Solving for Optimal Simple Rules in Rational Expectations Models*

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March, 2003

Abstract

This paper presents algorithms that solve for optimal simple monetary policy rules in rational expectations models with precommitment and discretion. The algorithms are applied to the models in Fuhrer (1997), Clarida, Galí, and Gertler (1999), and Rudebusch (2002) to examine the efficiency properties of operational policy rules. We show that optimized Taylor-type rules preform well in these models, but that, aside from the Fuhrer-Moore model, this result is sensitive to whether the central bank can respond to current period shocks. Taylor-type rules that are operational in the sense that they do not respond to current period information are found to be highly inefficient in the Rudebusch model and in the Clarida et al. (1999) model.

Keywords: Discretion, Precommitment, Operational Policy Rules.
JEL Classification: C61, C62, E58.

*I thank Charlie Bean, Warwick McKibbin, Adrian Pagan, Peter Stemp, Graeme Wells, participants at the 2001 SCE conference, and four anonymous referees for comments. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

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1 Introduction

This paper presents algorithms that solve for optimal simple rules in rational expectations models with precommitment and discretion. For the case where the central bank can precommit we present a solution algorithm that accommodates a broad class of simple rules. The solution for optimal simple rules with discretion is new to the literature. Applying these solution techniques to several US models, we show that simple rules, such as Taylor-type rules (Taylor, 1993), perform well relative to fully optimal policy, but that this result is sensitive to whether the rule allows the central bank to respond to current period shocks. When current period shocks are excluded from the central bank's information set – often necessary if a rule is to be operational (see McCallum 1994; 1999a) – the performance of Taylor-type rules is less impressive.

The economic environments that we consider are ones where the policymaker behaves optimally, subject to constraints imposed by the behavior of other economic agents (households and firms). But as Kydland and Prescott (1977) and Calvo (1978) showed, modeling policymakers as optimizing agents introduces complications in environments where the actions households and firms take today depend on how they expect the policymaker to behave in the future. Formally, Kydland and Prescott (1977) demonstrated that where private agents are forward-looking the policymaker's optimal policy is time-inconsistent. This time-inconsistency arises because the policymaker chooses its optimal policy today subject to constraints that, because they contain expectations of the future, only hold *ex ante*. As time passes these constraints no longer bind. Thus, if the policymaker reoptimizes in any subsequent period, then it will be bound by a different set of constraints and will choose not to continue with the policy that was previously chosen to be optimal.

In response to Kydland and Prescott (1977) one approach is to *assume* that the policymaker can precommit to never reoptimize and to solve for the optimal policy, building precommitment into the optimization problem.[1] Following Kydland and Prescott (1980), several studies (Oudiz and Sachs, 1985; Backus and Driffill,

[1] The source of the precommitment technology is invariably left unspecified. Some authors solve for the precommitment policy as a reputational equilibrium supported by trigger-strategies (Barro and Gordon, 1983). Alternatively, McCallum (1995) argues that recognizing the eventual futility of trying to exploit private agents' expectations, policymakers can "just do it."

1986; Currie and Levine, 1985; 1993; Söderlind, 1999; and Dennis 2001, for example) have developed algorithms that solve for fully optimal precommitment policies, although the resulting policies are non-standard because they respond to Lagrange multipliers in addition to state variables.[2] An alternative to precommitment is to allow policymakers to reoptimize over time and to solve for the optimal discretionary policy. With discretion the approach is to formulate the problem as a recursive optimization and to iterate backward through time to solve for the Markov-perfect Nash-equilibrium; the resulting equilibrium is time-consistent (Dockner, Jørgensen, Van Long, and Sorger, 2000, Theorem 4.3).[3]

With fully optimal precommitment rules or optimal discretionary rules, policy is implemented through a state-contingent decision rule in which *all* state variables enter the rule.[4] Thus, discretionary policymaking is completely consistent with a policymaker actively setting policy using a state-contingent rule (McCallum, 1999a). A third approach is to *assume* that the policymaker can precommit to not reoptimize over time, while restricting the information set, or variables, to which the policy rule responds.[5] The resulting rules are termed optimal (or optimized) simple precommitment rules. Optimal simple precommitment rules are attractive on several grounds, not least because they tend to be easier to solve for than fully optimal precommitment rules, especially in large systems.

[2] While the unobserved Lagrange multipliers enter the fully optimal precommitment rule they can be substituted out in a variety of ways. One way involves eliminating the Lagrange multipliers using their transition equation, which produces a rule that depends on the entire history of the state vector (Currie and Levine, 1993).

[3] Aside from the most basic model specifications it is usually impossible to solve for fully optimal precommitment rules or optimal discretionary rules analytically. Numerical algorithms are available, however, such as those described in Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1985, 1993), McKibbin and Sachs (1991), Söderlind (1999), and Dennis (2001). An alternative method of solving for optimal discretionary rules is illustrated through application in Krusell, Quadrini, and Rios-Rull (1997) and Cooley and Quadrini (1999). See also Hansen, Epple, and Roberds (1985).

[4] In the precommitment case, the Lagrange multipliers on the policy constraints also enter the rule. See footnote 2.

[5] A further alternative, recently developed by Woodford (1999a), is to *assume* that the policymaker sets policy according to a "timeless perspective." Under the timeless perspective the policymaker reoptimizes at each point in time, but "precommits" not to exploit private agents' expectations. In the resulting equilibrium the policymaker responds in each state of the world as it would have chosen to if it had optimized once at a time infinitely far in the past. While the timeless perspective circumvents problems associated with initial conditions, the resulting policy does not necessarily lead to outcomes that are superior to the time-consistent policy. See Svensson and Woodford (1999) for an in depth discussion of the timeless perspective approach to optimal policy.

A second reason why optimal simple rules are popular is that they provide a compact representation of the policymaker's decision rule, yet also tend to perform well relative to more complicated policy rules and relative to the fully optimal precommitment policy. In the Rudebusch and Svensson (1999) model, for example, the minimized loss function is raised only 1.7% - 3.7% when policy is set using an optimal Taylor-type rule instead of the fully optimal policy rule.[6] While the relative efficiency of Taylor-type rules in the Rudebusch and Svensson (1999) model is unusually good, Levin, Wieland, and Williams (2003) find that optimal forward-looking Taylor-type rules are about 85% as efficient as fully optimal precommitment policies, even in models as large as FRB/US.

A third reason why simple rules are attractive is that for a policy rule to be operational it cannot respond to information that is unavailable at the time policy decisions are formulated. On this issue, McCallum (1994, 1999a) and McCallum and Nelson (1999) argue that publishing lags mean that when central banks set policy they must rely on lagged data, and form expectations of current period outcomes based on lagged information. Orphanides' (1998) results on the disparity between real-time output gap estimates and final-data estimates underscore McCallum's point. Simple rules are a natural way to restrict the variables to which policymakers can respond.

Furthermore, Batini, Harrison, and Millard (2001) and Leitemo and Söderström (2001) argue on robustness grounds that central banks should not respond to exchange rates when setting policy (Bernanke and Gertler (1999) make a similar argument regarding stock market indices). These studies find that where the underlying model is mis-specified, policy rules that respond to asset prices may perform badly even though asset prices are observed in real time and may contain information useful for setting policy. Situations where the policymaker does not respond to asset prices are easily examined using (optimal) simple rules.

Importantly, each of these reasons for focusing on simple rules has merit independently of whether a technology or institutional framework exists that allows the

[6] In the Rudebusch and Svensson (1999) model private agents are not forward-looking so time-inconsistency issues do not arise. Because of this, the fully optimal policy rule can be solved using standard linear-quadratic dynamic programming methods. The loss function that Rudebusch and Svensson (1999) use is a linear combination of the unconditional variances of annual inflation, the output gap, and the change in the federal funds rate. Varying the weights on these three variances generates the 1.7% - 3.7% range quoted in the text.

policymaker to precommit. The fact that a policymaker cannot make commitments that are not credible does not mean that the policymaker can implement a non-operational policy rule. Moreover, there is no fundamental incompatibility between discretion and benevolence and thus there is no reason why a discretionary policymaker would not endeavor to implement a robust policy. In addition, agents and institutions that are external to the policy process may believe that policy is set with discretion while not having access to the same information set as the policymaker. In this situation an optimal simple discretionary rule can be used to approximate the policymaker's decision rule while retaining the view that policy is set with discretion. As such, it is just as interesting, relevant, and desirable to be able to solve for rules that are compact, robust, and operational with discretion as it is with precommitment.[7] In this paper, we show how to solve for optimal simple discretionary rules, conditional upon the variables that enter the policy rule. If all state variables enter the policy rule, then the algorithm solves for the optimal discretionary rule.

The structure of this paper is as follows. In the following section we describe the class of models to which the algorithms we develop can be applied. We outline the structural form used for the policy constraints and show how this form relates to the alternative state-space form that is often employed by control methods. Section 3 shows how to solve for a wide class of optimal simple rules with the assumption that the policymaker optimizes once while precommitting to never reoptimize. Section 4 turns to the discretionary case. The basic approach is to decouple the policy rule to be applied today from the rule to be applied in the future, thereby building in the condition that the coefficients chosen in today's policy rule do not constrain those that future policymakers may choose. The value function is expressed as a function of both the future rule and today's rule and is then optimized with respect to the coefficients in today's rule. Iterating "backward through time" we solve for the stationary time-consistent equilibrium. In this equilibrium, the rule that is

[7] A further argument that is sometimes made to motivate simple rules is that they are transparent and easily monitored, and, as such, can be used by policymakers as a vehicle to build reputation. Often this argument associates simple rules with precommitment, with the view that the policymaker is "precommitting to the simple rule." However, from an optimization perspective, precommitment refers to the idea that reoptimization does not occur in subsequent periods, i.e., the policymaker is precommitting to not reoptimize, and does not hold any implications for the policymaker's information set. In fact, this reputation argument further motivates solving for optimal simple discretionary rules; if the policymaker could precommit there would be no need to build reputation.

implemented is the same each period.

