Embodying Embodiment in a Structural, Macroeconomic Input-Output Model

This draft: October 25, 2001

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Abstract:

This paper describes an attempt to build a regression-based system of labor productivity equations that incorporate the effects of capital-embodied technological change into IDLIFT, a structural, macroeconomic input-output model of the U.S. economy. Builders of regression-based forecasting models have long had difficulty finding labor productivity equations that exhibit the Neoclassical or Solowian property that movements in investment should cause accompanying movements in labor productivity. Theory dictates that this causation is driven by the effect of traditional capital deepening as well as technological change embodied in capital. Lack of measurement of the latter has hampered the ability of researchers to properly estimate the productivity-investment relationship. Wilson (2001a), by estimating industry-level embodied technological change, has alleviated this difficulty. In this paper, I utilize those estimates to construct capital stocks that are adjusted for technological change which are then used to estimate Neoclassical-type labor productivity equations. It is shown that replacing IDLIFT's former productivity equations, based on changes in output and time trends, with the new equations results in a convergence between the dynamic behavior of the model and that predicted by Neoclassical production theory.

Keywords: Equipment-Embodied Technological Change, Input-Output, Productivity, Forecasting

JEL Codes: C3, C5, O3

* This paper was adapted from Chapters 4 and 5 of my dissertation at the University of Maryland. Much of the research for this paper was conducted while I was a researcher at the INFORUM research center. I would like to thank the staff of INFORUM, particularly Doug Meade, and its director (and my advisor) Clopper Almon for all of their support and advice.

1. Introduction

The hypothesis that much of technological progress is embodied in new capital goods, and therefore investment in new capital is necessary to foster productivity growth, is an old one -- tracing its roots at least as far back as Smith's *Wealth of Nations*, which attributed its source to the division of labor: "The invention of all those machines by which labour is so much facilitated and abridged, seems to have been originally owing to the division of labour" (Smith, 1776, p.9).[1] The basic hypothesis was refined and extended over time by Karl Marx, Joseph Schumpeter, and Robert Solow, among others.[2] Yet, obtaining independent measures of the rate(s) at which embodied (or "investment-specific") technological change has progressed has long eluded us. Absent knowledge of this rate, it is impossible to correctly measure the *productive capacity* of the economy's capital stock. The concept of the *productive capacity of capital*, or simply *productive capital* for short, is the theoretically correct (in terms of Neoclassical production theory) concept of capital to be used in production and productivity analyses. The productive capital stock, combined with information on the degree to which capital is being utilized, tells us the flow of capital services used in the production process. The flow of capital services to production is one of the main determinants of labor productivity. Thus, modeling long-run labor productivity growth relies on good measures of productive capital (as well as utilization rates). And, of course, modeling long-run productivity growth is *a*, if not *the*, key element to any forecasting model.

Yet surprisingly little research has focused on the measurement of capital and the implications of mismeasurement for modeling productivity.[3] The situation appears to be changing, however. Thanks in part to the rapid advances in equipment technology which have exacerbated and exposed the shortcomings of the current ways of measuring capital, researchers interested in productivity analysis and forecasting can no longer ignore these shortcomings in their empirical work.

The thesis work of Wilson (2001a) attempts to satisfy the need for a quantitative idea of the contribution these technological advances in equipment have on productive capital and productivity (and more importantly, on their growth rates).. In that work, I first develop a production-side approach to estimating equipment-embodied technological change.[4] The method

[1]See Scherer (1999), Chapters 2-4, for a discussion of the history of economic thought relating to technological change (particularly that which is embodied in machinery) and long-run productivity growth.

[2]In *The Communist Manifesto*, Marx argued that technological advances in machinery are a distinguishing feature of the "bourgeois" or capitalist system: "The bourgeoisie cannot exist without constantly revolutionizing the instruments of production, and thereby the relations of production, and with them the whole relations of society" (Marx and Engels, 1848).

[3]Unless otherwise indicated, *capital* will hereafter refer to *productive capital*.

[4]It should be noted that the general production-side approach and the estimation of embodied technological change for aggregate U.S. manufacturing was a collaborative effort with Plutarchos Sakellaris (see Sakellaris and Wilson (2001)) and was also presented in Chapter 2 of

estimates embodied technological change for U.S. manufacturing industries directly from observed production, input, and investment decisions at the plant level using the Longitudinal Research Database at the U.S. Census Bureau. Specifically, I estimate a gross-output production function in which the equipment capital input is a parameterized stream of all past investments net of physical depreciation. The vintage weights in this stream, or perpetual inventory, imply an estimable rate of embodied technological change.

The empirical results are shown in the top panel of Table 1. The estimates of embodied technological change generally clustered around 5-10%. 15 of the 24 industries had positive estimates with 9 being significant. Of the 9 negative estimates, 5 were significant. These estimates seem quite reasonable in their ordering, but the presence of negative estimates (which are at best counter-intuitive and at worst nonsensical) and of unrealistically high estimates for producers of Computers and Communications Equipment as well as the relative imprecision of the estimates leaves some skepticism regarding the usefulness of these estimates.

Evaluating the reasonableness of the estimated rates of embodied technological change and deriving analogous rates for nonmanufacturing industries, for which longitudinal plant-level data is not currently available, was the motivation for Wilson (2001b). In that study, I construct an index capturing the extent of research and development directed at the various capital goods that constitute a given industry's capital stock. Specifically, I combine (and adjust) data from the National Science Foundation and the Commerce Department to construct a weighted average of the R&D done on the equipment capital that an industry purchases for 62 industries that span the U.S. private economy. This industry-level index of capital-embodied R&D is shown to have a large, positive correlation with the manufacturing estimates of embodied technological change.

The estimated relationship in manufacturing between embodied technological change and the index of embodied R&D, along with the index values for nonmanufacturing industries, is used to impute nonmanufacturing rates of embodied technological change. These rates are shown in the bottom panel of Table 1. They range from 0 to 11%. It should be noted that the estimated coefficients in the imputation regression have large standard errors, thus the imputed rates have correspondingly large standard errors associated with them. Nonetheless, the magnitudes and the cross-sectoral ranking of these rates of embodied technological change are quite reasonable. For instance, the lowest imputed rates of embodied technological change are found in mining industries which, not surprisingly, invest mainly in mining equipment which has historically experienced very little R&D. The highest rate is in Communications Services which invests mainly in R&D-intensive equipment such as telecommunication equipment and computers.

In the current paper, I use the results of the above research to analyze the effect on the IDLIFT model of replacing the former labor productivity equations, which contain no influence from investment, with Neoclassical-type equations, estimated using correctly measured productive capital stocks. Section 2 briefly describes the structure, particularly of the labor productivity equations, of the IDLIFT input-output simulation and forecasting model which is maintained by

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my dissertation. Chapter 3 of my dissertation extended these empirical results to the industry-level and presented a separate method of deriving rates of embodied technological change for non-manufacturing industries. Chapter 3 formed the basis for Wilson (2001b).

the INFORUM research center.[5] In this section, I also discuss the shortcomings of the current labor productivity equations that motivated the proposed changes suggested in this paper. Then, armed with a full set of industry-level estimates of capital-embodied technological change spanning the economy from the aforementioned research, I construct *quality-adjusted*[6] equipment capital stocks in Section 3. The estimation of several alternative sets of productivity equations utilizing the constructed capital stocks is described in Section 4. Section 5 discusses the choice of which set of productivity equations to use in the new model and the incorporation of these equations into the programming framework of IDLIFT. In Section 6, Base forecasts are generated for both the current version of the model and the new, rival version. Two alternative scenarios, or "shocks," are then introduced to each model and the deviations from base are analyzed. Section 7 concludes.

2. Brief Overview of the IDLIFT model

A. The structure of IDLIFT

Since its founding in 1967 by Clopper Almon, Inforum has been building, and encouraging others to build, regression-based structural macroeconomic models based on input-output relationships between industries. The Inforum modeling philosophy differs from that of other large-scale macro models primarily in the input-output structure underlying the model.

Inforum's main model of the U.S. economy is IDLIFT, which is presently in the process of replacing its predecessor, LIFT (Long-term Interindustry Forecasting Tool).[7] In this section, I will discuss the general structure of the IDLIFT model as it currently stands. For a discussion of how IDLIFT differs from the LIFT model and planned future changes to the model (aside from those proposed in this paper), see Meade (1999).

The IDLIFT model forecasts output, employment, prices, exports, imports and interindustry flows for 97 commodity sectors; personal consumption expenditures (PCE) for 92 categories; equipment investment by 55 industries, construction spending for 19 categories; and the components of value-added for 51 industries. In addition, the model provides a full accounting of the macroeconomy. Macroeconomic variables such as the personal savings rate or the 3-month Treasury bill rate are estimated econometrically. Others are determined according to national accounting identities and still others are given to the model exogenously.

The overall structure of the model is based on the national accounting system embodied in

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[5]INFORUM stands for Interindustry Forecasting at the University of Maryland. It is a non-profit research center founded by Clopper Almon in 1967 which provides industry-level and macroeconomic forecasting and policy analysis. Douglas Meade has been largely responsible for the development of IDLIFT.

[6]In the context of the capital, the terms *quality* and *embodied technological change* should be thought of as synonymous.

[7]The "ID" in IDLIFT stands for Interdyme, the C++ framework developed at Inforum for building interindustry dynamic macroeconomic models (LIFT was built using Fortran).

the U.S. national income and product accounts (NIPA). There is a real side and a price side. On the real side, each component of final demand (i.e., the usual C+I+G+X-M) is modeled at the various levels of disaggregation mentioned above using structural behavioral equations. The disaggregate, sectoral equations have been estimated individually (as is the case with the labor productivity equations) or as a system (such as a demand system for consumption equations) using mainly industry-level time series data. Bridge matrices convert each of these final demand components from their particular level of disaggregation to the 97-sector commodity level. Sectoral (gross) output is then determined according to the fundamental input-output equation:

$$q = Aq + f\ , \tag{1}$$

where q is a 97×1 vector of output, A is the intermediate coefficient matrix (also called the input-output matrix or the requirements matrix), and f is the vector of final demand:

$$\begin{aligned} f_{97\times 1} = {} & H^{c}_{97\times 92} c_{92\times 1} + H^{eq}_{97\times 55} eq_{55\times 1} + H^{s}_{97\times 19} s_{19\times 1} \\ & + i_{97\times 1} + x_{97\times 1} - m_{97\times 1} + g_{97\times 1}. \end{aligned} \tag{2}$$

The subscripts indicate the dimension of each matrix or vector. Here *c* denotes the consumption vector, *eq* denotes equipment investment by purchaser, *s* structures investment (construction) by type of structure, *i* inventory change, *x* exports, *m* imports, and *g* government spending.[8] H^j is the bridge matrix for component j. All of the variables in equations (1) and (2) should rightly have time subscripts as well, including the A and H matrices which vary according to trends in the across-the-row totals. A detailed discussion of the equations or systems that forecast the components of the final demand vector is beyond the scope of this paper.[9]

Given the forecasted vector of output (q^*), employment (number of jobs) by sector is computed as:

$$n^* = q^* \cdot \left(\frac{1}{(q/\ell)^*} \right) \cdot \left(\frac{\ell}{n} \right)^* , \tag{3}$$

where ℓ is hours worked. An asterisk indicates that a variable is forecasted by the model. For instance, ℓ is not a variable in the model *per se* (it is determined by identity once $[\ell/n]^*$ and n^* are forecasted), but the average hours per job (ℓ/n) and labor productivity (q/ℓ) are. Employment forecasts, together with forecasts of the labor force, determine the unemployment rate, a key variable in the model. Aside from being extremely interesting in its own right, the unemployment rate affects many macroeconomic and industry equations on both the real and the income side of

[8]In the model, government spending is actually decomposed into 5 components such as state and local spending, defense spending, etc. The macro-level of these components are generally exogenous to the model; the exogenous macro values are shared-out to the 97 sector level using the sectors' shares of that component of government spending from the most recent year of available data.

[9]For such a discussion, see Meade (1999).

the model. By extension, then, it is evident that labor productivity is a key driver of the model (both through its effect on the model's unemployment rate and through its own direct presence in many model equations).

On the income/price side of the model, prices at the 97-sector level are determined according to equations modeling the markups over unit intermediate and labor costs. Given this forecasted price row vector p (1×97), value added by commodity sector is calculated as a residual using the dual of the fundamental input-output equation:

$$p = pA + v\,. \tag{4}$$

The components of value added (corporate profits, inventory valuation adjustment, capital consumption adjustment, net interest income, rental income, indirect taxes, government subsidies, and the big one: labor compensation) are each modeled separately. The forecasted values of the capital income components (everything except labor compensation) are then scaled to be consistent with equation (4) and the markup forecasts. Hourly labor compensation is modeled as a function of the growth in M2/GNP, the growth in labor productivity, and a supply shock (it is then multiplied by the forecast of the labor hours requirement, ℓ, from the real side). So we can see that labor productivity has an important influence on the income side of the model as well.

B. The Problem and the Need for Change

With its considerable influence on labor compensation on the income side and employment and the savings rate on the real side, it should be evident by now that labor productivity is one of the most important variables in the IDLIFT model (as well as virtually any other large-scale structural macro model). Currently, the IDLIFT model's labor productivity equations are determined essentially by time trends and the difference between industry output and its previous peak, and does not contain any factor inputs as explanatory variables:

$$ln(q^i / l^i) = \beta^i_0 + \beta^i_1 t_1 + \beta^i_2 t_2 + \beta^i_3 qup + \beta^i_4 qdown \tag{5}$$

where: t_1 = a linear time trend starting in the first year of data;

t_2 = a second time trend, starting in 1972;

$qup_t = dq_t$, when $dq_t > 0$, 0 otherwise;

$qdown_t = -dq_t$, when $dq_t < 0$, 0 otherwise;

$dq_t = \ln(q_t) - \ln(qpeak_{t-1})$;

$qpeak_t = q_t$, if $q_t > qpeak_{t-1}(1\text{-}spill)$, otherwise $= qpeak_{t-1}(1\text{-}spill)$;

$spill$ = depreciation rate of capacity;

and i indexes the 55 industries/sectors.

Inforum has long had difficulty building into its models a sensible relationship between investment and labor productivity. Given that labor productivity is the key driver of the long-run output growth behavior of the model, the lack of an influence from investment or capital stock is lamentable. Virtually any neoclassical-based growth model attributes a substantial share of output growth to the growth of capital. Its omission from Inforum models, IDLIFT in particular, is due neither to a disbelief in neoclassical production theory nor to a lack of effort.

Many valiant attempts have been made over the years to develop and estimate productivity equations based on firm optimization behavior that incorporate the effects of changes in capital stock. These attempts have generally been foiled by one of two problems. First, in industry-level time-series regressions (with which the IDLIFT equations are typically estimated), the capital coefficient is often found to be either negative or positive but very close to zero (particularly in service sectors). Second, because the investment equations in IDLIFT have always been of a *flexible accelerator*-type nature (i.e. driven largely by current and lagged changes in output), the introduction of investment (via capital stock) into the productivity equations provided a seed for the explosion of output in the model's forecast. Any exogenous positive shock to the model caused output to grow, which caused investment to grow, which caused labor productivity to grow, which caused output to grow (mainly through productivity's increasing of the wage rate which lowers the savings rate which thus spurs consumption, the largest component of final demand),...*ad infinitum*. The model has lacked a supply constraint (such as a nonconvex adjustment cost in the investment equations) to put the brakes on investment and stabilize output.

For these reasons, IDLIFT's labor productivity equations (as well as those of other Inforum-type models) have heretofore remained essentially a series of time trends. Inforum's discontent with this situation has been around since its inception, as demonstrated here by the words of Almon (1969) describing an early version of IDLIFT's predecessor, LIFT:

> Until recently, our model has used *exogenous projections of labor productivity which were based on simple extrapolations of past trend*. This practice left an awkward hole in the middle of the model. For on the one hand, the endogenous generation of investment by industry was one of the distinguishing features of the model; and on the other hand, the *growth in labor productivity essentially determines the overall growth projection given by the model*. Even the most casual observation suggests that capital investment has something to do with the increase in labor productivity. Therefore, the absence of any connection between the two in the model struck people as a clear indication of ineptitude, or at least indolence on our part.
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> The truth is that it is easier to recognize that there must be some connection than to measure the connection. We have made a number of false starts on the problem. ... At length, *we gave up the production approach to labor productivity* -- *although we retain it for capital investment* -- *because we couldn't make it work as well as the simple time trend equation*. (Italics added).

The above statement was quoted in Meade (1999) who went on to say: "Thirty years have passed since this remark, and we are no closer to a labor productivity equation that incorporates capital, research and development or any other significant influence we believe should be working."

The maintained hypothesis has been that one of the key problems with finding a successful Neoclassical equation has been mismeasurement of capital due to unobserved changes in embodied technology. It is well-known that classical measurement error causes an attenuation bias on the coefficient associated with the mismeasured independent variable. In fact, the problem is even worse. The measurement error in equipment capital that is caused by ignoring embodied technological change is not random; it is systematically related to the intertemporal investment distribution. The error will be greater the more an industry's capital is comprised of recent vintages. Recent investment will be positively correlated with other factor inputs such as labor. This will lead to an upward bias in the estimated labor elasticity. Furthermore, if constant returns to scale are imposed, this positive bias in labor elasticity implies a lower capital elasticity (in a value-added production function).

