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Nonlinearities in International Business Cycles*

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Abstract

This paper documents the dynamic properties of national output, its components, and the current account for five OECD countries. There is strong evidence of conditional volatility for almost all time series as well as significant deviations from normality. The deviations are detected particularly in GDP, net exports, investment time series.

JEL Codes:

Keywords: Conditional Volatility, Business Cycles, Seminonparametric Estimator

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1 Nonlinear International Business Cycles

This paper documents the most significant dynamic properties of the cyclical components of national output, its components, and the current account for five OECD countries. The study imposes few theoretical restrictions on the data. The econometric tool used, a seminonparametric (SNP) estimator (Gallant and Tauchen, 1989), is flexible and can capture several features of macroeconomic data that have not yet been studied under a unified framework. SNP can capture conditional volatility, asymmetry, thick tails, and higher order moments of the conditional distribution in time series.

The countries considered in this study are Japan, Mexico, South Korea, the United States and the United Kingdom. Univariate statistical analysis is performed for GDP for each of the five countries, while bivariate analysis is performed on each of the other time series together with domestic GDP. The results show that conditional volatility is present in almost all time series. Meanwhile, significant deviations from normality (Gaussianity) are common, but not as prevalent. The deviations are detected particularly in GDP, net exports, and investment time series.

This study is an outgrowth of business cycle research and extends work that studies the dynamic properties of international macroeconomic time series. Significant contributions are Backus and Kehoe (1992), Backus et al. (1992), and Backus et al. (1995), for industrial countries, and Mendoza (1995), for developing countries. These papers studied the dynamic properties of macroeconomic aggregates for several countries and reported selected first and second unconditional moments (e.g., means, variances, covariances). In a world of linear time series processes with constant normally distributed shocks, those selected moments completely characterize the distribution of the time series.

Beginning withe the seminal work of Neftci (1984), who found evidence of asymmetry for U.S. unemployment, the assumption of normality with homogeneous volatility, and in general the assumption of linearity of macroeconomic time series has been challenged.

Engle (1982) had previously found that conditional volatility was an important feature of U.K. inflation. Strong evidence shows that U.S. macroeconomic time series importantly deviate from linear, normally distributed time series with constant volatilities. Brock and Sayers (1988) and Ashley and Patterson (1989) report that industrial production is nonlinear, and Potter (1995) and Patterson and Ashley (1999) find the same for U.S. GDP. There is also strong evidence that nonlinearity, in the form of asymmetry, is important for other countries as well. Rothman (1996) finds evidence of asymmetry in output, prices, investment and the money supply for six OECD countries. Razzak (2001) finds evidence asymmetry in GDP in six OECD countries, but not in the U.S. The evidence of nonlinearity has not gone unchallenged. Hess and Iwata (1997) review work by Beaudry and Koop (1993), and find evidence of asymmetric persistence only in the U.S. and Italy GDP and not for the rest of the G-7 countries.

The paper here improves on previous work because it is a comprehensive study of the statistical properties of macroeconomic time series. It uses a flexible approach that allows for the study of several features of the data under a unified framework. The study takes an initially neutral stand as to which properties should be important for a particular series, and attempts to find whether any, some or all of conditional volatility, asymmetry and nonlinearities, in general, are important properties for that particular series.

It is important to characterize the most important dynamic properties of macroeconomic aggregates because these additional properties impose additional restrictions on economic models. If asymmetry and conditional volatility are found to be important statistical characteristics of, say, output, then a "good" model of output should produce this result. One easy way to produce asymmetry and conditional would be simply to assume a linear model with technological shocks that are asymmetric and have conditional volatility. However, Altug et al. (1999) find that in the U.S. Solow residuals (specified in alternative ways) are linear. Thus, the source of the deviation from linear Gaussian time series lies in the model. A deeper reason that it is important to characterize the nonlinearities in the time series is that there is evidence that the economy itself is nonlinear. Valderrama (2002b) shows that a canonical business cycle model that succeeds in matching first and second moments of macroeconomic aggregates does not capture the conditional volatility, asymmetry of U.S. investment and consumption. That study shows that calibrating the model to take into account these additional features of the data changes the parameters in economically significant ways. Moreover, as Kim and Kim (2000) show, conditional volatility and higher order terms are important for welfare considerations when risk is important. They showed that this is the case for international risk sharing in international business cycles.

2 Econometric Methodology

The statistical properties of the international macroeconomic time series are characterized using an SNP estimator. The SNP methodology estimates successively richer statistical models until a full statistical characterization is achieved. A preferred statistical model is selected taking into account that more complex statistical models will have higher likelihoods, but they will be penalized because of the larger number of parameters estimated.

The empirical macroeconomics literature has focused on a vector autoregressive (VAR) structures to summarize the statistical properties of data. The SNP hierarchy of statistical models nests a VAR. It also allows for richer statistical features, such as conditional volatility (i.e., conditional heteroskedasticity), asymmetric business cycles (i.e., skewness) and "excess volatility" or "thick tails" (i.e., excess kurtosis) and other nonlinearities. These are features of interest and are characteristic of many macroeconomic time series. The key to SNP is to take a flexible approach to estimating the statistical model. As more data are available, increasingly rich statistical models can be estimated.

