FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

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November 2023

Working Paper 2023-34

https://www.frbsf.org/economic-research/publications/working-papers/2023/34/

Suggested citation:

Afonso, Gara, Gabriele La Spada, Thomas M. Mertens, John C. Williams. 2023 "The Optimal Supply of Central Bank Reserves under Uncertainty," Federal Reserve Bank of San Francisco Working Paper 2023-34. https://doi.org/10.24148/wp2023-34

The Optimal Supply of Central Bank Reserves under Uncertainty*

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November 9, 2023

Abstract

This paper provides an analytically tractable theoretical framework to study the optimal supply of central bank reserves when the demand for reserves is uncertain and nonlinear. We fully characterize the optimal supply of central bank reserves and associated market equilibrium. We find that the optimal supply of reserves under uncertainty is greater than that absent uncertainty. With a sufficient degree of uncertainty, it is optimal to supply a level of reserves that is *abundant* (on the flat portion of the demand curve) absent shocks. The optimal mean spread between the market interest rate and administered rates under uncertainty may be higher or lower than that absent uncertainty. Our model is consistent with the observation that the variability of interest rate spreads is a function of the level of reserves.

*The views expressed here are solely those of the authors and do not necessarily represent those of the Federal Reserve Banks of New York and San Francisco, the Federal Open Market Committee, or the Federal Reserve System.

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1 Introduction

Central banks have multiple goals in supplying reserves to the banking system. They target a level of the policy rate and aim to minimize its high-frequency volatility around that target, but they also experience other costs and benefits related to the size of reserves. These goals may involve tradeoffs; for example,a larger supply of reserves reduces the mean level of the policy rate and its volatility but may also involve other costs not directly related to interest rate control. Since the introduction of large-scale asset purchases in 2008, the Federal Open Market Committee (FOMC) has changed the monetary policy operating framework from one of scarce reserves with daily interventions to one of ample reserves without regular interventions. This increase in the quantity of reserves has been accompanied by a reduction in the variability of short-term interest rates around the target rate, as shown in Figure 1.

The optimal supply of reserves when demand is uncertain remains an open question in the literature. A useful benchmark is based on the Friedman rule, which minimizes the cost of holding money by setting the interest rate to zero. Applying the same logic to reserves, the central bank should provide sufficient reserves such that the opportunity cost of holding reserves is zero; that is, the market rate at which banks trade their reserves with each other should coincide with the interest rate on reserves paid by the central bank. In the presence of market frictions or other distortions, however, the optimal spread between the market and administered rates may differ from zero.

This paper provides an analytically tractable framework for studying the optimal supply of central bank reserves when demand is uncertain and nonlinear. The analysis provides a full characterization of the optimal supply of central bank reserves and associated market equilibrium. It finds that demand uncertainty is a key determinant of the optimal reserves supply. Greater uncertainty about the demand for reserves unambiguously raises the optimal supply.

The central bank faces a demand curve for reserves that links the quantity of reserves to the spread between the policy rate implied by the market and the interest rate on reserves. This demand is the result of a tradeoff: banks receive benefits from holding reserves but also incur costs; since the net benefits decline with the quantity of reserves held, the curve is nonlinear and has a lower asymptote. We represent the resulting demand curve as a piecewise linear function characterized by three regions. For a scarce supply of reserves, the slope of the demand curve is steep, i.e., price elastic: a small change in the quantity of reserves results in a meaningful change in the spread. At a sufficiently large, or ample, supply of reserves, the price-elasticity declines, i.e., the demand curve flattens but is still downward sloped. At an even larger, or abundant, supply of reserves, demand becomes completely price inelastic, i.e., the demand curve is flat.

The central bank is assumed to target a level of the spread and a quantity of reserves consistent with ample supply. The choice of reserves minimizes a quadratic loss function over expected deviations from the target spread and deviations from the target level of reserves. The central bank chooses reserves under uncertainty about demand. After the quantity of reserves has been chosen, three types of shocks affect the realized spread. There is uncertainty about the location of the kink between the ample and abundant regions, about the minimum spread (i.e., the floor of the demand curve), and about the slope of the demand curve in the ample region.

We start by assuming only two regions in the demand curve, ample and abundant. We first solve for the optimal quantity of reserves absent uncertainty. The optimal level of reserves increases less than one-for-one with an increase in the target level of reserves and is decreasing in the target level of the spread. We then analyze the optimal supply of reserves under uncertainty. Uncertainty about the location of the kink in the demand curve implies that certainty equivalence does not apply, and that the optimal supply of reserves depends on the degree of uncertainty. We derive analytically the optimal supply of reserves, associated comparative statics, and the equilibrium distribution of the interest rate spread.

Our analysis yields five main findings. First, uncertainty about the location of the kink between ample and abundant reserves unambiguously increases the optimal supply of reserves. This result is due to the truncation of the effects of these shocks over the flat part of the demand curve. Second, uncertainty about the slope of the demand further increases the optimal supply of reserves. Third, even though the underlying demand is piecewise linear, the mean spread is a smooth, declining function of the level of reserves. Fourth, an increase in uncertainty may decrease or increase the optimal mean spread. Fifth, for a sufficiently high degree of uncertainty, it is optimal to supply a level reserves that exceeds the expected kink in the demand curve; that is, it is optimal to supply reserves that are on average in the abundant reserves region. These results are robust to adding a third, scarce reserves, region, or by assuming that the optimal level of reserves absent uncertainty is in the abundant reserves region.

This paper's primary contribution is in developing a theoretical framework for analyzing the optimal policy of reserves supply. It builds on the empirical evidence highlighting the nonlinearity of the demand for reserves and the multiple sources of uncertainty around it (Afonso et al., 2023, and references therein). It also connects to an extensive literature that examines the optimal provision of liquidity in an economy. One branch of this literature explores conditions under which the Friedman rule applies; in general, the presence of distortions or externalities may violate the optimality of the Friedman rule and the optimal spread between the market interest rate and the rate paid on reserves may be non-zero.[1] In this paper, we treat the optimal interest rate spread as a parameter and derive the implications for the optimal provision of central bank reserves under uncertainty.

2 Model

We conduct our analysis with a simple model of the demand for central bank reserves that captures the key features implied by theory. We represent the nonlinear nature of demand using a piecewise-linear function of the interest rate spread between the market rate and the interest rate paid on reserve balances. The demand for reserves is price sensitive in the ample reserves region and price insensitive in the abundant reserves region. In a later section, we introduce a third region of scarce reserves, in which the demand for reserves is more price sensitive than in the ample reserves region. We incorporate three sources of uncertainty about the demand for reserves highlighted in the empirical evidence of Afonso et al. (2023).

[1] See, for example, Goodfriend (2002), Keister et al. (2008), and Cúrdia and Woodford (2011), Kimbrough (1986), Guidotti and Vegh (1993), Correia and Teles (1996, 1999), Chari, Christiano, and Kehoe (1991) and (1996), Goodfriend and King (1997), Nicolini (1998), Schmitt-Grohe and Uribe, (2004b), Schmitt-Grohe and Uribe (2009), Williamson (2008), Weiss (1980), Freeman (1993), Smith (2002), and Bhattacharya et al. (2005), Stein (2012) and Kashyap and Stein (2012).

2.1 Demand for Reserves

The interest rate spread, S, between the market rate and the interest rate on reserve balances is assumed to depend on the level of reserves, X, according to the piecewise-linear demand function:

$$S = \begin{cases} \bar{S} + \nu - (\alpha + \eta)(X - \bar{X} - \epsilon) & \text{if } X < \bar{X} + \epsilon, \\ \bar{S} + \nu & \text{else,} \end{cases} \tag{1}$$

where $\bar{X} + \epsilon$ denotes the level of reserves above which the demand for reserves becomes flat (abundant reserves), $\alpha + \eta$ is the steepness of the demand curve if $X < \bar{X} + \epsilon$ (ample reserves), and $\bar{S} + \nu$ is the spread if reserves are abundant (floor of the demand curve). We assume that $\bar{X}$, $\alpha > 0$, and $\bar{S}$ are known to the central bank, whereas ϵ, η, and ν are uncertain. Absent uncertainty, the demand equation (1) is shown by the solid black lines in Figure 2.

Figure 2 also shows how the three elements of uncertainty affect the demand for reserves. Each source of uncertainty is represented by a mean-zero shock with finite second moments that is assumed to be uncorrelated with the other shocks. Specifically, the shock ϵ represents uncertainty about the location of the kink in the demand curve; it has a probability density function $g(\cdot)$, cumulative distribution function $G(\cdot)$, and variance σ_ϵ^2. The shock ν, with variance σ_ν^2, reflects uncertainty about the level of the minimum spread and affects the spread equally along the demand curve. That is, ϵ corresponds to a horizontal shift in the reserve demand curve, while ν represents a vertical shift. The shock η, with variance σ_η^2, represents uncertainty about the slope of the demand curve in the downward-sloping (ample reserves) region.