Thus, in order to correct this measurement problem, in the next section I construct quality-adjusted capital stocks using the estimated rates of embodied technological change in Table 1. In Section 4, I estimate various labor productivity equations, some of which attempt to avoid the measurement error either by using the quality-adjusted capital stocks or by including the stock of embodied R&D along with unadjusted capital stock as an independent variable.

3. Constructing Quality-Adjusted Capital Stocks

As I stated above, quality-adjusted capital stocks are needed in order to properly estimate labor productivity equations that are of a Neoclassically based specification. In addition, for a structural model that forecasts labor productivity based partially on forecasted capital, a capital stock formula must be built into the model such that capital can be updated in each future period using the model's forecast of investment. The capital stocks I construct in this paper are defined according to the usual perpetual inventory formula that aggregates current and past vintages of investment into a current real capital stock according to some weighting scheme (i.e., a distributed lag):

$$K_t = \sum_{s=1}^{T} I_{t-s} \cdot \underbrace{\left[\left(\frac{1}{p_{t-s}} \right) D_{t,t-s} (1+\gamma)^{t-s-t_0} \right]}_{Weight(t-s)} \qquad (6)$$

where K is the capital stock (either equipment or structures), I is nominal investment, p is the price deflator, γ is the rate of embodied technological change, and $D_{t,t-s}$ captures physical depreciation (i.e., wear and tear). $D_{t,t-s}$ gives the fraction of vintage t-s capital still in production in year t. In most capital stock data constructs, the quality (technology) change component is generally considered to be included in either the measure of depreciation (making it *economic* depreciation rather than *physical* depreciation) or the price deflator.[10] Unfortunately, most data sources of investment price deflators and economic depreciation do not adequately adjust for quality change in equipment. Thus, it is important to decompose the vintage weights in equation (6) into the three separate components of physical depreciation, quality change, and price change.

Using the definition of capital in equation (6), I construct separate industry-level capital stocks for structures and equipment. For structures, I assume that technological change is negligible and thus $\gamma = 0$. For equipment, the rates of embodied technological change come from Table 1. As for the price deflator, Hornstein and Krusell (1996) show that if embodied technological change is measured independently, one should deflate investment by a consumption deflator. I measure physical depreciation in structures as the inverse of the weighted average of the service lives of the structures assets owned by the industry. The weights are the industry's shares of capital in each asset type constructed from the capital flows tables supplied by the U.S. Bureau of Economic Analysis (BEA).

The Board of Governors of the Federal Reserve (FRB) and the U.S. Bureau of Labor Statistics (BLS) construct capital stocks using a methodology for capturing physical depreciation based on stochastic service lives and a nongeometric, "beta-decay" function. In Sakellaris and

[10] See Gort and Wall (1998) or Hulten and Wykoff (1981) for discussions of the distinction between economic and physical depreciation.

Wilson (2001), we back out the implied industry-level physical depreciation patterns, $D_{t,t-s}$, for equipment from the FRB capital stocks. However, the fact that the equipment stocks will need to be forecasted introduces a complication into how they must be constructed. The physical depreciation schedules constructed in Sakellaris and Wilson (2001) are functions of both year and age. In order to "forecast" physical depreciation for future years, one must make some assumption regarding how $D_{t,t-s}$ will vary over t in the future.

What is needed is a time-invariant physical depreciation pattern to apply to the forecasted investment flows. One would also like this pattern to match as closely as possible the FRB physical depreciation schedules since these schedules were used in estimating γ with the plant-level data. Thus, I use the average (over years and industries) age profile from those schedules.

The average profile is shown in Figure 1 by the line labeled "Actual." It has a reverse-S shape. What I needed was a function with a minimal number of parameters that could mimic this reverse-S shape. I found such a function in the "cascading buckets" concept which is frequently utilized by users of the G regression software package (the package used to estimate the time-series labor productivity equations in Section 4). A cascading buckets system is a combination of several "bucket" functions. A single bucket is created by the use of the *@cum* function in G. The statement, k_t = *@cum*(k_t, i_t, z), defines the variable k_t by the following equations:

$$k_0 = 0 \; ; \qquad k_t = (1 - z)\cdot k_{t-1} + i_t \qquad \forall t > 0 \tag{7}$$

The reverse-S shape can be obtained by a "cascading" of two or buckets, i.e. by having the outflow of the first bucket (here, $z \cdot k_{t-1}$) be the inflow (here, i_t) into the next bucket, then the outflow of the second bucket be the inflow into a third bucket, and so on.... The final function is the sum of these buckets.

In fact, even more variety of shape can be obtained by letting the inflow into the lower (i.e. second, third, ...) bucket "splatter out" or "miss" some of the lower bucket so that only $(z-\epsilon)\cdot k_{t-1}$ actually flows into it (and $\epsilon \cdot k_{t-1}$ is lost). Allowing some "splatter" turns out to be quite necessary for fitting the average physical depreciation schedule because without the splatter there would be no decrease in efficiency over the first N-1 years, where N is the number of buckets (i.e. without splatter, nothings falls out of the bucket system until there is no longer a lower bucket to catch the last bucket's outflow). A decrease in efficiency beginning in the first year is a property of the age-efficiency schedule I am trying to fit.

Using the following three-bucket system, I was able to very closely replicate the age profile implied by the average physical depreciation schedule shown in Figure 2-2:

b1 = *@cum(*b1, *drop, A)*

b2 = *@cum(*b2, b1[1]**B, C)*

b3 = *@cum(*b3, b2[1]**A, C)*

where *drop* is a variable that is one at age 0 and zero thereafter and the notation [1] indicates a lag of 1 period.[11] Allowing $B < A$ results in some of the outflow from b1 to splatter out or miss b2 allowing for efficiency loss immediately after the first year. I performed a grid search to find the parameters *A*,*B*, and *C* which resulted in the lowest sum of squared errors (SSE). The values *A*=.14, *B*=.129, and *C*=.3 led to a SSE < 0.001. Figure 1 shows the fitted values from this

[11]Actually, *drop(0)* is set equal to 0.989, the value of the average physical depreciation schedule at age 0. This value is slightly less than one due to the fact that the FRB allows for some wear-out in the first year of a capital good's life.

cascading bucket versus the actual depreciation schedule. Clearly, the fit is extremely close. This three-bucket system with the above parameter values became the $D_{t,t-s}$ used in the definition of the equipment capital stock given in equation (1). Now, rather than *drop* going into the first bucket, the actual equipment investment (adjusted for embodied technological change) flows in:

$vi = (eqicu/pced)*(1 + \gamma)^{t-t0}$
b1 = *@cum*(b1, *vi*, 0.14)
b2 = *@cum*(b2, b1[1]*0.129, 0.3)
b3 = *@cum*(b3, b2[1]*0.14, 0.3)
J = b1 + b2 + b3

where *eqicu* is equipment investment in current dollars, *pced* is the PCE deflator, *vi* is vintage equipment investment adjusted for embodied technological change assumed to take place at the rate γ, and *J* is the resulting quality-adjusted equipment capital stock.

4. Alternative Labor Productivity Equations

In this section, I perform a series of contests involving several alternative specifications for labor productivity equations. I begin with a set of general specifications and evaluate their performance in terms of average fit (over all sectors) and the signs and magnitudes of the coefficient estimates. Based on this evaluation, this set of specifications was pared down to a smaller set of candidate specifications. A series of modifications is applied to each specification which are then reestimated and their results evaluated. The modifications are excluding materials as a factor input, using an alternative method of adjusting for capacity utilization, and allowing for disembodied technological change.

This approach of estimating a number of specific equations that are special cases of a more general model and choosing a single equation for forecasting based on economic and statistical criteria, is similar to the general-to-specific modeling approach recommended by Hendry (2000).[12]

A. Equations in Log-Levels and Including Materials

In this subsection, I estimate 11 different specifications of a labor productivity equation for each of the 55 sectors in the IDLIFT investment sectoring scheme.[13] The average (over sectors, for each specification) adjusted R^2, average estimated coefficients, and percent of coefficients that are positive are shown in Figures 2 through 4. With the exception of the current IDLIFT specification, all of the specifications are derived from a standard Cobb-Douglas Neoclassical production function:

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[12]General-to-specific modeling is also known as the LSE methodology. For references to this literature, see Hendry (1997), Hendry (1995), Hendry and Clements (1996), Hoover and Perez (1999), Ericsson and Marquez (1998), and Cook and Hendry (1993). For a critique of general-to-specific modeling, see Faust and Whiteman (1997).

[13]Actually industries 6 (Construction) and 55 (Scrap and used equipment) are omitted due to lack of data.

$$Q_{it} = M_{it}^{\theta} L_{it}^{\beta} J_{it}^{\alpha} S_{it}^{\eta} \tag{8}$$

Table 2 gives a guide to the notation used in this equation as well as the other equations in this section.

Some specifications attempt to proxy for unobserved variation in capital utilization using the energy-capital ratio as was done in Sakellaris and Wilson (2001). The utilization rate of equipment is assumed to be an increasing function of the energy-equipment ratio (likewise for the utilization rate of structures). It is assumed that in order to increase utilization by 1%, one must increase the energy-equipment ratio by τ%. The special case $\tau = \infty$ means that there is no variation in utilization; $\tau = 1$ means energy use is perfectly proportional to capital services; and $\tau = 0$ means an infinitesimal change in the energy-equipment ratio will fully adjust utilization to the desired level.

The eleven specifications that I compare are as follows. The letter preceding each will be used hereafter as the specification's label.

(A) Standard Neoclassical, Cobb-Douglas Production Function in logs:

$$q - \ell = b_0 + (\beta - 1)\ell + \theta m + \alpha j + \eta s$$

(B) Standard and adjusting to control for utilization using energy:

$$\mathrm{q} - \ell = \mathrm{b}_0 + \left(\frac{\beta - 1}{\beta}\right)\ell + \left(\frac{\theta}{\beta}\right)\mathrm{m} + \left(\frac{\alpha(\tau - 1)}{\beta\tau}\right)\mathrm{j} + \left(\frac{\eta(\tau - 1)}{\beta\tau}\right)\mathrm{s} + \left(\frac{\alpha + \eta}{\tau}\right)\mathrm{e}$$

This equation is derived from assuming $U^J = \left(E/J\right)^{\frac{1}{\tau}}$ and $U^S = \left(E/S\right)^{\frac{1}{\tau}}$, and replacing j and s in specification A above with $log(U^J J)$ and $log(U^S S)$.

(C) Standard with constant returns to scale (RTS) imposed:

$$q - \ell = b_0 + \theta(m - \ell) + \alpha(j - \ell) + \eta(s - \ell)$$

This equation is derived by setting $\beta + \theta + \alpha + \eta = 1$.

(D) Standard with constant RTS and adjusting for utilization using energy:

$$q - \ell = b_0 + \theta(m - \ell) + \left(\frac{\alpha(\tau - 1)}{\tau}\right)(j - \ell) + \left(\frac{\eta(\tau - 1)}{\tau}\right)(s - \ell) + \left(\frac{\alpha + \eta}{\tau}\right)(e - \ell)$$

(E) Current IDLIFT equation:

$$\ell - q = b_0 + a_1 * t + a_2 * t^2 + a_3 * qup + a_4 * qdown$$

where *qup* and *qdown* are defined in equation (5).

(F) Same as A but with *J* not adjusted for embodied technological change (i.e., *J* is constructed with γ=0 for all sectors).
(G) Same as B but with *J* not adjusted for embodied technological change (i.e., *J* is constructed with γ=0 for all sectors).
(H) Same as C but with *J* not adjusted for embodied technological change (i.e., *J* is constructed with γ=0 for all sectors).
(N) Same as D but with *J* not adjusted for embodied technological change (i.e., *J* is constructed with γ=0 for all sectors).
(V) Same as H but also include the log of embodied R&D:

$$q - \ell = b_0 + \theta(m - \ell) + \alpha(j - \ell) + \eta(s - \ell) + \sigma(r - \ell)$$

Here I assume that factor payments must be made to embodied technology just as they are for traditional capital and any other *internal* factor of production (i.e., embodied R&D is not a public good or externality), therefore constant RTS now means $\beta + \theta + \alpha + \eta + \sigma = 1$.
(Z) Same as V but adjusting for utilization using energy

$$q - \ell = b_0 + \theta(m - \ell) + \left(\frac{\alpha(\tau - 1)}{\tau}\right)(j - \ell) + \left(\frac{\eta(\tau - 1)}{\tau}\right)(s - \ell) + \left(\frac{\alpha + \eta}{\tau}\right)(e - \ell) + \sigma(r - \ell)$$

It should be noted that in this equation the embodied R&D index, unlike the stocks of equipment and structures, is assumed to have a constant rate of utilization.

Figures 2 through 4 summarize the results of estimating these 11 equations for all of the 55 sectors in IDLIFT (spanning the U.S. private economy). Given that data mismeasurement is generally considered to be more serious in nonmanufacturing industries and that the estimated rates of embodied technological change used for constructing equipment stock in these industries are imputed, I also look separately at the results just for nonmanufacturing sectors.[14] In the following discussion, I will generally focus on the results for all sectors, though I will point out things that are substantially different in the nonmanufacturing subset.

Figure 2 shows the average adjusted-R^2 over all sectors for regressions corresponding to each specification above. Figure 3 gives the average estimated factor elasticities for each specification. The percentage of estimated elasticities that are positive for each specification is shown in Figure 4. Several important findings are apparent from the figures. First, I find that for

[14]Figures analogous to Figures 2 through 7 for the nonmanufacturing subset are not reported but are available from the author upon request.

the most part adjusting equipment capital for quality using the γ's from Table 1 substantially improves the fit and sensibility (in terms of average value and positivity of estimated factor elasticities) of the labor productivity equation in comparison to using an adjusted equipment capital stock (J_t). Second, despite some loss of fit, imposing constant RTS seems to greatly improve the sensibility of the estimates. The beneficial effects that imposing constant RTS has on α and η, in terms of increasing the percentage that are positive and raising their average values closer to a priori expectations based on income shares, seem to easily outweigh the cost of a slightly lowered fit. Finally, including the index of embodied R&D along with non-quality-adjusted J_t rather than just using a quality-adjusted J_t improves the average fit slightly but has a substantial deleterious effect on the capital elasticities. Furthermore, controlling for utilization using the energy-capital ratio improves the fit and raises the estimated elasticities of structures, but it reduces the elasticities of equipment.

Based on these findings, it seems reasonable to drop from our consideration all but specifications C, D, V, and Z. That is, we can feel comfortable hereafter imposing constant RTS and adjusting equipment capital by constructing the stock according to the γ's in Table 1 or by including embodied R&D as an additional independent variable (although these embodied R&D specifications do seem to yield less realistic estimates). Furthermore, adjusting for utilization seems to be a slight improvement over not controlling for it in terms of fit, so I will retain specification D for now despite its tendency to produce outlying unrealistic capital elasticities.

B. Equations Omitting Intermediate Inputs

I next analyze how the regression results for these four specifications change if we remove materials. It is often the case in production function or productivity regressions that intermediate inputs (materials) dominate the explanatory power of the independent variables and obscure the effects of the other inputs. This domination by materials appears to be the case in our regressions as well. Evidence of this domination is the very high average estimated materials elasticities and enormous mexval statistics (marginal explanatory power, not shown) for the coefficient on materials obtained in the regressions described above. Furthermore, all but specification E (the current IDLIFT equation, which does not include materials) have very high adjusted-R^2's.

Another problem with including materials in aggregate or industry-level production regressions is that data on materials is often inadequately measured. The measures on real materials used in the above regressions are constructed by taking the column sum of a constant dollar input-output flow matrix. That is, real materials for industry j is $m_{jt} = \sum_i a_{ijt} q_{jt}$ where a_{ijt} is element (i,j) in the intermediate coefficient matrix (A in equation (1)). The problem here is that we do not observe the true input-output coefficients, a_{ijt} (at least in the U.S. data). Or, more accurately, we do "observe" a_{ijt} but only every 5 years as the BEA constructs input-output tables on a quinquennial basis. Coefficients for years in between are simply interpolated between benchmark-year coefficients and are therefore essentially determined by q_t. Thus, shocks in q_t, which affect the dependent variable in a productivity regression and are part of the regression disturbance term, are transmitted to the regressor $(m_t-\ell_t)$ causing an upward bias in the estimator of its coefficient.[15]

Therefore, I re-ran the regressions corresponding to C, D, V, and Z omitting the $\theta(m-\ell)$ term. These new *sans*-materials specifications will hereafter be referred to as C', D', V', and Z'). The omission of materials can be justified theoretically by assuming that materials and value added have a Leontief relationship as is frequently done in the literature (e.g., Basu (1996) and Wilson (2000)). That is, $Y = min[M, F(J,S,L)]$. Assuming firms are optimizing, this implies $dlog(Y) = dlog(F(J,S,L))$. The $F(\,)$ function can be any of equations (1)-(11) after omitting the term $\theta(m-\ell)$.

