Open Market Operations[*]

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Abstract

Standard monetary theory is extended to incorporate liquid government bonds in addition to currency. This allows us to study different monetary policies, including OMO's (open market operations). We consider various specifications for market structure, and for the liquidity of money and bonds – i.e., their acceptability or pledgeability as media of exchange or collateral. Theory delivers sharp predictions, even when there are multiple equilibria, and can generate novel phenomena like negative nominal interest rates, endogenous market segmentation, liquidity traps and the appearance of sluggish nominal prices. Importantly, we show how to explain differences in asset acceptability or pledgeability using information frictions.

"Look at Rothschild – you'd think he was selling apples instead of government bonds." *The House of Rothschild* (1934 film)

1   Introduction

Monetary policy is believed by many to have important effects on the economy. Broadly speaking, governments issue two categories of paper, fiat currency and bonds, although there are subcategories, including currencies of different denominations and real or nominal bonds with different maturities. Monetary policy consists of controlling the amounts of these objects outstanding, or their growth rates, often in an attempt to target some nominal interest or inflation rate. There are different ways to change the supply of government-issued assets held by the public, including transfers and spending on goods or other assets. The conventional policy used to alter the mix is to buy or sell bonds for cash – an *open market operation*, or OMO. This project studies the effects of these kinds of policies through the lens of the New Monetarist framework.[1]

In this framework, at some points in time agents trade with each other in decentralized markets, as in search theory, while other times they trade in more centralized markets, as in general equilibrium theory. When they trade with each other, frictions in the environment make it interesting to ask how they trade: Do they use barter, credit or media of exchange? If they use credit, is it unsecured or secured? Which assets serve as media of exchange or constitute acceptable collateral? We spend some time analyzing why different assets, such as currency or bonds, may be more or less acceptable as media of exchange or pledgeable as collateral – i.e., why they may be more or less liquid. Both for cases where liquidity is exogenous and where it is derived endogenously, we

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[1]Recent expositions of this literature include Williamson and Wright (2010), Wallace (2010), Nosal and Rocheteau (2011) and Lagos et al. (2014). Our use of the approach in this exercise means that we do not impose sticky prices. While these models can accommodate or even endogenize nominal rigidities, there is a belief that we do not need this for interesting policy analyses. Still, as discussed below, prices here can *appear* sticky.

analyze what happens when monetary policy changes under various scenarios for market structure, including random or directed search, and including cases where the terms of trade are determined by bargaining or by posting.

One policy instrument is the money supply, the growth rate of which equals inflation in stationary equilibrium. While the growth rate matters, with flexible prices, the level of the money supply does not, so OMO's are effectively the same as changing the stock of outstanding bonds. Theory delivers sharp predictions for these effects, and novel phenomena including *negative nominal interest rates*, endogenous *market segmentation*, outcomes resembling *liquidity traps*, and the appearance of *sluggish nominal prices*. For some parameters, injecting cash by buying bonds reduces nominal bond returns and stimulates output through a straightforward but previously neglected channel. The effects do not follow from expanding the money supply, but from contracting the bond supply. Sluggish prices emerge as follows: Increasing the money supply has the direct effect of lowering the value of money so that real balances remain constant – classical neutrality. But having fewer bonds makes agents try to substitute into other forms of liquidity, which raises real balances. Hence, the quantity equation does not hold, as the value of money falls by less than the increase in supply.

For other parameters, the economy can get into circumstances where output is very low, OMO's have no impact, and nominal bond returns freeze at their lower bound. This is a liquidity trap, defined by Wikipedia (as good a source as any) to be a situation where "injections of cash ... by a central bank fail to decrease interest rates and hence make monetary policy ineffective" – exactly what happens here.[2] However, our lower bound for the nominal rate can be zero, positive or negative. While a lower bound of zero almost always appears in theory, negative nominal returns do occur in practice, and should be no surprise

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[2]If Wikipedia is not scholarly enough, consider (Keynes 1936): "after the rate of interest has fallen to a certain level, liquidity-preference may become virtually absolute in the sense that almost everyone prefers cash to holding a debt which yields so low a rate of interest. In this event the monetary authority would have lost effective control over the rate of interest."

when assets provide services in addition to returns. As a leading example, with travellers' checks the service is insurance against loss or theft. Here it is liquidity, not insurance, that is center stage, but in any case, once we explicitly model the role of assets in transactions negative nominal yields are possible.[3]

As regards market segmentation, there is a prior literature discussed in Section 6. For now, suffice it to say that our approach is quite different, and based on methods from search theory. However, as is well understood in modern monetary economics, search *per se* is not an essential ingredient; it is used here for convenience, and because it seems a natural way to think about market segmentation where buyers choose to visit sellers that accept particular payment instruments. Of course, it is important to ask why sellers might treat different assets differently. We follow a body of work (references given below) that has some agents less able to discern legitimate from fraudulent versions of certain assets. This means the agents may reject those assets outright, or accept them only up to endogenous thresholds. Now liquidity is not invariant to changes in policy, and it is easy to get multiple equilibria; still, theory delivers sharp predictions about the effects of inflation and OMO's.

To summarize the motivation, first, we are interested in the impact of monetary policy. While the effects of anticipated inflation are well understood, we think the effects of one-time monetary injections are actually not. A common modeling device is to inject cash by lump-sum transfers, which is classically neutral: the value of money falls in proportion while quantities, relative prices and real rates of return stay the same. This is somewhat uninteresting, and contrary to what some people think they see in reality. Therefore, it seems relevant to entertain alternative ways of increasing the money supply. An obvious alternative

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[3]For recent discussions of negative nominal rates in practice, see *Wall Street Journal* (Aug. 10, 2012), *The Economist* (July 14, 2014) and *NY Times* (December 18, 2014). By way of examples, the nominal yield on T-bills currently hovers around 0 in the US, the current interest rate on reserves at the ECB is −0.1%, nominal German bond yields are negative out to 3 years, and a similar situation exists in Switzerland, as discussed more below.

is to have central banks buy assets – maybe short or long government bonds, or even private securities – as they have been doing in a big way in recent years. A natural place to start is to have the authorities buy T-bills, a conventional OMO. Injecting currency in this way, it turns out, can lower real bond returns, can affect quantities in various ways, and, due to substitution between alternative sources of liquidity, can cause real balances to rise. To analyze this rigorously, we believe it is important to model liquidity in detail, and that is the object of the exercise.

Section 2 describes the environment. Section 3 considers equilibrium with random matching and bargaining taking liquidity as given. Section 4 endogenizes acceptability and pledgeability based on asymmetric information. Section 5 considers directed search and segmentation. Section 7 concludes.

2 Environment

Except for having bonds, the background model is standard (see the surveys in fn. 1 for references, but the basic assumptions come from Lagos and Wright 2005 and Rocheteau and Wright 2005). Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. Each period in the CM, a large number of infinitely-lived agents work, consume and adjust their portfolios. In the DM, some of these agents, called *sellers*, can produce a good different from the CM good, but do not want to consume, while others, called *buyers*, cannot produce but would like to consume the good. Generally, μ is the measure of buyers, and n is the ratio of the sellers to buyers in the DM, where they meet pairwise, with α denoting the probability a buyer meets a seller and α/n the probability a seller meets a buyer.

The within-period payoffs for buyers and sellers are given

$$\mathcal{U}(q, x, \ell) = u(q) + U(x) - \ell \text{ and } \widetilde{\mathcal{U}}(q, x, \ell) = -c(q) + U(x) - \ell, \qquad (1)$$

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where q is the DM good, x is the CM numeraire good and ℓ is labor supply. For sellers, $c(q)$ is a cost of production. For buyers, one can interpret $u(q)$ as the utility of consuming q, or as a production function taking q as an input and delivering output $u(q)$ in numeraire in the next CM (this follows because, as shown below, CM payoffs are linear in numeraire). The same formalization can therefore represent DM transactions as consumers acquiring output q, or producers investing in input q, which is relevant to the extent that liquidity considerations impinge on both households and firms.[4]

As usual, U, u and c are twice continuously differentiable with $U' > 0$, $u' > 0$, $c' > 0$, $U'' < 0$, $u'' < 0$ and $c'' \geq 0$. Also, $u(0) = c(0) = 0$, and there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q}) > 0$. Define the efficient q by $u'(q^*) = c'(q^*)$. Quasi-linearity in (1) simplifies the analysis because it leads to a degenerate distribution of assets across agents of a given type at the start of each DM, and makes CM payoffs linear in wealth.[5] There is a discount factor $\beta = 1/(1+r)$, $r > 0$, between the CM and DM, while any discounting between the DM and CM can be subsumed in the notation in (1). It is assumed that x and q are nonstorable, to rule out direct barter, and that agents are to some degree anonymous in the DM, to hinder unsecured credit. This is what generates a role for assets in the facilitation of intertemporal exchange.

There are two assets that can potentially serve in this capacity, money, and government bonds called T-bills. Their supplies are A_m and A_b. Their CM prices are ϕ_m and ϕ_b. The benchmark specification has short-term real bonds that,

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[4]To think about it as a market for reallocating investment goods, imagine everyone in the DM is a producer with capital k. They realize a shock making their production function either $f(k)$ or $g(k)$ with $g'(k) > f'(k)$. In meetings between low- and high-productivity producers, if the former gives q units of k to the latter, we can write $u(q) = g(q+k) - g(k)$ and $c(q) = f(k) - f(k-q)$ and keep everything else in the model the same.

[5]Wong (2012) shows the same results obtain for any $\hat{U}(x, 1-\ell)$ with $\hat{U}_{11}\hat{U}_{22} = \hat{U}_{12}^2$, which holds for quasi-linear utility, but also any $\hat{U}$ that is homogeneous of degree 1, including $\hat{U} = x^a(1-\ell)^{1-a}$ or $\hat{U} = [x^a + (1-\ell)^a]^{1/a}$. Alternatively, Rocheteau et al. (2008) show these same results obain for any $\tilde{U}$ if we incorporate indivisible labor à la Rogerson (1988). In any case, the implicit constraints $x \geq 0$, $q \geq 0$ and $\ell \in [0,1]$ are assumed slack.

somewhat like Arrow securities, are issued in one CM and pay 1 unit of numeraire in the next, but we show below how to accommodate nominal and long-term bonds. The real value of money and bonds per buyer are z_m and z_b. For money, $z_m = \phi_m A_m$; for real bonds, $z_b = A_b$; for nominal bonds, as discussed below, $z_b = \phi_m A_b$; and for long-term bonds, $z_b = (\phi_b + \gamma) A_b$ where γ is a real coupon paid each CM. These assets are *partially liquid*, in the sense that they are accepted in some DM transactions, at least up to some limit. There are two standard interpretations. Following Kiyotaki and Wright (1989,1993), sellers may accept some assets as media of exchange (immediate settlement). Following Kiyotaki and Moore (1997,2005), they may accept some assets as collateral securing promises of numeraire in the next CM (deferred settlement), with the idea being that those who renege on promises are punished by having assets seized.

A third interpretation concerns repos (repurchase agreements): buyers in the DM give assets to sellers, who give them back in the CM at a prearranged price. Of course, it may not be important to give back the same assets or prearrange a price when trading fungible assets in frictionless markets. Still, through repos , assets aid in intertemporal trade. As the results hold under all these interpretations, we are agnostic about the institutional niceties. To be clear, the point is not that there is anything deep about this discussion of settlement, collateral and repos; it is rather that different stories apply more or less interchangeably to the same formalization. In any case, only a fraction $\chi_j \in [0,1]$ of asset j can be used in the DM, either as a payment instrument or collateral, where $\chi_m > 0$ so that currency can be valued, and $\chi_b > 0$ so this does not collapse to a pure-currency economy. Under the deferred settlement interpretation, the pledgeability parameter χ_b describes the *haircut* one takes when using z_j as collateral, often motivated by saying debtors can abscond with a fraction $1 - \chi_j$ of an asset.[6]

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[6]Section 4 endogenizes χ_j by introducing counterfeiting. In particular, under the deferred settlement interpretation, and thinking of money broadly to include demand deposits, counterfeiting includes *bad checks*. While $\chi_j < 1$ is not critical, and most results go through with $\chi_j = 1$, there is no reason to impose that restriction at this point.

