Pre-commitment, the Timeless Perspective, and Policymaking from Behind a Veil of Uncertainty•

Richard Dennis*

Federal Reserve Bank of San Francisco

August 2001

JEL Classification: E52, C61

Abstract

Woodford (1999) develops the notion of a "timelessly optimal" pre-commitment policy. This paper uses a simple business cycle model to illustrate this notion. We show that timelessly optimal policies are not unique and that they are not necessarily better than the time-consistent solution. Further, we describe a method for constructing optimal pre-commitment rules in an environment where the policymaker does not know the initial state of the economy. This latter solution is useful for characterizing the benefits policymakers extract through exploiting initial conditions.

• The views expressed in this paper are those of the author and do not necessary reflect those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

* Address for correspondence: Richard Dennis, Economic Research Department, Mail Stop 1130, Federal Reserve Bank of San Francisco, 101 Market Street, San Francisco, CA 94105, USA. Email: Richard.Dennis@sf.frb.org.

1) Introduction

One of the more exciting developments in optimal policymaking is the “timelessly optimal” policy concept that Woodford (1999) introduced to the literature. Prior to Woodford (1999) optimal pre-commitment policies were solved using the methods first outlined in Kydland and Prescott (1980) and developed more fully in Currie and Levine (1993). The problem associated with these earlier techniques is that they come with an “initial period problem.” More precisely, they imply that policymakers behave very differently in the initial period than in subsequent periods. If the behavior in the initial period were repeated in later periods then the policy strategy would no longer be optimal. Thus Currie and Levine’s (1993) treatment of pre-commitment cannot be formulated as a recursive optimization problem.

Woodford’s (1999) innovation was to consider the optimization problem from a “timeless perspective,” an innovation directed at eliminating this initial period problem. The timeless perspective argues that a policymaker optimizing today should not exploit the initial state of the economy, but rather implement the policy that it would have chosen to implement today if it had been optimizing from a time period far in the past. As we shall see, while the timeless perspective overcomes the initial period problem, it does so at a cost. There is a continuum of solutions, each of which fully satisfies the requirements to be timelessly optimal; that is, the timeless perspective lends itself to indeterminacy.

Alongside this literature on the timeless perspective, other authors (such as King and Wolman, 1999, Khan et al, 2000, and Amato and Laubach, 2001) depart from Currie and Levine (1993) in that they assume that the economy is in steady state in the period prior to some initial optimization. With this assumption these authors rule out any exploitation of the initial state vector. However, as we show below, assuming the economy is in steady state in the initial period does not generally lead to a timelessly optimal policy because the initial period is still treated differently than subsequent periods.

In this paper we use a simple business cycle model to illustrate Woodford's (1999) timeless perspective, as described in Svensson and Woodford (1999). We compare timelessly optimal policies with the Currie and Levine (1993) solution and draw out the nature of the indeterminacy. We also motivate and describe a solution procedure that constructs optimal pre-commitment rule in the case where the policymaker does not observe, and therefore cannot exploit, the initial state of the economy.

This paper is structured as follows. In the following Section we describe the simple business cycle model that we use to illustrate the ideas in the paper. Section 3 describes the optimal pre-commitment solution, as developed in Currie and Levine (1985, 1993). In Section 4 we turn to Woodford's (1999) timeless perspective. We show that there is a continuum of solutions that are all timelessly optimal and we relate this indeterminacy to the problem of assigning value to past commitments. In Section 5 we motivate an alternative optimal policy in which the policymaker must choose its decision rule without knowing the past history of the state vector. We relate this solution to Svensson and Woodford (1999), to Currie and Levine (1993), and to King and Wolman (1999). Section 6 concludes.