In section 5 the algorithms developed in sections 3 and 4 are applied to three models of the US economy. The focus in section 5 is on whether the performance of optimal Taylor-type rules, which have been shown to perform well relative to more complicated policy rules (Levin, Wieland, and Williams, 1999; 2003), is sensitive to the assumption that the central bank can respond to current period shocks. We motivate this question using a popular new-Keynesian model (Clarida, Galí, and Gertler, 1999). Solving the model for a range of optimal simple rules (with both precommitment and discretion) we demonstrate that Taylor-type rules that do not respond to current period shocks are highly inefficient. Moreover, we show that the importance of being able to respond to demand shocks relative to supply shocks turns on whether the central bank can precommit. We then solve for optimal simple rules in Fuhrer's (1997) version of the Fuhrer-Moore model and in the Rudebusch (2002) model and show that allowing the central bank to respond to current period shocks is critical for the efficiency of Taylor-type rules in the Rudebusch model, but not in the Fuhrer-Moore model. Section 6 concludes.

2 The General Setup

2.1 Policy Constraints

To solve for optimal policy rules in rational expectations models, many solution methods require that the optimization constraints be written in state-space form[8]

$$\begin{bmatrix} \mathbf{x}_{1t+1} \\ E_t \mathbf{x}_{2t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{11} \\ \mathbf{B}_{12} \end{bmatrix} [\mathbf{u}_t] + \begin{bmatrix} \mathbf{v}_{t+1} \\ \mathbf{0} \end{bmatrix}, \qquad (1)$$

where $\mathbf{u}_t$ is the control vector, $\mathbf{v}_{t+1}$ is a vector of *iid* shocks with variance-covariance matrix $\mathbf{\Sigma}$, $\mathbf{x}_{1t}$ is a vector of state variables, and $\mathbf{x}_{2t}$ is a vector of jump variables. E_t represents the mathematical expectations operator conditional upon period t information. To write a model in state-space form requires, control variables and shocks aside, that every variable in the system be uniquely designated as either a state variable or a jump variable, making the vectors $\mathbf{x}_{1t}$ and $\mathbf{x}_{2t}$ mutually exclusive. A virtue of the state-space form is that it provides a clear delineation between state variables

[8] Such algorithms include those described in Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1985, 1993), and Söderlind (1999).

and jump variables in a system whose dynamics are governed by first-order difference equations.

But the state-space form is not the only form to which control methods can be applied, nor is it necessarily the most convenient form. Economic theories are invariably developed in terms of structural equations, and the manipulations required to transform an economic model from structural form to state-space form can be tedious and prone to error. Fortunately, numerous rational expectations solution algorithms exist to quickly solve first- and second-order structural forms and automated methods are available to reduce high-order structural models down to first- or second-order form (Binder and Pesaran, 1995). Particularly with medium- to large-scale models it can be more convenient to work with a model in structural form than to manipulate it into state-space form.

With these issues in mind, the solution algorithms presented in the following two sections apply to the following second-order structural form model

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{v}_t, \qquad \mathbf{v}_t \sim iid[\mathbf{0}, \mathbf{\Sigma}], \tag{2}$$

where $\mathbf{y}_t$ is an $n \times 1$ vector of endogenous variables, $\mathbf{u}_t$ is an $p \times 1$ vector of policy instruments, and $\mathbf{v}_t$ is an $n \times 1$ vector of stochastic innovations with (possibly singular) variance-covariance matrix $\mathbf{\Sigma}$. The matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ contain policy- and time-invariant coefficients, whose values are assumed to be known to all agents. The information set available to agents in period t consists of the infinite history of the predetermined variables, the controls, and the shocks, thus E_t is conditioned on the information set $I_t = \{\{\mathbf{y}_i\}_{-\infty}^{t}, \{\mathbf{u}_i\}_{-\infty}^{t}, \{\mathbf{v}_i\}_{-\infty}^{t}, \mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{\Sigma}\}$.

Equation (2) is considerably more general than it may first appear. Systems containing more general lead and lag structures in $\mathbf{y}_t$ can be manipulated into second-order structural form. Moreover, by redefining variables, expanding $\mathbf{y}_t$, and exploiting the law of iterated expectations, expectations of current and/or future variables conditional on period $t-s$ $(s > 0)$ information are also possible (Binder and Pesaran, 1995).

The connection between the state-space form and the structural form can be seen by writing equation (2) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{v}_{t+1} \\ \mathbf{y}_t \\ E_t\mathbf{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{A}_1 & -\mathbf{A}_0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_t \\ \mathbf{y}_{t-1} \\ \mathbf{y}_t \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{A}_3 \end{bmatrix} [\mathbf{u}_t] + \begin{bmatrix} \mathbf{v}_{t+1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{3}$$

Equation (3) shows that $\mathbf{y}_{t-1}$ and $\mathbf{v}_t$ define the state of the system. If $\mathbf{A}_2$ has full rank, then, setting $\mathbf{x}_{1t} = \begin{bmatrix} \mathbf{y}'_{t-1} & \mathbf{v}'_t \end{bmatrix}'$ and $\mathbf{x}_{2t} = [\mathbf{y}_t]$, it is possible to map directly between equations (1) and (3). When $\mathbf{A}_2$ is singular – a frequent occurrence – system reduction techniques are needed to eliminate redundant variables and deflate the system, which can then be manipulated into state-space form. One advantage of working with systems in structural form is that these additional steps are unnecessary.

2.2 Policy Objectives

Let $\mathbf{z}_t = \begin{bmatrix} \mathbf{y}'_t & \mathbf{u}'_t \end{bmatrix}'$. We assume that the policymaker's objective function is an infinite horizon discounted quadratic form[9]

$$L_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\mathbf{z}'_{t+i} \mathbf{W} \mathbf{z}_{t+i} \right] \qquad 0 < \beta < 1, \tag{4}$$

where $\mathbf{W}$ is a symmetric, positive semi-definite, time-invariant, matrix of known policy weights. A quadratic objective function is highly attractive on computational grounds because with linear constraints it leads to decision rules that feedback linearly off the state variables. Because the instrument vector enters $\mathbf{z}_t$ it is unnecessary to include a separate term to penalize movements in the instrument vector, or to include separate terms penalizing interactions between instruments and target variables; such terms are easily accommodated within $\mathbf{W}$.

3 Optimal Simple Precommitment Rules

The analysis in this section assumes that the policymaker precommits to optimize only once (in the initial period), with the chosen decision rule then applied in the

[9] Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1985, 1993), Söderlind (1999), Dennis (2001), and an enormous number of applied papers, all examine optimal policy with the assumption that policy objectives are quadratic. Although many of the models examined in this literature are not micro-founded, quadratic objectives can often be rationalized in terms of a second-order approximation to a representative agents' utility function (Woodford, 1999b).

initial period and in all subsequent periods. It is well established that where the policymaker precommits to never reoptimize, the fully optimal precommitment policy takes the form of a feedback rule on the state vector and the vector of Lagrange multipliers (Backus and Driffill, 1986); the latter represent the cost the policymaker incurs by honoring commitments made in the past. The fully optimal precommitment policy efficiently utilizes all available information. Simple rules, by contrast, restrict the policymaker's use of information and are formulated directly in terms of feedback rules. By construction simple rules are inefficient relative to the fully optimal precommitment policy.

With precommitment, simple rules are not limited to formulations where the policy instrument responds only to state variables.[10] Because the coefficients in the simple rule are chosen only once it is possible to analyze simple rules that "respond" to endogenous variables. Thus, outcome-based Taylor-type rules (Taylor, 1993), in which the policymaker sets the nominal interest rate based on current inflation and the current output gap, can be analyzed, although such formulations are often more accurately interpreted as equilibrium relationships between endogenous variables (Svensson, 2002) than as decisions rules. It is also possible to solve for rules that contain expectations of the future, such as the "forward-looking" rules analyzed in de Brouwer and O'Regan (1997), Batini and Nelson (2001), Clarida, Galí, and Gertler (2000), and Batini, Harrison, and Millard (2001), although these too are formulated as relationships between endogenous variables.

In this section we consider simple rules with the following general form

$$\mathbf{u}_t = \boldsymbol{\varphi}_1 \mathbf{y}_{t-1} + \boldsymbol{\varphi}_2 \mathbf{y}_t + \boldsymbol{\varphi}_3 E_t \mathbf{y}_{t+1} + \boldsymbol{\varphi}_4 \mathbf{v}_t, \tag{5}$$

for some restricted $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$, $\boldsymbol{\varphi}_3$, and $\boldsymbol{\varphi}_4$ matrices. The restrictions imposed on these feedback coefficient matrices arise from the question under study and are exogenous to the algorithm.

Working with the state-space form, Currie and Levine (1985, 1993) and Söderlind (1999) set $\mathbf{x}_t = \begin{bmatrix} \mathbf{x}_{1t}' & \mathbf{x}_{2t}' \end{bmatrix}'$ and define a simple rule to be one where $\mathbf{u}_t = \mathbf{F}_0 \mathbf{x}_t$, for

[10] One issue not discussed in this paper is that of solving for optimal policy rules in non-linear models. WinSolve, a model solution package written and published by Richard Pierse (http://www.econ.surrey.ac.uk/rpierse/winsolve/) can solve such problems, under the assumption that the policymaker can precommit.

some restricted $\mathbf{F}_0$. Thus equation (5) is more general than Currie and Levine (1985, 1993) and Söderlind (1999) in that it also accommodates forward-looking rules. In turn, Currie and Levine (1985, 1993) and Söderlind (1999) consider a wider class of formulations than Oudiz and Sachs (1985). In Oudiz and Sachs (1985) simple rules are formulated to have the instrument vector responding only to state variables, as in $\mathbf{u}_t = \mathbf{F}_1\mathbf{x}_{1t}$, for some restricted $\mathbf{F}_1$.