In the DM, α_m is the probability a buyer meets a seller that accepts only money; α_b is the probability he meets one that accepts only bonds; and α_2 is the probability he meets one that accepts both. Below we sometimes use $\alpha_j = \alpha n_j$ where n_j is the fraction of type-j sellers. Special cases include ones where everyone accepts cash, $\alpha_b = 0$; the assets are perfect substitutes, $\alpha_b = \alpha_m = 0$; and something that 'looks like' a CIA model, $\alpha_b = \alpha_2 = 0$.[7] Under the deferred settlement interpretation, since agents renege iff debt exceeds the value of collateral, trades are constrained by asset holdings just like they are in immediate settlement. To streamline the presentation we usually frame the discussion in terms of media of exchange, but it is good to keep in mind that it is basically a relabeling to switch between Kiyotaki-Wright money and Kiyotaki-Moore credit.

We focus on stationarity equilibria, in which $z_m = \phi_m A_m$ is constant, so the money growth rate π equals the inflation rate: $\phi_m/\phi_{m,+1} = A_{m,+1}/A_m = 1 + \pi$, where subscript $+1$ indicates next period. Stationarity also entails z_b constant, which means A_b is constant for real bonds, while the ratio $B = A_b/A_m$ is constant for nominal bonds. We restrict attention to $\pi > \beta - 1$, or the limit $\pi \to \beta - 1$, which is the Friedman rule (there is no monetary equilibrium with $\pi < \beta - 1$). The government (a consolidated monetary and fiscal authority) has the budget constraint

$$G + T - \pi \phi_m A_m + A_b(1 - \phi_b) = 0, \tag{2}$$

where the first term is their consumption of x, the second is a lump-sum transfer, the third is seigniorage, and the fourth is debt service.

It is crucial to distinguish between different interest rates. Define the return on an *illiquid nominal bond* – one that is never accepted in the DM – by the Fisher equation, $1 + \iota = (1 + \pi)/\beta$, where $1/\beta = 1 + r$ is the return on an *illiquid*

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[7]To motivate $\alpha_b > 0$, while it is true that few retailers take bonds and not currency, note that financial institutions regularly use government securities as collateral. Aleks Berentsen gave us the example of institutions that use Swiss bonds as collateral, where francs would not work (see also fn. 11). Just like $\chi_j < 1$, $\alpha_b > 0$ is not critical, and often we use the special case $\alpha_b = 0$ below, but there is no reason to impose that at this point.

real bond.[8] For liquid bonds the nominal yield is ρ. For a real liquid bond, we compute the nominal yield as the amount of cash you can get in the next CM by investing a dollar today, $1 + \rho = \phi_m / \phi_b \phi_{m,+1} = (1 + \pi) / \phi_b$. As in Silveira and Wright (2010) or Rocheteau and Rodriguez-Lopez (2014), it is convenient to define the *spread* by $s = (\iota - \rho) / (1 + \rho)$. In terms of economics, ι is the cost of the liquidity services provided by z_m, because rather than holding cash one could make an illiquid investment. Similarly, s is the cost of the liquidity services provided by z_b, because liquid bonds yield ρ while illiquid bonds yield ι.

We generally use ι as a policy instrument, although one can take this to be short-hand notation for inflation, by the Fisher equation $1 + \pi = \beta (1 + \iota)$. Note also that the Friedman rule is equivalent to $\iota = 0$. One policy experiment concerns a permanent change in ι. The other experiment, an OMO, concerns a change in A_b accompanied by a change in A_m to satisfy (2) at a point in time. In subsequent periods, the lump-sum tax T adjusts to satisfy (2). The change in the stock of bonds on the Central Bank's balance sheet is permanent, and the growth rate of the money supply π stays constant, with changes in interest payments covered by the lump-sum tax T. Given this, and the fact that policy changes are unanticipated, there are no transitional dynamics in the model, because there are is slowly-adjusting state variable, like capital. Hence, after a policy change the economy can go from one stationary state directly to another.

3   Random Matching

Our first specification involves random search and bargaining, because it is well understood in this literature. We first consider short-term real bonds, then discuss alternative forms of government debt.

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[8]Thus, $1 + \iota$ is the amount of cash required in the next CM to make you indifferent to giving up a dollar in this CM, while $1 + r$ is the amount of x required to make you indifferent to giving up a unit of x. Whether or not these trades are actually made, we can price them. As usual, the Fisher equation is simply a no-arbitrage condition for illiquid bonds.

3.1 Baseline: Short Real Bonds

A buyer's DM state is his portfolio (z_m, z_b), while in the CM all that matters is the sum $z = z_m + z_b$. Let the CM and DM value functions be denoted $W(z)$ and $V(z_m, z_b)$. Then

$$W(z) = \max_{x, \ell, \hat{z}_m, \hat{z}_b} \{U(x) - \ell + \beta V(\hat{z}_m, \hat{z}_b)\} \text{ st } x = z + \ell + T - (1+\pi)\hat{z}_m - \phi_b \hat{z}_b$$

where $\hat{z}_j$ is the real value of asset j taken out of the CM, and the real wage is $\omega = 1$ because we assume that 1 unit of ℓ produces 1 unit of x (this is easy to generalize). The FOC's for assets are $1 + \pi = \beta V_1(\hat{z}_m, \hat{z}_b)$ and $\phi_b = \beta V_2(\hat{z}_m, \hat{z}_b)$, while the envelope condition is $W'(z) = 1$.[9] In the DM

$$V(z_m, z_b) = W(z_m + z_b) + \alpha_m[u(q_m) - p_m] + \alpha_b[u(q_b) - p_b] + \alpha_2[u(q_2) - p_2]$$

where p_j are payments in type-j meetings and we use $W'(z) = 1$. Payments are constrained by $p_j \le \bar{p}_j$, where $\bar{p}_j$ is the buyer's *liquidity position* in a type-j meeting: $\bar{p}_m = \chi_m z_m$, $\bar{p}_b = \chi_b z_b$ and $\bar{p}_2 = \chi_m z_m + \chi_b z_b$.

In the baseline model the terms of trade are determined by bargaining: to get q you pay $p = v(q)$, where $v(\cdot)$ depends on the solution concept. Kalai's proportional solution, e.g., implies $v(q) = \theta c(q) + (1 - \theta) u(q)$, where θ is the buyer's bargaining power. However, other than $v(0) = 0$ and $v'(q) > 0$, all we need is the following: Let $p^* = v(q^*)$ be the payment required to get the efficient amount. Then $p^* \le \bar{p}_j \implies p_j = p^*$ and $q_j = q^*$, while $p^* > \bar{p}_j \implies p_j = \bar{p}_j$ and $q_j = v^{-1}(\bar{p}_j)$. This holds for Kalai and Nash bargaining (for the record, we prefer Kalai for the reasons discussed in Aruoba et al. 2007). It also holds for creatively designed bilateral trading mechanisms (Hu et al. 2009; Gu et al. 2014), and for Walrasian pricing (Rocheteau and Wright 2005) or auctions (Galenianos

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[9] There is a similar CM problem for sellers, but we can impose without loss in generality that they carry no assets into the DM: if assets are priced fundamentally, they are indifferent to carrying them; if assets bear a liquidity premium, they strictly prefer not carrying them. Hence the presentation downplays sellers, but note their marginal value of numeraire is also 1.

and Kircher 2008) when agents in the DM trade multilaterally. For now we use the generic form $p = v(q)$.

As is standard, $\iota > 0$ implies buyers pay all they can in type-m meetings and and are still constrained: $p_m = \chi_m z_m < p^*$. Since $\chi_m z_m < p^*$, in type-2 meetings buyers may as well pay all they can in cash before using bonds, because in these meetings agents are indifferent to any combination of z_m and z_b. Buyers use all the bonds they can in type-2 meetings iff $\bar{p}_2 \leq p^*$, and use all they bonds they can in type-b meetings iff $\bar{p}_b \leq p^*$. It is clear that $p_2 \geq p_b$. What must be determined is whether: 1. $p_2 = \bar{p}_2$ and $p_b = \bar{p}_b$ (buyers are constrained in all meetings); 2. $p_2 < \bar{p}_2$ and $p_b = \bar{p}_b$ (they are constrained in type-b but not type-2 meetings); or 3. $p_2 < \bar{p}_2$ and $p_b < \bar{p}_b$ (they are unconstrained in both). We consider each case in turn, assuming throughout that a monetary equilibrium exists.[10]

Case 1 is the most interesting, with buyers constrained in all meetings:

$$v(q_m) = \chi_m z_m, \ v(q_b) = \chi_b z_b, \text{ and } v(q_2) = \chi_m z_m + \chi_b z_b. \tag{3}$$

The Euler equations are derived by differentiating $V(z_m, z_b)$ using (3) and inserting the results into the FOC's for assets in the CM. The results are

$$1 + \pi = \beta \left[1 + \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2) \right] \tag{4}$$

$$\phi_b = \beta \left[1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2) \right], \tag{5}$$

where $\lambda(q_j) \equiv u'(q_j)/v'(q_j) - 1$ is the *liquidity premium* in a type-j meeting, i.e., the Lagrange multiplier on $p_j \leq \bar{p}_j$. If $\chi_b = 0$, bonds would be priced *fundamentally* at $\phi_b = 1/\beta$. In any case, (4)-(5) simplify to

$$\iota = \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2) \tag{6}$$

$$s = \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2), \tag{7}$$

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[10]It is routine to show $\alpha_m > 0$ implies monetary equilibrium exists iff $\iota < \bar{\iota}_m$, while $\alpha_m = 0$ implies it exists iff $\alpha_2 > 0$, $\chi_b A_b < p^*$ and $\iota < \bar{\iota}_2$, where $\bar{\iota}_m$ and $\bar{\iota}_2$ may or may not be finite.

using the nominal rate on illiquid bonds ι and the spread s defined above.

Conditions like (6) are standard in monetary economics. The LHS is ι the cost of holding currency, the return on illiquid assets, and the RHS is the benefit: with probability α_m a buyer is in a situation where relaxing the constraint $p_m \leq \bar{p}_m$ is worth $\lambda(q_m)$; with probability α_2 he is in a situation where relaxing the constraint $p_2 \leq \bar{p}_2$ is worth $\lambda(q_2)$; and in either case he can use a fraction χ_m of z_m in payment. Condition (7) is similar: s is the cost of holding liquid bonds, and the RHS is the benefit, similar to the RHS of (6). Recent work by Krishnamurphy and Vissing-Jorgenson (2012) documents that conditions such as (7) are empirically relevant, where s is measured by the difference between government and corporate bond yields. They take a reduced-form approach, inserting T-bills into utility functions, as a way to capture what they call a *convenience yield*. While we try to model assets' role in the exchange process in more detail, the spirit is similar. Additional evidence and discussion is contained in Nagel (2014). On the whole, we think that these papers provide considerable support for the class of models studied here.