The SNP estimator used here is adopted from Gallant and Tauchen (2001) and the following description is mostly taken from there. Define $\{y_t\}_{t=-\infty}^{\infty}$ as a possibly multivariate ($y \in \Re^m$) Markovian discrete time series in L lags. Define $p(y_t|x_{t-1})$, where $x_{t-1} = (y_{t-L}, \ldots, y_{t-1})$, as the conditional distribution of the time series, which completely characterizes the time series properties of y. $\tilde{y}_t$ is the observed stochastic process, that is, the quarterly observations of y.

The goal of the SNP procedure is to estimate the transition density function of the observed data that fully characterizes all the "true" statistical properties of the time series. SNP approximates the true distribution $p(y_t|x_{t-1})$ with a distribution $f(y_t \mid y_{t-L}, \ldots, y_{t-1}, \Omega)$ that describes the observed data, where Ω represents the parameter vector of the statistical model.

The SNP procedure makes parametric assumptions that are flexible enough to allow it to capture a very rich set of possible statistical properties. In particular, SNP assumes that the conditional density can be written in the following way:

$$f(y_t \mid y_{t-L}, \ldots, y_{t-1}, \Omega) = \left|R_{x_{t-1}}\right|^{-1} h\left[R_{x_{t-1}}^{-1}(y_t - \mu_{t-1})\right],$$

where $R_{x_{t-1}}$ is an upper triangular matrix, $R_{x_{t-1}} R'_{x_{t-1}} = \Sigma_{x_{t-1}}$, that is, the matrix square root of the conditional variance-covariance matrix; μ_{t-1} is the conditional mean of the process; and $e_t \equiv y_t - \mu_{t-1}$ is the error.

The conditional mean is assumed to be given by:

$$\begin{aligned}
\mu_{t-1} &= b_0 + B_1 y_{t-L_\mu} + B_2 y_{t-L_\mu+1} + \cdots + B_{L_\mu} y_{t-1} \\
\mu_{t-1} &= b_0 + B x_{t-1}
\end{aligned}$$

That is, the conditional mean can be described by a VAR. $\Psi = \text{vec}[b_0 \mid B_1 \cdots B_{L_\mu}]$ groups the parameters of the VAR structure in a parameter vector.

The conditional variance (scale) is assumed to be given by:

$$\text{vech}(R_{t-1}) = \rho_0 + \sum_{i=1}^{L_r} P_i |e_{t-1-L_r+i}| + \sum_{i=1}^{L_g} \text{diag}(G_i) R_{t-2-L_g+j}$$

The scale captures an ARCH structure through the P matrices and a GARCH structure through the G matrices. In general, SNP nests a GARCH(L_g, L_r) conditional volatility structure (L_r order for ARCH, and L_g order for GARCH). The parameters that describe the scale are arranged into the following vector $T = \text{vec}[\text{vec}\,\rho \mid \text{vech}\, P_1 \cdots P_{L_r} \mid \text{vech}\, G_1 \cdots G_{L_g}]$. The G and P matrices are assumed to be diagonal meaning that the conditional variance terms for consumption and investment only depend on their own lagged innovations or own lagged realizations, and not on the other series' lagged innovations or realizations.

The SNP estimator nests a non-Gaussian transition density and higher order heterogeneity because it assumes that the density of the error is a transformation of the normal distribution. A hermite polynomial is used for this purpose:

$$h\left[R_{x_{t-1}}^{-1}(y_t - \mu_{t-1})\right] \propto \left[P(z,x)\right]^2 \phi(t \mid \mu, R)$$
$$P_K(z,x) = \sum_{\alpha=0}^{K_z} \sum_{\beta=0}^{K_x} (a_{\beta\alpha} x^{\beta}) z^{\alpha}$$

where $z = R^{-1}(y_t - \mu_{t-1})$. By increasing the degrees of the polynomial $P(z,x)$, SNP attains increasingly rich statistical structures. If $K_z = 0$ and $K_x = 0$ then the statistical model has a Gaussian error structure. If $K_z > 0$, $K_x = 0$ then the statistical model has a semiparametric error structure. If $K_z > 0$, $K_x > 0$ then conditional distribution is fully non-parametric and depends on lags of the data. It is through the hermite expansion that one attains the full richness of SNP to capture a wide variety of possible statistical processes.[1] The parameters of the hermite polynomial, $a_{\beta\alpha}$, are collected in the matrix

$A = [a_{\beta\alpha}]$. The number of parameters grows rapidly as one expands through the hermite polynomial. Thus, SNP allows suppression of interactions between series through the control parameters I_z and I_x so that only the terms with interaction between different series of degree greater than $K_z - I_z$ and $K_x - I_x$ are estimated. The SNP parameter vector Ω is given by $\Omega = [A \mid \Psi \mid T]$. [2]