Figures 5 through 7 summarize the results of these regressions (ignore for now the specifications labeled D" and Z", these will be explained below). As with the previous regressions, I repeat the regressions for a nonmanufacturing subset to check for robustness. Except in the cases mentioned below, the nonmanufacturing subset yielded similar results to those of the full sample.

As expected, the adjusted R^2's fall, though not by much, when materials are left out (see Figure 5). Again the fits are higher when capital utilization is adjusted for (compare Z' to V' and D' to C'). The specifications that use the quality-adjusted equipment stocks (C' and D') yield quite reasonable factor elasticities, particularly the specification which does not include the energy-capital ratio (C'). Compared to D', the non-utilization adjusted specification (C') has a somewhat lower percentage of η's that are positive but a much higher percentage of positive α's. This result does not appear to be the case in nonmanufacturing though, where (D') dominates. When utilization is not adjusted for, there is also strong evidence that including embodied R&D causes the coefficients on unadjusted equipment to turn negative, particularly in nonmanufacturing.

Though not in the nonmanufacturing subset, the average estimated elasticities for specification C' over all sectors are almost exactly as one would expect. The generally accepted estimates of labor and capital's share in the economy's output is 2/3 and 1/3, respectively, when output is value added and 1/3 and 1/6 when output is gross output (with materials responsible for the other ½). The capital share is further broken down, generally, to be 2/3's equipment (which includes embodied R&D) and 1/3 structures. Thus, one would expect our estimates of the output elasticities with respect to each input to be somewhat close to these values. This means that when materials are included, we would expect $\alpha(+\sigma) \approx (1/6)*(2/3)=2/18 = 0.111$, $\eta \approx (1/6)*(1/3)=1/18 = 0.056$, $\beta = 0.33$ and $\Theta \approx 0.5$. When materials are excluded, we expect $\alpha(+\sigma) \approx 2/9 = 0.222$, $\eta \approx 1/9 = 0.111$, and $\beta = 0.66$. According to the average estimates obtained thus far, these *a priori* expectations are met more closely by the regressions which do <u>not</u> include materials.

Overall, as in the previous section where materials were included, specifications C' and D' seem to outperform V' and Z' here. However, before abandoning the idea of including embodied R&D as a separate regressor, I will explore another method of adjusting for utilization applied to both the embodied R&D specification (V') and the specification which uses quality-adjusted equipment stock (C').

C. *Alternative Adjustment for Unobserved Variation in Capacity Utilization*

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[15] In fact, exactly the same problem is true for our measures of real energy expenditures which are also constructed via slow-moving input-output coefficients multiplied by industry output.

Besides using the energy-capital ratio, another method that has been suggested to control for unadjusted variation in factor utilization is what is actually used in the current IDLIFT equation. Industry-level variation in utilization is captured by including the terms *qup* and *qdown* which are defined in equation (5). The method first measures capacity with the previous peak level of industry output less some "depreciation." The absolute value of the percentage difference between current output and capacity is then included as a regressor, with positive and negative differences treated asymmetrically. The rationale behind this method is that when current output is being stretched beyond the previous peak level, the economy will be pushing up against capacity constraints, and when output is much below the previous peak, there is excess capacity not being utilized.

There is the possibility, however, of reverse causation (i.e. simultaneity, or what Almon (1998) refers to as the "umbrella effect"[16]) here since industry-level (log) output is part of both the dependent variable and the regressors *qup* and *qdown*. If there is any measurement error in output, this may bias the coefficients on *qup* and *qdown* as well as artificially inflate the R^2's. This possibility is explored using a mixed empirical-Monte Carlo technique in the next subsection. For now, as an alternative to specifications D and Z, I estimate two analogous equations that are simply specifications C' and V' with *qup* and *qdown* as additional independent variables. Call these specifications D" and Z".

The results of these estimations are shown in Figures 5 through 7. Compared to their energy-intensity counterparts (D' and Z'), specifications D" and Z" have slightly lower fits but far more reasonable capital elasticities. Compared to their counterparts that do not adjust for variation in utilization (C' and V'), these equations are quite similar in fit and in the capital elasticities (with the exception of Z" which actually has even more reasonable capital elasticities than V').

At this point, it seems reasonable to drop from our consideration the specifications which attempt to adjust for unobserved variations in capital utilization using the energy-capital ratios (specifications D' and Z') due to their propensity to yield nonsensical capital elasticities and to the fact that including *qup* and *qdown* as explanatory variables seems to be a powerful alternative way of adjusting for utilization. I will also drop the specifications which include embodied R&D and an unadjusted equipment stock as separate explanatory variables (specifications V' and Z"). The rationale behind these specifications was that including embodied R&D separately may be superior in nonmanufacturing industries to using the imputed rates of embodied technological change to compute equipment capital. However, these specifications seem to actually perform much worse in the nonmanufacturing subset than they do overall. Therefore, hereafter I will consider only specifications C', D", and E.

D. Allowing for Disembodied Technological Change

It is possible that there is some spurious positive correlations between labor productivity and the factor inputs due to the fact that these variables are all trended upward. In other words, the above equations should probably also contain a Hicks-neutral productivity (or disembodied

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[16]Almon (1998) cautions against the use of "umbrella" variables, which in econometric parlance are simply endogenous variables, as explanatory variables. The name comes from the analogy to using "the number of people carrying umbrellas to explain rainfall." (p. 97).

technology) term that is sure to be highly trended.

The results of estimating equations C' and D" with a single linear time trend added are as follows. The adjusted R^2's for both of these specifications are now slightly better than that of the current IDLIFT equation (specification E) at 0.866, 0.867 and 0.853 for specifications C', D", and E, respectively. The average estimated capital elasticities decrease somewhat due to the introduction of the time trend though they are still reasonable. For specification C', the average α falls from 0.22 absent the time trend to 0.01 with it, while the average η rises from 0.15 to 0.17. Similarly, the percentage of α's that are positive falls from 80% to 52% and the percentage of η's that are positive rises from 52% to 63%. For specification D", α falls from 0.22 to 0.08 on average with the inclusion of the time trend and the average η remains at 0.18. The positivity of α falls from 80% to 59% and that of η drops from 61% to 57%. The results are quite similar in the nonmanufacturing subset.

From the results of this round of regressions, the most promising specification appears to be D" with a time trend. C' with a time trend also seems to be reasonable, though the average equipment elasticity is probably too low and the equipment elasticity is somewhat less likely to be positive under C' relative to D". Compared to the former IDLIFT equation, these specifications have as good a fit and obviously have far more economic appeal. Most importantly, they capture the productivity gains due to capital deepening (which, given how capital was constructed here, includes embodied technological change). Therefore, one of these two specifications, along with the coefficients found from estimating them, are used for each of the 55 sectors and can now be incorporated into the IDLIFT model. For a particular industry, which specification is used is chosen on a case-by-case basis, as described below, based on the criterion of best fit and most realistic coefficients. For the sake of clarity, let us explicitly write out specification C':

$$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) \tag{9}$$

and specification D":

$$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) + b^0 qup + b^1 qdown \tag{10}$$

E. Determining Industry-Specific Labor Productivity Equations

In the previous subsection, I evaluated many possible specifications for a general empirical model of labor productivity based on the criteria of average fit and the economic realism of the coefficients. The results of that evaluation have enabled us to now focus our attention on a small number of specifications in determining the "best" one for each particular industry (rather than simply the best on average). Obviously, the specification that yields the best results on average may not necessarily yield the best results for a particular industry. The choice of specification must be made on an industry-by-industry basis.

For each industry, I compare the results of estimating specifications C', D", and E. For a small number of industries, it was clear that the lagged values of the equipment and structures stocks had more explanatory power (with reasonable coefficients) than the current values and, thus, the lagged stocks were used instead. The improved explanatory power afforded by using lagged stocks can be explained by the industry having a time-to-build requirement greater than one year and/or by the presence of substantial learning-by-doing effects. For most industries, even the best specification yielded one or more unrealistic coefficients. For these industries it was necessary to "softly constrain" the coefficient estimates to lie inside a realistic range. "Soft constraining," also known as "Theil's mixed estimation" or "stochastic constraints," is a Bayesian regression technique that allows one to combine *a priori* theoretical beliefs on parameter values with the values estimated using the data. A soft constraint essentially adds artificial observations (or a fraction of an observation) in which the constraint holds with certainty. The *a priori* expectation for parameter values and the number of artificial observation to add are chosen by the econometrician. I only imposed soft constraints if the unconstrained estimated coefficient was outside the range of [0,0.4] for either capital elasticity (α and η), [0,1] for the coefficient on *qup*, and [0,-1] for the coefficient on *qdown*. The theoretically-based, *a priori* expected parameter values that I used as soft constraints were 0.18 for the elasticity of output with respect to the equipment stock, 0.17 for the structures elasticity, 0.5 for the coefficient on *qup*, and -0.5 for the coefficient on *qdown*.[17]

Table 3 shows the number of industries for which each of the four specifications was chosen (second column) as well as the number, within each specification, that required soft constraining (third column). Recall that the regressors in specification C' are a constant, time trend, log of the equipment-labor ratio, and the log of the structures-labor ratio. Specification D'' includes these same regressors in addition to *qup* and *qdown*. Specification E is the traditional (current) IDLIFT labor productivity equation. Let the specification which is equivalent to specification C' but with lagged capital stocks be denoted specification X.

Specification D'' was chosen in exactly one half of the industries. Overall, all but five industries required some type of soft constraint(s). In nearly all cases, the soft constraints were quite weak, amounting to only a fraction of an artificial observation. Thus, the equation fits suffered very little due to the use of soft constraints.

F. Mixed Empirical-Monte Carlo Test for Bias

As mentioned above, the fact that *qup* and *qdown* are constructed using q which is also part of the dependent variable for the above regressions, means that if there is measurement error in q, the coefficients on *qup* and *qdown* will be biased. This can be seen formally by assuming that there is an i.i.d. measurement error in q: $q^{true} = q^{measured} + \nu$, where $\nu \sim N(0, 2.5\times10^{-04})$. This says that the standard deviation in the measurement error of log output is assumed to be one half of one percent, which should be as large as is realistically possible. So our regression equation (10) becomes:

$$\begin{aligned}(q^{measured} - \ell)_t &= c^0 + c^1 t + \alpha(j-\ell)_t + \eta(s-\ell)_t \\ &\quad + b^0 qup_t^{measured} + b^1 qdown_t^{measured} + u_t \end{aligned} \tag{11}$$

Notice that ν will be contained in the dependent variable as well as *qup* and qdown resulting in spurious correlation between these two regressors and the dependent variable. The bias on the

[17]The rationale behind these *a priori* values for capital elasticities is explained in subsection B above. The *a priori* values for the coefficients on *qup* and *qdown* were chosen simply to be at the halfway point of their respective plausible ranges.

estimator of b^0 will be positive and that of b^1 will be negative.
To evaluate the seriousness of this problem, I perform a mixed empirical-Monte Carlo estimation procedure. In this procedure, I specify the data generating process (DGP) for the true dependent variable as:

$$\begin{aligned}(q-\ell)_t^{true} &= 2 + 0.01*t + 0.17*(j-\ell)_t + 0.16*(s-\ell)_t \\ &+0.1*qup_t^{true} - 0.1*qdown_t^{true} + \varepsilon_t\end{aligned} \tag{12}$$

where $\epsilon_t \sim N(0, 4\times10^{-06})$, so that the standard deviation of the i.i.d. shock to true productivity is 0.002. The 0.01 and -0.01 assumed coefficients represent the true relationship between *qup* and qdown and labor productivity, i.e. absent any spurious correlation due to the presence of measurement error in q. Using this DGP, I construct this "true" dependent variable, then regress it on t, $(j\text{-}\ell)$, $(s\text{-}\ell)$, $qup^{measured}$, and $qdown^{measured}$ each measured with actual historical time series. I repeat this procedure 2000 times and calculate the mean and standard deviation for each coefficient.[18]

The coefficient means and standard deviations are shown in Table 4. The estimated biases are all extremely close to zero. Thus, even assuming a very large variance for the measurement error in q, coefficient bias due to the presence of *qup* and *qdown* does not appear to be a problem.

5. Incorporating the Alternative Estimated Equations into IDLIFT

The equations summarized in Table 3 are incorporated into IDLIFT through a series of new C++ routines which take forecasted values of equipment investment, structures investment, and output and generate values for productivity, hours, and employment, which then get fed back into the model.

Incorporating these new labor productivity equations into the IDLIFT model turned out to far more complicated than it would seem at first. The task at hand was to use the new labor productivity equations to determine productivity and employment, at the 97-sector level of aggregation, which can feed back into the model. The model can then use the productivity and employment forecasts to help calculate various other components of the model such as the unemployment rate and hourly labor compensation.

The first complication was how to deal with having labor hours, which are calculated *using* the productivity equations, on the right-hand side of the productivity equations. There are at least

[18] I arbitrarily choose the "Printing and Publishing" industry for the historical data. The choice of industry should not affect the coefficient means (and therefore their biases) but may affect the standard deviations since the sample variance of a variable helps determine the variance of its coefficient (and, of course, the sample variance of a variable will be different across industries). To be sure, I repeated the procedure with a 2nd industry and obtained similar estimated biases.

three options for handling this problem. The first is to algebraically rearrange each of the specifications containing hours on the right-hand side so that output is on the right-hand side instead of labor and then estimate the equations in this form. For example, specification C' can be rearranged from:

$$q - \ell = c^0 + c^1 t + \alpha(j - \ell) + \eta(s - \ell) \tag{13}$$

to:

$$q - \ell = \left(\frac{1}{1-\alpha-\eta}\right)\{c_0 + c_1 t + \alpha j + \eta s - (\alpha+\eta)q\} \tag{14}$$

I tried this approach and found that the capital elasticities implied by the estimated coefficients were far less sensible than those estimated directly. As in Section 2 above, one could impose soft constraints to force the coefficients into a range that would imply reasonable capital elasticities. However, the constraints would have to be much stronger, i.e., the trade-off between *a priori* expectations of parameter values and those estimated by the data would have to lean far more towards the former. Another option would be to program the equations into the model with hours on the right-hand side, supply the model with starting values (a guess) for hours, let the productivity equations calculate new values for hours, and then let the model iterate until it converges. The third option is to use the estimated equation coefficients found in Section 2 above, but use them in the algebraically rearranged forms of the specifications (such as equation (14) above) which have output on the right-hand side. This option requires no iterative procedure since output has already been calculated earlier in the model and thus this is the option I used.

The next issue that needed to be dealt with was how to get forecasted values of structures investment at the 55-industry level, the level of disaggregation at which the productivity equations were estimated. Previously, the IDLIFT model generated only *equipment* investment by 55 industry and structures investment by *type*. The 25 types/categories of construction are listed in Appendix A. Rather than developing new structures investment equations by industry in IDLIFT, similar to the existing equipment investment equations, I instead exploited the fact that there is (approximately) a clear one-to-many mapping from some construction types to the industries that purchase those types. For instance, construction of "Farm buildings" (construction type 13) can be clearly attributed to the "Agriculture, forestry, and fisheries" investment industry (industry 1). This assumption can be supplied exogenously to the model through what is known as a "fix." Fixes are supplied by the model user and override or modify the equation results of endogenous variables. Thus, I fix structures investment in industry 1 to "follow" construction of farm buildings, starting from the last year of historical data for structures investment by industry (1997). That is, structures investment in year t, S_t, is determined by equalizing S_t/S_{1997} to C_t/C_{1997} for all t>1997, where C is construction in the corresponding type. Similarly, for cases where one type is associated with many industries, such as "Industrial" construction which is attributable to all of the manufacturing industries, I fix structures investment in each industry to follow the model's forecast for construction in that type. Again, industry structures investment does not *equal* the value of construction in that type; rather, it starts with the last historical data point and then moves forward at the same ratio of forecast year value to last data value that is the case in the forecasts of construction by type. For two industries (which each have very little investment in structures anyway), no clear match could be made to a construction type and so structures investment in those industries was assumed to simply follow aggregate nonresidential construction from their last data point on.[19]

Now, with forecast values for structures and equipment investment by 55 industry, one can calculate structures and quality-adjusted equipment capital stocks to be used in the productivity equations. This is done in the C++ routine, DANBKT.CPP, which is shown in Appendix B along with the other new routines. The routine takes in forecasted values of structures and equipment investment along with the exogenously supplied rates of embodied technological change and produces stocks. The stocks of structures are calculated using the traditional perpetual inventory method with depreciation rates computed as the reciprocal of the mean service life of structures in that industry (provided by the BEA). The quality-adjusted equipment capital stocks are calculated using the estimated rates of embodied technological change and the cascading bucket system, as described in Section 3.

