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Abstract

This paper quantifies the role of alternative shocks in accounting for the recent declines in Japanese saving rates and interest rates and provides some projections about their future course. We consider three distinct sources of variation in saving rates and real interest rates: changes in fertility rates, changes in survival rates, and changes in technology. The empirical relevance of these factors is explored using a computable dynamic OLG model. We find that the combined effects of demographics and slower total factor productivity growth successfully explain both the levels and the magnitudes of the declines in the saving rate and the after-tax real interest rate during the 1990s. Model simulations indicate that the Japanese savings puzzle is over.

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1 Introduction

One of the biggest distinctions between Japanese and U.S. households is that the Japanese save more. As recently as 1990 the gap between the personal saving rate in Japan and the United States exceeded 8 percentage points. This gap has spawned a large body of research that has documented these differences and tried to account for the Japanese saving puzzle (see, e.g., Hayashi (1997) and Horioka (1990) for reviews of this literature). Recently this gap has been narrowing. In 2002 the gap had fallen to less than 2 percentage points, leading some to predict that the Japanese saving rate is about to fall below the U.S. saving rate of 4 percent. Associated with this decline in the Japanese personal saving rate has been a concurrent decline in the after-tax real return on capital or *after-tax real interest rate*, from 6 percent in 1990 to 4 percent in 2000.[1]

This paper quantifies the role of alternative shocks in accounting for the recent declines in Japanese saving rates and interest rates and provides some projections about their future course. We start from the life-cycle hypothesis of Modigliani and Brumberg (1954). This choice is motivated by recent findings of Hayashi (1995) and Horioka, et.al. (2000). Hayashi estimates Engel curves for Japanese households and finds that they are inconsistent with the hypothesis that bequest motives are important. Horioka, et.al. (2000) argue, more generally, that survey evidence of Japanese households is much more consistent with the life-cycle hypothesis than the alternatives of altruistic or dynastic households.[2] Under the life-cycle hypothesis household saving varies with age. With the further assumption of overlapping generations, demographic changes such as the aging of a baby boom can have important implications for saving rates. In order to measure the magnitude of these effects we assume that households live for 80 years. Households are assumed to interact in perfectly competitive markets in a closed, general equilibrium

[1] Our measure of the after-tax real interest rate is from Hayashi and Prescott(2000)

[2] To check the robustness of our results, we also report simulations from a variant of the model populated by infinitely-lived, representative households.

economy.[3]

We focus our analysis on three distinct sources of variation in saving rates and real interest rates: changes in fertility rates, changes in survival rates, and changes in technology.[4] The interaction of fertility rates and survival rates jointly determines the age distribution of the population at any point in time. By varying fertility rates and survival rates, we can model the effects of a baby boom and increased longevity on the age distribution and thus on aggregate saving rates and interest rates. In a model calibrated to Spanish data, Rios-Rull (2002) found that permanent shocks to demographics have large effects on saving and interest rates.

Changes in the growth rate of productivity can also have large effects on saving and interest rates. Hayashi and Prescott (2002), for instance, have found that the the productivity slow-down in the 1990's produces big declines in after-tax real interest rates in a representative agent real business cycle model. Chen, İmrohoroğlu, and İmrohoroğlu (2005, 2006a, 2006b) found that changes in total factor productivity (TFP) growth alone can explain much of the variation in the Japanese saving rate over the last four decades of the twentieth century.[5]

We calibrate our model to Japanese data and conduct a dynamic analysis

[3]Japan is one of the largest economies in the world both in terms of aggregate and per capita GDP. Japan also has the smallest trade-to-GDP ratios for both goods and services in the OECD. For instance, in 2001 the trade-to-GDP ratio for goods was 9.3% in the United States and 8.4% in Japan and the ratio of services to GDP was 2.4% and 2.3% respectively. For these reasons we think it reasonable to assume that real interest rates are determined in the domestic market in Japan.

[4]In explaining the historical behavior of Japanese saving and interest rates, we also permit time variation in the depreciation rate and various indicators of fiscal policy, including government purchases, tax rates, the public debt, and the size of the public pension system.

[5]Changes in unemployment risk can also affect saving and interest rates. Unemployment rates in Japan rose from 2.2 percent in 1990 to 5.5 percent in 2003. Moreover, between 1990 and 2000 the median duration spell of unemployment rose from 3.5 to 5.5 months and the replacement rate fell from 0.84 to 0.68. If this risk is largely uninsurable then households will respond to it by increasing their demand for savings. The general equilibrium effects described in Aiyagari (1994) then imply that the real interest rate will fall. Although not reported here, we also simulated steady-state versions of our model incorporating unemployment risk and found that the measured increase in unemployment risk during the 1990s had a much smaller impact on saving and interest rates than either TFP or fertility rates. TFP and fertility rates had about equal sized affects on the saving rate. Because modeling unemployment risk when performing dynamic simulations substantially increases the computational burden, we chose to omit it.

starting from 1961 to trace out the evolution of saving and interest rates through the middle of the next century under alternative assumptions about productivity and fertility. We find that the model is reasonably successful in reproducing the observed year-to-year pattern of saving rates between 1961 and 2000. It also generates a substantial secular decline in interest rates over that period that roughly matches that seen in the data, although it does not produce the extraordinarily high real returns observed in the late 1960s and early 1970s. For the 1990s, the model predicts a decline in the saving rate from 0.136 to 0.055, which is reasonably close to the decline from 0.149 to 0.057 observed empirically. Over the same period, it predicts a decline in the interest rate of 240 basis points, compared with the observed decline of 210 basis points.

Using the National Institute for Population and Social Security's (IPSS) intermediate population projections and assuming that annual TFP growth recovers to 2 percent by 2010, the model predicts that the net national saving rate will fall to a low of 1.5 percent in 2051 and then remain below five percent indefinitely. The real interest rate gradually increases to about 5.7 percent by 2025 and remains in that neighborhood until at least the end of the century.

We also conduct a sensitivity analysis to assess the robustness of these conclusions to our conditioning assumptions about future demographics and TFP growth. Using the IPSS high population projections, the net national saving rate reaches a minimum of 2.2 percent in 2046 and then gradually rises to 4.5 percent after that. Under the low population projections, the saving rate falls to zero beginning in the year 2066. These alternative population assumptions have little effect on interest rate forecasts, however. To assess the role of our conditioning assumptions for TFP we use the intermediate population projections but assume that TFP growth remains at the levels of the 1990s. These conditioning assumptions yield forecasts of both saving and interest rates that are noticeably below the baseline case. The interest rate approaches a long-run value more than 100 basis points lower than the baseline forecast. The saving rate remains negative into the next century and eventually settles at a value below one percent, compared with 4.5 percent in the baseline specification.

Our work is related to research by Hayashi, Ito, and Slemrod (1988), who investigate the role of imperfections in the Japanese housing market in accounting for the Japanese saving puzzle in an overlapping generations endowment economy. They find that the combination of rapid economic growth, demographics, and housing market imperfections explains the level of Japanese saving rates in 1980. Their projections, which condition on an unchanged real interest rate, show declines in the saving rate of about 10 percent between 2000 and 2030.

The remainder of the paper is divided into six sections. In section 2 we describe the model economy, while section 3 reports its calibration. Section 4 evaluates the model's ability to explain the observed behavior of saving and interest rates since 1961 and section 5 reports our projections. Section 5 contains our conclusions.

2 Model

2.1 Demographic Structure

This economy evolves in discrete time. We will index time by t where $t \in \{..., -2, -1, 0, +1, +2, ...\}$. Households can live at most J periods and J cohorts of households are alive in any period t. They experience mortality risk in each period of their lifetime.

Let $\mu_{j,t}$ denote the number of households of age j in period t. Then the dynamics of population are governed by the first-order Markov process:

$$\mu_{t+1} = \begin{bmatrix} (1+n_{1,t})\psi_{1,t} & 0 & 0 & \dots & 0 \\ \psi_{2,t} & 0 & 0 & \dots & 0 \\ 0 & \psi_{3,t} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_{J,t} & 0 \end{bmatrix} \mu_t \equiv \Gamma_t \mu_t, \tag{1}$$

where μ_t is a $J \times 1$ vector that describes the population of each cohort in period t, $\psi_{j,t}$ is the conditional probability that a household of age $j-1$ in period t survives to period $t+1$, and $\psi_{1,t} = 1$ for all t. The growth rate of the number of age-1 households between periods t and $t+1$ is $n_{1,t}$, which we will henceforth refer to as the net fertility rate.[6] The aggregate population

[6]Note that this usage differs from other common definitions of the fertility rate and that the net fertility rate, as we have defined it, can be negative, indicating a decline in the size of the youngest cohort from one period to the next. We compute quantities analogous to $n_{1,t}$ from Japanese data and use these values to parameterize our model. We use our definition of the fertility rate to describe both the model quantities and their empirical counterparts.

in period t, denoted by N_t, is given by

$$N_t = \sum_{j=1}^{J} \mu_{j,t}. \tag{2}$$

The population growth rate is then given by $n_t = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t - j + 1$ to age $j > 1$ in period t is:

$$\pi_{j,t} = \psi_{j,t}\pi_{j-1,t-1} \tag{3}$$

where $\pi_{1,t-j+1} = 1$ for all t.

