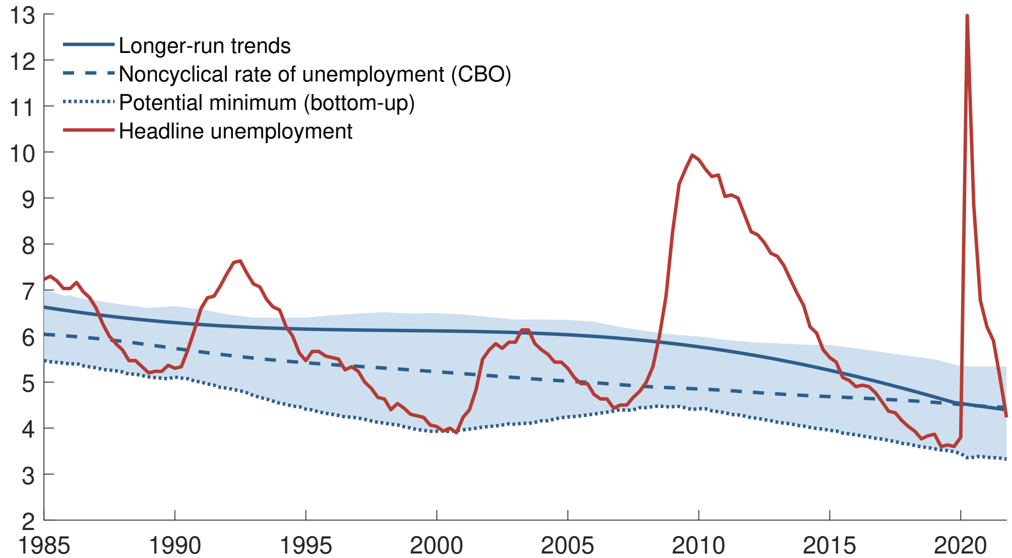

Figure 1: Estimates of U.S. longer-run rate of unemployment, 1985:Q1–2021:Q4

Note: Shaded area represents a range of estimates described in text and detailed in Appendix A; quarterly data. Sources: Bureau of Labor Statistics (BLS), CBO, and authors' calculations using CPS microdata.

different demographic groups based on recent business cycle peaks. Adapting a methodology used by DeLong and Summers (1988) to measure the economy's level of potential output, this approach obtains each demographic group's $u_{LR}(j,t)$ as

$$u_{LR}(j,t+1) \quad = \quad u_{LR}(j,t) + \min_{i=1,\ldots,k}\left[\frac{u(j,t+1) - u_{LR}(j,t)}{i}\right],$$

where the unemployment rates $u(j,t)$ are first smoothed, using an HP filter with smoothing parameter set to 10, in order to eliminate spurious minima in the series due to sampling noise. Setting $k = 4 \times 8 = 32$ (the number of future periods that determine potential minimum unemployment) with quarterly data, the result is the lowest contour (dotted blue line) in the range of estimates in Figure 1.[4] The approach suggests a longer-run rate of unemployment of 4.3% in 2005, slightly below the CBO's estimate during its base year. For the fourth quarter of 2021, this approach suggests a longer-run 'potential minimum" rate of unemployment of 3.3%.

[4]Our initial choice of 8 years corresponds to the length of a typical business cycle and aligns with the highest horizon parameter k in the range presented by DeLong and Summers (1988); Appendix A.3 presents alternative potential minimum estimates for different sets of horizon parameters.

3 Unemployment rates associated with no inflationary pressures $\left(u^*_{SP}\right)$

The second benchmark rate is meant to assess the degree of economic slack and inflationary pressures in the short and medium run. It is usually derived from an assumed relationship between price inflation and deviations of actual unemployment from this benchmark – the Phillips curve.

The most common approach to estimate this benchmark rate of unemployment is to cast the statistical relation as a state-space model (see Laubach, 2001) in which the noninflationary rate of unemployment is an unobserved variable with an assumed structure to its dynamics. The values of this state variable are then determined by the movements of observed unemployment and inflation rates via the Phillips curve, while simultaneously accounting for other factors, such as changes in production costs and currency exchange rates, that affect inflationary pressures in the economy.