Statistical model selection within the SNP procedure is done by expanding through the SNP hierarchy of candidate statistical models. At each step, the statistical model, $f(\tilde{y}_t|\tilde{x}_t, \hat{\Omega})$, is estimated for the observed data series by maximum likelihood, obtaining estimated parameters $\hat{\Omega}$. Gallant and Tauchen (2001) recommend the use of Schwarz's Bayesian Information Criterion (BIC) to help choose amongst different statistical models. BIC includes a term that penalizes larger statistical models. In practice, this study will follow suggestions by Gallant and Tauchen (2001) and will sometimes select a statistical model that does not minimize the BIC when the t-statistics for individual parameters suggest that a different model is preferred.[3] When this is the case, this is pointed out in the results.

3 Data Description

The countries considered in this study are Japan, Mexico, South Korea, the United States, and the United Kingdom. Japan, the United States, and the United Kingdom represent three of the world's largest economies, while Mexico and South Korea are two emerging markets that have received much attention recently. Moreover, these two countries have suffered through large crisis and SNP may be successful at capturing the

[1]SNP, as it is implemented by the program supplied in Gallant and Tauchen (2001), cannot explicitly take into account regime-switching behavior like that introduced by Hamilton (1989). However, it can take into account the asymmetrical time-varying behavior induced by Markov switching. Bansal and Zhou (2002) have estimated term structure models with regime shifts using SNP.

[2]Gallant and Nychka (1987) show that as the number of parameters increases with the sample size, SNP is a consistent estimator of the transition density.

[3]There is evidence that BIC is too conservative in selecting statistical models.

properties of their entire time series, including the periods of crisis, within a unified framework (Valderrama, 2002a).

This paper uses SNP to analyze quarterly observations of the following series: gross domestic product, private consumption, private investment, gross fixed capital formation (GFCF), government expenditures in goods and services, exports, imports, and the current account. Inventory investment and GFCF are added together to arrive at private investment. For Mexico, the U.S., and the U.K., consumer durables are added to GFCF and inventory investment to arrive at a measure of overall private investment. For these three countries, durables are are subtracted from private consumption series.

The national account data were collected from the OECD SourceOnline website, except for the U.S. data, which were collected from the Bureau of Economic Analysis website. The national accounts data collected are in real terms and are seasonally adjusted. The current account data were collected from the International Monetary Fund's IFS Statistics CD-ROM.

The time period under consideration is from 1955Q1 to 2001Q4. However, none of the series covers the entire span. Each of the tables with results in Section 4 details the span of each series. The analysis for each series, or pairs of series for the bivariate analysis, will be done using all data available.

The business cycle literature primarily focuses on the business cycle frequencies of the macroeconomic time series and it is common to filter the data to remove trending and seasonal effects, and this is done here. A band pass filter (BP) (Christiano and Fitzgerald, 1999; Baxter and King, 1999) is used to filter the data.[4] This application uses the BP filter by Christiano and Fitzgerald (1999). The BP filter minimizes the Mean Squared Error between the estimated spectral decomposition and the true spectral

[4] In the business cycle literature the Hodrick-Prescott (HP) filter is usually used (Hodrick and Prescott, 1980). The problem of using the HP filter is that it may distort sample second moments of the data in small samples (King and Rebelo, 1993).

decomposition of a particular process and works well for standard macroeconomic time series. The filter used in practice is symmetric (stationary), nonlinear and isolates the business cycle properties of the data between 6 and 32 quarters. This filter assumes that the raw data is close to having a unit root.

All the raw data, except for next exports and the current account, is put in log per capita levels before filtering.[5] For net exports, exports and imports series are filtered separately as described and then the difference is taken to represent net exports. Since the current account can take both positive and negative values one cannot take the natural logarithm of the series. As it is done in the literature when this is the case (Backus et al., 1992), the current account is reported as a percent of nominal GDP.

4 Results

The results are organized as follows: first, univariate analysis for each country's GDP; second, bivariate analysis for each component of GDP and domestic GDP for each country. A full statistical description of each Country's GDP and its components is obtained using SNP. For each time series in the univariate analysis, and for each pair of series in the bivariate analysis, the preferred model is obtained guided by use of the BIC criterion, but also aided by the t-statistics for each parameter of the statistical model. In this paper we report descriptive statistics for the preferred model described. The full set of estimates, with standard errors and corresponding t-values of each statistical model estimated (including the preferred model) are available from the author upon request.

An important reminder is that the order of the SNP model grows with the number of observations because of the way the models are chosen. So it is not entirely surprising that countries with more observations, Japan, the U.K. and the U.S., have the most complex

[5] The population series were collected from the IFS CD-ROM and quarterly observations were obtained by linear interpolation.

statistical behavior being captured by SNP. It is possible that there simply are not enough observations, especially in Mexico, to uncover the nonlinear behavior. Moreover, for Mexico, the observations from 1980Q1 to 1982Q2 are lost because of model selection. This was a period of crisis in Mexico, which might generate nonlinear movements in the time series.