To simplify our notation, in (1), we assume that the slope of the demand curve is zero in the abundant reserves region; our analysis and results, however, carry over if demand is price sensitive when reserves are abundant, as long as the mean slope in this region is smaller in magnitude than that in the ample reserves region, α.

2.2 Model Properties

We start with a description of key equilibrium properties of the model. The proofs build on methods developed in Bok et al. (2023) and are contained in the mathematical appendix.

For a given level of reserves, X, the cutoff value of the shock ϵ such that reserves become abundant, denoted by $\bar{\epsilon}(X)$, is given by:

$$\bar{\epsilon}(X) = X - \bar{X}.$$

For a given value of X, the probability of being in the flat part of the demand curve is given by:

$$\text{Prob}\Big(\epsilon < \bar{\epsilon}(X)\Big) = G\Big(\bar{\epsilon}(X)\Big),$$

where the derivative of this probability with respect to X is equal to $g(\bar{\epsilon}(X))$. To simplify notation, for the remainder of the paper, we suppress the argument in parenthesis and use $\bar{\epsilon}$ to indicate $\bar{\epsilon}(X)$.

In deriving model moments, it is useful to define two functions related to the cumulative distribution function $G(\cdot)$. First, define the super-cumulative distribution function, $\mathscr{G}(\cdot) \geq 0$, as:

$$\mathscr{G}(a) = \int_{-\infty}^{a} G(\epsilon) d\epsilon.$$

Since ϵ has zero mean, $\mathscr{G}(a) \geq a$ for all a. Second, define the super-super cumulative distribution function, $\mathbb{G}(\cdot) \geq 0$, as:

$$\mathbb{G}(a) = \int_{-\infty}^{a} \mathscr{G}(\epsilon) d\epsilon.$$

Note that $\mathbb{G}'(a) = \mathscr{G}(a)$ and $\mathscr{G}'(a) = G(a)$.

Result 1 (Mean and Variance of Spread)

For a given value of X, joint uncertainty about ϵ, ν, and η implies that:

1. *the mean spread under uncertainty, $\mathbb{E}S$, exceeds the spread absent uncertainty and is given by:*

$$\mathbb{E}S = \bar{S} + \alpha\Big(\mathscr{G}(\bar{\epsilon}) - \bar{\epsilon}\Big); \tag{2}$$

2. *the variance of the spread, $Var[S] = \mathbb{E}(S - \mathbb{E}S)^2$, is given by:*

$$Var[S] = \sigma_\nu^2 - \alpha^2\Big(\mathscr{G}(\bar{\epsilon}) - \bar{\epsilon}\Big)^2 + (\alpha^2 + \sigma_\eta^2)\Big(\bar{\epsilon}^2 + \sigma_\epsilon^2 - 2\mathbb{G}(\bar{\epsilon})\Big), \tag{3}$$

which decreases with the level of reserves because

$$\frac{dVar[S]}{dX} = -2\Big(\alpha^2 G(\bar{\epsilon}) + \sigma_\eta^2\Big)\Big(\mathscr{G}(\bar{\epsilon}) - \bar{\epsilon}\Big) \leq 0;$$

3. *in the special case of no uncertainty about ϵ, the variance of the spread is given by:*

$$Var[S] = \begin{cases} \sigma_\nu^2 + \sigma_\eta^2\bar{\epsilon}^2 & \text{if } X < \bar{X}, \\ \sigma_\nu^2 & \text{else}, \end{cases}$$

which decreases with the level of reserves for $X < \bar{X}$.

Uncertainty about the vertical location of the demand curve (ν) and uncertainty about the slope of the demand curve (η) have no effect on the mean spread for a given supply of reserves. For a given level of reserves, the mean spread is greater than or equal to the spread absent uncertainty due to the truncation of the shock ϵ along the flat portion of the reserve demand curve. This truncation of the distribution of ϵ smooths the relationships between the level of reserves and the mean and variance of the spread, despite the underlying piecewise-linear model. These relationships are illustrated in Figures 3 and 4, which are constructed assuming that ϵ and η follow Gaussian distributions (the variance of ν is set to zero). Figure 3 plots the spread as a function of reserves absent uncertainty about ϵ (the black line) and the expected spread as a function of reserves with uncertainty about ϵ (the red line). Figure 4 plots the corresponding relationships between the level of reserves and the standard deviation of the spread, with and without uncertainty about ϵ or η.

3 The Optimal Supply of Reserves

In this section, we analytically derive the optimal supply of reserves by the central bank given the demand equation (1). We begin with the deterministic version, in which the three shocks are set to zero. We then derive the optimal supply of reserves incorporating the three shocks.

3.1 The Central Bank Optimization Problem

Consistent with the theoretical considerations discussed above, we assume that the central bank aims to achieve a target spread, $\hat{S}$. In addition, we assume that it pays a penalty if the quantity of reserves deviates from a specified level, $\hat{X}$. In the stochastic problem, the central bank chooses the supply of reserves, X, before the realization of the three shocks.

We assume a quadratic objective function, denoted by $\mathscr{L}$. Specifically, the central bank aims to minimize the expected value of a linear combination of the squared difference between the realized spread and the target spread and the squared difference between the level of reserves and the reserve target, subject to the piecewise-linear demand for reserves in equation (1); that is,

$$\mathscr{L} = \min_X \frac{1}{2}\mathbb{E}\left[(S-\hat{S})^2 + \lambda(X-\hat{X})^2\right], \tag{4}$$

where $\lambda \geq 0$ is the relative weight given to deviations from the reserve target $\hat{X}$. This problem is mathematically identical to:

$$\mathscr{L} = \min_X \frac{1}{2}\left((\mathbb{E}S-\hat{S})^2 + \mathrm{Var}[S] + \lambda(X-\hat{X})^2\right),$$

where the loss function is written in terms of a linear combination of the squared difference between the mean spread and target spread, the variance of the spread, and the squared difference between the level of reserves and the target level.

As discussed above, the central bank's target level of the spread $\hat{S}$ is influenced by a number of factors, including monetary policy objectives and financial market frictions. The choice of the target reserve level $\hat{X}$ reflects central bank's considerations that are not directly related to the control of interest rates, such as the central bank's footprint in the financial system and possible disintermediation of the banking system, which could lead to a less efficient allocation of resources and inhibit market functioning.[2] If it is costly for the central bank to operate a large balance sheet due to reasons not directly associated with interest rate control, it may find optimal to set the target level of reserves below the satiation point, i.e., $\hat{X} < \bar{X}$.

In the following, we assume that, absent uncertainty, the target spread and target level of reserves are both in the ample reserves region. Specifically, we assume $\hat{S} > \bar{S}$ and $\hat{X} < \bar{X}$. Our analysis generalizes to the less-interesting case in which the target levels lie in the abundant reserves region. In principle, the value of $\hat{X}$ could be higher or lower than that implied by $\hat{S}$ (absent uncertainty). If $\hat{X} < \bar{X} - \frac{1}{\alpha}(\hat{S} - \bar{S})$, the penalty associated with the level of reserves is increasing in X for all $X \geq \hat{X}$.

3.2 Optimal Supply of Reserves absent Uncertainty

We first analyze the optimal supply of reserves in the deterministic version of the model that excludes the three shocks. Substituting the demand for reserves in the ample reserves region into the first-order condition yields the optimal level of reserves absent uncertainty, X^*:

$$X^* = \bar{X} - \frac{1}{\alpha^2 + \lambda}\Big(\alpha(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\Big) < \bar{X}.$$

The optimal level of reserves absent uncertainty increases less than one-for-one with the target level of reserves $\hat{X}$ and decreases with the target spread $\hat{S}$. Note that the optimal level of reserves X^* is invariant to the difference between $\bar{X}$ and $\hat{X}$ and the difference between $\bar{S}$ and $\hat{S}$.

The associated second-order condition for a minimum is $\alpha^2 + \lambda > 0$, which is true by assumption. The optimal level of the spread absent uncertainty, S^*, is given by:

$$S^* = \hat{S} - \frac{\lambda}{\alpha^2 + \lambda}\Big((\hat{S} - \bar{S}) - \alpha(\bar{X} - \hat{X})\Big).$$

The spread under the optimal supply of reserves absent uncertainty equals the target spread if $\lambda = 0$

[2]See Borio (2023).

and increases less than one-for-one with the target spread if $\lambda > 0$.

3.3 Optimal Supply of Reserves under Uncertainty

We now turn to the derivation of the optimal supply of reserves under uncertainty. Throughout we assume that the degree of uncertainty is fixed and there is no ability to "learn" from past behavior. We first consider the conditions for the optimal supply of reserves and then analyze its properties.

After substituting the demand model (1) into the optimization problem (4), the central bank's objective function is given by:

$$\min_{X}\left\{ \left((\bar{S}-\hat{S})^2+\sigma_\nu^2\right) G(\bar{\epsilon}) + \mathbb{E}\left[\left(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}-\epsilon)-\hat{S}\right)^2 \middle| \epsilon > \bar{\epsilon}\right] + \lambda(X-\hat{X})^2\right\}.$$

Note that certainty equivalence does not hold due to the multiplicative uncertainty associated with the slope parameter α and because the effect of realized horizontal shocks, ϵ, depends on the level of reserves X (via the truncation of their distribution).