2.2 Firm's Problem

Firms combine capital and labor using a Cobb-Douglas constant returns to scale prodution function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{4}$$

where Y_t is the output which can be used either for consumption or investment, K_t is the capital stock, L_t is effective aggregate labor input and A_t is total factor productivity.[7] Total factor productivity grows at the rate $\gamma_t = A_{t+1}^{1/(1-\alpha)}/A_t^{1/(1-\alpha)}$. We will assume that the the market for goods and the markets for the two factor inputs are competitive. Then labor and capital inputs are chosen according to

$$r_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{5}$$
$$w_t = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}, \tag{6}$$

where r_t is the rental rate on capital and w_t is the wage rate per effective unit of labor.

[7] As described below, labor efficiency is assumed to vary with age, so that changes in the age distribution of the population alter the average efficiency of the labor force. This effect is measured by L_t, while changes in efficiency due to technical progress are captured by A_t.

2.3 Household's Problem

All households have one adult but may have one or more children. The utility function for a household born (and thus of age 1) in period s is given by

$$U_s = \sum_{j=1}^{J} \beta^{j-1} \pi_{j,s+j-1} \eta_j u(c_{j,s+j-1}/\eta_j, \ell_{j,s+j-1}), \tag{7}$$

where β is the preference discount rate, $c_{j,s+j-1}$ is total household consumption for a household of age j in period $s+j-1$, and η_j is the scale of a household of age j. This specification of preferences makes it possible for the size of a household to vary with the age of the adult member but imposes the restriction that the distribution of household size, $[\eta_1, ..., \eta_J]$, remain fixed over time.

Households of age j and cohort s supply labor of $1 - \ell_{j,s+j-1}$ in period $s+j-1$. They receive labor income that consists of an efficiency-weighted wage rate $w_{s+j-1}\varepsilon_j$ per unit of labor supplied, where w_t denotes the market wage rate per unit of effective labor in period t and ε_j denotes the time-invariant efficiency of an age-j worker. The efficiency index ε_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the retirement age. The budget constraint for a household of age j in period t is:

$$c_{j,t} + a_{j,t} \leq R_t a_{j-1,t-1} + w_t \varepsilon_j (1 - \ell_{j,t}) + b_{j,t} + \xi_t - \theta_{j,t} \tag{8}$$

where $a_{j,t}$ denotes assets held at the end of period t (with $a_{0,t} = 0$ for all t), $\theta_{j,t}$ are taxes imposed by the government, $b_{j,t}$ denotes public pension (social security) benefits, ξ_t is a uniform, lump-sum government transfer to all individuals alive in period t, and $R_t = 1 + r_t - \delta_t$. Here, δ_t denotes the depreciation rate of capital in period t. The pension benefit $b_{j,t}$ is assumed to be zero before age J_r and a lump-sum payment thereafter. Households are also subject to a borrowing constraint that rules out negative holdings of assets: $a_{j,t} \geq 0$.

Taxes imposed by the government are given by

$$\theta_{j,t} = \tau_t^c c_{j,t} + \tau_t^a (R_t - 1) a_{j-1,t-1} + \tau_t^\ell + w_t \varepsilon_j (1 - \ell_{j,t}) \tag{9}$$

where τ^c is the tax rate on consumpation and τ^a and τ^ℓ are the tax rates on income from labor and capital, respectively.

2.4 Household's Decision Rules

Suppose that a household's asset holdings take on a finite number of values $a \in \{0, ..., \bar{a}\}$. Then we can summarize the situation of an age-j household in period t with the state variable $x_{j,t}$. The individual state consists solely of asset holdings $a_{j-1,t-1}$: $x_{j,t} = \{a_{j-1,t-1}\}$. The period-$t$ wealth distribution describes the measure of households in each individual state: $\lambda_j(x_{j,t})$, $j = 1, ...J$. The aggregate state of the economy, denoted X_t, is composed of the aggregate capital stock, K_t, total factor productivity, A_t, the wealth distribution, λ_t, and the population distribution, μ_t,: $X_t = \{K_t, A_t, \lambda_t, \mu_t\}$. Finally, define the government policy rule in period t as Ψ_t. It will be convenient when solving the household's problem to assume that households know the entire future path of government policies, $\mathbf{\Psi}^t = \{\Psi_i\}_{i=t}^{\infty}$ and the entire future path of total factor productivity. With these various definitions and assumptions in hand, we can now state Bellman's equation for a typical household:

$$\begin{aligned} & V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t) \\ = & \max u(c_{j,t}/\eta_j, \ell_{j,t}) + \beta\psi_{j+1}V_{j+1}(x_{j+1,t+1}; X_{t+1}, \mathbf{\Psi}^{t+1}) \end{aligned} \tag{10}$$

subject to

$$c_{j,t} + a_{j,t} \leq R(X_t)a_{j-1,t-1} + w(X_t)\varepsilon_j(1 - \ell_{j,t}) + b_{j,t} + \xi_t - \theta_{j,t} \tag{11}$$

$$a_{j,t} \geq 0, \quad c_{j,t} \geq 0 \tag{12}$$

$$\mu_{t+1} = \Gamma_t \mu_t \tag{13}$$

and the law of motion of the aggregate wealth distribution and the law of motion for the aggregate capital stock given by $K_{t+1} = K(X_t)$. Since households die at the end of period J, $V_{J+1,t} = 0$ for all t. A solution to the household's problem consists of a sequence of value functions: $\{V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}_{j=1}^{J}$ for all t, and policy functions:$\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), \ell_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}_{j=1}^{J}$ for all t. The law of motion for the wealth distribution is computed using forward recursion on the following sum:

$$\lambda_{j+1}(x_{j+1,t+1}) = \sum_{a_{j-1,t-1} \in \Lambda_{j,t}} \lambda_j(x_{j,t}) \tag{14}$$

where the set $\Lambda_{j,t}$ is $\{a_{j-1,t-1} | a_{j,t} \in a_{j,t}(x_{j,t}, X_t; \mathbf{\Psi}^t)\}$. The recursion starts from the initial conditions of a newly-born household with zero assets, $\lambda_0(0) = 1$ and otherwise $\lambda_0(\cdot) = 0$.

2.5 Government

The government raises revenue by taxing consumption and income from labor and capital at the flat rates τ^c, τ^ℓ, and τ^a, respectively. It receives additional revenue by imposing a 100-percent tax on all accidental bequests. Total accidental bequests in period t are:

$$Z_t = \sum_{j=2}^{J} \sum_{a} (1 - \psi_{j,t}) R(X_t) a_{j-1,t-1}(x_{j-1,t-1}) \lambda_{j-1}(x_{j-1,t-1}) \mu_{j-1,t-1} \tag{15}$$

and total tax government is

$$T_t = \sum_{j=1}^{J} \sum_{a} \theta_{j,t}(x_{j,t}) \lambda_j(x_{j,t}) \mu_{j,t} + Z_t \tag{16}$$

Note that $\theta_{j,t}$ depends on $x_{j,t}$ since it is a function of $a_{j-1,t-1}$ by (10).

Total government expenditure is the sum of government purchases, public pension benefits, interest on the public debt, and lump-sum transfers. Government purchases are set exogenously to G_t. Aggregate pension benefits are given by

$$B_t = \sum_{j=J_r} \mu_{j,t} b_{j,t}. \tag{17}$$

The public debt is set exogenously and evolves according to

$$D_{t+1} = R_t D_t + G_t + B_t + \Xi_t - T_t. \tag{18}$$

Aggregate lump-sum transfers, Ξ_t, are set so as to satisfy this equation, and the per capita transfer, ξ_t, is determined from the equation

$$\Xi_t = \sum_{j=1}^{J} \xi_t \mu_{j,t}. \tag{19}$$

2.6 Recursive Competitive Equilibrium

Given this description of the economy we can now define a recursive competitive equilibrium.

Definition 1: Recursive Competitive Equilibrium

Given government policy rules $\{\mathbf{\Psi}^t\}_t$ and a law of motion for population $\{\Gamma_t\}_t$, a recursive competitive equilibrium is a set of value functions $\{V_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t\}_{j=1}^J$ for all t, policy functions: $\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t),$ $\ell_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}_{j=1}^J$ for all t, a wealth distribution λ_t, factor prices $\{w(X_t), r(X_t)\}$ for all t, a law of motion for aggregate capital $K_{t+1} = K(X_t)$ and a function for the average efficiency of labor input $h_t = h(X_t)$ such that:

- Given the functions of factor prices $\{w(X_t), R(X_t)\}$ and the law of motion for aggregate capital $K(X_t)$ and the function for average efficiency of labor input $h(X_t)$, the set of household policy functions $\{a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t), \ell_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\}$ solve the household's dynamic program (11).

- The factor prices are competitively determined so that (5) and (6) hold and $R_t = r_t + 1 - \delta_t$.

- The commodity market clears:

$$Y_t = C_t + I_t + G_t$$

  where $C_t = \sum_j \sum_a c_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\lambda_j(x_{j,t})\mu_{j,t}$ is aggregate consumption and I_t is aggregate investment, and G_t is government purchases.