The estimates of u^*_{SP} for which we present results here specify the observation equation, or the Phillips curve, as a relation between the growth in price inflation $\Delta\pi_t$ and the gap between unemployment and non-inflationary unemployment with slope γ. It also controls for past inflation and a set of other potential factors that affect inflation contained in $\mathcal{E}_t$:[5]

$$\begin{aligned}
\Delta\pi_t &= \rho_\pi \Delta\pi_{t-1} + \gamma\left(u_{t-1} - u^*_{SP,t-1}\right) + \delta\Delta\mathcal{E}_t + \sigma_\pi \varepsilon^\pi_t && (1)\\
\left(u_t - u^*_{SP,t}\right) &= \phi\left(u_{t-1} - u^*_{SP,t-1}\right) + \sigma_{\widehat{u}_{SP}} v_t^{\widehat{u}_{SP}} && (2)\\
u^*_{SP,t} &= u^*_{SP,t-1} + \sigma_{u^*_{SP}} v_t^{u^*_{SP}} && (3)
\end{aligned}$$

The unemployment gap associated with change in inflationary pressures, $\left(u_t - u^*_{SP,t}\right)$, is assumed to follow a stationary AR(1) process while the unobserved $u^*_{SP,t}$ is assumed to follow a random walk in equations (2) and (3), respectively. The disturbances to the observation equation ϵ^π_t are assumed to be drawn from an i.i.d. normal distribution $\mathcal{N}\left(0, \sigma^2_\pi\right)$, just as the disturbances $v_t^{\widehat{u}_{SP}}$ and $v_t^{u^*_{SP}}$ to the unemployment gap and the inflationary rate of unemployment. The latter are drawn from the distributions $\mathcal{N}\left(0, \sigma^2_{\widehat{u}_{SP}}\right)$ and $\mathcal{N}\left(0, \sigma^2_{u^*_{SP}}\right)$, respectively.

[5] See, e.g., Laubach (2001) or more recently Crump et al. (2019, 2022) who combine the two concepts, u^*_{LR} and u^*_{SP}, in a joint estimation framework. The relation, inspired by the work of Phillips (1958) showing wages tend to increase when the unemployment rate is low, is often associated with Friedman (1968) where it is more precisely expressed in growth rates of inflation, capturing the accelerationist hypothesis. In that case the unobserved short run $u^*_{SP,t}$ is the non-accelerating inflation rate of unemployment (NAIRU). Expressed in levels it is closer conceptually to a non-inflationary rate of unemployment (NIRU).