Result 2 (Optimal Level of Reserves and Expected Spread)

The first-order condition yields the following equation for the optimal level of reserves, X^{**}:

$$X^{**} = \bar{X} + \frac{1}{\alpha^2+\sigma_\eta^2+\lambda}\left(\alpha\left(G(\bar{\epsilon})-1\right)(\hat{S}-\bar{S}) + (\alpha^2+\sigma_\eta^2)\mathscr{G}(\bar{\epsilon}) - \lambda(\bar{X}-\hat{X})\right), \tag{5}$$

which implies the inequality $X^{**}-\bar{X} < \mathscr{G}(\bar{\epsilon})$.

The corresponding second-order condition is given by:

$$(\alpha^2+\sigma_\eta^2)\left(1-G(\bar{\epsilon})\right) - \alpha(\hat{S}-\bar{S})g(\bar{\epsilon}) + \lambda > 0.$$

Appendices A.1 and A.2 derive these results.

Result 2 lays out the necessary and sufficient conditions for the existence of local minima. Note that vertical shocks to the demand for reserves, ν, do not affect the optimal choice of reserves X^{**} because vertical shocks simply add to the variance of the spread without affecting its mean and do not change how the supply of reserves affects the spread (as opposed to ϵ and η).

Unlike the case of no uncertainty, the first-order condition is an implicit function of X^{**} that may have multiple solutions, some of which represent local minima and others local maxima depending on the sign of the second-order condition. That said, the continuity of the loss function along with extreme values of reserves being suboptimal (due to the penalty associated with deviations from the target reserve level) guarantees the existence of a global minimum. In our analysis, we rely on the second-order condition holding at the global minimum, while recognizing that there may be other local minima.

Result 3 (Uncertainty Increases Optimal Supply of Reserves)

The optimal supply of reserves under uncertainty is characterized as follows.

1. *In the case of joint uncertainty about ϵ, ν, and η, the optimal reserve level, denoted by X^{**}, is given by:*

$$X^{**} = X^{*} + \frac{1}{\alpha^2 + \sigma_\eta^2 + \lambda}\Big((\alpha^2 + \sigma_\eta^2)\mathscr{G}(\bar{\epsilon}) + \alpha(\hat{S} - \bar{S})G(\bar{\epsilon})\Big) + \left(\frac{\sigma_\eta^2}{(\alpha^2 + \lambda)(\alpha^2 + \sigma_\eta^2 + \lambda)}\right)\Big(\alpha(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\Big) \geq X^{*},$$

where the terms on the right hand side of the equation are positive by the assumption that the target spread, $\hat{S}$, exceeds the floor spread, $\bar{S}$, and the target value of reserves is below the kink between the ample and abundant reserves regions ($\hat{X} < \bar{X}$);

2. *and the associated optimal mean spread under uncertainty, $\mathbb{E}S^{**}$, is given by:*

$$\mathbb{E}S^{**} = S^{*} + \alpha(\mathscr{G}(\bar{\epsilon}) + X^{*} - X^{**}) \lesseqgtr S^{*};$$

see Appendix A.3 for details.

3. *In the special case of no uncertainty about ϵ, optimal reserves, denoted by X^{*}_{η}, and the associated optimal*

means spread, S^*_η, are given by:

$$X^*_\eta = \bar{X} - \frac{1}{\alpha^2 + \sigma^2_\eta + \lambda}\Big(\alpha(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\Big) \geq X^*,$$

$$S^*_\eta = \hat{S} - \frac{\lambda}{\alpha^2 + \sigma^2_\eta + \lambda}\Big((\hat{S} - \bar{S}) - \alpha(\bar{X} - \hat{X})\Big).$$

Rearranging terms in the equation for optimal reserves under uncertainty provides greater insight into the effects of uncertainty on the optimal supply of reserves:

$$\begin{aligned} X^{**} = X^* &+ \frac{\alpha^2 + \sigma^2_\eta}{\alpha^2 + \sigma^2_\eta + \lambda}\mathscr{G}(\bar{\epsilon}) + \frac{\alpha}{\alpha^2 + \lambda}(\hat{S} - \bar{S})G(\bar{\epsilon}) \\ &+ \frac{\sigma^2_\eta}{(\alpha^2 + \lambda)(\alpha^2 + \sigma^2_\eta + \lambda)}\left(\alpha(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\right). \end{aligned} \tag{6}$$

Uncertainty associated with horizontal shifts in the demand increases the optimal supply of reserves because the kink in the demand curve truncates the effect of reserves on the spread. The larger supply of reserves has the benefit of both reducing the direct effect of uncertainty on the mean spread and increasing the probability of being on the flat part of the demand curve, where the deviations from the target spread are truncated. Uncertainty about the slope of the demand curve in the ample reserves region increases the optimal level of reserves because the magnitude of the effect of this source of uncertainty depends on the distance between X and $\bar{X}$. The logic is the same as in Brainard (1967), where multiplicative uncertainty calls for attenuation of the response of the policy instrument.

Uncertainty may increase or decrease the optimal mean spread due to two opposing effects: the direct positive effect of higher uncertainty (for any given value of reserves) and the indirect effect of a higher optimal level of reserves. In the special case of $\lambda = \sigma^2_\eta = 0$, the optimal supply of reserves fully offsets the direct effect of uncertainty on the mean spread, and $\mathbb{E}S^{**} < S^*$. Figure 3 illustrates the optimal choice of reserves. The black square denotes the optimal choice of reserves and the corresponding spread absent uncertainty. The red and blue squares show the optimal choice of reserves when there is uncertainty about the kink (ϵ shocks) and the slope (η shocks), respectively.

The green square corresponds to the optimal choice of reserves for both sources of uncertainty combined. Uncertainty about the floor of the demand curve (ν shocks) does not affect the mean spread or the optimal reserve supply.

3.4 Factors Affecting the Optimal Supply of Reserves

The optimal supply of reserves in equation (5) reflects the key parameters of the model as well as the uncertainty around the demand for reserves. In this section, we explore how the optimal reserve supply changes in response to shifts in some of these parameters.

A direct implication of equation (5) is that an equally-sized joint increase in $\bar{X}$ and $\hat{X}$ yields a one-for-one increase in the optimal supply of reserves. Similarly, an equally-sized joint increase in $\bar{S}$ and $\hat{S}$ has no effect on the optimal supply of reserves. More generally, the main findings regarding the sensitivity of the optimal supply of reserves to model parameters are summarized in the following result.

Result 4 (Comparative Statics for Optimal Reserves)
In the presence of joint uncertainty about ϵ, ν, and η, and assuming the second-order condition holds, the following comparative statics are obtained, where $\Gamma > 0$ denotes the second-order condition:

1. *optimal reserves are increasing in the target level of reserves, $\hat{X}$:*

$$\frac{dX^{**}}{d\hat{X}} = \frac{\lambda}{\Gamma} \geq 0;$$

2. *optimal reserves may be increasing or decreasing in the mean level of reserves at the kink, $\bar{X}$:*

$$\frac{dX^{**}}{d\bar{X}} = \frac{\Gamma - \lambda}{\Gamma};$$

3. *optimal reserves are decreasing in the target level of the spread, $\hat{S}$:*

$$\frac{dX^{**}}{d\hat{S}} = -\frac{\alpha(1 - G(\bar{\epsilon}))}{\Gamma} \leq 0;$$

4. *optimal reserves are increasing in the uncertainty about the slope of the demand curve in the ample reserves region:*

$$\frac{dX^{**}}{d\sigma_\eta^2} = \frac{\mathcal{G}(\bar{\epsilon}) - (X^{**} - \bar{X})}{\Gamma} \geq 0.$$

Perhaps not surprisingly, the optimal level of reserves is increasing in the target level of reserves (if $\lambda > 0$) and decreasing in the target spread. Interestingly, the sign of the relationship between the optimal level of reserves and the kink in the demand curve is indeterminate if $\lambda > 0$; that is, if the central bank targets a level of reserves for reasons unrelated to rate control. In the special case of $\lambda = 0$, the optimal supply of reserves moves one-for-one with the location of the kink in the demand curve. Finally, greater uncertainty about the slope of the demand curve in the ample reserves region increases the optimal supply of reserves. Intuitively, greater uncertainty about the slope of the demand curve raises the odds of higher equilibrium spreads. Since spreads are asymmetrically distributed with a floor in the abundant reserves region, the central bank can reduce its average loss by supplying more reserves on the margin.

3.5 Can it be Optimal to Supply Abundant Reserves?

We have assumed that the central bank targets a spread and a level of reserves in the ample reserves region, i.e., $\hat{X} < \bar{X}$ and $\hat{S} > \bar{S}$. We show that, absent uncertainty, the optimal reserves supply is indeed ample, $X^* < \bar{X}$. Under uncertainty, however, the optimal level can exceed $\bar{X}$ if the degree of uncertainty is sufficiently large relative to the penalty on the quantity of reserves.

