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When Bonds Matter: Home Bias in Goods and Assets*

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Abstract

Recent models of international equity portfolios exhibit two potential weaknesses: 1) the structure of equilibrium equity portfolios is determined by the correlation of equity returns with real exchange rates; yet empirically equities don't appear to be a good hedge against real exchange rate risk; 2) Equity portfolios are highly sensitive to preference parameters. This paper solves both problems. It first shows that in more general and realistic environments, the hedging of real exchange rate risks occurs through international bond holdings since relative bond returns are strongly correlated with real exchange rate fluctuations. Equilibrium equity positions are then optimally determined by the correlation of equity returns with the return on non-financial wealth, conditional on the bond returns. The model delivers equilibrium portfolios that are well-behaved as a function of the underlying preference parameters. We find reasonable empirical support for the theory for G-7 countries. We are able to explain short positions in domestic currency bonds for all G-7 countries, as well as significant levels of home equity bias for the US, Japan and Canada.

Keywords : International risk sharing, International portfolios, Home equity bias

JEL codes: F30, F41, G11

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1 Introduction

The current international financial landscape exhibits two critical features. First, the last twenty years have witnessed an unprecedented increase in cross-border financial transactions. Second, despite this massive wave of financial globalization, international portfolios remain heavily tilted toward domestic assets (see table 5 in appendix, as well as French and Poterba (1991) and Tesar and Werner (1995)). The importance of these two features has not gone unnoticed, and has generated renewed interest for the theory of optimal international portfolio allocation.[1]

An important strand of literature, launched into orbit by the influential contribution of Obstfeld and Rogoff (2000), sets out to explore the link between the allocation of consumption expenditures and optimal portfolios in frictionless general equilibrium models à la Lucas (1982).[2] One popular approach, initially developed by Baxter et al. (1998) and extended by Coeurdacier (2008), consists in characterizing the constant equity portfolio that –locally– reproduces the complete market allocation. Agents in these models achieve locally-perfect risk sharing solely through trades in claims to domestic and foreign equities.

As emphasized by Coeurdacier (2008) and Obstfeld (2007), the structure of these optimal portfolios reflects the hedging properties of relative equity returns against real exchange rate fluctuations.[3] For instance, with Constant Relative Risk Aversion (CRRA) preferences, the optimal equity position is related to the covariance between the excess return on domestic equity (relative to foreign equity), and the rate of change of the real exchange rate. When the CRRA coefficient exceeds unity, home equity bias arises when excess domestic equity returns are positively correlated with the real exchange rate (measured as the foreign price of the domestic basket of goods, so that an increase in the real exchange rate represents an appreciation). In that case, efficient risk sharing requires that domestic consumption expenditures increase as the real exchange rate appreciates. If domestic equity returns are high precisely at that time, domestic equity provides the appropriate hedge against real exchange rate risk, and the optimal equity portfolio exhibits home portfolio bias. Seen in this light, most of the theoretical literature mentioned above represents a search for the '*right*' correlation between relative equity returns and real exchange rate fluctuations.

This line of research faces two serious challenges. First, as shown convincingly by van Wincoop and Warnock (2006), the empirical correlation between excess equity returns and the real exchange rate is low, too low to explain observed equity home bias. Further, most of the fluctuations in the real exchange rate represent movements in the nominal exchange rate, so once forward currency markets are introduced, the conditional correlation between equity

[1] Some of that literature dates back to the early 1970s or 1980s. See Adler and Dumas (1983) for a survey.

[2] A chronological but non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Baxter and Jermann (1997), Baxter, Jermann and King (1998), Coeurdacier (2008), Obstfeld (2007), Kollmann (2006), Heathcote and Perri (2007a), Coeurdacier, Kollmann and Martin (2007) and Collard, Dellas, Diba and Stockman (2007).

[3] A result also emphasized in the earlier, partial equilibrium literature. See Adler and Dumas (1983).

returns and real exchange rates disappears. This casts a serious doubt on the ability of this class of models to quantitatively explain the home equity bias. Second, as shown initially by Coeurdacier (2008) and Obstfeld (2007), the equilibrium equity portfolios are extremely sensitive to the values of preference parameters. Whether the coefficient of relative risk aversion is smaller, bigger than or equal to unity, whether domestic and foreign goods are substitute or complements, equity portfolios can exhibit home, foreign, or no bias. In other words, this class of models predict delivers equity portfolios that are *unstable*.

This paper addresses both issues simultaneously. We argue that many of the results in the previous literature are not robust to the introduction of bonds denominated in different currencies. Of course, bonds are redundant in the previous set-up since risk-sharing is locally efficient with equities only. This creates an obvious and uninteresting indeterminacy. This indeterminacy is lifted once we allow for additional sources of risk that perturbates equity returns, bond returns, and nonfinancial income. That asset returns and income are subject to more than one source of uncertainty strikes us as eminently realistic. This additional risk factor can take many forms that cover many cases of interest: redistributive shocks, fiscal shocks, investment shocks, preference shocks, nominal shocks..... In presence of these additional risks, locally-efficient risk sharing will typically require simultaneous holdings of equities and bonds.

The important economic insight here is that in many models of interest, equilibrium relative bond returns are strongly positively correlated with the real exchange rate. As a consequence, it is optimal for investors to use bond positions to hedge real exchange rate risks. All that will be left for equities is to hedge the impact of additional sources of risk on their total wealth. Of course, the precise form of the additional sources of risk matters for optimal portfolio holdings. We explore this question systematically using a simple extension of Coeurdacier (2008)'s model. We confirm our intuition and find that the optimal equity portfolio takes an extremely simple expression. First, unlike the previous literature, optimal equity holdings do not depend on the correlation between equity returns and the real exchange rate. Moreover, this optimal equity portfolio does not depend upon the preferences of the representative household and is therefore *stable*. Equivalently, optimal equity positions coincide with the equity positions of a *log-investor* who doesn't care about hedging the real exchange rate risk.

This simple results has important empirical implications. First, since equity positions are not driven by real exchange rate risk, home equity bias can only arise from hedging demands *other* than the real exchange rate. This simultaneously validates van Wincoop and Warnock (2006)'s result and establishes its limits. In particular, we show that home equity bias arises if the correlation between non-financial return and equity return, *conditional on bond returns*, is negative (a generalization of both Baxter et al. (1998), and Heathcote and Perri (2007b)).[4]

[4]In independent work, Engel and Matsumoto (2006) develop similar results in a specific model with nominal rigidities. The february 2008 version of their paper, available at

Is this case relevant in practice? The answer is yes. We show that bond returns hedge real exchange rate risk in equilibrium when the additional source of risk represents redistributive shocks, fiscal shocks, investment shocks, or nominal shocks in the presence of price rigidities. The polar case is one where bond returns do not provide a perfect hedge for fluctuations in the (welfare-based) real exchange rate. This arises in two situations: in the presence of preference/variety shocks similar to Coeurdacier et al. (2007) or Pavlova and Rigobon (2003), with nominal shocks as in Lucas (1982), or in Obstfeld (2007)'s version of Engel and Matsumoto (2006)'s sticky price model. In both cases, the new source of risk simply perturbates bond returns, leaving equities, consumption expenditures and non-financial income unchanged. It is then optimal not to hold bonds in equilibrium, which brings us back to the results of the equity-only model.

In the presence of taste/quality/variety shocks, our results break down for the following reason: total consumption expenditures vary with the welfare-based real exchange rate, while bond returns vary with the real exchange rate measured by the statistical agency. Both exchange rates move in opposite directions in response to a positive preference shock that increases the demand for domestic goods: the unit price of domestic goods increases (a real appreciation of the measured real exchange rate) while the demand-adjusted price declines (a depreciation of the welfare-based real exchange rate). Hence relative bonds do not provide a good hedge against fluctuations in the relevant relative price. In the context of nominal shocks, nominal bonds (as opposed to real bonds) allow perfect hedging of a nominal shocks, with no effect on the real allocation of resources.

While theoretically restoring the results from the earlier literature, we argue that these two additional sources of shocks are unlikely to be too relevant in practice. First, we observe that nominal and real bonds returns are strongly correlated in industrial economies, limiting the extent to which nominal bonds are unable to hedge fluctuations in total nominal expenditures. Second, to the extent that welfare-based real exchange rates differ from actual ones, we claim that these shocks cannot account for the home-equity bias. We establish the argument a contrario in two steps. First, we argue that for these shocks to be consistent with home equity bias requires a positive correlation between equity returns and the *unobserved* welfare-based real exchange rate. But, and this is the second step in the argument, if risks are (locally) efficiently shared, the unobserved welfare-based real exchange rate is related to *observed* consumption expenditures through the well-known Backus and Smith (1993) condition. Generalizing the results of van Wincoop and Warnock (2006), we show that the correlation between equity returns and consumption expenditures is too low for reasonable values of the coefficient of relative risk aversion. Consequently, these types of shocks cannot play a substantive role in explaining observed equity portfolios. Equivalently, we show that the welfare-based real exchange rates recovered under the assumption or efficient risk sharing are very correlated with observed real exchange rates, under reasonable assumptions about the value of the coefficient of relative risk aversion.

http://www.ssc.wisc.edu/~cengel/working_papers.htm also draws a similar connection on the impact of forward trades, or bond trading on optimal equity positions, and on the importance of nontradable risks, conditional on bond returns, for optimal equity positions.

We evaluate the robustness of our results to two extensions. First, we introduce non-traded goods as in Obstfeld (2007) and Collard et al. (2007). In presence of non-traded goods, real bonds still load on the real exchange rate while domestic equities (in traded and nontraded goods) still hedge the remaining sources of risks. We show that the overall home equity bias (across traded and non-traded equities) is independent of preferences. However, the optimal holdings of traded and non-traded domestic equity depend upon their hedging properties of movements in the terms of trade. Second, we allow for multiple sources of risks, effectively making markets incomplete, using Devereux and Sutherland (2006) and Tille and van Wincoop (2007)'s local methods of solving for portfolios in incomplete market settings.

The model also provides tight predictions about equilibrium bond holdings. These reflect the optimal hedge for fluctuations in real exchange rates, as well as a hedge for the implicit real exchange rate exposure arising from equilibrium equity holdings. This allows us to establish two results. First, we show that while these bond portfolios typically vary with investors' preferences, they do so smoothly. In other words, the portfolio instability of earlier models is not simply transferred to bond portfolios. Second, the model predicts that a country's bond position in it's own currency falls as the home equity bias increases. The reason is that an increase in domestic equity holdings increases the implicit domestic currency exposure. Investors optimally undo this exposure by shorting the domestic currency bond. The overall domestic bond position reflects the balance of these two effects. We find that for plausible values, it is possible for a country to be short or long in its own debt, i.e. to have short or long domestic currency debt positions.

The last part of the paper establishes the empirical relevance of our theory. We use quarterly data on equity, bond and non-financial returns for the G-7 countries since 1980 to estimate the parameters of the models. We find that the model predicts short positions in domestic currency bonds, and generates reasonable estimates of home equity bias for the US, Japan and Canada.

Section 2 follows Coeurdacier (2008) and develops the basic model with equities only. Section 3 constitutes the core of the paper. It introduces bonds and an additional source of risk, then characterizes the efficient equity and bond positions under different risk structures. Section 4 extends the model to non-tradables and incomplete markets. Finally, section 5 presents our empirical results.

2 A Benchmark Model

2.1 Goods and preferences

Consider a two-period ($t = 0, 1$) endowment economy similar to Coeurdacier (2008). There are two symmetric countries, Home (H) and Foreign (F), each with a representative household. Each country produces one tradable good. Agents consume both goods with a preference towards the local good. In period $t = 0$, no output is produced and no consumption takes place, but agents trade financial claims (stocks and bonds). In period $t = 1$, country i

receives an exogenous endowment y_i of good i. Countries are symmetric and we normalize $E_0(y_i) = 1$ for both countries, where E_0 is the conditional expectation operator, given date $t = 0$ information. Once stochastic endowments are realized at period 1, households consume using the revenues from their portfolio chosen in period 0 and their endowment received in period 1.

The country i household has the standard CRRA preferences, with a coefficient of relative risk aversion σ:

$$U_i = E_0 \left[\frac{C_i^{1-\sigma}}{1-\sigma} \right], \tag{1}$$

where C_i is an aggregate consumption index in period 1. For $i, j = H, F$, C_i is given by:

$$C_i = \left[a^{1/\phi} c_{ii}^{(\phi-1)/\phi} + (1-a)^{1/\phi} c_{ij}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}; \text{ with } i \neq j \tag{2}$$

where c_{ij} is country i's consumption of the good from country j at date 1. ϕ is the elasticity of substitution between the two goods and $a > 1/2$ represents preference for the home good (mirror-symmetric preferences).

The ideal consumer price index that correspond to these preferences is for $i = H, F$:

$$P_i = \left[a p_i^{1-\phi} + (1-a) p_j^{1-\phi} \right]^{1/(1-\phi)}; \text{ with } i \neq j \tag{3}$$

where p_i denotes the price of the country $i's$ good in terms of the numeraire.

Resource constraints are given by:

$$c_{ii} + c_{ji} = y_i; \text{ with } i \neq j \tag{4}$$

We denote Home terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good, by q:

$$q \equiv \frac{p_H}{p_F} \tag{5}$$

An increase in q represents an improvement Home's terms of trade.

2.2 Financial markets

Trade in stocks and bonds occurs in period 0. In each country there is one stock à la Lucas (1982). A share δ of the endowment in country i is distributed to stockholders as dividend, while a share $(1-\delta)$ is not capitalized and is distributed to households of country i. At the simplest level, one can think of the share $1-\delta$ as representing 'labor income', but more general interpretations are also possible. At a generic level, $1-\delta$ represents the share of output that cannot be capitalized into financial claims. This could be due to domestic financial frictions, capital income taxation or poor property right enforcement. In our symmetric setting, δ is common to both countries.[5] The supply of each type of share is normalized at unity. We

[5]See Caballero, Farhi and Gourinchas (2008) for a model where δ differs across countries.

assume also that agents can trade a CPI-indexed bond in each country denominated in the composite good of country i. Buying one unit of the Home (Foreign) bond in period 0 gives one unit of the Home composite (Foreign) good at $t = 1$. Both bonds are in zero net supply.

Each household fully owns the local stock of tradable and the local stock of non-tradable, at birth, and has zero initial foreign assets. The country i household thus faces the following budget constraint at $t = 0$:

$$p_S S_{ii} + p_S S_{ij} + p_b b_{ii} + p_b b_{ij} = p_S, \quad \text{with} \quad j \neq i \tag{6}$$

where S_{ij} is the number of shares of stock j held by country i at the end of period 0, and b_{ij} represents claims (held by i) to future unconditional payments of the good j. p_S is the share price of both stocks, identical due to symmetry.[6]

Market clearing in asset markets for stocks and bonds requires:

$$S_{ii} + S_{ji} = 1; \;\; b_{ii} + b_{ji} = 0; \;\; \text{with } i \neq j \tag{7}$$

Symmetry of preferences and distributions of shocks implies that equilibrium portfolios are symmetric: $S_{HH} = S_{FF}$, $b_{HH} = b_{FF}$, and $b_{FH} = b_{HF}$. In what follows, we denote a country's holdings of local stock by S, and its holdings of bonds denominated in its local composite good by b. The vector $(S; b)$ thus describes international portfolios. $S > \frac{1}{2}$ means that there is equity home bias on stocks, while $b < 0$ means that a country issues bonds denominated in its local composite good, and simultaneously lends in units of the foreign composite good.

2.3 Characterization of world equilibrium

We characterize first the equilibrium with locally complete markets. As shown below, markets are locally complete in our model when the number of shocks is at least equal to the number of assets. In a world with just endowment shocks, markets will be complete but portfolios will be indeterminate (i.e. the number of assets is larger than the dimension of the shocks).

2.3.1 Efficient consumption and relative prices

After the realization of uncertainty in period 1, the representative consumer in country i maximizes $\frac{C_i^{1-\sigma}}{1-\sigma}$ subject to a budget constraint (for $j \neq i$):

$$P_i C_i = p_i c_{ii} + p_j c_{ij} \leq I_i \qquad (\lambda_i)$$

where I_i represent the (given) total income of the representative agent in country i and λ_i is the Lagrange-Multiplier associated with the budget constraint.
The intratemporal equilibrium conditions are as follows:

$$c_{ii} = a \left(\frac{p_i}{P_i}\right)^{-\phi} C_i; \; c_{ij} = (1-a) \left(\frac{p_j}{P_i}\right)^{-\phi} C_i; \;\; \text{with } i \neq j \tag{8}$$

[6] Bond prices are also identical due to symmetry.

Using equations (8) for both countries and market-clearing conditions for both goods (4) gives:

$$q^{-\phi}\Omega_a\left[(\frac{P_F}{P_H})^{\phi}\frac{C_F}{C_H}\right]=\frac{y_H}{y_F} \tag{9}$$

where $\Omega_u(x)$ is a continuous function of two variables (u,x) such that: $\Omega_u(x)=\frac{1+x(\frac{1-u}{u})}{x+(\frac{1-u}{u})}$.

2.3.2 Budget constraints

Recall that each household holds shares S and $1-S$ of local and foreign stocks, while b denotes her holding of bonds denominated in her local good; also, stock $j's$ dividend is p_jy_j. The period 1 budget constraints of countries H and F are thus:

$$P_iC_i = S\delta p_iy_i + (1-S)\delta p_jy_j + P_ib - P_jb + (1-\delta)p_iy_i; \text{ with } i\neq j \tag{10}$$

where the last term represents non-financial income.