- The laws of motion for aggregate capital and the effective labor input are given by:

$$K(X_t) = \sum_j \sum_a a_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t)\lambda_j(x_{j,t})\mu_{j,t}$$

$$L(X_t) = \sum_j^{J_r-1} \sum_a \varepsilon_j(1 - \ell_{j,t}(x_{j,t}; X_t, \mathbf{\Psi}^t))\lambda_j(x_{j,t})\mu_{j,t}.$$

- The measure of households λ_t is generated by (15).

- The government budget constraint is satisfied in each period:

$$D_{t+1} - T_t = R_t D_t + G_t + B_t + \Xi_t$$

In our simulations we assume that the economy eventually approaches a stationary recursive competitive equilibrium. Before we can define a stationary recursive competitive equilibrium we need to define some of the building blocks.

Definition 2: Stationary population distribution

Suppose that the fertility rate and the conditional survival probabilities are constant over time: $n_{1,t} = n_1$ for all t and $\psi_{j,t} = \psi_j$ for all t and j. Then a stationary population distribution, μ_t^*, satisfies $\mu_{t+1}^* = \Gamma^* \mu_t$ and $\mu_{t+1}^* = (1+n_1) \cdot \mu_t^*$ where

$$\Gamma^* = \begin{bmatrix} (1+n_1)\psi_1 & 0 & 0 & \dots & 0 \\ \psi_2 & 0 & 0 & \dots & 0 \\ 0 & \psi_3 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_J & 0 \end{bmatrix}$$

A stationary population distribution has two desirable properties. First, cohort shares in the total population are constant over time: $\mu_{j,t+1}/N_{t+1} = \mu_{j,t}/N_t$ for all t. Second, the aggregate population growth rate is time-invariant: $n_t = N_{t+1}/N_t = n_1$ for all t. This allows us to convert the growth economy into a stationary economy using the following transformations:

$$\tilde{c}_{j,t} = \frac{c_{j,t}}{A_t^{1/(1-\alpha)}}, \quad \tilde{a}_{j,t} = \frac{a_{j,t}}{A_t^{1/(1-\alpha)}}$$

Other per-capita variables in the household budget constraint are transformed in same way. Aggregate variables in period t are transformed by dividing by $A_t^{1/(1-\alpha)} N_t$ except for aggregate labor input, which is transformed by dividing by N_t.

Definition 3: Stationary recursive competitive equilibrium

Suppose the population distribution is stationary and the growth rate of total factor productivity is constant over time: $\gamma_t = \gamma^*$ for all t. Then a stationary recursive competitive equilibrium is a recursive competitive equilibrium that satisfies:

$$\tilde{c}_{j,t} = \tilde{c}_j^*, \quad \tilde{a}_{j,t} = \tilde{a}_j^*, \quad \tilde{\ell}_{j,t} = \ell_j^*$$

for all t and j, i.e., the factor prices are constant over time: $\{r_t, \tilde{w}_t\} = \{r^*, \tilde{w}^*\}$ for all t where $\tilde{w} = w/A^{1/(1-\alpha)}$.

This completes the description of the model.

3 Calibration

The model is calibrated to Japanese data. We assume that each household has one adult member. New households are formed when individuals reach the age of 21 and households die no later than the end of the 100th year of life, i.e., $J = 80$. The labor efficiency profile, ε_j, is constructed from Japanese data on employment, wages, and weekly hours following the methodology described in Hansen (1993). The family scale, η_j, is calibrated to Japanese data following the methodology of Cubeddu and Rios-Rull (1996). The vectors representing both labor efficiency and family scale as a function of age are assumed to be time-invariant. The net fertility rate, $n_{1,t}$, is calibrated to data on the growth rate of 21-year-olds for the period 1961-2000, and the series is extended to 2150 using various projections that will be described in more detail below. A similar procedure is used to calibrate the conditional survival probabilities, $\psi_{j,t}$.[8]

We assume that the utility function is isoelastic: is given by

$$\frac{[c_{h,i,j}^{\phi}(1-\ell_{h,i,j})^{1-\phi}]^{1-\sigma}}{1-\sigma}, \tag{20}$$

We choose σ, the risk aversion coefficent, to be 0.617. This choice produces a hump-shaped life-cycle consumption profile. The preference discount factor is chosen to reproduce the average capital-output ratio observed in Japanese data over the period 1984-2001. This yields a value of $\beta = 0.9655$. The preference parameter on leisure, is calibrated to reproduce average hours in Japanese data over the period 1984-2001. This results in a value of $\phi = 0.652$. And the capital share parameter, α, is calibrated to reproduce capital's share of output over the same sample period 0.3625.[9]

Dynamic simulations require values for the initial state of the economy in 1961 and for the entire future time path of the exogenous elements of the state vector. The aggregate state vector X_t consists of the aggregate capital stock, total factor productivity, the age distribution of the population, and total asset holdings of each cohort. Of these, TFP is taken as exogenous. The age distribution of the population at each date is determined by the exogenous sequences of the net fertility rate, $n_{1,t}$, and the survival probabilities, $\psi_{j,t}$, so

[8] See the data appendix for more details.

[9] We chose to limit the sample period for calibrating the parameters to 1984-2001 because we believe this sample period to be more representative of the long-run properties of the Japanese economy than the 1961-2001 sample period.

that knowledge of these sequences is also required. Finally, it is necessary to know the path of future government policies, $\mathbf{\Psi}^t$, and the depreciation rate, δ_t.

Total factor productivity is constructed using the same methodology as Hayashi and Prescott (2002) for the period 1961 through 1999. In our baseline model, we assume a linear increase in the TFP growth rate to 2 percent between 2000 and 2010, after which TFP is assumed to grow at a constant rate of 2 percent per year. We also report analyses using alternative projections for TFP growth. In some variants of our model, the depreciation rate is taken from Hayashi and Prescott up through 2001 and is assumed to remain at the 2001 value of 0.077 thereafter. In our baseline model, however, we adopt the simpler assumption that the depreciation rate is constant at 0.0848, the average of the Hayashi-Prescott series.

We use actual data on the growth rate of 21-year-olds, $n_{1,t}$, and survival probabilities $\psi_{j,t}$ to construct the age distribution of the population that that would arise in a closed economy. We use historical data for the years 1961-2000 and projections of these series by the National Institute for Population and Social Security Research (IPSS) up through 2050. These data are described in more detail in the Appendix. In our baseline model, we assume that survival probabilities remain unchanged after 2050 and that the net fertility rate increases linearly to zero between 2050 and 2060 and remains at zero thereafter. These assumptions about fertility and survival rates imply an age distribution of the population at each date.

Figure 1 shows the implications of our baseline demographic assumptions for the time path of fractions of different age groups in total population. The figure also displays the actual cohort shares and the official IPSS open-economy projections. These are quite close to the model series. Our demographic assumptions imply that the Japanese population falls by about 50 percent over the next 100 years.

In some variants of our model, government purchases as a fraction of GDP as well as the tax rates on income from labor and capital are set to the values reported in Hayashi and Prescott (2002) for the period 1961-2001, after which they are assumed to remain constant at their values for 2001. Similarly, the net public debt as a fraction of GDP is taken from Broda and Weinstein (2004) for 1978-2004 and from government sources for 1961-1977. Each of these fiscal variables is then assumed to remain constant indefinitely at its more recently observed value. In our baseline model, however, the government purchases ratio and the tax rates are assumed to be constant from 1961 onward at the average values reported by Hayashi and Prescott. Specifically, government purchases are set to 14.5 percent of GDP and the tax rates on labor and capital are set to 0.245 and 0.445, respectively. The baseline model also assumes the public debt to be zero in all periods. All variants of the model reported here set the consumption tax rate to zero. series. In addition, all variants of the model assume public pension benefits to be equal to 17 percent of average earnings up through 1976 and 40 percent thereafter.

We take the value of the capital stock in 1961 from Hayashi and Prescott (2002). We do not have Japanese data on asset holdings by age in 1961. Therefore, we assume a uniform distribution by age in our baseline model.

4 Explaining Historical Behavior

The Japanese net national saving rate and after-tax real interest rate exhibited substantial variation during the decades following 1960. The saving rate peaked in excess of 25 percent in the late 1960s, then fluctuated between 10 and 15 percent from the early 1970s until 1990, and fell to about 5 percent during the 1990s. The after-tax real return on capital fell averaged 17.2 percent from 1961 to 1970 and fell below 4 percent during some years in the 1990s. Before using our model to explain this recent decline in Japanese saving and interest rates or to project the saving rate saving and interest rates in coming decades, it is useful to document that the same model can generate large fluctuations in the saving and interest rates that are related to those observed in Japanese data in the decades after 1960.

Chen, İmrohoroğlu, and İmrohoroğlu (2005) found that a model similar to that used here, but with constant birth and death rates over time, could account for much of the variation in the Japanese saving rate after 1961. The success of their model contrasts sharply with pervious findings that standard economic theory was incapable of accounting for the observed swings in Japanese saving. (See, for example, Christiano 1989.) The major reason for this success was the inclusion of time-varying TFP growth.

More recently, Chen, İmrohoroğlu, and İmrohoroğlu (2006a, 2006b) incorporated time-varying birth and death rates into their model, as in the analysis reported here. They found that the model continued to perform well in accounting for historical saving behavior but that inclusion of demographic variation resulted in little increase in explanatory power as compared with a model that included only time-varying TFP growth. This conclusion contrasts with our preliminary finding based on a comparison of steady states calibrated to Japanese data from 1990 and 2000 and mentioned in the introduction. As noted there, demographics and TFP growth seem to be roughly equally important in accounting for the declines in saving and interest rates predicted by our model.