Result 5 (Optimality of Abundant Reserves)

It is optimal to supply an abundant amount of reserves greater than $\bar{X}$ if:

$$-\alpha\Big(1 - G(\bar{\epsilon})\Big)(\hat{S} - \bar{S}) + (\alpha^2 + \sigma_\eta^2)\mathcal{G}(\bar{\epsilon}) - \lambda(\bar{X} - \hat{X}) > 0.$$

Note that a strong penalty on the quantity of reserves raises the hurdle to meet this condition.

4 The Distribution of Spreads: Theory and Evidence

In this section, we discuss how the model in Section 2 replicates salient features of observed spreads in the federal funds market. The critical assumption of the model is that the policymaker faces a nonlinear demand curve with uncertain parameters at the time of setting the amount of reserves. This uncertainty arises from variation in the location of the kink, the slope, and the floor of the demand curve.

The Federal Reserve has implemented monetary policy with different frameworks. Prior to 2009, the Federal Reserve operated a scarce reserves regime in which control of the federal funds rate was maintained through daily changes in the supply of reserves provided to the market. Since then, it has aimed to operate an ample reserves regime in which administered interest rates affect the federal funds rate by changing banks' opportunity costs of lending in the federal funds market. A key distinguishing feature of these two regimes is the level of reserves relative to the size of the banking system. Reserves increased from 0.1% of bank assets during the scarce reserves regime, to 12.3% on average during the second quarter of 2008 until the fourth quarter of 2019. After the COVID-19 pandemic (2020:Q3-2023:Q2), this ratio increased to 15.3%.

In the presence of horizontal shocks to the demand for reserves (ϵ), the model predicts a positive, nonlinear relationship between the level of reserves and the probability that the interest rate spread does not change from one day to the next. In contrast, if only vertical shocks are present (ν), this probability should be the same at all levels of reserves.

The evidence supports the basic assumptions of our model of a nonlinear demand curve for reserves and the importance of high-frequency horizontal shifts in the demand for reserves. Figure 1 shows the percentage of days within a quarter on which the daily change in the spread between the federal funds rate and a measure of the target rate is zero, from 2006 to 2023: the percentage strongly increases with the amount of reserves in the banking system, as measured by the reserves to asset ratio, and flattens around 100% when reserve are sufficiently large. As the supply of reserves increases, the interest rate spread remains more stable. This conclusion is also supported by the analysis in Afonso et al. (2023), who find that the sensitivity of the interest rate spread to shocks is negatively related to the quantity of reserves as long as reserves are below a given threshold (i.e., demand satiation point).

5 Optimal Reserves Supply Including a Scarce Reserves Region

So far, we have considered optimal reserves supply in the presence of ample and abundant reserves regions. In this section, we recognize that the demand curve displays a higher degree of nonlinearity, and there might be a second kink when reserves reach lower levels, $X < \bar{X}_2 < \bar{X}_1$. In this scarce reserves region, the demand curve becomes steeper.

Specifically, the demand for reserves is given by

$$S = \begin{cases} \bar{S} + \nu - (\alpha + \eta)(X - \bar{X}_1 - \epsilon) - (\beta + \varphi)(X - \bar{X}_2 - \epsilon) & \text{if } \epsilon > X - \bar{X}_2, \\ \bar{S} + \nu - (\alpha + \eta)(X - \bar{X}_1 - \epsilon) & \text{if } X - \bar{X}_2 \geq \epsilon > X - \bar{X}_1, \\ \bar{S} + \nu & \text{if } \epsilon < X - \bar{X}_1, \end{cases} \tag{7}$$

where $\beta \geq 0$ is the additional slope in the scarce reserves region relative to the ample reserves region (whose slope is α). φ is a mean zero shock with variance σ_φ^2 that represents uncertainty about the parameter β. In the special case of $\beta = 0$ and $\sigma_\varphi^2 = 0$, this setup is identical to the two-region model described above. As before, we assume that the optimal supply of reserves absent uncertainty lies in the ample reserves region, that is $\bar{X}_2 < X^* < \bar{X}_1$. This condition is satisfied if: $0 < \alpha(\hat{S} - \bar{S}) + \lambda(\bar{X}_1 + \hat{X}) < (\alpha^2 + \lambda)(\bar{X}_1 - \bar{X}_2)$.

In the absence of uncertainty, the equation for the optimal reserves is the same as in the two-region model, with $\bar{X}_1$ replacing $\bar{X}$. With uncertainty, the first-order condition for the optimal supply of reserves, denoted X^{***}, leads to

$$\begin{aligned} X^{***} = & \frac{\alpha^2 + \sigma_\eta^2 + \lambda}{((\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} X^{**} + \\ & + \frac{\beta(1 - G(\bar{\epsilon}_2))}{((\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} (\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}) + \frac{2\alpha\beta + \beta^2 + \sigma_\varphi^2}{((\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} (\bar{X}_2 + \mathscr{G}(\bar{\epsilon}_2)), \end{aligned}$$

where $\bar{\epsilon}_1 = \bar{\epsilon}(\bar{X}_1)$ and $\bar{\epsilon}_2 = \bar{\epsilon}(\bar{X}_1)$. Note that $S = \bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) > \hat{S}$ is the spread at $X = \bar{X}_2$ in the deterministic version of the model.

Result 6 (Higher Optimal Reserves in the Three-Region Model)

The existence of the scarce reserves region increases the optimal reserves supply:

$$X^{***} \geq X^{**} + \frac{\beta(1 - G(\bar{\epsilon}_2))}{\alpha^2 + \sigma_\eta^2 + \lambda}\Big(\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}\Big).$$

Appendix C contains details on the derivation. Under the assumption that the optimal supply of reserves absent uncertainty lies in the ample reserves region, the second term on the right-hand side is clearly positive. As a result, the addition of a third, scarce reserves region increases the optimal supply of reserves relative to that in the two-region model under uncertainty. The existence of a scarce reserves region thus reinforces our results on the effects of uncertainty on the optimal supply of reserves. An important implication is that adding a scarce reserves region relaxes the condition for optimal reserves to lie in the abundant reserves region (see Result 5).

6 Conclusion

This paper analyzes the optimal supply of central bank reserves under uncertainty about the demand for reserves. Relative to the case of no uncertainty, both uncertainty about the level of reserves that satiates demand and uncertainty about the slope of the demand curve below the satiation point increase the optimal supply of reserves. With a sufficiently high degree of uncertainty, it is optimal to supply a level of reserves that exceeds the mean value of the satiation point; that is, the optimal supply of reserves lies in the abundant reserves region.

A key advantage of this analysis is that it is analytically tractable, and we are able to derive closed-form solutions. Potential extensions of the model include adding intertemporal aspects of the supply and demand for reserves, structural changes, and the role of experimentation and learning in the design of optimal policy. In addition, this model provides a structure that can be empirically implemented, allowing for the estimation of the effects of uncertainty on the optimal supply for reserves.

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Appendix

A General Problem

The central bank minimization problem under uncertainty (4) can be written as

$$\min_X \Bigg\{ \int_{-\infty}^{\bar{\epsilon}} \int_{-\infty}^{\infty} (\bar{S} + \nu - \hat{S})^2 f(\nu) g(\epsilon) d\nu d\epsilon$$
$$+ \int_{-\infty}^{\infty} \int_{\bar{\epsilon}}^{\infty} \int_{-\infty}^{\infty} \left(\bar{S} + \nu - (\alpha + \eta)(X - \bar{X} - \epsilon) - \hat{S}\right)^2 f(\nu) g(\epsilon) h(\eta) d\nu d\epsilon d\eta + \lambda (X - \hat{X})^2 \Bigg\},$$

where f, g, and h are the probability density functions of the shocks ν, ϵ, and η, respectively. These density functions can have either finite or infinite support.