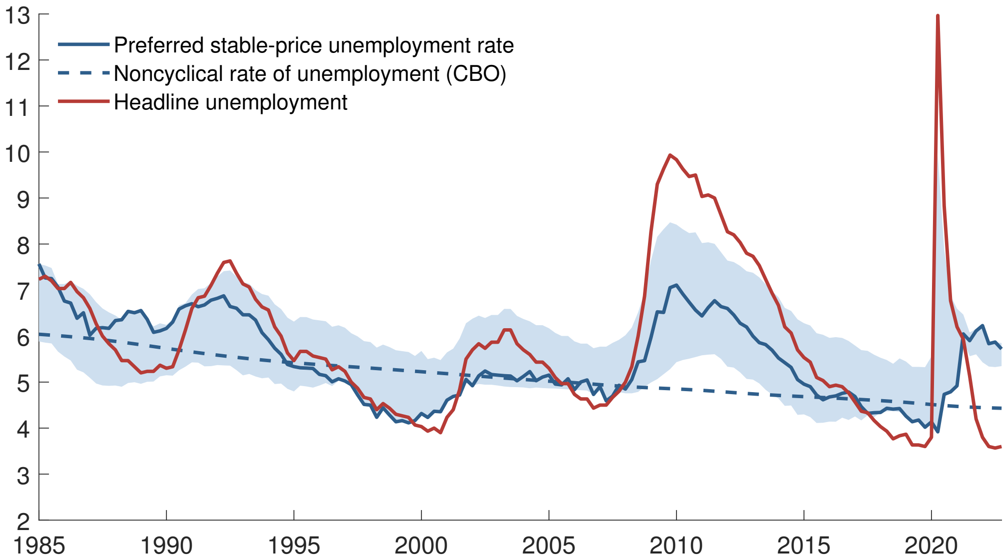

Figure 2: Estimates of U.S. stable-price rate of unemployment, 1985:Q1–2022:Q4

Note: Shaded area represents the full range of estimates from set of sources described in text detailed in Appendix B; quarterly data. Sources: BLS and authors' calculations using CPS microdata and estimates reviewed in Crump, Nekarda and Petrosky-Nadeau (2020).

The model parameters are estimated by maximum likelihood applying the Kalman filter to evaluate the likelihood function on quarterly data up to the onset of the pandemic, from 1985:Q1 to 2019:Q4. All model parameters are estimated save the variance of the nonstationary process which is fixed outside the estimation to avoid a "pile-up problem" (Stock, 1994). In particular, we set $\sigma_{u^*_{SP}}$ to 0.04 in the baseline estimates below, the lower range of estimates discussed in Laubach (2001). We then filter estimated model to obtain estimates of $u^*_{SP,t}$ from 1985:Q1 to 2022:Q4. Inflation is measured as year-over-year growth in the core PCE price index, the observed unemployment rate u_t is the headline U-3 rate, and the additional factor $\mathcal{E}_t$ in our baseline includes the year-over-year growth rate of the nominal broad U.S. dollar index. A more detail discussion of the data and baseline model parameter estimates along with a set of alternative specifications is provided in Appendix B.

Figure 2 plots a range (shaded area) of alternative state-space model estimates of the noninflationary rate of unemployment from 1985:Q1 through 2022:Q4. The solid blue line highlights our preferred approach to addressing the unique challenges from the COVID-19 pandemic, which we will discuss in greater detail next. The figure also includes the CBO's estimate of the longer-run rate of unemployment (blue dashed line) for reference. The figure highlights the degree to which estimates of the noninflationary rate of unemployment fluctuate with the actual rate of unemployment. It also shows that, over longer periods, the noninflationary rate tends to converge back towards the level of the longer-run rate of unemployment.

4 The noninflationary rate of unemployment during the pandemic

Estimating the noninflationary rate of unemployment has been challenging due to the exceptionally large and rapid movements in the unemployment rate during the second quarter of 2020, reaching nearly 15% within two months. In the Phillips curve framework for a given level of the noninflationary rate of unemployment, such a rise in the unemployment rate warrants a more pronounced slowdown in inflation than actually occurred. As a result, models that use the period just before the onset of the pandemic as a baseline imply a sharp increase in the noninflationary rate of unemployment to fit the sharp increase in actual unemployment without a commensurately large decline in price pressures. This is illustrated by the dashed blue line in Figure 3, where the noninflationary rate of unemployment rises sharply to just over 8% in the second quarter of 2020.

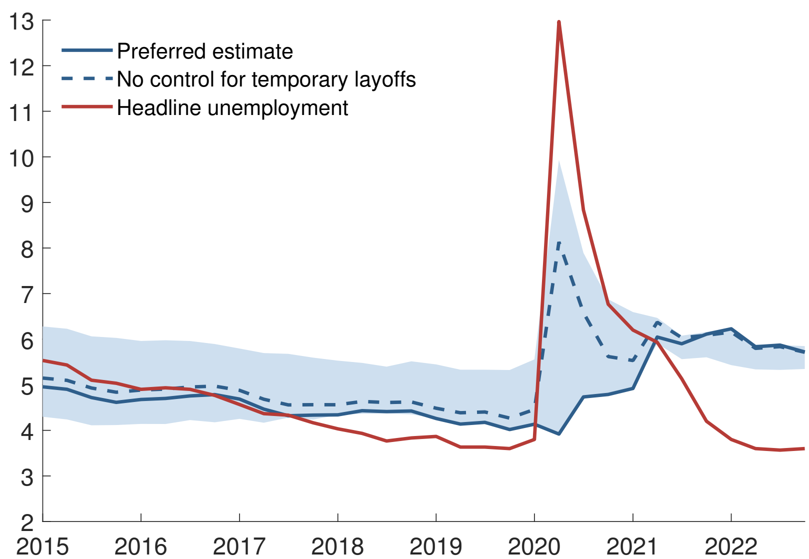

Figure 3: Estimates of stable-price unemployment through the pandemic, 2015:Q1–2022:Q4