Given $1 + \rho = (1 + \pi)/\phi_b$, (4)-(5) immediately imply

$$\rho = \frac{\alpha_m \chi_m \lambda(q_m) - \alpha_b \chi_b \lambda(q_b) + (\chi_m - \chi_b)\alpha_2 \lambda(q_2)}{1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)}. \tag{8}$$

In contrast to conventional theory, we can get $\rho < 0$, in two ways: if $\chi_m = \chi_b$ then $\rho < 0$ iff $\alpha_m \lambda(q_m) < \alpha_b \lambda(q_b)$; and if $\alpha_m \lambda(q_m) = \alpha_b \lambda(q_b)$ then $\rho < 0$ iff $\chi_m < \chi_b$ and $\alpha_2 \lambda(q_2) > 0$. Related models by Williamson (2012,2014a,b), Rocheteau and Rodriguez-Lopez (2014), Dong and Xiao (2014) and Han (2014) have $\alpha_b = 0$ and $\chi_b = \chi_m = 1$, making $\rho < 0$ impossible. Our generalizations describe logically how to get $\rho < 0$, and according to *The Economist* (July 14, 2014) the logic may well be relevant: "Not all Treasury securities are equal; some are more attractive for repo financing than others. With less liquidity in the market, those desirable Treasuries can be hard to find: some short-term debt can trade on a negative

yield because they are so sought after." This is exactly what the theory is trying to capture.[11]

Note that $\rho < 0$ does not defy standard no-arbitrage conditions, because while agents can issue bonds – i.e., borrow – they cannot guarantee claims against them will be liquid – i.e., circulate in the DM. This is similar to agents accepting negative nominal returns on travellers' checks, or demand deposits if we count fees, that are less susceptible than cash to loss and theft. That does not violate no-arbitrage if agents cannot guarantee the security of their paper without incurring some cost, as is presumably incurred with travellers' checks or deposit banking. In He et al. (2008) or Sanches and Williamson (2010) where cash is subject to theft, the lower bound can be negative for this reason. In Andolfatto (2013) it can be positive because imperfect enforcement limits government's ability to tax and deflate. Here it is purely liquidity considerations at work.

A stationary monetary equilibrium is a list (q_m, q_b, q_2, z_m, s) solving (3)-(7) with $z_m > 0$ and $z_b = A_b$. To characterize it, use (3) to rewrite (6) as

$$\iota = \alpha_m \chi_m L \left(\chi_m z_m \right) + \alpha_2 \chi_m L \left(\chi_m z_m + \chi_b z_b \right), \tag{9}$$

where $L \left(\cdot \right) \equiv \lambda \circ v^{-1} \left(\cdot \right)$. Given $z_b = A_b$, under standard assumptions (see fn. 10), a solution $z_m > 0$ to (9) exists, is generically unique, and entails $L' \left(\cdot \right) < 0$, by the argument in Wright (2010). From z_m, (3) determines (q_m, q_b, q_2). Then (7) determines s, (8) determines ρ, etc. Clearly, a one-time change in A_m is neutral, because ϕ_m adjusts to leave $z_m = \phi_m A_m$ and other real variables the same. Hence, an OMO that swaps A_b for A_m has the same real impact as changing only A_b, assuming changes in the CM price level $1/\phi_m$ are not artificially impeded. Still, as we soon show, even with flexible prices the quantity equation may not hold.

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[11]Relatedly, the Swiss National Bank (2013) says: "With money market rates persistently low and Swiss franc liquidity still high, trading activity on the repo market remained very slight ... [but] the secured money market did not grind to a complete halt, due to the demand for high-quality securities. The increased importance of these securities is reflected in the trades on the interbank repo market which were concluded at negative repo rates."

To save space, write $L'_m = L'(\chi_m z_m)$, $L'_2 = L'(\chi_m z_m + \chi_b z_b)$ etc. Then $\partial z_m/\partial\iota = 1/D_R < 0$, where $D_R \equiv \alpha_m \chi_m^2 L'_m + \alpha_2 \chi_m^2 L'_2 < 0$, so as usual a higher nominal rate on illiquid bonds (or inflation or money growth) reduces real balances. In terms of quantities,

$$\frac{\partial q_m}{\partial\iota} = \frac{\chi_m}{v'_m D_R} < 0, \ \frac{\partial q_b}{\partial\iota} = 0 \ \text{and} \ \frac{\partial q_2}{\partial\iota} = \frac{\chi_m}{v'_2 D_R} < 0,$$

where $v'_m = v'(q_m)$ etc. In terms of financial variables,

$$\frac{\partial s}{\partial\iota} = \frac{\alpha_2 \chi_m \chi_b L'_2}{D_R} > 0$$
$$\frac{\partial \phi_b}{\partial\iota} = \beta\frac{\alpha_2 \chi_m \chi_b L'_2}{D_R} > 0$$
$$\frac{\partial \rho}{\partial\iota} = \frac{\alpha_m L'_m + \alpha_2 [1 - (1+\rho)\chi_b/\chi_m] L'_2}{(1+s)(\alpha_m L'_m + \alpha_2 L'_2)} \gtrless 0,$$

assuming $\alpha_2 > 0$. If $\alpha_2 = 0$ then $\partial s/\partial\iota = \partial\phi_b/\partial\iota = 0$, since $\alpha_2 = 0$ means there is no substitution between z_m and z_b in DM meetings, but if $\alpha_2 > 0$ then higher ι raises s and ϕ_b as agents try to move out of cash and into bonds. The one ambiguous effect is $\partial\rho/\partial\iota$, which is as it should be: intuitively, inflation raises nominal bond returns by the Fisher (1930) effect, but lowers real returns by the Mundell (1963) effect.

For OMO's, assuming $\alpha_2 > 0$, increasing the stock of outstanding bonds reduces real balances: $\partial z_m/\partial A_b = -\alpha_2 \chi_m \chi_b L'_2/D_R < 0$. In terms of quantities,

$$\frac{\partial q_m}{\partial A_b} = -\frac{\alpha_2 \chi_b L'_2}{v'_m D_R} < 0, \ \frac{\partial q_b}{\partial A_b} = \frac{\chi_b}{v'_b} > 0 \ \text{and} \ \frac{\partial q_2}{\partial A_b} = \frac{\alpha_m \chi_b L'_m}{v'_2 D_R} > 0,$$

Higher A_b decreases z_m and q_m because liquidity is less scarce in type-2 meetings, so agents economize on cash, which comes back to haunt them in type-m meetings. Hence OMO's have different effects on q in different meetings, and an ambiguous impact on DM output $\Sigma_j\alpha_j q_j$. Given $\chi_b > 0$, one can also check $\partial s/\partial A_b < 0$, $\partial\phi_b/\partial A_b < 0$ and $\partial\rho/\partial A_b > 0$.

While Case 1 is the most interesting, for completeness, consider Case 2 with buyers unconstrained in type-2 meetings. The equilibrium conditions are similar except $q_2 = q^*$, so $\lambda(q_2) = 0$ and z_m is determined as in a pure-currency economy. An increase in ι lowers z_m and q_m, does not affect q_b or q_2, and increases s as agents again try to shift from z_m to z_b. An increase in A_b does not affect z_m, q_m or q_2, increases q_b and decreases s. Similarly, in Case 3 with buyers unconstrained in type-2 and type-b meetings, $q_b = q_2 = q^*$. Now bonds provide no liquidity at the margin, so $s = 0$. Hence, increases in ι reduce z_m and q_m but otherwise affect nothing, while increases in A_b affect nothing.
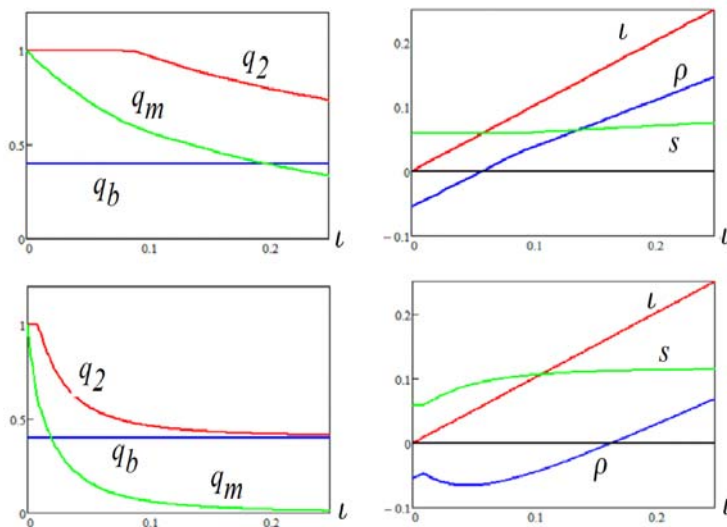

Figure 1: Effects of ι (nominal illiquid interest rate)

Which case obtains? If $\chi_b A_b \geq v(q^*)$ (bonds are abundant) it is Case 3. If $\chi_b A_b < v(q^*)$ (bonds are scarce) it is Case 2 when ι is small and Case 1 when ι is big. In Figure 1, Case 1 (Case 2) obtains to the left (right) of the ι at which the curves kink.[12] Effects to note are: q_m can be above or below q_b; ρ can be negative; and ρ can be nonmonotone in ι. Figure 2 shows the impact of changing

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[12] Figure 1 uses $u(q) = 2\sqrt{q}$, $c(q) = q$, $v(q) = c(q)$ (buyer-take-all bargaining), $\beta = 0.95$, $A_b = 0.4$, $\alpha_m = 0.3$, $\alpha_2 = 0.2$ and $\alpha_b = 0.1$. The upper panels use $\chi_m = \chi_b = 1$; the lower panels use $\chi_m = 0.1$ and $\chi_b = 1$. Figure 2 is similar but $\chi_m = \chi_b = 1$ and $i = 0.04$. These are not meant to be realistic, only to illustrate possibilities.

A_b, where Case 1 obtains to the left of the point where q_2 kinks, Case 3 to the right of the point where q_b kinks, and Case 2 in between. While this is not an empirical paper, we mention that some people argue real-world markets suffer from a scarcity of high-quality liquid assets.[13] In our stylized model, a shortage of such liquid assets corresponds to Case 1.
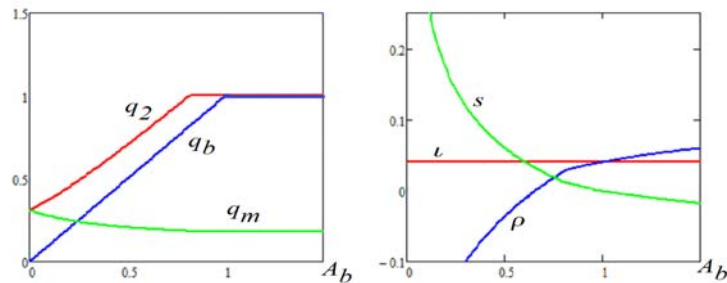

Figure 2: Effects of ι (nominal illiquid interest rate)

Issuing currency and buying bonds – an OMO depicted as a move to the left in Figure 2 – lowers ρ and, at least over some range, increases q_m while it decreases q_2 and/or q_b. Again, this has nothing to do with increasing the money supply *per se*, which is neutral. The real effects are due to decreasing A_b, which stimulates demand for real money balances as an alternative source of liquidity. Thus, an OMO decreases the T-bill rate ρ not by putting more currency in the hands of the public, but by raising the bond price through a contraction in supply. Issuing currency to finance the OMO is irrelevant because, absent *ad hoc* restrictions on the ability of prices to adjust, the direct effect is merely to lower ϕ_m so that z_m stays the same. Note however that due to indirect effects it *looks like* nominal prices are sluggish: in general equilibrium, the net impact of an OMO that injects cash is to raise z_m, which means that the value of money ϕ_m goes down (the price level goes up) by less than the increase in A_m. This is the New Monetarist anatomy of an OMO.

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[13]On this see BIS (2001), Caballero (2006), Caballero and Krishnamurthy (2006), Gourinchas and Jeanne (2012), IMF (2012) and Gorton and Ordonez (2013,2014)

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To close this part of the discussion, note that similar results can emerge if one were to simply insert bonds as goods in utility functions, like apples. We do not endorse this, however, because bonds are not apples, they are assets valued for their returns plus liquidity services, and the value of liquidity is not a primitive the way the utility of apples may be. We think it is better to model liquidity explicitly than treat it like fruit juice. Now, some assets are similar to apples, like apple trees, and it is only fiat objects that have no intrinsic value. But to the extent that assets provide liquidity, we want to try to take this seriously.

3.2   Variations: Nominal or Long Bonds

Consider nominal bonds issued in one CM and paying a dollar in the next. For stationarity, let money and bonds grow at the same rate π, so $B = A_b/A_m$, $z_m = \phi_m A_m$ and $z_b = B z_m$ are constant over time. Households' CM budget constraint becomes $x = z + \ell + T - (1+\pi)\hat{z}_m - (1+\pi)\hat{z}_b/(1+\rho)$, but otherwise things are similar and we can emulate the method for real bonds.