These constraints imply:

$$P_HC_H - P_FC_F = [\delta\,(2S-1)+(1-\delta)](p_Hy_H - p_Fy_F) + 2b(P_H-P_F) \tag{11}$$

which says that the difference between countries' consumption spending equals the difference between their incomes.

2.3.3 Log-linearization of the model and locally complete markets.

Henceforth, we write $y\equiv y_H/y_F$ to denote relative outputs in both countries. We log-linearize the model around the symmetric steady-state where y equal unity, and use Jonesian hats ($\widehat{x}\equiv log(x/\overline{x})$) to denote the log-deviation of a variable x from its steady state value $\overline{x}$.

The log-linearization of the Home country's real exchange rate $RER\equiv P_H/P_F$ gives:

$$\widehat{RER}=\widehat{\frac{P_H}{P_F}}=(2a-1)\widehat{q}. \tag{12}$$

As shown in the appendix, if 1) the dimensionality of the shocks equals the number of available independent assets and 2) shock innovations do not leave asset pay-off unaffected, one can replicate the efficient consumption allocation up-to the first order. This implies that, abstracting from second-order deviations (terms homogenous to $\widehat{x}^2$), the equilibrium allocation is the one that prevails in a world with effectively complete markets. This property turns out to simplify the portfolio problem: one just needs to find the portfolio that replicates *locally* the efficient allocation.[7] In particular, when these two conditions are verified, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the following familiar Backus and Smith (1993) condition (in log-linearized terms):

$$-\sigma(\widehat{C_H}-\widehat{C_F})=\widehat{\frac{P_H}{P_F}}=(2a-1)\,\widehat{q} \tag{13}$$

[7] In the appendix, we show that such a portfolio is the one chosen by a utility-maximizing investor.

Hence, any shock that raises Home aggregate consumption relative to Foreign aggregate consumption must be associated with a Home real exchange rate depreciation. Thus, under (locally) complete markets, the log-linearization of (9) gives:

$$\widehat{y} = -\phi\widehat{q} + (2a-1)(\phi - 1/\sigma)\widehat{\frac{P_H}{P_F}} \tag{14}$$

Using (12), (14) implies:

$$\widehat{y} = -\lambda\widehat{q} \tag{15}$$

where $\lambda \equiv \phi\left(1-(2a-1)^2\right) + \frac{(2a-1)^2}{\sigma}$. Note that $\lambda > 0$ as $1/2 < a < 1$. A relative increase in the supply of the home good ($\hat{y} > 0$) is always associated with a worsening of the terms of trade ($\hat{q} < 0$) with an elasticity $-1/\lambda$. Note that without home bias in preferences ($a = \frac{1}{2}$), λ is simply the elasticity of substitution between Home and Foreign goods (ϕ).

Note also that from (15), we get that relative equity returns $\widehat{R}_e$ (relative dividends) are equal to:

$$\widehat{R}_e = \widehat{q} + \widehat{y} = (1-\lambda)\widehat{q} \tag{16}$$

When $\lambda > 1$, an increase in relative output is associated with an improvement in relative equity returns. Conversely, when $\lambda < 1$, an increase in Home relative output is associated with a relative decrease in Home dividends. This happens when either $\phi < 1$ or the preference for the home good is sufficiently strong.[8]

We next log-linearize equation (11); using (13) we obtain:

$$\widehat{P_H C_H} - \widehat{P_F C_F} = \left(1 - \frac{1}{\sigma}\right)(2a-1)\,\widehat{q} = \left[\delta\,(2S-1) + (1-\delta)\right](\widehat{q} + \widehat{y}) + 2b\,(2a-1)\,\widehat{q} \tag{17}$$

The first equality shows the Pareto optimal reaction of relative consumption spending to a change in the welfare-based real exchange rate. This reaction depends on the coefficient of relative risk aversion σ. In a Pareto-efficient equilibrium, a shock that appreciates the (welfare-based) real exchange rate of country H, induces an increase in country H relative consumption expenditures when $\sigma > 1$ (as assumed in the analysis here). The risk-sharing condition (13) shows that when the (welfare based) real exchange rate appreciates by 1%, then relative aggregate country H consumption (C_H/C_F) decreases by $1/\sigma$ %. Hence, efficient relative consumption spending by H ($P_H C_H / P_F C_F$) increases by $(1 - \frac{1}{\sigma})$%. The expression to the right of the second equal sign in (17) shows the change in country H relative income (compared to the income of F) necessary to obtain the Pareto-optimal allocation. Given $\sigma > 1$, the efficient portfolio has to be such that a real appreciation (welfare based) is associated with an increase in relative spending and income.

2.4 The Instability of Optimal Equity Portfolios

Financial markets are locally complete when there exists a portfolio (S, b) such that (15) and (17) both hold for arbitrary realizations of the relative shocks $\widehat{y}$. Clearly, here portfolios

[8] Specifically, when $\phi > 1$ and $\sigma > 1$ (the empirically plausible case), we need: $a > \frac{1}{2}\left[1 + \left(\frac{1-\phi}{\frac{1}{\sigma}-\phi}\right)^{1/2}\right]$

are undetermined since the dimension of 'relative' shocks exceeds the dimension of 'relative assets'. Much of the literature focuses on the case where bonds are not available and efficient risk sharing is implemented with equities only (Coeurdacier (2008), Obstfeld (2007), Kollmann (2006)).

Substituting $b = 0$ into (17), we obtain the optimal equity portfolio position:

$$S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}{\delta(\lambda - 1)}\right] \tag{18}$$

When $\delta = 1$, this expression coincides with the equilibrium equity position of Coeurdacier (2008) and Obstfeld (2007). In the more general case where $\delta < 1$, the optimal equity portfolio has two components. The first term inside the brackets represents the position of a log-investor ($\sigma = 1$). As in Baxter and Jermann (1997), the domestic investor is already endowed with an implicit equity position equal to $(1 - \delta)/\delta$ through non-financial income. Offsetting this implicit equity holding and diversifying optimally implies a position $S = (2\delta - 1)/2\delta < 1/2$ for $\delta < 1$. As is well known, this component of the optimal portfolio impart a *foreign* equity bias.

The second component of the optimal equity portfolio is a hedge against real exchange rate fluctuations. It only applies when $\sigma \neq 1$, i.e. when total consumption expenditures fluctuate with the real exchange rate. Looking more closely at the structure of this hedging component calls for a number of observations. First, this hedging demand is a complex and non-linear function of the structure of preferences summarized by the parameters σ, ϕ and a. As Obstfeld (2007) and Coeurdacier (2008) note, for reasonable parameter values, this hedging demand can contribute to home equity bias only when $\lambda < 1$, i.e. when the terms of trade impact of relative supply shocks is large.[9]

This model faces three main problems. First, the non-linearity in (18) implies that small changes in preferences can have a large impact on this hedging demand. This is most apparent if we consider the optimal portfolio in the neighborhood of $\lambda = 1$. As figure 1 makes clear, small and reasonable changes in σ, ϕ or a have a large and disproportionate impact on optimal portfolio holdings, from large foreign bias ($S < 0$) to unrealistically high domestic bias ($S > 1$). To the extent that we don't know precisely what value these parameters take, one is left with the unescapable conclusion that this model does not provide enough guidance to pin down equity portfolios, or a-fortiori, explain the home portfolio bias. As emphasized by Obstfeld (2007), and as the figures make clear, things are even worse since the benchmark model cannot deliver home equity holdings between $S = 1 - 1/2\delta < 0.5$ and $S = 1$ thus excluding the relevant empirical range.

Second, the model also implies that equity pay-offs are perfectly correlated with terms of trade and real exchange rates in all states of nature (see equation 16). This feature is quite unrealistic, as argued by van Wincoop and Warnock (2006). Indeed, in the case of the US, these authors show that relative equity returns are poorly correlated with the real exchange rate, and unable to account for the observed home portfolio bias.

[9] When $\lambda = 1$, this component is indeterminate since the relative return on equities is independent of the real exchange rate (and constant). This case is similar to Cole and Obstfeld (1991).

Third, given the constant sharing rule δ, the model also predicts a perfect correlation between equity returns and non-financial income. While this correlation might be positive, it is hard to believe that it is perfect and many papers found it pretty low (see Fama and Schwert (1977) for earlier work and Bottazzi, Pesenti and van Wincoop (1996), Julliard (2003, 2004), Lustig and Nieuwerburgh (2005)).[10]

3 Equity and Bond Equilibrium Portfolios

This paper's main objective is to show that optimal equity portfolios are in fact stable and well defined once we introduce bonds. Of course, introducing bonds in the model of the previous section yields an indeterminacy since markets are already locally complete. We approach this issue by adding one additional source of uncertainty in the model. With one additional shock, and one additional asset (the bonds), the markets remain locally complete and we can use an extension of the previous method to characterize optimal portfolio holdings. This calls for three remarks. First, since relative endowment or supply shocks are unlikely to represent the only source of uncertainty in the economy, adding other sources of uncertainty is quite realistic and general. Second, we maintain in this section the assumption that markets are locally complete. We do this by adding only one additional source of uncertainty. This is mainly for tractability. Section 4.2 will cover the more general case where markets are incomplete (even locally). Lastly, going from the general to the particular, we show how to map our results in specific models where the additional source of risk arises from redistributive shocks, shocks to government expenditures or investment, from demand shocks, or from nominal shocks.

3.1 A general representation with an additional source of risk

Assume that a shock ε_i affects country i in period $t = 1$. Again, denote $\varepsilon = \varepsilon_H/\varepsilon_F$ the relative shock and assume $E_0(\varepsilon) = 1$. The only assumption we make is that the stochastic properties of ε_i are symmetric across countries and that $\widehat{\varepsilon} = \ln \varepsilon$ is not perfectly correlated with $\widehat{y}$. To characterize optimal portfolio, we only need to specify how this additional shock impacts equity returns $\hat{R}_e$, bond returns $\hat{R}_b$ and non-financial income $\hat{w}$. That is, we assume the following:

$$\widehat{R}_e = (1-\bar{\lambda})\widehat{q} + \gamma_e \widehat{\varepsilon} \tag{19}$$

$$\widehat{R}_b = (2a-1)\widehat{q} + \gamma_b \widehat{\varepsilon} \tag{20}$$

$$\widehat{w} = (1-\bar{\lambda})\widehat{q} + \gamma_w \widehat{\varepsilon} \tag{21}$$

where $\bar{\lambda}$ is a positive number ($\bar{\lambda}$ is model dependant but will be closely related to the previous λ and reflect preference parameters; see the examples below). The parameters γ_k, that can be positive or negative, represents the impact of $\hat{\varepsilon}$ on equity returns, bond returns and non-financial income. Different models will have different implications on what γ_k and

[10] See Baxter and Jermann (1997) for an opposite view.

$\bar{\lambda}$ should be, and will be explored in more details in the next section. For this section, the only restriction we impose on the model is $\gamma_e \neq 0$. We focus on this case as it will be the relevant one empirically but the case $\gamma_e = 0$ will be explored in details in section 3.2.2.

3.1.1 Equilibrium Portfolios

Under the assumption -verified below- that markets are *locally*-complete, the budget constraint (17) can be rewritten as follows:

$$(1-\frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,((1-\bar{\lambda})\widehat{q}+\gamma_e\widehat{\varepsilon})+(1-\delta)((1-\bar{\lambda})\widehat{q}+\gamma_w\widehat{\varepsilon})+2b\,((2a-1)\widehat{q}+\gamma_b\widehat{\varepsilon}) \tag{22}$$

Financial markets are still *locally*-complete since one can always find a portfolio (S, b) such that (22) holds for arbitrary realizations of the shocks $\widehat{y}$ and $\widehat{\varepsilon}$. Clearly, here portfolios are uniquely determined since the dimension of 'relative shocks' equals the dimension of 'relative assets'. The unique portfolio (S^*, b^*) that satisfies (22) for all realization of shocks is (for $\gamma_e \neq 0$):

$$\begin{aligned} b^* &= \frac{1}{2}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)(\gamma_w/\gamma_e-1)}{(2a-1)-\gamma_b/\gamma_e\left(1-\bar{\lambda}\right)} \\ S^* &= \frac{1}{2}\left[1-\frac{1-\delta}{\delta}\frac{\gamma_w}{\gamma_e}-\frac{\gamma_b}{\gamma_e}\frac{2b}{\delta}\right] \end{aligned} \tag{23}$$

3.1.2 Equilibrium Loadings

It is informative to rewrite the equilibrium bond and equity portfolios in terms of the equilibrium asset return loadings on the real exchange rate $(2a-1)\,\hat{q}$ and on non-financial income $\hat{w}$. To do this, let's first manipulate equations (19)-(21) to eliminate $\hat{\varepsilon}$:

$$\begin{aligned} R\hat{E}R &= (2a-1)\,\hat{q} = (2a-1)\,\psi\hat{R}_b-(2a-1)\,\psi\frac{\gamma_b}{\gamma_e}\hat{R}_e \qquad (24) \\ &\equiv \beta_{RER,b}\hat{R}_b+\beta_{RER,e}\hat{R}_e \\ \hat{w} &= \left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_w}{\gamma_e}\right)\psi\hat{R}_b+\left(\frac{\gamma_w}{\gamma_e}-\left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_w}{\gamma_e}\right)\psi\frac{\gamma_b}{\gamma_e}\right)\hat{R}_e \qquad (25) \\ &\equiv \beta_{w,b}\hat{R}_b+\beta_{w,e}\hat{R}_e \end{aligned}$$

where $\psi = \left[(2a-1)-\left(1-\bar{\lambda}\right)\gamma_b/\gamma_e\right]^{-1}$.

The advantage of the formulation above is twofold. First, the loadings $\beta_{w,i}$ and $\beta_{RER,i}$ have the interpretation of conditional covariance-variance ratios. It is immediate to see that

$$\beta_{w,i} = \frac{cov_{\hat{R}_j}\left(\hat{w},\hat{R}_i\right)}{var_{\hat{R}_j}\hat{R}_i};\beta_{RER,i} = \frac{cov_{\hat{R}_j}\left(R\hat{E}R,\hat{R}_i\right)}{var_{\hat{R}_j}\hat{R}_i}$$

where $i, j = e, b$. Second, since these loadings are expressed in terms of observables, they have an intuitive empirical counterpart, independently of the specifics of the model and of the source of the shock $\hat{\varepsilon}$. They can be readily estimated from a multivariate regression.

We can now express the optimal portfolios in terms of these equilibrium loadings:

$$\begin{aligned} b^* &= \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}(1-\delta)\beta_{w,b} \\ S^* &= \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\beta_{w,e} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{RER,e}\right] \end{aligned} \tag{26}$$

Let's consider the equilibrium bond portfolio first. Equation (26) indicates that it contains two terms. The first term represents the hedging of real exchange rate risk through bond holdings. When $\sigma > 1$, the household's relative consumption expenditures increase when the real exchange rate appreciates. If domestic bonds deliver a high return precisely when the currency appreciates, then domestic bonds constitute a good hedge against real exchange rate risk. This component disappears for the log investor ($\sigma = 1$). Since we expect the conditional correlation between relative bond returns and real exchange rates to be positive, this term should be positive. The second term represents the hedging of non-financial income risk. When domestic bonds and relative non-financial income are conditionally positively correlated ($\beta_{w,b} > 0$), investors want to short the domestic bond to hedge the implicit exposure from their non-financial income. This term disappears when there is no non-financial income ($\delta = 1$). Equation (26) indicates that investors will go long or short in their domestic bond holdings depending on the strength of these two effects.

Let's now turn to the equilibrium equity position in (26). The first term inside the brackets represents the symmetric risk sharing equilibrium of Lucas (1982): $S = 1/2$. The second term inside the brackets determines how this symmetric equilibrium is affected when non-financial income and equity returns are correlated. In the case of Baxter and Jermann (1997), $\beta_{w,e} = 1$ and the equilibrium equity position becomes $S = (2\delta - 1)/2\delta < 1$. In general, the correlation between non-financial income and equity returns is less than perfect. In particular, home equity bias can arise if $\beta_{w,e} < 0$. Importantly for the empirical exercises we conduct below, what matters is the covariance-variance ratio between non-financial income and equity returns *conditional on the bond returns*. To our knowledge, this condition has not yet been investigated in the literature.[11]

Finally, the last term inside the brackets is the van Wincoop and Warnock (2006) term that has been emphasized in the literature so far. It represents the demand for domestic equity that arises from the correlation between equity returns and the real exchange rate, *conditional on the bond returns*, $\beta_{RER.e}$. If this correlation is positive, domestic equities represent a good hedge against movements in real exchange rates that affect relative consumption expenditures when $\sigma \neq 1$. We know from their paper that this correlation is close to zero, especially after we condition on the bond returns.

[11] Engel and Matsumoto (2006) also note that this is the relevant condition in presence of bond holdings, or forward exchange contracts.

To summarize, our model indicates that equity home bias can arise, even if equities are a poor hedge for exchange rate risk ($\beta_{RER,e} = 0$), as long as non-financial income and equity returns are negatively conditionally correlated: $\beta_{w,e} < 0$. The model can also potentially account for short positions in domestic bond market if we find that $(1 - 1/\sigma)\,\beta_{RER,b} < (1 - \delta)\,\beta_{w,b}$ for plausible values of the intertemporal elasticity of substitution σ.