Note: Shaded area represents the full range of estimates from set of sources described in text and detailed in Appendix B; quarterly data. Sources: BLS and authors' calculations using CPS microdata and estimates reviewed in Crump, Nekarda and Petrosky-Nadeau (2020).

However, much of the rise in unemployment during this period was driven by people on temporary layoff who were expected to return to work. Indeed, the share of unemployed people on temporary layoff rose from 14% before the pandemic to 78% in April 2020 (see Wolcott et al., 2020), only to return to its pre-pandemic level by mid-2021. This contrasts with past recessions, when the share of the unemployed on temporary layoff did not depart significantly from its historical average.

Temporary layoffs do not contribute to inflationary pressures in the same way as permanent job losses: employers tend to maintain ties with these workers so they can quickly bring them back and ramp up production as demand returns. Following this insight, our preferred estimate (solid blue line in Figure 3) controls for the spike in temporary layoffs and results in a limited increase in the noninflationary rate of unemployment at the start of the pandemic.[6] That said, as the share of temporary layoffs reverted to its historical level and PCE price inflation gained momentum in 2021, our estimated noninflationary rate of unemployment progressively rose to 6% by the fourth quarter of 2021, equaling the model that does not control for temporary layoffs (dashed blue line).

5 Conclusions

Two benchmark natural rates of unemployment can serve as useful guides in assessing the current state of the labor market, particularly relative to the Federal Reserve's goals of maximum employment and price stability. We outline various approaches for estimating both the longer-run rate of unemployment and the rate of unemployment associated with price stability. The unprecedented economic conditions during the pandemic created unique challenges for estimating the latter benchmark. Though longer-run and noninflationary rates of unemployment typically do not coincide at a point in time, any gap between the two benchmark rates tends to close over time. As such, the current sizable gap following the disruptions to the economy from the pandemic is likely to close as the FOMC follows an expected path of removing policy accommodation, intended to slow inflation to levels consistent with its price stability goals.

[6] The unemployment rate controlling for temporary layoffs that enters our preferred model for the pandemic is the headline unemployment rate through 2019:Q4, but for all dates from 2020:Q1 onward, the difference between temporary layoffs and their 2019:Q4 value is subtracted from the unemployed stock before dividing by the labor force to obtain the unemployment rate. See appendix additional details.

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Appendix

A Estimating the longer-run unemployment rate (u^*_{LR})

The range of estimates for $u^*_{LR,t}$ in Figure 1 are from the following sources: CBO noncyclical rate of unemployment; Crump et al. (2022); Hornstein and Kudlyak (2019); Morris, Rich and Tracy (2019); Tasci (2012); Tüzemen (2019); our "longer-run trends" approach; and our measure of potential minimum unemployment.

For our estimates of $u_{LR}(j,t)$, we consider demographic groups based on age (16 to 24, 25 to 34, 35 to 44, 45 to 54, 55 and older), sex, race and ethnicity (Black, Hispanic, White, and other) and education (high school or less, some college or associate's degree, and bachelor's degree or beyond), and three approaches to building a longer run u^*_{LR} for which with provide a brief description of the accompanying programs used to obtain the estimates.

Description of data preparation and general steps for estimation of u^*_{LR}

For all of our estimates of the longer-run unemployment rate, we perform the following general steps for data preparation:

- Extract CPS basic monthly microdata files and harmonize over time the classification of educational attainment.
- Partition the population into subgroups by the characteristics listed above, where each subgroup is indexed by j.
- For each month t, use CPS sample weights to calculate monthly estimates of stocks of employed $E(j,t)$ and unemployed $U(j,t)$ for each group j; calculate the labor force $L(j,t)$ for each group j as the sum of the employed and unemployed: $L(j,t) = E(j,t) + U(j,t)$; and sum across groups to obtain the aggregate labor force: $L(t) = \sum_j L(j,t)$. We then seasonally adjust these stocks and take quarterly averages.
- Compute labor force shares $\omega^{lf}(j,t)$ for each group j as the ratio of the group-specific labor force to the aggregate labor force: $\omega^{lf}(j,t) = L(j,t)/L(t)$. Compute group-specific unemployment rates $u(j,t)$ as the ratio of the group-specific unemployment stock to the group-specific labor force: $u(j,t) = U(j,t)/L(j,t)$.