In Case 1, similar to the benchmark model, $\partial z_m/\partial \iota = 1/D_N < 0$ where $D_N < 0$. Also

$$\frac{\partial q_m}{\partial \iota} = \frac{\chi_m}{v'_m D_N} < 0, \ \frac{\partial q_b}{\partial \iota} = \frac{B\chi_b}{v'_b D_N} < 0 \text{ and } \frac{\partial q_2}{\partial \iota} = \frac{\chi_m + B\chi_b}{v'_2 D_N} < 0.$$

The only qualitative difference from real bonds is that ι now affects q_b. Higher B reduces real balances: $\partial z_m/\partial B = -\alpha_2 \chi_m \chi_b z_m L'_2/D_N < 0$. For quantities,

$$\frac{\partial q_m}{\partial B} = -\frac{\alpha_2 CL'_2}{v'_m D_N} < 0, \ \frac{\partial q_b}{\partial B} = \frac{C\left(\alpha_m L'_m + \alpha_2 L'_2\right)}{v'_b D_N} > 0 \text{ and } \frac{\partial q_2}{\partial B} = \frac{\alpha_m CL'_m}{v'_2 D_N} > 0,$$

where $C > 0$. One can also derive effects on ρ, s and ϕ_b, study Cases 2 and 3, etc. As the results are similar to Section 3.1, we focus below on real bonds.[14]

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[14]We clarify an additional point: A one-time increase in A_m reduces ϕ_m through the usual channel, and if we hold constant the nominal bond supply A_b, the real supply $\phi_m A_b$ falls, which has real effects. So, one might say money is not neutral, but that is because $B = A_b/A_m$ changes. Increasing A_m holding B constant is neutral.
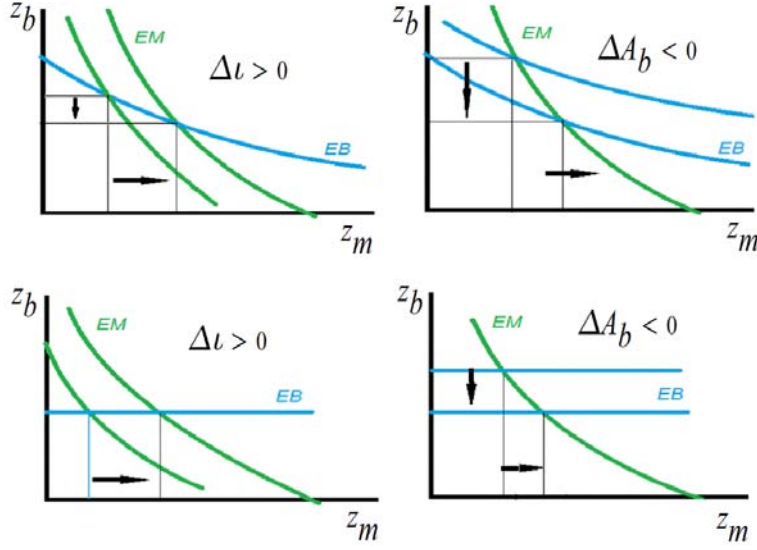
x

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Figure 3: Increase in ι and decrease in A_b with long and short bonds

Now consider long-term bonds, say consols paying in perpetuity γ in numeraire. In Case 1, the Euler equations for money and bonds are

$$\iota = \alpha_m \chi_m L(\chi_m z_m) + \alpha_2 \chi_m L(\chi_m z_m + \chi_b z_b) \tag{10}$$

$$r = \frac{\gamma(1+r)A_b}{z_b} + \alpha_b \chi_b L(\chi_b z_b) + \alpha_2 \chi_b L(\chi_m z_m + \chi_b z_b). \tag{11}$$

Appendix A derives the effects of i and A_b on z_m and z_b, as well as the q's and financial variables. Since the results are qualitatively similar, we revert below to short-term bonds. However, first it is useful to depict conditions (10)-(11) in (z_m, z_b) space as the EM and EB curves in Figure 3. It is easy to verify that both are downward sloping, but EB has a greater slope, and so they cross just once. An increase in ι shifts EM to the southwest but does not affect EB, while a decrease in A_b shifts EB to the southwest but does not affect EM. For comparison, the bottom two panels show the analogous results in the model with one-period bonds, where $z_b = A_b$ is exogenous.

After an OMO, the direct effect of higher A_m on real variables is nil, since ϕ_m adjusts to keep z_m constant. But the effect of lower A_b is to stimulate demand for

z_m, so ϕ_m goes down by less than A_m goes up. In the lower right panel of Figure 3, with short-run bonds, EB shifts down after an increase A_m and decrease in A_b, as shown by the arrows. Since z_m increases, prices respond by less than the increase in A_m. This looks like sluggish prices. The upper right panel, with long-term bonds, is similar but has an additional multiplier effect: after EB shifts down there is a rise in z_m given z_b, then the rise in z_m further reduces z_b etc. This looks like more sluggish prices. But in neither case are prices actually sticky – e.g., lump-sum injections of A_m decrease ϕ_m proportionally.

3.3 A Liquidity Trap

A recurring theme here is that there can emerge outcomes resembling liquidity traps, where OMO's do not affect ρ, or the allocation, which involves inefficiently low q's. While Figure 2 shows changes in A_b are neutral when bonds are abundant, that is because agents can get satiated in bond liquidity. Now, for something completely different, consider incorporating heterogeneous buyers, where type-j have probabilities α_m^j, α_b^j and α_2^j of type-m, type-b and type-2 meetings.

Let μ_j be the fraction of type-j buyers, and suppose there is a type j with $\alpha_m^j = \alpha_b^j = 0 < \alpha_2^j$. If they choose $\hat{z}_m^j > 0$ and $\hat{z}_b^j > 0$, where superscripts here indicate type, their Euler equations are

$$1 + \pi = \beta \left[1 + \alpha_2^j \chi_m \lambda(q_2^j) \right] \tag{12}$$

$$\phi_b = \beta \left[1 + \alpha_2^j \chi_b \lambda(q_2^j) \right] \tag{13}$$

As a special case of (8), special because for this type of buyer $\alpha_m^j = \alpha_b^j = 0$, we get $\rho = \underline{\rho}$ where

$$\underline{\rho} = \iota \left(\chi_m - \chi_b \right) / \left(\chi_m + \iota \chi_b \right),$$

which pins down the nominal T-bill rate as a function of ι and the χ's. Intuitively, for type-j, bonds and money are perfect substitutes because one unit of $\hat{z}_m$ in

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the DM always gets them the same as χ_b/χ_m units of $\hat{z}_b$. Hence, if $\hat{z}_m^j > 0$ and $\hat{z}_b^j > 0$, the assets must have the same return after adjusting for pledgeability.

Consider two types: type-m buyers have $\alpha_b = \alpha_2 = 0 < \alpha_m$; type-2 buyers have $\alpha_m = \alpha_b = 0 < \alpha_2$. Type-$m$ hold $\hat{z}_m^m > 0$, and we can set $\hat{z}_b^m = 0$ because, just like sellers, type-m buyers are indifferent to holding bonds if priced fundamentally, and strictly prefer not to if there is a liquidity premium. For these buyers, $\iota = \alpha_m \chi_m L(\chi_m z_m^m)$ determines z_m and $q_m = v^{-1}(\chi_m z_m^m)$. Type-2 buyers hold all the bonds, $\hat{z}_b^2 = A_b/\mu_2 > 0$, and maybe some cash, $z_m^2 \geq 0$. There are three possibilities. If bonds are plentiful, $A_b \geq A_b^*$ where $\chi_b A_b^*/\mu_2 = v(q^*)$, then $\hat{z}_m^2 = 0$ and $q_2 = q^*$. If $A_b < A_b^*$, there are two subcases. One has $\hat{z}_m^2 = 0$ even though $q_2 < q^*$, and occurs if $A_b \geq \bar{A}_b$ where $\iota = \alpha_2 \chi_m L(\chi_b \bar{A}_b/\mu_b)$. In this subcase type-2 cannot get q^*, but q_2 is big enough they choose not to bear the cost ι of topping up their liquidity with cash. The other subcase has $\hat{z}_m^2 > 0$, occurs if $A_b < \bar{A}_b$, and implies $\iota = \alpha_2 \chi_m L(\chi_m z_m^2)$. In this subcase total liquidity for type-2 is independent of A_b because, at the margin, it's money that matters.
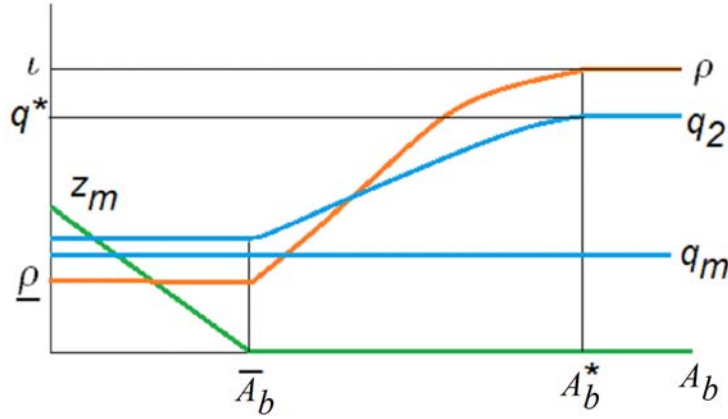

Figure 4: Effects of A_b, with a liquidity trap in $(0, \bar{A}_b)$

As shown in Figure 4, $\hat{z}_m^2 > 0$ occurs for $A_b \in (0, \bar{A}_b)$. As A_b increases over this interval, there is one-for-one crowding out of z_m^2 until it hits 0 at $\bar{A}_b$. Over the range $(0, \bar{A}_b)$ we get a liquidity trap, where OMO's are ineffectual, because changes in A_b induce changes in real balances such that total liquidity stays the

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same. This is related to Wallace's (1981) result for OLG models, although (in our notation) he had $\alpha_m = \alpha_b = 0$ and $\alpha_2 = \chi_m = \chi_b = 1$, meaning that money and bonds are always perfect substitutes. To clarify why we have two types, we want some agents to go through different regimes as A_b increases, from using money and getting $q < q^*$, to not using money but still getting $q < q^*$, to getting $q = q^*$. This is the role of type-2. But we do not want monetary equilibrium to collapse for $A_b \geq A_b^*$. Thus we include type-m, who always need cash.

4   Endogenous Liquidity

To endogenize α_j and χ_j, here we follow a long tradition (see Lagos et al. 2014, Sec. 11) appealing to *recognizability*, by introducing asymmetric information.

4.1   Acceptability

As in Lester et al. (2012), assume: 1. some sellers cannot distinguish high- from low-quality versions of certain assets; 2. low-quality assets have 0 value; and 3. they can be produced on the spot for free (Section 4.2 changes this). To guarantee low-quality assets are worth 0, think of them as fraudulent or counterfeit, and assume anyone getting stuck with one in the DM has it authenticated and confiscated in the next CM, similar to Nosal and Wallace (2007). Then sellers unable to recognize quality must reject an asset outright, since if they were to accept it, buyers would simply hand over worthless paper.[15] Assume all sellers can recognize cash, but to recognize bonds they must pay an individual-specific cost κ, with CDF $F(\kappa)$ (this can be a cost of information or a counterfeit detection technology). Let n_2 be the measure that pay this cost and $n_m = 1 - n_2$ the measure that do not and hence accept only cash.

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[15]This is extreme, but convenient relative to having buyers choose asset quality before knowing if they will meet an informed or uninformed counterparty (Williamson and Wright 1994; Berentsen and Rocheteau 2004). In those settings, sellers accept unrecognized assets with some probability; here they reject them outright, so we avoid bargaining under private information.