Equation (5) is in fact closely related to the Oudiz and Sachs (1985) formulation. If equation (5) leads to the system having a unique stable equilibrium, as we will require in the sequel, then in equilibrium the endogenous variables, $\mathbf{y}_t$, and expectations of future endogenous variables, $E_t\mathbf{y}_{t+1}$, are uniquely related to the state variables $\mathbf{y}_{t-1}$ and $\mathbf{v}_t$. In general, then, there exist multiple feedback matrices in $\mathbf{u}_t = \boldsymbol{\varphi}_1\mathbf{y}_{t-1} + \boldsymbol{\varphi}_2\mathbf{y}_t + \boldsymbol{\varphi}_3 E_t\mathbf{y}_{t+1} + \boldsymbol{\varphi}_4\mathbf{v}_t$ that in equilibrium produce the same state-contingent rule ($\mathbf{u}_t = \mathbf{F}_2\mathbf{y}_{t-1} + \mathbf{F}_3\mathbf{v}_t$).

Let $\boldsymbol{\varphi} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \boldsymbol{\varphi}_3 & \boldsymbol{\varphi}_4 \end{bmatrix}$, to obtain unique feedback matrices we simply require that sufficient restrictions be placed on $\boldsymbol{\varphi}$ such that each row of $\boldsymbol{\varphi}$ contains fewer free parameters than there are state variables in the system.[11]

Augmenting equation (2) with equation (5) gives

$$\begin{bmatrix} \mathbf{A}_0 & -\mathbf{A}_3 \\ -\boldsymbol{\varphi}_2 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{y}_t \\ \mathbf{u}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \boldsymbol{\varphi}_1 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{u}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_2 & \mathbf{0} \\ \boldsymbol{\varphi}_3 & \mathbf{0} \end{bmatrix} E_t \begin{bmatrix} \mathbf{y}_{t+1} \\ \mathbf{u}_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\varphi}_4 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_t \\ \mathbf{0} \end{bmatrix}, \tag{6}$$

which in obvious notation can be written as

$$\mathbf{B}_0\mathbf{z}_t = \mathbf{B}_1\mathbf{z}_{t-1} + \mathbf{B}_2 E_t\mathbf{z}_{t+1} + \mathbf{B}_4\boldsymbol{\eta}_t, \tag{7}$$

where $\boldsymbol{\eta}_t = \begin{bmatrix} \mathbf{v}_t' & \mathbf{0}' \end{bmatrix}'$ has variance-covariance matrix $\boldsymbol{\Phi}$. Equation (7) can be solved directly using the rational expectations (RE) solution methods described in Anderson and Moore (1985), Binder and Pesaran (1995), McCallum (1983, 1999b), or Uhlig (1999). Alternatively, equation (7) can be rearranged into first-order form

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ E_t\mathbf{z}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{B}_1 & \mathbf{B}_0 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_t \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{B}_4 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_t \\ \mathbf{0} \end{bmatrix}. \tag{8}$$

[11] Similar considerations apply to the rules that Currie and Levine (1985, 1993) and Söderlind (1999) consider, for, in equilibrium, any endogenous variables, $\mathbf{x}_{2t}$, that enter their rules are uniquely related to the predetermined variables, $\mathbf{x}_{1t}$.

Because $\mathbf{B}_2$ is singular, equation (8) cannot be solved directly using the methods in Blanchard and Kahn (1980). However, it can be solved using many other solution methods, such as those derived in Klein (2000), Sims (2002), and Christiano (2002). Provided feedback matrices $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$, $\boldsymbol{\varphi}_3$, and $\boldsymbol{\varphi}_4$ exist such that the number of eigenvalues in the system with modulus greater than one equals the number of jump variables in $\mathbf{z}_t$, then the system has a unique stationary equilibrium. The solution to equation (7) or (8) yields an expression describing how the predetermined variables evolve over time and how the jump variables respond to the predetermined variables and the shocks. These equations can be used to show that $\mathbf{z}_t$ evolves according to the VAR(1) process (Uhlig, 1999), where $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ embed the cross-equation restrictions implied by rational expectations.

$$\mathbf{z}_t = \boldsymbol{\theta}_1 \mathbf{z}_{t-1} + \boldsymbol{\theta}_2 \boldsymbol{\eta}_t. \tag{9}$$

Using equation (9), the value of the loss function, conditional upon a given (stable) policy rule, is[12]

$$L_t^{pc}(\boldsymbol{\varphi}) = \sum_{j=0}^{\infty} \left[\beta^j \mathbf{z}_t' \boldsymbol{\theta}_1'^j \mathbf{W} \boldsymbol{\theta}_1^j \mathbf{z}_t + \sum_{k=j}^{\infty} \beta^{k+1} tr \left(\boldsymbol{\theta}_2' \boldsymbol{\theta}_1'^j \mathbf{W} \boldsymbol{\theta}_1^j \boldsymbol{\theta}_2 \boldsymbol{\Phi} \right) \right], \tag{10}$$

where "*tr*" is the trace operator. By construction the spectral radius of $\boldsymbol{\theta}_1$ is less than one and the recursive equilibrium law of motion, equation (9), is stable, which (with $0 < \beta < 1$) ensures that the geometric sums in equation (10) converge. Now, let $\mathbf{P}$ be the fix-point of

$$\mathbf{P} = \mathbf{W} + \beta \boldsymbol{\theta}_1' \mathbf{P} \boldsymbol{\theta}_1, \tag{11}$$

then equation (10) can be simplified to

$$\begin{aligned} L_t^{pc}(\boldsymbol{\varphi}) &= \mathbf{z}_t' \mathbf{P} \mathbf{z}_t + \frac{\beta}{(1-\beta)} tr \left[\boldsymbol{\theta}_2' \mathbf{P} \boldsymbol{\theta}_2 \boldsymbol{\Phi} \right] \\ &= \begin{bmatrix} \mathbf{z}_{t-1}' & \boldsymbol{\eta}_t' \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_1' \mathbf{P} \boldsymbol{\theta}_1 & \boldsymbol{\theta}_1' \mathbf{P} \boldsymbol{\theta}_2 \\ \boldsymbol{\theta}_2' \mathbf{P} \boldsymbol{\theta}_1 & \boldsymbol{\theta}_2' \mathbf{P} \boldsymbol{\theta}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1} \\ \boldsymbol{\eta}_t \end{bmatrix} \\ &\quad + \frac{\beta}{(1-\beta)} tr \left[\boldsymbol{\theta}_2' \mathbf{P} \boldsymbol{\theta}_2 \boldsymbol{\Phi} \right]. \end{aligned} \tag{12}$$

With $\mathbf{W}$ known, $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ known conditional on $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$, $\boldsymbol{\varphi}_3$, and $\boldsymbol{\varphi}_4$, $0 < \beta < 1$, and the spectral radius of $\boldsymbol{\theta}_1$ less than one, standard fix-point solution methods can

[12] In terms of notation, θ_1^j represents the matrix product of j θ_1 matrices.

be applied to equation (11) to solve for $\mathbf{P}$. Equation (12) can then be evaluated and minimized with respect to the free parameters in the feedback matrices $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$, $\boldsymbol{\varphi}_3$, and $\boldsymbol{\varphi}_4$. Computationally, we solve for optimal simple precommitment rules as follows:

1. Choose initial values for the free parameters in $\boldsymbol{\varphi} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \boldsymbol{\varphi}_3 & \boldsymbol{\varphi}_4 \end{bmatrix}$.
2. Given $\boldsymbol{\varphi}$, solve for $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ using any standard RE solution method.
3. Given $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, evaluate $L_t^{pc}(\boldsymbol{\varphi})$ according to equations (11) and (12).
4. Minimize $L_t^{pc}(\boldsymbol{\varphi})$ with respect to the free parameters in $\boldsymbol{\varphi}$.

In general, the term $\begin{bmatrix} \mathbf{z}_{t-1}' & \boldsymbol{\eta}_t' \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1 & \boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_2 \\ \boldsymbol{\theta}_2'\mathbf{P}\boldsymbol{\theta}_1 & \boldsymbol{\theta}_2'\mathbf{P}\boldsymbol{\theta}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1} \\ \boldsymbol{\eta}_t \end{bmatrix}$ in equation (12) implies that the feedback coefficients in the optimal simple precommitment rule depend on the initial state of the economy (see Currie and Levine, 1993, pp155). Typically, the feedback coefficients also depend on the variance-covariance matrix of the shocks, $\boldsymbol{\Phi}$ (i.e., certainty equivalence fails). The unconditional variance-covariance matrix for $\mathbf{z}_t$, $\boldsymbol{\Omega}$, can be obtained by applying standard fix-point solution methods to[13]

$$\boldsymbol{\Omega} = \boldsymbol{\theta}_1\boldsymbol{\Omega}\boldsymbol{\theta}_1' + \boldsymbol{\theta}_2\boldsymbol{\Phi}\boldsymbol{\theta}_2'.$$

An important special case to this analysis is the limiting case where β tends to one. Appendix A shows that $lim_{\beta\rightarrow1}(1-\beta)L_t^{pc} \rightarrow \widehat{L}_t^{pc} = tr[\mathbf{W}\boldsymbol{\Omega}]$. In this limiting case the feedback coefficients in the optimal simple precommitment rule are invariant to the initial state, although they still depend of the variance-covariance matrix of the shocks, $\boldsymbol{\Phi}$, but the latter is time-invariant.