The important insight of van Wincoop and Warnock (2006) was to note that any general equilibrium model must be consistent with the partial equilibrium implications of the portfolio problem. Going back to equation (24), we see that estimates of $\beta_{RER.e} = 0$ require that $\gamma_b/\gamma_e = 0$, i.e. bond returns are unaffected by the additional source of risk $\hat{\varepsilon}$. In this case, of great importance empirically, equilibrium portfolio holdings simplify substantially. Substituting $\gamma_b/\gamma_e = 0$ in (23), we obtain:

$$
\begin{aligned}
S^* &= \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\frac{\gamma_w}{\gamma_e}\right] \\
b^* &= \frac{1}{2}\left(1 - \frac{1}{\sigma}\right) + \frac{1}{2}\left(2a-1\right)^{-1}\left(\bar{\lambda} - 1\right)\left(1 - \delta\right)\left(1 - \frac{\gamma_w}{\gamma_e}\right)
\end{aligned}
\tag{27}
$$

Let's concentrate on each term in turn. The optimal equity portfolio (27) presents a number of interesting characteristics. First, and contrary to most of the literature, it does not depend on the 'tradability' of goods in consumption, as measured by a.[12] Second, it is also independent of preference parameters such as the elasticity of substitution across goods ϕ or the degree of risk aversion σ. Hence the complex and non-linear dependence of optimal equity portfolios as a function of preferences *disappears* once we introduce trade in bonds and a genuine additional source of uncertainty that impacts both equity and non-financial income. This independence of equity positions from preference parameters implies that the optimal equity holdings would be the same for a log-investor ($\sigma = 1$). Hence our result has the simple interpretation that the optimal equity portfolio is the portfolio of the log-investor when $\gamma_b/\gamma_e = 0$.

Since we know that log-investors do not care about fluctuations in the real exchange rate, what determines optimal equity holdings is *not* the correlation between equity returns and the real exchange rate. Instead, optimal equity holdings insulate total income (both financial and non-financial) from the $\widehat{\varepsilon}$ shocks only. Conditional on relative bond returns, the domestic investor is endowed with an implicit equity exposure through the impact of the $\hat{\varepsilon}$ shock on nonfinancial income, equal to $\gamma_w\left(1-\delta\right)/\delta$. Offsetting this implicit conditional equity position and diversifying optimally implies a position $S^* = 0.5\left(1 - \gamma_w/\gamma_e\left(1-\delta\right)/\delta\right)$. When $\gamma_w/\gamma_e = 0$, so that the implicit conditional exposure is zero, the optimal equity portfolio is perfectly diversified: $S^* = 0.5$. More generally, for equity portfolio holdings to exhibit home bias requires a negative γ_w/γ_e, i.e. a negative covariance between non-financial

[12] For models where the equity portfolio share depends on the preference for the home good or trade costs in goods, see Coeurdacier (2008), Kollmann (2006), Obstfeld (2007). See section 4 for an extension of this result with non-tradable goods.

and financial income, conditional on bond returns. This result echoes the partial equilibrium finding above since when $\gamma_b/\gamma_e = 0$, we obtain that $\beta_{w,e} = \gamma_w/\gamma_e$.

A couple of remarks are necessary at this stage. First, as we will show shortly, a negative γ_w/γ_e arises naturally when the additional shock reallocates income between its financial and nonfinancial components. This occurs with redistributive shocks but also with shocks to government/investment expenditures (as in Heathcote and Perri (2007b)). Second, and more importantly, it is obvious that this invalidates the results of much of the previous literature that emphasized the hedging properties of equity returns for real exchange rate risk. In particular, in our model, home portfolio bias can arise independently of the correlation between equity returns and the real exchange rate. The finding that $\beta_{RER,e} = 0$, as emphasized by van Wincoop and Warnock (2006), has no bearing on the optimal portfolio holdings. Instead, equity portfolio bias arises only when $\beta_{w,e} = \gamma_w/\gamma_e < 0$, a condition that has not been looked at in the empirical literature.

Since our results are so different from the previous literature, one is entitled to wonder why the optimal equity portfolio in (27) isn't loading on the real exchange rate? After all, (19) shows that relative equity returns fluctuate with the terms of trade, or equivalently with the real exchange rate? The answer is that real exchange rate risk is best taken care of through bond holdings since the latter load perfectly on the real exchange rate, and not on the $\widehat{\varepsilon}$ shocks. Intuitively, real bond trading is equivalent here to trading in forward real exchange contracts that remove perfectly real exchange rate risk. Hence, once the $\widehat{\varepsilon}$ shocks have been hedged by equity positions, the bond portfolio will be structured such that financial and non-financial income have the appropriate exposure to real exchange rate changes.

Looking at the bond position in (27), we can decompose the optimal bond portfolio as the sum of two components. The first term on the right hand side of (27) is the optimal hedge for fluctuations in total consumption expenditures when $\sigma \neq 1$ (the term $\frac{1}{2}\left(1 - \frac{1}{\sigma}\right)$). Investors more risk averse than the log-investor want to have a positive exposure of their incomes to real exchange rate changes. They do so by increasing their holding of Home bonds (and decreasing their holdings of Foreign bonds) since Home bonds have higher pay-offs when the real exchange rate appreciates.[13]

The second term on the right hand side represents the bond portfolio of the log-investor (term $(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\gamma_w/\gamma_e)$). This term represents a hedge for the *implicit* real exchange rate exposure arising from the optimal equity position and non-financial income. The log-investor wants to neutralize the exposure of his total income to real exchange movements. It does so by structuring his bond portfolio such that any capital gains on financial and non-financial incomes are offset by capital losses on the bond portfolio. To understand this result, consider a combination of shocks that leads to a 1% increase in the Home terms-of-trade.[14] Given (19) and (22), relative equity returns and non-financial incomes changes

[13] This result is closely related to Adler and Dumas (1983) and Krugman (1981).

[14] Of course in this model, terms-of-trade are endogenous but it is always possible to find a combination of shocks that leads to a 1% increase in the Home terms-of-trade.

are equal to $(1-\bar{\lambda})\%$. At the optimal equity portfolio (S^*), capital gains/losses on equity positions and non-financial incomes for the Home investor (relative to the Foreign one) are equal to $(\bar{\lambda}-1)[\delta\,(2S^*-1)+(1-\delta)]\% = (\bar{\lambda}-1)(1-\delta)(1-\gamma)\%$. In these states of the world, Home bond excess returns over Foreign bonds are equal to $(2a-1)\%$.[15] Then, holding $b=\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\gamma)$ Home bonds and $(-b)$ Foreign bonds generates capital gains/losses on the bond position necessary to insulate relative incomes from real exchange rate changes.

This intuition helps understand why the model predicts a specific relationship between domestic equity and bond holdings. Expressing γ_w/γ_e in terms of S^* and substituting the result into (27) one obtains:

$$\begin{aligned} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)-\frac{1}{2}\left(2a-1\right)^{-1}\left(1-\bar{\lambda}\right)\left(1-\delta+\delta\left(2S^*-1\right)\right) \qquad (28) \\ &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)-\frac{1}{2}\frac{\beta_{w,b}}{1-\beta_{w,e}}\left(1-\delta+\delta\left(2S^*-1\right)\right) \end{aligned}$$

The slope of this relationship is controlled by the sign of $\beta_{w,b}/\left(1-\beta_{w,e}\right)$, which can be estimated empirically. In the empirically plausible case where $\beta_{w,b}>0$ and $\beta_{w,e}<1$, we would expect a negative relationship between home equity bias $(2S^*-1)$ and domestic bond holdings: the investor optimally hedges the real exchange risk implicit in holdings of domestic equity holdings and nonfinancial income, by shorting the domestic currency bond.

Finally, notice that the bond portfolio depends upon preference parameters σ, a and potentially $\bar{\lambda}$ in a complex and non-linear way. A natural question then, is whether this bond portfolio inherits the instability of the equity portfolio of the previous model. To answer this question requires that we flesh out some of the details of the model, as we do next.

3.2 Examples

We want to show how fully specified general equilibrium models are nested in the reduced-form model given by the system of equations (19), (20) and (21). To do so, we need to specifiy the additional source of uncertainty necessary to pin-down bond and equity portfolios. We provide two series of polar cases: the first one corresponds to the case $\gamma_b=0$ and $\gamma_e\neq 0$, i.e relative bond returns perfectly load on the real exchange rate; the second one corresponds to the case of $\gamma_b\neq 0$ and $\gamma_e=0$, i.e relative equity returns load perfectly on the real exchange rate but relative bond returns do not. While these are polar cases, we believe they illustrate well one of the key message of the paper: depending on which financial asset is used to hedge real exchange rate fluctuations, conclusions in terms of portfolios are drastically different. The empirical part will then provide strong evidence that the first case is the most relevant.

[15] Since the real exchange rate appreciation following a 1% increase in the Home terms-of-trade is $(2a-1)\%$.

3.2.1 Case I: Relative bond returns load perfectly on the real exchange rate ($\gamma_b = 0$, $\gamma_e \neq 0$)

Redistributive shocks The distribution of total income between financial and non-financial income is controlled by the parameter δ. Variations in δ redistribute income from its financial to non-financial components or vice versa. If we interpret non-financial income as labor income, shocks to δ affect the labor share of total income. Fluctuations in the labor share can occur in a model where capital and labor enter into the production function with a non-unit elasticity in presence of capital *and* labor augmenting productivity shocks or in presence of *biased* technical change in the sense of Young (2004) (see also Rios-Rull and Santaeulalia-Llopis (2006)). Alternatively, movements in the labor share occur if we move away perfect competition in the goods markets and firm profits are shared between shareholders and workers based on their (stochastic) bargaining power or reservation utility.

In terms of the previous set-up, we can interpret ε_i as shocks to the share that his distributed as dividend, with $E_0(\varepsilon_i) = \delta$. One can verify that financial and non-financial incomes satisfy:

$$\widehat{R}_e = (1-\lambda)\widehat{q} + \widehat{\varepsilon} \tag{29}$$

$$\widehat{R}_b = (2a-1)\widehat{q} \tag{30}$$

$$\widehat{w} = (1-\lambda)\widehat{q} - \frac{\delta}{1-\delta}\widehat{\varepsilon} \tag{31}$$

This system of equation is a specific case of the general representation described above (equations (19), (20) and (21)) where $\gamma_e = 1$ and $\gamma_w = -\frac{\delta}{1-\delta}$.

Then, the optimal portfolio can be easily derived from (27) with $\frac{\gamma_w}{\gamma_e} = -\delta/\left(1-\delta\right)$:

$$\begin{aligned} S^* &= 1 \\ b^* &= \frac{1}{2}(1-\frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\lambda-1) \end{aligned} \tag{32}$$

The implications for portfolios are similar to Coeurdacier et al. (2007). Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. This perfectly offsets the impact of $\hat{\varepsilon}$ shocks on total income. The equity portfolio exhibits full equity home bias. Once the redistributive shocks have been hedged by holding local equity, the bond portfolio takes care of the exposure of incomes to real exchange rate movements.[16]

The bond position is negative when $\lambda < 1 - (1-\frac{1}{\sigma})(2a-1)$ and positive otherwise. A negative bond position (borrowing in domestic bonds and investing in foreign bonds) is possible only for sufficiently low values for λ. This condition echoes the condition for home equity bias in the equity only model of section 2. However, unlike (27) inspection of (32) reveals that the optimal bond positions are nicely behaved as a function of the underlying

[16]Notice that this result does not depend upon the size of the redistributive shock: even a very small amount of redistributive variation leads to full equity home bias (as long as changes in labor shares are not negligible in first-order approximation of the model).

preference parameters (for $\sigma > 1$ and $a > 0.5$). Figure 2 reports the variation in b^*. Hence, unlike equity positions in the equity only model (see figure 1), the portfolios positions vary smoothly with preferences parameters. This implies that uncertainty about the true preference parameters translates into uncertainty of the same order regarding optimal portfolio positions.

Government expenditures shocks/Investment expenditures shocks Government expenditures shocks constitute another potential source of uncertainty. They break the link between private consumption and output and can also affect revenues from both financial and non-financial incomes depending on the way fiscal expenditures are financed. This will also severe the link between the real exchange rate and relative equity returns *net of taxes*.

Assume that in each country i, a government must finance period 1 government expenditures $E_{G,i}$ equal to $P_{g,i}G_i$, where G_i is the aggregate consumption index of the government and $P_{g,i}$ is the price index for government consumption, potentially different from the price index for private consumption. G_i is stochastic and symmetrically distributed, with $E_0(G_i) = \bar{G}$.

We denote $E_G = \frac{P_{g,H}G_H}{P_{g,F}G_F}$ the ratio of Home to Foreign government expenditures and $\widehat{E_G}$ the log deviation from its steady-state symmetric value of one.

Preferences of the government are similar to that of the consumers:[17]

$$G_i = \left[a_G^{1/\phi} \left(g_{ii}\right)^{(\phi-1)/\phi} + (1-a_G)^{1/\phi} \left(g_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{33}$$

where g_{ij} is country i government's consumption of the good from country j in period 1 and $a_g > 1/2$ represents the preference for the home good of the government (mirror-symmetric preferences) that may differ from the bias in household preferences ($a_G \neq a$).

Government expenditures in country $i = \{H, F\}$ are financed through taxes on financial income (for a share δ_g), $T_{R,i} = \delta_g E_{G,i}$, and through taxes on non-financial incomes (for a share $(1-\delta_g)$), $T_i^w = (1-\delta_g)E_{G,i}$, so as to ensure budget balance in period 1.

Market-clearing conditions for both goods are now:

$$c_{ii} + c_{ji} + g_{ii} + g_{ji} = y_i. \tag{34}$$

Following similar steps as before, relative demand of Home over Foreign goods by governments ($y_G = (g_{HH} + g_{FH}) / (g_{HF} + g_{FF})$) satisfies (in log-linearized terms):

$$\widehat{y_G} = -\lambda_G \widehat{q} + (2a_G - 1)\widehat{E_G} \tag{35}$$

where $0 \leq \lambda_G = \phi(1-(2a_g-1)^2)+(2a_g-1)^2 \leq \phi$ represents the impact of fluctuations in the terms of trade on relative government consumption, after controlling for relative expenditures $\widehat{E_G}$.

[17] One can also allow for a different elasticity of substitution between Home and Foreign goods for government consumption. This extension is straightforward and does not add much substance.

Relative demand of Home over Foreign goods by consumers ($y_C = (c_{HH} + c_{FH}) / (c_{HF} + c_{FF})$) still satisfies equation (15) since the private allocation across goods has not changed:

$$\widehat{y_C} = -\lambda \widehat{q} \tag{36}$$

Equation (35) and (36) together with market clearing conditions of both goods (**??**) and (34) implies the following equilibrium on the goods market:

$$\widehat{y} = s_C \widehat{y_C} + s_G \widehat{y_G} = -\bar{\lambda} \widehat{q} + s_G (2a_G - 1) \widehat{E_G} \tag{37}$$

where s_C (resp. $s_G = 1 - s_C$) is the steady-state ratio of consumption spending (resp. government spending) over GDP and $\bar{\lambda} = s_C \lambda + s_G \lambda_G$. Note that, intuitively, efficient terms-of-trade $\widehat{q}$ are decreasing with the relative supply of goods $\widehat{y}$ (with an elasticity $1/\bar{\lambda}$) and increasing with relative government expenditure shocks (due to the presence of government home bias in preferences a_G), which act as relative demand shocks in this set-up.

This gives the following relative financial incomes and non-financial incomes (net-of-taxes)[18]:

$$\begin{aligned}
\widehat{R}_e &= (1 - \bar{\lambda}) \widehat{q} + s_G (2a_G - 1 - \frac{\delta_G}{\delta}) \widehat{E_G} && (38) \\
\widehat{R}_b &= (2a - 1) \widehat{q} && (39) \\
\widehat{w} &= (1 - \bar{\lambda}) \widehat{q} + s_G (2a_G - 1 - \frac{1 - \delta_G}{1 - \delta}) \widehat{E_G} && (40)
\end{aligned}$$

Direct inspection of the returns reveals that in general markets are complete and that the system (38)-(39)-(40) is similar to (19)-(20)-(21), where:[19]

$$\begin{aligned}
\widehat{\varepsilon} &= \widehat{E_G} \ ; \gamma_e = s_G (2a_G - 1 - \frac{\delta_g}{\delta}); \gamma_b = 0 \ ; \gamma_w = s_G (2a_G - 1 - \frac{1 - \delta_G}{1 - \delta}) && (41) \\
\frac{\gamma_w}{\gamma_e} &= \frac{2a_G - 1 - \frac{1 - \delta_G}{1 - \delta}}{2a_G - 1 - \frac{\delta_G}{\delta}} = -\frac{\delta}{1 - \delta} \left(1 - \frac{2 (1 - a_G)}{2 (1 - a_G) \delta - (\delta - \delta_G)} \right) && (42)
\end{aligned}$$

The impact of the fiscal shock on relative equity returns and non-financial incomes depends on the fluctuations in relative government expenditures $\widehat{E_G}$, as well as the government preferences for the home good a_G, the steady state share of government expenditures in output $(1 - s_C)$, and the relative fiscal incidence of the shocks δ_G/δ. Importantly, the parameter (γ_w/γ_e) does not depend upon the preferences of the representative household, only on the preferences of the government in terms of consumption (a_G) and taxation (δ_G).

[18] Implicitly, we assume that taxes are raised on capital (profits) and non-financial (labor) incomes and not on bond returns as we wish to illustrate a case where $\gamma_b = 0$.