In this section, we use our model to simulate the Japanese saving rate from 1961 to 2000. Like Chen, İmrohoroğlu, and İmrohoroğlu, we find that the model does a reasonably good job of accounting for observed variation in Japanese saving. In the next section, we use the model to generate projections of the saving rate into the next century. These findings suggest that demographic considerations will play a larger role going forward than was the case up until about 1990 and that the evolution of the saving rate also depends crucially on assumptions about future TFP growth.

Figure 2 displays our baseline results for the period 1961-2001. The upper panel of each figure shows the behavior of the net national saving rate and the lower panel shows the after-tax real interest rate.

The model tracks the observed saving rate reasonably well, particularly at the endpoints of the sample. The empirical saving rate reaches its maximum value of 0.266 in 1970. The simulated series reaches its maximum of 0.243 in the same year. In general, the simulated series is below the actual one, and the discrepancy exceeds 0.05 during 1983-1987.[10] The observed series declines from 0.149 in 1990 to 0.057 in 2000, while the simulated series declines from 0.136 to 0.055.

The model does less well in capturing the year-to-year variation in the after-tax real interest rate, although it does reproduce the secular decline from 14 percent in 1962 to about 4 percent in 2000. The model does not generate the extraordinarily high real return to capital seen in the late 1960s and early 1970s, and the simulated series is slightly above the observed one from 1974 onward. The model predicts a decline of 240 basis points during the 1990s, which is quite close to the observed decline of 210 basis points.

[10]Our model does a somewhat better job in matching the saving rate reported by Hayashi (1997). That series is below the Hayashi-Prescott series in every year after 1973, and the difference between the two series averages 0.014.

5 Projections

Given the success of our model in reproducing much of the year-to-year pattern of saving rates as well as the long-term decline in interest rates, it is interesting to explore its implications for the future. Figure 3 displays these projections under alternative assumptions about demographics and TFP growth.[11]

The single most important fact about Japanese saving in the post-World War II period has been its magnitude. Our results indicate that the Japanese saving "puzzle" is a historical artifact. The results reported in Figure 3 predict that Japan's net saving rate will never exceed 10 percent again. Saving rates fall to a low of -1.5 percent in 2009 and eventually rise to a new steady-state value of 4.5 percent by the year 2140. This pattern is not monotonic, however. The saving rate increases to 2.7 percent in 2027 as a result of the echo of the baby boom. It then falls again to 1.5 percent in 2051 before increasing gradually to the new steady state. It is worth pointing out that this pattern is driven by persistent shocks to demographics and total factor productivity. These shocks are permanent in the sense that the age structure of the population is permanently altered from that observed historically and the TFP growth rate is assumed never to return to the values seen during the 1960s. The assumed decline in TFP growth is transitory, however, when compared to the average value for the 1970s and 1980s.

Hayashi, et.al., (1988) also provide long-run projections for saving rates. Their model predicts a decline in the saving rate of 10 percentage points between 2000 and 2030, close to the decline of 11 percentage points in our baseline model. The decline in our model is concentrated in the period between 1990 and 2010, however, and as noted above, the saving rate in 2030 is temporarily high because of transitory demographics. Moreover, the projections of Hayashi, et. al., are conditioned on a very different macroeconomic environment with a constant real interest rate and annual growth rates of output of 4 percent per year.

[11] We do not explore the implications of changes in fiscal policy or the depreciation rate. We are pursuing the effects of changes in fiscal policy, including those arising from demographic change, in separate work, and we have no basis for forecasting depreciation rates significantly different from those observed in recent data. Because time variation in these variables does not improve the in-sample fit of our models, we have held them constant in the baseline version.

The baseline results in Figure 3 suggest further that after-tax real interest rates have temporarily bottomed out. They reach a minimum value of 4.1 percent from 2003 to 2007 and then gradually rise to 5.7 percent in 2025. After that they fluctuate within a range of 35 basis points.

How sensitive are these predictions to our assumptions about total factor productivity and demographic factors? In order to answer this question we report three other simulations in Figure 3. Two of these variants maintain our baseline assumptions for TFP growth but use either the high or low IPSS population projections rather than the intermediate projections used in our baseline model. The third variant retains the baseline population projections but assumes that the TFP growth rate does not recover and instead remains at 0.33 percent per year, its average value for the 1990s.

Consider first the results for alternative demographic assumptions. These assumptions have no discernible effect on the saving rate either in the very long run, by which point they all yield the same age structure of the population, or up until the local peak associated with the echo of the baby boom around 2025. Over intermediate forecast horizons, however, demographics exert a noticeable influence on saving. The saving rate under the high population assumption is uniformly above the baseline projection, and the decline after the local peak in 2029 is muted. The corresponding decline under the low population assumption is quite pronounced, however, with the saving rate falling to zero in 2066-2068.

Demographic assumptions have much smaller effects on interest rates. The low (high) population assumption results in interest rates that are below (above) those predicted by the baseline model during much of the transition to the new steady state. Because the differences are generally too small to be noticeable, these alternative projections are not shown in Figure 3. The differences from the baseline projection are largest during the years 2035-2087, when they range between five and twenty basis points.

The results would look very different if the low growth rate of TFP of the 1990s is assumed to be permanent while the demographic variables are set to their baseline values. The assumption of permanently low total factor productivity growth is also maintained by Hayashi and Prescott (2002).

Consider first the real interest rate. This plot has two noteworthy features. First, observe that the recovery of real interest rates after 2007 that occurs under the baseline parameterization is predicated on a recovery of total factor productivity growth. If instead total factor productivity growth remains low, the real interest rate stays in the neighborhood of 4 percent until 2040. In addition, the new steady-state interest rate is also lower − 4.7 percent versus 5.8 percent in the baseline case.

We see similar patterns in the net saving rate, which remains negative into the next century and eventually approaches a new long-run value below one percent, as compared with 4.5 percent in the baseline specification. Note, however, that the saving rate with low TFP growth is above the baseline case for the years 2001-2011. This is because an anticipated recovery of TFP depresses saving in the short term.

From this analysis we see that both changing demographics and lower productivity growth contribute to reproducing the observed decline in the interest rate from 6 percent in 1990 to 3.6 percent by the year 2000. These results also indicate that observed and projected changes in fertility rates produce very persistent responses in the saving rate, but much smaller responses in the after-tax real interest rate. Sustained but temporary shocks to total factor productivity growth have large contemporaneous effects but do not produce much propagation over time in the model. A large permanent decline in TFP growth, on the other hand, causes large and permanent declines in both the net saving rate and the real interest rate.

6 Conclusion

In this paper we have shown that the measured declines in saving rates and real interest rates in Japan during the 1990s are consistent with the predictions of theory. Both low total factor productivity growth and the life cycle hypothesis play important roles in accounting for these facts. Our theory also has sharp implications for the future evolution of saving rates and interest rates. It provides a quantitative confirmation of previous claims that the Japanese saving puzzle is over. According to our projections, Japanese saving rates will remain below 5 indefinitely. Moreover, this finding is reasonably robust to alternative assumptions about demographics and future TFP growth. The population distribution, which is a key determinant of saving, changes only gradually over time in a highly predictable way. Thus, even when we posit a robust recovery in total factor productivity, saving rates remain low by historical standards.

Appendix (to be revised)

A.1 Data set

Efficiency units by age

We construct efficiency units by age from time-series data that covers the period from 1990 to 2000 following the methodology of Hansen(1993). For each year an (11×2) age-sex array of data is available. Each of the 22 groups in the array is denoted by subscript i in each year t. [12]

- $MEFT$: Monthly contractual earnings for employed full-time wage and salary workers
- $HEPT$: Hourly scheduled cash earnings for part-time wage and salary workers
- $AEFT$: Annual special cash earnings for employed full-time wage and salary workers[13]
- $AEPT$: Annual special cash earnings for employed part-time wage and salary workers
- NFT : Number of wage and salary workers who work full-time
- NPT : Number of wage and salary workers who work part-time
- HFT : Monthly actual numebr of scheduled hours worked for employed full-time wage and salary workers
- HPT : Daily actual number of scheduled hours worked for employed part-time wage and salary workers
- $NDPT$: Monthly Actual number of days worked for employed part-time wage and salary workers

[12] The data source is *Basic Survey in Wage Structure* by the Ministry of Health, Labor and Welfare.

[13] The annual special cash earnings, $AEFT$ and $AEPT$, are reported for the previous year e.g. special cash earnings for 1990 refer to earnings received in 1989. For this reason we treat the annual special cash earnings reported at $t + 1$ as those in t.