4 Optimal Simple Discretionary Rules

In the previous section the treatment assumed that the policymaker precommits to optimize just once. Although time passes the optimization problem is never revisited

[13] An analytical solution for these unconditional variances is available: $vec[\Omega] = [I - \theta_1 \otimes \theta_1]^{-1} vec\left[\theta_2\Phi\theta_2'\right]$, where "vec" is the vector stacking operator and $\otimes$ represents the Kronecker product. However, for large systems this analytic expression requires inverting what can potentially be a very large matrix. An alternative to is solve for the unconditional variances numerically using iterative methods such as the doubling algorithm. The doubling algorithm can also be applied to equation (11), and to equations (17) and (18) in the following section.

and the feedback coefficients chosen in the initial period are employed in every period. The fact that the optimization problem is never revisited is taken into account when the feedback coefficients are chosen.

This section considers discretion. With discretion policymakers choose the feedback coefficients in the rule period-by-period while acknowledging that policymakers in all subsequent periods behave likewise. The policymaker optimizing today recognizes that the values it chooses for the feedback coefficients do not constrain those that subsequent policymakers may choose. At the same time the policymaker appreciates that the rule it chooses impacts the economy's evolution through time. Thus discretionary policy is not to be confused with a myopic policy in which a policymaker only accounts for outcomes that occur during the period in which that policymaker's rule is applied.

The solution procedure developed below solves the asymptotic optimization problem directly (with iterations occurring in meta-time), but conceptually the approach involves backward induction. Because backward induction is employed, and because policymakers reoptimize each period, only state variables are permitted to enter the rule. We *assume* that the variables that enter the policy rule are exogenously given, determined by the question at hand, and seek a stationary time-consistent equilibrium *conditional* upon these variables. Thus, if one wants to assume that monetary policy is set according to a Taylor-type rule (say), then the solution procedure allows successive policymakers to reoptimize over the free parameters in the Taylor-type rule, iterating "backward through time" to solve for the optimal simple Taylor-type rule with discretion. This optimal simple discretionary rule can then be compared to the optimal simple precommitment rule to assess how the absence of precommitment affects economic outcomes in the context of a specific policy rule.

From the perspective of a policymaker optimizing in period t, the parameters that determine how the economy evolves from $\mathbf{z}_t$ are outside its control. Thus the policymaker optimizing in period t can assume that

$$\mathbf{z}_{t+i+1}=\widehat{\mathbf{\Pi}}_1\mathbf{z}_{t+i}+\widehat{\mathbf{\Pi}}_2\boldsymbol{\eta}_{t+i+1}, \quad \text{for all } i \geq 0 \tag{13}$$

and treat $\widehat{\mathbf{\Pi}}_1$ and $\widehat{\mathbf{\Pi}}_2$ as parametric. Given a policy rule of the form

$$\mathbf{u}_t= \boldsymbol{\psi}_1\mathbf{y}_{t-1}+\boldsymbol{\psi}_4\mathbf{v}_t, \tag{14}$$

equations (13), (14), and (2) yield

$$\mathbf{z}_{t+i} = \left[\mathbf{C}_0 - \mathbf{C}_2\widehat{\mathbf{\Pi}}_1\right]^{-1}\left[\mathbf{C}_1\mathbf{z}_{t+i-1} + \mathbf{C}_4\boldsymbol{\eta}_{t+i}\right] \equiv \mathbf{\Pi}_1\mathbf{z}_{t+i-1} + \mathbf{\Pi}_2\boldsymbol{\eta}_{t+i}, \quad \text{for all } i \geq 0 \quad (15)$$

where $\mathbf{C}_0 = \begin{bmatrix} \mathbf{A}_0 & -\mathbf{A}_3 \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \mathbf{C}_1 = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \boldsymbol{\psi}_1 & \mathbf{0} \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} \mathbf{A}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{C}_4 = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\psi}_4 & \mathbf{0} \end{bmatrix}$. Given equation (15), the loss function, $L_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\mathbf{z}'_{t+i}\mathbf{W}\mathbf{z}_{t+i}\right]$, conditional upon $\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1$, and $\widehat{\mathbf{\Pi}}_2$, can be written as

$$\begin{aligned} L_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right) &= \mathbf{z}'_t\mathbf{M}\mathbf{z}_t + \frac{\beta}{1-\beta} tr\left[\mathbf{\Pi}'_2\mathbf{M}\mathbf{\Pi}_2\Phi\right] \\ &= \begin{bmatrix} \mathbf{z}'_{t-1} & \boldsymbol{\eta}'_t \end{bmatrix} \begin{bmatrix} \mathbf{\Pi}'_1\mathbf{M}\mathbf{\Pi}_1 & \mathbf{\Pi}'_1\mathbf{M}\mathbf{\Pi}_2 \\ \mathbf{\Pi}'_2\mathbf{M}\mathbf{\Pi}_1 & \mathbf{\Pi}'_2\mathbf{M}\mathbf{\Pi}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1} \\ \boldsymbol{\eta}_t \end{bmatrix} \\ &\quad + \frac{\beta}{1-\beta} tr\left[\mathbf{\Pi}'_2\mathbf{M}\mathbf{\Pi}_2\Phi\right], \end{aligned} \quad (16)$$

where

$$\mathbf{M} = \mathbf{W} + \beta\mathbf{\Pi}'_1\mathbf{M}\mathbf{\Pi}_1. \quad (17)$$

The policymaker chooses $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_4$ to minimize $L_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right)$, as given by equation (16).

As it currently stands, however, in each period t, the feedback parameters chosen are (in general) a function of the system's state variables. But because the state variables evolve over time and policymakers optimize at different points in time a stationary time-consistent equilibrium does not generally exist. However, for all $0 < \beta < 1$, the feedback parameters that the period t policymaker chooses to minimize $L_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right)$ also minimize $(1-\beta)\, L_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right)$, and in the limit as $\beta \to 1$, $(1-\beta)\, L_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right) \to \widehat{L}_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right) = tr\left[\mathbf{\Pi}'_2\mathbf{M}\mathbf{\Pi}_2\mathbf{\Phi}\right] = tr\left[\mathbf{W}\mathbf{\Omega}\right]$, which is invariant to the state vector.[14] Thus, in this limiting case the feedback parameters chosen in period t are invariant to the state vector, for all t. With this source of non-stationarity removed we can solve for optimal simple discretionary rules as follows.[15]

1. Initialize $j \leftarrow 0$, let ϵ be an arbitrarily small convergence tolerance, and choose initial values for $\widehat{\mathbf{\Pi}}_1$ and $\widehat{\mathbf{\Pi}}_2$ subject to the condition that $\widehat{\mathbf{\Pi}}_1$ has spectral radius less than one.

[14] The solution for the relevant variance-covariance matrix, $\mathbf{\Omega}$, is given by equation (18).
[15] Experience suggests that it is sometimes advisable to use dampening at step 6 of the algorithm.

2. Given $\widehat{\mathbf{\Pi}}_1$ and $\widehat{\mathbf{\Pi}}_2$, solve for ${}^j\mathbf{\Pi}_1$ and ${}^j\mathbf{\Pi}_2$ using equation (15).

3. Given ${}^j\mathbf{\Pi}_1$ and ${}^j\mathbf{\Pi}_2$, evaluate $\widehat{L}_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right) = tr\left[{}^j\mathbf{\Pi}_2^{\prime}\mathbf{M}{}^j\mathbf{\Pi}_2\mathbf{\Phi}\right]$ using equation (17) to solve for $\mathbf{M}$.

4. Minimize $\widehat{L}_t^d\left(\boldsymbol{\psi}_1, \boldsymbol{\psi}_4, \widehat{\mathbf{\Pi}}_1, \widehat{\mathbf{\Pi}}_2\right) = tr\left[{}^j\mathbf{\Pi}_2^{\prime}\mathbf{M}{}^j\mathbf{\Pi}_2\mathbf{\Phi}\right]$ with respect to the free parameters in $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_4$, conditional on $\widehat{\mathbf{\Pi}}_1$ and $\widehat{\mathbf{\Pi}}_2$. The solution to this minimization problem yields matrices ${}^{j+1}\mathbf{\Pi}_1$ and ${}^{j+1}\mathbf{\Pi}_2$.

5. If $\parallel {}^{j+1}\mathbf{\Pi}_1 - \widehat{\mathbf{\Pi}}_1 \parallel < \epsilon$, then Stop.

6. Set $\widehat{\mathbf{\Pi}}_1 \leftarrow {}^{j+1}\mathbf{\Pi}_1$ and $\widehat{\mathbf{\Pi}}_2 \leftarrow {}^{j+1}\mathbf{\Pi}_2$.

7. $j \leftarrow j + 1$ and return to Step 2.

As with the precommitment solution, the unconditional variance-covariance matrix of $\mathbf{z}_t$, $\mathbf{\Omega}$, is easily obtained by solving for the fix-point of

$$\mathbf{\Omega} = \mathbf{\Pi}_1\mathbf{\Omega}\mathbf{\Pi}_1^{\prime} + \mathbf{\Pi}_2\mathbf{\Phi}\mathbf{\Pi}_2^{\prime}. \tag{18}$$

Note that $\mathbf{\Pi}_1 = \widehat{\mathbf{\Pi}}_1$ in the time consistent equilibrium and that by construction the spectral radius of $\widehat{\mathbf{\Pi}}_1$ is less than one. Consequently, $\mathbf{\Omega}$ can be solved from equation (18) using standard fix-point solution methods.

5 Operational Policy Rules and the Value of New Information

In this section the algorithms developed above are employed to investigate the performance of optimal simple precommitment rules relative to fully optimal policy and the performance of optimal simple discretionary rules relative to the optimal discretionary policy. Of particular interest is McCallum's argument that, to be operational, a policy rule should not respond to information that is unavailable at the time policy decisions are made. The analysis takes two quarterly macroeconometric models of the US economy and uses these models to investigate the performance of various optimal simple Taylor-type rules. As is standard in the literature, however, we do not constrain the information sets that households and firms possess to be the same as the policymaker's.