2) A Simple Business Cycle Model

To ensure that our analysis is as transparent as possible we employ a very simple business cycle model. Firms operate in a monopolistically competitive environment. They employ labor for production and choose the price of their good subject to a downward sloping demand curve and a real cost of adjusting their price level. An aggregator, takes the firms' outputs and bundles them into a single consumption good that it sells to households in a perfectly competitive market. Households choose their level of consumption, holdings of real money balances, and stock of bonds to take into the next period, subject to their budget constraint. Further, households make a labor-leisure choice. The government chooses the level of the nominal interest rate, supplies households with whatever level of nominal money balances they demand, and remits any seignorage revenues to households through lump sum transfers. As shown in Walsh (2001), when log-linearized about its deterministic steady state, this business cycle model can be represented as

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + g_t \tag{1}$$

$$\pi_t = \pi^* + \beta(E_t \pi_{t+1} - \pi^*) + \alpha c_t + u_t . \tag{2}$$

In equations (1) and (2), c_t is consumption, π_t is inflation, i_t is the nominal interest rate, g_t is a demand shock, and u_t is a cost push shock. Turning to the parameters, ρ is the rate of time preference, β is the discount factor, σ is the inter-temporal elasticity of substitution, and α indexes firms' price markup over marginal costs. π^* is the steady-state inflation rate whose value is determined by the interaction of the government's interest rate rule with equations (1) and (2). The shocks, g_t and u_t are independent zero mean finite variance processes.

2.1) Policy Objectives

Equation (1) is a necessary condition arising from the households' constrained optimization problem. It is derived from an additively separable constant elasticity of substitution utility function. Woodford (2000) derives conditions under which the following objective function can correctly be employed as a second order approximation to the representative agent's utility function.

$$Loss(0,\infty) = E_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t - \hat{\pi})^2 + \omega(c_t - \hat{c})^2 + \nu(i_t - \hat{\pi} - \rho)^2] \tag{3}$$

In equation (3), $\hat{\pi}$ and $\hat{c}$ ($\hat{c} > 0$) are the target values for inflation and consumption that are chosen by the government. The parameters ω, $\nu \geq 0$, are in principle determined by the preference and technology parameters characterizing the model. However, in applications these policy preference parameters are more usually interpreted as parameters that determine the type of the central banker appointed by the government. The government specifies the target values for consumption and inflation, but delegates the job of achieving these targets to an instrument independent central banker. Because of monopolistic competition output in deterministic steady state will be less than the perfectly competitive level. The parameter $\hat{c}$ can be thought of as reflecting this lost output. Under the time-consistent policy $\hat{c}$ > 0 leads to an inflation bias (see Clarida et al. 1999), effectively because the central banker has been directed to alter the real side of the economy in a model that is super-neutral.

3) The Optimal Pre-commitment Rule

Let the period at which optimization is taking place be period s. Construct the Lagrangian

$$L = E_s \sum_{t=s}^{\infty} \beta^{t-s} [(\pi_t - \hat{\pi})^2 + \omega(c_t - \hat{c})^2 + \nu(i_t - \hat{\pi} - \rho)^2$$

$$- 2\lambda_t^1 (c_t - \sigma^{-1}\rho - \sigma^{-1} E_t \pi_{t+1} - E_t c_{t+1} + \sigma^{-1} i_t - g_t)$$

$$- 2\lambda_t^2 (\pi_t - \alpha c_t - (1-\beta)\pi^* - \beta E_t \pi_{t+1} - u_t)\,.$$

Minimizing the Lagrangian with respect to λ_t^1, λ_t^2, c_t, π_t, and i_t, the first-order conditions for optimality are

$$\frac{\partial L}{\partial \pi_t} = (\pi_t - \hat{\pi}) + \sigma^{-1}\beta^{-1}\lambda_{t-1}^1 + \lambda_{t-1}^2 - \lambda_t^2 = 0 \qquad \forall\ t \geq s \qquad (4)$$

$$\frac{\partial L}{\partial c_t} = \omega(c_t - \hat{c}) + \beta^{-1}\lambda_{t-1}^1 - \lambda_t^1 + \alpha\lambda_t^2 = 0 \qquad \forall\ t \geq s \qquad (5)$$

$$\frac{\partial L}{\partial i_t} = \nu(i_t - \hat{\pi} - \rho) - \sigma^{-1}\lambda_t^1 = 0 \qquad \forall\ t \geq s \qquad (6)$$

$$\frac{\partial L}{\partial \lambda_t^1} = c_t - \sigma^{-1}\rho - \sigma^{-1}E_t\pi_{t+1} - E_t c_{t+1} + \sigma^{-1}i_t - g_t = 0 \qquad \forall\, t \geq s \qquad (7)$$