- $OHFT$: Monthly actual number of overtime hours worked for employed full-time wage and salary workers
- Age : Average age of the each age group

Following Hansen(1993), let small letters denote the real values defined as $x = X/P$ for any nominal variable X, where P is the GDP deflator. From the data series obtained in above we can construct the following series:

- $weft$: Average weekly earnings for employed full-time wage and salary workers
- $wept$: Average weekly earnings for employed part-time wage and salary workers
- $AHFT$: Average weekly hours for employed full-time wage and salary workers
- $AHPT$: Average weekly hours for employed part-time wage and salary workers

where

$$\begin{aligned}
&weft = meft/4 + aeft/48 \\
&wept = hept \cdot HPT \cdot NDPT/4 + aept/48 \\
&AHFT = (HFT + OHFT)/4 \\
&AHPT = HPT \cdot NDPT/4
\end{aligned}$$

From these data an weekly measure of hourly earnings for each subgroup at time t (HE_{it}) was constructed as:

$$HE_{it} = \frac{weft_{it} \cdot NFT_{it} + wept_{it} \cdot NPT_{it}}{AHFT_{it} \cdot NFT_{it} + AHPT_{it} \cdot NPT_{it}}$$

Let HE_i be the average of HE_{it} over the t-year sample. The efficiency units for each age-sex subgroup i are formed as follows:

$$\epsilon_i = HE_i/HE$$

where

$$HE = \frac{\sum(NFT_i + NPT_i)HE_i}{\sum(NFT_i + NPT_i)}$$

where NFT_i and NPT_i are average over t. We next construct, ϵ, an (11 $\times$ 1 vector) of efficiency units by age. Each element of this vector is a weighted average of the ϵ_i's for males and females in that age group, where the weights are the numbers of male and female workers. From these 11 values we derive the ϵ_j for each age $j = 21, ..., 65$ using interpolation by polynomials.

Replacement rate

Our model is an annual one, whereas most of the Japanese regulations and data relating to unemployment are categorized in terms of unemployment duration measured in months. For example, an employed person who loses his/her job gets at most 3 months of unemployment benefits. The nominal replacement ratio varies between 50% to 80% depending on the age and the salary of the person as prescribed by the Social Security Law. We define the model's nominal replacement ratio as $\bar{m} = 0.65$.

Define, p_0 to be the probability of becoming unemployed in any month given that one was employed the previous month and let p_i be the probability of being unemloyed at least i months, given that one has been unemployed for at least $i-1$ months. We estimate these conditional probabilities from data on the duration of unemployment; 1 month, 1-3 month, 3-6 month, 6-12 month, 12-24 month and over 24 months using linear interpolation.[14]

Given the p_j, let $x_i = Pi_{j=1}^{i} p_j$ be the probability of being unemployed for at least j months, given that an unemployment spell has begun. Thus, x_i is the probability that than an unemployment spell, once begun, will last at least i months. Then $z_i = x_i - x_{i+1}$ is the probability that an unemployment spell lasts exactly i months, given that an unemployment spell has begun.

Then expected income in the first year of unemployment is calculated as $m = \sum_{j=1}^{11} z_j m_j + x_{12} m_{12}$ where m_j is annual income for an individual with a duration spell of unemployment of exactly j periods. Thus a person with a duration of unemployment spell of one month has annual income of $0.65/12 + 11/12 = m_1$ This defines the replacement ratio in our annual model. It is 0.842 in 1990 and 0.683 in 2000.

[14] The data source is *Report on the Special Survey of the Labor Force Survey* by the Statistics Bureau, Management and Coordination Agency, Government of Japan.

Transition probability matrix

We model employment as a stationary, two-state, first-order Markov process with transition matrix of the form:

$$\begin{bmatrix} Prob(s_1|s_1) & Prob(s_2|s_1) \\ Prob(s_1|s_2) & Prob(s_2|s_2) \end{bmatrix} = \begin{bmatrix} 1-P_0 & P_0 \\ 1-P_1 & P_1 \end{bmatrix} \tag{21}$$

where s_1 means employed, s_2 means unemployed and $Prob(s_k|s_j)$ is the conditional probability of being the state s_k from the state s_j for $j,k = 1,2$. We measure P_1, the probability of being unemployed this year conditional on being unemployed last year, by $\Pi_{j=1}^{13} p_j$, the probability that an unemployment spell, once begun, will last for at least 13 months. We use the unemployment rate, which is given by $\nu = (1-\nu)P_0 + \nu P_1$, to measure the remaining transition probability P_0. The transition matrices for 1990 and 2000 are respectively given by:

$$P^{1990} = \begin{bmatrix} 0.981 & 0.019 \\ 0.931 & 0.069 \end{bmatrix}, \quad P^{2000} = \begin{bmatrix} 0.956 & 0.044 \\ 0.836 & 0.164 \end{bmatrix} \tag{22}$$

Demographic parameters

The exogeneous demographic parameters consist of the initial population distribution μ_{1990}, the fertility rate $n_{1,t}$, $t = 1990, ..., 2000$, and the conditional surviving probability $\{\psi_j\}_{j=1}^{J}$, $t = 1990, ..., 2000$. We have data on *Population* and *Deaths* for $t = 1990, ..., 2001$ and for $j = 1, ..., J$. [15] Note that the model age $j = 1$ means age 21 and J is set to 65 in our simulations. Then the maximum age is 85, which is the maximum available age data in *Population*. This is the reason why we set $J = 65$.

The initial population distribution μ_{1990} is given by the Japanese population distribution for 1990 in our steady-state analysis and the Japanese population distribution in 1985 in our dynamic analysis. The fertility rate and the conditional surviving probability are calculated as: for $t = 1985, ..., 2000$

$$n_{1,t} = (Population_{1,t+1} - Population_{1,t})/Population_{1,t}$$
$$\psi_{j,t} = 1 - Dealths_{j-1,t-1}/Population_{j-1,t-1}$$

[15] The data source: *Population* is the *Annual Report on Current Population Estimates* by the Ministry of Public Management, Home Affairs, Posts and Telecommunications; *Deaths* are the *Vital Statistics* by the Ministry of Health, Labor and Welfare.

The other exogeneous parameters

We follow Hayashi and Prescott(2002) in calibrating the other parameters with the exceptions of the time preference parameter β and the relative risk aversion σ. The value of β is chosen to reproduce the 1990 value of the after-tax real interest rate in the data.[16] The values of the other parameters are as follows.

Calibrated parameters

α	0.36
β	0.97(0.98)
δ	0.089
τ	0.48
σ	2

A.2 Simulation Methodology

Stationary equilibrium

Given data on conditional survival probabilities $\{\psi_j\}$ we next need to translate the economy into the economy with variables in per-capita efficiency units. Define a cohort share in total population by $\tilde{\mu}_j = \mu_j/N$. Then the stationary population distribution, cohort shares $\{\tilde{\mu}_j^*\}_{j=1}^J$ are calculated by $\tilde{\mu}_j^* = \psi_j \tilde{\mu}_{j-1}^*/(1+n_1)$ such that $\sum_j \tilde{\mu}_j^* = 1$ holds. [17] In a stationary equilibrium all variables in per-capita efficiency units are constant over time, so we can ignore the time subscript t. Then the transformed Bellman's equation is expressed as:

$$V_j(\tilde{x}_j) = \max\{u(\tilde{c}_j/\eta_j) + \tilde{\beta}\psi_{j+1}\sum_{s_{j+1}} V(\tilde{x}_{j+1})P(s_{j+1}, s_j)\}$$

subject to

$$\begin{aligned}&\tilde{c}_j + \tilde{a}_j = R\tilde{a}_{j-1}/\gamma + \tilde{w}\epsilon_j s_j - \tilde{\theta}_j + m\tilde{w}\epsilon_j(1-s_j) + \tilde{\xi}\\ &a_{j,t} \geq 0, \quad c_{j,t} \geq 0\end{aligned}$$

[16] In the steady-state analysis the value of β is set to 0.978 and in the dynamic analysis it is set to 0.985.

[17] Note that we need only the conditional survival probabilities $\{\psi_j\}$ to calculate the stationary population distribution cohort shares. Next define $\mu_j = \Pi_{i=1}^j \psi_i$ as $\psi_1 = 1$ and $N = \sum_{j=1}^J \mu_j$. Then we can calculate $\{\tilde{\mu}_j^*\}$ as described above.

where $\tilde{x_j} = \{\tilde{a}_{j-1}, s_j\}$, $\tilde{\beta} = \beta\gamma^{(1-\sigma)}$ and $R = 1 + r - \delta$. Since $\tilde{\theta}_j = \tau_a(R - 1)\tilde{a}_{j-1}/\gamma$ by (10), the budget constraint becomes:

$$\tilde{c}_j + \tilde{a}_j = (1 + \hat{r})\tilde{a}_{j-1}/\gamma + \tilde{w}\epsilon_j s_j + m\tilde{w}\epsilon_j(1 - s_j) + \tilde{\xi}$$

where $\hat{r} = (1 - \tau_a)(r - \delta)$ is the equilibrium after-tax real interest rate.

Let $\epsilon_1 > 0$ and $\epsilon_2 > 0$ denote the convergence criteria for the after-tax real interest rate and lump-sum transfers in per-capita efficiency units, respectively. Computing a stationary equilibrium requires finding a fixed point in the after-tax real interest rate, $\hat{r}^*$, and the lump-sum transfers in per-capita efficiency units, $\tilde{\xi}^*$. This is accomplished in the following way. First set the smoothing parameter $\rho \in (0, 1)$ and guess $\hat{r}_0$ and $\tilde{\xi}_0$. Then iterate on the following steps:

1. Compute the average efficiency of labor input $h = (1 - \nu)\sum_{j=1}^{J_r-1} \tilde{\mu}_j^* \epsilon_j$ and the real wage in efficiency unit $\tilde{w}_0$ using the first-order condition of the firms maximization problem.