The basic issues that are examined in this section can be illustrated using the following simple dynamic optimization problem

$$\min E_t \sum_{j=0}^{\infty} \beta^j \left[\lambda \pi_{t+j}^2 + (1-\lambda) y_{t+j}^2\right] \qquad 0 \leq \lambda \leq 1, \tag{19}$$

subject to

$$y_t = \theta y_{t-1} + (1-\theta) E_t y_{t+1} - \varphi \left[i_t - E_t \pi_{t+1}\right] + \epsilon_t, \qquad \epsilon_t \sim iid\left[0, \sigma_\epsilon^2\right] \tag{20}$$

$$\pi_t = \phi \pi_{t-1} + (1-\phi) \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t, \qquad \varepsilon_t \sim iid\left[0, \sigma_\varepsilon^2\right]. \tag{21}$$

The constraints in this intertemporal optimization problem are taken from Clarida, Galí, and Gertler (1999, section 6), who derive equations (20) and (21) from micro-founded models for household and firm behavior, respectively. In terms of notation, y_t represents the output gap, π_t represents inflation, and i_t denotes a short-term nominal interest rate, which serves as the monetary policy instrument. Demand and supply shocks are given by ϵ_t and ε_t, respectively.

The optimization problem summarized in equations (19) - (21) is sufficiently complicated that, except in special cases, neither the fully optimal precommitment rule, nor the optimal discretionary rule can be solved analytically. Thus, to solve for either solution we must assign values to the model's parameters. For illustration purposes, then, we parameterize the model to the quarterly frequency as follows: $\beta = 1.0, \lambda = 0.5, \theta = 0.5, \phi = 0.5, \varphi = 0.8, \alpha = 0.1$, and $\sigma_\epsilon = \sigma_\varepsilon = 1$. Beginning with the Phillips curve, equation (21), we set $\beta = 1.0$ and $\phi = 0.5$, because these values are consistent with two-period overlapping relative real-wage contracts, and we set $\alpha = 0.1$, based on the empirical estimate in Christiano, Eichenbaum, and Evans (2001). In the optimizing IS curve, we assume that $\varphi = 0.8$. Very few estimates of θ are available in the literature, but we set $\theta = 0.5$, based on the estimate in Smets (2000). For the variances of the demand and supply shocks, we set both variances to one and note that the relative performance of optimal policy rules depends on the ratio of these two variances rather than on the variances themselves.

The Phillips curve and the IS curve constrain the central bank's optimization problem. To place these constraints in structural form, we simply define $\mathbf{y}_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}, \mathbf{v}_t = \begin{bmatrix} \epsilon_t \\ \varepsilon_t \end{bmatrix}, \mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -\alpha & 1 \end{bmatrix}, \mathbf{A}_1 = \begin{bmatrix} \theta & 0 \\ 0 & \phi \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} (1-\theta) & \varphi \\ 0 & (1-\phi)\beta \end{bmatrix}$, and $\mathbf{A}_3 = \begin{bmatrix} -\varphi \\ 0 \end{bmatrix}$. With the vectors and matrices so defined, the algorithms developed in sections 2 and 3 can be applied directly.

With the parameter values above, the fully optimal precommitment rule (PC) and the optimal discretionary rule (D) are[16]

$$i_t = 0.514\pi_{t-1} + 0.625y_{t-1} + 1.250\epsilon_t + 1.029\varepsilon_t - 0.057\mu_{t-1} \quad \text{(PC)} \qquad (22)$$

$$i_t = 0.805\pi_{t-1} + 0.625y_{t-1} + 1.250\epsilon_t + 1.609\varepsilon_t, \quad \text{(D)} \qquad (23)$$

where μ_t is the Lagrange multiplier on equation (21) in the precommitment formulation.

Crucially, these rules contain the current period shocks, ϵ_t and ε_t. However, as McCallum argues, central banks do not observe economic shocks in real time, and consequently they must rely on dated information when they set policy. In fact, even when historical data is extensively filtered and analyzed, general agreement on shocks in often not reached. Instead, McCallum (1995, 1999a) argues that, even if private agents have full information, for a policy rule to be operational it should only allow policymakers to respond to information that is available when policy decisions are made. In the context of the optimization problem above, McCallum's point, in essence, implies that rather than setting interest rates according to either equation (22) or equation (23), a more appropriate, and operational, policy rule would be

$$i_t = \varphi_1\pi_{t-1} + \varphi_2 y_{t-1}. \qquad (24)$$

Table 1 presents the optimal feedback coefficients for equation (24) with precommitment and discretion, and compares them to the feedback coefficients in equations (22) and (23). Also shown are the feedback coefficients and the minimized loss function for rules containing the demand and/or the supply shock.

[16] To solve for the fully optimal precommitment rule and the optimal discretionary rule the algorithms in Dennis (2001) were used.

Table 1	Optimal Feedback Coefficients: $\lambda = 0.5$.					
	π_{t-1}	y_{t-1}	ϵ_t	ε_t	μ_{t-1}	Loss
PC	0.514	0.625	1.250	1.029	-0.057	2.242
PC	0.514	0.338	1.250	1.029	-	2.242
D	0.805	0.625	1.250	1.609	-	3.049
PC	0.742	0.281	1.250	-	-	2.597
D	0.805	0.625	1.250	-	-	4.075
PC	1.215	0.895	-	0.953	-	2.980
D	0.805	0.625	-	1.609	-	3.668
PC	1.210	0.736	-	-	-	3.195
D	0.805	0.625	-	-	-	4.693
	$i_t = \varphi_1 \pi_{t-1} + \varphi_2 y_{t-1} + \varphi_3 \epsilon_t + \varphi_4 \varepsilon_t$					

Several results from Table 1 stand out. First, in this model the Lagrange multiplier, μ_t, is linearly related to y_t,[17] which is why the fully optimal precommitment rule leads to the same value for the minimized loss function as the optimized precommitment rule containing the state variables $\pi_{t-1}, y_{t-1}, \epsilon_t$, and ε_t. Second, relative to the fully optimal precommitment rule the optimal discretionary rule results in a 36% increase in expected loss, which is in line with the results in Dennis and Söderström (2002). Third, when the parameters in equation (24) are optimized, the minimized loss rises 43% with precommitment and 54% with discretion; these are substantial increases, and larger than those typically reported for other models (Rudebusch and Svensson, 1999; Levin et al. 2003). Fourth, for the optimal simple discretionary rules in Table 1, which contain all of the endogenous state variables in the system, certainty equivalence holds; the feedback coefficients on the endogenous state variables are independent of the shocks. However, even when the rule contains all endogenous state variables, certainty equivalence does not hold for the optimal simple precommitment rules. As a consequence, the feedback parameters in the optimal simple precommitment rules are sensitive to the covariance matrix of the shocks. Fifth, with precommitment, if one had to choose between knowing the demand shock or knowing the supply shock one would choose the demand shock; the opposite holds with discretion.[18] Where the central bank can precommit, not knowing the supply

[17] Backus and Driffill (1986) show that a linear relationship always exists between the Lagrange multipliers (μ_t here) and the state vector. In this application the linear relationship is particularly simple, with the Lagrange multiplier depending only on the output gap.

[18] In these New-Keynesian models, supply shocks produce a variance-trade-off between output and inflation. This trade-off is worse with discretion than with precommitment (in the sense that a

shock leads to the minimized loss increasing 16%, whereas not knowing the demand shock raises the minimized loss 33%.

While these results are only intended to be indicative, they support McCallum's point and raise the possibility that the reason that simple outcome-based or simple forward-looking policy rules – such as Taylor-type rules – perform well (relative to the fully optimal policy) is that they allow the central bank to respond to current period shocks. But of course, central banks, invariably, cannot respond to current period shocks when they set policy because current period outcomes are unavailable. In the remainder of this section we use the macroeconometric models estimated in Fuhrer (1997) and Rudebusch (2002) to examine this issue in greater detail.

5.1 Data-Consistent Models

5.1.1 Fuhrer (1997)

Fuhrer (1997) estimates a version of the Fuhrer-Moore model. In the Fuhrer-Moore model monetary policy operates through the term-structure. Movements in the short-term interest rate, i_t, are transmitted into the long-term real interest rate, ρ_t, which in turn impacts the output gap, y_t. Prices and wages are jointly determined, with a proportion of workers given the opportunity to renegotiate their wage contract each quarter. These workers and firms bargain over the nominal wage rate, w_t, that these workers will be paid during the coming year. While they bargain over nominal wages, workers negotiate keeping in mind what that nominal wage rate implies in terms of their real wage, $\bar{w}_t$. Across all workers, the average real wage received is v_t. Once agreement is reached on the contract wage rate, the level of aggregate demand determines the number of hours worked each quarter. Because hours worked is demand determined, if workers anticipate rising aggregate demand, then they negotiate a higher contract wage rate. Prices, p_t, are set as a constant mark-up over costs, which are determined by the nominal wage rate paid on existing

larger inflation variance is associated with any given output variance) thus it is not surprising that the supply shock matters more with discretion than with precommitment.

contracts. The model (estimated on quarterly data) is

$$
\begin{aligned}
y_t &= 1.45y_{t-1} - 0.47y_{t-2} - 0.34\rho_{t-1} + \epsilon_t \\
\rho_t &= \frac{40}{41}E_t\rho_{t+1} + \frac{1}{41}\left(i_t - E_t\pi_{t+1}\right) \\
p_t &= 0.42w_t + 0.31w_{t-1} + 0.19w_{t-2} + 0.08w_{t-3} \\
\pi_t &= 4\left(p_t - p_{t-1}\right) \\
v_t &= 0.42\bar{w}_t + 0.31\bar{w}_{t-1} + 0.19\bar{w}_{t-2} + 0.08\bar{w}_{t-3} \\
\bar{w}_t &= 0.42v_t + 0.31E_tv_{t+1} + 0.19E_tv_{t+2} + 0.08E_tv_{t+3} \\
&\quad +0.002\left[0.42y_t + 0.31E_ty_{t+1} + 0.19E_ty_{t+2} + 0.08E_ty_{t+3}\right] + \varepsilon_t,
\end{aligned}
$$

where the shock vector $\mathbf{v}_t = \begin{bmatrix} \epsilon_t \\ \varepsilon_t \end{bmatrix}$ is distributed with mean $\mathbf{0}$ and variance-covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} 0.36 & -0.03 \\ -0.03 & 0.03 \end{bmatrix}$.