[19] The exception is the very peculiar case where $2a_g = 1 + \delta_g/\delta$. In that case, government expenditures shocks do not modify equity returns conditionally on bond returns and then cannot be hedged perfectly. This rules out the case where government expenditures fall entirely on the domestic good ($a_G = 1$) and the fiscal incidence is equally distributed on financial and non-financial income ($\delta_G = \delta$) .

In this set-up, (22) needs to be slightly modified since private consumption in steady-state does not equal total consumption. (22) can be rewritten as follows where γ_e and γ_w are defined in (41):

$$s_C(1-\frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,((1-\bar{\lambda})\widehat{q}+\gamma_e\widehat{\varepsilon})+(1-\delta)((1-\bar{\lambda})\widehat{q}+\gamma_w\widehat{\varepsilon})+2b(2a-1)\widehat{q} \quad (43)$$

Equilibrium portfolios are given by:

$$\begin{aligned} S^* &= \frac{1}{2}\left(1-\frac{\gamma_w}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \quad (44) \\ b^* &= \frac{1}{2}s_C(1-\frac{1}{\sigma})+\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\frac{\gamma_w}{\gamma_e}) \end{aligned}$$

Once again, portfolios are uniquely determined and the equity portfolio is independent from consumer preferences (ϕ, σ and a). Note that here, we have restricted ourselves to cases where the *marginal* and *average* shares of taxes on financial and non-financial income in total fiscal revenues are the same (and equal to δ_G and $1-\delta_G$ respectively). However, what matters for equity portfolios is how marginal changes in government expenditures are financed, not how they are financed on average. So δ_G must be understood as the contribution of taxes on financial income to finance a marginal increase in government expenditures.

While optimal equity portfolio are independent from household preferences, they depend on government preferences (a_G and δ_G) through γ_w/γ_e. Some specific calibrations of the parameters help to understand the equity portfolio.

When $a_G = 1$, *i.e* government expenditures are fully biased towards local goods, the equity portfolio is fully biased towards local stocks and $S = 1$.[20] The reason is simple, from (37), a 1% increase in Home government expenditures raises Home dividends and Home non-financial income before taxes by s_G%. With a portfolio fully biased towards local equity, the Home investor will have an increase of taxes of s_G%. Then, such a portfolio insulates completely consumption expenditures from changes in government expenditures (and taxes) and allow efficient risk-sharing of government expenditures shocks. Notice that in this case, government expenditures shocks act as redistibutive shocks since $\gamma_w/\gamma_e = -\delta/\left(1-\delta\right)$.[21]

For $a_G < 1$, the equity portfolio depends on the incidence of taxes. When $\delta_G = \delta$, *i.e* when increases in government expenditures fall on financial incomes proportionally to their share in gross GDP, $\gamma_w/\gamma_e = 1$ and the equity portfolio is the one of Baxter and Jermann (1997); in particular, investors exhibit foreign bias in equities.

$$S^* = \frac{1}{2}\frac{2\delta-1}{\delta} \quad (45)$$

The reason is simple. Conditionally on relative bond returns, shocks to Home government expenditures exactly decrease Home equity returns and Home labor incomes by the same

[20]This is true except for a unique knife-edge case where equity and bonds have the same pay-offs, which occurs here when $\delta_G = \delta$. In that case, the portoflio is inderterminate.

[21]Although in that case $\bar{\lambda} = s_C\lambda + (1-s_C)\lambda_G$

amount, making financial and non financial incomes perfectly correlated. Being over-exposed on government expenditures shocks due to their non financial incomes, investors will reduce their holdings of local stocks and increase their holdings of foreign stocks to share optimally government expenditures risks.

When $\delta_G = 1$, *i.e* changes in government expenditures are entirely financed by taxes on financial incomes, $\frac{\gamma_w}{\gamma_e} = \frac{2a_G-1}{2a_G-1-\frac{1}{\delta}}$ and the equity portfolio is:

$$S^* = \frac{1}{2}\left[1 + \frac{(2a_G-1)(1-\delta)}{1-\delta(2a_G-1)}\right] > \frac{1}{2} \tag{46}$$

The equity portfolio always exhibits Home bias for $a_G > \frac{1}{2}$.. Holding bond returns constant, an increase in Home government expenditures decreases dividends net of taxes at Home and raises Home non-financial incomes by raising the relative demand for Home goods (see (38) and (40) for $\delta_G = 1$). Conditional on bond returns, relative equity returns and relative non-financial incomes move in opposite directions and investor favors local equities to hedge non-finanical. incomes. In other words, because higher Home government expenditures are increasing Home non-financial incomes, the burden of taxes must primarly fall on Home households to preserve efficient risk-sharing, the reason why they hold most of local equity.

Models with shocks to investment expenditures: Note that the mechanism described above is very similar to the one described in Heathcote and Perri (2007b) and Coeurdacier, Kollmann and Martin (2008). Here, government expenditures play the exact same role as (endogenous) investment in these papers: increases in Home investment raise Home wages (non-financial incomes) due to Home bias in investment spending but decrease Home dividends (net of the financing of investment). This imply a negative covariance between Home wages and Home relative equity returns (holding bond returns constant). Hence, to hedge fluctuations in wages generated by changes in invesment across countries, investors exhibit Home equity bias. Because invesment is entirely financed by shareholders, their model is isomorphic to ours when government expenditures are entirely financed on financial incomes ($\delta_G = 1$); hence, the equity portfolios of (46) is identical to the one described in Heathcote and Perri (2007b).and Coeurdacier et al. (2008) if we replace Home bias in government expenditures by the degree of Home bias in invesment expenditures (see the appendix for a full derivation of a transposition of Heathcote and Perri (2007b) and Coeurdacier et al. (2008) in a static model).

While equilibrium portfolios do depend on the assumptions regarding the additional source of uncertainty, some common features are robust across models when relative bond returns load perfectly on the real exchange rate:

- First, the equity portfolios is driven by the covariance between relative equity returns and non-financial incomes conditional on real exchange rate changes. This is so because fluctuations in equity portfolio returns and non-financial incomes that are correlated the real exchange rate will be hedged using the bond positions.

- Second, optimal equity positions will be 'robust' to changes in household preferences:

  1. Changing the risk aversion induces a change in the exposure of total consumption expenditures to the real exchange rate (the left hand side of (22) but this is optimally taken care of by bonds holdings (term $\frac{1}{2}s_C(1-\frac{1}{\sigma})$).
  2. Changing the elasticity of substitution across goods changes the response of the real exchange rate to output shocks ($\bar{\lambda}$) but his will be also taken care of by optimal bond holdings in our model (term $\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\frac{\gamma_w}{\gamma_e})$).

One can find some other relevant additional source of uncertainty but we believe that bond and equity portfolios will share most of the features described in the previous examples when bond returns *perfectly* track the real exchange rate.

3.2.2 Case II: Relative equity returns load perfectly on the real exchange rate ($\gamma_b \neq 0$, $\gamma_e = 0$)

Our previous results hinge on the key-assumption that bond returns differential across countries load *perfectly* on the real exchange rate. In practice, this might not be true for at least two reasons: first, real bonds might not exist in practice.[22] Most bonds available to investors are nominal and nominal bonds returns differential across countries might not load perfectly on the real exchange rate in presence of nominal shocks. While nominal bonds may load pretty well on the real exchange rate in practice (see section 5 for some evidence), one might still want to know what are the predictions of our benchmark model in presence of nominal shocks. Second, even in the absence of nominal shocks, the bond return differential might not load perfectly on the *welfare-based* real exchange rate, the one that matters from the investor's point of view. This happens for instance in presence of shocks to the quality of goods (or equivalently changes in the number of varieties available to consumers) as in Corsetti, Martin and Pesenti (2005) or Coeurdacier et al. (2007).

We will explore these two cases sequentially. However, because we will assume $\gamma_e = 0$, the portfolio cannot be described by equations (23) as in the previous cases. So, we first solve for portfolios in a generic reduced-form model where relative equity retuns load perfectly on the (welfare-based) real exchange rate ($\gamma_e = 0$) but bond returns do not ($\gamma_b \neq 0$).

Equilibrium Portfolios when $\gamma_e = 0$ and $\gamma_b \neq 0$ We keep the same generic representation ignoring any additional source of risk on relative equity returns. This gives the following set of equations for the efficient terms-of-trade, relative equity returns and relative non-financial incomes (see section 2):

$$\widehat{R_e} = (1-\overline{\lambda})\widehat{q} \tag{47}$$
$$\widehat{R_b} = (2a-1)\widehat{q} + \gamma_b\widehat{\varepsilon} \tag{48}$$
$$\widehat{w} = (1-\overline{\lambda})\widehat{q} + \gamma_w\widehat{\varepsilon} \tag{49}$$

[22] Note that hey do exist in most developed markets: US, Euro zone, UK, Sweden are some well known examples were inflation-indexed bonds have been created.

The real exchange rate is still defined by the following equation:

$$\widehat{RER} = (2a - 1)\widehat{q} \tag{50}$$

Under the maintained hypothesis that markets are locally complete, the relative budget constraint (17) becomes:

$$(1 - \frac{1}{\sigma})(2a - 1)\widehat{q} = \delta\,(2S - 1)\,(1 - \overline{\lambda})\widehat{q} + (1 - \delta)\left((1 - \overline{\lambda})\widehat{q} + \gamma_w\widehat{\varepsilon}\right) + 2b((2a - 1)\widehat{q} + \gamma_b\widehat{\varepsilon}) \tag{51}$$

Financial markets are still locally complete given that the representative investor still has two 'relative assets' to hedge two 'relative shocks'. Note that in this set-up, changes in relative incomes due to capital gains and losses on bond return differentials are not purely driven by changes in the real exchange rate. Since in turn relative equity returns load perfectly on the real exchange rate but bonds do not (due to $\widehat{\varepsilon}$), portfolios will be unique since the two 'relative assets' do not have the same pay-offs in all states of nature. Moreover, equities will be used to hedge changes in relative consumption expenditures and real exchange risk, contrary to bonds that will be used to hedge the shocks $\widehat{\varepsilon}$. The optimal portfolio then satisfies:

$$\begin{aligned} S^* &= \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1 - \frac{1}{\sigma} - (1 - \delta)\,\frac{\gamma_w}{\gamma_b}\right)(2a - 1)}{\delta\left(\overline{\lambda} - 1\right)}\right] \\ b^* &= -\frac{1}{2}(1 - \delta)\frac{\gamma_w}{\gamma_b} \end{aligned} \tag{52}$$

The equity portfolio shares the same difficulties as in previous literature: it is highly dependent on preference parameters and involves for most parameter values shorting Home or Foreign equities. Note that in the specific case of $\gamma_w = 0$, the equity portfolio is identical to (18) and bonds are not used in equilibrium ($b = 0$) to insulate relative consumption expenditures from $\widehat{\varepsilon}$ shocks.

The case of nominal shocks Following Obstfeld (2007) and Engel and Matsumoto (2006), we add money in our benchmark model by assuming that money enters the utility function. To simplify matters, we assume that consumption and real money balances are separable in the utility function. The expected utility at date 0 of a representative agent in country i is now:

$$U_i = E_0\left[\frac{C_i^{1-\sigma}}{1 - \sigma} + \chi log(\frac{M_i}{P_i})\right] \tag{53}$$

where M_i denotes money holdings in country i by agent i, χ is a positive parameter. We assume a log- utility for real money balance to simplify some of the calculus but our main results are essentially unaffected by this assumption.[23]

[23]Supposing CRRA utility in money generates an additional hedging demand in the portfolio that goes towards zero once we converge towards a cashless economy.

Nominal shocks will be simply shocks to the money supply M_i in country i. We introduce $M = \frac{M_H}{M_F}$ the relative money supply and $\widehat{M}$ its deviation from its steady state value.

Bonds in country i are nominal bonds that pay one unit of country i's currency. s is the nominal exchange rate, defined as the number of Foreign currency units per unit of Home currency. A rise in s represents a nominal appreciation of the Home currency. We express all variables in Home currency terms. Without loss of generality, we assume that $E_0(M) = 1$ and $E_0(s) = 1$. We denote $\widehat{s}$ the deviations of the nominal exchange rate from its steady-state value of one.

The log-linearization of the Home country's real exchange rate $RER \equiv \frac{sP_H}{P_F}$ gives:

$$\widehat{RER} = \widehat{\frac{sP_H}{P_F}} = (2a-1)\widehat{q} \tag{54}$$

where $\widehat{q} = \widehat{\frac{sp_H}{p_F}}$ denotes the Home terms-of-trade and p_i is now the price of good i in units of currency i. Note that an increase in the Home terms-of-trade is an appreciation of the Home real exchange rate.

With only relative nominal shocks ($\widehat{M}$) and relative output shocks ($\widehat{y}$) and two 'relative assets' (stocks and nominal bonds), markets will still be (locally) complete:

$$-\sigma(\widehat{C_H} - \widehat{C_F}) = \widehat{\frac{sP_H}{P_F}} \tag{55}$$

First-order conditions for the demand for money are as follows (in log-linearized terms):

$$\sigma(\widehat{C_H} - \widehat{C_F}) = \widehat{M} - (\widehat{P_H} - \widehat{P_F}) \tag{56}$$

Using (55) and (56), we get that the rate of depreciation of the nominal exchange rate $(-\hat{s})$ is equal to relative money supply shocks:

$$-\hat{s} = \widehat{M} \tag{57}$$

In our benchmark flex-price model, the efficient consumption allocation is unchanged[24] and so are the efficient terms-of-trade and consequently (15) still holds:

$$\widehat{y} = -\lambda\widehat{q} \tag{58}$$

And relative equity returns $\widehat{R}_e$, relative non-financial incomes $\widehat{w}$ and bond returns differentials $\widehat{R_b}$ can be rewritten as:

$$\begin{aligned}
\widehat{R}_e &= (1-\lambda)\widehat{q} && (59)\\
\widehat{R_b} &= \widehat{s} = (2a-1)\widehat{q} + \widehat{\frac{P_H}{P_F}} && (60)\\
\widehat{w} &= (1-\lambda)\widehat{q} && (61)
\end{aligned}$$

[24] Nominal shocks have no real effects here because prices are flexible. An extension with price rigidities gives a convex combination of this example and the example with redistributive shocks (as output shocks will act as redistributive shocks in presence of price rigidities). See Engel and Matsumoto (2006).

Introducing $\widehat{\varepsilon} = \widehat{\frac{P_H}{P_F}}$, we are back to our general representation (equations (47), (48) and (49)) with $\gamma_w = 0$ and $\gamma_b = 1$. The equilibrium portfolios is then:

$$\begin{aligned} S^* &= \frac{1}{2}\left[1 - \frac{1-\delta}{\delta} - \frac{1}{\delta}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)}{(1-\lambda)}\right] \\ b^* &= 0 \end{aligned} \tag{62}$$

In particular, as might have been expected, the bond return differential fails to load on the real exchange because of the inflation differentials across countries $\widehat{P_H/P_F}$. This additional source of risk on bond returns shifts bond position towards zero (to hedge inflation risk). It then remains for equities to hedge real exchange rate exposure efficiently. While potentially restoring the difficulties of previous literature, we can safely argue that this example is not relevant empirically. Indeed, it would contradict the empirical evidence provided by van Wincoop and Warnock (2006) (and confirmed in our empirical section; see section 5) as it would imply the correlation between equity returns and the real exchange rate, *conditional on nominal bond returns* ($\beta_{RER,e}$) is non-zero (while the correlation between bond returns and the real exchange rate, *conditional on equity returns* ($\beta_{RER,b}$) should be close to zero).

The case of changes in quality/preference shocks We follow Coeurdacier et al. (2007) by adding preference shocks to the utility provided by Home goods and Foreign goods to the consumers of both countries. In that case, the aggregate consumption index C_i, for $i = H, F$, is now given by:

$$C_i = \left[a^{1/\phi}\left(\Psi_i c_{ii}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_j c_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{63}$$

where Ψ_i, $i = H, F$ with $E_0(\Psi_i) = 1$ is an exogenous worldwide shocks to the (relative) preference for the country i good. Note that the shock Ψ_i can also have a more supply oriented interpretation, as a shock to the quality of good i. We denote $\Psi \equiv \frac{\Psi_H}{\Psi_F}$ the relative preference shocks and $\widehat{\Psi}$ its deviation from its steady-state value of one.

As shown by Coeurdacier et al. (2007), the welfare-based real exchange rate in this case is equal to (up to the first-order):

$$\widehat{RER} = (2a-1)\left(\widehat{\frac{p_H}{p_F}} - \widehat{\frac{\psi_H}{\psi_F}}\right) \tag{64}$$

We adjust the terms-of-trade for quality/preference shocks by scaling q as follows: $q = \frac{p_H/\psi_H}{p_F/\psi_F}$. Then, we still have the following equality:

$$\widehat{RER} = (2a-1)\,\widehat{q} \tag{65}$$

As shown by Coeurdacier et al. (2007), under the assumption of complete financial markets, efficient terms-of-trade (adjusted for quality/preference shocks) satisfy:

$$\widehat{\psi y} = -\lambda \widehat{q} \tag{66}$$

where $\widehat{\psi y}$ are relative endowment adjusted for quality/preference shocks. Relative equity returns and relative non-financial incomes still load perfectly on the real exchange rate adjusted for the quality/preference shocks (see also Coeurdacier et al. (2007)):

$$\widehat{R}_e = (1-\lambda)\widehat{q} \tag{67}$$
$$\widehat{w} = (1-\lambda)\widehat{q} \tag{68}$$

However, if we assume that CPI-indexed bonds returns are not adjusted for quality (a case where the observed real exchange rate deviates from the welfare-based one), then bond returns differentials can be rewritten as follows:

$$\widehat{R_b} = (2a-1)\widehat{\frac{p_H}{p_F}} = (2a-1)\left(\widehat{q}+\widehat{\psi}\right) \tag{69}$$

Then, introducing $\widehat{\varepsilon} = (2a-1)\,\widehat{\psi}$, we are back to our model in his reduced form (with $\gamma_w = 0$) and equity and bond portfolios will be again the same one (see (63)). In particular, as it might have been expected, bond returns differential fails to load on the welfare-based real exchange because of relative change in quality/preference shocks $\widehat{\psi}$. This additional source of risk on bond returns shift bond position towards zero and risk-sharing is done by equities only.