Gross Domestic Product

Table 1 gives the results for the univariate analysis of domestic GDP. The first column gives the name of the country. The second column gives the autoregressive order selected for the preferred model. For U.K. GDP, for example, the preferred model has six autoregressive terms for the conditional mean. The third and fourth columns give the highest order for the ARCH and GARCH terms, respectively. Those columns list, next to each term, the significance level for the highest order term. For Mexico's GDP, for example, the third order ARCH term is significant at the 5% level, and there is no GARCH term considered in the preferred model. The next to last column gives the order of the hermite polynomial, which captures the departure from normality, and next to that the significance level for the highest order term is listed. For Japan's GDP, a fifth order hermite polynomial term is significant at the 1% level in the preferred model. The last column lists the BIC-preferred model if this is different from the overall preferred model listed in that column. Again for Japan, the preferred model is a VAR(6), ARCH(1), with a hermite polynomial of the *fourth* degree.

For GDP, all five countries exhibit important conditional volatility, all significant at the 5% level, and two significant at the 1% level. For Mexico, the conditional volatility process is even ARCH(3), significant at the 5% level. For four countries there is evidence of non-normality. For Mexico and the U.K. the highest degree of the hermite polynomial is three, which roughly corresponds to the third moment (asymmetry) of the time series.

This indicates that for Mexico and the U.K. GDP exhibits a significant asymmetry. For South Korea, the preferred model has a significant fourth degree polynomial, which roughly corresponds to the fourth moment (kurtosis) of the time series. Thus, for South Korea, not only is asymmetry important, the significant term indicates excess kurtosis or "thick tails" (sometimes also called excess volatility). Japan, where the fifth degree term of the hermite polynomial is significant, also shows evidence of thick tails. The thick tails refer to large deviations from the mean that have a higher probability than that predicted by the normal distribution. For Japan and Korea this characteristic is significant even after taking into account the conditional volatility. For the United States, the lack of asymmetry is surprising given the work by Patterson and Ashley (1999), Rothman (1996), and Hess and Iwata (1997), although Razzak (2001) also fails to find asymmetry. One reason the procedure here fails to capture the asymmetry: SNP is trying to estimate both an ARCH(1) term and a the terms for the hermite polynomial. Given the (relatively) large number of parameters, it might not be possible to capture both features of the data with one statistical model. However, it is striking that the higher order terms were significant for Mexico and South Korea even as they have fewer observations available.

Private Consumption

For the remainder of the section, the SNP analysis is bivariate. This paper reports the joint behavior of each component of GDP, together with GDP. For each country, this study obtains an estimate of the joint conditional distribution of the two series under consideration (private consumption and GDP here). It is important to do this because it is possible that once one takes account of the nonlinear behavior of each country's GDP there would not be any nonlinear features left in the components. However, this is not the case for most of the series considered.

Table 2 gives the results of the SNP bivariate analysis between private consumption and GDP for each of the five countries. The table is similar to Table 1 but there are two rows now for each country; there is one for private consumption (C) and one for GDP (Y). The third column gives the order of the VAR in the preferred model for each pair of series. For SNP, this paper assumes the conditional volatility terms are diagonal. That is, one series's conditional depends on its own innovations (ARCH) or its own lagged values (GARCH). Thus, this paper reports the significance of the conditional volatility terms for each series. Similarly, the hermite polynomial term often depends on one series' values (C, C^2), except for the rare occurrence that there are significant interaction terms (Y, C). Taking the U.S. as an example, the SNP preferred model for the joint conditional distribution of private consumption and GDP is as follows: the conditional mean is given by a VAR(6); the conditional variance is given by an ARCH(1), but only the ARCH(1) term for consumption is significant at least at the 10% level (it is significant at the 1% level); a hermite polynomial of the fourth degree is needed, but only the hermite terms for consumption are significant at least at the 10% level (the quartic term, C^4, is significant at the 1% level). There are no interaction terms to consider.

For all countries except Mexico there is strong evidence of conditional volatility. For those four countries, the ARCH(1) terms corresponding to consumption are all significant at the 1% level. For Japan, the GARCH(1) term is also significant at the 1% level for both consumption and GDP. The evidence of deviations from normality is weaker. For Mexico, the cubic term for consumption, which captures asymmetry, is significant at the 10% level. For South Korea and the U.S., the quartic terms for consumption are significant at the 1% level. For the United Kingdom, the deviation from normality is even stronger. Not only are the quartic terms significant for both consumption and GDP at the 1% level, but many lagged terms that enter the polynomial are significant at the 1% level. This indicates that the shape of the conditional distribution changes through time as a function of past realizations of both consumption and GDP, which is highly nonlinear behavior.