Then, the first-order condition is

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\bar{S} + \nu - \hat{S})^2 f(\nu) h(\eta) d\nu d\eta g(\bar{\epsilon}) \frac{d\bar{\epsilon}}{dX}$$
$$- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\bar{S} + \nu - (\alpha + \eta)(X^{**} - \bar{X} - \bar{\epsilon}) - \hat{S}\right)^2 f(\nu) h(\eta) d\nu d\eta g(\bar{\epsilon}) \frac{d\bar{\epsilon}}{dX}$$
$$+ \int_{-\infty}^{\infty} \int_{\bar{\epsilon}}^{\infty} \int_{-\infty}^{\infty} -(\alpha + \eta)(\bar{S} + \nu - (\alpha + \eta)(X^{**} - \bar{X} - \epsilon) - \hat{S}) f(\nu) g(\epsilon) h(\eta) d\nu d\epsilon d\eta + \lambda (X^{**} - \hat{X}) = 0,$$

where the first two terms cancel out when substituting in $\bar{\epsilon} = X^{**} - \bar{X}$. As a result, the first-order condition can be simplified to

$$\int_{-\infty}^{\infty} \int_{\bar{\epsilon}}^{\infty} \int_{-\infty}^{\infty} -(\alpha + \eta)(\bar{S} + \nu - (\alpha + \eta)(X^{**} - \bar{X} - \epsilon) - \hat{S}) f(\nu) g(\epsilon) h(\eta) d\nu d\epsilon d\eta + \lambda (X^{**} - \hat{X}) = 0. \quad \text{(A.1)}$$

A.1 Optimal Supply of Reserves

Before solving for the optimal level of reserves, we first derive a few useful results. The cumulative distribution function is defined as

$$\int_{-\infty}^{\bar{\epsilon}} g(\epsilon) d\epsilon = G(\bar{\epsilon}),$$

which we use in computing the following integral

$$\int_{-\infty}^{\bar{\epsilon}} \epsilon g(\epsilon)d\epsilon = \int_{-\infty}^{\bar{\epsilon}} \bar{\epsilon} g(\epsilon)d\epsilon - \int_{-\infty}^{\bar{\epsilon}} (\bar{\epsilon} - \epsilon)g(\epsilon)d\epsilon = \bar{\epsilon}G(\bar{\epsilon}) - [(\bar{\epsilon} - \epsilon)G(\epsilon)]_{-\infty}^{\bar{\epsilon}} - \int_{-\infty}^{\bar{\epsilon}} G(\epsilon)d\epsilon$$
$$= \bar{\epsilon}G(\bar{\epsilon}) - \mathscr{G}(\bar{\epsilon}). \tag{A.2}$$

Furthermore, since the shocks are independent and have zero mean,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\alpha + \eta)\nu f(\nu)h(\eta)d\nu d\eta = 0$$

and

$$\begin{aligned}
\int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty} (\alpha + \eta)^2 \epsilon g(\epsilon)h(\eta)d\nu d\eta &= \int_{-\infty}^{\infty} (\alpha + \eta)^2 h(\eta)d\eta \int_{\bar{\epsilon}}^{\infty} \epsilon g(\epsilon)d\nu \\
&= \int_{-\infty}^{\infty} (\alpha + \eta)^2 h(\eta)d\eta \left(\int_{-\infty}^{\infty} \epsilon g(\epsilon)d\nu - \int_{-\infty}^{\bar{\epsilon}} \epsilon g(\epsilon)d\nu \right) \\
&= -(\alpha^2 + \sigma_\eta^2)(\bar{\epsilon}G(\bar{\epsilon}) - \mathscr{G}(\bar{\epsilon})).
\end{aligned}$$

We can use these results to simplify the first-order condition in equation (A.1) to

$$\begin{aligned}
0 &= \int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty} -(\alpha + \eta)(\bar{S} - (\alpha + \eta)(X^{**} - \bar{X} - \epsilon) - \hat{S})g(\epsilon)h(\eta)d\epsilon d\eta + \lambda(X^{**} - \hat{X}) \\
&= (1 - G(\bar{\epsilon})) \left(-\alpha(\bar{S} - \hat{S}) + (\alpha^2 + \sigma_\eta^2)(X^{**} - \bar{X}) \right) - \int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty} (\alpha + \eta)^2 \epsilon g(\epsilon)h(\eta)d\epsilon d\eta + \lambda(X^{**} - \hat{X}) \\
&= (1 - G(\bar{\epsilon})) \left(-\alpha(\bar{S} - \hat{S}) + (\alpha^2 + \sigma_\eta^2)(X^{**} - \bar{X}) \right) + (\alpha^2 + \sigma_\eta^2)(\bar{\epsilon}G(\bar{\epsilon}) - \mathscr{G}(\bar{\epsilon})) + \lambda(X^{**} - \hat{X}).
\end{aligned}$$

Using the definition $\bar{\epsilon} = X^{**} - \bar{X}$, we get

$$-\alpha(1 - G(\bar{\epsilon}))(\bar{S} - \hat{S}) + (\alpha^2 + \sigma_\eta^2 + \lambda)(X^{**} - \bar{X}) - (\alpha^2 + \sigma_\eta^2)\mathscr{G}(\bar{\epsilon}) + \lambda(\bar{X} - \hat{X}) = 0,$$

which we can rearrange as

$$X^{**} = \bar{X} - \frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) - \frac{\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}(\bar{X} - \hat{X}) + \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathscr{G}(\bar{\epsilon}),$$

or alternatively as

$$X^{**} = X^{*} + \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda} G(\bar{\epsilon})(\hat{S}-\bar{S}) + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}\mathscr{G}(\bar{\epsilon}). \tag{A.3}$$

A.2 Second-order condition

Differentiating the left-hand side of the first-order condition in equation (A.1) yields

$$\begin{aligned}
&\frac{d}{dX}\int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty}\int_{-\infty}^{\infty} -(\alpha+\eta)(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}-\epsilon)-\hat{S})f(\nu)g(\epsilon)h(\eta)d\nu d\epsilon d\eta + \lambda(X-\hat{X})\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\alpha+\eta)(\bar{S}+\nu-\hat{S})f(\nu)g(\bar{\epsilon})h(\eta)d\nu d\eta + \int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty}\int_{-\infty}^{\infty}(\alpha+\eta)^2 f(\nu)g(\epsilon)h(\eta)d\nu d\epsilon d\eta + \lambda\\
&=-\alpha(\hat{S}-\bar{S})g(\bar{\epsilon}) + (1-G(\bar{\epsilon}))(\alpha^2+\sigma_\eta^2)+\lambda.
\end{aligned} \tag{A.4}$$

A.3 Expected spread

The expected spread is

$$\begin{aligned}
\mathbb{E}S &= \int_{-\infty}^{\infty}\int_{-\infty}^{\bar{\epsilon}}\int_{-\infty}^{\infty}(\bar{S}+\nu)f(\nu)g(\epsilon)h(\eta)d\nu d\epsilon d\eta +\\
&\quad+\int_{-\infty}^{\infty}\int_{\bar{\epsilon}}^{\infty}\int_{-\infty}^{\infty}(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}-\epsilon))f(\nu)g(\epsilon)h(\eta)d\nu d\epsilon d\eta\\
&=\bar{S}-\int_{\bar{\epsilon}}^{\infty}\int_{-\infty}^{\infty}(\alpha+\eta)(X-\bar{X}-\epsilon))g(\epsilon)h(\eta)d\epsilon d\eta\\
&=\bar{S}-\alpha\int_{\bar{\epsilon}}^{\infty}(X-\bar{X}-\epsilon)g(\epsilon)d\epsilon.
\end{aligned}$$

Using equation (A.2), we can write the expected spread at the optimal reserve level X^{**} as

$$\begin{aligned}
\mathbb{E}S^{**} &= \bar{S}-\alpha(X^{**}-\bar{X})+\alpha\mathscr{G}(\bar{\epsilon})\\
&=\bar{S}-\alpha(X^{*}+\Delta X^{**}-\bar{X})+\alpha\mathscr{G}(\bar{\epsilon})\\
&=S^{*}+\alpha(\mathscr{G}(\bar{\epsilon})-\Delta X^{**}),
\end{aligned}$$

where $\Delta X^{**} = X^{**}-X^{*} \geq 0$. Since $X^{*} \leq \bar{X}$, we have $\Delta X^{**} \geq X^{**}-\bar{X} = \bar{\epsilon}$. Since $\mathscr{G}(\bar{\epsilon})-\bar{\epsilon} \geq 0$, we cannot sign $\mathscr{G}(\bar{\epsilon})-\Delta X^{**}$. Thus, we get $\mathbb{E}S^{**} \lesseqgtr S^{*}$.

Substituting in for X^{**} delivers

$$\begin{aligned}
\mathbb{E}S^{**} &= \bar{S} - \frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}\left(-\alpha(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) - \lambda(\bar{X} - \hat{X}) + (\alpha^2 + \sigma_\eta^2)\mathcal{G}(\bar{\epsilon})\right) + \alpha\mathcal{G}(\bar{\epsilon}) \\
&= \bar{S} - \frac{\alpha^2}{\alpha^2 + \sigma_\eta^2 + \lambda}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \frac{\alpha\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}(\bar{X} - \hat{X}) - \frac{\alpha(\alpha^2 + \sigma_\eta^2)}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathcal{G}(\bar{\epsilon}) + \alpha\mathcal{G}(\bar{\epsilon}) \\
&= \bar{S} - \frac{\alpha^2}{\alpha^2 + \sigma_\eta^2 + \lambda}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \frac{\alpha\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}(\bar{X} - \hat{X}) + \frac{\alpha\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathcal{G}(\bar{\epsilon}).
\end{aligned}$$

