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Abstract

Consumption-habits have become an integral component in new Keynesian models. However, consumption-habits can be modeled in a host of different ways and this diversity is reflected in the literature. I examine whether different approaches to modeling consumption habits have important implications for business cycle behavior. Using a standard new Keynesian business cycle model, I show that, to a first-order log-approximation, the consumption Euler equation associated with the additive functional form for habit formation encompasses the multiplicative function form. Empirically, I show that whether consumption habits are internal or external has little effect on the model's business cycle characteristics.

Keywords: *Habit formation, Business cycles.*

JEL Classification: E52, E58.

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1 Introduction

Consumption-habits are a key component in modern new Keynesian business cycle models.[1] They are relied on to explain movements in aggregate consumption data and to generate the "hump-shaped" impulse responses widely recognized to characterize the responses of output and consumption to demand and supply shocks. With consumption-habits there is complementarity between consumption in successive periods, the household utility function is time-inseparable, and the marginal rate of substitution between consumption today and consumption at any point in the future depends on the path consumption is expected to take in the interim. Although consumption habit formation appears useful empirically, there appears to be little consensus on how habit formation should be modeled. Thus, while McCallum and Nelson (1999), Amato and Laubach (2004), and Christiano, Eichenbaum, and Evans (2005) assume that the habit formation is *internal* to households, Smets and Wouters (2003), and Ravn, Schmitt-Grohé, and Uribe (2006) assume that the habit formation is *external*. Similarly, where Fuhrer (2000), McCallum and Nelson (1999), and Amato and Laubach (2004) assume that it is consumption *relative* to the habit stock that enters utility (multiplicative habits), Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) assume that what matters is the *difference* between consumption and the habit stock (additive habits).[2]

In light of the host of different ways that habit formation can be modeled, I examine whether the particular modeling choice has material consequences for the business cycle properties of a standard new Keynesian model. To this end, I present a new Keynesian business cycle model, typical of those used to analyze monetary policy, and I derive the consumption Euler equations that arise from different modeling assumptions regarding habit formation. I consider both internal and external habit formation and both multiplicative and additive functional forms and, for a popular class of utility functions, find, surprisingly, that to a log-linear approximation whether habits are additive or multiplicative, internal or external, appears largely unimportant for business cycle behavior. This is not to say that different approaches to modeling consumption-habits are innocuous more generally. It is well-known, for example, that, in the absence of an optimally designed consumption tax, the competitive equilibrium

[1] Consumption habits have a long history in macroeconomics. Duesenberry (1949) argues that habit formation can arise through a desire to advance socially or to acquire high quality goods, desires prompted by the "*...inferiority feelings that are aroused by unfavorable comparisons between living standards.*" Similarly, Ryder and Heal (1973) argue that the "*... satisfaction that a man derives from consuming a given bundle of goods depends not only on that bundle, but also on his past consumption and on his general social environment.*"

[2] Schmitt-Grohé and Uribe (2005) provide a useful overview of these different approaches to modeling habit formation.

is Pareto inefficient when the habit formation is external. Moreover, due to the effect it has on a firm's pricing decision, the choice of additive or multiplicative habits can have important implications when habits are modeled at the goods level (Ravn, Schmitt-Grohé, and Uribe 2006). In addition, Abel (1990) has shown that asset prices, and the magnitude of the equity premium, depend on how habit formation is introduced.

Habit formation has been used to explain a (Granger) causal relationship from high growth to a high saving rate (Carroll, Overland, and Weil 2000), why recessions are so feared, despite their short durations (Campbell and Cochrane 1999), and as a mechanism to capture the inertial, humped-shaped, impulse responses for output that are generated by structural VAR models (Fuhrer 2000). Habit formation has also been used to help understand why current accounts are so volatile (Gruber 2004), to explain why exchange rate pegs tend to be associated with rising consumption and appreciating real exchange rates (Uribe 2002) and, because it breaks the link between the coefficient of relative risk aversion and the (inverse) elasticity of intertemporal substitution, to study the equity premium puzzle (Abel 1990; Constantinides 1990; Campbell and Cochrane 1999). Consumption habits are also attractive because they improve the characteristics of real business cycle models (Boldrin, Christiano, and Fisher 2001) and because they make consumption endogenously persistent, an important feature of estimated consumption Euler equations.

The remainder of the paper is structured as follows. In the following section the model describing household behavior is outlined and estimable first-order conditions are presented and discussed. Section 3 shows that, to a log-linear approximation, the consumption Euler equations with additive habits encompass those with multiplicative habits. Section 4 uses an estimated new Keynesian model to compare the business cycle properties of internal and external habits. Section 5 concludes.