Here we set $\chi_j = 1$ and use Kalai bargaining, $v(q) = \theta c(q) + (1-\theta) u(q)$. Then the marginal seller is one with $\kappa = \Delta$, where

$$\Delta = \alpha (1-\theta) [u(q_2) - c(q_2) - u(q_m) + c(q_m)] = \frac{\alpha (1-\theta)}{\theta} [u(q_2) - u(q_m) - \hat{z}_b]$$

is the increase in profit from being informed. Equilibrium entails $n_2 = F(\Delta)$, with $\Delta = \Delta(z_m)$ because the q's depend in z_m. In Figure 5 $n_2 = F \circ \Delta(z_m)$ defines a curve in (n_2, z_m) space called IA for *information acquisition*, that slopes down and shifts right with A_b. Assuming buyers are constrained in all meetings, the Euler equation defines a curve called RB, for *real balances*, that slopes down and shifts down as A_b increases. Also note that increases in ι shift RB down but do not affect IA.
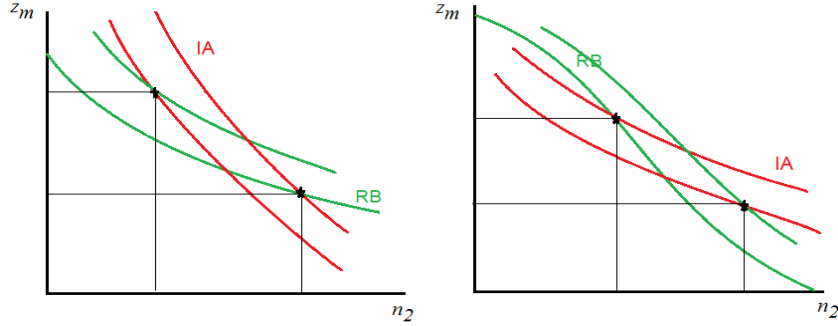

Figure 5: Equilibria with endogenous α's

As shown, RB can cut IA from below or from above. In either case, $n_2 = F \circ \Delta(z_m)$ is decreasing in z_m via IA and z_m is decreasing in n_2 via RB. Hence, equilibrium involves $n_2 = F \circ \Delta \circ z_m(n_2) \equiv \Phi(n_2)$ where $\Phi : [0,1] \to [0,1]$ is increasing. Existence follows by Tarski's fixed-point theorem, even when $F(\cdot)$ is not continuous, as when there is a mass of sellers with the same κ. Since Φ is increasing, *multiplicity* can easily emerge. Intuitively, higher n_2 decreases z_m, this raises the relative profitability of recognizing bonds, and that increases the fraction of sellers investing in information. This can also lead to *fragility*, with small changes in parameters causing jumps in (z_m, n_2). Also, while we are assum-

ing κ must be paid every period, it is not hard to alternatively make it a one-time cost. Then the model generates *hysteresis*, as is commonly thought to characterize dollarization, where higher domestic inflation leads to more locals learning to use foreign currency, but subsequent disinflations do not lead to reversals, since they tend not to forget.

Using '$x \asymp y$' to mean 'x and y take the same sign' we have

$$\frac{\partial z_m}{\partial \iota} = \frac{v'_m v'_2}{D_\alpha} \asymp D_\alpha, \; \frac{\partial q_m}{\partial \iota} = \frac{v'_2}{D_\alpha} \asymp D_\alpha \text{ and } \frac{\partial q_2}{\partial \iota} = \frac{v'_m}{D_\alpha} \asymp D_\alpha,$$

and in terms of the number of the number of informed sellers,

$$\frac{\partial n_2}{\partial \iota} = \frac{\alpha (1 - \theta) (c'_m u'_2 - c'_2 u'_m) F'}{D_\alpha} \asymp -D_\alpha$$

where $D_\alpha = \alpha^2 (1 - \theta)(\lambda_2 - \lambda_m)(c'_m u'_2 - c'_2 u'_m) F' + \alpha n_m \lambda'_m v'_2 + \alpha n_2 \lambda'_2 v'_2$. Notice $D_\alpha < 0$ iff RB cuts IA from below, in which case higher ι by shifting RB down decreases z_m and increases n_2 along FE. As usual, with multiple equilibria, the results alternate across them. While intuitive results (i.e., ones qualitatively similar to what we had with the α's fixed) follow from $D_\alpha < 0$, it not obvious that this is the only interesting case (e.g., sometimes there is a unique equilibrium and it has $D_\alpha > 0$). For financial variables, one can check $\partial s / \partial \iota \asymp \partial \phi_b / \partial \iota \asymp -D_\alpha$, while $\partial \rho / \partial \iota$ is ambiguous for reasons discussed in Section 3.

The effects of OMO's are given by

$$\frac{\partial z_m}{\partial A_b} = -\frac{\alpha v'_m n \left[{}_2\lambda'_2 + \alpha (1 - \theta) F' (u'_2 - c'_2)(\lambda_2 - \lambda_m) \right]}{n D_\alpha} \asymp D_\alpha$$

$$\frac{\partial q_m}{\partial A_b} = -\frac{\alpha \left[n_2 \lambda'_2 + \alpha (1 - \theta) F' (u'_2 - c'_2)(\lambda_2 - \lambda_m) \right]}{n D_\alpha} \asymp D_\alpha$$

$$\frac{\partial q_2}{\partial A_b} = \frac{\alpha \left[(n - n_2) \lambda'_m + \alpha (1 - \theta) F' (u'_m - c'_m)(\lambda_2 - \lambda_m) \right]}{n D_\alpha} \asymp -D_\alpha$$

$$\frac{\partial n_2}{\partial A_b} = \frac{\alpha \alpha (1 - \theta) F' \left[(n - n_2)(u'_2 - c'_2) \lambda'_m + n (u'_m - c'_m) \lambda'_2 \right]}{n D_\alpha} \asymp -D_\alpha.$$

One can also derive the effects on financial variables to conclude $\partial s / \partial A_b \asymp$

$\partial \phi_b / \partial A_b \simeq -\partial \rho / \partial A_b$. In terms of Figure 5, if $D_\alpha < 0$, as in the left panel, an OMO that retires bonds shifts RB up and FE left, thus increasing q_m and decreasing q_2. This was true with fixed α's, too, but here the effects are multiplied: lower A_b initially raises z_m and q_m, then n_2 falls, which leads to a further rise in z_m, which leads to a further fall in n_2, etc. Then again, if $D_\alpha > 0$, the results are reversed, but that is to be expected when there are multiple equilibria, as naturally can arise in the choice of payment instruments. Hence we are content reporting results contingent on D_α.

4.2   Pledgeability

The next step is to endogenize the χ's. As in Rocheteau (2011) or Li et al. (2012), suppose agents can produce low-quality assets at costs proportional to their values: for counterfeit currency the cost of producing 1 unit is $\tilde{\gamma}_m \phi_m$, and for fake bonds it is $\tilde{\gamma}_b$. Also, all sellers are now uninformed, and the decision to produce fraudulent assets is made in the CM before visiting the DM, but still fraudulent assets are confiscated in the next CM.[16]

As in standard signaling models, here we use bargaining with $\theta = 1$, and focus for now on $\alpha_2 > 0 = \alpha_m = \alpha_b$ (but see below). So there is only one kind of meeting, but we need to distinguish payments made in money and bonds, say d_m and d_b, not just the sum. This leads to the incentive condition

$$(\phi_{m,-1} - \beta \phi_m) a_m + \beta \alpha_2 \phi_m d_m \leq \tilde{\gamma}_m \phi_m a_m, \tag{14}$$

(see Li et al. 2012, Appendix B). The RHS is the cost of counterfeiting a_m dollars; the LHS is the cost of acquiring a_m genuine units $(\phi_{m,-1} - \beta \phi_m) a_m$, plus the cost of giving up d_m with probability α_2. Sellers rationally believe buyers would not

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[16]As mentioned above, instead of information frictions one can try to justify $\chi_j < 1$ by letting borrowers abscond with a fraction of the collateral. As pointed out by Ricardo Cavalcanti, however, making this rigorous requires additional assumptions on institutions monitoring compliance and seizing assets. That can be harsher than assumptions on the physical environment to render low-quality assets worthless (e.g., counterfeits fully depreciate each period).

pay with fraudulent assets if (14) holds – who would spend \$20 to pass \$10 worth of counterfeits? One can also interpret $d_m < a_m$ as over-collateralization, or *asset retention*, as a signal of quality. Similarly, for d_b

$$\left(\phi_{b,-1} - \beta\right) a_b + \beta\alpha_2 d_b \leq \tilde{\gamma}_b a_b. \tag{15}$$

There are now multiple DM constraints: bargaining implies $c(q_2) = \phi_m d_m + d_b$; feasibility implies $\phi_m d_m \leq z_m$ and $d_b \leq z_b$; and (14)-(15) imply $d_j \leq \chi_j z_j$ with

$$\chi_m = \frac{\gamma_m - \iota}{\alpha_2} \text{ and } \chi_b = \frac{\gamma_b - s}{\alpha_2}, \tag{16}$$

where $\gamma_j = \tilde{\gamma}_j / \beta$. Notice $\partial\chi_j / \partial\alpha_2 < 0$, because fraud is more tempting when there are more opportunities to pass bad assets. Also, $\partial\chi_m / \partial\iota < 0$ and $\partial\chi_b / \partial s < 0$, so pledgeability like acceptability is not invariant to economic conditions. There are again different cases, depending on which conditions bind.

Supposing first that $\chi_j \in (0,1)$, $j = m, b$, we reduce (6) and (7) to

$$\iota = (\gamma_m - \iota) \lambda(q_2) \text{ and } s = (\gamma_b - s) \lambda(q_2), \tag{17}$$

using (16). The first equality yields q_2. Then the second yields $s = \iota\gamma_b / \gamma_m$, which further implies $\chi_b = \gamma_b (\gamma_m - \iota) / \alpha_2 \gamma_m$, and that tells us this case, with $\chi_j \in (0,1)$, obtains iff $\gamma_m > \iota$, $\gamma_m < \iota + \alpha_2$ and $\gamma_b < \alpha_2 \gamma_m / (\gamma_m - \iota)$. Notice

$$\frac{\partial q_2}{\partial\iota} = \frac{1 + \lambda_2}{(\gamma_m - \iota) \lambda'_2} < 0,$$

which is different from what we found with the χ's fixed because now they respond to policy. From (8), the nominal rate ρ satisfies $1 + \rho = \chi_m / \chi_b = \gamma_m / \gamma_b$. Thus, $\rho < 0$ iff cash is easier to counterfeit than bonds. In any case, OMO's affect neither q_2 nor ρ because, once again, z_m and z_b are perfect substitutes.[17]

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[17]We mean perfect substitutes after adjusting for pledgeability, of course.. Also, as always we assume money is valued, which here requires $\iota < (\gamma_m - \iota) \lambda(\tilde{q})$ with $\tilde{q}$ defined by $c(\tilde{q}) = \gamma_b z_b (\gamma_m - \iota) / \alpha_2 \gamma_m$. This is implicit in what follows and we do not mention it again.