B Comparative statics

This section derives the comparative statics for the optimal level of reserves in Result 4. We start with comparative statics for the optimal reserve holdings with respect to the variance of the slope, σ_η^2. Differentiating equation (5) yields

$$\begin{aligned}
\frac{dX^{**}}{d\sigma_\eta^2} &= \frac{d}{d\sigma_\eta^2}\left(\bar{X} - \frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) - \frac{\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}(\bar{X} - \hat{X}) + \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathcal{G}(\bar{\epsilon})\right) \\
&= \frac{\alpha}{(\alpha^2 + \sigma_\eta^2 + \lambda)^2}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \frac{\lambda}{\alpha^2 + \sigma_\eta^2 + \lambda}(\bar{X} - \hat{X}) + \frac{\alpha^2 + \sigma_\eta^2}{(\alpha^2 + \sigma_\eta^2 + \lambda)^2}\mathcal{G}(\bar{\epsilon}) + \\
&\quad + \frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}g(\bar{\epsilon})(\hat{S} - \bar{S})\frac{d\bar{\epsilon}}{d\sigma_\eta^2} + \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}G(\bar{\epsilon})\frac{d\bar{\epsilon}}{d\sigma_\eta^2},
\end{aligned}$$

where $\bar{\epsilon} = \bar{\epsilon}(X^{**}) = X^{**} - \bar{X}$. Therefore, we can write the above expression as

$$\begin{aligned}
&\left(1 - \frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}g(\bar{\epsilon})(\hat{S} - \bar{S}) - \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}G(\bar{\epsilon})\right)\frac{dX^{**}}{d\sigma_\eta^2} \\
&\quad = \frac{\alpha}{(\alpha^2 + \sigma_\eta^2 + \lambda)^2}(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \frac{\lambda}{(\alpha^2 + \sigma_\eta^2 + \lambda)^2}(\bar{X} - \hat{X}) + \frac{\alpha^2 + \sigma_\eta^2}{(\alpha^2 + \sigma_\eta^2 + \lambda)^2}\mathcal{G}(\bar{\epsilon}).
\end{aligned}$$

The right-hand side of this expression is positive because $\mathcal{G}(\bar{\epsilon}) \geq 0$ and because we assume that the target spread and reserves are in the ample region, i.e., $\hat{S} > \bar{S}$, $\hat{X} < \bar{X}$. We can sign the bracket on the left-hand side via the second-order condition in equation (A.4): it is positive at a minimum and negative at a maximum. To see this, take the second-order condition at a minimum, which is

$$-\alpha(\hat{S}-\bar{S})g(\bar{\epsilon}) + (1-G(\bar{\epsilon}))(\alpha^2+\sigma_\eta^2) + \lambda > 0.$$

This inequality is equivalent to

$$\alpha^2+\sigma_\eta^2+\lambda > \alpha(\hat{S}-\bar{S})g(\bar{\epsilon}) + (\alpha^2+\sigma_\eta^2)G(\bar{\epsilon})$$

and thus

$$1 - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(\hat{S}-\bar{S})g(\bar{\epsilon}) - \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon}) > 0.$$

Therefore, $\frac{dX^{**}}{d\sigma_\eta^2} \geq 0$.

Next we derive the comparative statics for the optimal reserve holdings with respect to the target level of reserves, $\hat{X}$.

$$\begin{aligned}
\frac{dX^{**}}{d\hat{X}} &= \frac{d}{d\hat{X}}\left(\bar{X} - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(1-G(\bar{\epsilon}))(\hat{S}-\bar{S}) - \frac{\lambda}{\alpha^2+\sigma_\eta^2+\lambda}(\bar{X}-\hat{X}) + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}\mathcal{G}(\bar{\epsilon})\right) \\
&= 1 + \frac{\lambda}{\alpha^2+\sigma_\eta^2+\lambda} + \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S})\frac{d\bar{\epsilon}}{d\hat{X}} + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon})\frac{d\bar{\epsilon}}{d\hat{X}}.
\end{aligned}$$

Rearranging terms,

$$\left(1 - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S}) - \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon})\right)\frac{dX^{**}}{d\hat{X}} = 1 + \frac{\lambda}{(\alpha^2+\sigma_\eta^2+\lambda)^2},$$

and thus $\frac{dX^{**}}{d\hat{X}} \geq 0$.

To derive comparative statics with respect to the target spread, $\hat{S}$, we differentiate the optimal level of reserves in equation (5) and obtain

$$
\begin{aligned}
\frac{dX^{**}}{d\hat{S}} &= \frac{d}{d\hat{S}}\left(\bar{X} - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(1-G(\bar{\epsilon}))(\hat{S}-\bar{S}) - \frac{\lambda}{\alpha^2+\sigma_\eta^2+\lambda}(\bar{X}-\hat{X}) + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}\mathcal{G}(\bar{\epsilon})\right) \\
&= -\frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(1-G(\bar{\epsilon})) + \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S})\frac{d\bar{\epsilon}}{d\hat{S}} + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon})\frac{d\bar{\epsilon}}{d\hat{S}}
\end{aligned}
$$

Rearranging terms,

$$
\left(1 - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S}) - \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon})\right)\frac{dX^{**}}{d\hat{S}} = -\frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(1-G(\bar{\epsilon})),
$$

and thus $\frac{dX^{**}}{d\hat{S}} \leq 0$.

Finally, comparative statics with respect to the location of the kink, $\bar{X}$ (i.e., demand satiation point), can be obtained as follows

$$
\begin{aligned}
\frac{dX^{**}}{d\bar{X}} &= \frac{d}{d\bar{X}}\left(\bar{X} - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}(1-G(\bar{\epsilon}))(\hat{S}-\bar{S}) - \frac{\lambda}{\alpha^2+\sigma_\eta^2+\lambda}(\bar{X}-\hat{X}) + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}\mathcal{G}(\bar{\epsilon})\right) \\
&= 1 - \frac{\lambda}{\alpha^2+\sigma_\eta^2+\lambda} - \frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S}) - \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon}) \\
&\quad + \left(\frac{\alpha}{\alpha^2+\sigma_\eta^2+\lambda}g(\bar{\epsilon})(\hat{S}-\bar{S}) + \frac{\alpha^2+\sigma_\eta^2}{\alpha^2+\sigma_\eta^2+\lambda}G(\bar{\epsilon})\right)\frac{dX^{**}}{d\bar{X}},
\end{aligned}
$$

where we used the definition $\bar{\epsilon} = X^{**} - \bar{X}$. Rearranging terms, we obtain

$$
\frac{dX^{**}}{d\bar{X}} = \frac{\Gamma - \lambda}{\Gamma},
$$

where $\Gamma = (\alpha^2+\sigma_\eta^2)(1-G(\bar{\epsilon})) - \alpha(\hat{S}-\bar{S})g(\bar{\epsilon}) + \lambda > 0$ is the second-order condition for a minimum.

B.1 Comparative statics of the variance of spreads

Taking the derivative of the variance of spreads in equation (3), we get

$$\begin{aligned}\frac{d\text{Var}[S]}{dX} &= -2\alpha^2(\mathcal{G}(\bar{\epsilon}) - \bar{\epsilon}) + (\alpha^2 + \sigma_\eta^2)(2\bar{\epsilon} - 2\mathcal{G}(\bar{\epsilon})) \\ &= 2(\alpha^2 G(\bar{\epsilon}) + \sigma_\eta^2)(\bar{\epsilon} - \mathcal{G}(\bar{\epsilon})) > 0,\end{aligned}$$

where the inequality follows from $\mathcal{G}(\bar{\epsilon}) > \bar{\epsilon}$ for all reserve values.

C Extension to Scarce Reserves Region

When the demand curve includes a region of scarce reserves, the central bank's optimization problem is given by

$$\begin{aligned}\min_X &\int_{-\infty}^{\bar{\epsilon}_1}\int_{-\infty}^{\infty}(\bar{S}+\nu-\hat{S})^2 f(\nu)g(\epsilon)d\nu d\epsilon + \int_{\bar{\epsilon}_1}^{\bar{\epsilon}_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}_1-\epsilon)-\hat{S})^2 f(\nu)g(\epsilon)h(\eta)d\nu d\eta d\epsilon + \\ &+\int_{\bar{\epsilon}_2}^{\infty}\int_{-\infty}^{\infty}\int_{\underline{\varphi}}^{\bar{\varphi}}\int_{-\infty}^{\infty}\left(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}_1-\epsilon)-(\beta+\varphi)(X-\bar{X}_2-\epsilon)-\hat{S}\right)^2 f(\nu)g(\epsilon)h(\eta)k(\varphi)d\nu d\varphi d\eta d\epsilon \\ &+\lambda(X-\hat{X})^2\end{aligned}$$

Denote the objective function in the two-region model by $\mathcal{L}_1$ and the state vector by $\theta = \{\epsilon, \nu, \eta, \varphi\}$, the state space by Θ, and the joint density function by $\gamma(\theta)$. We further denote the partition of the state space for ϵ between $\bar{\epsilon}_2$ and ∞ (with all other variables unrestricted) by Θ_2. Then we use that

$$\begin{aligned}&\int_{\bar{\epsilon}_2}^{\infty}\int_{-\infty}^{\infty}\int_{\underline{\varphi}}^{\bar{\varphi}}\int_{-\infty}^{\infty}\left(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}_1-\epsilon)-(\beta+\varphi)(X-\bar{X}_2-\epsilon)-\hat{S}\right)^2 f(\nu)g(\epsilon)h(\eta)k(\varphi)d\nu d\varphi d\eta d\epsilon \\ &= \int_{\bar{\epsilon}_2}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}_1-\epsilon)-\hat{S})^2 f(\nu)g(\epsilon)h(\eta)d\nu d\eta d\epsilon + \\ &\quad+\int_{\theta\in\Theta_2}\left(-2(\bar{S}+\nu-(\alpha+\eta)(X-\bar{X}_1-\epsilon)-\hat{S})(\beta+\varphi)(X-\bar{X}_2-\epsilon)+(\beta+\varphi)^2(X-\bar{X}_2-\epsilon)^2\right)\gamma(\theta)d\theta\end{aligned}$$