2 Households

The representative household maximizes a utility function defined over consumption, real money balances, and labor. Households are infinitely lived, they rent their labor to firms in a perfectly competitive market, and they transfer wealth through time either by purchasing one-period nominal bonds or by holding nominal money balances. With c_t denoting household consumption, m_t denoting household nominal money balances, l_t denoting household labor supply, P_t denoting the aggregate price level, and H_t denoting the habit stock, the representative household's decision problem is to choose $\{c_t, m_t, l_t\}_0^\infty$ to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[e^{g_t} u(c_t, H_t) + m\left(\frac{m_t}{P_t}\right) - \frac{l_t^{1+\chi}}{1+\chi} \right], \quad \chi > 0, \tag{1}$$

subject to the budget constraint

$$c_t + \frac{m_t}{P_t} + \frac{b_t}{P_t} = w_t l_t + \frac{m_{t-1}}{P_t} + \frac{(1+R_{t-1})\, b_{t-1}}{P_t} + \frac{d_t}{P_t}, \tag{2}$$

where b_t and d_t denote household bond holdings and lump-sum payments (dividends and seigniorage revenues), respectively, w_t denotes the real wage, and R_t denotes the nominal return on bonds. Movements in g_t, an aggregate consumption preference shock, alter the marginal utility of consumption, affecting the household's willingness to substitute between consumption and leisure and between consumption at different points in time. When the habit formation is internal, H_t depends on the household's own consumption; when the habit formation is external, H_t depends on aggregate consumption.

Separate to whether the habit formation is internal or external, models of habit formation differ with respect to functional form. Although there are other possibilities, by far the most common assumptions are that the habit formation enters utility either *multiplicatively* or *additively.*

2.1 Two common functional forms

Assuming that the utility function is of the isoelastic form, with multiplicative habits the period utility function is

$$u(c_t, H_t) = \frac{e^{g_t}\left(\frac{c_t}{H_t}\right)^{1-\sigma}}{1-\sigma}, \tag{3}$$

where $\sigma \in (1, \infty)$. Note that σ is restricted to be greater than one; this restriction may seem unusual, but it is straightforward to show that it is a necessary condition for joint concavity of the utility function. Moreover, $\sigma > 1$ is sufficient to ensure both that the elasticity of intertemporal substitution is correctly signed, i.e., that the expected change in consumption be inversely related to the *ex ante* real interest rate, and that the coefficient on lagged consumption is positive for all $\gamma \in [0, 1)$.

To model the habit stock, I follow Abel (1990) and assume that H_t evolves according to

$$H_t = \left(c_{t-1}^D C_{t-1}^{1-D}\right)^\gamma, \tag{4}$$

where $\gamma \in [0, 1)$ and $D \in \{0, 1\}$. With C_t denoting aggregate consumption, several special cases of equations (3) and (4) are worth commenting upon. First, when $\gamma = 0$, equation

(3) simplifies to the standard time-separable specification. Second, when $D = 0$ the habit formation is external to the household, i.e., is of the "catching up with the Joneses" type. With external habits household consumption depends on what other households are consuming and the desire to catch up to what others are consuming leads to a consumption externality whereby households overconsume. Finally, when $D = 1$ it is the household's own consumption that affects the habit stock, the habit formation is internal, and the habit formation does not create a consumption externality.

Given equations (1) – (4), I show in Appendix A that the log-linearized consumption-Euler equation takes the form

$$\begin{aligned} \mathrm{E}_t \Delta \widehat{c}_{t+1} &= \frac{\gamma(\sigma-1)}{[\sigma+\gamma\beta D(\gamma(\sigma-1)-1)]} \mathrm{E}_t [\Delta \widehat{c}_t + \beta D \Delta \widehat{c}_{t+2}] \\ &+ \frac{1}{[\sigma+\gamma\beta D(\gamma(\sigma-1)-1)]} [(1-\gamma\beta D)(R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho) - g_t], \end{aligned} \tag{5}$$

where $\rho \equiv -\ln(\beta)$ and Δ is the difference operator.

When $D = 0$, the habit formation is external and equation (5) simplifies to

$$\widehat{c}_t = \frac{\gamma(\sigma-1)}{\sigma+\gamma(\sigma-1)} \widehat{c}_{t-1} + \frac{\sigma}{\sigma+\gamma(\sigma-1)} \mathrm{E}_t \widehat{c}_{t+1} - \frac{1}{\sigma+\gamma(\sigma-1)} (R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho - g_t), \tag{6}$$

whereas when $D = 1$ the habit formation is internal and equation (5) becomes

$$\begin{aligned} \mathrm{E}_t \Delta \widehat{c}_{t+1} &= \frac{\gamma(\sigma-1)}{[\sigma+\gamma\beta(\sigma\gamma-1-\gamma)]} \mathrm{E}_t [\Delta \widehat{c}_t + \beta \Delta \widehat{c}_{t+2}] \\ &+ \frac{1}{[\sigma+\gamma\beta(\sigma\gamma-1-\gamma)]} [(1-\gamma\beta)(R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho) - g_t]. \end{aligned} \tag{7}$$

Separate to whether the habit formation is internal or external, a consequence of habit formation is that the elasticity of intertemporal substitution depends on parameters other than σ, the coefficient of relative risk aversion. Comparing equations (6) and (7), a key difference between them is that with internal habits current consumption depends not only on next period's expected consumption, but also on expected consumption two periods ahead.