The routine DANPROD.CPP then takes in these stocks along with the model's forecasted values of output by 55 industries (which are aggregated from the 97-sector level) and the coefficient estimates for the productivity equations (including the estimate of ρ, the autocorrelation coefficient) and calculates both productivity and hours for each industry. Since other stages of the model require productivity and hours at the 97-sector level, these had to be disaggregated to that level. To split 55-industry hours to the 97-sector level, I used a one-to many mapping key. The shares used to split one industry to many sectors were taken from the 97-by-1 hours vector forecasted by the old IDLIFT productivity equations. Thus, the old productivity equations were left operational in the model solely for the purpose of providing time-varying shares for this disaggregation. Productivity at the 97-sector level was then calculated by simply dividing the output (already generated by the model at this level) by the 97-sector level hours. Employment at the 97-sector level was calculated by dividing hours by the model's forecasts of average annual hours per worker. The disaggregation and the calculation of productivity and employment can be seen in the routine REMPLOY.CPP in Appendix B.

6. Forecast and Simulation Results

With these new, alternative routines incorporated into the model (along with the estimates for the productivity equations), one can produce a base forecast that is stable, i.e. a forecast that does not cause any variable to spiral out of control. In addition, these new routines were programmed into the model in such a way as to allow the model to calculate productivity, hours, and employment using both the new set of equations and the old set of equations. The model user can specify which set of equations he or she would like to feed back into the model. That is, the user can have the model calculate productivity and hours using the new equations but have those calculated values in no way affect the rest of the model, and the same for the old equations. This allows one to generate a base forecast for both the old model (i.e. the model set to have the

[19]The two industries are Construction (6) and Air transportation (40).

current IDLIFT equations' forecasts feed back into the model) and the new model (having the new equations feed back into the model).[20]

Since what we are interested in is how the behavior of the two models differs in response to changes in economic activity, such as variations in equipment investment, comparing the two base forecasts to one another is of little interest. What will be of interest to us in this section is comparing and contrasting the responses of each model to some exogenous shock to the system. The behavior of each model in response to such an experiment is the only way to illuminate the effect of changing the IDLIFT's productivity equations. Since the key difference between the two models is the presence of a direct influence of capital stock on productivity in the new model, the interesting shocks to investigate will naturally involve investment.

Moreover, given IDLIFT's dependence on many exogenous, user-supplied assumptions ("fixes"), one cannot fairly compare a forecast from the old model with one from the new model. The existing fixes, which either override or modify the endogenous forecasts produced by the model's equations, were specified in such a way as to produce the most sensible forecast using the *current* model. Alternatively, these fixes could be specified so as to optimize the sensibility of the new model. However, having each model have its own optimal fixes would confuse the differences in the models' results due to different productivity equations with those due to different sets of fixes. Yet, many of these fixes must be given values for the model to run at all, therefore turning off all fixes is not an option either. Thus, I run both models using the fixes in place for the most recent semi-annual Inforum forecast using IDLIFT (see Inforum (2001)). One important exception is the exclusion of all fixes on industry-level productivity, industry-level employment, and the aggregate unemployment rate. Thus, again, comparison of the two models must be between the models' *differences* from their own base forecast to a simulation forecast in which a shock was imposed, and not between the models' base forecasts.

To produce base forecasts, I ran each model out to 2015. 1997 was the last year of historical data for most of the industry-level variables in the model, yet much of the aggregate data is available through 2000 (or at least through 1998 or 1999) and this data is imposed on the model through fixes (with the exception of the unemployment rate as mentioned above).[21] The new functions generally result in lower labor productivity and thus higher hours and employment in the base forecast. This result is true even if the output of these functions is not fed back into the model, but it is stronger when feedback does occur. However, this difference in productivity between the base forecasts is largely due to fixes that act to boost productivity in the current model and thus is not very revealing.

For each model, I evaluate the response of the model to a shock in equipment investment. Specifically, with a set of fixes on equipment investment, I override the models' forecasted

[20]The model using the new, alternative productivity equations will be referred to as the "new" model in this section while the old/current/pre-existing IDLIFT model will be referred to as the "old" model.

[21]For instance, NIPA data is available on aggregate equipment investment and residential and nonresidential structures through 2000.

vectors of equipment investment with the investment vector from the base forecast plus 2%. That is, for each industry I multiply the equipment investment values from the base forecast by 1.02 and force the model to use these new values in all of the functions that make use of equipment investment. Because aggregate equipment investment is known (from NIPA data) through 2000, I impose this fix for the years 2001 through 2015.

Figures 8 through 17 graph, for each model, the deviations over the forecast period of key macro variables relative to each model's base forecast. In both models, real GDP rises by about a quarter of a percent relative to the base in the first year in which the 2% higher equipment investment is imposed. From then on the models diverge substantially. The old model falls to near the base level in the second year, oscillates between 0.05% and 0.2% over base through 2008, then seems to settle at about 0.08% over base. The new model also comes back down closer to base in 2002 but then rises relative to base almost monotonically until the end of the forecast where it stands at 0.31% over base. This Solowian response of real GDP, i.e. higher and less variable, to permanently higher equipment investment is what one would have expected and hoped for from the new model. The increase in labor productivity induced by higher investment also reduces unit labor costs and this reduction lowers the GDP deflator. The GDP deflator rises in the old model in response to the demand stimulus of higher investment. Because of this, the deviation from base in *nominal* GDP is actually higher in the old model. The different responses of the price level also has an effect on the Treasury bill rate: the deviation from base is generally lower and less volatile in the new model. The lower interest rates in the new model cause, in part, a smaller deviation in the savings rate.

In both models, the unemployment rate goes down relative to base due to the substantial demand stimulus caused by the increase in investment. However, the deviation is smaller on average in the new model because its increase in labor productivity has an immediate negative effect on employment. This Ricardian (or Luddite) effect would have occurred in the old model as well had labor productivity increased substantially, which it did not.[22] This difference in labor productivity deviations can be seen in Figure 17. Labor productivity in the new model grows steadily to almost 0.4% above its base level by the end of the forecast. This is compared to the old model in which productivity oscillates until it converges to about 0.04% over base. In short, in the new model, the long-run effect of investment on productivity is ten times what it was in the old model.

The deviations in labor productivity by industry for the new model are shown in Table 5 below, along with each industry's estimated elasticity of output with respect to equipment capital stock. Listed in parentheses after each industry name is the label identifying the equation type

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[22]In Ricardo's later works, he developed the notion that the introduction of machinery can, under certain circumstances such as the sudden introduction of a new type of machinery, have an adverse effect on employment. In his *Notes* to Malthus's *Principles*, he states:

> *It might be possible to do almost all the work performed by men with horses, would the substitution of horses in such case, even if attended with a greater produce, be advantageous to the working classes, would it not on the contrary very materially diminish the demand for labor?*

used for that industry. As one would expect, the largest deviations can be found in industries which have the largest elasticities of equipment stock. The correlation between this elasticity and the deviation in labor productivity is approximately zero in 2001 but rises to 0.96 by 2015.

Next I impose a one-time shock on each model of 10% higher equipment investment (relative to that which is forecast by the model's equations) in 2001. Determination of equipment investment returns to IDLIFT's investment equations from 2002 on. The shock is assumed to take place in every industry. Figures 18 through 29 show the deviations relative to the base for the same macro variables as in the earlier figures as well as equipment investment (Figure 28) and quality-adjusted equipment stock (Figure 29).[23] Both models have an initial response of between 1.2 and 1.3 percent in real GDP. After oscillating for several years, the old model returns nearly to its base level. The new model, however, quickly reaches a steady state at approximately three-tenths of a percent above its base. As with the previous experiment, the GDP deflator's deviation is lower in the new model than in the old model. The GDP deflator converges to the base level over time in the old model whereas it falls steadily relative to the base in the new model. Interest rates deviations move similarly in the two models though they are somewhat less volatile in the new model. The same is true for their savings and unemployment rates. In both models, unemployment initially drops dramatically in response to the shock, then jumps dramatically, and finally begins to converge to its base level around 2005. The new model has less of a drop and subsequent jump because the positive demand stimulus of raising investment is partially offset by the increase in productivity which has a negative effect on employment in the short-run (the Ricardian effect), though this is dominated by the stimulus as can be seen in Figure 24.

As expected, labor productivity in the old model, after oscillating for several periods, returns to its base level by 2010 and stays there whereas productivity in the new model, after also oscillating for a few years, is permanently above its base levels. This permanent increase in productivity in response to a temporary increase in investment is the key difference in the behavior of the two models. In the old model, a one-time jump in aggregate investment only affects labor productivity by directly increasing every industry's final demand, which directly increases their output, which increases their *qup* which increases their labor productivity. The next year, when equipment investment comes back down, output will likely be below its previous peak making *qdown* go up which will lower labor productivity. This cycle will fade away over time returning labor productivity to its base level. In the new model, on the other hand, labor productivity in every industry jumps initially because of both the jump in *qup* <u>and</u> the jump in the equipment stock. In the following year, productivity comes back down due to the jump in *qdown* in the following period but this decline is offset somewhat by the still-present higher level of equipment stock. There is also a strong and long-lasting positive effect on equipment investment itself from the initial shock. This effect has two causes. First, the 2001 jump in investment causes the following year's desired capital stock (constructed and used in the model's investment equations)

[23]Equipment investment here is not adjusted for embodied technological change. Also, note that though quality-adjusted equipment capital is shown for both models in Figure 29, it only has an effect on the other variables (as well as its own future values through the investment equations) in the new model.

to rise which increases the forecast of investment for that year which then increases desired capital and investment for the next year, and so on. Furthermore, the increase in final demand in 2001 raises the 2000-2001 change in output. Distributed lags in the change in output are part of the model's investment equations. Thus, the increased change in output in 2001 directly increases investment for the following four years (there are four lags of output change in the investment equations).

The continuing though depreciating presence of that extra 10% of equipment purchased in 2001, combined with the long-lasting increase in equipment investment due to the positive feedback from the initial demand stimulus, keeps the quality-adjusted equipment stock about 2% above its base level from 2005 through the end of the forecast (see Figure 29). The physical depreciation and obsolescence of the extra 10% of vintage-2001 equipment begins to dominate any positive feedback remaining from the initial stimulus by 2009 and a very slow decline in the equipment stock begins. Shortly thereafter, labor productivity thus begins to decline very slowly.

The labor productivity deviations from the base forecast of the new model are shown for each industry in Table 6 below, along with each industry's estimated elasticity of output with respect to equipment capital. As was the case with the permanent shock, the largest deviations are in industries with large elasticities of equipment stock. The correlation between the estimated elasticity and the deviation in productivity is -0.07 in 2001 but rises to 0.82 by 2015.

7. Conclusion

The preceding experiments show that the introduction of the new labor productivity equations into IDLIFT do have substantial effects on the general equilibrium behavior of the model. With the new equations operating, the macroeconomic variables of the model exhibit behavior in response to changes in investment that is more in line with that predicted by the Solow growth model. Importantly, we do not see the model spiral out of control in terms of output or prices when the new equations are introduced as was feared due to the lack of a supply constraint in the investment equations. In general, the macroeconomic situation of the economy is *permanently* and substantially improved by an increase in equipment investment, even if it is only a one-time shock, according to the new model. In contrast, the macroeconomy of the IDLIFT model without the new equations exhibits a smaller long-run benefit due to a permanent investment increase and little or no long-run benefit from a temporary increase. The permanent and reasonable response of the new model to increases in investment is one of the main contributions of this paper.

This Neoclassical response in the new model was accomplished through the use of new labor productivity equations that account for both traditional capital deepening (more units of capital per worker) and embodied technological change (higher quality units per worker). It is important to note, however, that these new equations were not simply chosen *ad hoc* and forced into the model but rather were shown to fit the historical data more closely that the preexisting IDLIFT equations.

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Appendix A – Construction Categories

1. 1 unit residential structures
2. 2 or more unit structures
3. Mobile homes
4. Additions & alterations
5. Hotels, motels, dormitories
6. Industrial
7. Offices
8. Stores, restaurants, garages
9. Religious
10. Educational
11. Hospital & institutional
12. Miscellaneous NR bldg.
13. Farm buildings
14. Mining exploration shafts & wells
15. Railroads
16. Telephone & telegraph
17. Electric light & power
18. Gas & petroleum pipes
19. Other structures
20. Highways & streets
21. Military facilities
22. Conservation
23. Sewer systems
24. Water supply facilities
25. Brokers' commission

Appendix B

New C++ Routines for Calculating Capital Stocks, Productivity, and Hours

The core program of IDLIFT is **model.cpp**. This program contains the main model loop within which is the real-side loop and the price-side loop. Within the real side loop is the investment-output loop and within this loop productivity and employment are calculated. The schematic below shows the organization of **model.cpp**:

For t = godate until t= stopdate:

Model loop: load vectors and matrices, read fixes, initialize output and prices

Real-side loop:

Call to PCE function

Call to exports function

Investment-Output loop:

Call to equipment investment function

Call to construction function

Call to government spending function

Solve for output given results of above functions

***Calls to productivity, hours, and employment functions** (see detail below)*

Price/income-side loop:

Calls to functions calculating value added components

Solve for prices given results of value added functions

Calculate aggregate variables

End of model loop: t = t + 1

Within the calls to the productivity, hours, and employment functions, I placed the new routines (called UpdateKBuckets(), DanProductivity(), and ReviseEmploy() below) *after* calls to the preexisting functions for productivity, average annual hours, and employment. This section of **model.cpp** is excerpted below (the new functions are in bold):

```
// (*****) Productivity and Employment :
if(t>=prd.LastData() ) {
      update(out,Outlag);
      Productivity(hrs, prd, Outlag, qpeak, Qpeaklag, eqi, caphat, Caphatlag,
            pdm, iag56, ProductivityEquations, prdtrnd);
      }
// (*****) Average Hours Worked function:
if(t>=yhr.LastData() ) {
      AvgHours(yhr,Outlag,AverageHoursEquations);
      othrsf();  // dom serv., govt. ent.
      }
// Call Employ to calculate employment, and various identities:
p5 = pdm[5];
if(t>=emp.LastData() ) {
      Employ(emp,hrs,prd,yhr,out);
```

```
        }
    if(t>dprd.LastData() ) {
        str.fix(t);
        UpdateKBuckets(vi, vbk1_, vbk2_, vbk3_, qastk, vbk1lag,
            vbk2lag, vbk3lag, eqicu, DanProductivityEquations,
            str, strcap, Strcap, Qastk);
        }
    if(t>=dprd.LastData() ) {
        DanProductivity(dhrs, dprd, qag, qagpeak, Qagpeaklag, qastk, Qastk,
        strcap, Strcap, iag56, DanProductivityEquations, hrsag);
        }
    // By setting dhrs.LastData > last forecast year, we can get model
    //  to compute dprd and dhrs BUT WITHOUT feeding them back into the
    //  model.
    if(t>=emp.LastData() && t>=dprd.LastData() && t>=dhrs.LastData() ) {
        ReviseEmploy(emp, dhrs, hrs, yhr, prd, out, iag56);
        }
```

The vectors calculated by the functions called in this section are: productivity by 97 sectors (**prd**), average annual hours by 97 sectors (**yhr**), employment by 97 sectors (**emp**), productivity by 55 industries (**dprd**), structures capital by 55 industries (**strcap**), quality-adjusted equipment capital by 55 industries (**qastk**), and hours by 55 industries (**dhrs**). A vector name followed by ".LastData()" returns the year which is the last year for which there is historical data (this information is stored in a separate file). When or if the new routines feed back into the model can be controlled by setting the last data year. Setting the last data year to the first year of the model run will fully incorporate the new routines into the model. Setting the last data year to a year greater than the last year of the forecast will allow the model to calculate **dprd** but will not allow this vector to affect the rest of the model; rather, the rest of the model will use the old productivity, hours, and employment vectors.

Thus, the old productivity and employment functions are called whether or not the new vectors are feeding back into the model. Besides making the turning on and off of the new routines extremely simple, having the old functions always called provides a convenient and time-varying vector, namely **hrs**, to be used as a "split vector" for disaggregating **dhrs** (55×1) to the 97-sector level.