The result of this data preparation is a set of quarterly time series on labor force shares and unemployment rates for each group, which are compiled in the spreadsheet `data.xlsx` that serves as the input to the MATLAB programs used to estimate u^*_{LR}, described below. Note that labor force shares ω^{lf} are denoted by `omegaLF` and unemployment rates u are denoted by `u` in the code.

A.1 "Longer-run trends"

The long-run estimates $u_{LR}(j,t)$ and $\omega^{lf}_{LR}(j,t)$ are obtained by extracting slow moving trends in the group specific $\omega^{lf}(j,t)$ and $u(j,t)$ with a bandpass filter on quarterly data between 1985:Q1 and 2019:Q4 at a horizon of 40 years or more. The trends beyond 2019 are obtained from a linear projection of the group-specific trend over the last two decades.

Description of programs for estimation of u^*_{LR} - "Longer-run trends" approach

- `extract_bpf_trend.m` estimates $\omega^{lf}_{LR}(j,t)$ and $u_{LR}(j,t)$ using a bandpass filter on the quarterly time series for labor force shares $\omega^{lf}(j,t)$ and unemployment rates $u(j,t)$.[7]
- Then the MATLAB script `main_uLR.m` produces a quarterly time series of estimates of the aggregate longer-run rate of unemployment $u^*_{LR}(t)$ following our "Longer-run trends" approach by computing the weighted sum over groups of these estimates of the group-specific longer-run unemployment rates $u_{LR}(j,t)$ using the corresponding estimates of the longer-run labor force shares $\omega^{lf}_{LR}(j,t)$ as the weights:

$$u^*_{LR}(t) = \sum_j \omega^{lf}_{LR}(j,t) \times u_{LR}(j,t)$$

A.2 CBO's noncyclical rate of unemployment

The CBO's approach builds a counterfactual rate of unemployment applying each group's rate of unemployment as observed during a reference period: $u_{LR}(j) = u(j,t_0)$ and $u^*_{LR,t} = \sum_j \omega^{lf}(j,t) \times u_{LR}(j)$. Choosing 2005 as the base period (i.e. setting $t_0 = 2005$) and the demographic groups described above replicates the CBO's noncyclical rate of unemployment.

Description of programs for estimation of u^*_{LR} - "CBO-inspired" approach

We replicate the CBO estimates of u^*_{LR} following the approach outlined in Shackelton, 2018, Appendix B:

[7] We use the optimal random walk filter of Christiano and Fitzgerald (2003).