Combining the terms results in the objective function

$$\mathscr{L}_1 + \int_{\theta\in\Theta_2} \left(-2(\bar{S}+\nu-\hat{S})(\beta+\varphi)(X-\bar{X}_2-\epsilon)+2(\alpha+\eta)(\beta+\varphi)(X-\bar{X}_1-\epsilon)(X-\bar{X}_2-\epsilon)+(\beta+\varphi)^2(X-\bar{X}_2-\epsilon)^2\right)\gamma(\theta)d\theta$$
$$= \mathscr{L}_1 + \int_{\theta\in\Theta_2} \left(2(\hat{S}-\bar{S})\beta(X-\bar{X}_2-\epsilon)+2\alpha\beta(X-\bar{X}_1-\epsilon)(X-\bar{X}_2-\epsilon)+(\beta+\varphi)^2(X-\bar{X}_2-\epsilon)^2\right)\gamma(\theta)d\theta$$

First-order condition (as before, multiplied by 1/2 and integral for derivatives of boundaries in Leibniz rule are zero):

$$\begin{aligned}
&\frac{d\mathscr{L}_1}{dX} + \int_{\theta\in\Theta_2} (\beta(\hat{S}-\bar{S})+2\alpha\beta X^{***}-\alpha\beta(\bar{X}_1+\bar{X}_2)-2\alpha\beta\epsilon+(\beta+\varphi)^2(X^{***}-\bar{X}_2-\epsilon))\gamma(\theta)d\theta = 0\\
\Leftrightarrow&\frac{d\mathscr{L}_1}{dX} = -\int_{\theta\in\Theta_2}\left((2\alpha\beta+(\beta+\varphi)^2)X^{***}+\beta(\hat{S}-\bar{S})-\alpha\beta(\bar{X}_1+\bar{X}_2)-(2\alpha\beta+(\beta+\varphi)^2)\epsilon-(\beta+\varphi)^2\bar{X}_2\right)\gamma(\theta)d\theta\\
\Leftrightarrow&\frac{d\mathscr{L}_1}{dX} + (2\alpha\beta+\mathbb{E}[\beta+\varphi)^2])(1-G(\bar{\epsilon}_2))X^{***}\\
&\quad+\left(\beta(\hat{S}-\bar{S})-\alpha\beta(\bar{X}_1+\bar{X}_2)-\mathbb{E}[(\beta+\varphi)^2]\bar{X}_2\right)(1-G(\bar{\epsilon}_2))-(2\alpha\beta+\mathbb{E}[\beta+\varphi)^2])\int_{\bar{\epsilon}_2}^{\infty}\epsilon g(\epsilon)d\epsilon = 0
\end{aligned}$$

Using that $\int_{\bar{\epsilon}_2}^{\infty}\epsilon g(\epsilon)d\epsilon = \mathscr{G}(\bar{\epsilon}_2)-\bar{\epsilon}_2 G(\bar{\epsilon}_2)$ and $\bar{\epsilon}_2 = X^{***}-\bar{X}_2$, we get

$$(2\alpha\beta+\mathbb{E}[\beta+\varphi)^2])(1-G(\bar{\epsilon}_2))X^{***}+\left(\beta(\hat{S}-\bar{S})-\alpha\beta(\bar{X}_1+\bar{X}_2)-\mathbb{E}[(\beta+\varphi)^2]\bar{X}_2\right)(1-G(\bar{\epsilon}_2))+(2\alpha\beta+\mathbb{E}[(\beta+\varphi)^2])(\bar{\epsilon}_2G(\bar{\epsilon}_2)-\mathscr{G}(\bar{\epsilon}_2))$$
$$= (2\alpha\beta+\mathbb{E}[\beta+\varphi)^2])(X^{***}-\bar{X}_2)+\left(\beta(\hat{S}-\bar{S})-\alpha\beta(\bar{X}_1-\bar{X}_2)\right)(1-G(\bar{\epsilon}_2))-(2\alpha\beta+\mathbb{E}[(\beta+\varphi)^2])\mathscr{G}(\bar{\epsilon}_2)$$

Taken together, we get the combined first-order condition

$$(\alpha^2+\sigma_\eta^2+\lambda)(X^{***}-\bar{X}_1)+\alpha(1-G(\bar{\epsilon}_1))(\hat{S}-\bar{S})+\lambda(\bar{X}_1-\hat{X})-(\alpha^2+\sigma_\eta^2)\mathscr{G}(\bar{\epsilon}_1)+$$
$$+(2\alpha\beta+\beta^2+\sigma_\varphi^2)(X^{***}-\bar{X}_2)+\beta(1-G(\bar{\epsilon}_2))(\hat{S}-\bar{S})-\alpha\beta(1-G(\bar{\epsilon}_2))(\bar{X}_1-\bar{X}_2)-(2\alpha\beta+\beta^2+\sigma_\varphi^2)\mathscr{G}(\bar{\epsilon}_2)=0$$

which simplifies to

$$\begin{aligned}
X^{***} =& \frac{\alpha^2+\sigma_\eta^2+\lambda}{(\alpha+\beta)^2+\sigma_\eta^2+\sigma_\varphi^2)+\lambda}X^{**}+\\
&+\frac{\beta(1-G(\bar{\epsilon}_2))}{((\alpha+\beta)^2+\sigma_\eta^2+\sigma_\varphi^2)+\lambda}(\bar{S}+\alpha(\bar{X}_1-\bar{X}_2)-\hat{S})+\frac{2\alpha\beta+\beta^2+\sigma_\varphi^2}{((\alpha+\beta)^2+\sigma_\eta^2+\sigma_\varphi^2)+\lambda}(\bar{X}_2+\mathscr{G}(\bar{\epsilon}_2)).
\end{aligned}$$

$S = \bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) > \hat{S}$ is the spread at $X = \bar{X}_2$ with $\epsilon = 0$, $\nu = 0$, and $\eta = 0$. To show that $X^{***} > X^{**}$, we use that $\mathcal{G}(\bar{\epsilon}_2) \geq \bar{\epsilon}_2 = X^{***} - \bar{X}_2$. Consequently, $\mathcal{G}(\bar{\epsilon}_2) + \bar{X}_2 \geq X^{***}$ and thus the last term on the right-hand side can be replaced:

$$\begin{aligned} X^{***} \geq & \frac{\alpha^2 + \sigma_\eta^2 + \lambda}{(\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} X^{**} + \\ & + \frac{\beta(1 - G(\bar{\epsilon}_2))}{((\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} (\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}) + \frac{2\alpha\beta + \beta^2 + \sigma_\varphi^2}{((\alpha+\beta)^2 + \sigma_\eta^2 + \sigma_\varphi^2) + \lambda} X^{***}. \end{aligned}$$

Solving for X^{***} results in

$$X^{***} \geq X^{**} + \frac{\beta(1 - G(\bar{\epsilon}_2))}{\alpha^2 + \sigma_\eta^2 + \lambda} (\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}).$$

Figures

Figure 1: **Bank Reserves and Fraction of Days With No Change in Daily Spread.** The solid black line represents the percentage of days within 60-day rolling windows on which the change in the daily interest rate spread is zero. The spread is the difference between the daily volume-weighted median federal funds rate and a measure of the target rate; consistent with the changes in monetary policy implementation during the sample period, the target rate is measured as the federal funds target rate for 10/02/2006-12/15/2008, the interest rate on excess reserves for 12/16/2008-07/28/2021, and the interest rate on reserves balances for 07/29/2021-10/22/2023. The 60-business-day windows (roughly a quarter) roll every 20 business days (roughly a month). The gray shaded area shows central bank reserves relative to commercial banks' assets. Data are from October 2006 to October 2023. The daily volume-weighted median federal funds rate is publicly available from Federal Reserve Bank of New York (10/2/2006-02/29/2016) and from the Federal Reserve Economic Data, FRED ("EFFR," 03/01/2016-10/23/2023). The daily federal funds target rate, interest rate on excess reserve, and interest rate on reserves balances are available from FRED ("DFEDTAR," "IOER," and "IORB"). Weekly data on reserves and total assets of U.S. commercial banks and U.S. branches and agencies of foreign banks are available from FRED ("WRESBAL" and "TLAACBW027SBOG").