$$\frac{\partial L}{\partial \lambda_t^2} = \pi_t - \alpha c_t - (1-\beta)\pi^* - \beta E_t\pi_{t+1} - u_t = 0 \qquad \forall\, t \geq s \qquad (8)$$

along with the initial conditions $\lambda_{s-1}^1 = \lambda_{s-1}^2 = 0$ (see Currie and Levine, 1993). The nature of the policy strategy is to exploit initial conditions in the first period, but to promise never to do so in the future. The current value of promises not to exploit the initial state are given by λ_{s-1}^1 and λ_{s-1}^2. It is the very fact that policymakers "optimally" exploit initial conditions, making $\lambda_{s-1}^1 = \lambda_{s-1}^2 = 0$ part of the optimal policy program, which provides the initial conditions necessary to close the system.

Because the system has been log-linearized around its deterministic steady state, it is a simple matter to extract from equations (4) – (8) the deterministic steady state of the system and thereby illustrate the relationship between the deterministic steady-state values and the targeted values. Working through the system it is clear that $c^* = 0$, (from equation (8)), and $i^* = \pi^* + \rho$ (from equation (7)). Then, combining equation (4) with equation (6) gives $\pi^* = \hat{\pi}$ and $\lambda^{1*} = 0$. Finally, equation (5) implies $\lambda^{2*} = \frac{\omega\hat{c}}{\alpha}$. We will use these steady-state values in what follows.

Before leaving this Section it is useful to outline from an implementation point of view the nature of the equilibrium just described. It is envisaged that optimization takes place only once, in period s (making period s special). Subsequent to period s the policymaker's hands are tied (the policymaker has pre-committed). The solution to the optimization problem provides a state contingent rule that describes how the instrument is to be set in every possible state of the world. Moreover, the feedback parameters in this state contingent rule are explicit functions of the parameters in the model and objective function. Therefore, if a change to one or more parameters occurs, the policy rule automatically reflects this change; there is no need for re-optimization to take place.

4) The "Timeless Perspective" and "Timelessly" Optimal Pre-commitment Rules[1]

In the solution described above the initial values for the two Lagrange multipliers are driven to zero as a consequence of optimal policy behavior. Policymakers exploit the fact that when they set their instrument in the first period, agents have already formed their expectations and the policymaker is unconstrained by past commitments. While they exploit expectations in the first period, this exploitation comes with promises never to do so again. Woodford (1999), among others, finds the nature of this optimal policy program troublesome, for if policymakers exploit expectations in the first period what is to stop them from doing so at some point in the future. However, if optimizing the objective function is not the criteria used to determine the initial values for these Lagrange multipliers, then how are the policymaker's actions to be constrained in the first period? Or, equivalently, how are λ_{s-1}^{1} and λ_{s-1}^{2} to be determined?

The solution to this problem that Woodford (1999) and Svensson and Woodford (1999) arrive at is for policymakers to

> "…adopt, not the pattern of behavior from now on that it would now be optimal to choose, taking previous expectations as given, but rather the pattern of behavior *to which it would have wished to commit itself to at a date far in the past*, contingent upon the random events that have occurred in the meantime." (Woodford, 1999, pp293, italics in original)

> "…select a policy rule for behavior in periods t ≥ s to which it would have been optimal to commit oneself in a period far in the past." (Svensson and Woodford, 1999, pp17)

They term this approach the "timeless perspective." To illustrate the timeless perspective to pre-commitment, consider the special case where ν = 0, which leads to $\lambda_{t}^{1} = 0$, ∀ t.