An alternative way of introducing habit formation is to assume that the habit stock enters the utility function additively, i.e., that utility depends on the difference between consumption and the habit stock. Retaining the isoelastic utility function, with additive habits $u(c_t, H_t)$ is specified to be

$$u(c_t, H_t) = \frac{e^{g_t}(c_t - H_t)^{1-\alpha}}{1-\alpha}, \tag{8}$$

where the habit stock, H_t, now obeys

$$H_t = \eta \left(c_{t-1}^{D} C_{t-1}^{1-D} \right), \tag{9}$$

with $\alpha \in (0, \infty)$ and $\eta \in [0, 1)$. Because consumption must always be greater than the habit stock, additive habits are closely related to the notion that there is a subsistence level below which a household's consumption cannot fall without catastrophe.

With these functional forms, utility maximization produces the following log-linear consumption-Euler equation (see Appendix A)

$$\begin{aligned} \mathrm{E}_t \Delta \widehat{c}_{t+1} &= \frac{\eta}{(1+\eta^2 \beta D)} \mathrm{E}_t \left[\Delta \widehat{c}_t + \beta D \Delta \widehat{c}_{t+2} \right] \\ &+ \frac{(1-\eta)(1-\eta \beta D)}{\alpha (1+\eta^2 \beta D)} \left(R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho \right) - \frac{(1-\eta)}{\alpha (1+\eta^2 \beta D)} g_t. \end{aligned} \tag{10}$$

In the special case where $D = 0$, equation (10) simplifies to

$$\widehat{c}_t = \frac{\eta}{1+\eta} \widehat{c}_{t-1} + \frac{1}{1+\eta} \mathrm{E}_t \widehat{c}_{t+1} - \frac{(1-\eta)}{\alpha (1+\eta)} \left(R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho - g_t \right), \tag{11}$$

whereas when $D = 1$, equation (10) becomes

$$\begin{aligned} \mathrm{E}_t \Delta \widehat{c}_{t+1} &= \frac{\eta}{(1+\eta^2 \beta)} \mathrm{E}_t \left[\Delta \widehat{c}_t + \beta \Delta \widehat{c}_{t+2} \right] \\ &+ \frac{(1-\eta)(1-\eta \beta)}{\alpha (1+\eta^2 \beta)} \left(R_t - \mathrm{E}_t \widehat{\pi}_{t+1} - \rho \right) - \frac{(1-\eta)}{\alpha (1+\eta^2 \beta)} g_t. \end{aligned} \tag{12}$$

In contrast to multiplicative habits, joint concavity of the utility function does not place any restriction on the coefficient of relative risk aversion and $\alpha > 0$ is sufficient to ensure that the coefficients on lagged consumption and the *ex ante* real interest rate are correctly signed.

3 Additive versus multiplicative habits

Although it is clear that there are similarities between the log-linear consumption-Euler equations derived assuming multiplicative and additive habits, in this section I present two propositions (proved in Appendix B) that show that these similarities make it (almost) impossible to distinguish between them.

Proposition 1: With external habits, the log-linear consumption-Euler equation and the log-linear labor supply equation derived under multiplicative habits are encompassed by those derived under additive habits, but the converse is not true.

Proposition 1 establishes that when the habit formation is external additive and multiplicative habits are almost observationally equivalent with respect to consumption and leisure decisions, and to the extent that they are not observationally equivalent, additive habits encompass multiplicative habits.

Proposition 2: With internal habits, the log-linear consumption-Euler equation derived under multiplicative habits is a special case of the equation derived under additive habits.

Proposition 2 establishes that when the habit formation is internal it is always possible to parameterize a model with additive habits that replicates to a log-linear approximation the consumption Euler equation derived from a model with multiplicative habits.

4 Internal versus external habits

I now examine the relationship between internal habits and external habits. To this end, I place separately equations (11) and (12) within a small-scale New Keynesian business cycle model and use full information maximum likelihood (FIML) to estimate the two versions of the model. I show that the coefficient estimates are similar and that the model behaves similarly regardless of whether the habit formation is internal or external.

To close the model I require equations for inflation and the short-term nominal interest rate. For the inflation equation, I employ the partial-indexation specification developed by Smets and Wouters (2003)

$$\widehat{\pi}_t = \frac{\omega}{1+\omega\beta}\widehat{\pi}_{t-1} + \frac{\beta}{1+\omega\beta}E_t\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{(1+\omega\beta)\theta}\widehat{c}_t + v_t, \tag{13}$$

in which $1-\theta$, $\theta \in (0,1]$, describes the share of monopolistically competitive firms that are able to optimally set their price each quarter and $\omega \in [0,1]$ is an indexation parameter describing the proportion of last period's inflation that the non-optimizing firms use to update their price. With respect to the short-term nominal interest rate, I assume that the central bank conducts monetary policy according to

$$R_t = (1-\phi_3)(\phi_0 + \phi_1 \mathrm{E}_t\pi_{t+1} + \phi_2\widehat{c}_{t-1}) + \phi_3 R_{t-1} + \varepsilon_t, \tag{14}$$

which is a standard forward-looking Taylor-type rule specification.