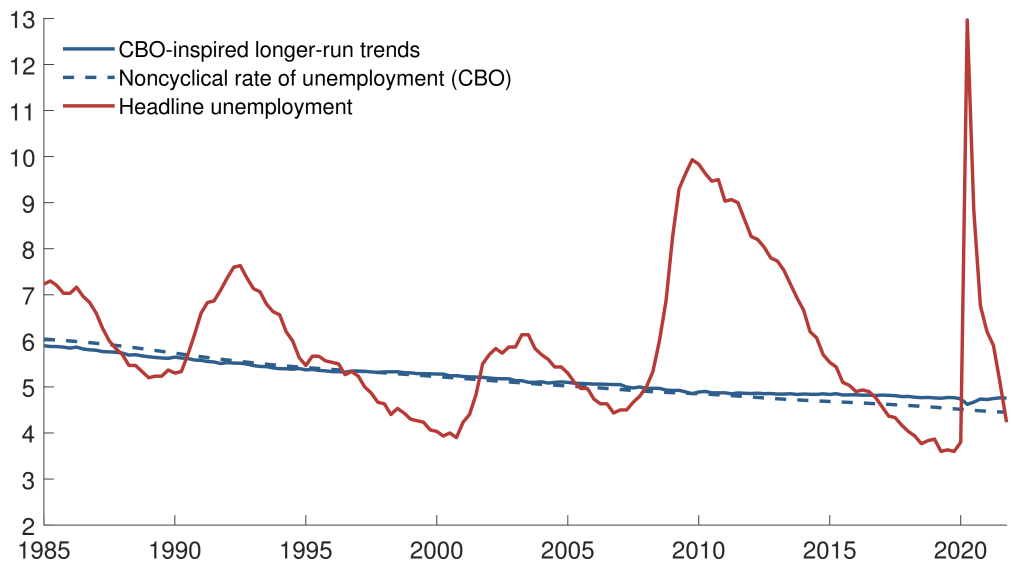

Figure A.1: Comparison of CBO-inspired longer-run trends estimate with published CBO noncyclical rate of unemployment

Note: Quarterly data. Sources: CBO and authors' calculations using CPS microdata.

- The MATLAB script `main_uLR.m` sets 2005 as the base year ($t_0 = 2005$), following the CBO methodology, and produces a quarterly time series of estimates of the aggregate longer-run rate of unemployment $u^*_{LR}(t)$ following our "CBO-inspired" approach by computing the weighted sum over groups of group-specific unemployment rates in the base year $u(j, t_0)$ using the time series of observed labor force shares $\omega^{lf}(j, t)$ as the weights:

$$u^*_{LR}(t) = \sum_j \omega^{lf}(j, t) \times u(j, t_0)$$

A.3 Minimum potential unemployment

We adapt the concept of potential output proposed by DeLong and Summers (1988) to let recent business cycle peaks inform a rate of unemployment approximating having reached full employment. We first smooth the group series $u(j, t)$ to eliminate spurious minima due to sampling noise, using a Hodrick-Prescott filter with a smoothing parameter set to 10. Then this approach defines $u_{LR}(j, t)$ as

$$u_{LR}(j, t+1) \quad = \quad u_{LR}(j, t) + \min_{i=1,\ldots,k} \left\{ \frac{u(j, t+i) - u_{LR}(j, t)}{i} \right\}$$

The length of the forward-looking window, k, is set to $8 \times 4 = 32$ in our baseline using quarterly data. This choice is guided by the length of a typical business cycle of 8 years, which is a reasonable assumption for the horizon over which unemployment should return to its potential minimum rate. [ADD explanation of extrapolation for T-k+1 to T: estimate slope of PMU between T-k and previous]

Figure A.2 plots potential minimum unemployment rate estimates for values of k set to 5 years and 8 years.

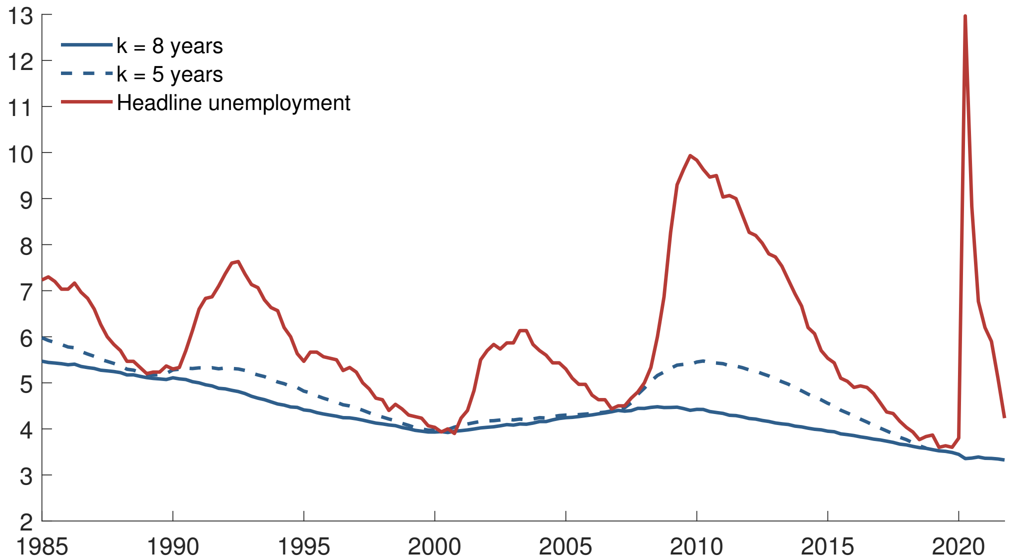

Figure A.2: Comparison of potential minimum unemployment rate estimates for different values of horizon parameter k