[1] Our description of the timeless perspective follows that presented in Svensson and Woodford

Combining equations (4), (5), and (8), and using the result that $\pi^* = \hat{\pi}$ we obtain the second order equation

$$\left(1+\beta+\frac{\alpha^2}{\omega}\right)\lambda_t^2 = \beta E_t \lambda_{t+1}^2 + \lambda_{t-1}^2 + \alpha\hat{c} + u_t$$

for which the solutions to the quadratic component are

$$\mu = \frac{\left(1+\beta+\frac{\alpha^2}{\omega}\right)}{2\beta} \pm \frac{\sqrt{\left(1+\beta+\frac{\alpha^2}{\omega}\right)-4\beta}}{2\beta}.$$

By inspection, one root is greater than one, the other less than one. Denote the stable root μ_s, then the process for λ_t^2 is given by

$$\lambda_t^2 - \lambda^{2*} = \mu_s(\lambda_{t-1}^2 - \lambda^{2*}) + \mu_0 u_t, \qquad \forall\ t \geq s \qquad (9)$$

where $\lambda^{2*} = \frac{\omega\hat{c}}{\alpha}$ is the steady-state value of λ_t^2 and $\mu_0 = \left(1+\beta+\frac{\alpha^2}{\omega}\right)^{-1}$. In the approach outlined in the previous Section, equation (9) comes with the initial condition $\lambda_{s-1}^2 = 0$, and this initial condition, together with the saddle-point property, implies that the system has a unique stable solution. But under the timeless perspective the policymaker does not necessarily set policy in the initial period such that $\lambda_{s-1}^2 = 0$. Without a unique initial value for λ_{s-1}^2, equations (4) – (8) have multiple solutions, with these solutions indexed by λ_{s-1}^2.

Now imagine that at some earlier point in time, say period k, the policymaker solves for the optimal pre-commitment rule, exploiting expectations and ignoring past commitments

(1999).

at that time. Then, employing the condition $\lambda_{k-1}^2 = 0$, the evolution of the system ∀ t ≥ k is given by

$$\lambda_t^2 = \left(1-\mu_s^{t-k+1}\right)\lambda^{2*} + \mu_0 \sum_{j=0}^{t-k} \mu_s^j u_{t-j} \tag{10}$$

$$\pi_t = \hat{\pi} + \left(\mu_s - 1\right)\mu_s^{t-k}\lambda^{2*} + \mu_0 \sum_{j=0}^{t-k} \mu_s^j u_{t-j} \tag{11}$$

$$c_t = -\frac{\alpha}{\omega}\mu_s^{t-k+1}\lambda^{2*} + \frac{\alpha}{\omega}\mu_0 \sum_{j=0}^{t-k} \mu_s^j u_{t-j} \ . \tag{12}$$

It is clear from equations (10) – (12) that the state of the system in period t depends explicitly on the length of time that has passed since the period k optimization took place: t-k. Svensson and Woodford (1999) term the solution given in equations (10) – (12) the "k-optimal" equilibrium[2] to indicate that it describes the equilibrium that is optimal given that initial conditions are exploited in period k. If we now take the limit of equations (10) – (12) as k → -∞, then we arrive at the stationary optimal equilibrium:

$$\lambda_t^2 = \lambda^{2*} + \mu_0 \sum_{j=0}^{\infty} \mu_s^j u_{t-j} \tag{13}$$

$$\pi_t = \hat{\pi} + \mu_0 \sum_{j=0}^{\infty} \mu_s^j u_{t-j} \tag{14}$$

$$c_t = \frac{\alpha}{\omega}\mu_0 \sum_{j=0}^{\infty} \mu_s^j u_{t-j} \ . \tag{15}$$

Comparing equations (10) – (12) with equations (13) – (15), notice that in this limit the intercepts in equations (10) – (12) tend to $\frac{\omega\hat{c}}{\alpha}$, $\hat{\pi}$, and 0, respectively. In each case these intercepts equal the steady-state value of the variable in question. We are now in a position to describe the conditions required for a policy rule to be timelessly optimal. A policy rule is timelessly optimal if provided it has been followed for a sufficient length of time, and provided agents expect the rule to be followed in the future, the system converges to the stationary optimal equilibrium. Alternatively, for a policy to be timelessly optimal it must satisfy the following two criteria:

- If the economy has arrived at its present state according to the stationary optimal equilibrium, then the policy rule chosen today must ensure that it continues to evolve according to the stationary optimal equilibrium.
- If the economy has *not* arrived at its present state according to the stationary optimal equilibrium, then the policy rule chosen today must ensure that the economy converges asymptotically to the stationary optimal equilibrium.