The first new function, UpdateKBuckets, takes in the equipment investment and structures forecasts and calculates capital stocks. Here is the code for this function:

```
// Apply Structures fixes here so as to exogenously supply str with values
str.fix(t);

// Private Structures Buckets:
arith("In UpdateKBuckets, before STR:",t);
for(i=1;i<=NEQI;i++) {
      if(i>=55) continue;
   tempstr = str[i];
   lagstrcap = Strcap[1][i];
   tempsp = sp[i];
      tempstrcap = (1.-sp[i])*Strcap[1][i] + str[i];
      strcap[i] = (1.-sp[i])*Strcap[1][i] + str[i];
      }
```

```
// Private Equipment Buckets:
//arith("In UpdateKBuckets, before EQI:",t);
for(k=1;k<=NEQI;k++) {
       if(k>=55) continue;
   tempapc = apc[t];
   tempeqicu = eqicu[k];
   tempgamma = P[k][7];
   temp = (eqicu[k]/apc[t])*exp((t-1987)*safelog(1.+P[k][7]));
   vi[k] =  (eqicu[k]/apc[t])*exp((t-1987)*safelog(1.+P[k][7]));
   vbk1_[k] = (1.-0.14)*vbk1lag[1][k] + vi[k];
   vbk2_[k] = (1.-0.3)*vbk2lag[1][k] + 0.129*vbk1lag[1][k];
   vbk3_[k] = (1.-0.3)*vbk3lag[1][k] + 0.3*vbk2lag[1][k];
   qastk[k] = vbk1_[k] + vbk2_[k] + vbk3_[k];
       }
```

The quality-adjusted equipment capital stock (**qastk**) and structures capital stock (**strcap**) vectors are then passed (along with the 55-level output vector (**qag**) and the coefficients for the productivity equations, which are stored in a separate file) to the "DanProductivity" function which calculates productivity and hours at the 55-industry level. The main section of code for this function is below:

```
t1 = t-1900;
if (t>=1972)
   t2 = t-1971;
else t2 = 0;
if (t>=1992)
   t3 = t-1991;
else t3 = 0;

n = P.neq;
/* For each number i, for each equation, gives us the number of sectors*/
for(i = 1; i <= n; i++){
       j = P.sec(i);
       which = P.type(i);
       if(j==86)
              Oliver=small;

       // i is the equation #, j is the sector #
       if(j<=0)
              continue;
       if(which>='a' && which <= 'f') {
              arith(" in Danprod before dq calculations, sector:",j);
              Qup=Qdown=0.0;
              curqag=qag[j];
              peakqag=Qagpeak[0][j];
              peaklag=Qagpeak[1][j];
              if(curqag <= 0. || peaklag <= 0) {
                     cprintf("\r\n\r\nIn Danprod:Negative or zero output in sector
                            %d:"   " curqag=%12.1f lagqagpeak=%12.1f\n\r",
                            j,qag[j],Qagpeak[1][j]);
                     trouble(NEGOUT);
                     dymetap();
                     continue;
                     }
              else dq = safelog(curqag) - safelog(peaklag);
              /* Difference of log of Output and Peak output  */
```

```
        arith(" in Danprod after dq, sector:",j);
        if(dq<0)
              Qdown=-1.0*dq;  //thats right, in my eqns qdown is always
              positive

        else Qup=dq;
        #ifdef DBG_PRD
        #endif
        }
 if(which=='a') {
        stuff = 1/(1-P[i][2]-P[i][3]);
        depend = stuff*( P[i][1] + P[i][2]*safelog(qastk[j])
              + P[i][3]*safelog(strcap[j])
              + P[i][6]*t1 - (P[i][2] + P[i][3])*safelog(curqag) );
}
 if(which=='b') {
        stuff = 1/(1-P[i][2]-P[i][3]);
        depend = stuff*( P[i][1] + P[i][2]*safelog(qastk[j])
              + P[i][3]*safelog(strcap[j]) + P[i][4]*Qup + P[i][5]*Qdown
              + P[i][6]*t1 - (P[i][2] + P[i][3])*safelog(curqag) );
}
 if(which=='c') {
        stuff = 1/(1-P[i][2]-P[i][3]);
        depend = stuff*( P[i][1] + P[i][2]*safelog(Qastk[1][j])
              + P[i][3]*safelog(Strcap[1][j])
              - (P[i][2] + P[i][3])*safelog(curqag) );
}
 if(which=='d') {
        stuff = 1/(1-P[i][2]-P[i][3]);
        depend = stuff*( P[i][1] + P[i][2]*safelog(Qastk[1][j])
              + P[i][3]*safelog(Strcap[1][j])
              + P[i][6]*t1 - (P[i][2] + P[i][3])*safelog(curqag) );
}
 if(which=='e') {
        depend = P[i][1] + P[i][3]*t2 + P[i][4]*Qup + P[i][5]*Qdown +
              P[i][6]*t1;
}
 if(which=='f') {
        stuff = 1/(1-P[i][2]-P[i][3]);
        depend = stuff*( P[i][1] + P[i][2]*safelog(qastk[j])
              + P[i][3]*safelog(strcap[j]) + P[i][4]*t3
              + P[i][6]*t1 - (P[i][2] + P[i][3])*safelog(curqag) );
}

 if(depend>=7 || depend<=0) {
        cprintf("In Danprod, Sector %d  depend is CRAAAAAZY!: %12.1f\n",
              j,depend);
        //continue;
        }
 Calc = exp(depend);
 Act = dprd[j];
 RCalc = P.rhoadj(Calc,dprd[j],i);
 dprd[j] = RCalc;
 #ifdef DBG_PRD
 fprintf(chk,"Calc = %9.2f Actual = %9.2f RCalc = %9.2f dhrs = %9.2f\n\n",
Calc,Act,RCalc,dhrs[j]);
 #endif
 }
```

```
dprd.fix(t);
// Calculate 55-industry hours (DHRS):
dhrs = ebediv(qag,dprd);
return(n);
```

Finally, the “ReviseEmploy” function simply disaggregates **dhrs** (55×1) to the 97-sector level. The 97×1 vector **hrs**, which is calculated using the old productivity equations, provides the shares to be used to split out **dhrs** to the more disaggregate level when there is a one-to-many mapping from the 55-industry level to the 97-sector level. The resulting 97×1 vector is now called **hrs** (overwriting the former **hrs** vector) and 97-sector **prd** is now recalculated as **out** (97×1) divided by **hrs**. From this point on, ReviseEmploy takes **hrs** and **prd** and calculates employment just as the old “Employ” function would have with the old vectors and the rest of the model proceeds with these new vectors for **hrs**, **prd**, and **emp**.

Table 1 - Estimates of Embodied Technological Change

Sector	Rate (std. error)
Manufacturing	
Food & Tobacco	-0.056 (0.021)
Textiles and knitting	0.098 (0.030)
Apparel	0.004 (0.025)
Paper	-0.064 (0.027)
Printing & publishing	-0.053 (0.023)
Chemicals	-0.004 (0.024)
Petroleum refining & Fuel Oil	0.017 (0.039)
Rubber & Plastic products	0.084 (0.026)
Shoes & leather	-0.046 (0.052)
Lumber	0.007 (0.023)
Furniture	-0.056 (0.028)
Stone, clay & glass	0.006 (0.026)
Primary metals	0.080 (0.029)
Metal products	-0.005 (0.022)
Industrial Equipment, except computers & office equipment	0.031 (0.024)
Computers & other office equipment	2.927 (0.202)
Electrical equipment except communications and elec. components	0.049 (0.029)
Communication equipment	0.141 (0.044)
Electronic components	0.766 (0.059)
Motor vehicles & parts	-0.064 (0.028)
Non-motor vehicle transportation equipment	0.098 (0.033)
Scientific Instruments	-0.023 (0.034)
Other instruments	0.087 (0.039)
Miscellaneous manufacturing	0.029 (0.032)
Nonmanufacturing	
Agriculture, forestry, and fisheries	0.009
Metal mining	0.027
Coal mining	-0.006
Natural Gas and Crude Petroleum extraction	0.013
Non-metallic mining	-0.001
Construction	0.026
Railroads	0.024
Air transport	0.109
Other transportation	0.058
Communication services	0.112
Electric utilities	0.056
Gas utilities, and water and sanitary services	0.033
Wholesale trade	0.066
Retail trade, and restaurant and bars	0.044
Finance and Insurance	0.065

Table 1 continued on next page...

Real Estate	0.065
Hotels, and personal and repair services (exc. auto)	0.061
Business services	0.078
Automobile services	0.063
Movies and amusement parks	0.037
Medical services	0.062
Education, social services, membership organizations	0.062

Table 2 - Notation Guide

Variable	Abbreviation	Elasticity of Output with respect to the variable
Real Output (log)	*Q (q)*	--
Real Materials, including Energy (log)	*M (m)*	Θ
Labor Hours (log)	*L* (ℓ)	β
Real Equipment Stock (log)	*J (j)*	α
Real Structures Stock (log)	*S (s)*	η
Index of Equipment-Embodied R&D (log)	*R (r)*	σ
Real Energy Expenditures (log)	*E (e)*	--
Elasticity of Energy:Capital Ratio w.r.t. Utilization	τ	--

Table 3 Specification Choice

Specification	Number of industries	Number requiring soft constraints
C’	19	18
D"	27	25
E	4	2
X	4	4
Total	54	49

Table 4 Mixed Empirical-Monte Carlo Results

Coefficient	True value	Mean Estimate	Std. Deviation	Estimated Bias
c^0	2	1.99092	0.23680	-0.00908
c^1	0.01	0.00968	0.00842	-0.00032
α	0.17	0.18120	0.28050	0.0112
η	0.16	0.15852	0.03292	-0.00148
b^0	0.1	0.10118	0.35719	0.00118
b^1	-0.1	-0.08530	0.37322	0.0147

Table 5 -- Deviations in Labor Productivity (Permanent Shock)

Industry	Equipment Elasticity	Percent Deviations from Base 2001	2005	2010	2015
1 Agriculture, forestry and fisheries (C’)	0.288	0.02	0.54	0.83	0.97
2 Metal mining (X)	0.094	-0.30	-0.09	0.00	0.03
3 Coal mining (C’)	0.295	-0.05	0.52	0.87	1.05
4 Crude petroleum and natural gas (C’)	0.359	-0.04	0.69	1.05	1.21
5 Non-metallic mining (D")	0.065	0.02	0.06	0.10	0.12
6 Construction (D")	0.318	0.08	0.56	0.87	1.02
7 Food and tobacco products (D")	0.206	0.06	0.32	0.48	0.57
8 Textile mill products (D")	0.060	0.09	0.08	0.13	0.14
9 Apparel and other textile products (D")	0.131	0.02	0.13	0.22	0.28
10 Paper and allied products (D")	0.133	0.09	0.11	0.21	0.27
11 Printing and publishing (D")	0.140	0.22	0.09	0.17	0.22
12 Chemicals (X)	0.239	-0.12	0.34	0.63	0.76
13 Plastics and synthetic materials (X)	0.207	-0.20	0.18	0.44	0.54
14 Petroleum refining (D")	0.025	0.03	0.01	0.03	0.05
15 Rubber and miscellaneous plastics (D")	0.081	0.31	0.08	0.13	0.14
16 Footwear and leather products (C’)	0.326	0.03	0.35	0.63	0.79
17 Lumber and wood products (C’)	0.054	-0.04	0.06	0.12	0.12
18 Furniture (D")	0.026	0.36	-0.04	-0.03	-0.02
19 Stone, clay and glass products (D")	0.058	0.05	0.06	0.11	0.14
20 Primary iron and steel (D")	0.095	0.16	0.09	0.15	0.16
21 Primary nonferrous metals mfg. (D")	0.055	0.17	0.06	0.09	0.10
22 Metal products (D")	0.064	0.15	0.05	0.09	0.11
23 Engines and turbines (C’)	0.123	-0.16	0.04	0.13	0.16
24 Agricultural, construction & mining mach. (C’)	0.061	-0.18	-0.11	-0.07	-0.06
25 Metalworking machinery (E)	N/A	0.56	0.00	0.00	0.01
26 Special industry machinery (E)	N/A	0.15	0.01	0.00	0.02
27 General and miscellaneous industrial mach. (C’)	0.062	-0.19	-0.07	-0.01	0.03
28 Computers and office equipment (C’)	0.125	-0.23	-0.04	0.04	0.08
29 Service industry machinery (D")	0.083	0.43	0.08	0.11	0.12
30 Electrical industrial equipment and app. (D")	0.088	0.40	0.03	0.08	0.10
31 Household appl., elec lighting & wiring (D")	0.306	0.29	0.48	0.72	0.81
32 Audio, video and comm. equipment (C’)	0.073	-0.17	-0.06	0.00	0.03
33 Electronic components (D")	0.215	0.01	0.32	0.45	0.49
34 Motor vehicles and equipment (C’)	0.086	-0.12	0.02	0.10	0.12
35 Aircraft and parts (C’)	0.195	0.00	0.25	0.37	0.40

Table 5 continued on next page...

36 Ships and other transportation equipment (D")	0.074	-0.03	0.02	0.05	0.06
37 Instruments (C’)	0.158	-0.07	0.12	0.22	0.26
38 Miscellaneous manufacturing (D")	0.297	0.06	0.44	0.61	0.65
39 Railroad transportation (X)	0.029	-0.08	-0.04	-0.02	-0.01
40 Air transportation (C’)	0.330	0.11	0.82	1.09	1.16
41 Trucking and other transport (C’)	0.094	-0.07	0.05	0.11	0.13
42 Communications services (C’)	0.183	-0.02	0.37	0.48	0.50
43 Electric utilities (C’)	0.349	0.08	0.66	0.93	1.02
44 Gas, water and sanitary services (C’)	0.095	-0.07	0.05	0.09	0.11
45 Wholesale trade (D")	0.089	0.10	-0.05	0.03	0.06
46 Retail trade, restaurants & bars (E)	N/A	0.02	0.00	0.00	0.00
47 Finance and insurance (D")	0.036	0.04	0.02	0.01	0.03
48 Real estate and rental (E)	N/A	0.04	0.00	0.00	0.00
49 Hotels, repairs except auto (D")	0.132	0.11	0.14	0.20	0.22
50 Business and professional services (D")	0.214	0.15	0.08	0.25	0.32
51 Automotive repair and services (D")	0.031	0.11	0.01	0.04	0.06
52 Movies and amusements (D")	0.161	0.14	0.23	0.31	0.34
53 Health services (C’)	0.348	0.17	0.71	0.92	0.98
54 Educational and social services and NPO (C’)	0.147	0.01	0.23	0.29	0.31

Table 6 -- Deviations in Labor Productivity (One-Time Shock)

Industry	Equipment Elasticity	Percent Deviations from Base 2001	2005	2010	2015
1 Agriculture, forestry and fisheries (C’)	0.288	0.32	0.74	0.97	0.96
2 Metal mining (X)	0.094	-1.46	-0.02	0.06	0.08
3 Coal mining (C’)	0.295	-0.15	1.07	1.49	1.63
4 Crude petroleum and natural gas (C’)	0.359	0.04	1.11	1.47	1.56
5 Non-metallic mining (D")	0.065	0.08	0.20	0.16	0.16
6 Construction (D")	0.318	0.45	1.12	1.32	1.37
7 Food and tobacco products (D")	0.206	0.32	0.42	0.51	0.54
8 Textile mill products (D")	0.060	0.47	0.17	0.12	0.10
9 Apparel and other textile products (D")	0.131	0.07	0.23	0.21	0.22
10 Paper and allied products (D")	0.133	0.42	0.17	0.22	0.22
11 Printing and publishing (D")	0.140	1.12	0.27	0.20	0.21
12 Chemicals (X)	0.239	-0.56	0.55	0.63	0.61
13 Plastics and synthetic materials (X)	0.207	-1.01	0.40	0.41	0.37
14 Petroleum refining (D")	0.025	0.12	0.02	0.02	0.04
15 Rubber and miscellaneous plastics (D")	0.081	1.03	0.18	0.14	0.13
16 Footwear and leather products (C’)	0.326	0.19	0.52	0.74	0.79
17 Lumber and wood products (C’)	0.054	-0.21	-0.02	0.14	0.13
18 Furniture (D")	0.026	1.83	0.16	-0.01	-0.01
19 Stone, clay and glass products (D")	0.058	0.25	0.15	0.12	0.11
20 Primary iron and steel (D")	0.095	0.79	0.20	0.17	0.16
21 Primary nonferrous metals mfg. (D")	0.055	0.86	0.23	0.12	0.09
22 Metal products (D")	0.064	0.78	0.16	0.11	0.11
23 Engines and turbines (C’)	0.123	-0.81	0.15	0.18	0.17
24 Agricultural, construction & mining mach. (C’)	0.061	-0.86	0.03	0.04	0.04
25 Metalworking machinery (E)	0.000	2.70	0.24	-0.02	0.00
26 Special industry machinery (E)	0.000	0.72	-0.05	0.00	0.00
27 General and miscellaneous industrial mach. (C’)	0.062	-0.94	0.03	0.04	0.06
28 Computers and office equipment (C’)	0.125	-1.13	0.21	0.20	0.17
29 Service industry machinery (D")	0.083	2.16	0.29	0.16	0.15
30 Electrical industrial equipment and app. (D")	0.088	1.94	0.15	0.14	0.14
31 Household appl., elec lighting & wiring (D")	0.306	1.30	0.87	0.80	0.77
32 Audio, video and communication equipment (C’)	0.073	-0.85	0.04	0.06	0.06
33 Electronic components (D")	0.215	0.02	0.43	0.38	0.35
34 Motor vehicles and equipment (C’)	0.086	-0.65	0.13	0.16	0.17
35 Aircraft and parts (C’)	0.195	-0.01	0.41	0.39	0.37
36 Ships and other transportation equipment (D")	0.074	-0.15	0.36	0.13	0.12
37 Instruments (C’)	0.158	-0.36	0.31	0.32	0.31
38 Miscellaneous manufacturing (D")	0.297	0.28	0.78	0.66	0.57
39 Railroad transportation (X)	0.029	-0.41	-0.03	-0.01	-0.01
40 Air transportation (C’)	0.330	0.51	1.13	1.07	0.95
41 Trucking and other transport (C’)	0.094	-0.36	0.07	0.10	0.08

Table 6 continued on next page...