To estimate the two specifications I use FIML. Specifically, for a given specification and given parameter values, $\Gamma = \{\gamma, \sigma, \rho, \theta, \omega, \phi_0, \phi_1, \phi_2, \phi_3\}$, I solve for the rational expectations equilibrium, which, for both models, has the form

$$\mathbf{z}_t = \mathbf{h}(\Gamma) + \mathbf{H}(\Gamma)\mathbf{z}_{t-1} + \mathbf{G}(\Gamma)\mathbf{v}_t, \tag{15}$$

where $\mathbf{z}_t = \begin{bmatrix} \pi_t & \widehat{c}_t & R_t \end{bmatrix}'$ and $\mathbf{v}_t = \begin{bmatrix} v_t & g_t & \varepsilon_t \end{bmatrix}'$. From equation (15), the concentrated log-likelihood function is

$$\log L_c\left(\Gamma; \{\mathbf{z}_t\}_1^T\right) \propto (T-1)\ln\left(\left|\mathbf{G}(\Gamma)^{-1}\right|\right) - \frac{(T-1)}{2}\ln\left(\left|\widehat{\mathbf{\Omega}}(\Gamma)\right|\right), \tag{16}$$

where

$$\widehat{\mathbf{\Omega}}(\Gamma) = \sum_{t=2}^{T} \frac{\left[\mathbf{G}(\Gamma)^{-1}\left(\mathbf{z}_t - \mathbf{h}(\Gamma) - \mathbf{H}(\mathbf{\Gamma})\,\mathbf{z}_{t-1}\right)\right]\left[\mathbf{G}(\Gamma)^{-1}\left(\mathbf{z}_t - \mathbf{h}(\Gamma) - \mathbf{H}(\mathbf{\Gamma})\,\mathbf{z}_{t-1}\right)\right]'}{T-1}.$$

The coefficient estimates for the two (additive habit) specifications are summarized in Table 1, with standard errors in parenthesis.[3]

Table 1: Estimates assuming additive habits

	Parameter								
Habits	ρ	η	α	θ	ω	$\overline{\pi}$	ϕ_1	ϕ_2	ϕ_3
Internal	2.358	0.832	3.180	0.882	0.682	3.399	1.445	0.799	0.867
	(0.851)	(0.098)	(4.651)	(0.036)	(0.075)	(2.009)	(1.177)	(0.880)	(0.040)
External	2.383	0.824	5.647	0.883	0.685	3.356	1.486	0.790	0.866
	(0.820)	(0.084)	(5.359)	(0.035)	(0.075)	(1.711)	(1.143)	(0.866)	(0.040)

The estimates of ρ imply a value for the quarterly discount factor, β, of about 0.994. With either internal or external habits, η is estimated to be about 0.8. Assuming internal habits, Fuhrer (2000) estimates η to be either 0.8 or 0.9, depending on the estimator used, McCallum and Nelson (1999) calibrate η to 0.8, while Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum, and Evans (2005) estimate η to be 0.73 and 0.63, respectively. Gruber's (2004) estimate for the United States place η at 0.816. I estimate α to be about 3 with internal habits and and 5.5 with external habits.

On the pricing side, I estimate θ to be about 0.88, which suggests that firms reoptimize their prices relatively infrequently. Among those firms that cannot change their price, I estimate that their indexation parameter is about 0.68. Although the estimates are imprecise, the Taylor rule coefficients imply that the Taylor principle holds over the sample period and reveal considerable interest rate inertia. Interestingly, the greatest difference between the two sets of estimates lies in the coefficient of relative risk aversion, α, which is somewhat higher when the habit formation is external.

4.1 Business cycle characteristics

Table 1 shows that the parameter estimates with internal habits are very similar to those with external habits, but since these parameters enter the model differently (because the consumption Euler equations differ) this does not necessarily mean that the two specifications

[3] For each specification, $\widehat{c}_t$ is measured by detrending the log of PCE consumption using the Hodrick-Prescott filter, π_t is measured using the annualized quarterly percent change in the PCE price index, and R_t is measured using the quarterly average of the federal funds rate expressed at an annual rate. The sample period begins in 1983.Q1 and ends in 2004.Q2, covering the low inflation period that followed Paul Volcker's appointment, but excluding the period of reserves targeting that occurred in the early 1980s.

have similar reduced forms, nor does it necessarily imply that they generate similar business cycle behavior.