With Currie and Levine's (1993) solution optimization takes place only once, in period s, and initial conditions are exploited at that time. Consequently, $\lambda_{s-1}^2 = 0$, indicating that when optimizing in period s the policymaker places no value on past commitments made to households. Under the timeless perspective the optimization problem is a recursive one with re-optimizations occurring at every point in time. This contrasts with the previous Section in which optimization takes place only once. Thus the problem for the timeless perspective is to come up with a mechanism describing how a policymaker optimizing today should value past commitments. There is no unique way of valuing these past commitments. For a policymaker optimizing in period s possible valuation rules that are all timelessly optimal include (staying with the case where $\nu = 0$):

- $\lambda_{s-1}^2 = \lambda^{2*} + \mu_0 \sum_{j=0}^{\infty} \mu_s^j u_{s-1-j}$.
- $\lambda_{s-1}^2 = -\frac{\omega}{\alpha}\left(c_{s-1} - \hat{c}\right)$;
- $\lambda_{s-1}^2 = \pi_{s-1} - \hat{\pi} - \frac{\omega}{\alpha}\left(c_{s-2} - \hat{c}\right)$;
- $\lambda_{s-1}^2 = \pi_{s-1} - \hat{\pi} + \pi_{s-2} - \hat{\pi} + -\frac{\omega}{\alpha}\left(c_{s-3} - \hat{c}\right)$;

---

[2] Actually, Svensson and Woodford (1999) term this "t_o-optimal" in their notation.

- $\lambda_{s-1}^{2} = p_{s-1} - \hat{p}_{s-1}$.[3]

To underscore that the problem the policymaker faces is to find a way of appropriately tying its hands during successive re-optimizations, we have deliberately expressed each prescription in terms of a setting for λ_{s-1}^{2}. Each of these methods for valuing past commitments, which is to be applied period by period, leads to a timelessly optimal equilibrium because each of these valuation rules "…implies *eventual* convergence to the stationary optimal equilibrium ... regardless of the initial conditions when the policy is adopted." (Svensson and Woodford, 1999 pp18, italics in original). If the economy is evolving according to the stationary optimal equilibrium, then each of these valuation rules leads to identical valuations of past commitments. But these rules will typically provide different valuations if the economy has arrived at period s-1 along an out of "stationary optimal equilibrium" path.

Of course, there are many ways of tying the policymaker's hands in addition to the few possibilities listed above. Applications of the timeless perspective (such as Walsh, 2001 and McCallum and Nelson, 2000) typically assume that past commitments are valued according to the second rule in the list above, but this is only one of many possible assumptions. Finally, notice that because past commitments are not valued optimally when viewed from any given period, it is not necessarily the case that timelessly optimal rules lead to better outcomes than the time-consistent rule.

5) Optimal Policy from Behind a Veil of Uncertainty

Assessing the benefit that accrues to having a pre-commitment mechanism simply requires comparing the pre-commitment solution (Section 3) with the time-consistent solution (see Dennis 2001). In this Section we will develop an approach that allows us to assess the benefit that accrues to being able to exploit the initial state of the economy. As

[3] Here p_s denotes the price level in period s and $\hat{p}_{s-1}$ ($\hat{p}_{s-1} = \hat{p}_{s-2} + \hat{\pi}$) is the price level target.

we will see, the solution we develop has much in common with that employed in King and Wolman (1999), Khan et al., (2000), and Amato and Laubach (2001), although it is not assumed that their approach is motivated on the following construct.

Consider a policymaker in the following hypothetical situation; the policymaker is behind a veil of uncertainty. While it knows the policy objective function, the policy constraints, the ex ante distribution of the shocks and the current date, the policymaker does *not* know the state of the economy. From behind this veil the policymaker must construct and implement its optimal policy. After the policy has been in effect for one period the veil is lifted and the state of the economy is revealed. From then on the policymaker implements the continuation policy, fully aware of, and accounting for, any shocks that have occurred. In the setting described, uncertainty about the state of the economy affects how the policymaker sets policy in the first period.