Private Investment

Table 3 shows the results of the bivariate SNP analysis for the joint behavior of private investment and GDP. The evidence of conditional volatility being an important characteristic of private investment is very strong. For all countries, the ARCH(1) term for private investment is significant at the 10% level, and for three of the countries it is significant at the 1% level. The evidence of deviations from normality is more mixed. Only for Mexico and South Korea are the hermite polynomial terms significant. However, for both countries the quartic terms are significant at the 1% level. For both countries, there is strong evidence of skewness and thick tails in investment. These significant terms might be a result of the international crises that both of these countries have suffered. The crises led to amazing booms and busts in private investment. The skewness in the distribution might be a result of the "sudden stop" phenomenon (Calvo, 1998; Mendoza, 2001).

Gross Fixed Capital Formation

Table 4 shows the results of the bivariate SNP analysis for the joint behavior of GFCF and GDP. For five countries, the preferred model includes at least an ARCH(1) term for conditional volatility. Except for the U.S., the ARCH(1) term for conditional volatility is significant at the 1% level. For South Korea, the ARCH(2) term for GFCF is significant at the 1% level. For the U.S. the ARCH(1) term for investment is significant at the 20% level. For Japan, the U.K., and the U.S. the GFCF fourth degree term of the hermite polynomial is significant that the 1% level. For Mexico and South Korea hermite polynomial terms for GFCF are not significant. Taken together with Table 3 this seems to indicate that, for Mexico and South Korea, the deviations from normality for investment arise from the behavior of changes in inventories and consumer durables (for Mexico), and not from the behavior of GFCF.

Government Expenditures

Table 5 shows the results of the bivariate SNP analysis for the joint behavior of government expenditures in goods and services and GDP. The evidence of conditional volatility is very strong. For all five countries the ARCH(1) conditional volatility term for government expenditures is significant at the 5% level, and for three it is significant at the 1% level. Japan's government expenditures have an additional GARCH(1) term significant at the 5% level. The evidence of deviation from normality is more mixed. South Korea is the only country in which the government fourth degree term for the polynomial is significant at the 1% level. Thus it is the country with the strongest evidence of thick tails.[6] For the U.K. the fourth degree term is significant at the 5% level.[7] For the U.S. it is significant at the 10% level. The weak evidence of non normality for government expenditures is reinforced by the fact that for three of the countries (Japan, Mexico, the U.K.) the BIC-preferred model does not include nonlinearity terms, and for a fourth (the U.S.) some of the terms are only significant at the 10% level.

Net Exports

Table 6 shows the results of the bivariate SNP analysis for the joint behavior of net exports and GDP. For the five countries, there is strong evidence of conditional volatility for net exports, at least at the 5% level. For Japan, while the ARCH(1) term for net exports is not significant at the 10% level, the GARCH(1) term is significant at the 1% level.[8] The evidence of non normality for net exports is mixed. For Japan, Mexico, and

[6]Although this result is not show in the Table, the third degree term for government expenditures is not significant at the 20% level so there is no evidence of asymmetry.

[7]Like South Korea, for the U.K. there is no evidence of asymmetry.

South Korea the evidence, the hermite polynomial is significant to the fourth degree at least at the 5% level.[9]

Current Account

Table 7 shows the results of the bivariate SNP analysis for the joint behavior of the current account (as a percent of GDP) and GDP. The evidence of conditional volatility is very strong. For all countries except Japan, the ARCH(1) term is significant at the 1% level. For Japan, the ARCH(2) term is significant at the 5% level. The evidence of non normality is mixed. For Mexico, asymmetry in the current account, captured by the third degree term of the polynomial, is significant at the 1% level. Again, this might capture the "sudden stop" phenomenon. For the U.K. and the U.S. the current account fourth degree term of the polynomial, capturing thick tails, is also significant at least at the 5% level.[10] For South Korea, there is not evidence for either asymmetry or for thick tails, although the conditional volatility term is very significant. The results for the bivariate SNP analysis for the joint behavior of private consumption and the current account are also reported in Table 8. Here, the study isolates the impact of consumption on the dynamics of the current account. As for the analysis for the current account and GDP, there is strong evidence of conditional volatility. However, the evidence of significant deviations from normality is poorer. The strongest case, again, is Mexico, for which the current account fourth degree term of the polynomial is significant at the 1% level.

[8] For Japan, the SNP specification with ARCH(1), no GARCH, and a fourth degree hermite polynomial has a highly significant ARCH(1) term for net exports.

[9] None of the three countries have a significant net exports third order term in the polynomial, the term associated with asymmetry.

[10] The third degree term of the polynomial, associated with asymmetry, is not significant for either country.