42 Communications services (C’)	0.183	-0.08	0.48	0.48	0.42
43 Electric utilities (C’)	0.349	0.49	1.03	1.12	1.09
44 Gas, water and sanitary services (C’)	0.095	-0.28	0.08	0.15	0.15
45 Wholesale trade (D")	0.089	0.02	0.08	0.09	0.09
46 Retail trade, restaurants & bars (E)	0.000	0.10	0.02	0.00	0.00
47 Finance and insurance (D")	0.036	0.17	0.00	0.04	0.07
48 Real estate and rental (E)	0.000	0.20	0.00	0.00	0.00
49 Hotels, repairs except auto (D")	0.132	0.58	0.22	0.20	0.19
50 Business and professional services (D")	0.214	2.33	1.03	0.41	0.38
51 Automotive repair and services (D")	0.031	0.71	0.01	0.07	0.07
52 Movies and amusements (D")	0.161	0.76	0.38	0.34	0.33
53 Health services (C’)	0.348	0.93	0.92	0.96	0.87
54 Educational and social services and NPO (C’)	0.147	0.14	0.22	0.28	0.26

Figure 1, Gridsearch Results of Matching FRB Physical Depreciation Patterns

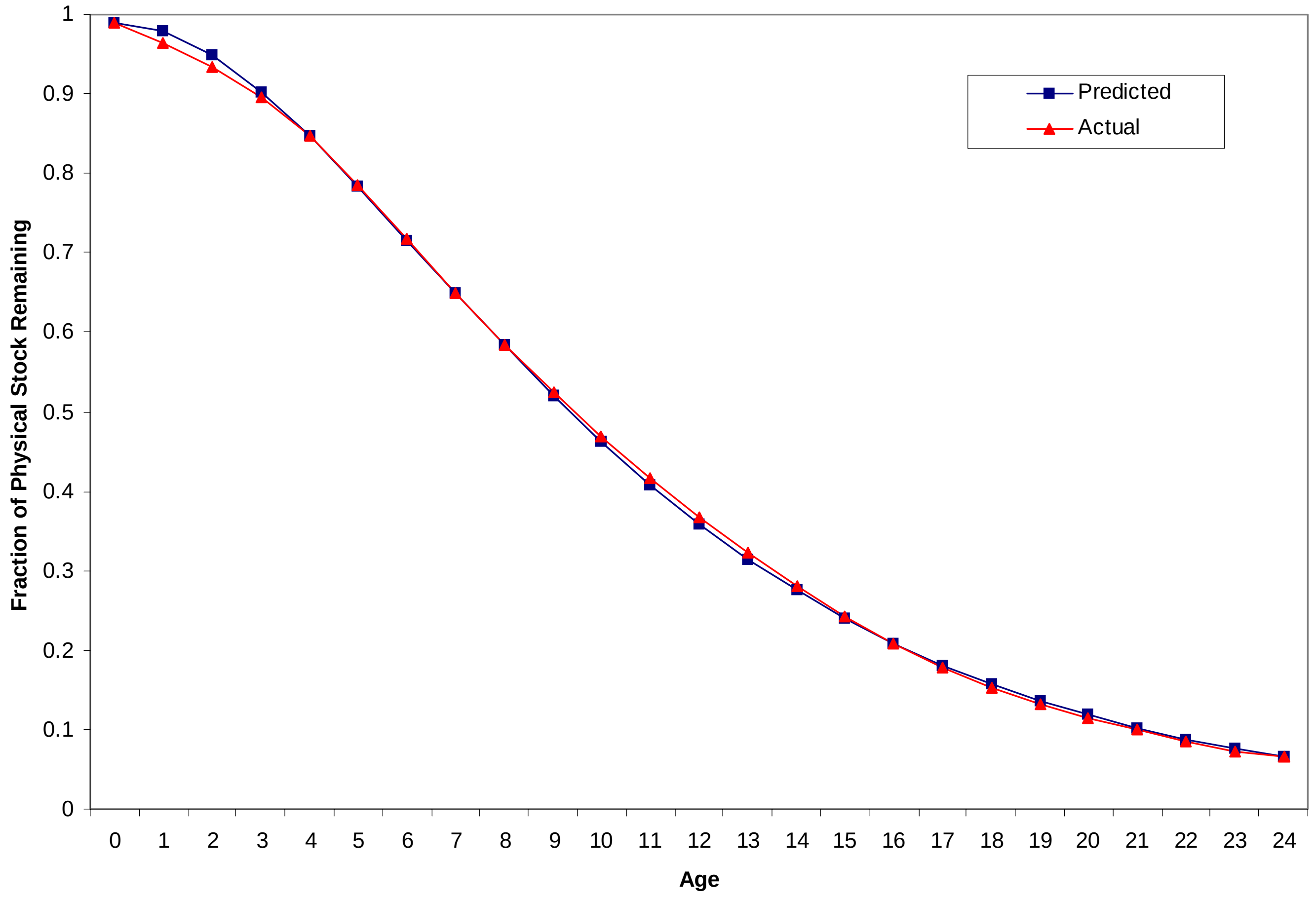


Figure 2, Average Adjusted R-squared

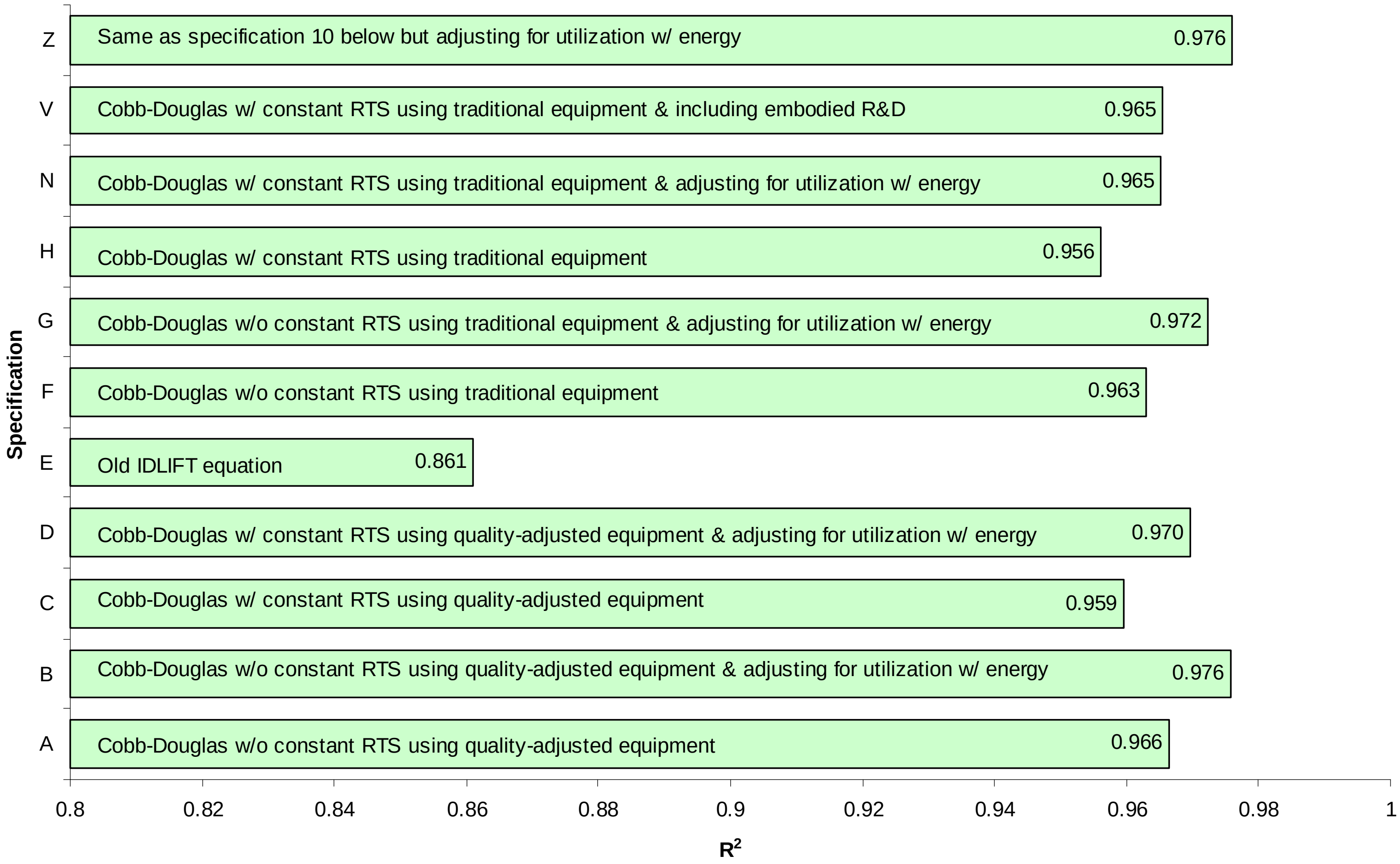


Figure 3, Average Elasticities

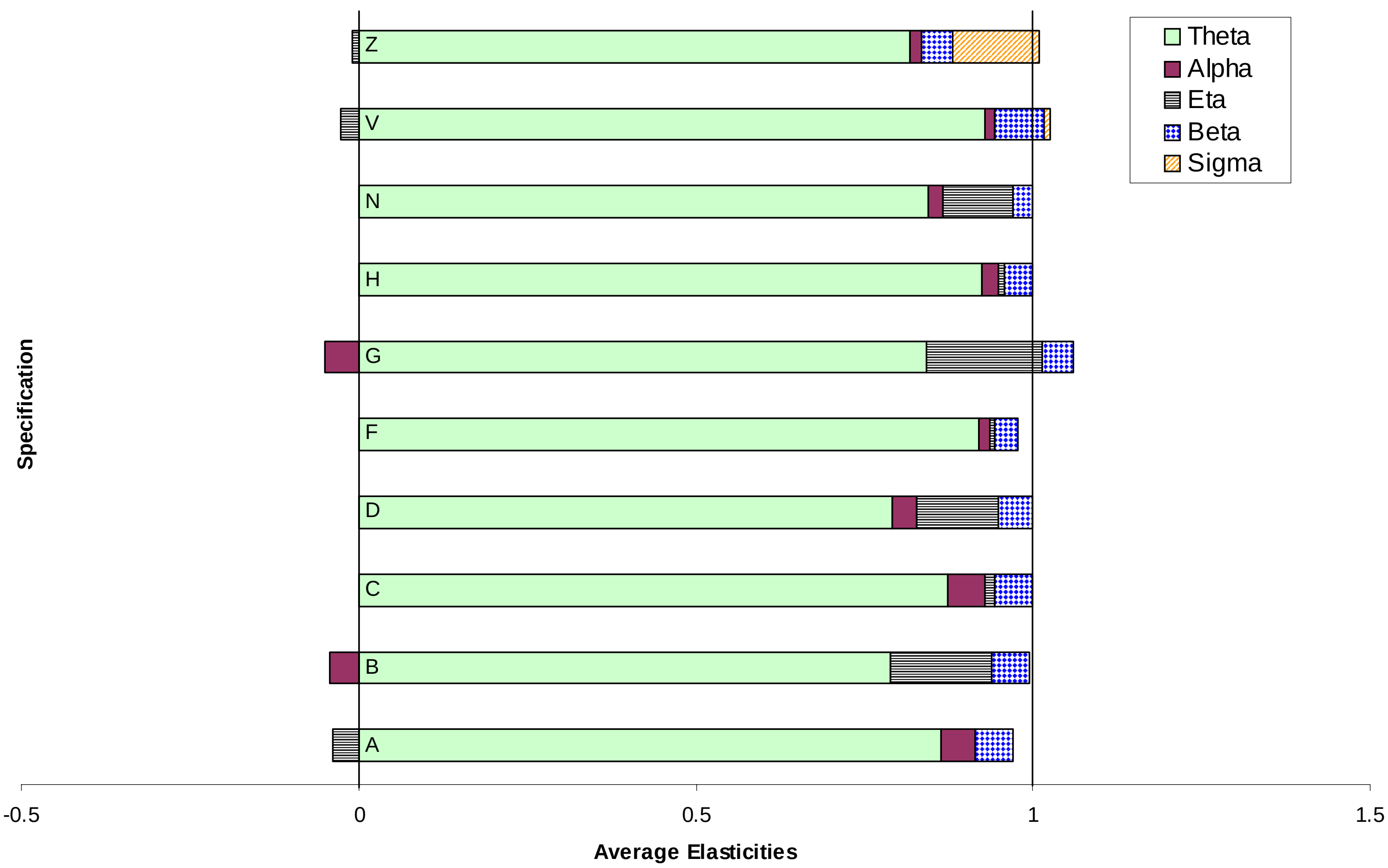


Figure 4, Percent of Elasticities that are Positive

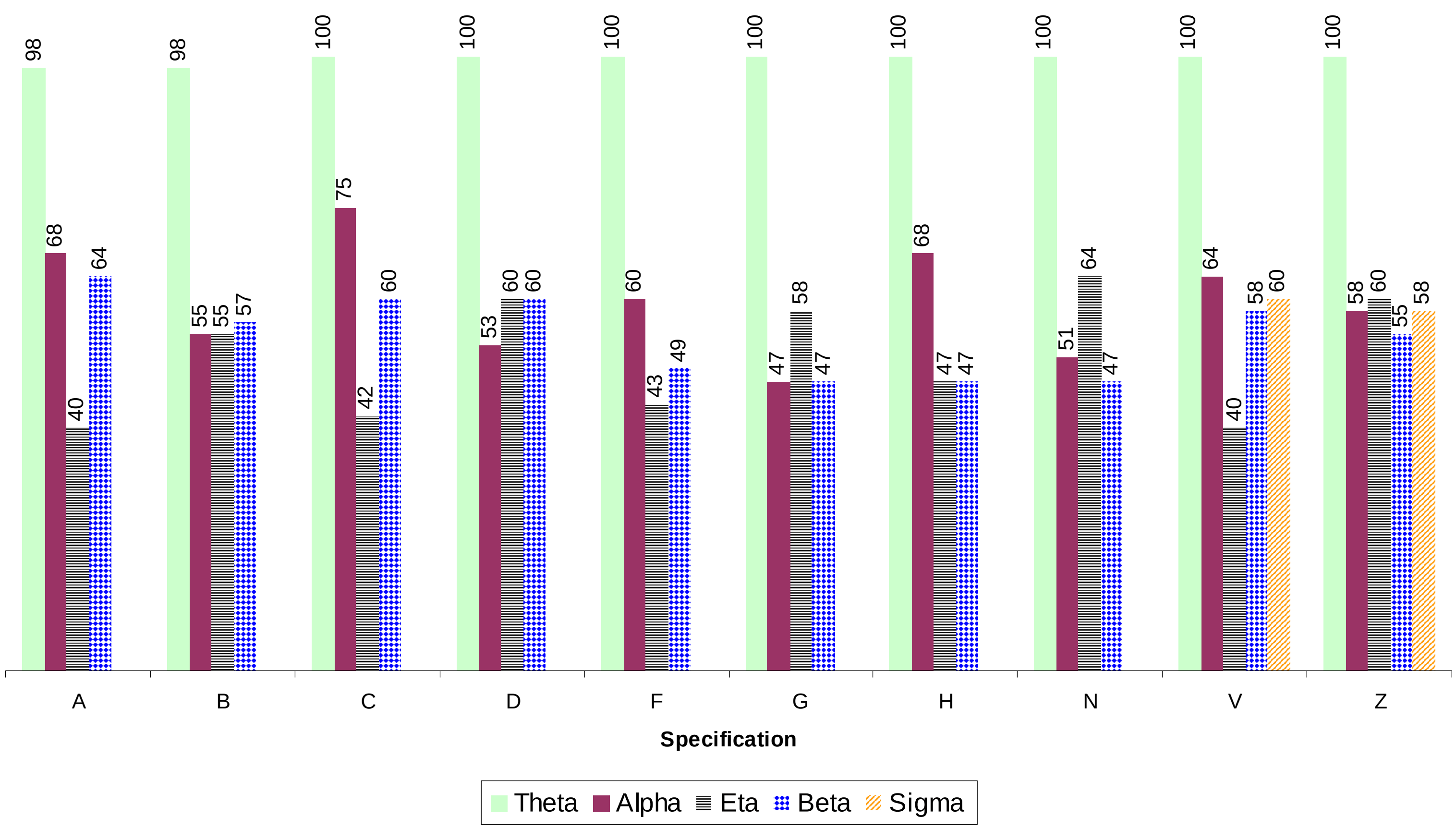


Figure 5, Average Adjusted R-squared -- Specifications Not Including Materials

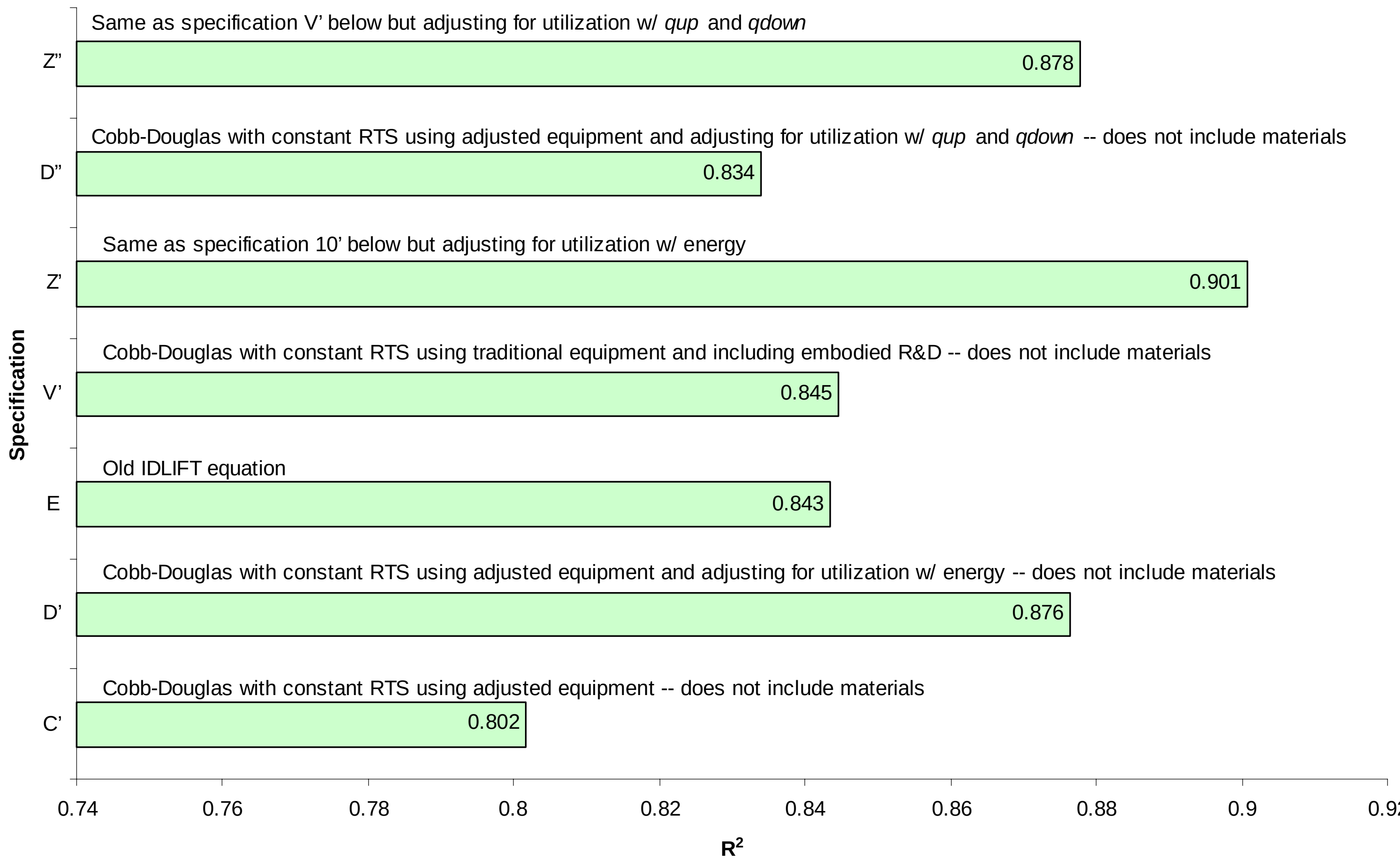