The effect of the uncertainty is to force the policymaker to take a guess at what the state of the economy is prior to the veil of uncertainty being removed. This guess will have welfare implications and the policymaker will take as its guess the state vector that optimizes the objective function. In what follows we will implement this procedure in two stages. In the first stage we construct the Euler equations for optimal policy; in the second stage we evaluate the policy objective function - for a guess at the initial state vector - and then optimize the objective function with respect to this guess. This two-stage solution procedure makes full use of the property that the parameters in the optimal feedback rule are independent of the state vector. Notice that, as with Section 3, the nature of the solution is for the policymaker to optimize just once and for the state-contingent rule developed at that time to be implemented from then on. Thus the only difference between this solution and that in Section 3 is how the policymaker values past commitments in the initial period; the solution we develop is very unlikely to be timelessly optimal and is not intended to be so.

From behind the veil of uncertainty the Lagrangian for the optimization problem is

$$L^U = E\sum_{t=s}^{\infty}\beta^t[(\pi_t - \hat{\pi})^2 + \omega(c_t - \hat{c})^2 + \nu(i_t - \hat{\pi} - \rho)^2$$

$$- 2\lambda_t^1(c_t - \sigma^{-1}\rho - \sigma^{-1}E_t\pi_{t+1} - E_t c_{t+1} + \sigma^{-1}i_t - g_t)$$

$$- 2\lambda_t^2(\pi_t - \alpha c_t - (1-\beta)\pi^* - \beta E_t\pi_{t+1} - u_t)\,.$$

Relative to the Lagrangian in Section 3, we have replaced a conditional expectation with an expectation operator that reflects the policymaker's subjective probabilistic beliefs about the state vector's distribution. Differentiating this Lagrangian with respect to π_t, c_t, i_t, λ_t^1, and λ_t^2 gives

$$\frac{\partial L}{\partial \pi_t} = (\pi_t - \hat{\pi}) + \sigma^{-1}\beta^{-1}\lambda_{t-1}^{1g} + \lambda_{t-1}^{2g} - \lambda_t^2 = 0 \qquad t = s$$

$$\frac{\partial L}{\partial c_t} = \omega(c_t - \hat{c}) + \beta^{-1}\lambda_{t-1}^{1g} - \lambda_t^1 + \alpha\lambda_t^2 = 0 \qquad t = s$$

$$\frac{\partial L}{\partial i_t} = \nu(i_t - \hat{\pi} - \rho) - \sigma^{-1}\lambda_t^1 = 0 \qquad t = s$$

$$\frac{\partial L}{\partial \lambda_t^1} = c_t - \sigma^{-1}\rho - \sigma^{-1}E_t\pi_{t+1} - E_t c_{t+1} + \sigma^{-1}i_t - g_t = 0 \qquad t = s$$

$$\frac{\partial L}{\partial \lambda_t^2} = \pi_t - \alpha c_t - (1-\beta)\pi^* - \beta E_t\pi_{t+1} - u_t = 0 \qquad t = s$$

and

$$\frac{\partial L}{\partial \pi_t} = (\pi_t - \hat{\pi}) + \sigma^{-1}\beta^{-1}\lambda_{t-1}^{1} + \lambda_{t-1}^{2} - \lambda_t^2 = 0 \qquad \forall\ t > s$$

$$\frac{\partial L}{\partial c_t} = \omega(c_t - \hat{c}) + \beta^{-1}\lambda_{t-1}^{1} - \lambda_t^1 + \alpha\lambda_t^2 = 0 \qquad \forall\ t > s$$

$$\frac{\partial L}{\partial i_t} = \nu(i_t - \hat{\pi} - \rho) - \sigma^{-1}\lambda_t^1 = 0 \qquad \forall\ t > s$$

$$\frac{\partial L}{\partial \lambda_t^1} = c_t - \sigma^{-1}\rho - \sigma^{-1}E_t\pi_{t+1} - E_t c_{t+1} + \sigma^{-1}i_t - g_t = 0 \qquad \forall \text{ t} > \text{s}$$

$$\frac{\partial L}{\partial \lambda_t^2} = \pi_t - \alpha c_t - (1-\beta)\pi^* - \beta E_t\pi_{t+1} - u_t = 0 \qquad \forall \text{ t} > \text{s}$$

We denote the policymaker's guesses of the two state variables λ_{s-1}^{1g} and λ_{s-1}^{2g}.