5 Conclusions

This paper is a step towards documenting nonlinearities in international business cycles. First, this paper shows how the SNP methodology can be used to characterize the dynamic properties of macroeconomic time series under a unified framework that can capture different features. Second, this paper shows that there are important properties of international time series that significantly deviate from the usual assumptions. Using SNP analysis, this paper shows that conditional volatility is a prevalent property of all macroeconomic time series for five OECD countries and should be considered an important characteristic to be explained by macroeconomic models. Additionally, this paper shows that there are significant deviations from normality for the different macroeconomic aggregates, but that they are not as prevalent as the conditional volatility found. Nevertheless, for all macroeconomic aggregates there were at least two countries whose series exhibit important deviations from normality. Thus, while previous literature found that countries share certain characteristics captured by first and second moments (volatility of investment is higher than the volatility of GDP, which is in turn greater than the volatility of consumption), this paper also shows that the same time series in two different countries have different behavior with respect to higher order terms. This is bad news and good news. This is bad news because it means that it will be hard to come up with models that explain all features of economic dynamics for all countries, as countries are not as similar as once thought. At the same time it is good news because the results indicates that there are important economic processes that lead to differences across countries that remain to be explored. This paper provides a tool for measuring these additional features of the data under a unified framework.

This paper does not venture into what are the economic processes that produce conditional volatility or that lead to important deviations from normality. For each component of GDP, and for GDP itself, there are many possible economic stories that lead to deviations from the usual linear Gaussian assumption. However, up to this point, there has not been much work comparing the predictions from economic models to the nonlinear characteristics of macroeconomic time series. Much of the work has been limited because it is common to compare the prediction of models with the behavior of the data only for selected first and second moments.

References

Altug, S., Ashley, R. A., Patterson, D. M., December 1999. Are technology shocks nonlinear? Macroeconomic Dynamics 3 (4), 506–533.

Ashley, R. A., Patterson, D. M., August 1989. Linear versus nonlinear macroeconomies: A statistical test. International Economic Review 30 (3), 685–704.

Backus, D. K., Kehoe, P. J., September 1992. International evidence on the historical properties of business cycles. The American Economic Review 82 (4), 864–888.

Backus, D. K., Kehoe, P. J., Kydland, F. E., August 1992. International real business cycles. Journal of Political Economy 100 (4), 745–775.

Backus, D. K., Kehoe, P. J., Kydland, F. E., 1995. International business cycles: Theory and evidence. In: Cooley, T. F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, N.J., Ch. 11, pp. 331–356.

Bansal, R., Zhou, H., October 2002. Term structure of interest rates with regime shifts. Journal of Finance 57 (5), 1997–2043.

Baxter, M., King, R. G., November 1999. Measuring business cycles: Approximate band-pass filters for economic time series. Review of Economics Studies 81 (4), 575–93.

Beaudry, P., Koop, G., June 1993. Do recessions permanently change output. Journal of Monetary Economics 31 (2), 665–688.

Brock, W. A., Sayers, C. L., 1988. Is the business cycle characterized by deterministic chaos. Journal of Monetary Economics 22, 71–90.

Calvo, G. A., November 1998. Capital flows and capital market crises: The simple economics of sudden stops. Journal of Applied Economics 1 (1), 35–54.

Christiano, L. J., Fitzgerald, T. J., July 1999. The band pass filter, working Paper 7257: NBER, Cambridge, MA.

Engle, R. F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation. Econometrica 50, 987–1007.

Gallant, A. R., Nychka, D. W., 1987. Semi-nonparametric maximum likelihood estimation. Econometrica 55, 363–390.

Gallant, A. R., Tauchen, G., 1989. Seminonparametric estimation of conditionally constrained heterogeneous processes: Asset pricing applications. Econometrica 57, 1091–1120.

Gallant, A. R., Tauchen, G., February 2001. SNP: A program for nonparametric time series analysis, version 8.8, User's Guide, University of North Carolina at Chapel Hill. Available along with code and worked example by anonymous ftp at site ftp.econ.duke.edu in directory pub/arg/snp.

Hamilton, J. D., March 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57 (2), 357–384.

Hess, G. D., Iwata, S., December 1997. Asymmetric persistence in GDP? a deeper look at depth. Journal of Monetary Economics 40 (3), 535–554.

Hodrick, R., Prescott, E. C., 1980. Post-war U.S. business cycles. Working paper, Carnegie Mellon University, reproduced.

Kim, J., Kim, S. H., December 2000. Spurious welfare reversals, Unpublished Manuscript.

King, R. G., Rebelo, S. T., 1993. Low frequency filtering and real business cycles. Journal of Economic Dynamics and Control 17, 207–231.

Mendoza, E. G., February 1995. The terms of trade, the real exchange rate, and economic fluctuations. International Economic Review 36 (1), 101–137.

Mendoza, E. G., June 2001. Credit, prices, and crashes: Business cycles with a sudden stop. Working Paper 8338, National Bureau of Economic Research, Cambridge, MA.

Neftci, S. N., April 1984. Are economic time series asymmetric over the business cycle? Journal of Political Economy 92 (2), 307–328.

Patterson, D. M., Ashley, R. A., 1999. A Nonlinear Time Series Workshop: A Toolkit for Detecting and Identifying Nonlinear Series Sependence. Dynamic Modeling and Econometrics in Economics and Finance, Volume 2. Kluwer Academic Publishers, Boston.