5.1.2 Rudebusch (2002)

Rudebusch's (2002) model consists of three equations. The first equation is a dynamic IS curve in which the output gap, y_t depends on two lags of itself and on the (lagged) ex ante real interest rate. The second equation is an expectations augmented Phillips curve for (annualized) inflation, π_t. The final equation is an identity defining annual inflation, $\overline{\pi}_t$. The model (estimated on quarterly data) is

$$
\begin{aligned}
y_t &= 1.15y_{t-1} - 0.27y_{t-2} - 0.09\left(i_{t-1} - E_{t-1}\overline{\pi}_{t+3}\right) + \epsilon_t \\
\pi_t &= 0.29E_{t-1}\overline{\pi}_{t+3} + 0.71\left(0.67\pi_{t-1} - 0.14\pi_{t-2} + 0.40\pi_{t-3} + 0.13\pi_{t-4}\right) \\
&\quad +0.13y_{t-1} + \varepsilon_t \\
\overline{\pi}_t &= \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right),
\end{aligned}
$$

where the shock vector $\mathbf{v}_t = \begin{bmatrix} \epsilon_t \\ \varepsilon_t \end{bmatrix}$ is distributed with mean $\mathbf{0}$ and variance-covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} 0.69 & 0.00 \\ 0.00 & 1.02 \end{bmatrix}$. The monetary policy instrument is the nominal interest rate, i_t.

5.2 The Value of New Information

To assess the force of McCallum's criticism, we analyze the performance of four optimal simple rules as λ, the weight on inflation in the loss function, varies between

0.0 and 1.0 in the following policy objective function[19]

$$\begin{aligned} Loss &= \lim_{\beta \to 1} (1-\beta) E_t \sum_{j=0}^{\infty} \beta^j \left[\lambda \pi_{t+j}^2 + (1-\lambda) y_{t+j}^2 \right] \\ &= \lambda Var(\pi_t) + (1-\lambda) Var(y_t), \qquad 0 \leq \lambda \leq 1. \end{aligned} \tag{25}$$

The four simple rules that we analyze are

$$i_t = \phi_1 \pi_{t-1} + \phi_2 y_{t-1} \qquad \text{(lagged Taylor)} \tag{26}$$

$$i_t = \phi_1 \pi_{t-1} + \phi_2 y_{t-1} + \phi_3 \epsilon_t \qquad \text{(lagged Taylor + demand)} \tag{27}$$

$$i_t = \phi_1 \pi_{t-1} + \phi_2 y_{t-1} + \phi_4 \varepsilon_t \qquad \text{(lagged Taylor + supply)} \tag{28}$$

$$i_t = \phi_1 \pi_{t-1} + \phi_2 y_{t-1} + \phi_3 \epsilon_t + \phi_4 \varepsilon_t. \qquad \text{(lagged Taylor + shocks)} \tag{29}$$

Equation (26) is the standard Taylor rule formulation, but with lagged inflation and the lagged output gap entering the rule in place of contemporaneous values. The following three equations augment the lagged Taylor rule with the demand shock, the supply shock, and both shocks, respectively. Each rule's relative performance allows us to assess whether the assumption that the central bank can respond to each shock is important for economic outcomes.[20]

[19] A very small weight (0.01) is also placed on interest rate stabilization in order to prevent the feedback parameters in the optimal simple rules from blowing up to enormous magnitudes in the Fuhrer-Moore model. Aside from damping the magnitude of the feedback coefficients, this small weight on interest rate stabilization in no way affects our findings.

[20] While these four simple rules are not certainty equivalent in either of the two models, we analyze their properties using the shock variance-covariance matrices that are estimated along with the other model parameters. As such, the results that follow depend on all the parameters in the model, rather than on just those parameters that effect the model's conditional forecasts.

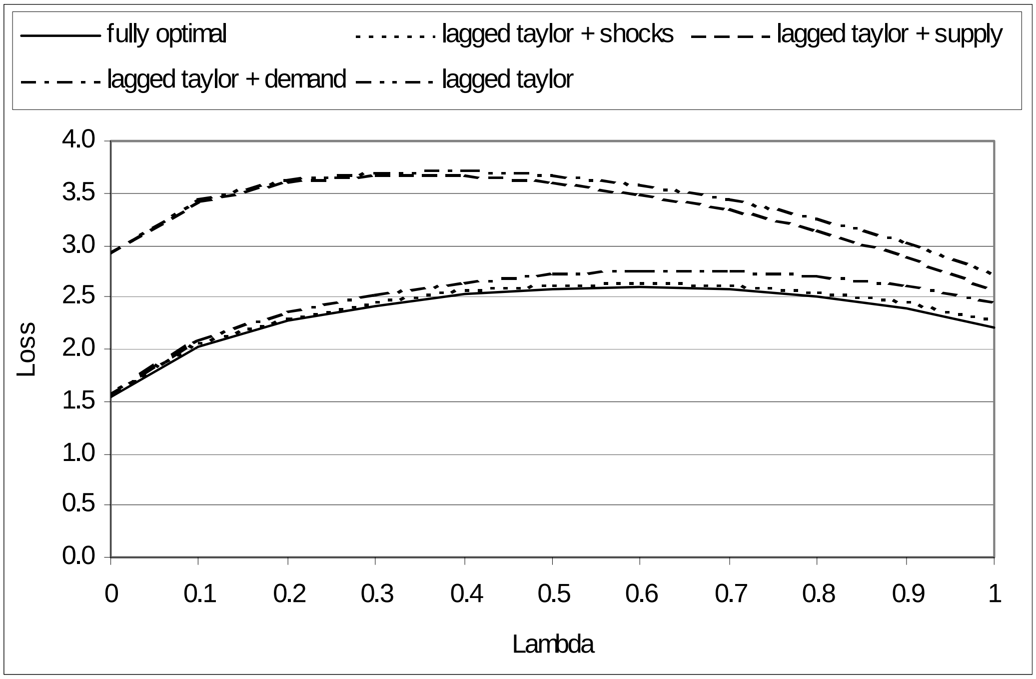

Fig. 1. Rudebusch Model under Precommitment

Figure 1 shows how these four rules perform in the Rudebsuch model, assuming that the central bank can precommit. Also shown is the performance of the fully optimal precommitment rule. By construction, the four optimal simple rules are inferior to the fully optimal precommitment rule, which efficiently responds to all available information. Two results come from Figure 1. The first result is that the two rules that contain the demand shock are nearly as efficient as the fully optimal precommitment rule. Only when λ is high, with the central bank focusing on stabilizing inflation, does the performance of the lagged Taylor + demand rule (Eqn. 27) become noticably inferior to the fully optimal precommitment rule. The second, related, result, is that not responding to the demand shock, i.e., rules (26) and (28), leads to a significant drop in performance, for all λ. In situations where λ is small, not being able to respond to the current period demand shock almost doubles the minimized loss function. Clearly, allowing the central bank to respond to the demand shock is critical for the Taylor rule's performance in the Rudebusch model.

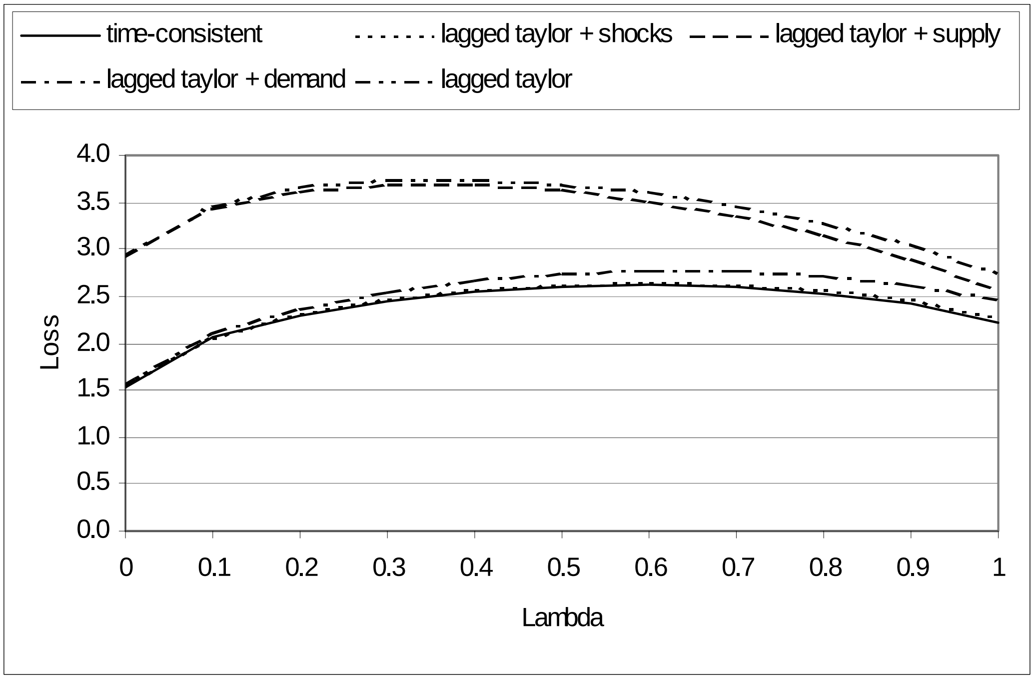

Fig. 2. Rudebusch Model under Discretion

In the case where the central bank has discretion, Figure 2 reveals a similar picture to the precommitment case. In Figure 2, however, the benchmark against which the four optimal simple rules are compared is the optimal discretionary rule (time-consistent rule), which responds to all state variables. Like the precommitment case, knowing and responding to the demand shock is critical for performance. The similarities between the precommitment rules and the discretionary rules can be seen in Table 2. Table 2 shows that for a given λ, the optimal simple precommitment rules and the optimal simple discretionary rules are very similar and that the time-inconsistency problem is miniscule. But, as expected, the minimized loss function with discretion is larger than that with precommitment.