[to be completed to show how this case is not very relevant in practice, following the steps decribed in the introduction]

4 Extensions

4.1 The Role of Nontradable [preliminary]

Recent work by Obstfeld (2007) and Collard et al. (2007) put forward the presence of non-traded goods as key to understand international equity portfolios.[25] But in these models, equity portfolios are also driven by the hedging of the real exchange rate coming from changes in the relative price of non-traded goods. Consequently their portfolios are also strongly affected by slight changes in preferences. Like in our benchmark model, their findings might be altered by trade in bonds in presence of an additional source of risk. For simplicity, we will focus on the case where relative bond returns load perfectly on the real exchange rate.[26] Under this assumption, we uncover that our findings are robust to the addition of non-traded goods. In particular, contrary to existing literature, the equity portfolio (aggregated across the traded and non-traded sector) will be independent on preferences and driven by the

[25] See also Dellas and Stockman (1989), Baxter et al. (1998) for earlier work on the role of non-traded goods. See also Matsumoto (2007).

[26] As in the previous cases, this assumption is important as bonds will do a better job than equity to hedge real exchange rate fluctuations. As shown in the next section, this is the empirically relevant case.

hedging of non-financial incomes conditional on bond returns. We show this in an extension of Obstfeld (2007) 's set-up with non-financial incomes.

We consider the same two-period ($t = 0, 1$) endowment economy with symmetric countries, Home (H) and Foreign (F). Each country now produces two goods, a tradable (T) and a non-tradable good (NT): at $t = 1$, country i receives an exogenous endowment y_i^T of the tradable good i and an exogenous endowment y_i^{NT} of the non-tradable good i. $E_0(y_i^T) = E_0(y_i^{NT}) = 1$ holds for both countries, where E_0 is the conditional expectation operator, given date $t = 0$ information. Like in the benchmark case, a share δ of the endowment in each sector is distributed to shareholders while a share $(1-\delta)$ is not capitalized and is distributed to households of country i. There is no output (and no consumption) at $t = 0$, but agents trade claims (stocks and bonds) at $t = 0$.

The aggregate consumption index C_i, for $i = H, F$ is given by:

$$C_i = \left[\eta^{1/\theta}\left(c_i^T\right)^{(\theta-1)/\theta} + (1-\eta)^{1/\theta}\left(c_i^{NT}\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{70}$$

where c_i^T is the consumption of a composite tradable goods using Home and Foreign tradable goods and c_i^{NT} is the consumption of non-tradable goods. θ is the elasticity of substitution between tradable and non-tradable goods.

Consumption of the tradable good is defined in the following way:

$$c_i^T = \left[a^{1/\phi}\left(c_{ii}^T\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(c_{ij}^T\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{71}$$

where c_{ij}^T is country $i\prime s$ consumption of the tradable good from country j. ϕ is the elasticity of substitution between the two tradable goods. We assume a preference bias for local goods, $\frac{1}{2} < a < 1$. The consumer price index that correspond to these preferences is for $i = H, F$:

$$P_i = \left[\eta\left(P_i^T\right)^{(1-\theta)} + (1-\eta)\left(P_i^{NT}\right)^{1-\theta}\right]^{1/(1-\theta)} \tag{72}$$

where P_i^T is the price index over tradable goods in country i and P_i^{NT} is the price of non-tradable goods.

The tradable-goods price index is defined by:

$$P_H^T = \left[a\left(p_H^T\right)^{1-\phi} + (1-a)\left(p_F^T\right)^{1-\phi}\right]^{1/(1-\phi)} \tag{73}$$

$$P_F^T = \left[(1-a)\left(p_H^T\right)^{1-\phi} + a\left(p_F^T\right)^{1-\phi}\right]^{1/(1-\phi)}, \tag{74}$$

where p_H^T and p_F^T are the prices of the tradable good H and F, respectively.

Resource constraints over tradable and non-tradable goods are given by:

$$c_{ii}^T + c_{ji}^T = y_i^T \tag{75}$$

$$c_H^{NT} = y_H^{NT} \; ; \; c_F^{NT} = y_F^{NT} \tag{76}$$

We still denote Home terms of trade by q:

$$q \equiv \frac{p_H^T}{p_F^T} \tag{77}$$

We denote the Home price of non-tradable over the Foreign price of non-tradable by P^{NT}:

$$P^{NT} \equiv \frac{P_H^{NT}}{P_F^{NT}} \tag{78}$$

4.1.1 Financial markets

There is trade in stocks and bonds, in period 0. In both countries there are two different stocks, stocks of tradable and stocks of non-tradable. Each stock is a Lucas tree that gives a a share δ of the future endowments (tradable or non-tradable). The supply of each type of share is normalized at unity. There is a CPI bond denominated in the Home composite good, and a CPI bond denominated in the Foreign composite good. Buying one unit of the Home (Foreign) bond in period 0 gives one unit of the Home (Foreign) good at $t = 1$. Both bonds are in zero net supply. Each household fully owns the local stock of tradable and the local stock of non-tradable, at birth, and has zero initial foreign assets. The country i household thus faces the following budget constraint, at $t = 0$:

$$p_S^T S_{ii}^T + p_S^T S_{ij}^T + p_S^{NT} S_{ii}^{NT} + p_S^{NT} S_{ij}^{NT} + p_b b_{ii} + p_b b_{ij} = p_S^T + p_S^{NT}, \quad \text{with} \;\; j \neq i \tag{79}$$

where S_{ij}^T is the number of shares of stock of tradable j held by country i at the end of period 0, S_{ij}^{NT} is the number of shares of stock of non-tradable j held by country i, while b_{ij} represents claims (held by i) to future unconditional payments of the composite good j. p_S^T (resp. p_S^{NT}) is the share prices of stock of tradable (resp. non-tradable), p_b is the bond price. Asset prices of each type are identical across countries due to symmetry[27].

Market clearing in asset markets for the for stocks and the two bonds requires:

$$\begin{aligned} S_{ii}^T + S_{ji}^T &= S_{ii}^{NT} + S_{ji}^{NT} = 1 & (80) \\ b_{ii} + b_{ji} &= 0 & (81) \end{aligned}$$

Symmetry of preferences and shock distributions implies that equilibrium portfolios are symmetric: $S_{HH}^T = S_{FF}^T$, $S_{FH}^T = S_{HH}^T$, $S_{HH}^{NT} = S_{FF}^{NT}$, $S_{FH}^{NT} = S_{HH}^{NT}$, $b_{HH} = b_{FF}$ and $b_{FH} = b_{HF}$. In what follows, we denote a country's holdings of local stock of tradable (resp. non-tradable) by S^T (resp. S^{NT}), and its holdings of CPI bonds denominated in its local composite good by b. The vector $(S^T; S^{NT}; b)$ thus describes international portfolios. $S^k > \frac{1}{2}$ means that there is equity home bias on stocks of sector k for $k = \{N, NT\}$, and $b > 0$ means that the country lending in units of the loca composite good and borrowing in units of the foreign composite good.

[27] Bond prices are also identical due to symmetry.

4.1.2 Equity and bond portfolios with locally complete markets

Like in the benchmark model, portfolios are undetermined with endowment shocks in both sectors in presence of bonds but this knife-edge result breaks down when we add and additional source of risk which pins down the international bond and equity portfolios in both sectors. We do show the resolution of the model and presents directly his reduced form (see appendix for the derivation of the equations).

We write $y^T \equiv \frac{y_H^T}{y_F^T}, y^{NT} \equiv \frac{y_H^{NT}}{y_F^{NT}}$ to denote relative outputs in both sectors. We log-linearize the model around the symmetric steady-state where y^T and y^{NT} equal unity, and use $\widehat{x} \equiv log(x/\bar{x})$ to denote the log deviation of a variable x. Finally, like in the benchmark case, we add an additional source of risk denoted $\widehat{\varepsilon}$ on equity returns and non-financial incomes. We do it in the case where relative bond returns load perfectly on the real exchange rate.[28]

4.1.3 Reduced form of the model

The Home real exchange rate is defined as follows:

$$\widehat{RER} = \widehat{\frac{P_H}{P_F}} = \eta(2a-1)\widehat{q} + (1-\eta)\widehat{P^{NT}} \tag{82}$$

where $P^{NT} = P_H^{NT}/P_F^{NT}$ is the relative price of Home non-tradable goods over Foreign non-tradable goods and η is the steady-state share of spending devoted to tradable goods.[29] Note also that the relative price of the tradable composite in both countries verifies: $P_H^T/P_F^T = (2a-1)\widehat{q}$.

Relative consumption expenditures are equal to relative financial and non-financial incomes; thus, assuming locally complete markets, we have:

$$\begin{aligned} \widehat{P_H C_H} - \widehat{P_F C_F} &= (1-\frac{1}{\sigma})\widehat{RER} \\ &= \eta\delta\left(2S^T-1\right)\widehat{R_e^T} + (1-\eta)\delta\left(2S^{NT}-1\right)\widehat{R_e^{NT}} + (1-\delta)\widehat{w} + 2b\widehat{RER} \end{aligned} \tag{83}$$

where $\widehat{R_e^k}$ denotes Home excess equity return in sector $k = \{T, NT\}$ and $\widehat{w}$ denote Home excess non-financial income (Home over Foreign aggregated over both sectors).

The additional source of risk $\widehat{\varepsilon}$ is assumed to pertubate equity returns (in both sectors) and non-financial incomes and to leave bond returns unchanged. In equilibrium, relative

[28] Relaxing this assumption is straightforward but we prefer to focus on this case as this will be the relevant one empirically.

[29] Here to simplify notations, we assume that the share of spending devoted to tradable goods is the same as the weight of tradable goods in the consumption index. This is true only if in the steady state tradable and non tradable goods have the same price: $p^{T*} = p^{NT*}$. This assumption is however irrelevant for equity portfolios (see Obstfeld (2007)).

financial and non-financial incomes verify (see appendix):[30]

$$\begin{aligned}
\widehat{R_e^T} &= (1-\lambda)\widehat{q} + (1-\eta)(\theta - \frac{1}{\sigma})(2a-1)\widehat{P^{NT}} + \gamma_e \widehat{\varepsilon} && (84)\\
\widehat{R_e^{NT}} &= [(1-\theta) + (1-\eta)(\theta - \frac{1}{\sigma})]\widehat{P^{NT}} + (\theta - \frac{1}{\sigma})\eta(2a-1)\widehat{q} + \gamma_e \widehat{\varepsilon} && (85)\\
\widehat{R_b} &= \widehat{RER} = \eta(2a-1)\widehat{q} + (1-\eta)\widehat{P^{NT}} && (86)\\
\widehat{w} &= \eta\widehat{R^T} + (1-\eta)\widehat{R^{NT}} + (\gamma_w - \gamma_e)\widehat{\varepsilon} && (87)
\end{aligned}$$

where $\lambda \equiv \phi(1-(2a-1)^2) + (2a-1)^2((1-\eta)\theta + \frac{\eta}{\sigma})$. Note again that $\lambda > 0$ as $1/2 < a < 1$

4.1.4 Equilibrium Equity and Bond Portfolios

The method to solve for portfolios is essentially identical to our benchmark model: we derive the portfolio that reproduce the complete market allocation for consumption locally. To save space, we do not show the details of the derivations (see appendix) and only present

The equity portfolio averaged across sector satisfies (see appendix for detailed equity positions across sectors):

$$\eta S^T + (1-\eta)S^{NT} = \frac{1}{2}\left(1 - \frac{\gamma_w}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \quad (88)$$

The averaged equity bias is identical to section 3 when $\gamma_b = 0$ as we have assumed here that bond returns load perfectly on the real exchange rate. As a consequence, the average equity portfolio is used to hedge the additional shock $\widehat{\varepsilon}$. Contrary to Obstfeld (2007) (see also Collard et al. (2007)), the overall equity bias in the economy does not depend on preferences at all (equation (88)).[31]

The bond position satisfies (with $\Omega = \frac{(\theta-1)(2a-1)}{\phi-1+(2a-1)^2(\theta-\phi)}$):

$$b = \frac{1}{2}(1-1/\sigma) - \frac{1}{2}\frac{(1-\gamma)(1-\delta)}{\eta\Omega + 1 - \eta}(1-\theta+(\theta-1/\sigma)(1-\eta+\eta(2a-1)\Omega)) \quad (89)$$

As in our benchmark model, the bond portfolio is the sum of two terms: the first one $\frac{1}{2}(1-1/\sigma)$ is the optimal hedge for fluctuations in total consumption expenditures when $\sigma \neq 1$. the second term $\left(\frac{1}{2}\frac{(1-\gamma)(1-\delta)}{\eta\Omega+1-\eta}(1-\theta+(\theta-1/\sigma)(1-\eta+\eta(2a-1)\Omega))\right)$ corresponds to the hedge of real exchange rate movements that are correlated with non-financial incomes and financial incomes arising from the optimal equity portfolios. Hence, the implications of the model with non-traded goods are very similar to our benchmark case.

[30] We assume here that the impact of the $\widehat{\varepsilon}$ shock on relative equity returns in the same across sectors. Extending the model with different γ_e^i for $i = \{T, NT\}$ is straightforward but testing empirically such a distinction will not be possible.

[31] But the way this equity bias is shared across sectors does depend on the two elasticities of substitution θ and ϕ. In appendix, we show how the overall equity bias is shared between the two sectors. If θ and ϕ are both larger (or smaller) than unity (such that $(\theta-1)(2a-1)$ and $\phi - 1 + (2a-1)^2(\theta-\phi)$ have the same sign), then portfolio is well balanced across sectors. If $\theta < 1$ and $\phi > 1$, then equity portfolios will involve short position in one sector (see appendix). The equity positions across sectors never depend on σ.

4.2 The General Case: Bond and Equity Holdings under Incomplete Markets

[To be written]

Where we use the Devereux and Sutherland (2006) approach (see also Tille and van Wincoop (2007)) to characterize the optimal equity and bond positions when markets are incomplete (i.e. there are more than one ε type shocks). Our conjecture is that what matters is the relative variance of the additional shocks, conditional on the welfare-based real exchange rate. We would expect equities to be used more heavily to hedge real exchange rate fluctuations when equity returns load 'more' on total consumption expenditures and the opposite in the case where bond returns do the job.

5 Empirical Analysis: What do we know about the hedging properties of bond and equity returns?

[Preliminary results]

The results of the previous sections indicate that the pivotal factor is the ability of bond returns to hedge fluctuations in real exchange rates. When this is the case, i.e. in presence of redistributive, government or fiscal shocks, equity portfolios are independent of preference parameters and are driven by the correlation between equity returns and non financial income, conditional on bond returns. When this is not the case, i.e. in the presence of nominal, quality or preference shocks, bond holdings are essentially useless and equity holdings will be driven by the correlation between equity returns and real exchange rate changes.

As discussed previously, we can assess the model empirically by estimating the reduced-form loading factors $\beta_{RER,i}$ and $\beta_{w,i}$ for $i = e, b$. We estimate these loading factors for the members of the G-7, using quarterly data over the period 1980-2007.

5.1 Description of the data

We use quarterly data over the period 1980-2007 for the G-7 countries. To estimate the empirical counterpart of the general specification of section 3.1, we construct for each country a series for (relative) non-financial income, (relative) equity market returns, (relative) bond returns and real exchange rate changes. To do so, we use data on 1) income measures of national income, to construct payments on the non-financial asset (non-financial income); 2) market returns to construct relative equity and bond returns; 3) nominal exchange rates and CPI to construct real exchange rate changes. For each G-7 country, the series are constructed relative to the Foreign counterpart, where the Foreign country is made of the 6 remaining countries. Data on national income and on CPI are taken from the OECD database. Data on nominal exchange rate, bond and equity returns are from Global Financial Data.