Potter, S. M., April–June 1995. A nonlinear approach to U.S. G.N.P. Journal of Applied Econometrics 10 (2), 109–125.

Razzak, W., January 2001. Business cycle asymmetries: International evidence. Review of Economic Dynamics 4 (1), 230–243.

Rothman, P., 1996. International evidence of business-cycle nonlinearity. In: Barnett, W. A., P., K. A., Salmon, M. (Eds.), Nonlinear dynamics and economics: Proceedings of the Tenth International Symposium in Economic Theory and Econometrics. International Symposia in Economic Theory and Econometrics. Cambridge University Press, Cambdridge, UK, Ch. 14, pp. 333–341.

Valderrama, D., September 2002a. The impact of financial frictions on a small open economy: When current account borrowing hits a limit. Working Paper 2002-15, Federal Reserve Bank of San Francisco.

Valderrama, D., September 2002b. Statistical nonlinearities in the business cycle: A challenge for the canonical RBC model. Working Paper 2002-13, Federal Reserve Bank of San Francisco.

Table 1: **Gross Domestic Product**

COUNTRY	AR Order	Conditional Vol.		Higher Order Terms	BIC-preferred
		ARCH	GARCH		
Japan	6	1^{***}		$K_z = 5^{***}$	$K_z = 4$
Mexico	5	3^{**}		$K_z = 3^{**}$	$K_z = 0$
South Korea	3	1^{**}		$K_z = 4^{*}$	$K_z = 5$
United Kingdom	5	1^{***}		$K_z = 3^{**}$	$K_z = 0$
United States	6	1^{***}			

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. GDP in real, log, per capita terms, detrended using BP filter. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 2: **Private Consumption**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	Conditional Vol. GARCH	Higher Order Terms	BIC-preferred
Japan	C	5	1^{***}	1^{***}	$K_z = 4^{**}$§	
	Y		1^{***}	1^{***}	$K_z = 4$	
Mexico	C	3			$K_z = 3^{*}$	$K_z = 0$
	Y				$K_z = 3$	
South Korea	C	3	1^{***}		$K_z = 4^{***}$	
	Y		1^{*}		$K_z = 4^{***}$	
United Kingdom	C	5	1^{***}		$K_z = 4^{***}, K_x = 1^{***}$	$K_z = 0$
	Y		1^{***}		$K_z = 4^{***}, K_x = 1^{***}$	
United States	C	6	1^{***}		$K_z = 4^{***}$	
	Y		1		$K_z = 4$	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 5% level. VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. C is private consumption is in real, log, per capita terms, detrended using BP filter. Private consumption does not include purchases of durables, except for Japan and South Korea. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 3: **Private Investment**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	Conditional Vol. GARCH	Higher Order Terms	BIC-preferred
Japan	I	6	1^{***}		$K_z = 4$	
	Y		1^{***}		$K_z = 4^{**}$§	
Mexico	I	4	1^{**}		$K_z = 4^{***}$, Y, I int.	
	Y		1^{***}		$K_z = 4$	
South Korea	I	3	1^{***}		$K_z = 4^{***}$	
	Y		1		$K_z = 4^{***}$	
United Kingdom	I	5	1^{***}	1^{***}	$K_z = 4$	
	Y		1^{***}	1^{***}	$K_z = 4^{***}$	
United States	I	4	1^{*}		$K_z = 4$	
	Y		1^{***}		$K_z = 4^{***}$§§	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 5% level. §§Quadratic term highest order term significant at the 1% level. Y, I int. represents an interaction term in the hermite polynomial ($K_z = 4, I_z = 2$). VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. I is private investment is in real, log, per capita terms, detrended using BP filter. Private investment is the sum of gross fixed capital formation, changes in inventories, and purchases of consumer durables. Consumer durables are not included in Japan and South Korea. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 4: **Gross Fixed Capital Formation**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	Conditional Vol. GARCH	Higher Order Terms	BIC-preferred
Japan	FCF	5	1^{***}		$K_z = 4^{***}$	
	Y		1^{***}		$K_z = 4^{*}$§	
Mexico	FCF	4	1^{***}		$K_z = 4^{*}$§	$K_z = 0$
	Y		1^{**}		$K_z = 4$	
South Korea	FCF	5	2^{***}		$K_z = 4$	
	Y		2^{*}		$K_z = 4^{***}$	
United Kingdom	FCF	5	1^{***}		$K_z = 4^{***}$	$K_z = 0$
	Y		1^{***}		$K_z = 4$	
United States	FCF	6	1		$K_z = 4^{***}$	
	Y		1^{***}		$K_z = 4$	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 10% level. VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. FCF is gross fixed capital formation is in real, log, per capita terms, detrended using BP filter. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 5: **Government Expenditures**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	GARCH	Higher Order Terms	BIC-preferred
Japan	G	5	1^{***}	1^{**}		
	Y		1^{***}	1		
Mexico	G	4	1^{**}		$K_z = 4^{*}$ §	$K_z = 0$
	Y		1^{*}		$K_z = 4$	
South Korea	G	3	1^{**}		$K_z = 4^{***}$	$K_z = 5$
	Y		1^{**}		$K_z = 4$	
United Kingdom	G	5	1^{***}		$K_z = 4^{**}$	$K_z = 0$
	Y		1^{***}		$K_z = 4^{***}$	
United States	G	5	1^{***}		$K_z = 4^{*}$	
	Y		1^{***}		$K_z = 4^{**}$	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 10% level. VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. G is government expenditures in goods and services is in real, log, per capita terms, detrended using BP filter. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 6: **Net Exports**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	Conditional Vol. GARCH	Higher Order Terms	BIC-preferred
Japan	NX	5	1	1^{***}	$K_z = 4^{***}$	
	Y		1^{***}	1^{***}	$K_z = 4^{***}$	
Mexico	NX	6	1^{***}		$K_z = 4^{***}$	
	Y		1^{***}		$K_z = 4^{***}$	
South Korea	NX	6	1^{***}		$K_z = 4^{**}$	
	Y		1^{***}		$K_z = 4^{***}$	
United Kingdom	NX	5	1^{**}		$K_z = 4$	$K_z = 0$
	Y		1^{***}		$K_z = 4^{**}$	ARCH(2)
United States	NX	5	1^{***}		$K_z = 4^{*}$	$K_z = 0$
	Y		1^{***}		$K_z = 4^{***}$	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 10% level. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. NX is net exports, the difference of exports minus imports; each is in real, log, per capita terms, detrended using BP filter. Span: Japan 1955Q2–2000Q2 (181 observations), Mexico 1980Q1–2001Q3 (87), South Korea 1970Q1–2001Q3 (127), United Kingdom 1955Q1–2001Q3 (186), United States 1959Q1–2001Q4 (172). First 8 observations are reserved for model selection.