For some guess at the state vector in the initial period $z_{s-1}^g = \begin{bmatrix} \lambda_{s-1}^{1g} \\ \lambda_{s-1}^{2g} \\ c_{s-1}^g \\ \pi_{s-1}^g \\ i_{s-1}^g \end{bmatrix}$, and a target vector

$\hat{z} = \begin{bmatrix} 0 \\ 0 \\ \hat{c} \\ \hat{\pi} \\ \hat{\pi}+\rho \end{bmatrix}$, the evolution of the system $\forall$ t $\geq$ s is governed by

$$z_t - z^* = H(z_{t-1} - z^*) + Gv_t\,,$$

which implies

$$z_t - \hat{z} = H(z_{t-1} - \hat{z}) + (H - I)(\hat{z} - z^*) + Gv_t\,.$$

where $z^* = \begin{bmatrix} 0 \\ \alpha^{-1}\omega\hat{c} \\ 0 \\ \hat{\pi} \\ \hat{\pi}+\rho \end{bmatrix}$. Now writing the policy objective function as

$$E[Loss(s,\infty)] = E\sum_{t=s}^{\infty} \beta^{t-s}(z_t - \hat{z})' W(z_t - \hat{z}),$$

where $W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \nu \end{bmatrix}$, and exploiting the fact that the spectral radius of H is less than one, (a condition that must hold if a stable solution exists), the objective function evaluated along the optimal path starting from z^g_{s-1} is:

$$Loss(s,\infty) = E[(z^g_{s-1} - z_{s-1})' H' PH(z^g_{s-1} - z_{s-1})] + E[(z_{s-1} - \hat{z})' H' PH(z_{s-1} - \hat{z})],$$
$$+\frac{1}{1-\beta}[(\hat{z} - z^*)'(H-I)' P(H-I)(\hat{z} - z^*)] + \frac{1}{1-\beta} tr(G' PG\Omega) \qquad (16)$$

where $P = W + \beta H' PH$ is positive semi-definite. Equation (16) partitions the loss function into four distinct terms. The first term is the loss that occurs through incorrectly guessing the state vector. The second term reflects the loss arising because the state vector differs from the target vector. The third term captures the loss arising because the target vector is incompatible with the non-stochastic steady state. Finally, the fourth term summarizes the loss generated because the system is continually hit by shocks.

The final step in the procedure is for the policymaker to choose its guess of the initial state vector to maximize the expected loss function. Differentiating equation (16) with respect to z^g_{s-1} gives

$$H' PH[z^g_{s-1} - E(z_{s-1})] = 0. \qquad (17)$$

A sufficient condition for equation (17) to hold is $z^g_{s-1} = E(z_{s-1}) = z^*$. When $\dim[\ker(H' PH)] > 0$ it is not necessary for $z^g_{s-1} = z^*$ to hold for equation (17) to be satisfied. This case corresponds to the situation where some elements in the state vector are redundant and can be eliminated from the solution. Thus choosing $z_{s-1}^{g} = z^{*}$ is without loss of generality. For the model of this paper equation (17) yields the solution $c_{s-1}^{g} = 0$, $\pi_{s-1}^{g} = \hat{\pi}$, $i_{s-1}^{g} = \rho + \hat{\pi}$, $\lambda_{s-1}^{2g} = \frac{\omega \hat{c}}{\alpha}$, and $\lambda_{s-1}^{1g} = 0$.

Alternatively, if the policymaker knows the state of the economy in period s-1, then the unconditional expectation in equation (16) can be replaced with the conditional expectation operator and then choosing the state vector optimally leads to: $c_{s-1} = c_{s-1}$, $\pi_{s-1} = \pi_{s-1}$, $i_{s-1} = i_{s-1}$, $\lambda_{s-1}^{2} = 0$, and $\lambda_{s-1}^{1} = 0$, which is, of course, the Currie and Levine (1993) solution.