5.1.1 Relative non-financial incomes

We use GDP by income to construct the empirical counterpart of relative payments to the non-financial asset (relative non-financial incomes) and of the average share of financial income in total income δ. For each country i, we express income measures in a common currency (US Dollar), converting local currency measures at current nominal exchange rates. According to national accounting, in country i, at date t, GDP_{it} (by income, in USD) can be decomposed in the following way:

$$GDP_{it} = Comp_{it} + Mixed_{it} + Profits_{it} + T_{it}$$

where $Comp_{it}$ denotes the compensation of employees, $Mixed_{it}$ denotes mixed incomes, $Profits_{it}$ denotes operating profits and T_{it} denotes taxes on production. Our measure of the non-financial income w_{it} depends on the allocation of mixed incomes. Mixed incomes includes self-employed incomes as well as proprietary incomes, most of which is not capitalized in the financial market. Consequently, we include a fraction λ_{it} of mixed incomes $Mixed_{it}$ to the compensation of employees $Comp_{it}$ to construct our estimate of non-financial income. The share λ_{it} is constructed under the assumption that the nonfinancial component of $Mixed_{it}$ is the same as in the $Comp_{it} + \Pr ofits_{it}$. This implies: $\lambda_{it} = \frac{Comp_{it}}{Comp_{it}+Profits_{it}}$.[32] We then calculate an equivalent measure for the rest of the world by summing non-financial incomes (in USD) for the 6 remaining countries. Our final measure of relative financial income in country i at date t is (in log) :

$$w_{it} = \log(Comp_{it} + \lambda_{it} Mixed_{it}) - \log(\sum_{j \neq i} Comp_{jt} + \lambda_{jt} Mixed_{jt})$$

Since what matters is private incomes (GDP net of taxes), we define the average share of non-financial (resp. financial) income in country i, $1 - \delta_i$ (resp. δ_i) as follows:

$$1 - \delta_i = \frac{1}{T} \sum_t \frac{Comp_{it} + \lambda_{it} Mixed_{it}}{GDP_{it} - T_{it}}$$

where T is the number of observations in our sample ($T = 109$).

Table **??** summarizes our estimates of δ for our sample of G7 countries. Note that the share of incomes that is capitalized in financial markets is smaller than the usual measure of 1/3 for most countries as we have assumed that a large share of mixed incomes are not capitalized in financial markets. Our results are in line with previous estimates (see Gollin (2002)).

[32] We tried alternative measures but our results were essentially unaffected. In particular, we also attributed all mixed incomes to non-financial incomes ($\lambda_{it} = 1$, for all i, t) or all mixed incomes to financial incomes ($\lambda_{it} = 0$, for all i, t).

5.1.2 Relative bond and equity market returns

We use data on quarterly bond and equity returns to estimate relative bond and equity returns ($\hat{R}_b$ and $\hat{R}_e$). We use returns on a 3-months (nominal) government bond in local currency and quarterly equity returns on the main stock-index of the country. All returns are expressed in US dollars by converting local currency returns in USD (using realized currency returns over the quarter). In each country i, returns on both assets are expressed relative to the remaining 6 countries, using constant GDP-weights.

Hence, relative bond returns $\hat{R}_{b,it}$ in country i at date t are defined as (in log-terms):

$$\hat{R}_{b,it} = \log(R_{b,it}) - \log(\sum_{j\neq i} \alpha_j^i R_{b,jt})$$

where : α_j^i is the averaged GDP-weight over the period 1980-2007 of country j among the remaining 6 countries:

$$\alpha_j^i = \frac{1}{T}\sum_t GDP_{jt}/(\sum_{k\neq i} GDP_{kt}) \tag{90}$$

$R_{b,it}$ is the gross return on a (local currency) 3 months T-bill converted in USD dollars: $R_{b,it} = \left(1 + r_{it}^b\right)(1 + \Delta s_{it})$ where r_{it}^b is the 3-months interest rate and Δs_{it} is the appreciation rate of the local currency with respect to the USD.over the quarter (between $t-1$ and t).

Similarly, relative equity returns $\hat{R}_{e,it}$ in country i at date t are defined as (in log-terms):

$$\hat{R}_{e,it} = \log(R_{e,it}) - \log(\sum_{j\neq i} \alpha_j^i R_{e,jt})$$

where the GDP-weights α_j^i are defined above. $R_{e,it}$ is the gross quarterly return on the main stock market of country i converted in USD: $R_{e,it} = (1 + r_{it}^e)(1 + \Delta s_{it})$ where r_{it}^e is the quarterly equity returns in local currency and Δs_{it} is the appreciation rate of the local currency with respect to the USD over the quarter (between $t-1$ and t).

5.1.3 Real exchange rate

We define the real exchange rate of country i at date t as follows (in levels):

$$RER_{it} = P_{it}/\sum_{j\neq i} \alpha_j^i P_{jt}$$

where the GDP-weights α_j^i are defined by (90); P_{it} is the consumer price index of country i at date t, expressed in USD dollar[33]: $P_{it} = P_{it}^{LC} S_{it}$ where P_{it}^{LC} is the CPI of country i in local currency of country i at date t and S_{it} is the nominal exchange rate at date t (USD per unit of local currency of country i).

Hence, quarterly real exchange rate changes between $t-1$ and t are defined (in log-terms) as follows:

$$R\hat{E}R_{it} = \log(\frac{RER_{it}}{RER_{it-1}})$$

[33]Price indices in USD are normalized to 100 in 2001Q1 for each countries: $P_{i,2001Q1} = 100$

5.2 Estimates of the loadings on real exchange rate changes

A key implication of our theoretical model is that portfolios are drastically different depending on the financial asset that is used to hedge real exchange rate changes. As shown in section 3.1, the moments that matters for equilibrium portfolios are the loading factors of relative bond and equity returns on real exchange rate changes $\beta_{RER,i}$ for $i = e, b$. They can be estimated for each country by the simple following regression for each country i of the G7-countries:

$$R\hat{E}R_{it} \equiv c + \beta^{i}_{RER,b}\hat{R}_{b,it} + \beta^{i}_{RER,e}\hat{R}_{e,it} + u_{it} \tag{91}$$

where u_{it} is attributed to measurement error on the real exchange rate (potentially due to mismeasured *welfare-based* inflation).

Results of the regression (91) for each countries are displayed in table **??**. Our empirical results confirm the results of van Wincoop and Warnock (2006) for all the countries considered in the sample: relative bond returns capture most of the variations of the real exchange rate and *conditional on bond returns*, the correlation between equity returns and the real exchange rate is zero. Hence, equity will essentially not be used to hedge real exchange rate changes and households will prefer bonds for that purpose. From a theoretical standpoint, the reduced-form model with $\gamma_b = 0$ seems to provide a reasonable approximation of the data (recall that when $\gamma_b = 0$, the theoretical model implies $\beta_{RER,b} = 1$ and $\beta^{i}_{RER,e} = 0$, which is consistent with our empirical estimates).

5.3 Estimates of the loadings on returns to non-financial wealth

5.3.1 Estimating the returns to non-financial wealth

We now turn to the properties of equities to hedge nontradable risks. Before assessing the hedging properties of bonds and stocks, we need to estimate the (relative) returns to non-financial wealth using our measure of (relative) non-financial incomes (the dividends payments associated to non-financial wealth). To do so, we follow the method of Campbell and Shiller (1988) as detailed in Campbell (1996). A similar approach has been used by Baxter and Jermann (1997) and Julliard (2003).[34] If we denote $r_{w,t+1}$ the relative (log) return on the non-financial asset and $r_{e,t+1}$ the (log) relative return on equities in a given country (we abstract from country i indices for simplicity). We obtain from the above method (see Campbell (1996)), under the assumption that $E_t r_{w,t+1} = E_t r_{e,t+1}$:

$$r_{w,t+1} - E_t r_{w,t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^{j}\Delta w_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^{j} r_{e,t+1+j} \tag{92}$$

[34] Baxter and Jermann (1997) also estimates the returns to non-financial wealth. Compared to their paper, note that we do not need to impose a cointegration relationship. Since we measure everything in relative terms, our variables of interest (relative non-financial incomes/relative equity returns) are all stationnary.

where Δw_{t+1+j} is the growth rate of relative non-financial income between time $t+j$ and time $t+j+1$; ρ is a constant strictly smaller than one.[35] This estimation method makes clear that the return on non-financial wealth depends upon expectations of future non-financial income growth (the first term on the right hand side of (92)), as well as expectations of changes in expected future returns on non-financial wealth (the second term in (92)).

In order to construct the empirical counterpart of the left hand side of equation (92), we estimate a first-order VAR of the following form:[36]

$$z_{t+1} = Az_t + \epsilon_{t+1}$$

where $z_t' = (r_{e,t}, \Delta w_t, x_t)$ and x_t represents other controls that helps to predict the (relative) growth rate of non-financial incomes and relative equity returns. In practice, we used the (log) relative consumption expenditures and (log) relative non-financial income. Details of the estimations of the VAR are presented in the appendix.

With estimates of $\hat{A}$ and $\hat{\epsilon}_{t+1}$ in hand, we can construct an empirical counterpart to $r_{w,t+1} - E_t r_{w,t+1}$ as:

$$\hat{w}_{t+1} = r_{w,t+1} - E_t r_{w,t+1} = \left(e_2' - \rho e_1' \hat{A}\right)\left(I - \rho \hat{A}\right)^{-1} \hat{\epsilon}_{t+1}$$

These are the innovations to the returns on non-financial wealth that will be used to estimates the loadings of market returns on returns to non-financial wealth.

5.3.2 Market returns loadings on returns to non-financial wealth

We use the innovations $\hat{w}_{i,t}$ estimated previously for each country i to estimate the following equation:

$$\hat{w}_{i,t} = c + \beta^i_{w,b} \hat{R}_{b,it} + \beta^i_{w,e} \hat{R}_{e,it} + v_{it} \tag{93}$$

where v_{it} is attributed to measurement error in the return to the non-financial asset.

This regression estimates the loadings of (relative) bond and equity returns for each country i of our sample. It gives insights about the hedging capacity of bonds and equities against fluctuations in non-financial wealth. Recall in particular that following our generic specification (and assuming $\gamma_b = 0$ in line with our previous estimates), $\beta_{w,e}$ is the empirical counterpart of γ_w/γ_e, which is the only parameter that has implications for the equity portfolio.

Results of the regression (93) for each countries are shown in table **??**. We find that $\beta_{w,e}$ is significantly negative (economically large) for three countries in our sample: Canada, the US and Japan. Hence, for these countries, our model predicts that local equities will be used to hedge returns to non-financial wealth. Our results differ strongly from Baxter

[35] ρ depends simply on the mean dividend-price ratio for financial and non-financial incomes. We will use the value of $\rho = 0.95$ in line with standard estimates in the literature. Our results are robust to changes in the value of ρ.

[36] Standard Akaike and Schwarz lag-selection criteria indicate that a VAR(1) is preferrable for all countries.

and Jermann (1997) since they do not condition for bond returns. In fact, the next to last row of the table reports the unconditional correlation between the return on non-financial income and equity returns. For all G-7 countries, this correlation is large and positive. Hence controlling for the bond returns is essential to assess the capacity of the model to generate realistic equity positions.[37]

The positive loadings of (relative) bond returns $\beta_{w,b}$ imply that shorting the local currency bond and going long in the foreign currency bond is a good hege against fluctuations in returns to non-financial wealth. Note that this result is not theoretically surprising. In our model, a (potentially large) part of non-financial and financial incomes comove with the real exchange rate and we know from the previous section that relative bond returns track almost perfectly on the real exchange rate.

5.4 Implied bond and equity portfolios

The previous estimates allow us to back out the implied equity and bond positions using equations (27) and (26). Note however, that these equations hold for countries of symmetric size. Allowing for different country sizes, we get that (27) and (26) must be rewritten in the following way (under the assumption verified empirically that $\beta_{RER,e} = 0$; see appendix):

$$
\begin{aligned}
S_i^* &= \omega_i - \frac{1-\delta}{\delta}\beta_{w,e}(1-\omega_i) \\
b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}\left(1-\delta\right)\beta_{w,b}
\end{aligned}
$$

where ω_i is the relative size of country i in world GDP (assumed to be equal to its relative wealth in the symmetric non-stochastic equilibrium).

The implied equity bias and bond portfolios are summarized in table **??**. The model is successfull in predicting a significant degree of equity home bias for the US, Japan and Canada but not for European countries (note however that the biases for these countries are not statisitcally different from zero). The model can predict short positions in local currency bonds for sufficiently low risk aversion. **[to be completed]**

6 Conclusion

[To be written]

[37] An additional difference between Baxter and Jermann (1997) and our results is that we express everything in relative terms. In so doing, we make sure that aggregate shocks that affect Home and Foreign equity returns in the same way (and hence cannot be insured) are not affecting our results. Julliard (2003) argues that Baxter and Jermann (1997)'s implicit assumption that returns on financial and nonfinancial income are uncorrelated across countries is violated empirically.

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	CAN	FRA	GER	ITA	JPN	UK	US	Average
Share of non-financial income δ	28.2%	28.7%	25.1%	35.8%	35.6%	28.2%	24.4%	29.4%

Table 1: Estimates of the share of financial incomes δ across countries.

	CAN	FRA	GER	ITA	JPN	UK	US
$\beta_{RER,b}$	1.021^{***}	0.965^{***}	0.988^{***}	0.965^{***}	1.004^{***}	0.967^{***}	0.967^{***}
	(0.021)	(0.016)	(0.015)	(0.018)	(0.015)	(0.024)	(0.017)
$\beta_{RER,e}$	-0.013	-0.002	-0.010	0.018^{**}	-0.005	-0.011	0.003
	(0.012)	(0.007)	(0.007)	(0.007)	(0.007)	(0.015)	(0.011)
R^2	0.97	0.98	0.97	0.97	0.98	0.95	0.98

Table 2: Loadings on real exchange rate changes: $R\hat{E}R_{it} \equiv \beta^i_{RER,b}\hat{R}_{b,it} + \beta^i_{RER,e}\hat{R}_{e,it} + u_{it}$. Standard errors are in parenthesis. (**) (resp (*)) indicates significance at the 1% level (resp. 5%). Constants are not reported.

	CAN	FRA	GER	ITA	JPN	UK	US
$\beta_{w,b}$	1.938^{***}	1.313^{***}	0.960^{***}	1.779^{***}	2.319^{***}	0.733^{***}	2.101^{***}
	(0.071)	(0.095)	(0.104)	(0.069)	(0.099)	(0.024)	(0.083)
$\beta_{w,e}$	-0.132^{***}	0.072	0.055	-0.030	-0.273^{***}	0.073^{***}	-0.374^{***}
	(0.012)	(0.042)	(0.049)	(0.028)	(0.048)	(0.026)	(0.056)
$\beta_{w,e}$, unc	0.43	0.41	0.30	0.24	0.36	0.61	0.29
R^2	0.90	0.70	0.49	0.87	0.86	0.84	0.87

Table 3: Loadings on returns to non-financial wealth: $\hat{w}_{i,t} = c + \beta^i_{w,b}\hat{R}_{b,it} + \beta^i_{w,e}\hat{R}_{e,it} + v_{it}$ Standard errors are in parenthesis. (**) (resp (*)) indicates significance at the 1% level (resp. 5%). Constants are not reported.

		CAN	FRA	GER	ITA	JPN	UK	US
Implied Equity Home Bias $\frac{1-\delta}{\delta}\beta_{w,e}(1-\omega_i)$		32%	-16%	-14%	5%	39%	-17%	64%
Implied Bond Positions $\frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b}-\frac{1}{2}\left(1-\delta\right)\beta_{w,b}$	$\sigma=2$	-21%	-14%	-12%	-13%	-17%	-8.0%	-27%
	$\sigma=4$	3.1%	24%	0%	9.6%	12.2%	1.1%	-1.0%

Table 4: Implied equity bias $(S_i^* - \omega_i)$ and bond positions b_i for each country i. Positions are calculated under the assumption that $\beta_{RER,e} = 0$. Calculations are are done using the observed share of financial income δ of each country and for $\sigma = \{2; 4\}$.

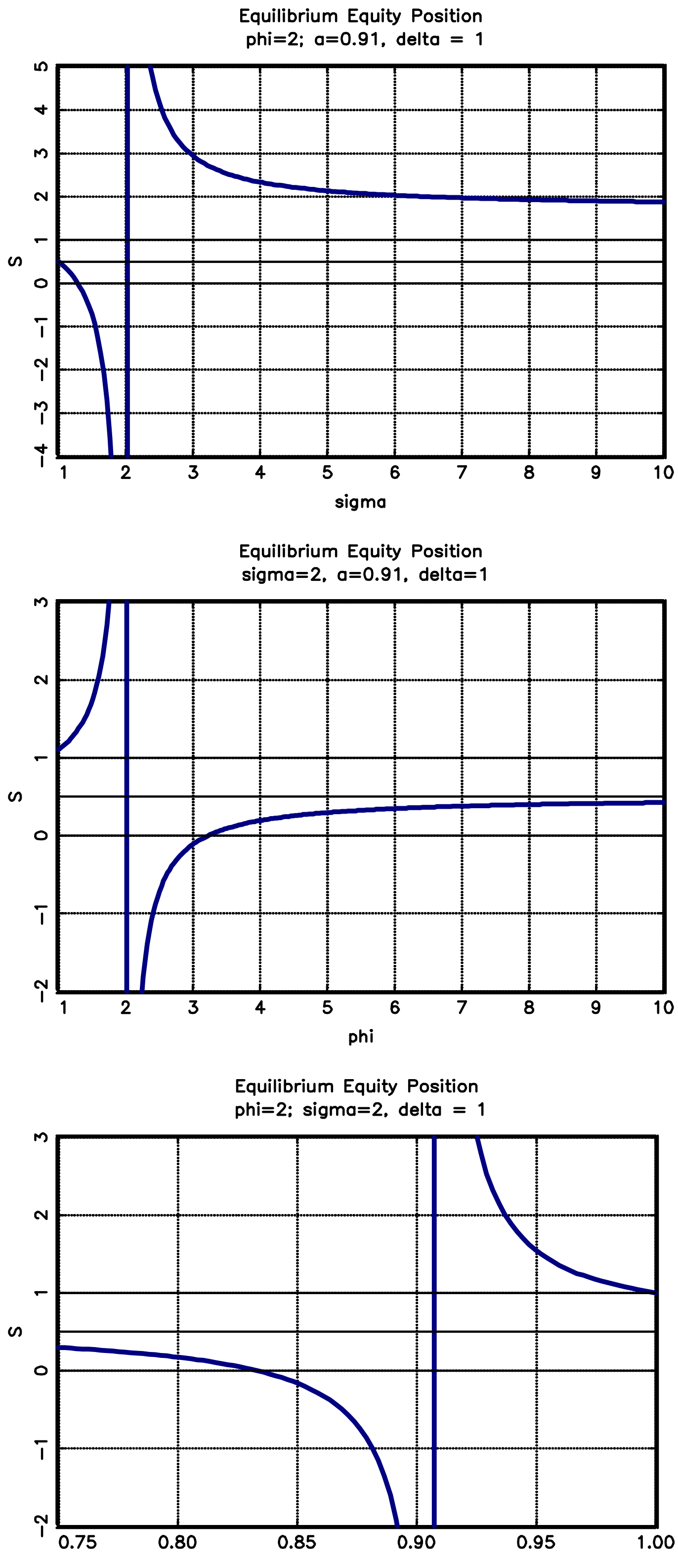

Figure 1: The instability of optimal equity position as a function of preference parameters.