Note: Quarterly data. Sources: Authors' calculations using CPS microdata.

Description of Programs for Estimation of u^*_{LR} - "Minimum potential unemployment" approach

- `functions/potential_minimum.m` implements an adaptation of the DeLong and Summers (1988) algorithm on the smoothed $u(j,t)$ to estimate a quarterly time series of potential minimum unemployment rates $u_{LR}(j,t)$ for each group j.

- Then the MATLAB script `main_uLR.m` estimates a quarterly time series of u^*_{LR} following our "Minimum potential unemployment" approach by computing the weighted sum over groups of these estimates of the group-specific longer-run unemployment rates $u_{LR}(j,t)$ using the corresponding observed labor force shares $\omega^{lf}(j,t)$ as the weights:

$$u_{LR}^*(t) = \sum_j \omega^{lf}(j,t) \times u_{LR}(j,t)$$

B Estimating the stable-price unemployment rate (u_{SP}^*)

The range of estimates for $u_{SP,t}^*$ in Figures 2 and 3 are from the following sources: Aaronson et al. (2015); CBO, short-run; Crump et al. (2022); our baseline model; our preferred pandemic specification; and additional variations on our state-space model described next. These variations to our model include the baseline model with demographic adjustment to the unemployment rate, joint estimation of a wage Phillips curve and a price Phillips curve, and joint estimation of $u_{SP,t}^*$ and underlying trend inflation. The range also includes $u_{SP,t}^*$ estimates using data through 2021:Q4 to estimate model parameters, and various alternative controls during the pandemic subsample – specifically adding the goods share of consumption expenditures and commodity price inflation as an additional explanatory variables.

The point estimates for the slope of the Phillips curve with the change in inflation are in a range of 0 to -0.028 across all specifications estimated through 2019, consistent with Crump et al. (2019). The uncertainty around the estimates of $u_{SP,t}^*$ is sufficiently large that it is not possible to statistically distinguish the different specifications. Replacing lagged inflation with inflation expectations in the Phillips curve gives a Phillips-curve slope coefficient point estimate of -0.0011, with 0.28 as the estimated coefficient on the change in inflation expectations.[8]

Description of programs for estimation of u_{SP}^*

To obtain our estimates of u_{SP}^*, we specify each state-space model described above through Excel files called by functions in MATLAB and follow the notation in MATLAB's Econometrics Toolbox documentation for `ssm` class. We estimate a given model via maximum likelihood on quarterly time series data from 1985:Q1 through 2019:Q4. Then we filter on data through 2021:Q4.

- The MATLAB script `main_uSP.m` estimates specified models and plots output
- The Excel workbook `data.xlsx` contains the data for estimation, downloaded from FRED

[8] We use 4-quarter ahead median inflation expectations on CPI from the Survey of Professional Forecasters, since this and the GDP deflator are the only inflation expectations series available for the duration of our estimation sample.)

- The Excel workbook `models/spec_<model_name>.xlsx` contains the specification for estimation of a particular model, with the following worksheets:
  - `calibration` - set values for parameters that are specified exogenously, i.e. not estimated based on data, namely $\sigma_{u^*_{SP}}$ in the state-space model equations below.
  - `x` - state variables, mapped to x_t in the state-space model equations below, with `type` specifying the process the variable is assumed to follow: 0 for ARMA (stationary), 2 for random walk (nonstationary).
  - `y` - observables, mapped to y_t in the state-space model equations below.
  - `z` - exogenous regressors, mapped to z_t in the state-space model equations below.
  - `A`, `B`, `C`, `D`, `Beta`, `v`, `e` - specification of state-space system in terms of constants and parameters, mapped to A, B, C, D, β, v_t, e_t respectively in the state-space model equations below.
- The Excel worksheet `parameters.xlsx` contains the user-specification of initial values (`intival`), lower bounds (`lb`), and upper bounds (`ub`) for parameters to be estimated, namely γ, ϕ, ρ_π δ, $\sigma_{\hat{u}_{SP}}$, and σ_π in the state-space model equations below. (Note: `intival` must be specified, but `lb` or `ub` can be left blank which results in a particular parameter being unbounded below or above, respectively.)