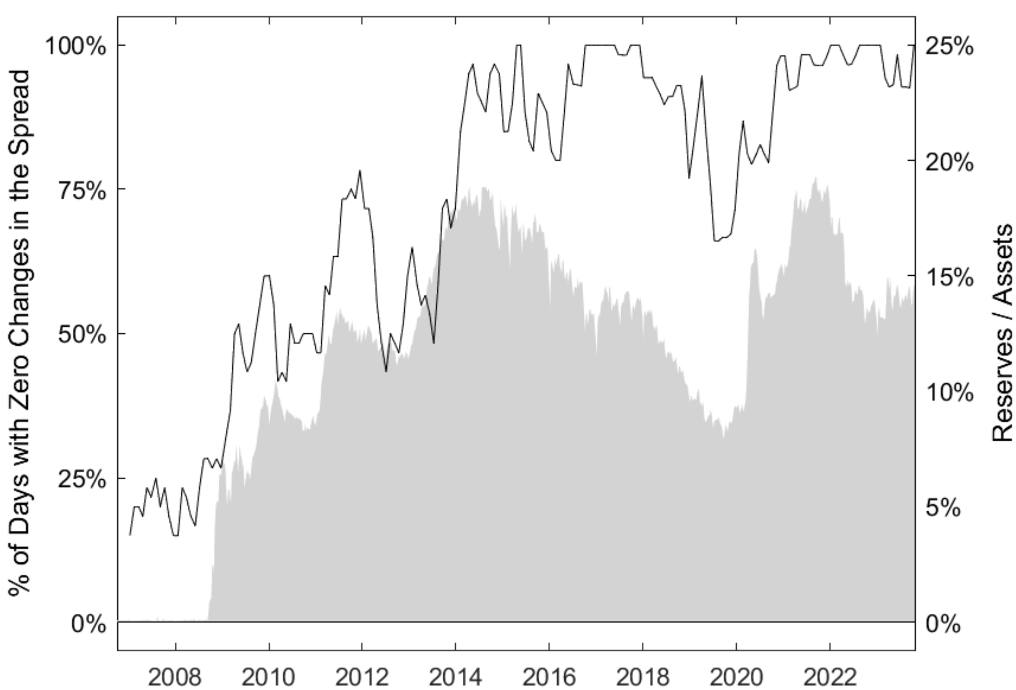
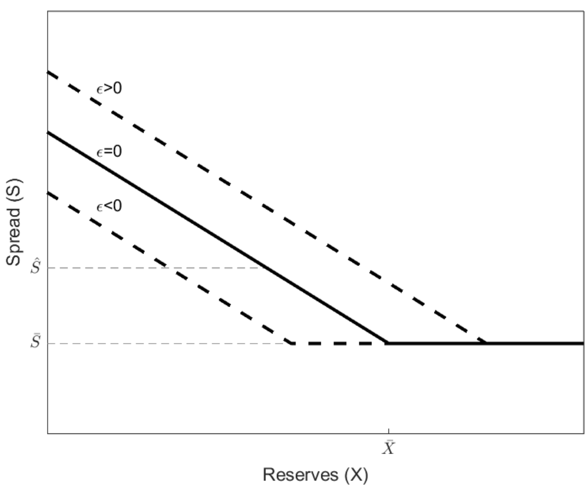
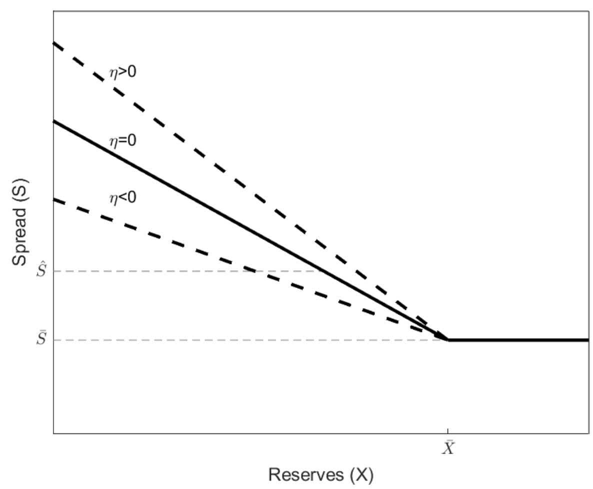


Figure 2: **A Graphical Representation of Uncertainty about the Demand for Reserves.** These figures show the effects of different types of uncertainty on the demand for reserves in equation (1). Panel (a) shows the effect of uncertainty about the location of the kink $\bar{X}$ (i.e., point of demand satiation), represented by the ϵ shock; panel (b) shows the effect of uncertainty about the slope in the ample reserves region α, represented by the η shock; and panel (c) shows the effect of uncertainty about the floor of the demand curve $\bar{S}$, represented by the ν shock.

(a) Uncertainty about the location of the kink (ϵ shock)



(b) Uncertainty about the slope (η shock)



(c) Uncertainty about the floor (ν shock)

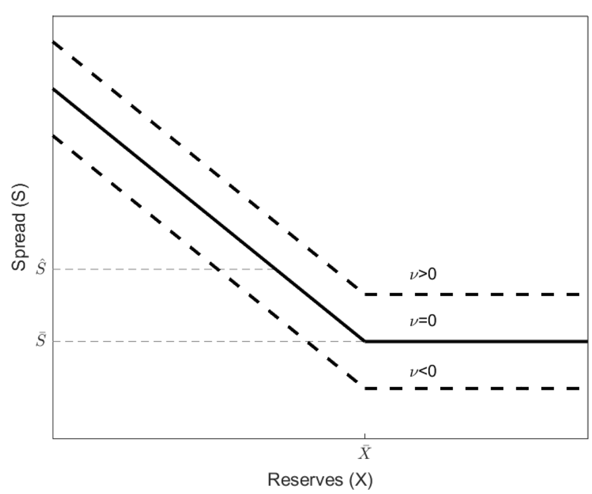


Figure 3: **Expected Interest Rate Spread and Optimal Reserves.** This figure shows the expected spread between the market-implied policy rate and the interest rate on reserve balances, both absent uncertainty and in the presence of different types of uncertainty about the reserve demand curve. The black line represents the deterministic case as well as the case with uncertainty about the slope in the ample reserves region (η shocks), but without uncertainty about the location of the kink between ample and abundant reserves (i.e., the satiation point; ϵ shocks). The red line represents the case with uncertainty about the location of the kink. The squares represent the expected spreads at the optimal reserve levels in the different specifications: black for the deterministic case, blue for the case with uncertainty about the slope but without uncertainty about the location of the kink, red for the case with uncertainty about the location of the kink but without uncertainty about the slope, and green for the case with uncertainty about both the location of the kink and the slope. In these figures, the shocks are assumed to be independent, normally distributed, and with zero mean. Note that the expected spread and optimal reserves when there is only uncertainty about the floor of the demand curve (ν shocks) are the same as in the deterministic case.

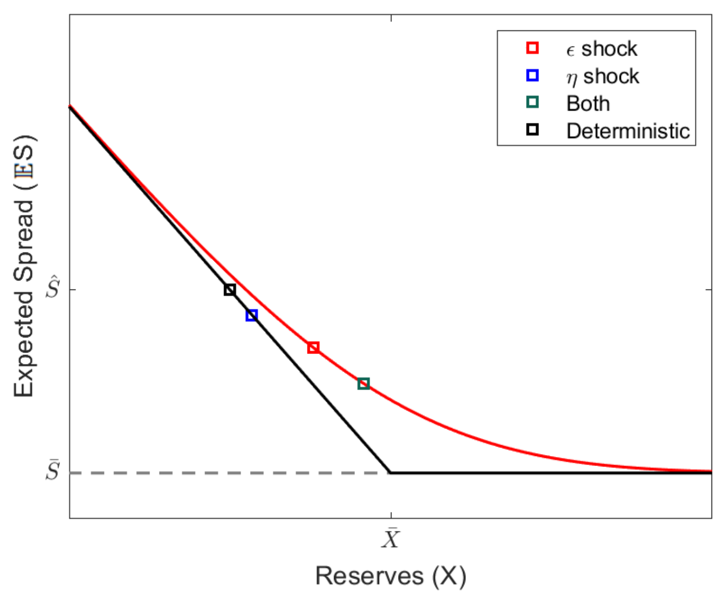


Figure 4: **Variability of the Interest Rate Spread and Optimal Reserves.** This figure shows the standard deviation of the spread between the market-implied policy rate and the interest rate on reserve balances, both absent uncertainty and in the presence of different types of uncertainty about the reserve demand curve. The black line is for the deterministic case; the blue line is for the case with uncertainty about the slope in the ample reserves region (η shocks) but without uncertainty about the location of the kink between ample and abundant reserves (i.e., the satiation point; ϵ shocks); the red line is for the case with uncertainty about the location of the kink but without uncertainty about the slope; and the green line is for the case with uncertainty about both the location of the kink and the slope. The squares represent the spread standard deviations at the optimal reserve levels in the different specifications. In these figures, the shocks are independent, normally distributed, and with zero mean. Note that, when there is only uncertainty about the floor of the demand curve (ν shocks), the standard deviation of the spread is a positive constant for all reserve values (i.e., a vertical upward shift relative to the deterministic case), and the corresponding optimal reserve level is the same as in the deterministic case.