Table 2: Unconditional (co)variances

	Internal			External		
	π_t	$\widehat{c}_t$	R_t	π_t	$\widehat{c}_t$	R_t
π_t	1.989	0.408	0.476	1.973	0.392	0.450
$\widehat{c}_t$	–	0.571	0.194	–	0.563	0.191
R_t	–	–	2.553	–	–	2.548

To compare their business cycle characteristics, I report in Table 2 the unconditional (co)variances for inflation, consumption, and the federal funds rate implied by the two models. These (co)variances reveal how, and to what extent, the two models differ in terms of their unconditional moments. Strikingly, although the (co)variances are not identical, they are extremely similar. So similar, in fact, that an appeal to one form of habit formation over another is unlikely to better fit these data, or to help resolve puzzles involving the correlations between these variables.[4]

To further explore the similarities between internal and external habits, Figure 1 presents impulse response functions showing how the two models respond to demand and supply shocks. Also shown are the responses from a recursively identified VAR(2) model.[5]

[4] In an econometric sense, the fact that the (co)variances differ for the two models implies that the parameter D is identified. At the same time, the similarities between the (co)variances mean that the identification is weak.

[5] A transformation has been applied to the two new Keynesian models to allow their impulse response functions to be compared to those from the VAR. The details of this transformation, which are straightforward to apply, can be found in Dennis (2004).

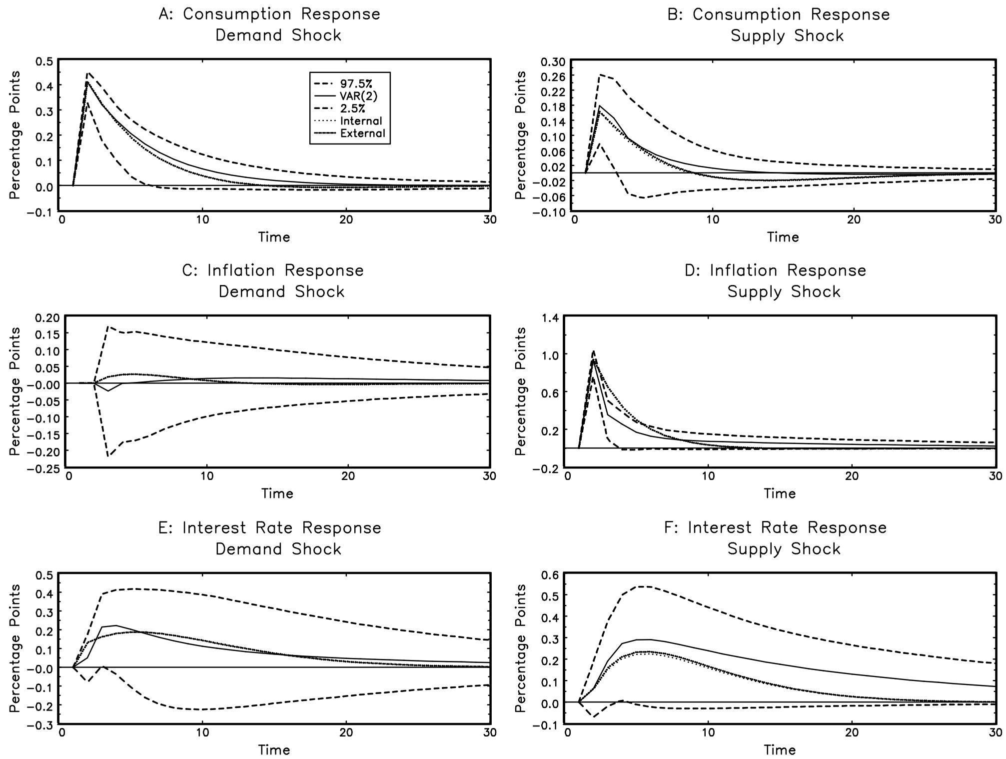

Fig. 1: Responses to demand and supply shocks

There are two important points to take away from Figure 1. The first point is that the impulse responses from the new Keynesian models are in broad agreement with the VAR model, especially following a demand shock. The second point is that the differences between the two specifications are both qualitatively and quantitatively very small. In effect, although the consumption Euler equation with internal habits has a structure that differs importantly from that with external habits, the decision rules that describe equilibrium consumption behavior are very similar.

Some intuition for the results in Table 2 and Figure 1 can be found by focusing on the consumption Euler equations. Using the values estimated in Table 1, the consumption Euler equations for internal and external habits, respectively, are

$$\widehat{c}_t = 0.330\widehat{c}_{t-1} + 0.998\mathrm{E}_t\widehat{c}_{t+1} - 0.328\mathrm{E}_t\widehat{c}_{t+2} - 0.001\left(R_t - \mathrm{E}_t\widehat{\pi}_{t+1} - 2.358\right) + \widetilde{g}_t, \quad (17)$$

$$\widehat{c}_t = 0.452\widehat{c}_{t-1} + 0.548\mathrm{E}_t\widehat{c}_{t+1} - 0.001\left(R_t - \mathrm{E}_t\widehat{\pi}_{t+1} - 2.383\right) + \widetilde{g}_t. \quad (18)$$