Summarizing, if the policymaker knows the initial state of the economy, then we saw in Section 3 that the welfare maximizing policy sets $\lambda_{s-1}^{1} = \lambda_{s-1}^{2} = 0$. In this Section we have the result that if the policymaker does not know the initial state of the economy then the policy that maximizes expected welfare sets $\lambda_{s-1}^{1g} = 0$ and $\lambda_{s-1}^{g} = \frac{\omega \hat{c}}{\alpha}$. In other words, when the initial state is unobserved, the best that the policymaker can do is to assume that the economy is in steady state and set policy for the initial period accordingly. Setting the Lagrange multipliers equal to their steady state values was one of the approaches taken in King and Wolman (1999) and Khan et al., (2000), but this approach is *not* timelessly optimal unless the Lagrange multipliers are constant in the stationary optimal equilibrium (cf. McCallum and Nelson, 2000, pp5-6). If the above approach were followed recursively, then in each period the Lagrange multipliers would be set equal to their steady-state values. Alternatively, if the initial period were treated in the same way as subsequent periods, then the Lagrange multipliers in the initial period would be a function of the history of the state vector.

6) Conclusions

In this paper we have presented and discussed a recent development - the timeless perspective - that has taken place in the optimal monetary policy rules literature. We used a simple business cycle model to illustrate the derivation and characteristics of timelessly optimal pre-commitment policies, and compared these policies with the "standard" pre-commitment solution. An advantage of the timeless approach to pre-commitment is that it transforms a one-shot optimization problem (the standard pre-commitment solution) into a recursive problem with optimization occurring each period. But because of their recursive formulation timelessly optimal pre-commitment policies are indeterminate with different policies arising as a function of how the current policymaker values past commitments. We demonstrated this indeterminacy and described the particular solution applications of the timeless perspective have tended to focus on.

Finally, in Section 5 we introduced a construct that allows us to assess the benefits that arise because the policymaker knows the initial state of the economy when optimizing. This construct led to a solution in which the policymaker optimally assumes that the economy is in its deterministic steady state in the initial period, mimicking the solution examined in King and Wolman (1999), Khan et al., (2000) and Amato and Laubach (2001). We emphasize, however, that assuming that the lagrange multipliers equal their deterministic steady state values in the initial period does not generally lead to a recursive optimization problem, nor does it typically lead to a timelessly optimal solution (cf. McCallum and Nelson, 2000, and Amato and Laubach, 2001).

References

Amato, J., and T. Laubach, (2001), "Implications of Habit Formation for Optimal Monetary Policy," Bank for International Settlements, mimeo.

Clarida, R., Gali, J., and M. Gertler, (1999), "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37, pp1661-1707.

Currie, D., and P. Levine, (1993), *Rules, Reputation and Macroeconomic Coordination*, Cambridge University Press.

Dennis, R., (2001), "Optimal Policy in Rational Expectations Models: New Solution Algorithms," Federal Reserve Bank of San Francisco Working Paper, 2001-09.

Kydland, F., and E. Prescott, (1980), "Dynamic Optimal Taxation, Rational Expectations and Optimal Control," *Journal of Economic Dynamics and Control*, 2, pp79-91.

Khan, A., King, R., and A. Wolman, (2000), "Optimal Monetary Policy," Federal Reserve Bank of Richmond Working Paper.

King, R., and A. Wolman, (1999), "What should the Monetary Authority do when Prices are Sticky," in Taylor, J., (ed), *Monetary Policy Rules*, University of Chicago Press, Chicago.

McCallum, B., and E. Nelson, (2000), "Timeless Perspective vs Discretionary Monetary Policy in Forward-Looking Models," *National Bureau of Economic Research*, Working Paper #7915.

Svensson, L., and M. Woodford, (1999), "Implementing Optimal Policy through Inflation-Forecast Targeting," mimeo.

Walsh, C., (2001), "The Output Gap and Optimal Monetary Policy," University of California Santa Cruz, mimeo.

Woodford, M., (1999), "Commentary on: How should Monetary Policy be Conducted in an Era of Price Stability?," in *New Challenges for Monetary Policy*, A Symposium Sponsored by the Federal Reserve Bank of Kansas City.

Woodford, M., (2000), "Interest and Prices," Princeton University, mimeo.