The following routines in the `functions` subdirectory are called when running `main_uSP.m`:

- The `load_data.m` function loads data from `data.xlsx` into a `timetable` object.
- The `construct_dataset.m` function transforms raw data into transformed series that enter the state-space model, with transformations specified iteratively in the `transformation` column of the `x`, `y`, and `z` worksheets of the model specification workbook.
- The `load_spec.m` and `map_calibration.m` functions load a given model specification from `models/spec_<model_name>.xlsx` into a `struct` object composed of `table` objects for each worksheet, and sets any calibrated parameter values.
- The `load_parameters.m` function loads initial values and bounds for parameters to be estimated from `parameters.xlsx` into a `table` object
- The `ParamMap.m` and `map_parameters.m` functions map the model specification structure `spec` into MATLAB's required format for input in state-space model estimation

Estimation u^*_{SP} - Phillips curve state-space framework

Our baseline state-space model specification is as follows:
(Δ denotes the first-difference operator)

$$\begin{cases} \Delta\pi_t = \rho_\pi \Delta\pi_{t-1} + \gamma \widehat{u}_{SP,t-1} + \delta \Delta\mathcal{E}_t + \sigma_\pi \varepsilon^\pi_t, \quad \varepsilon^\pi_t \sim \mathcal{N}(0,1) \\ \widehat{u}_{SP,t} = u_t - u^*_{SP,t} \\ \widehat{u}_{SP,t} = \phi \widehat{u}_{SP,t-1} + \sigma_{\widehat{u}_{SP}} v^{\widehat{u}_{SP}}_t, \quad v^{\widehat{u}_{SP}}_t \sim \mathcal{N}(0,1) \\ u^*_{SP,t} = u^*_{SP,t-1} + \sigma_{u^*} v^{u^*}_t, \quad v^{u^*}_t \sim \mathcal{N}(0,1) \end{cases}$$

The following matrix equations map this Phillips curve framework into MATLAB's Econometrics Toolbox state-space model notation and setup (as outlined in the documentation for specifying a state-space model and proceeding with estimation). It is in this format that we specify a model in the Excel workbook infrastructure contained in the files `models/spec_<model_name>.xlsx`, with various worksheets corresponding to specific vectors or matrices in the MATLAB state-space model setup and named accordingly.

Description of data for estimation u^*_{SP}

The state-space model is specified at a quarterly frequency. All untransformed time series data are quarterly averages of monthly values.

- u_t = unemployment rate: `baseline` model
  = counterfactual unemployment rate adjusted for the rise in temporary layoffs: `pandemic` model
  - FRED series `UNRATE`, optionally using additional series `UNEMPLOY`, `LNS13023653`, and `CLF16OV` to control for the share of the unemployed on temporary layoff from 2020 onward relative to their 2019:Q4 share[9]
- π_t = core PCE price index, year-over-year percent change
  - FRED series `PCEPILFE`
- $\mathcal{E}_t$ = nominal USD exchange rate, year-over-year percent change

[9]Specifically, the unemployment rate controlling for temporary layoffs that enters our preferred model for the pandemic is the headline unemployment rate, i.e. the ratio of the unemployed to the labor force, for all dates through 2019:Q4; then for all dates from 2020:Q1 onward, the difference between temporary layoffs and their 2019:Q4 value is subtracted from the unemployed stock before dividing by the labor force to obtain the unemployment rate.

- FRED series `TWEXBMTH` and `TWEXBGSMTH` spliced together after transforming the series