Conditioning on the processes for the real interest rate and the demand shock, the stable eigenvalues implied by these two consumption equations are 0.829 for internal habits (equation 17) and 0.825 for external habits (equation 18). Because these eigenvalues are so similar, for internal and external habits to generate different consumption behavior effectively requires interaction between consumption and the processes for the real interest rate and/or the demand shock. However, empirically, the importance of these interactions is damped by the small elasticity of intertemporal substitution. In effect, regardless of whether the habit formation is internal or external, it is the relationship between consumption and its lag, rather than the relationship between consumption and the other state variables, that drives the estimation. At the same time, with expected consumption two periods ahead entering equation (17) and not equation (18), it is possible that behavioral differences, at least with respect to demand shocks, may arise if the demand shock is highly persistent. In a similar vein, news in the form of anticipated future shocks will have an earlier effect on consumption behavior when the habit formation is internal than when it is external, a point emphasized by Uribe (2002). To determine whether behavioral differences due to demand shock persistence are likely to be important in practice, I re-estimated the models allowing g_t to follow an AR(1) process.[6] The estimates of the autoregressive coefficient were 0.009 and 0.004 for internal and external habits, respectively. Thus, a role for serially correlated demand shocks, while potentially important, does not seem to be important on this occasion.

5 Conclusion

This paper has examined whether different approaches to modeling habit formation have important business cycle implications in new Keynesian models. The paper showed that to a log-linear approximation the differences among the main ways that habit formation is modeled in the new Keynesian literature are essentially innocuous for business cycle behavior. In particular, the paper showed that the consumption-Euler equation derived from additive habits encompasses the consumption-Euler equation derived from multiplicative habits, a result that holds regardless of whether the habit formation is internal or external to the household. This result suggests that higher order approximations will be necessary if substantive differences between additive habits and multiplicative habits are to be obtained in empirical contexts. Further, using an estimated small-scale new Keynesian model, the paper showed that internal habits and external habits generate very similar business cycle characteristics, explaining why the new Keynesian business cycle literature is so varied in its application of habit formation.

[6] For this exercise, I also re-derived the consumption Euler equations, which depend on the process for the demand shock; see Appendix A.

A Appendix A

The log-linear Euler equations that I examine are obtained from the following first-order equations

$$\left[\frac{e^{g_t}\partial u\left(c_t,H_t\right)}{\partial c_t}+\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left(\frac{e^{g_{t+i}}\partial u\left(c_{t+i},H_{t+i}\right)}{\partial H_{t+i}}\frac{\partial H_{t+i}}{\partial c_t}\right)\right] = \lambda_t, \tag{A.1}$$

$$-\frac{\lambda_t}{P_t}+\beta\left(1+R_t\right)\mathrm{E}_t\frac{\lambda_{t+1}}{P_{t+1}} = 0, \tag{A.2}$$

$$-l_t^{\chi}+\lambda_t\frac{W_t}{P_t} = 0. \tag{A.3}$$

As is well-known, a log-linear approximation to A.2 about a zero-inflation nonstochastic steady state yields

$$\widehat{\lambda}_t=\mathrm{E}_t\widehat{\lambda}_{t+1}+\left(\widehat{R}_t-\mathrm{E}_t\widehat{\pi}_{t+1}\right). \tag{A.4}$$

With multiplicative habits and the specifications for utility and the habit stock that are provided in the text, A.1 can be written as

$$\mathrm{E}_t\left[e^{g_t}c_t^{-\sigma}c_{t-1}^{\gamma(\sigma-1)}-\gamma\beta De^{g_{t+1}}c_{t+1}^{1-\sigma}c_t^{\gamma(\sigma-1)-1}\right]=\lambda_t, \tag{A.5}$$

which when log-linearized leads to

$$\frac{g_t-\left(\sigma+\gamma\beta D\left(\gamma\left(\sigma-1\right)-1\right)\right)\widehat{c}_t+\gamma\left(\sigma-1\right)\widehat{c}_{t-1}-\gamma\beta D\left(1-\sigma\right)\mathrm{E}_t\widehat{c}_{t+1}}{\left(1-\gamma\beta D\right)}=\widehat{\lambda}_t. \tag{A.6}$$

Combining A.4 and A.6 yields equation (7) in the text.

Similarly, with additive habits and the specifications for utility and the habit stock that are given in the text, A.1 becomes

$$\mathrm{E}_t\left[e^{g_t}\left(c_t-\eta c_{t-1}\right)^{-\alpha}-\eta\beta De^{g_{t+1}}\left(c_{t+1}-\eta c_t\right)^{-\alpha}\right]=\lambda_t, \tag{A.7}$$

which when log-linearized becomes

$$\frac{-\alpha\left(\widehat{c}_t-\eta\widehat{c}_{t-1}\right)+\eta\alpha\beta D\left(\mathrm{E}_t\widehat{c}_{t+1}-\eta\widehat{c}_t\right)+\left(1-\eta\right)g_t}{\left(1-\eta\right)\left(1-\eta\beta D\right)}=\widehat{\lambda}_t. \tag{A.8}$$

Combining A.4 and A.8 gives equation (12) in the text.