2. Compute the decision rules $\{\tilde{a}_j(\tilde{x}_j), \tilde{c}_j(\tilde{x}_j)\}_{j=1}^J$ by completing a backward induction from the age J to the age 1 with the terminal condtion $a_J = 0$, and the wealth distribution $\lambda = \{\lambda_j(\tilde{x}_j)\}_{j=1}^J$ by completing a forward recursion from the age 1 to the age J with the initial condtion (16).

3. Compute the new capital stock in per-capita efficiency unit using the law of motion for capital $\tilde{k} = \sum_j \sum_a \sum_s a_j(\tilde{x}_j)\lambda_j(\tilde{x}_j)\tilde{\mu}_j^* / \ gamma$ and derive the new after-tax real interest rate, $\hat{r}_1$, from the above $\tilde{k}$ using the first-order condition of the firms profit maximization problem. Next compute the new lump-sum transfers in per-capita efficiency unit. By transforming the equations (17),(18) and (20) into the per-capita efficiency unit form we have

$$\tilde{\xi}_1 = \tilde{b} + \tilde{tt} - \sum_j \sum_a m\tilde{w}\epsilon_j\lambda_j(\tilde{x}_j)\tilde{\mu}_j^*$$

where [18]

$$\tilde{b} = \sum_j \sum_a \sum_s (1 - \psi_j)R_0\tilde{a}_{j-1}(\tilde{x}_{j-1})\lambda(\tilde{x}_{j-1})\tilde{\mu}_{j-1}^*$$

[18] Since the subscript t is used for indicating time, we define the per capita capital income tax as $tt = T/N$.

$$\tilde{tt} = \sum_j \sum_a \sum_s (1-\psi_j)\tilde{\theta}_j(\tilde{x}_{j-1})\lambda_{j-1}(\tilde{x}_{j-1})\tilde{\mu}^*_{j-1}$$

$$R_0 = 1 + \hat{r}_0/(1-\tau_a)$$

4. If $|\hat{r}_1 - \hat{r}_0|/|\hat{r}_0| < \epsilon_1$ and $|\tilde{\xi}_1 - \tilde{\xi}_0|/|\tilde{\xi}_0| < \epsilon_2$, stop. Then we have got the equilibrium:$\hat{r}^* = \hat{r}_0$ and $\tilde{\xi}^* = \tilde{\xi}_0$. If not, compute $\hat{r}_2 = \rho\hat{r}_0 + (1-\rho)\hat{r}_1$ and $\tilde{\xi}_2 = \rho\tilde{\xi}_0 + (1-\rho)\tilde{\xi}_1$. Set $\hat{r}_0 = \hat{r}_2$ and $\tilde{\xi}_0 = \tilde{\xi}_2$, and go to step 1.

Transitional dynamics without unemployment risk

Assume that the economy starts from the period $t = 1$. Agents have all the information available at the end of $t = 0$. The agents have perfect foresight about the entire future paths of total factor productivity, $\{A_t\}_{t=1}^{\infty}$, of the government policies,$\{\Psi_t\}_{t=1}^{\infty}$, and of the demographics $\{\Gamma_t\}_{t=1}^{\infty}$. Note that there is no employment risk. The individual state is simply $\tilde{x}_{j,t} = \{\tilde{a}_{j-1,t-1}\}$. The transformed Bellman's equation is:

$$\begin{aligned} & V_{j,t}(\tilde{x}_{j,t}; X_t, \mathbf{\Psi}^t) \\ =& \max\{u(\tilde{c}_{j,t}/\eta_j) + \tilde{\beta}_t\psi_{j+1}V_{j+1,t+1}(\tilde{x}_{j+1,t+1}; X_{t+1}, \mathbf{\Psi}^t)\} \end{aligned}$$

subject to

$$\begin{aligned} & \tilde{c}_{j,t} + \tilde{a}_{j,t} = R_t\tilde{a}_{j-1,t-1}/\gamma_{t-1} + \tilde{w}_t\epsilon_j - \tilde{\theta}_{j,t} + \tilde{\xi}_t \\ & \tilde{a}_{j,t} \geq 0, \quad \tilde{c}_{j,t} \geq 0 \\ & \tilde{\mu}_{t+1} = \mu_{t+1}/N_{t+1}, \quad N_{t+1} = \sum_j \mu_{t+1}, \quad \mu_{t+1} = \Gamma_t\mu_t \end{aligned}$$

and the law of motion for the per-capita-efficiency-unit wealth distribution, and the law of motion for the capital stock in per-capita efficiency unit. We will derive an equilibrium transition path from the initial condition to the final stationary equilibrium. Computing the equilibrium transition path requires finding paths of the after-tax real interest rate, $\{\hat{r}^*_t\}_{t=1}^{\infty}$, and of the lump-sum transfers in per-capita efficiency units. $\{\tilde{\xi}^*_t\}_{t=1}^{\infty}$, This is done using the following steps. First set $\epsilon_1, \epsilon_2 > 0$ and $\rho \in (0,1)$.

1. Set the initial conditions $\{a_{j,0}\}_{j=1}^{J}$, γ_0 and μ_1. [19] Next set $\{\gamma_t, \Psi_t, \Gamma_t\}_{t=1}^{\infty}$ such that for $t_1 < \infty$ we have $\{\gamma_t, \Psi_t, \Gamma_t\} = \{\gamma_{t_1}, \Psi_{t_1}, \Gamma_{t_1}\}$ for all $t \geq t_1$. Compute the final stationary equilibrium, $\hat{r}^{**}$ and $\tilde{\xi}^{**}$, assuming that $\{\gamma, \Psi, \Gamma\} = \{\gamma_{t_1}, \Psi_{t_1}, \Gamma_{t_1}\}$.

2. Guess the paths of the after-tax real interest rate and the lump-sum transfers in per-capita efficiency units $\{\hat{r}_{0,t}, \tilde{\xi}_{0,t}\}_{t=1}^{\infty}$ such that $\{\hat{r}_{0,t}, \tilde{\xi}_{0,t}\}=\{\hat{r}^{**}, \tilde{\xi}^{**}\}$ for all $t \geq T$ where T is sufficiently large that we are close to the final stationary equilibrium, say, $T = t_1 + 130$.

3. Compute the paths of the average efficiency of labor input $\{h_t\}_{t=1}^{T}$ where $h_t = \sum_{j=1}^{J_r-1} \tilde{\mu}_j^* \epsilon_j$ and of the real wage in efficiency units $\{\tilde{w}_{0,t}\}_{j=1}^{T}$ using the first-order condtion of the firms maximization problem.

4. Compute the decision rules of the households alive at $t = 1$ and derive the path of life-time asset holdings, $\{a_{J-i+t,t}\}_{t=1}^{i}$ for $i = 1, 2, .., J$ using the initial conditions $\{a_{j,0}\}_{j=1}^{J}$ where i denote the maximum life-time period left for each household alive at $t = 1$. Compute the decision rules for the household born at $t = 2, 3, ..., T$ and derive the path of life-time asset holdings, $\{a_{j,j+t-1}\}_{j=1}^{J}$ for $t = 2, 3, ..., T$ using the initial asset holdings $\tilde{a}_{0,t} = 0$ for all t.

5. Compute the capital stock in per-capita efficiency unit $\tilde{k}_t = \sum_{j=1}^{J} \tilde{a}_{t-1,j-1}\tilde{\mu}_{j-1,t-1}/\gamma_{t-1}$ and derive the new after-tax real intrest rate, $\hat{r}_{1,t}$ for all $t = 1, 2, ..., T$. Next compute the new lump-sum transfers in per-capita efficiency unit $\tilde{\xi}_{1,t} = \tilde{b}_t + \tilde{tt}_t$ for all $t = 1, 2, ..., T$ where

$$\tilde{b}_{t+1} = \sum_j (1-\psi_{j+1,t+1}) R_{0,t+1} \tilde{a}_{j,t} \tilde{\mu}_{j,t}, \quad \tilde{tt}_{t+1} = \sum_j (1-\psi_{j+1,t+1}) \tilde{\theta}_{j+1,t+1} \tilde{\mu}_{j,t}$$

6. If $\sum_{t=1}^{T} |\hat{r}_{1,t} - \hat{r}_{0,t}|/|\hat{r}_{0,t}| < \epsilon_1$ and $\sum_{t=1}^{T} |\tilde{\xi}_{1,t} - \tilde{\xi}_{0,t}|/|\tilde{\xi}_{0,t}| < \epsilon_2$, stop. Then we have got the equilibrium transition path $\hat{r}_t^* = \hat{r}_{0,t}$ and $\tilde{\xi}_t^* = \tilde{\xi}_{0,t}$ for all $t = 1, 2, ..., T$. Otherwise, compute $\hat{r}_{2,t} = \rho\hat{r}_{0,t} + (1 - \rho)\hat{r}_{1,t}$ and $\tilde{\xi}_{2,t} = \rho\tilde{\xi}_{0,t} + (1 - \rho)\tilde{\xi}_{1,t}$ for all $t = 1, 2, ..., T$. Set $\hat{r}_{0,t} = \hat{r}_{2,t}$ and $\tilde{\xi}_{0,t} = \tilde{\xi}_{2,t}$ for all $t = 1, 2, ..., T$ and go back to step 3.