Table 7: **Current Account, GDP**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	GARCH	Higher Order Terms	BIC-preferred
Japan	CA	4	2**		$K_z = 3$, Y, CA int.	$K_z = 4$,
	Y		2**		$K_z = 3^{**}$	$I_z = 4$
Mexico	CA	3	1***		$K_z = 3^{***}$	$K_z = 0$
	Y		1		$K_z = 3$	
South Korea	CA	3	1***			
	Y		1**			
United Kingdom	CA	5	1***		$K_z = 4^{**}$	
	Y		1***		$K_z = 4^{***}$	
United States	CA	5	1***		$K_z = 4^{***}$	VAR(6),
	Y		1***		$K_z = 4$	$K_z = 0$

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 10% level. Y, CA int. represents an interaction term in the hermite polynomial ($K_z = 4, I_z = 2$). VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. Y is GDP in real, log, per capita terms, detrended using BP filter. CA is the current account, expressed as a percent of GDP and then filtered using BP filter. Span: Japan 1977Q1–2000Q2 (94 observations), Mexico 1981Q1–2000Q4 (80), South Korea 1976Q1–2001Q3 (103), United Kingdom 1970Q1–2001Q3 (126), United States 1960Q1–2001Q4 (168). First 8 observations are reserved for model selection.

Table 8: **Current Account, Private Consumption**

COUNTRY	Series	VAR Order	Conditional Vol. ARCH	Conditional Vol. GARCH	Higher Order Terms	BIC-preferred
Japan	CA	6	1^{**}		$K_z = 4^{*}$	
	C		1^{***}		$K_z = 4^{***}$	
Mexico	CA	5	1^{***}		$K_z = 4^{***}$	
	C		1		$K_z = 4$	
South Korea	CA	6	1^{**}			
	C		1^{***}			
United Kingdom	CA	5	1^{***}		$K_z = 4$	$K_z = 0$
	C		1^{***}		$K_z = 4^{**}$	
United States	CA	3	1^{**}		$K_z = 4^{*}$/S	$K_z = 0$
	C		1^{***}		$K_z = 4$	

* Significant at the 10% confidence level (CL). ** Significant at the 5% CL. *** Significant at the 1% CL. Significance refers to highest order term, unless otherwise specified. §Quadratic term highest order term significant at the 10% level. VAR order column gives highest significant VAR lag. ARCH column gives highest order ARCH term at the 10% significance level or above. GARCH column gives highest order GARCH term at the 10% significance level or above. Higher Order Terms column gives order of Hermite polynomial. BIC-preferred column gives the BIC-preferred model if it is not the overall preferred model. CA is the current account, expressed as a percent of GDP and then filtered using BP filter. C is private consumption is in real, log, per capita terms, detrended using BP filter. Private consumption does not include purchases of durables, except for Japan and South Korea. Span: Japan 1977Q1–2000Q2 (94 observations), Mexico 1981Q1–2000Q4 (80), South Korea 1976Q1–2001Q3 (103), United Kingdom 1970Q1–2001Q3 (126), United States 1960Q1–2001Q4 (168). First 8 observations are reserved for model selection.