Table 2	The Optimal Feedback Coefficients Rudebusch Model: $\lambda = 0.5$.			
	π_{t-1}	y_{t-1}	ϵ_t	ε_t	Loss
PC	2.29	1.29	7.11	1.91	2.622
D	2.09	1.26	7.21	2.22	2.633
PC	2.75	1.63	6.70	-	2.729
D	2.56	1.65	6.71	-	2.745
PC	2.96	2.50	-	1.44	3.610
D	2.66	2.46	-	1.75	3.635
PC	3.12	2.54	-	-	3.695
D	2.82	2.52	-	-	3.702
$i_t = \phi_1\pi_{t-1} + \phi_2 y_{t-1} + \phi_3\epsilon_t + \phi_4\varepsilon_t$					

Turning to the Fuhrer-Moore model, Figure 3 shows how these four optimal simple (precommitment) rules perform relative to the fully optimal precommitment policy. The main feature of Figure 3 is that all four optimal simple rules produce a numerically large efficiency loss, and that whether or not the central bank responds to the demand shock and/or supply shock has little effect on policy efficiency. In contrast to the Rudebusch model, therefore, setting monetary policy using the lagged Taylor rule rather than using the lagged Taylor + shocks rule leads to very little additional lost efficiency.

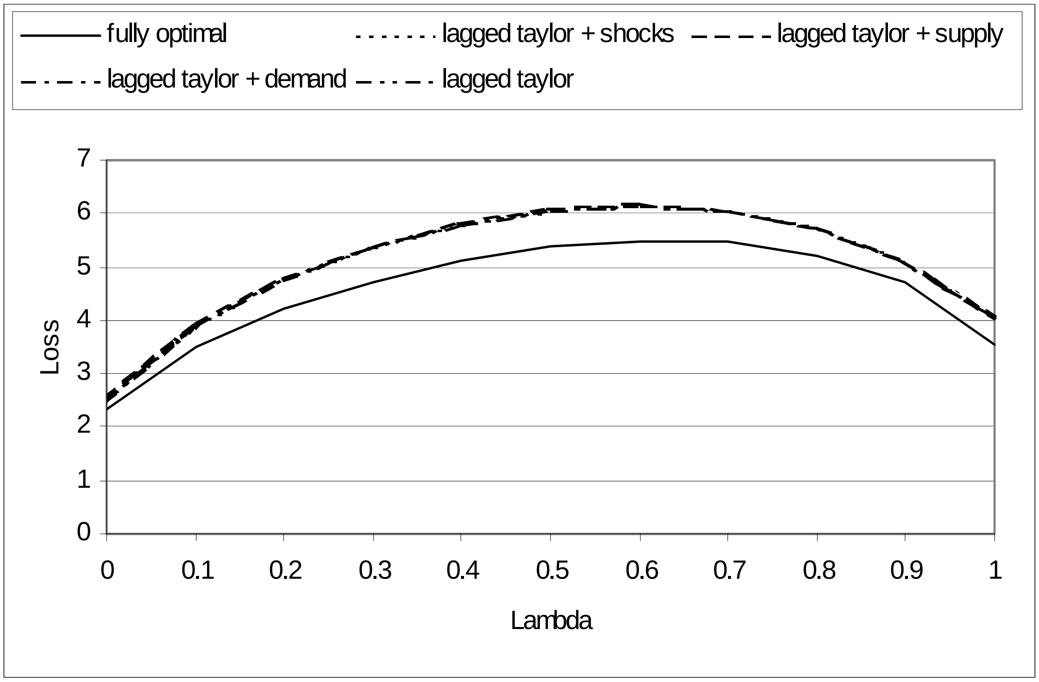

Fig. 3. Fuhrer-Moore Model under Precommitment

In the discretionary case, three results stand out. First, similar to the precommitment case, responding to the demand and/or the supply shock has very little effect on the minimized loss function. This can be seen from the fact that the curves generated by each policy rule largely coincide with each other. Second, not responding to the entire state vector, while inefficient, is not as inefficient with discretion as it is with precommitment. The performance difference between the optimal simple discretionary rules and the optimal discretionary rule (time-consistent rule) is not as large as that between the equivalent optimal simple precommitment rules and the fully optimal precommitment policy. Finally, the time-inconsistency problem is far more pronounced in the Fuhrer-Moore model than it is in the Rudebusch model. For all λ, the minimized loss function is considerably higher with discretion than it is with precommitment.

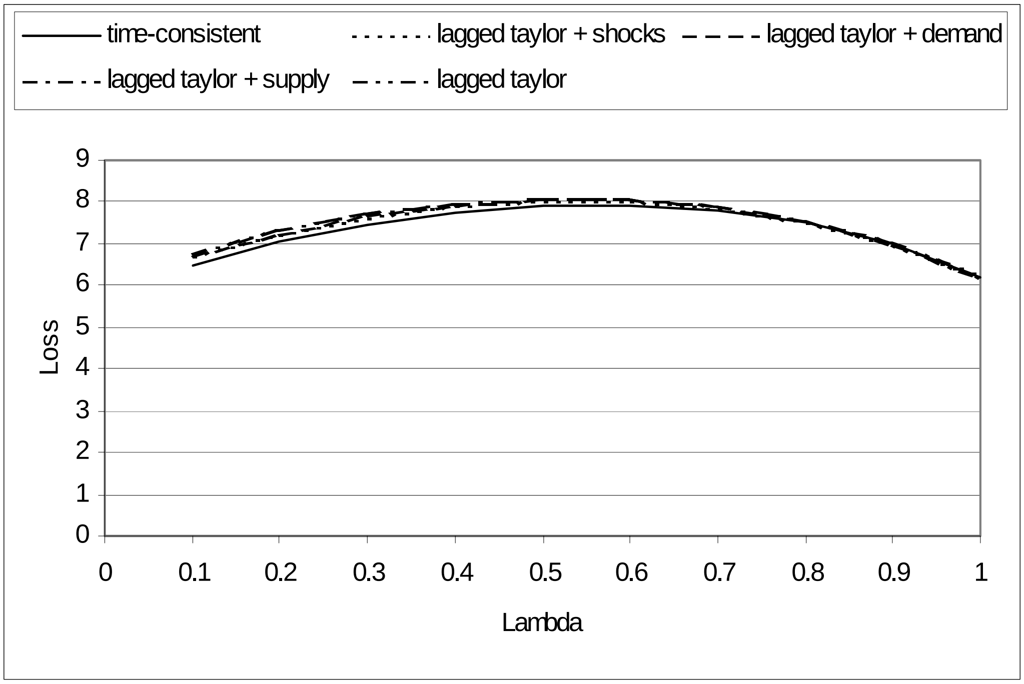

Fig. 4. Fuhrer-Moore Model under Discretion

Table 3 presents the feedback coefficients and the minimized loss function with precommitment and discretion for the case where $\lambda = 0.5$. Unlike the Rudebusch model, for the Fuhrer-Moore model the feedback coefficients in the optimized policy rule differ greatly between precommitment and discretion. With $\lambda = 0.5$, the central bank's inability to precommit leads to the minimized loss function rising approximately 33%, whichever optimal simple rule is used.

Table 3	The Optimal Feedback Coefficients Fuhrer-Moore Model: $\lambda = 0.5$.				
	π_{t-1}	y_{t-1}	ϵ_t	ε_t	Loss
PC	3.13	2.64	2.55	1.23	6.072
D	1.34	1.69	3.65	9.29	8.052
PC	3.13	2.63	2.46	-	6.072
D	1.34	1.70	2.97	-	8.071
PC	3.16	2.69	-	-1.20	6.094
D	1.34	1.70	-	5.78	8.102
PC	3.16	2.69	-	-	6.095
D	1.34	1.70	-	-	8.108
$i_t = \phi_1 \pi_{t-1} + \phi_2 y_{t-1} + \phi_3 \epsilon_t + \phi_4 \varepsilon_t$					

5.3 Discussion

McCallum's concerns about policy rule operationally appear to apply with force to Taylor-type rules in the Rudebusch model, but not in the Fuhrer-Moore model. In the Rudebusch model, optimal outcome-based Taylor-type rules and optimal forecast-based Taylor-type rules are highly efficient. However, this efficiency is achieved through the central bank responding to current period demand and supply shocks. Focusing on the case where $\lambda = 0.5$ and where the policymaker can precommit, the minimized loss functions under the fully optimal precommitment rule, the lagged Taylor + shocks rule, and the lagged Taylor rule are 2.578, 2.622, and 3.695, respectively. Thus the lagged Taylor + shocks rule is only 1.7% less efficient than the fully optimal precommitment rule, while the lagged Taylor rule (without the shocks) is 40% less efficient than the fully optimal precommitment policy.[21]

Very different results were obtained from the Fuhrer-Moore model. In the Fuhrer-Moore model, the minimized loss function is largely invariant to whether the central bank responds to the demand and/or the supply shock directly. This finding is consistent with the results in Levin et al. (1999), although they assume interest rate smoothing and place the lagged nominal interest rate in the policy rule.[22] The lagged Taylor-type rule's inefficiency in the Fuhrer-Moore model arises primarily because it excludes endogenous state variables and not because it excludes current period shocks.