[19] These values are obtained directly from the data or by computing the initial stationary equilibrium assuming some conditions.

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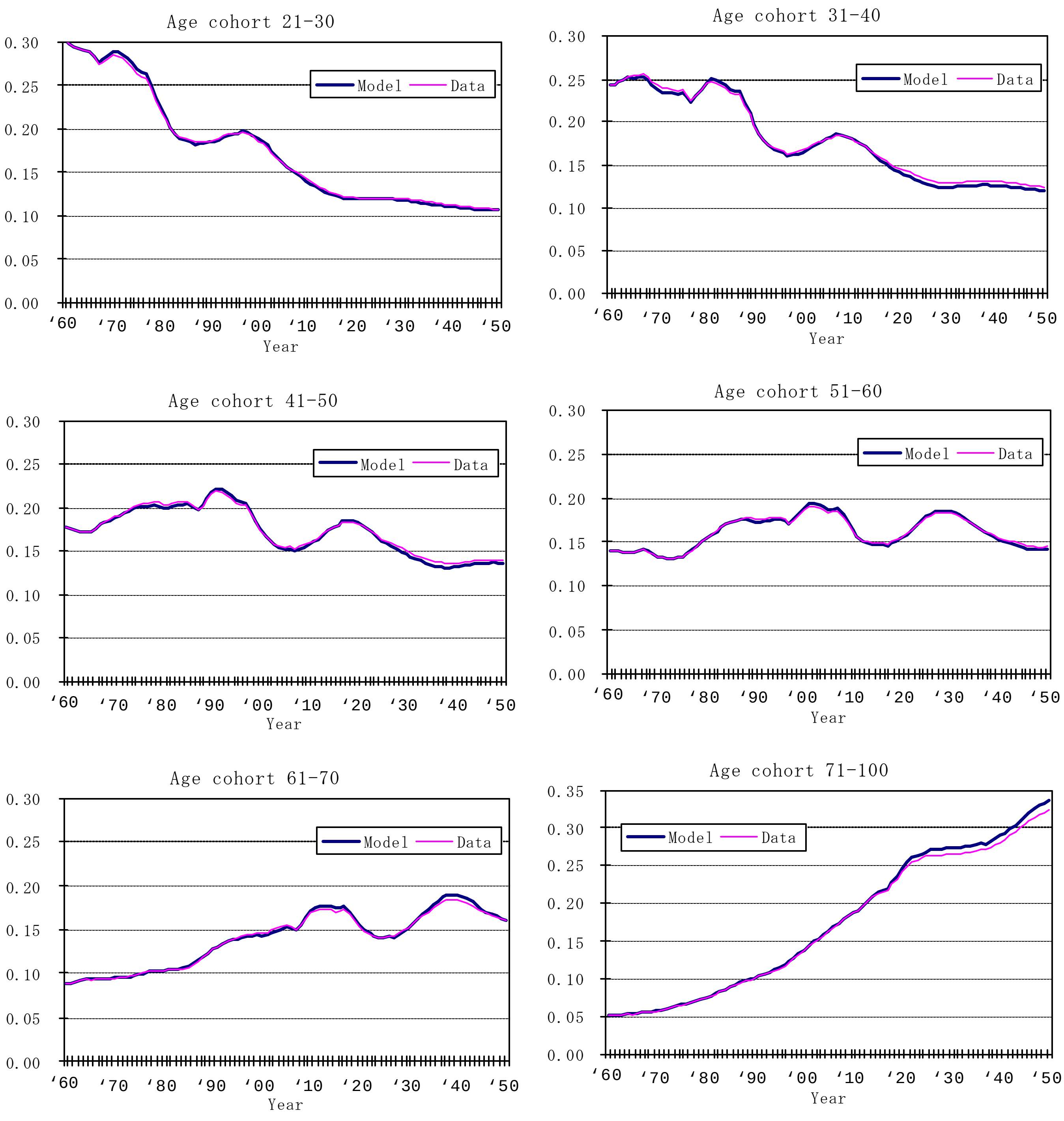
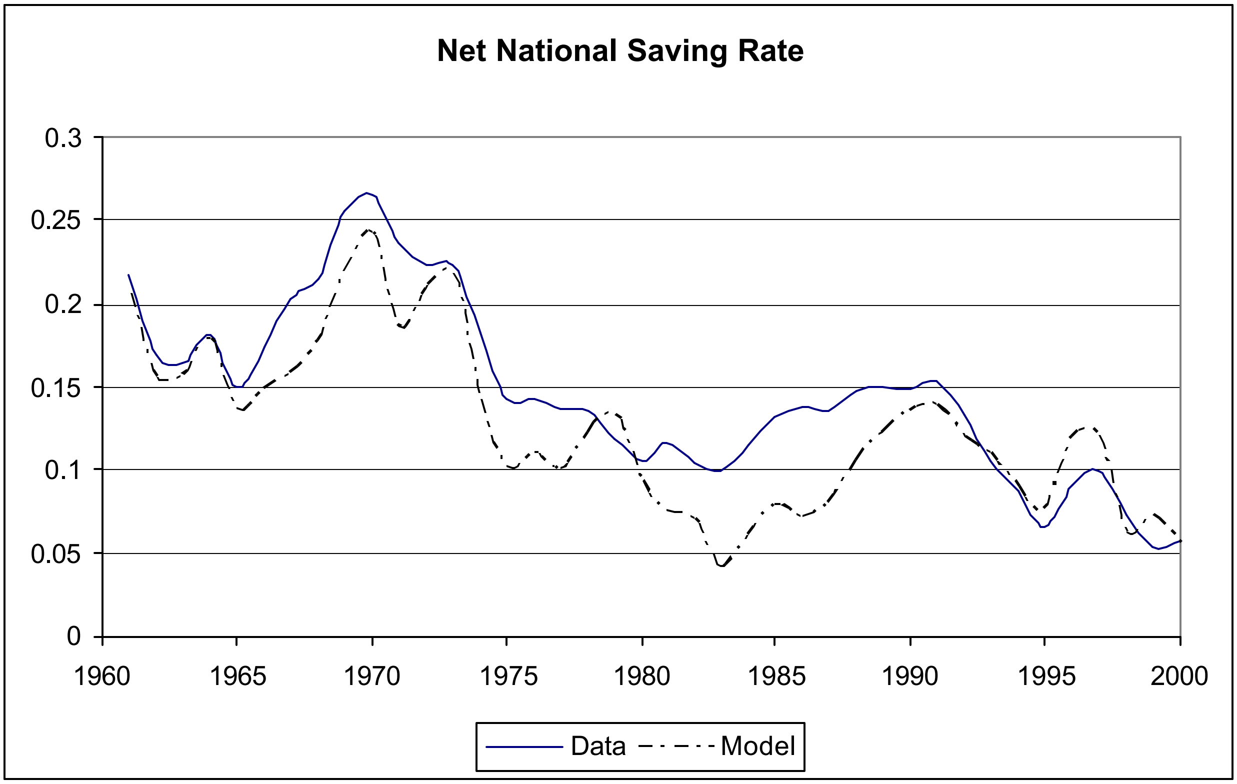