Moving to the next regime, $\chi_m = 1$ and $\chi_b \in [0, 1)$, we have

$$\iota = \alpha_2 \lambda(q_2) \text{ and } s = (\gamma_b - s) \lambda(q_2). \tag{18}$$

Now $\partial q_2 / \partial \iota$ is the same as it would be if χ_m were fixed, because it is fixed, at $\chi_m = 1$. Also, $s = \iota \gamma_b / (\iota + \alpha_2)$, so the regime $\chi_m = 1$ and $\chi_b \in [0, 1)$ requires $\gamma_m > \iota + \alpha_2$ and $\gamma_b < \iota + \alpha_2$. Moreover, $\rho = \iota (\iota + \alpha_2 - \gamma_b) / (\iota + \alpha_2 + \iota \gamma_b)$. One can similarly consider the regime $\chi_b = 1$ and $\chi_m \in [0, 1)$, or the regime $\chi_m = \chi_b = 1$. Also, while there is no monetary equilibrium if $\gamma_m \leq \iota$, there are nonmonetary equilibria where A_b is the only means of payment. Figure 6 displays regions of (γ_b, γ_m) space where the different equilibria exist, using Φ to mean any number in $(0, 1)$.
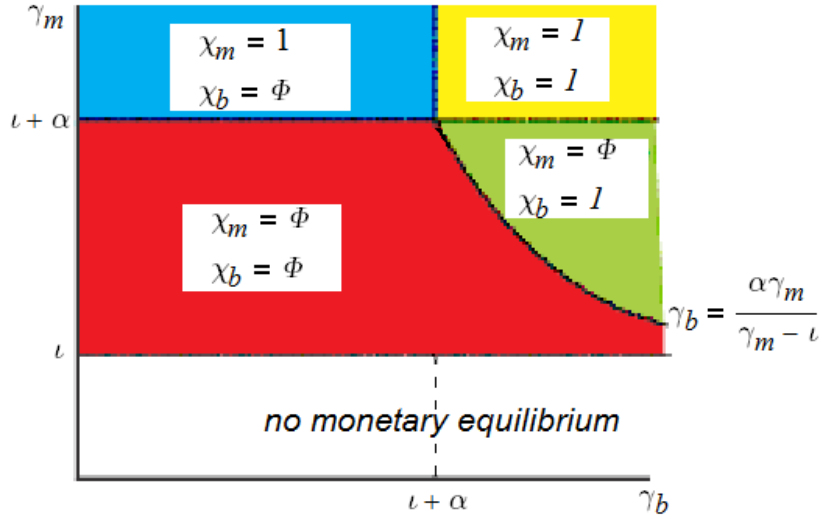

Figure 6: Different regimes with endogenous χ's

In the above discussion, OMO's are neutral, because bonds and currency are perfect substitutes. To change this, we now make them imperfect substitutes. Consider the regime $\chi_m = 1$, $\chi_b \in (0, 1)$, and let $\alpha_m > 0$, so that bonds are not accepted in some meetings. The equilibrium conditions are (6)-(7), with $\chi_b = (\gamma_b - s) / \alpha_2$, which implies $s = \gamma_b \lambda_2 / (1 + \lambda_2)$ and $\chi_b = \gamma_b / \alpha_2 (1 + \lambda_2)$, so

$\chi_b < 1$ requires $\gamma_b < \alpha_2 (1 + \lambda_2)$. The system can be reduced to

$$\iota = \alpha_m L (z_m) + \alpha_2 L (z_m + \chi_b z_b)$$

$$\gamma_b = \alpha_2 \chi_b [1 + L (z_m + \chi_b z_b)],$$

defining two curves in (χ_b, z_m) space labeled RB and IC in Figure 7. While RB slopes down, IC can be nonmonotone,

$$\frac{\partial z_m}{\partial \chi_b}_{|IC} \backsimeq \Theta \equiv 1 + L_2 + \chi_b z_b L_2'. \tag{19}$$

It is easy to build examples with multiple equilibria. Intuitively, if agents believe χ_b is low then q_2 is low, so λ_2 and s are high and is the incentive for fraud – which makes χ_b low.

The results depend on $D_\chi = (1 + L_2) (\alpha_m L'_m + \alpha_2 L'_2) + \alpha_m \chi_b z_b L'_2 L'_m$. From (19), $\Theta > 0$ implies $D_\chi < 0$, and there are three relevant configurations: (i) $\Theta > 0$ implies IC is upward sloping and cuts RB from below, as at point a in the left panel of Figure 7; $\Theta < 0$ implies IC is downward sloping and either (ii) cuts RB from below, as at e in the right panel, or cuts it from above, as at point c or g. An increase in ι shifts RB down. This can make $\partial z_m / \partial \iota = \Theta / D_\chi > 0$ when $\Theta < 0$, as in the move from e to f, or can more naturally make $\partial z_m / \partial \iota < 0$, as in the other cases. Similarly, $\partial \chi_b / \partial \iota \backsimeq D_\chi$.
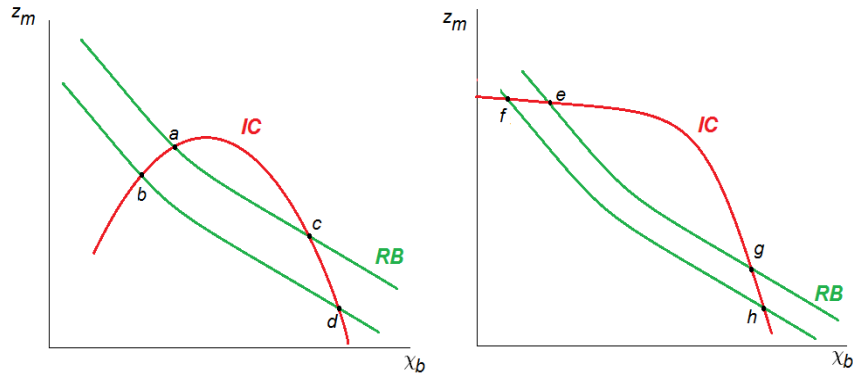

Figure 7: Different configurations with endogenous χ's

Thus the sign of $\partial z_m/\partial\iota \simeq \partial q_m/\partial\iota$ depends on the configuration of RB and IC. One can also check

$$\frac{\partial q_2}{\partial\iota} \simeq D_o, \; \frac{\partial \chi_b}{\partial\iota} \simeq D_o, \; \frac{\partial s}{\partial\iota} \simeq -D_o,$$

where $D_o = \alpha_m \alpha_2 \chi_b z_b \lambda'_m \lambda'_2 + \alpha_2 (1+\lambda_2)(\alpha_m \lambda'_m v'_2 + \alpha_2 \lambda'_2 v'_m)$ depends on the configuration. Similarly, $\partial \phi_b/\partial\iota \simeq -D_o$ but $\partial \rho/\partial\iota$ is ambiguous for reasons discussed in Section 3. In terms of OMO's,

$$\frac{\partial z_m}{\partial A_b} = -\frac{\alpha_2^2 \chi_b (1+\lambda_2) v'_m \lambda'_2}{D_o} \simeq D_o, \; \frac{\partial q_m}{\partial A_b} = -\frac{\alpha_2^2 \chi_b (1+\lambda_2) \lambda'_2}{D_o} \simeq D_o$$

$$\frac{\partial q_2}{\partial z A_b} = \frac{\alpha_m \alpha_2 \chi_b (1+\lambda_2) \lambda'_m}{D_o} \simeq -D_o, \; \frac{\partial \chi_b}{\partial A_b} = -\frac{\alpha_m \alpha_2 \chi_b \lambda'_m \lambda'_2}{D_o} \simeq -D_o.$$

Also, $\partial s/\partial A_b \simeq D_o$, $\partial \phi_b/\partial A_b \simeq D_o$ and $\partial \rho/\partial A_b \simeq -D_o$. Again, theory delivers sharp conditional predictions. A bigger point is we can study policy, based on liquidity considerations, with pledgeability and/or acceptability endogenous.

5 Directed Matching

Suppose now buyers can direct their search. Here we set $\chi_j = 1$, and consider two types of sellers: a measure n_m accept only money; and a measure $n_2 = n - n_m$ accept money and bonds. For now, normalize $n = 1$ and fix n_j. While a buyer can search for a given type of seller, whether he finds one is random. Define submarket j by a set of type-j sellers, with measure n_j, and a set of buyers that choose to look for them with measure μ_j. Let SM be the submarket where only money is accepted and $S2$ the one where both assets are accepted. The measure of buyers is $\mu_m + \mu_2 = \mu$, assumed not too large, so all want to participate. The matching technology is the same in each submarket and satisfies constant returns, so what matters is not market size but tightness, $\sigma_j = n_j/\mu_j$. The probability a buyer meets a seller is $\alpha(\sigma_j)$, and the probability a seller meets a buyer is $\alpha(\sigma_j)/\sigma_j$, with the former increasing and the latter decreasing in σ_j.

5.1 Bargaining

We start by assuming the terms of trade are negotiated using Kalai bargaining, $v(q) = \theta c(q) + (1 - \theta)u(q)$. As in Section 3.3, buyers going to SM bring only cash, $\hat{z}_m^m > 0$ and $\hat{z}_b^m = 0$, while those going to $S2$ bring bonds plus maybe cash, $\hat{z}_b^2 = A_b/\mu_2 > 0$ and $\hat{z}_m^2 \geq 0$, where superscripts indicate submarkets. Bargaining implies $\hat{z}_m^m = v(q_m)$ and $\hat{z}_b^2 + \hat{z}_m^2 \geq v(q_2)$, where the latter holds with equality iff $q_2 < q^*$ and $s > 0$ (when bonds are scarce they command a premium). When $q_2 < q^*$, we may have $\hat{z}_m^2 = 0$ or $\hat{z}_m^2 > 0$, with $\iota > s$ in the former case and $\iota = s$ in the latter (buyers hold both assets only if they are perfect substitutes).

Since a buyer now can meet only one type of seller, the Euler equations imply

$$\iota = \alpha(\sigma_m) \left[\lambda(q_m) - 1\right] \text{ and } s = \alpha(\sigma_2) \left[\lambda(q_2) - 1\right]. \tag{20}$$

If SM and $S2$ are both open, buyers must be indifferent between them:

$$\alpha(\sigma_m) \left[u(q_m) - v(q_m)\right] - \iota z_m = \alpha(\sigma_2) \left[u(q_2) - v(q_2)\right] - s z_b. \tag{21}$$

Since the total measure of buyers is μ,

$$n_m/\sigma_m + n_2/\sigma_2 = \mu. \tag{22}$$

A monetary equilibrium is a list (q_j, σ_j, s) solving (20)-(22).

Again there are three regimes: bonds are scarce, so type-2 carry some cash; bonds are less scarce, so type-2 carry no cash even though $q_2 < q^*$; bonds are plentiful, so type-2 get $q_2 = q^*$ with no cash. The first regime, where type-2 carry cash, entails $\rho = 0$ and $s = \iota$. Then (21)-(22) imply $\alpha(\sigma_2) = \alpha(\sigma_m)$ and $\sigma_m = \sigma_2 = \sigma = 1/\mu$. From (20) $q_m = q_2 = q$, where $\iota = \alpha(\sigma) \left[\lambda(q) - 1\right]$. As z_b and z_m are perfect substitutes in this regime, the two submarkets are essentially the same, and increases in A_b merely crowd out real money balances in $S2$. This is consistent with equilibrium when $A_b \leq \underline{A}_b = n_2 \mu v(q)$.