B Appendix B

Proof of proposition 1: Beginning with the consumption-Euler equation, consider equation (6), which was derived assuming multiplicative habits and is given by

$$\widehat{c}_t=\frac{\gamma\left(\sigma-1\right)}{\sigma+\gamma\left(\sigma-1\right)}\widehat{c}_{t-1}+\frac{\sigma}{\sigma+\gamma\left(\sigma-1\right)}\mathrm{E}_t\widehat{c}_{t+1}-\frac{1}{\sigma+\gamma\left(\sigma-1\right)}\left(R_t-\mathrm{E}_t\widehat{\pi}_{t+1}-\rho-g_t\right). \tag{B.1}$$

Multiplying and dividing both sides of equation B.1 by σ yields

$$\widehat{c}_t=\frac{\gamma\frac{(\sigma-1)}{\sigma}}{1+\gamma\frac{(\sigma-1)}{\sigma}}\widehat{c}_{t-1}+\frac{1}{1+\gamma\frac{(\sigma-1)}{\sigma}}\mathrm{E}_t\widehat{c}_{t+1}-\frac{\frac{1}{\sigma}}{1+\gamma\frac{(\sigma-1)}{\sigma}}\left(R_t-\mathrm{E}_t\widehat{\pi}_{t+1}-\rho-g_t\right). \tag{B.2}$$

Now let η and α be defined to satisfy

$$\eta \equiv \frac{\gamma(\sigma-1)}{\sigma}, \tag{B.3}$$
$$\alpha \equiv \sigma-\gamma(\sigma-1), \tag{B.4}$$

then equation B.2 is equivalent to

$$\widehat{c}_t = \frac{\eta}{1+\eta}\widehat{c}_{t-1} + \frac{1}{1+\eta}\mathrm{E}_t\widehat{c}_{t+1} - \frac{(1-\eta)}{\alpha(1+\eta)}\left(R_t - \mathrm{E}_t\widehat{\pi}_{t+1} - \rho - g_t\right). \tag{B.5}$$

Importantly, equation B.5 has a structure that is identical to equation (11), which was derived assuming additive habits. Moreover, for all $\sigma \in (1,\infty)$ and $\gamma \in [0,1)$, equations B.3 and B.4 satisfy $\alpha \in (0,\infty)$ and $\eta \in [0,1)$, the admissible parameter spaces for additive habits. Similarly, by setting $D = 0$ in equations A.6 and A.8 and then substituting equations B.3 and B.4 into equation A.8, it is straightforward to verify that equations A.8 and A.6 are equivalent. It follows that when the habit formation is external multiplicative habits is a special case of additive habits.

To see that the converse is not true, note that the transformations from the additive habits specification to the multiplicative habits specification are given by

$$\gamma \equiv \frac{\alpha\eta}{\alpha+\eta-1}, \tag{B.6}$$
$$\sigma \equiv \frac{\alpha}{1-\eta}, \tag{B.7}$$

and that when $\alpha \in (0,1]$, a subset of its parameter space, the values for σ and γ do not satisfy $\sigma \in (1,\infty)$ and $\gamma \in [0,1)$ for all $\eta \in [0,1)$.

Now consider the labor supply decision. The (log-linear) labor supply equations for multiplicative and additive habits are

$$\chi\widehat{l}_t = \widehat{w}_t + g_t - \sigma\widehat{c}_t + \gamma(\sigma-1)\widehat{c}_{t-1}, \tag{B.8}$$
$$\chi\widehat{l}_t = \widehat{w}_t + g_t - \frac{\alpha}{1-\eta}\left(\widehat{c}_t - \eta\widehat{c}_{t-1}\right), \tag{B.9}$$

respectively. Employing equations (B.6) – (B.7) in equation (B.8) or equations (B.3) – (B.4) in equation (B.9), it is not difficult to see that equations (B.8) and (B.9) are isomorphic. ■

Proof of proposition 2: Recall that the consumption-Euler equation derived assuming additive habits is

$$\begin{aligned}\mathrm{E}_t\Delta\widehat{c}_{t+1} = {} & \frac{\eta}{(1+\eta^2\beta)}\mathrm{E}_t\left[\Delta\widehat{c}_t + \beta\Delta\widehat{c}_{t+2}\right] \\ & + \frac{(1-\eta)}{\alpha(1+\eta^2\beta)}\left[(1-\eta\beta)\left(R_t - \mathrm{E}_t\widehat{\pi}_{t+1} - \rho\right) - g_t\right],\end{aligned} \tag{B.10}$$

and the one derived assuming multiplicative habits is

$$\begin{aligned}\mathrm{E}_t\Delta\widehat{c}_{t+1} = {} & \frac{\gamma(\sigma-1)}{[\sigma+\gamma\beta(\sigma\gamma-1-\gamma)]}\mathrm{E}_t\left[\Delta\widehat{c}_t + \beta\Delta\widehat{c}_{t+2}\right] \\ & + \frac{1}{[\sigma+\gamma\beta(\sigma\gamma-1-\gamma)]}\left[(1-\gamma\beta)\left(R_t - \mathrm{E}_t\widehat{\pi}_{t+1} - \rho\right) - g_t\right].\end{aligned} \tag{B.11}$$