Figure 1: Population Projections



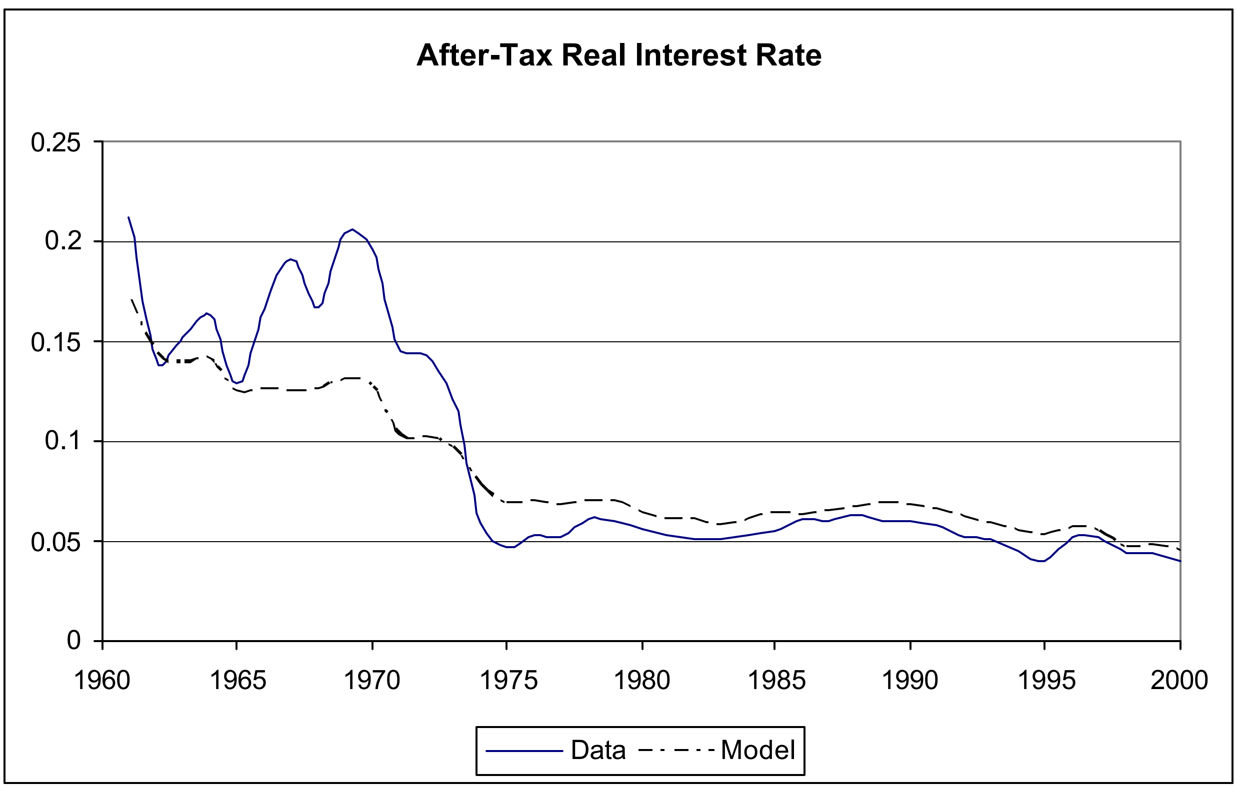


Figure 2: In-Sample Performance

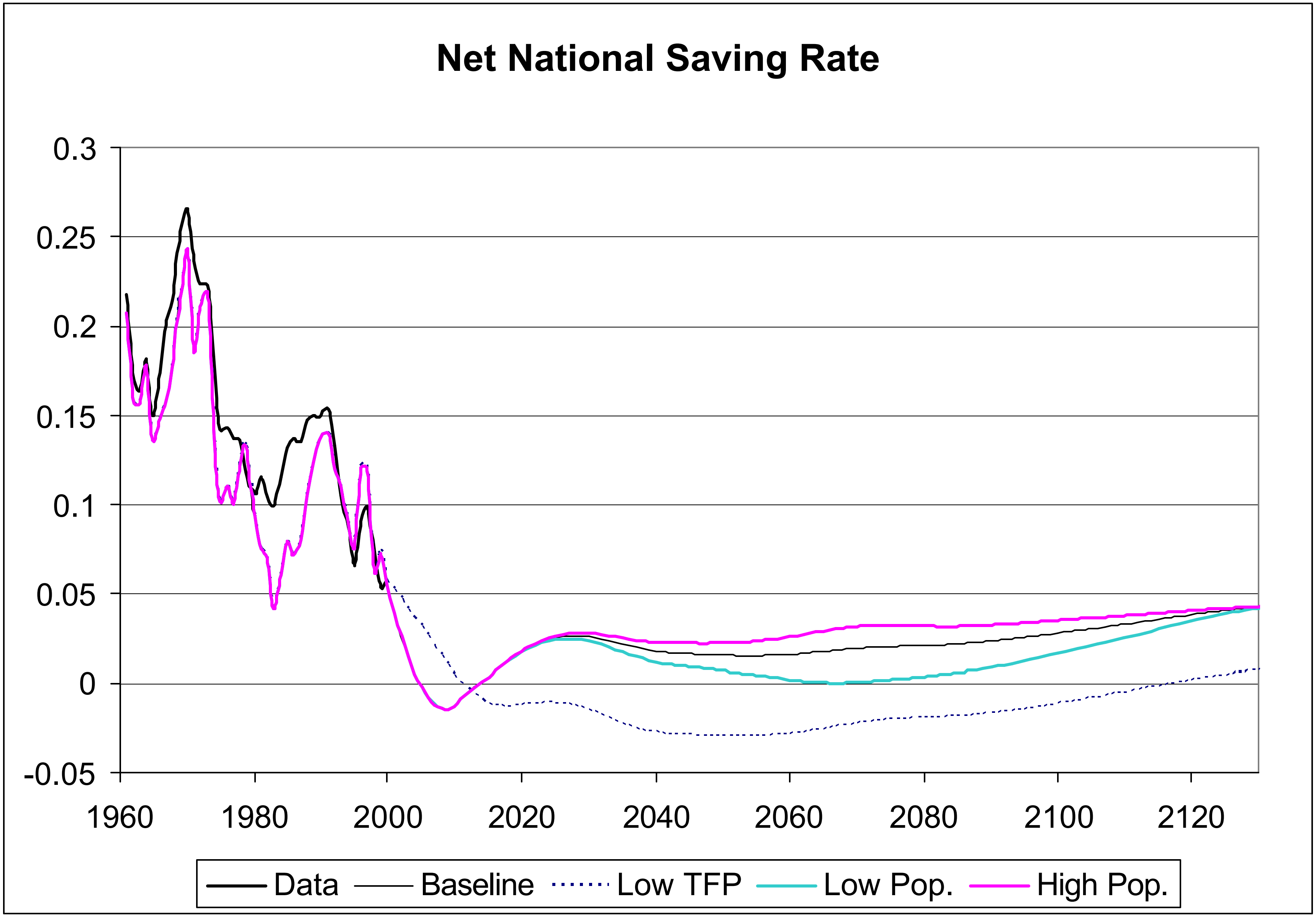


Figure 3a: Projections (Saving Rate)

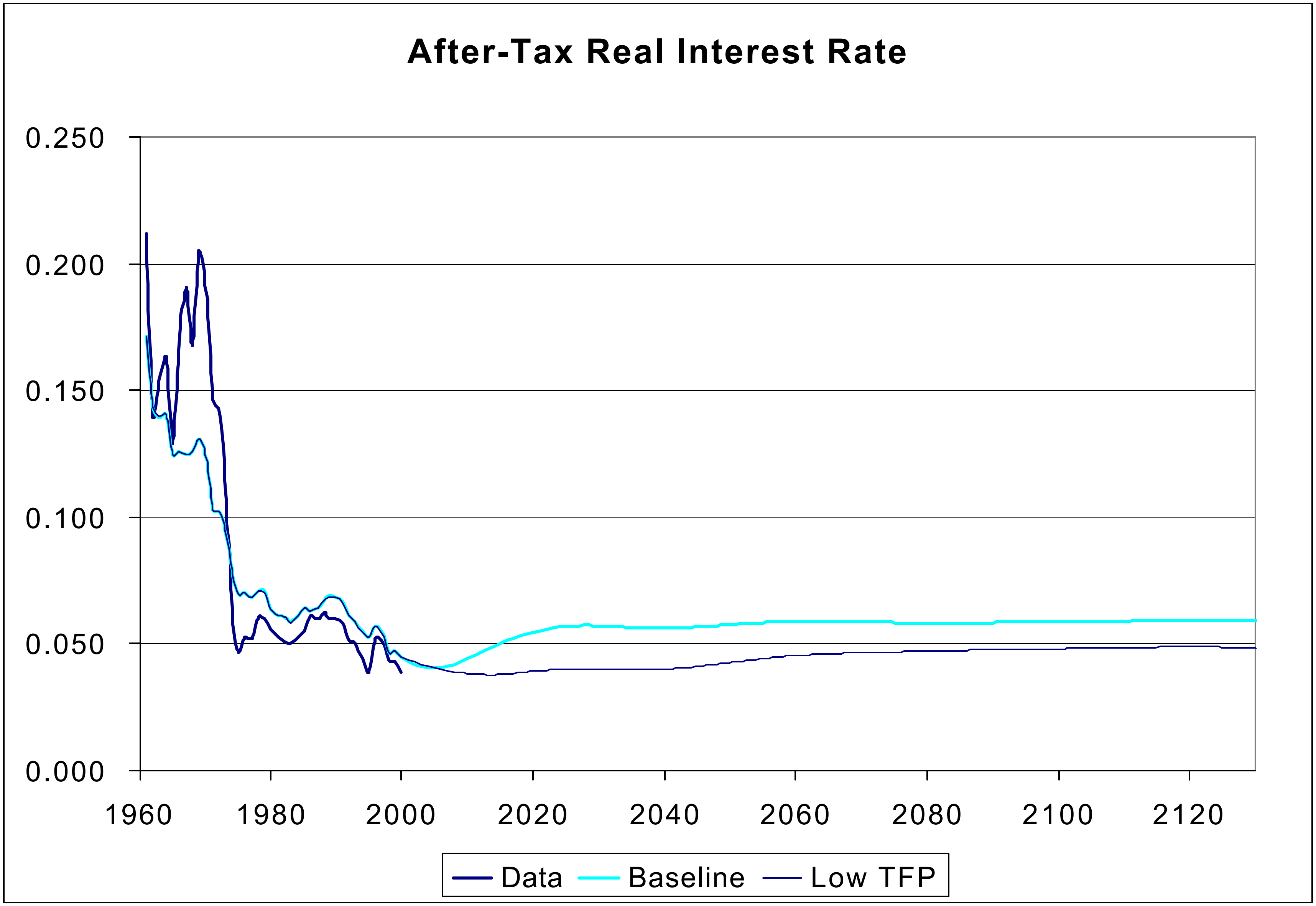


Figure 3b: Projections (Interest Rate)