Figure 6, Average Elasticities -- Specifications Not Including Materials

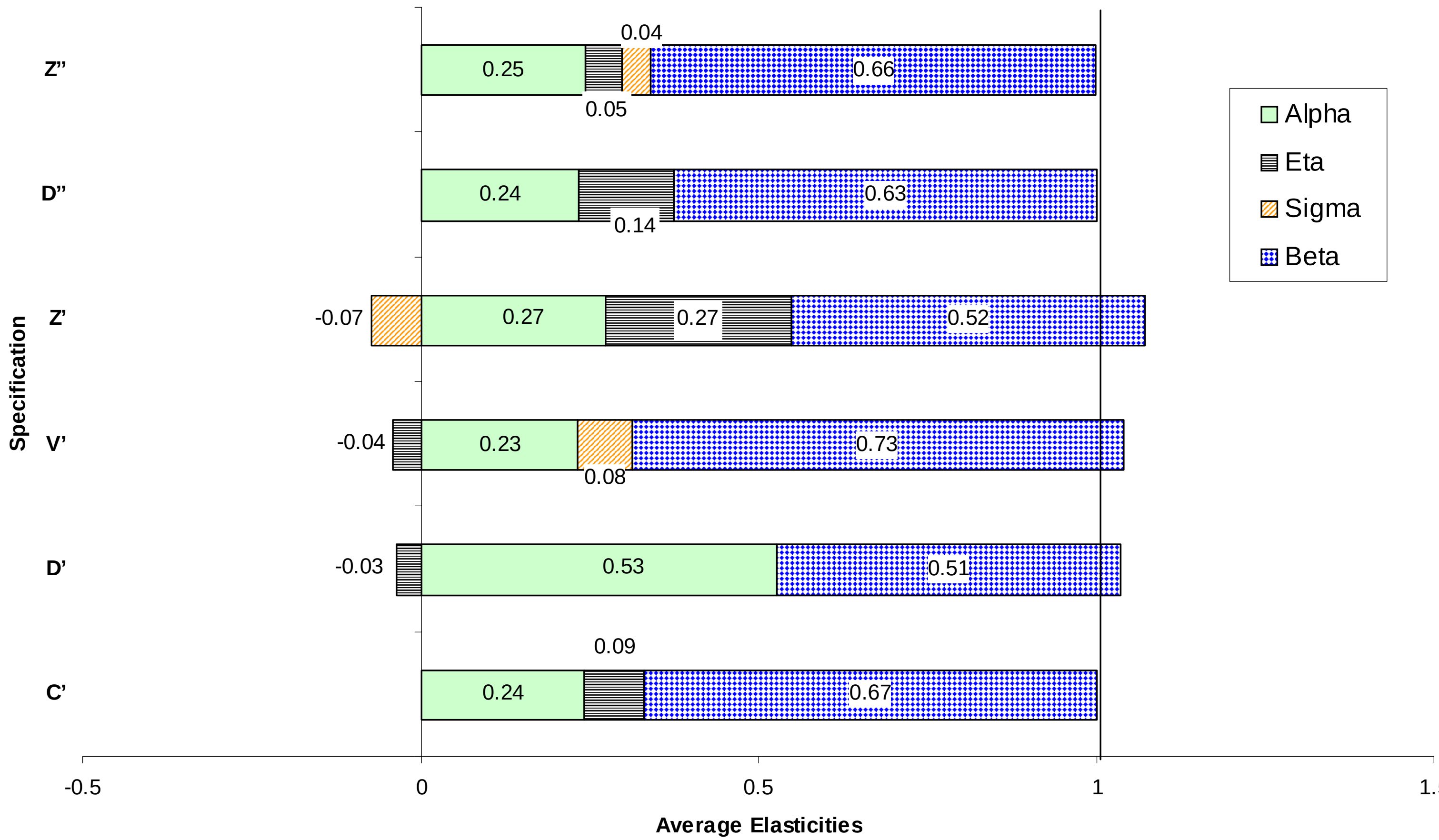
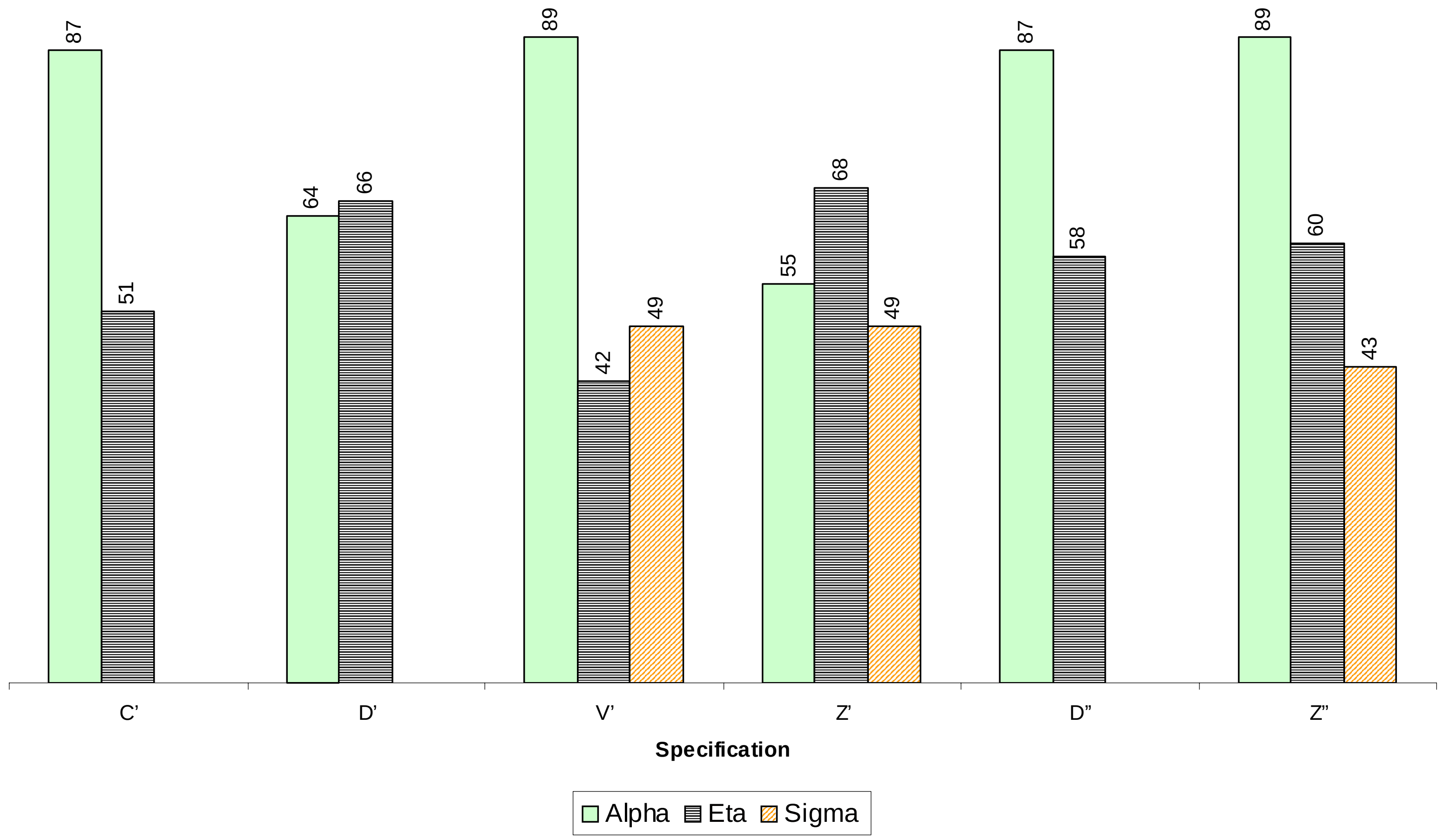


Figure 7, Percent of Elasticities that are Positive -- Specifications Not Including Materials



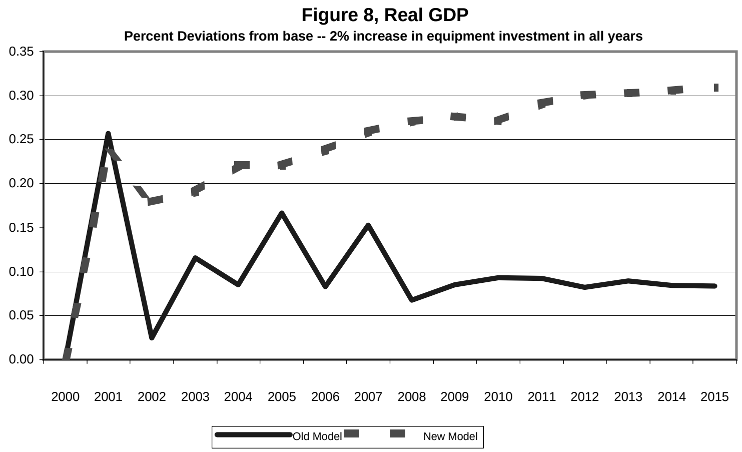


Figure 8, Real GDP
Percent Deviations from base -- 2% increase in equipment investment in all years
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

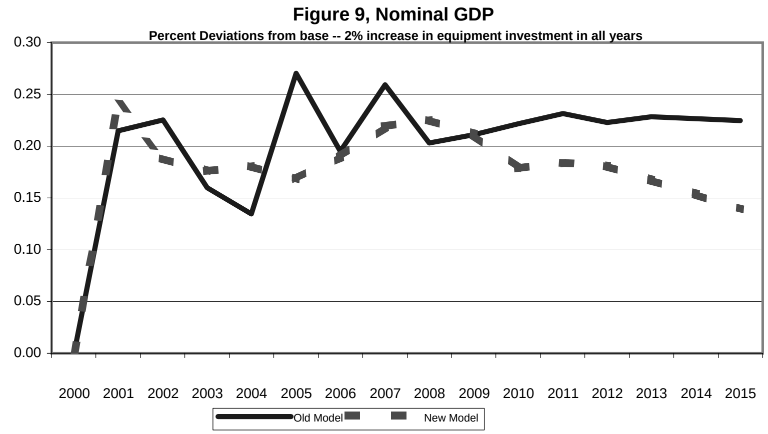


Figure 9, Nominal GDP
Percent Deviations from base -- 2% increase in equipment investment in all years
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

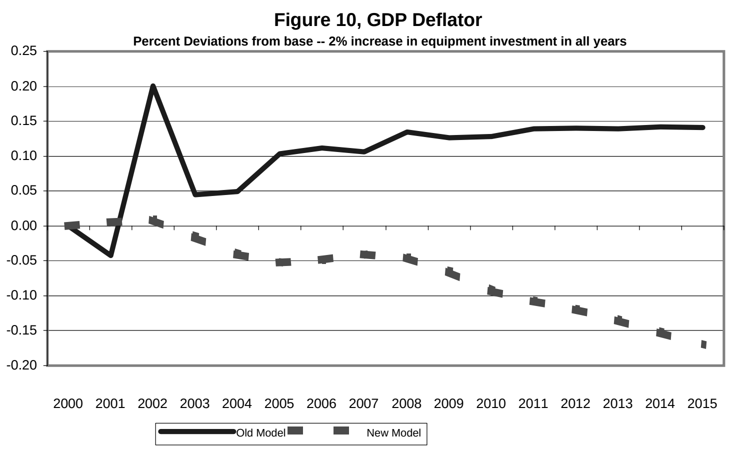


Figure 10, GDP Deflator
Percent Deviations from base -- 2% increase in equipment investment in all years
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

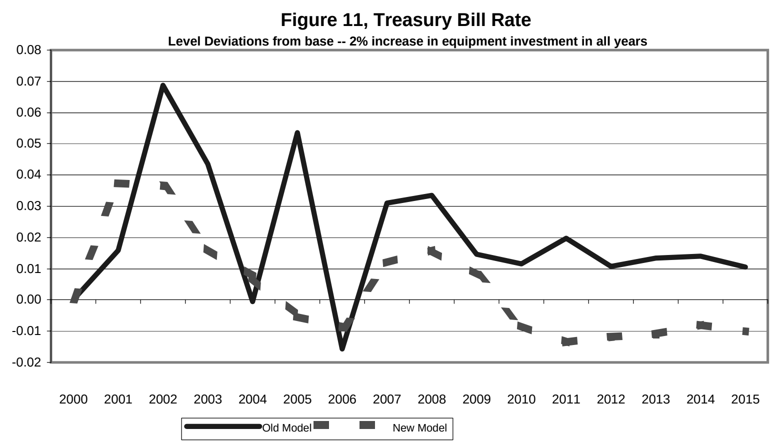


Figure 11, Treasury Bill Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

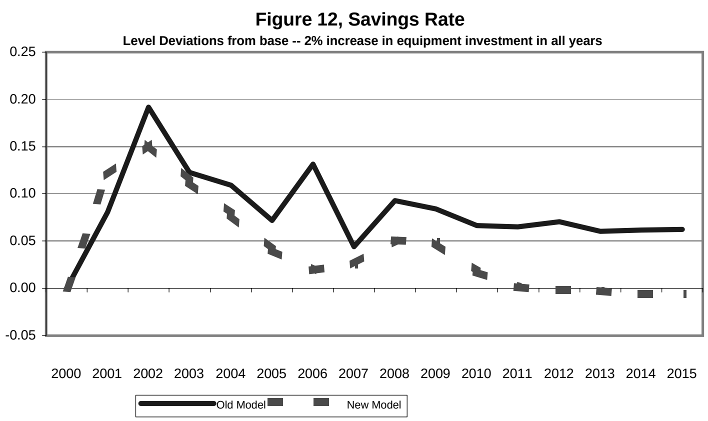


Figure 12, Savings Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

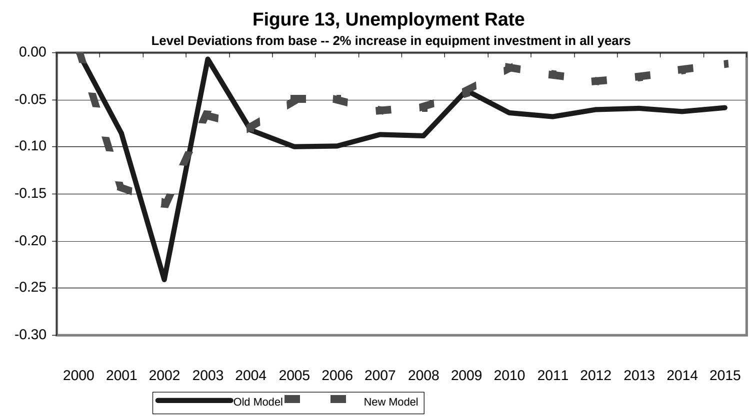


Figure 13, Unemployment Rate
Level Deviations from base -- 2% increase in equipment investment in all years
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