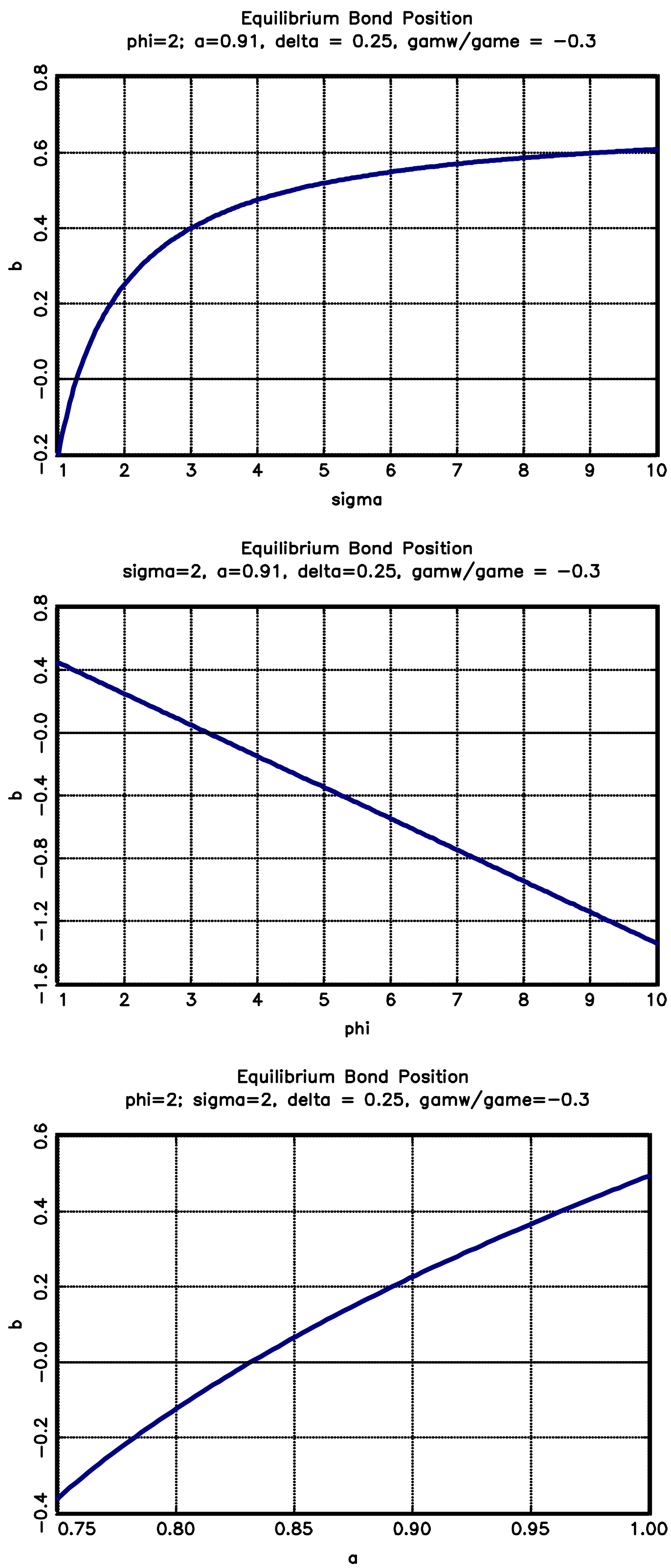

Figure 2: The stability of optimal bond positions as a function of preference parameters.

A Appendix

A.1 Optimal portfolios with locally complete markets

We assume symmetric countries. Due to the symmetry, one can simply look at relative returns across asset types. To simplify our exposition, we consider only bonds and equities and two sources of shocks as in our simple framework but this can be easily extended. Our strategy of resolution is very related to Devereux and Sutherland (2006) and Tille and van Wincoop (2007). Essentially, we show that the zero-order (or static) portfolio in Devereux and Sutherland (2006) and Tille and van Wincoop (2007) is the one that locally replicates the efficient consumption allocation. To do so, one need to show that the portfolio is consistent: with: 1) First-order approximation of non-portfolio equations (here intratemporal allocation across Home and Foreign goods and budget constraints in both countries) 2) Second-order approximation of the Euler equations.

Due to symmetry of the countries, we just need to verify these equations in relative terms. Let us rewrite the two key non-portfolio equations:

- One is related to intratemporal allocation across goods:.

  $$q^{-\phi}\Omega_a\left[(\frac{P_F}{P_H})^{\phi}\frac{C_F}{C_H}\right] = y_C$$

  where y_C the ratio of consumption of Home goods and Foreign goods by households.

  The first-order approximation is:

  $$\widehat{y_C} = \left[-\phi + (2a-1)^2(\phi-1)\right]\widehat{q} + (2a-1)\widehat{PC} \tag{94}$$

  where $\widehat{PC}$ denotes relative consumption expenditures ($\widehat{P_HC_H} - \widehat{P_FC_F}$).

- One is the relative budget constraint (log-linearized):

  $$\widehat{P_HC_H} - \widehat{P_FC_F} = (1-\delta)\widehat{w} + \delta\left(2S-1\right)\widehat{R} + 2b\widehat{R_b} \tag{95}$$

  where $\widehat{w}$ denotes relative non-financial incomes, $\widehat{R}$ denotes Home equity excess returns (over Foreign) and $\widehat{R_b}$ Home excess bond returns.

The Euler equations for equity holdings in country $i = H, F$ is:

$$\lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i}R_H]\ ;\ \lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i}R_F] \tag{96}$$

where $\lambda_{i,0}$ denotes the Lagrange-multiplier of the budget constraint in period $t = 0$ in country $i = H, F$. In relative terms across countries (remind that R denotes Home excess equity returns):

$$E_0\left[\left(\frac{C_H^{-\sigma}}{P_H} - \frac{C_F^{-\sigma}}{P_F}\right)R\right] = 0 \tag{97}$$

Following similar steps, we have a similar expression for Home bond excess returns:

$$E_0\left[\left(\frac{C_H^{-\sigma}}{P_H}-\frac{C_F^{-\sigma}}{P_F}\right)R_b\right]=0 \tag{98}$$

The second-order apprximation of equations (97) and (**??**) yields:

$$\begin{aligned} cov(\widehat{PC},\widehat{R}) &= (1-1/\sigma)cov(\widehat{RER},\widehat{R}) \quad (99)\\ cov(\widehat{PC},\widehat{R}_b) &= (1-1/\sigma)cov(\widehat{RER},\widehat{R}_b) \quad (100)\end{aligned}$$

As explained in Devereux and Sutherland (2006) and Tille and van Wincoop (2007), the optimal portfolio $(S; b)$ must be such that: 1) First-order approximations of non-portfolio equations are satisfied (equations (94) and (95)); 2) Second-order approximation of Euler equations are verified (equations (99) and (100)).

It should be clear that a portfolio $(S; b)$ such that $\widehat{PC} = (1-1/\sigma)\widehat{RER} = (1-1/\sigma)(2a-1)\widehat{q}$ satisfies the two (second-order) approximations of the Euler equations. Let us a assume that it is possible to find such a portfolio. Then, this is the same thing as saying that relative consumption expenditures are linked to the real exchange by the expression (13) or equivalently markets are (locally) complete.

If such a portfolio exists, it must also satisfy the first order non-portfolio equations (94) and (95). These can be rewritten (see the equivalent expressions (15) and (17) in the benchmark model):

$$\begin{aligned} \widehat{y_C} &= -\lambda\widehat{q} \quad (101)\\ (1-1/\sigma)(2a-1)\widehat{q} &= (1-\delta)\widehat{w}+\delta\left(2S-1\right)\widehat{R}+2b\widehat{R_b} \quad (102)\end{aligned}$$

The portfolio choice only affects equation (101) because of its impact on equity and bond Home excess returns (through its impact on $\widehat{q}$); so as long as the asset returns computed are consistent with equation (101), then the first-order approximation of (94) is verified. The key question is wether one can verify (102) in all states of nature (for any possible realizations of the shocks considered). Because we have two instruments (S and b), we must have at most two sources of risk. This is the first condition. Call $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ the two innovations (expressed in relative terms) arising from these two sources of risk and assume that our four endogenous variables $\{\widehat{q}; \widehat{w}; \widehat{R}; \widehat{R}_b\}$ are driven by $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ according to the following expression in matrix form (where we assume that Home bond excess load perfectly on the real exchange rate, as in our benchmark case):[38]

$$\begin{pmatrix}\widehat{q}\\ \widehat{w}\\ \widehat{R}\\ \widehat{R}_b\end{pmatrix}=\begin{pmatrix}a_{1,q} & a_{2,q}\\ a_{1,w} & a_{2,w}\\ a_{1,R} & a_{2,R}\\ (2a-1)a_{1,q} & (2a-1)a_{2,q}\end{pmatrix}\begin{pmatrix}\widehat{\varepsilon}_1\\ \widehat{\varepsilon}_2\end{pmatrix}$$

[38]We do not need additionnal assumptions on the stochastic properties except that they are not perfectly correlated.

Then, (102) is verified for all possible realizations of the shocks if and only if the following equality holds in matrix form (projections on the set of shocks $(\widehat{\varepsilon}_1; \widehat{\varepsilon}_2)$):

$$(1-1/\sigma)(2a-1)\begin{pmatrix} a_{1,q} \\ a_{2,q} \end{pmatrix} = (1-\delta)\begin{pmatrix} a_{1,w} \\ a_{2,w} \end{pmatrix} + \underbrace{\begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}}_{M} \begin{pmatrix} \delta\,(2S-1) \\ 2b(2a-1) \end{pmatrix}$$

The second condition for the portfolio to be unique and determined is that $\det(M) = \left(a_R^1 a_q^2 - a_R^2 a_q^1\right) \neq 0$. This equivalent as assuming that Home excess equity and Home bond excess returns are not perfectly correlated. In that case, the portfolio $(S; b)$ is unique and determined as follows:

$$\begin{pmatrix} \delta\,(2S-1) \\ 2b(2a-1) \end{pmatrix} = \begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}^{-1} \begin{pmatrix} \left[(1-1/\sigma)(2a-1)\right] a_{1,q} - (1-\delta)a_{1,w} \\ \left[(1-1/\sigma)(2a-1)\right] a_{2,q} - (1-\delta)a_{2,w} \end{pmatrix}$$

One can rewrite the same proof by changing the basis of shocks as in our examples by using a projection on $\widehat{q}$ and $\widehat{\varepsilon} = \widehat{\varepsilon}_2$ (providing that $a_{1,q} \neq 0$, i.e. $\hat{q}$ and $\widehat{\varepsilon}$ are not collinear); We obtain the results of section 3 under the assumption that $a_{2,q} = 0$ (i.e. $\gamma_b = 0$) and with an evident change of notation:

$$\begin{pmatrix} \widehat{q} \\ \widehat{w} \\ \widehat{R_e} \\ \widehat{R_b} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (1-\bar{\lambda}) & \gamma_w \\ (1-\bar{\lambda}) & \gamma_e \\ (2a-1) & 0 \end{pmatrix} \begin{pmatrix} \widehat{q} \\ \widehat{\varepsilon} \end{pmatrix}$$

and the optimal portfolio satisfies:

$$\begin{pmatrix} \delta\,(2S-1) \\ 2b(2a-1) \end{pmatrix} = \begin{pmatrix} -\frac{\gamma_w}{\gamma_e}(1-\delta) \\ \left(1-\frac{1}{\sigma}\right) + (2a-1)^{-1}\left(\bar{\lambda}-1\right)(1-\delta)\,(1-\gamma) \end{pmatrix}$$

A.2 Example with shocks to investment expenditures

Like government spending, investment breaks the link between private consumption and output; moreover, when dividends are reinvested, the link between contemporary dividends and wages is also broken. This is another potential candidate to solve the indeterminacy. However, our two-period model is not very convenient to model investment and a dynamic model with endogenous investment would be more appropriate. We will use a short cut for investment in our set-up by assuming some exogenous sunk costs that have to be financed out of dividends in period 1 in order to produce. This exogenous sunk-cost I_i in country i is stochastic (symmetric distribution) with equal mean in both countries ($E_0(I_i) = I$) and represents shocks to investment expenditures. The sunk costs in both countries are paid using a constant fraction of local and foreign goods: we assume that in country i, a share a_I of the sunk cost I_i is spent on goods i (and a share $(1-a_I)$ on foreign goods). We assume $a_I \geq \frac{1}{2}$ to model some bias in investment expenditures towards local inputs (as in Heathcote and Perri (2007b)).

We denote $I = \frac{I_H}{I_F}$ the ratio of Home over Foreign sunk costs (relative investment expenditures) and by $\widehat{I}$ the deviation from its steady state-value of one.

From a portfolio perspective, these stochastic sunk costs are isomorphic in a dynamic set-up to stochastic changes of investment that might be driven by investment specific technological change (Greenwood, Hercowitz and Krusell (1997)), changes in the depreciation rate or some stochastic 'news' about future productivity (Cochrane (1994), Beaudry and Portier (2006)). One can show that the structure of steady-state (time-invariant) portfolios would be the same in a dynamic model where investment is affected by some shocks that are not fully correlated with contemporaneous productivity shocks (as long as financial assets available are similar to the ones of this model.

The ratio of investment spending on Home goods relative to Foreign goods ($q\frac{y_{I,H}}{y_{I,F}}$) depends only on the relative sunk cost across countries:

$$q\frac{y_{I,H}}{y_{I,F}} = \frac{a_I I_H + (1-a_I)I_F}{a_I I_F + (1-a_I)I_H} = \Omega_{a_I}\left[\frac{I_F}{I_H}\right] \tag{103}$$

Or in log-linearized terms:

$$\widehat{y_I} = -\widehat{q} + (2a_I - 1)\widehat{I} \tag{104}$$

As before, relative demand of Home over Foreign goods by consumers (y_C) satisfies equation (15):

$$\widehat{y_C} = -\lambda\widehat{q} \tag{105}$$

Like for government expenditures, equation (104) and (36) together with market clearing conditions of both goods implies the following equilibrium on the goods market:

$$\widehat{y} = (1-s_I)\widehat{y_C} + s_I\widehat{y_I} = -\bar{\lambda}\widehat{q} + s_I(2a_I - 1)\widehat{I} \tag{106}$$

where s_I is the steady-state ratio of investment expenditures over GDP and $\bar{\lambda} = (1-s_I)\lambda + s_I$. Note that like for government expenditures, efficient terms-of-trade $\widehat{q}$ are decreasing with the relative supply of goods $\widehat{y}$ and increasing with shocks to relative sunk costs (due to $a_I \geq \frac{1}{2}$). Higher sunk costs at home crowds out Home goods for private consumption and raises their relative price.

This gives the following relative equity returns (net of the financing of sunk costs) and non-financial incomes:

$$\widehat{R} = (1-\bar{\lambda})\widehat{q} + s_I(2a_I - 1 - \frac{1}{\delta})\widehat{I} \tag{107}$$

$$\widehat{w} = (1-\bar{\lambda})\widehat{q} + s_I(2a_I - 1)\widehat{I} \tag{108}$$

Hence the model with investment shocks is isomorphic to the model with government expenditures in the case where government expenditures are financed entirely from financial incomes ($\delta_g = 1$). Then, obviously, we are back to a system similar to equations (19) and (21) where:

$$\widehat{\varepsilon} = s_I(2a_I - 1 - \frac{1}{\delta})\widehat{I} \tag{109}$$

$$\gamma = \frac{2a_I - 1}{2a_I - 1 - \frac{1}{\delta}} \tag{110}$$

and the equilibrium equity and bond portfolios are:

$$S^* = \frac{1}{2}[1+\frac{(2a_I-1)(1-\delta)}{1-\delta(2a_I-1)}] \quad (111)$$

$$b^* = (1-s_I)(1-\frac{1}{\sigma})+(2a-1)^{-1}(\bar{\lambda}-1)[\delta\,(2S^*-1)+(1-\delta)] \quad (112)$$

As noted above, the optimal portfolio have exactly the same structure as the portfolio with government shocks financed out of dividends. This is very intuitive since sunk costs act here as a tax on dividends. Moreover, as previously, in presence of bias towards local inputs ($a_I \geq \frac{1}{2}$), an increase in investment expenditures at Home raises Home wages while decreasing Home dividends. Consequently, an equity portfolio biased towards Home equity insulates consumption expenditures from changes in investment expenditures.