The differences between the results from the Fuhrer-Moore model and those from the Rudebusch model and the Clarida et al. (1999) model are striking, but also intuitive. For a central bank that aims to stabilize inflation and the output gap the advantages to knowing the demand and supply shocks come in one of two forms. First, knowing the shocks is essential if the policymaker is to offset the shocks before they impact the economy. Second, knowing the shocks provides the policymaker with information that it can use to help forecast future outcomes. In the Clarida et al. (1999) model the main advantage to knowing the shocks – particularly the demand

[21] With discretion, the equivalent efficiency losses are 1% for the lagged Taylor + shocks rule and 42% for the lagged Taylor rule that excludes the shocks. In the discretionary case the efficiency losses are measured relative to the time-consistent, or optimal discretionary, rule.

[22] For all $0 < \lambda < 1$, Levin et al. (1999) find that outcomes under the Taylor-type rules are "nearly indistinguishable" from the rule that responds to all state variables. However, they: 1) assume interest rate smoothing and place the lagged federal funds rate in their simple policy rule in addition to the output gap and inflation; and 2) use an optimal rule that contains all state variables as their benchmark, rather than the fully optimal precommitment rule.

shock – is that monetary policy can reduce or offset their impact as they occur. However, for the Rudebusch and Fuhrer-Moore models control lags mean that this is infeasible. Instead, for these two models, knowing the shocks is important to the extent that they help the policymaker to forecast current and future outcomes. In the Fuhrer-Moore model, observing the shocks does not greatly benefit the policymaker because the variances of the shocks are small and it is their propagation through time that policymakers must largely contend with. For the Rudebusch model, the variances of the shocks are larger – particularly that for the demand shock – and, as a consequence, not observing the shocks leads to larger forecast errors and a deterioration in policy performance.

Table 4 compares the performance of the optimal simple rules with discretion to those with precommitment. In line with Clarida et al. (1999), economic outcomes with discretion are consistent with output being over-stabilized and inflation being under-stabilized relative to precommitment, which suggests that appointing a conservative central banker (higher λ) would improve outcomes (Jensen, 2002). This "stabilization bias" is reflected in the unconditional variances of inflation and the output gap. For the Rudebusch model, time-inconsistency and stabilization bias have a relatively small effect on the variances of inflation and the output gap. However, for the Fuhrer-Moore model, while discretionary policymaking lowers the variance of the output gap the variance of inflation is raised considerably.

Table 4	Comparing Precommitment with Discretion ($\lambda = 0.5$)							
	Rudebusch				Fuhrer-Moore			
	Precommitment		Discretion		Precommitment		Discretion	
Rule	$Var[\pi]$	$Var[y]$	$Var[\pi]$	$Var[y]$	$Var[\pi]$	$Var[y]$	$Var[\pi]$	$Var[y]$
Eqn. 26	2.93	3.61	3.24	3.42	6.56	4.80	12.10	3.70
Eqn. 27	2.53	1.97	2.73	1.83	6.56	4.80	12.09	3.57
Eqn. 28	2.81	3.63	3.06	3.49	6.55	4.75	12.04	3.72
Eqn. 29	2.33	1.94	2.42	1.85	6.55	4.75	12.00	3.59

Dennis and Söderström (2002) show that one reason why the stabilization bias is small in the Rudebusch model is that private agents form their expectations using period $t-1$ information, which affects the policymaker's ability to influence economic outcomes through expectations.

6 Conclusion

A large portion of the monetary policy rules literature examines the properties of optimal simple rules. Where these rules have been constructed and evaluated in forward-looking models the usual approach has been to cast the model in state-space form and to assume that the policymaker can precommit. This paper extends the literature on solving for optimal simple rules to the case where the policymaker cannot precommit, and formulates the precommitment and the discretionary optimization problems with the optimization constraints written in structural form. The structural form can be advantageous because economic models are invariably formulated in terms of structural equations.

In the precommitment case, a general solution algorithm was developed that can solve for a wide class of optimal simple rules, including forward-looking rules, which contain variables that are not predetermined. With discretion we focused on the limiting case where the discount factor in the objective function tends to one. In this limiting case, we showed that the optimization problem could be formulated and solved recursively, and that the feedback coefficients in the resulting policy rule were independent of the state vector, as is necessary for time-consistency.

Having developed the solution algorithms, section 5 applied these algorithms to investigate whether operational Taylor-type rule, i.e., rules that preclude the central bank from responding to current period outcomes, lead to large increases in the value of the minimized loss function. Through analyzing the macroeconomic models estimated in Fuhrer (1997) and Rudebusch (2002), and a calibrated version of the Clarida et al. (1999) model, we showed that in the Rudebusch model and the Clarida et al. (1999) model, setting policy using an operational Taylor-type rule leads to a large increase in the value of the minimized loss function. In the benchmark case with $\lambda = 0.5$, using an operational Taylor-type rule, the minimized loss function in the Clarida et al. (1999) model increased 43 percent with precommitment and 54 percent with discretion. In the Rudebusch model similar results were obtained with precommitment and discretion: the minimized loss was 40 percent higher in the case with $\lambda = 0.5$, rising to around 90 percent higher as greater weight was placed on output stabilization (smaller values for λ); much of this efficiency decline occurred because the policymaker could not respond to current period demand shocks. Only in the Fuhrer-Moore model did we find that the value of the minimized loss function was largely invariant to whether an operational Taylor-type rule is used instead of an outcome-based rule or a forward-looking rule.

Comparing precommitment with discretion, we showed that time-inconsistency problems did not have a large impact on economic outcomes in the Rudebusch model, but that time-inconsistency was important in the Fuhrer-Moore model. In the Fuhrer-Moore model, discretionary policymaking lowered the variance of the output gap, but raised the variance of inflation considerably.

A Appendix - The Limiting Form of the Policy Objective Function as $\beta \to 1$

In this appendix we establish that in the limit as $\beta \to 1$ equation (4), scaled by $(1-\beta)$ converges to $tr\left[\mathbf{W\Omega}\right]$. Recall that with precommitment the value function, with discounting, was

$$L_t = \mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \frac{\beta}{(1-\beta)} tr\left[\boldsymbol{\theta}_2'\mathbf{P}\boldsymbol{\theta}_2\boldsymbol{\Phi}\right], \tag{30}$$

with $\mathbf{P}$ the solution to

$$\mathbf{P} = \mathbf{W} + \beta\boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1. \tag{31}$$

Recall also that the unconditional variance-covariance matrix of $\mathbf{z}_t$ is the fixed point of

$$\boldsymbol{\Omega} = \boldsymbol{\theta}_1\boldsymbol{\Omega}\boldsymbol{\theta}_1' + \boldsymbol{\theta}_2\boldsymbol{\Phi}\boldsymbol{\theta}_2'. \tag{32}$$

Analogous relationships hold under discretion. Multiplying equation (31) through by $(1-\beta)$ gives

$$\begin{aligned}
(1-\beta)L_t &= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\boldsymbol{\theta}_2'\mathbf{P}\boldsymbol{\theta}_2\boldsymbol{\Phi}\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\mathbf{P}\boldsymbol{\theta}_2\boldsymbol{\Phi}\boldsymbol{\theta}_2'\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\mathbf{P}\left(\boldsymbol{\Omega} - \boldsymbol{\theta}_1\boldsymbol{\Omega}\boldsymbol{\theta}_1'\right)\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\mathbf{P}\boldsymbol{\Omega}\right] - \beta tr\left[\mathbf{P}\boldsymbol{\theta}_1\boldsymbol{\Omega}\boldsymbol{\theta}_1'\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\mathbf{P}\boldsymbol{\Omega}\right] - \beta tr\left[\boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1\boldsymbol{\Omega}\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\left(\mathbf{P} - \boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1\right)\boldsymbol{\Omega}\right] \\
&= (1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + \beta tr\left[\left(\mathbf{W} - (1-\beta)\boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1\right)\boldsymbol{\Omega}\right]
\end{aligned}$$

In the limit as $\beta \to 1$, $(1-\beta)L_t \to tr\left[\mathbf{W\Omega}\right]$, provided $(1-\beta)\mathbf{z}_t'\mathbf{P}\mathbf{z}_t \to 0$ and $(1-\beta)\,tr\left[\boldsymbol{\theta}_1'\mathbf{P}\boldsymbol{\theta}_1\right] \to 0$. These two terms converge to zero provided the solution to equation (32) is bounded as $\beta \rightarrow 1$. It is clear that the fixed-point solution to equation (32) will be bounded when the spectral radius of $\boldsymbol{\theta}_1$ is less than one, which from Hamilton (1994, pp186) amounts to the requirement that when subject to control $\mathbf{z}_t$ be weakly stationary and ergodic. In control theory terms, $\mathbf{z}_t$ must be stabilizable. But the condition that the spectral radius of $\boldsymbol{\theta}_1$ be less than one arises naturally from the requirement that the model has a unique stationary equilibrium. Consequently, the optimization problem is well defined in the limiting case where $\beta \rightarrow 1$ and the objective function can be expressed in terms of a linear combination of unconditional variances.

Note that the term $\mathbf{z}_t^{'}\mathbf{P}\mathbf{z}_t$ in equation equation (30) describes how the feedback coefficients in the optimal simple rule depend on the initial state vector. In the limiting case where $\beta \rightarrow 1$ the dependence of the policy rule on the initial state vanishes.

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