I seek to show that for all $\sigma \in (1,\infty)$ and $\gamma \in [0,1)$ there are values for α and η that satisfy $\alpha \in (0,\infty)$ and $\eta \in [0,1)$ such that

$$\frac{\eta}{(1+\eta^2\beta)} = \frac{\gamma(\sigma-1)}{[\sigma+\gamma\beta(\gamma(\sigma-1)-1)]}, \tag{B.12}$$

$$\frac{(1-\eta)(1-\eta\beta)}{\alpha(1+\eta^2\beta)} = \frac{(1-\gamma\beta)}{[\sigma+\gamma\beta(\gamma(\sigma-1)-1)]}. \tag{B.13}$$

It follows from equations B.12 and B.13 that finding such values for α and η implies that the discount factors are also equal across the two specifications.

First, consider the case where $\gamma = 0$. When $\gamma = 0$, equations B.12 and B.13 imply that $\eta = 0$ and $\alpha = \sigma$, which clearly satisfy $\eta \in [0,1)$ and $\alpha \in (0,\infty)$ for all $\sigma \in (1,\infty)$.

Turning to the case where $\gamma \in (0,1)$, there are two values for η that satisfy equation B.12 and these values are given implicitly by the quadratic

$$\gamma(\sigma-1)\beta\eta^2 - [\sigma+\gamma\beta(\gamma(\sigma-1)-1)]\eta + \gamma(\sigma-1) = 0. \tag{B.14}$$

From equations B.12 and B.13, η and α must also satisfy the quadratic

$$\gamma(\sigma-1)\beta\eta^2 - [\gamma(\sigma-1)(1+\beta)+(1-\gamma\beta)\alpha]\eta + \gamma(\sigma-1) = 0. \tag{B.15}$$

Equating coefficients between equations B.14 and B.15 gives

$$\alpha = \sigma - \gamma(\sigma-1). \tag{B.16}$$

By inspection, equation B.16 satisfies $\alpha \in (0,\infty)$ for all $\gamma \in (0,1)$ and $\sigma \in (1,\infty)$.

Next, the solutions to equation B.14 are

$$\eta = \frac{(\sigma+\gamma\beta(\gamma(\sigma-1)-1))}{2\gamma(\sigma-1)\beta} \pm \frac{\sqrt{(\sigma+\gamma\beta(\gamma(\sigma-1)-1))^2 - 4\gamma^2(\sigma-1)^2\beta}}{2\gamma(\sigma-1)\beta}. \tag{B.17}$$

To see that it is the negative root

$$\eta = \frac{(\sigma+\gamma\beta(\gamma(\sigma-1)-1))}{2\gamma(\sigma-1)\beta} - \frac{\sqrt{(\sigma+\gamma\beta(\gamma(\sigma-1)-1))^2 - 4\gamma^2(\sigma-1)^2\beta}}{2\gamma(\sigma-1)\beta}, \tag{B.18}$$

that is the appropriate one, observe that in the limit as either $\sigma \downarrow 1$ or $\gamma \downarrow 0$ the positive root tends to ∞. Now let

$$\mu \equiv \frac{(\sigma+\gamma\beta(\gamma(\sigma-1)-1))}{2\gamma(\sigma-1)\beta} = \frac{(1+\gamma^2\beta)}{2\gamma\beta} + \frac{(1-\gamma\beta)}{2\gamma\beta(\sigma-1)}, \tag{B.19}$$

where equation B.19 has been written to make it clear that $\mu \in \left(\frac{1+\beta}{2\beta},\infty\right)$ for all $\gamma \in (0,1)$ and $\sigma \in (1,\infty)$. To see this, observe that both $\frac{(1+\gamma^2\beta)}{2\gamma\beta}$ and $\frac{(1-\gamma\beta)}{2\gamma\beta(\sigma-1)}$ are positive and that $\frac{(1+\gamma^2\beta)}{2\gamma\beta}$ is greater than $\frac{1+\beta}{2\beta} = 1 + \frac{1-\beta}{2\beta}$, which is greater than $\frac{1}{\beta}$. With μ defined by equation B.19, equation B.18 can be written as

$$\eta = \mu - \left(\mu^2 - \frac{1}{\beta}\right)^{\frac{1}{2}}, \tag{B.20}$$

and it follows immediately that $\mu \in \left(\frac{1+\beta}{2\beta},\infty\right)$ rules out complex-valued solutions for η. Finally, by inspection, equation B.20 satisfies $\eta \in [0,1)$ for all $\mu \in (1,\infty)$, which completes the proof. ■

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