A.3 Detailed derivation for the model with non-tradable goods

A.3.1 Intratemporal allocation across goods

In period 1 (after the realization of productivity shocks), a representative consumer in country (i) maximizes:

$$\left[\frac{(C_i)^{1-\sigma}}{1-\sigma}\right]$$

subject to a budget constraint (for $j \neq i$):

$$\begin{aligned} p_i^T c_{ii}^T + p_j^T c_{ij}^T + p_i^{NT} c_i^{NT} &\leq I_i \qquad (\lambda_H) \\ P_i C_i &\leq I_i \qquad (\lambda_H) \end{aligned}$$

where I_i are total asset incomes of the representative agent in country (i), λ_i is the Lagrange-Multiplier associated to the budget constraint. At this point, I take portfolios chosen in period 0 as given.
The first-order conditions are:
For consumption:

$$1 = \lambda_i P_i C_i^{\sigma} \quad (113)$$

Intratemporal allocation across goods:

$$c_{ii}^T = a\left(\frac{p_i^T}{P_i^T}\right)^{-\phi} c_i^T \quad (114)$$

$$c_{ij}^T = (1-a)\left(\frac{p_j^T}{P_i^T}\right)^{-\phi} c_i^T \quad (115)$$

$$c_i^T = \eta\left(\frac{P_i^T}{P_i}\right)^{-\theta} C_i \quad (116)$$

$$c_i^{NT} = (1-\eta)\left(\frac{P_i^{NT}}{P_i}\right)^{-\theta} C_i \quad (117)$$

Using equations (114) and (115) for both countries and market-clearing conditions for tradable goods (??) and (??) gives:

$$q^{-\phi}\Omega_a\left[(\frac{P_F^T}{P_H^T})^{\phi}\frac{c_F^T}{c_H^T}\right]=\frac{y_H^T}{y_F^T} \tag{118}$$

where $\Omega_u(x)$ is a continuous function of two variables (u,x) such that: $\Omega_u(x)=\frac{1+x(\frac{1-u}{u})}{x+(\frac{1-u}{u})}$

Then, using (116), we get:

$$q^{-\phi}\Omega_a\left[(\frac{P_F^T}{P_H^T})^{\phi-\theta}\frac{P_F^{\theta}C_F}{P_H^{\theta}C_H}\right]=\frac{y_H^T}{y_F^T} \tag{119}$$

Using equations (117) and market-clearing conditions for non-tradable goods for both countries (??) and (??), we get:

$$(\frac{P_H^{NT}}{P_F^{NT}})^{-\theta}\frac{P_H^{\theta}C_H}{P_F^{\theta}C_F}=\frac{y_H^{NT}}{y_F^{NT}} \tag{120}$$

A.3.2 Budget constraints

Recall that each household holds shares S^k and $1-S^k$ of local and foreign stocks in sector $k=\{T,NT\}$, respectively, while b denotes her holding of bonds denominated in her local composite good; also, 'tradable' stock $j's$ dividend is $p_j^T y_j^T$ and 'non-tradable' stock $j's$ dividend is $P_j^{NT}y_j^T$. The period 1 (relative) budget constraints of countries H and F are thus:

$$\begin{aligned} P_HC_H-P_FC_F &= \big(\delta(2S^T-1\big)+(1-\delta))(p_H^Ty_H^T-p_F^Ty_F^T) \\ &+\big(\delta(2S^{NT}-1\big)+(1-\delta))(P_H^{NT}y_H^{NT}-P_F^{NT}y_F^{NT})+2b(P_H-P_F) \end{aligned} \tag{121}$$

which says that the difference between countries' consumption spending equals the difference between their incomes.

A.3.3 Log-linearization of the model

Henceforth, we write $y^T\equiv\frac{y_H^T}{y_F^T}, y^{NT}\equiv\frac{y_H^{NT}}{y_F^{NT}}$ to denote relative outputs in both sectors. We log-linearize the model around the symmetric steady-state where y^T and y^{NT} equal unity, and use $\widehat{x}\equiv log(x/\bar{x})$ to denote the log deviation of a variable x from its steady state value $\overline{x}$.

The log-linearization of the Home country's real exchange rate $RER\equiv\frac{P_H}{P_F}$ gives:

$$\widehat{RER}=\widehat{\frac{P_H}{P_F}}=\eta(2a-1)\widehat{q}+(1-\eta)\widehat{P^{NT}}. \tag{122}$$

where $P^{NT}=P_H^{NT}/P_F^{NT}$ is the relative price of Home non-tradable goods over Foreign non-tradable goods and η is the steady-state share of spending devoted to tradable goods.[39] Note also that the relative price of the tradable composite in both countries verifies: $P_H^T/P_F^T=(2a-1)\widehat{q}$.

[39] Here to simplify notations, we assume that the share of spending devoted to tradable goods is the same as the weight of tradable goods in the consumption index. This is true only if in the steady state tradable and non tradable goods have the same price: $p^{T*}=p^{NT*}$. This assumption is however irrelevant for equity portfolios (see Obstfeld [2007]).

When markets are locally complete, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the following, familiar condition:

$$-\sigma(\widehat{C_H}-\widehat{C_F})=\widehat{RER}=\eta(2a-1)\widehat{q}+(1-\eta)\widehat{P^{NT}}. \tag{123}$$

Log-linearizing (120) and using (123) implies:

$$\widehat{y^{NT}}=-\theta\widehat{P^{NT}}+(\theta-\frac{1}{\sigma})\widehat{RER} \tag{124}$$

Similarly, log-linearizing (119) and using (123) implies:

$$\begin{aligned}\widehat{y^T} &= -\left[\phi\left(1-(2a-1)^2\right)+(2a-1)^2\left((1-\eta)\theta+\eta\frac{1}{\sigma}\right)\right]\widehat{q}+(1-\eta)(\theta-\frac{1}{\sigma})(2a-1)\widehat{P^{NT}}\\ &= -\lambda\widehat{q}+(1-\eta)(\theta-\frac{1}{\sigma})(2a-1)\widehat{P^{NT}} \end{aligned}\tag{125}$$

where $\lambda\equiv\phi(1-(2a-1)^2)+(2a-1)^2\left((1-\eta)\theta+\frac{\eta}{\sigma}\right)$. Note that $\lambda>0$ as $1/2<a<1$.

We next log-linearize equation (121); using (123) and we obtain:

$$\begin{aligned}\widehat{P_HC_H}-\widehat{P_FC_F} &= \eta\delta\left(2S^T-1\right))\widehat{R_e^T}+(1-\eta)\delta\left(2S^{NT}-1\right)\widehat{R_e^{NT}}+(1-\delta)\widehat{w}+2b\widehat{RER}\\ &= (1-\frac{1}{\sigma})\widehat{RER}\end{aligned}\tag{126}$$

where $\widehat{R^k}$ denotes Home excess return in sector $k=\{T,NT\}$ and $\widehat{w}$ denote relative non-financial income (Home over Foreign aggregated over both sectors).

Following the benchmark model and adding an additional source of uncertainty $\widehat{\varepsilon}$, we have the following relationships:

$$\begin{aligned}\widehat{R_e^T} &= \widehat{q}+\widehat{y^T}+\gamma_e\widehat{\varepsilon}\\ \widehat{R_e^{NT}} &= \widehat{P^{NT}}+\widehat{y^{NT}}+\gamma_e\widehat{\varepsilon}\\ \widehat{w} &= \eta\widehat{R^T}+(1-\eta)\widehat{R^{NT}}+(\gamma_w-\gamma_e)\widehat{\varepsilon}\end{aligned}\tag{127}$$

Using (124) and (125), this gives under efficient risk-sharing:

$$\begin{aligned}\widehat{R^T} &= (1-\lambda)\widehat{q}+(1-\eta)(\theta-\frac{1}{\sigma})(2a-1)\widehat{P^{NT}}+\gamma_e\widehat{\varepsilon}\\ \widehat{R^{NT}} &= (1-\theta)\widehat{P^{NT}}+(\theta-\frac{1}{\sigma})\widehat{RER}+\gamma_e\widehat{\varepsilon}\\ \widehat{w} &= \eta\widehat{R^T}+(1-\eta)\widehat{R^{NT}}+(\gamma_w-\gamma_e)\widehat{\varepsilon}\end{aligned}\tag{128}$$

The financial market is effectively complete (up to a first order approximation) when there exists a portfolio (S^T,S^{NT},b) such that (124), (125) and relative budget constraint hold for arbitrary realizations of the relative shocks $\widehat{y^T},\widehat{y^{NT}}$ and $\widehat{\varepsilon}$.

A.3.4 Equilibrium portfolios

Projection of equation (126) on $\widehat{\varepsilon}$ gives the averaged equity portfolio across sectors:

$$\eta S^{T}+(1-\eta)S^{NT}=\frac{1}{2}\left(1-\frac{\gamma_w}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \tag{129}$$

Projections on $\widehat{q}$ and $\widehat{P^{NT}}$ give the following relationships (assuming $a\neq 1/2$) and rearranging terms give the following sharing rule for the equity bias across sectors:

$$\frac{2S^{T}-1+(1-\delta)/\delta}{2S^{NT}-1+(1-\delta)/\delta}=\frac{(\theta-1)(2a-1)}{\phi-1+(2a-1)^{2}(\theta-\phi)}=\Omega \tag{130}$$

Solving further for equity portfolio using (88) and (130) gives:

$$\begin{aligned} S^{T} &= \frac{1}{2}\left(1-\frac{1-\delta}{\delta}+\frac{1-\delta}{\delta}\frac{\Omega(1-\gamma)}{\eta\Omega+1-\eta}\right) \\ S^{NT} &= \frac{1}{2}\left(1-\frac{1-\delta}{\delta}+\frac{1-\delta}{\delta}\frac{1-\gamma}{\eta\Omega+1-\eta}\right) \end{aligned}$$

The bond position satisfies:

$$b=\frac{1}{2}(1-1/\sigma)-\frac{1}{2}\frac{(1-\gamma)(1-\delta)}{\eta\Omega+1-\eta}\left(1-\theta+(\theta-1/\sigma)(1-\eta+\eta(2a-1)\Omega)\right) \tag{131}$$

A.4 Estimation of returns to non-financial wealth: VAR results

[to be done]

A.5 Countries of different sizes

[need to be generalized to the case $\gamma_b\neq 0$]

We extend our benchmark model by allowing different country sizes. We assume taht expected production in period $t=1$ is not equal across countries: $E_0(y_H)=\overline{y_H}$ and $E_0(y_F)=\overline{y_F}$. We denote by ω_i the relative size of country i: $\omega_i=\frac{\overline{y_i}}{\overline{y_i}+\overline{y_j}}$, with $\omega_H+\omega_F=1$.

Both countries also differ in their consumption Home bias:

$$C_i=\left[a_i^{1/\phi}\left(c_{ii}\right)^{(\phi-1)/\phi}+(1-a_i)^{1/\phi}\left(c_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)}$$

We assume that is the non-stochastic equilibrium, the following relationship holds:

$$(1-a_H)\,y_H=(1-a_F)y_F$$

This ensure that in the strade balance is zero and terms-of-trade q are equal to unity in the non-stochastic equillibrium.

Following the benchmark model, we assume that the reduced-form of returns on equities and on non-financial income satisfies for $i = \{H, F\}$:

$$\begin{aligned}
\widehat{R_{ei}} &= \widehat{p_i} + \widehat{y_i} + \gamma_e \widehat{\varepsilon_i} \\
\widehat{w_i} &= \widehat{p_i} + \widehat{y_i} + \gamma_w \widehat{\varepsilon_i}
\end{aligned}$$

Keeping the same notations as in the symmetric case, log-linearization around the non-stochastic equilibrium implies the following relationships since one can easily verify that markets are still locally complete:

$$\begin{aligned}
\widehat{RER} &= (a_H + a_F - 1)\widehat{q} \\
\widehat{PC} &= (1 - \frac{1}{\sigma})\widehat{RER} = (1 - \frac{1}{\sigma})(a_H + a_F - 1)\widehat{q}
\end{aligned}$$

$$\widehat{y} = -\lambda \widehat{q}$$

where λ is now such that: $\lambda = \phi(1 - (a_H + a_F - 1)^2) + \frac{(a_H + a_F - 1)^2}{\sigma}$.

Bond returns are still indexed on the country CPI such that Home bond excess returns $\widehat{R_b}$ are related to the the real exchange rate as follows (assuming $\gamma_b = 0$):

$$\widehat{R_b} = \widehat{RER} = (a_H + a_F - 1)\widehat{q}$$

Log-linearization of the budget constraint in country i gives (using market clearing conditions in the asset market) for $i \neq j$:

$$\widehat{P_i C_i} = (1-\delta)\widehat{w_i} + \delta S_{ii}\widehat{d_i} + \frac{\omega_j}{\omega_i}\delta\left(1 - S_{jj}\right)\widehat{d_j} + b_{ii}\widehat{P_i} - b_{jj}\widehat{P_j}$$

Taking the difference across countries, we get:

$$\begin{aligned}
\widehat{PC} &= (1 - \frac{1}{\sigma})\left(\widehat{P_H} - \widehat{P_F}\right) \\
&= (1-\delta)\left(\widehat{w_H} - \widehat{w_F}\right) + \delta\widehat{d_H}\left(S_{HH} - \frac{\omega_H}{\omega_F}(1 - S_{HH})\right) \\
&\quad -\delta\widehat{d_F}\left(S_{FF} - \frac{\omega_F}{\omega_H}(1 - S_{FF})\right) + 2b_{HH}\widehat{P_H} - 2b_{FF}\widehat{P_F}
\end{aligned}$$

Projection on $\widehat{\varepsilon_H}$ gives the following holdings of Home stocks by Home households:

$$\begin{aligned}
0 &= \omega_F(1-\delta)\gamma_w + \delta\gamma_e\left(\omega_F S_{HH} - \omega_H(1 - S_{HH})\right) \\
S_{HH} &= \omega_H - \frac{1-\delta}{\delta}\frac{\gamma_e}{\gamma_w}(1 - \omega_H)
\end{aligned}$$

Simlarly, holdings of Foreign stocks by Foreign households satisfy:

$$S_{FF} = \omega_F - \frac{1-\delta}{\delta}\frac{\gamma_e}{\gamma_w}(1 - \omega_F)$$

Bond holdings must satisfy:

$$b^{*}= b_{HH}= b_{FF}=\frac{1}{2}(1-\frac{1}{\sigma})+\frac{1}{2}(1-\delta)(1-\frac{\gamma_e}{\gamma_w})(\lambda-1)(a_H+a_F-1)^{-1}$$

Because the model in reduced form can be rewritten as follows:

$$\begin{aligned}
\widehat{R}_e &= (1-\bar{\lambda})\widehat{q}+\gamma_e\widehat{\varepsilon} \\
\widehat{R}_b &= (a_H+a_F-1)\widehat{q} \\
\widehat{w} &= (1-\bar{\lambda})\widehat{q}+\gamma_w\widehat{\varepsilon}
\end{aligned} \tag{132}$$

We can rewrite the previous system in terms of loadings:

$$\begin{aligned}
\widehat{w} &= (1-\frac{\gamma_w}{\gamma_e})(1-\bar{\lambda})(a_H+a_F-1)^{-1}\widehat{R}_b+\frac{\gamma_w}{\gamma_e}\widehat{R}_e \\
&= \beta_{w,b}\widehat{R}_b+\beta_{w,e}\widehat{R}_e \\
\widehat{q} &= \beta_{RER,b}\widehat{R}_b \text{ with } \beta_{RER,b}=1
\end{aligned}$$

In terms of loadings this gives (under the maintained assumption that $\gamma_b = 0$).

$$\begin{aligned}
S_i^{*} &= \omega_i-\frac{1-\delta}{\delta}\beta_{w,e}(1-\omega_i) \\
b^{*} &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b}-\frac{1}{2}\left(1-\delta\right)\beta_{w,b}
\end{aligned}$$

where S_i^{*} denotes the Holdings of stocks in country i by households of country i.

A.6 Home Bias in Equities

Source Country	Domestic Market in % of World Market Capitalization (1)	Share of Portfolio in Domestic Equity in % (2)	Degree of Home Bias $= HB_i$ (3)
Australia	1.9	83.6	0.832
Austria	0.3	58.5	0.583
Belgium	0.7	49.8	0.494
Canada	3.5	76.6	0.757
Denmark	0.4	62.7	0.625
Finland	0.5	63.3	0.631
France	4.2	68.8	0.674
Germany	2.9	57.5	0.562
Greece	0.3	93.4	0.933
Italy	1.9	57.1	0.562
Japan	13.2	91.9	0.906
Netherlands	1.4	32.1	0.311
New-Zealand	0.1	59.8	0.597
Norway	0.5	52	0.517
Portugal	0.2	77.8	0.777
Spain	2.3	86.3	0.859
Sweden	1	59.4	0.589
Switzerland	2.2	59.9	0.589
United Kingdom	7.3	65	0.622
United States	40.5	82.2	0.700
Average	3.67	70.58	0.70

Table 5: Home Bias in Equities in 2005 (from Sercu and Vanpee (2007); source CPIS).HB_i =1-(Share of Foreign Equities in Country i Equity Holdings)/(Share of Foreign Equities in the World Market Portfolio). By definition HB_i is equal to zero if the share of domestic equities in country i's portfolio is equal to the share of domestic equities in the world market portfolio and HB_i is equal to 1 if there is full equity home bias.