Consider next $\hat{z}_m^2 = 0$, with $s = 0$ and $q_2 = q^* > q_m$. From (21)

$$\alpha(\sigma_2) = \frac{\max_q \{-\iota v(q) + \alpha(\sigma_m) [u(q) - v(q)]\}}{u(q^*) - v(q^*)},\tag{23}$$

so $\sigma_2 < \sigma_m$ given $\iota > 0$, which say buyers trade with a lower probability in $S2$. It is easy to check there is a unique (σ_m, σ_2) solving (22)-(23), and this regime arises iff $A_b \geq A_b^* \equiv n_2 v(q^*)/\sigma_2$. Finally, consider $z_m^2 = 0$, $0 < s < \iota$ and $q_2 < q^*$. Buyer indifference now means

$$-\iota z_m + \alpha(\sigma_m) [u(q_m) - v(q_m)] = -s z_b + \alpha(\sigma_2) [u(q_2) - v(q_2)]$$
$$> -\iota z_m + \alpha(\sigma_2) [u(q_m) - v(q_m)],$$

where the inequality follows from the fact that bonds are less costly to hold then cash, and $q_m < q_2$ from (20). Consequently, $\alpha(\sigma_m) > \alpha(\sigma_2)$, and the probability of trade is higher in SM. This regime arises when $A_b \in (\underline{A}_b, A_b^*)$.
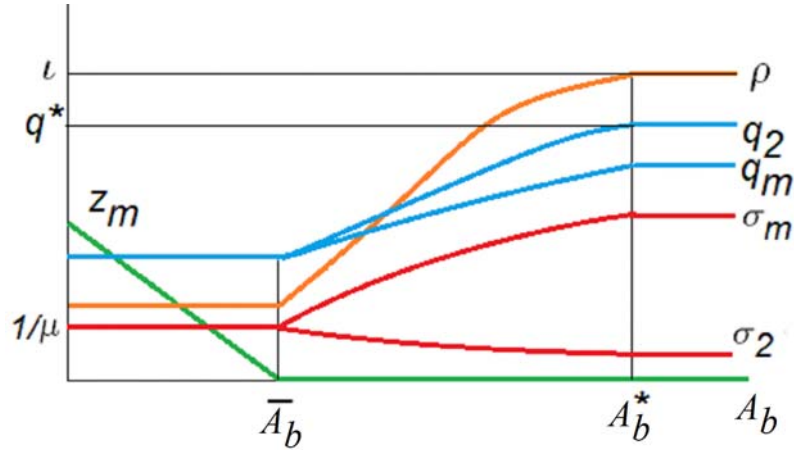

Figure 8: Effects of A_b with directed search

In Figure 8, as in the random-search model, OMO's are neutral when A_b is high or low, but not in the intermediate range. But now σ_2 and σ_m depend on A_b because this affects the measures of buyers in $S2$ and SM. Also, now q_m varies with A_b. So some insights are similar, and others change, when we segment the market endogenously based on tightness. In particular, when A_b increases,

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buyers in *SM* get better terms of trade and a higher probability of trade, even though bonds are not used in *SM*. Hence we can have both q_b and q_m increasing in A_b, while with random search q_b increases and q_m decreases. The intuition with random search is that higher A_b tends to raise q_2, and this lowers demand for z_m because it is a substitute for z_b in type-2 meetings, and that lowers q_m.

5.2 Posting

Now assume that search can be directed along two dimensions: the means of payment accepted by sellers; and the terms of trade. As in Moen (1997), suppose third parties called *market makers* set up submarkets in the DM to attract buyers and sellers, who then meet bilaterally according to a standard matching technology (it is equivalent here to have sellers or buyers post, instead of market makers). In the CM, they post $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j\right)$ for the next DM, where traders commit to swapping q_j for a portfolio $\left(\hat{z}_m^j, \hat{z}_b^j\right)$ if they meet.

In *SM*, market makers design $(q_m, \hat{z}_m, \sigma_m)$ to maximize buyers' surplus given sellers must get a surplus Π_m, with these surpluses in equilibrium dictated by the market. Assuming Π_m is not too big, so the market can open, the problem is

$$U^b(\iota, \Pi_m) = \max_{q,\hat{z},\sigma} \left\{\alpha\left(\sigma\right)\left[u\left(q\right) - \hat{z}\right] - \iota\hat{z}\right\} \text{ st } \frac{\alpha\left(\sigma\right)}{\sigma}\left[\hat{z} - c\left(q\right)\right] = \Pi_m. \qquad (24)$$

The expected payoff of the buyer, $U^b(\iota, \Pi_m)$, is decreasing in both ι and Π_m. Generically (24) has a unique solution (for some values of Π_m there may be multiple solutions, but they are payoff equivalent). Hence, we proceed assuming all submarkets are the same, or, by constant returns, there is just one. Then use the constraint to eliminate $\hat{z}_m$ and take FOC's wrt q_m and σ_m to get

$$\frac{u'\left(q\right)}{c'\left(q\right)} - 1 = \frac{\iota}{\alpha\left(\sigma\right)} \qquad (25)$$

$$\alpha'\left(\sigma\right)\left[u\left(q\right) - c\left(q\right)\right] = \Pi_m \left\{1 + \frac{\iota\left[1 - \varepsilon\left(\sigma\right)\right]}{\alpha\left(\sigma\right)}\right\}, \qquad (26)$$

where $\varepsilon(\sigma) \equiv \sigma\alpha'(\sigma)/\alpha(\sigma) \in (0,1)$ is the elasticity of matching.

We can similarly analyze $S2$, with ι replaced by s, Π_m replaced by Π_2, and $U^b(\iota, \Pi_m)$ by $U^b(s, \Pi_2)$. A monetary equilibrium is a list $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j, s_j, \Pi_j\right)$ such that: $\left(q_j, \hat{z}_m^j, \hat{z}_b^j, \sigma_j\right)$ solves the market-maker problem; $s_m = \iota$ and $s_2 = s$ are determined by market clearing; (22) holds; and $U^b(i, \Pi_m) = U^b(s, \Pi_2)$. As in similar models, technical complications arise due to the fact that the objective function in (24) is not necessarily concave. Hence, it is hard to prove monotonicity wrt exogenous variables, or even continuity, but in examples the outcomes look like Figure 8. In particular, when A_b is low SM and $S2$ have the same (q, σ), and the economy is stuck in a trap, where OMO's do not move q or ρ from their lower bounds, because again at the margin it's money that matters.

While Appendix B proceeds more generally, consider here a special matching function that allows a simple characterization, $\alpha(\sigma) = \min\{1, \sigma\}$.[18] Also, now suppose $\mu > n_m + n_2$, so not all buyers participate in the DM – they participate in submarket j up to the point where

$$\min\{1, \sigma_j\}\left[u(q_j) - \hat{z}_m^j - \hat{z}_b^j\right] - \iota\hat{z}_m^j - s\hat{z}_b^j = 0.$$

In SM, matching probabilities for buyers and sellers are $\alpha(\sigma_m) = \min\{1, \sigma_m\}$ and $\alpha(\sigma_m)/\sigma_m = \min\{1/\sigma_m, 1\}$. One can show in equilibrium $\sigma_m = 1$ and $\mu_m = n_m$.[19] From the buyer's participation constraint, $\hat{z}_m^m = u(q_m)/(1+\iota)$. Substituting this into the seller's surplus and maximizing wrt q_m, we get $u'(q_m)/c'(q_m) = 1+\iota$, the usual condition except that now buyers trade with probability 1.

In $S2$, similarly, $\sigma_2 = 1$ and $\mu_2 = n_2$. Again, if $z_m^2 > 0$ then $\rho = 0$, $q_2 = q_m$ and the submarkets are essentially identical. This regime is an equilibrium iff

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[18] Nosal and Rocheteau (2011) study this version in a one-asset model. It can be interpreted as eliminating search frictions by having everyone on the short side of the market match with probability 1 while the long side is rationed.

[19] Tightness $\sigma_m > 1$ is inconsistent with equilibrium because we can increase sellers' expected surplus, $\min\{1/\sigma_m, 1\}[\hat{z}_m^m - c(q_m)]$, by attracting additional buyers without harming those who are already there. Similarly, $\sigma_m < 1$ is inconsistent with equilibrium since we can raise sellers' expected surplus by attracting fewer buyers but asking for a higher payment.

$z_b = A_b/n_2 \leq u(q_m)/(1+\iota)$. If instead $z_m^2 = 0$ then q_2 solves $u'(q_2)/c'(q_2) = 1 + s$ and $z_b = A_b/n_2 = u(q_2)/(1+s)$ makes buyers indifferent. Then $q_2 = q^*$ obtains if $s = 0$, which requires $A_b/n_2 \geq u(q^*)$. If $u(q_m)/(1+\iota) < A_b/n_2 < u(q^*)$ then $q_2 < q^*$ and $s \in (0, \iota)$. So the outcome is like Figure 4 from the model with random search, instead of Figure 8, because $\sigma_m = \sigma_2 = 1$ and q_m are independent of z_b with this matching technology. However, in this version buyers know which assets are accepted in the submarkets they visit.

We now endogenize the measures of sellers across submarkets by having them pay κ to recognize bonds. Sellers participate in $S2$ iff $\kappa \leq \Delta = \Pi_2 - \Pi_m$, and $n_2 = F(\Delta)$, as in Section 4.1. However, equilibrium here is *unique*.[20] Also, equilibrium with $\iota = s$ and $\rho = 0$ now cannot exist if $F(0) = 0$ – if it is costly for all sellers to acquire information, none choose $S2$ when $\Pi_2 = \Pi_m$. If $F(0) > 0$, there can be an equilibrium with $\iota = s$ and $n_2 = F(0)$, which looks like Figure 8. For low A_b sellers participate in $S2$ iff $\kappa = 0$, in which case $\iota = s$ and $\rho = 0$, so the assets are perfect substitutes. As A_b increases, Δ becomes positive and some sellers with $\kappa > 0$ join $S2$. Output in both markets increases, but it increases more in $S2$. The reason q_m increases is that tightness in SM rises. In contrast, q_2 increases because the interest on bonds goes up, even though σ_2 decreases. As before, OMO's affect tightness and hence matching probabilities in the submarkets, so the impact of changing A_b spill over across submarkets.

6 Related Models

There is a prior literature on segmented markets and monetary policy. These papers, including Alvarez et al. (2001,2002,2009), impose CIA constraints, and model segmentation by assuming that not all agents are active in some asset markets because of transactions costs, generally, or sometimes because of a fixed

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[20] This is because market makers internalize complementarities between sellers' information acquisition and buyers' portfolio decisions (related to results in Rocheteau and Wright 2005 and Faig and Huangfu 2007).

cost to transfer resources between the asset and goods markets. They are inter-ested in getting temporary changes in real interest rates and economic activity when the money supply increases, as well as a negative relation between expected inflation and real interest rates, or persistent liquidity effects on interest and ex-change rates. See Kahn (2006) for a broader review, and Chiu (2014) for a more recent contribution.

Segmentation here is in terms of the assets that are accepted in different submarkets. Our agents can freely participate in any submarket, and face no CIA constraint, in the sense that they can always choose a submarket where cash is not needed. In equilibrium, they hold different portfolios depending on the submarket they choose, but these differences are not due to exogenous restrictions on participation. Our results have to do with market tightness. While this is complementary to the other literature, a big difference can be seen whenever we mention events like 'a type-i buyer meets a type-j seller' or 'a buyer chooses to look for sellers in submarket-j.' There is no notion of who meets or trades with whom in those other models, where agents interact only with their budget lines augmented by CIA or other such constraints.

More closely related is work by Williamson (2012,2014a,b), who also uses New Monetarist models to study policy. There are several differences. He only consid-ers take-it-or-leave-it offers, while we have general bargaining and posting. This helps us understand the robustness of results, and moreover, as is well known, there are interesting effects that are ruled out by take-it-or-leave-it offers. Also, while Williamson has shocks equivalent to random matching, we consider both random and directed matching, the latter leading to endogenous market segmen-tation.[21] Also, he has type-m and type-2 meetings, but no type-b meetings, which we found useful for getting negative nominal interest rates. Similarly, he does not

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[21]In general, directed search is a promising setting for future research including, e.g., models with adverse selection on asset quality. Guerrieri et al.(2010), Chang (2012), Shao (2013) and Guerrieri and Shimer (2014) make progress along these lines, but more can be done.

consider pledgeability, which is another device that helps deliver negative nominal rates. Moreover, he does not mention the ostensibly sluggish nominal price adjustment that we highlight. And, perhaps most importantly, he takes liquidity as exogenous, as in our baseline model, but we also endogenize acceptability and pledgeability using information frictions.[22]

Williamson models some details carefully, including intermediation, the cost of operating the currency system, and private debt, while to focus on other phenomenal we simplify along these dimensions. In particular, the liquidity trap in Section 3.3 assumes types are either permanent, or revealed before the CM closes, so agents can tailor their portfolios appropriately. In Williamson (2012), types are realized after the CM closes, so agents need banks to rebalance their portfolios. While banking is interesting, our results show that it is not necessary to get a liquidity trap. Similarly, we abstract from many details of policy implementation. We take ι as the policy instrument, e.g., but one can alternatively target the T-bill rate ρ, or use combinations of policies to target multiple variables. While our representation of policy institutions is stylized, as in several recent papers using related models, one can delve further into this.[23]

To sum up, there is some overlap with prior work, but many results are new. In particular, while we have an extensive description of what OMO's can do, we also characterize situations where they are ineffective. By way of review, OMO's clearly do not matter when bonds are illiquid, $\alpha_b = \alpha_2 = 0$ or $\chi_b = 0$. When they are liquid, in Section 3 with random matching and exogenous liquidity, OMO's do not matter in a liquidity trap with $A_b < \bar{A}_b$, nor when bonds are plentiful in the sense $A_b > A_b^*$. In Section 4, with endogenous liquidity, there are also

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[22]While information frictions are useful in modeling liquidity, there are other approaches. One studies pairwise-efficient trading mechanisms that treat assets asymmetrically, as is sometimes socially efficient (Zhu and Wallace 2007; Nosal and Rocheteau 2013; Hu and Rocheteau 2013,2014). Another assumes some sellers, interpretable as government agents, use particular trading strategies, interpretable as policy choices (Aiyagai et al. 1996; Li and Wright 1998).