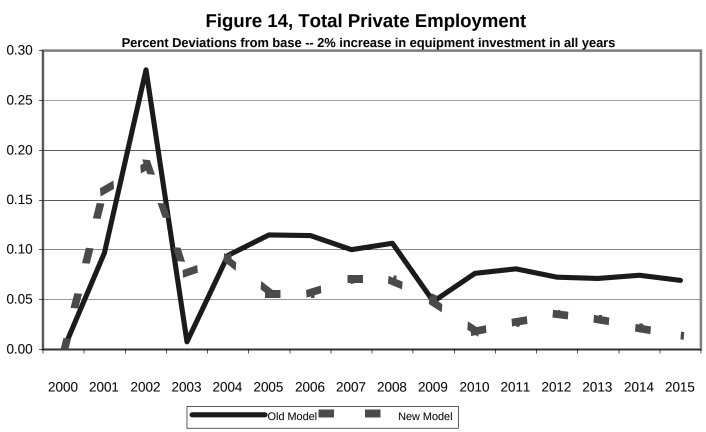


Figure 14, Total Private Employment
Percent Deviations from base -- 2% increase in equipment investment in all years
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

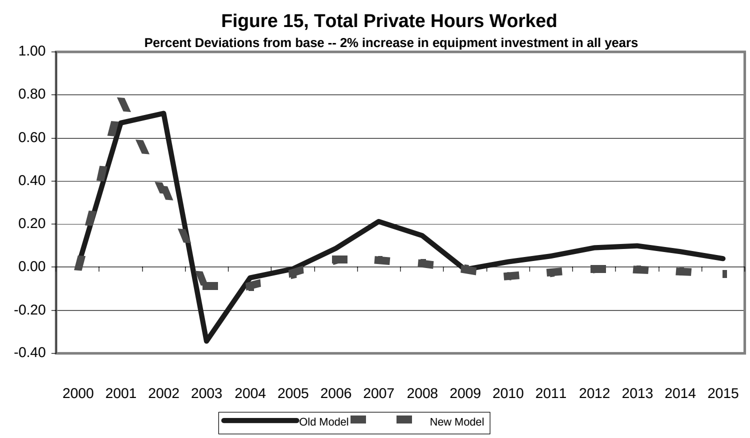


Figure 15, Total Private Hours Worked
Percent Deviations from base -- 2% increase in equipment investment in all years
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
Old Model
New Model

Figure 16, Average Real Wage

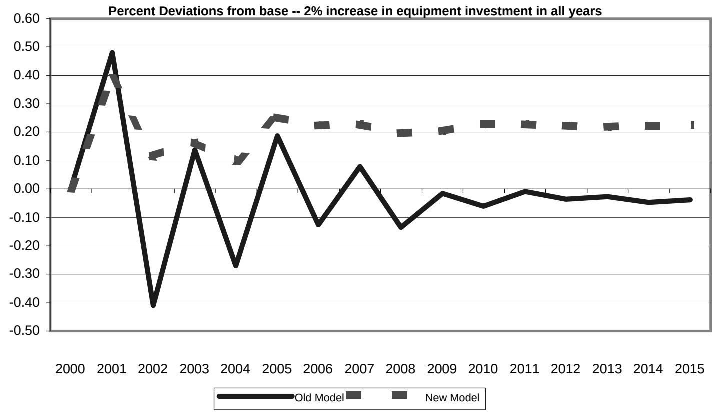


Figure 17, Labor Productivity

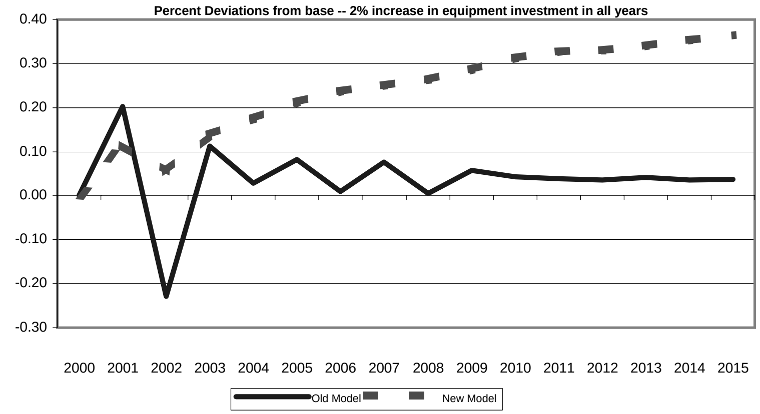


Figure 18, Real GDP

Percent Deviations from base -- 10% increase in equipment investment in 2001

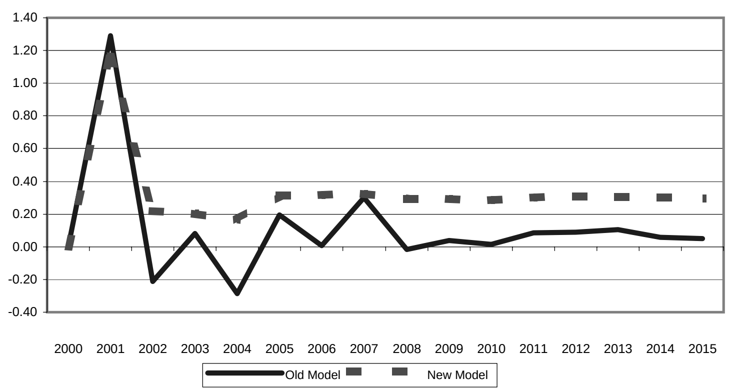


Figure 19, Nominal GDP

Percent Deviations from base -- 10% increase in equipment investment in 2001

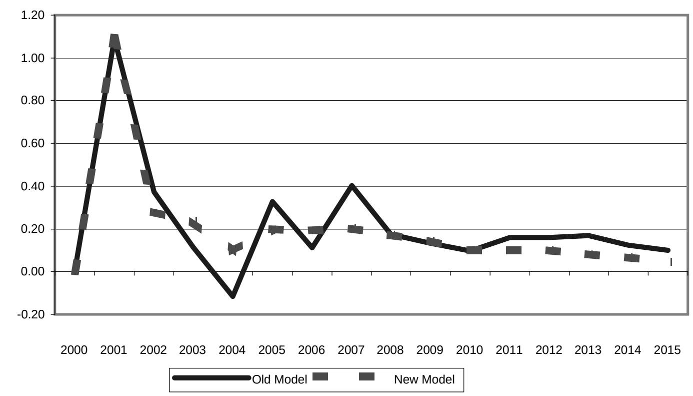


Figure 20, GDP Deflator

Percent Deviations from base -- 10% increase in equipment investment in 2001

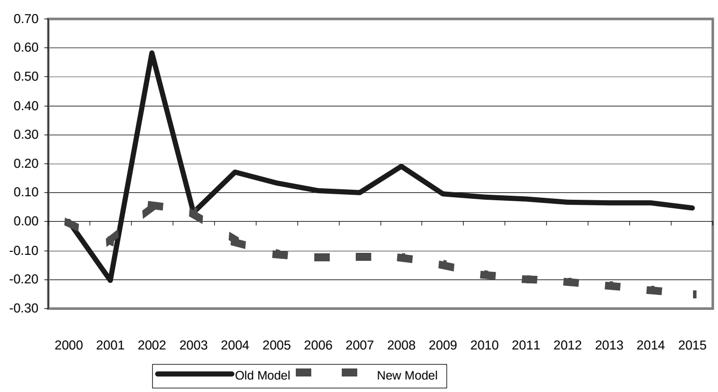


Figure 21, Treasury Bill Rate

Level Deviations from base -- 10% increase in equipment investment in 2001

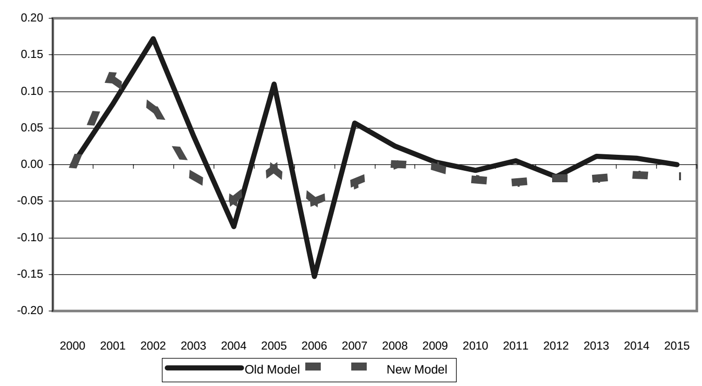


Figure 22, Savings Rate

Percent Deviations from base -- 10% increase in equipment investment in 2001

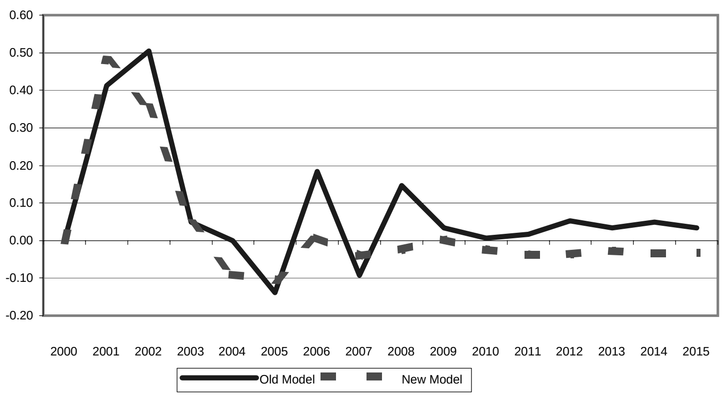


Figure 23, Unemployment Rate

Level Deviations from base -- 10% increase in equipment investment in 2001

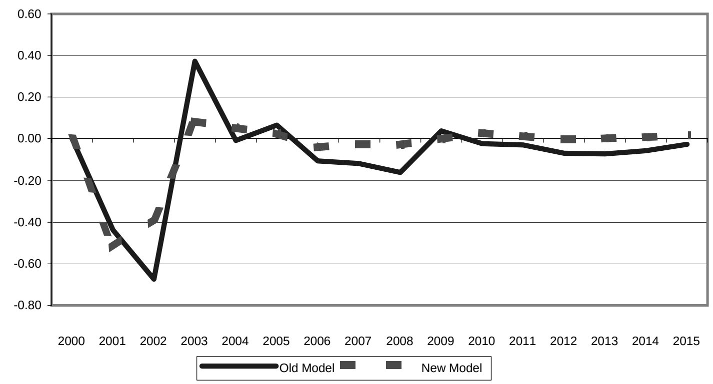


Figure 24, Total Private Employment

Percent Deviations from base -- 10% increase in equipment investment in 2001

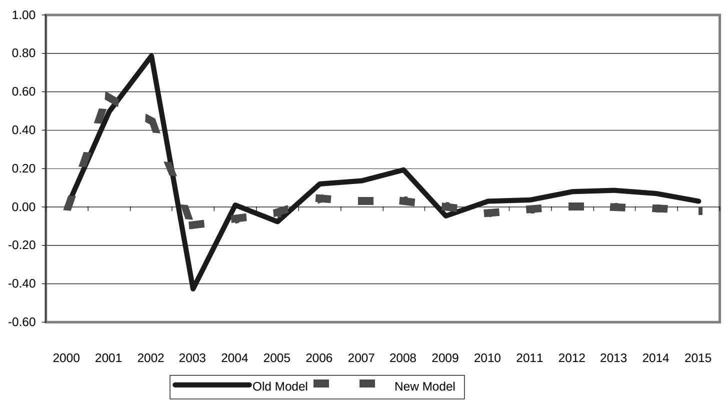


Figure 25, Total Private Hours Worked

Percent Deviations from base -- 10% increase in equipment investment in 2001

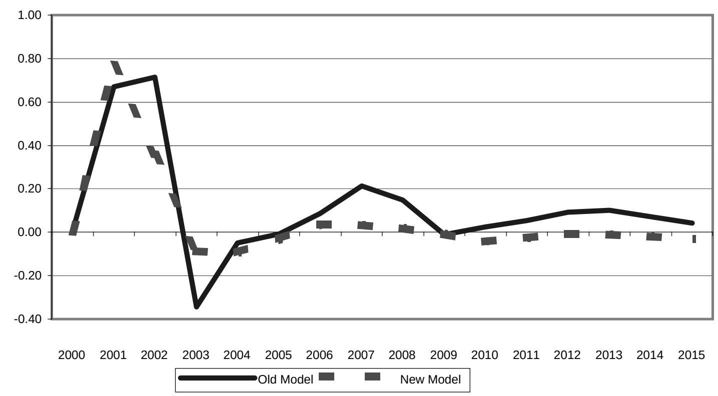


Figure 26, Average Real Wage

Percent Deviations from base -- 10% increase in equipment investment in 2001

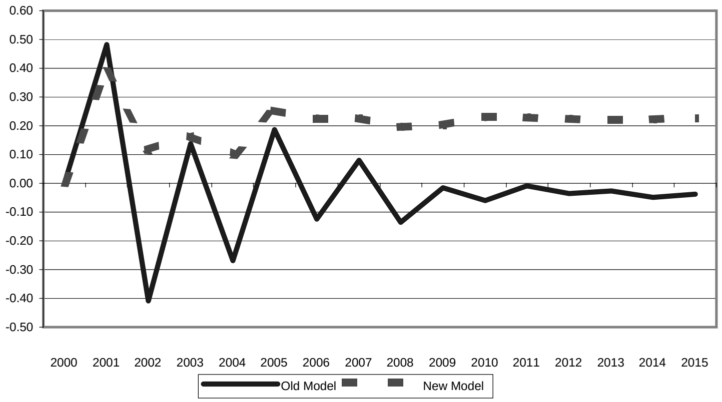


Figure 27, Labor Productivity

Percent Deviations from base -- 10% increase in equipment investment in 2001

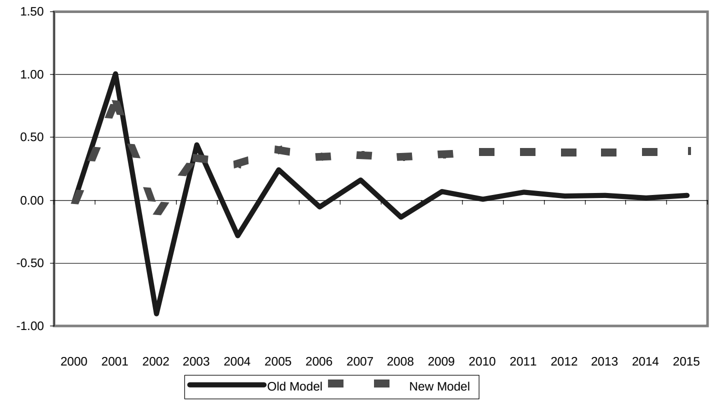


Figure 28, Equipment Investment

Percent Deviations from base -- 10% increase in equipment investment in 2001

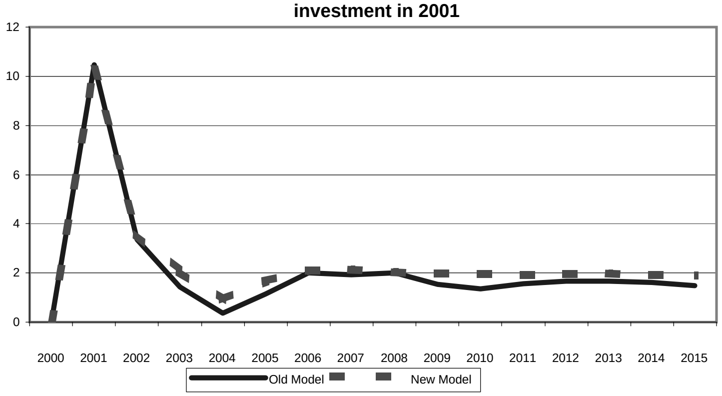


Figure 29, Equipment Capital

Percent Deviations from base -- 10% increase in equipment investment in 2001