[23]See Berentsen and Monnet (2008), Berentsen and Waller (2011), Afonso and Lagos (2013), Bech and Monnet (2014), Berentsen et al. (2014) and Chiu and Monnet (2014).

several cases where OMO's do not matter. Basically, these cases involve money and bonds being perfect substitutes (in the spirit of Wallace 1981, even if his OLG model is very different). In Section 5, with directed matching, OMO's are again ineffective either in a liquidity trap or when bonds are plentiful, but that model delivers something extra: because agents choose to participate in submarkets where bonds are liquid or where they are not, changing A_b affects even submarkets where bonds are not liquid, which does not happen with random matching. One lesson from all of this is that details can make a big difference, which is why it is important to model the exchange process carefully.

7 Conclusion

This paper studied OMO's, the textbook monetary policy, with agents interacting in markets where assets can be essential. We derived differences in asset liquidity based on information frictions, and considered various specifications for market structure, including random or directed search and bargaining or posting. Theory delivered sharp predictions, some of which are consistent with conventional wisdom. It also generated novel outcomes, like negative nominal rates, market segmentation, liquidity traps and ostensibly sluggish prices. Of course, there may be other effects that the model was not designed to capture but can be addressed in future work. One potentially interesting extension would be to extend the analysis to study short-run distributional dynamics, related to recent work by Jin and Zhu (2014), Chiu and Molico (2014) or Rocheteau et al. (2015). Without trying to include everything, the paper illustrated plainly how OMO's can (sometimes) affect interest rates and allocations, not by putting cash in the hands of the public, but by pulling bonds out.

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Appendix A: Long Bonds (not necessarily for publication)

With long-term bonds, as in Section 3.2, if the constraints bind in all meetings the Euler equations are given by (10)-(11). From these we derive

$$\frac{\partial z_m}{\partial \iota} = \frac{\chi_b^2(\alpha_b L'_b + \alpha_2 L'_2) - \gamma(1+r)A_b/z_b^2}{D_l} < 0$$

$$\frac{\partial z_b}{\partial \iota} = -\frac{\chi_m \chi_b \alpha_2 L'_2}{D_l} > 0$$

$$\frac{\partial q_m}{\partial \iota} = \frac{\chi_b^2(\alpha_b L'_b + \alpha_2 L'_2) - \gamma(1+r)A_b/z_b^2}{v'_m D_l} < 0$$

$$\frac{\partial q_b}{\partial \iota} = -\frac{\chi_m \chi_b \alpha_2 L'_2}{v'_b D_l} > 0$$

$$\frac{\partial q_2}{\partial \iota} = \frac{\chi_m \chi_b^2 \alpha_b L'_b - \gamma(1+r)A_b/z_b^2}{v'_2 D_l} < 0,$$

where $D_l = \chi_m^2 \left(D_m \alpha_m L'_m + D_2 \alpha_2 L'_2\right) > 0$, with $D_m = \chi_b^2 \left(\alpha_b L'_b + \alpha_2 L'_2\right) - \gamma(1+r)A_b/z_b^2$ and $D_2 = \chi_b^2 \alpha_b L'_b - \gamma(1+r)A_b/z_b^2$. For financial variables,

$$\frac{\partial s}{\partial \iota} = -\frac{\gamma(1+r)A_b \chi_m \chi_b \alpha_2 L'_2}{z_b^2 D_l} > 0$$

$$\frac{\partial \rho}{\partial \iota} = \frac{1}{1+s} + \frac{(1+\rho)\gamma(1+r)A_b \chi_m \chi_b \alpha_2 L'_2}{z_b^2(1+s)D_l} \gtrless 0$$

$$\frac{\partial \phi_b}{\partial \iota} = -\frac{\chi_m \chi_b \alpha_2 L'_2}{A_b D_l} > 0.$$

The only ambiguous result is $\partial \rho / \partial \iota$, which as in the baseline model depends on the Fisher and Mundell effects. The effects of A_b are

$$\frac{\partial z_m}{\partial A_b} = \frac{\chi_m \chi_b \alpha_2 L'_2 \gamma(1+r)}{z_b D_l} < 0$$

$$\frac{\partial z_b}{\partial A_b} = -\frac{\chi_m^2 \gamma(1+r)(\alpha_m L'_m + \alpha_2 L'_2)}{z_b D_l} > 0$$

$$\frac{\partial q_m}{\partial A_b} = \frac{\chi_m \chi_b \alpha_2 L'_2 \gamma(1+r)}{z_b v'_m D_l} < 0$$

$$\frac{\partial q_b}{\partial A_b} = -\frac{\chi_m^2 \gamma(1+r)(\alpha_m L'_m + \alpha_2 L'_2)}{z_b v'_b D_l} > 0$$

$$\frac{\partial q_2}{\partial A_b} = -\frac{\chi_m^2 \chi_b \gamma(1+r)\alpha_m L'_m}{z_b v'_2 D_l} > 0.$$

The effects of A_b on financial variables are

$$\frac{\partial s}{\partial A_b} = -\frac{\chi_m^2 \chi_b^2 \gamma(1+r)\left[\alpha_m L_m'(\alpha_b L_b' + \alpha_2 L_2') + \alpha_b \alpha_2 L_b' L_2'\right]}{z_b D_l} < 0$$

$$\frac{\partial \rho}{\partial A_b} = \frac{\chi_m^2 \chi_b^2 \gamma(1+r)\left(1+\rho\right)\left[\alpha_m L_m'(\alpha_b L_b' + \alpha_2 L_2') + \alpha_b \alpha_2 L_b' L_2'\right]}{z_b \left(1+s\right) D_l} > 0$$

$$\frac{\partial \phi_b}{\partial A_b} = -\frac{\phi_b \chi_m^2 \chi_b^2 \gamma(1+r)\left(1+\rho\right)\left[\alpha_m L_m'(\alpha_b L_b' + \alpha_2 L_2') + \alpha_b \alpha_2 L_b' L_2'\right]}{z_b \left(\rho - \pi\right)\left(1+s\right) D_l} < 0.$$

Appendix B: Directed Search (not necessarily for publication)

Since Section 5.2 focused mainly on examples, here we present a more general directed search model with posting when there is one asset z, with a spread s between the return on it and on an illiquid bond; a special case is fiat money where $s = \iota$. Market makers post $(q, \hat{z}, \sigma)$ to solve a version of (24), with s instead of ι and Π instead of Π_m. Generically there is a unique solution, with $U^b(s, \Pi)$ decreasing in both s and Π (we assume Π is not too big, so the market can open). The FOC's wrt q and σ are given by (25)-(26) except with s and Π instead of ι and Π_m. This generates a correspondence $\sigma(\Pi)$, similar to a demand correspondence, with σ the quantity and Π the price, and one can show $\sigma(\Pi)$ is decreasing (Rocheteau and Wright 2005, Lemma 5).

Let us normalize the measure of buyers to $\mu = 1$. One approach in the literature assumes that n is fixed, and therefore in equilibrium $\sigma = n$ (the seller-buyer ratio in the representative submarket is the population ratio). Then $\sigma(\Pi) = n$ pins down Π. In this case,

$$\frac{\partial q}{\partial s} = \frac{c'}{\alpha u'' - (\alpha + s) c''} < 0, \text{ and } \frac{\partial q}{\partial n} = -\frac{\alpha'(u' - c')}{\alpha u'' - (\alpha + s) c''} > 0.$$

Also, suppose ε is constant, as it is with a Cobb-Douglas matching function (truncated to keep probabilities between 0 and 1). Then we derive

$$\frac{\partial \hat{z}}{\partial s} = \frac{\alpha\left\{u'c'\left[\alpha + s(1-\varepsilon)\right] - \varepsilon(1-\varepsilon)(u-c)\left[\alpha u'' - (\alpha + s) c''\right]\right\}}{\left[\alpha + s(1-\varepsilon)\right]^2 \left[\alpha u'' - (\alpha + s) c''\right]} < 0$$

$$\frac{\partial \hat{z}}{\partial n} = \frac{\iota \alpha'\left\{\varepsilon(1-\varepsilon)(u-c)\left[\alpha u'' - (\alpha + s) c''\right] - u'c'\left[\alpha + s(1-\varepsilon)\right]\right\}}{\left[\alpha + s(1-\varepsilon)\right]^2 \left[\alpha u'' - (\alpha + s) c''\right]} > 0.$$

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Another approach in the literature assumes a perfectly-elastic supply of homogeneous sellers, with fixed cost of entry κ, so that in equilibrium $\Pi = \kappa$ and $\sigma = \sigma(\kappa)$ is endogenous. In this case,

$$\frac{\partial q}{\partial s} = \frac{c'\alpha''(u-c)}{D} < 0, \text{ and } \frac{\partial q}{\partial \kappa} = -\frac{\alpha'[1 + s(1-\varepsilon)/\alpha](u'-c')}{D} < 0.$$

with $D = [\alpha u'' - (\alpha + s)c''][\alpha''(u-c) + s\kappa(1-\varepsilon)\alpha'/\alpha^2] - \alpha'^2(u'-c')^2 > 0$ (while D cannot be signed globally, except in special cases like $\iota = 0$, in equilibrium $D > 0$ by the SOC's). Also, if ε is constant, then

$$\frac{\partial \sigma}{\partial s} = \frac{[\alpha u'' - (\alpha + s)c'']\kappa(1-\varepsilon)/\alpha - \alpha'(u'-c')c'}{D} < 0$$

$$\frac{\partial \sigma}{\partial \kappa} = \frac{[\alpha u'' - (\alpha + s)c''][1 + s(1-\varepsilon)/\alpha]}{D} < 0$$

$$\frac{\partial \hat{z}}{\partial s} = \frac{\kappa(1-\varepsilon)^2[\alpha u'' - (\alpha + s)c''] + c'^2[\alpha(u-c)\alpha'' - s\kappa(1-\varepsilon)\alpha']}{\alpha^2 D} < 0$$

$$\frac{\partial \hat{z}}{\partial \kappa} = -\frac{\iota\alpha'\{u'[\alpha + s(1-\varepsilon)] + \varepsilon(1-\varepsilon)c[\alpha u'' - (\alpha + s)c'']\}}{\alpha[\alpha + s(1-\varepsilon)]D} \gtrless 0.$$

Finally, to briefly mention efficiency, the FOC's imply $q = q^*$ iff $s = 0$. With entry, $s = 0$ also implies $\alpha'(\sigma)[u(q) - c(q)] = \kappa$. Hence, $s = 0$ implies $\sigma = \sigma^*$, where (q^*, σ^*) solves the planner problem $\max_{q,\sigma} \{\alpha(\sigma)[u(q) - c(q)] - \sigma\kappa\}$. For comparison, with Kalai bargaining, $s = 0$ implies $q = q^*$, but $\sigma = \sigma^*$ iff $1 - \theta = \varepsilon(\sigma^*)$, which is the Hosios (1990) condition, saying that bargaining shares should equal the elasticity of matching wrt participation. Since directed search yields (q^*, σ^*) automatically at $s = 0$, it is sometimes said that it satisfies the Hosios condition endogenously.