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# Asset Pricing with Concentrated Ownership of Capital and Distribution Shocks<sup>\*</sup>

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#### Abstract

This paper develops a production-based asset pricing model with two types of agents and concentrated ownership of physical capital. A temporary but persistent "distribution shock" causes the income share of capital owners to fluctuate in a procyclical manner, consistent with U.S. data. The concentrated ownership model significantly magnifies the equity risk premium relative to a representative-agent model because the capital owners' consumption is more-strongly linked to volatile dividends from equity. With a steady-state risk aversion coefficient around 4, the model delivers an unlevered equity premium of 3.9% relative to short-term bonds and a premium of 1.2% relative to long-term bonds.

Keywords: Asset Pricing, Equity Premium, Term Premium, Distribution Shocks, Income Inequality.

JEL Classification: D31, E13, E25, E32, E44, G12.

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### 1 Introduction

The distribution of wealth in the U.S. economy is highly skewed. The top decile of U.S. households owns approximately 80 percent of financial wealth and about 70 percent of total wealth including real estate.<sup>1</sup> Shares of corporate stock are an important component of financial wealth, representing claims to the tangible and intangible capital of firms. As recently as 1995, the lowest 75% of U.S. households sorted by wealth owned less than 10% of stocks.<sup>2</sup>

While the degree of wealth inequality in the U.S. economy has remained relatively steady over time (Kopczuk and Saez, 2004), measures of pre-tax income inequality display a large amount of volatility. Over the sample period from 1918 to 2012, the share of total pre-tax income including capital gains going to the top decile of U.S. households exhibits a mean of 40% and a standard deviation of 5.6% (top left panel of Figure 1).<sup>3</sup> Capital's share of income from 1929 to 2012 exhibits a mean of 37% and a standard deviation of 2% (bottom left panel of Figure 1).<sup>4</sup>

The right-hand panels of Figure 1 show that the U.S. historical equity premium is positively correlated with annual changes in both of the income share variables.<sup>5</sup> In both panels, the correlation coefficient is around 0.3 and statistically significant. Given the concentration of financial wealth in the top decile, fluctuations in the income share variables would be expected to impact stockholder consumption. A presumed link between stockholder consumption and equity prices is the foundation of consumption-based asset pricing models. While Figure 1 is suggestive, a recent empirical study by Greenwald, Lettau, and Ludvigson (2014) finds that temporary but persistent "factor share shocks" that redistribute income between stockholders and non-stockholders are an important driver of U.S. stock prices. Motivated by these observations, this paper develops a production-based model of asset pricing with the following features: (1) a stable but highly-skewed distribution of physical capital wealth, and (2) temporary but persistent fluctuations in income shares.

### 1.1 Overview

The framework for the analysis is a real business cycle model with two types of agents, called capital owners and workers. Capital owners represent the top decile of earners in the economy.

<sup>&</sup>lt;sup>1</sup>See Wolff (2006), Table 4.2, p. 113.

<sup>&</sup>lt;sup>2</sup>See Heaton and Lucas (2000), Figure 3, p. 224.

 $<sup>^3 \</sup>mathrm{See}$  Piketty and Saez (2003, 2013). Updated annual data are available from The World Top Incomes Database.

<sup>&</sup>lt;sup>4</sup>Capital's share is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. Both series are from the Bureau of Economic Analysis, NIPA Table 1.14, lines 1 and 4.

<sup>&</sup>lt;sup>5</sup>The U.S. equity premium is measured as the difference between the real return on equity and the real return on short-term bills, from Dimson, Marsh, and Staunton (2002), updated through 2012.

These agents own 100% of the productive capital stock—a setup that roughly approximates the highly skewed distribution of U.S. financial wealth. I associate capital owners in the model with U.S. stockholders. The consumption of the capital owners is funded from dividends and wage income. I associate workers in the model with U.S. non-stockholders. The consumption of the workers is funded only from wage income. Since workers do not save, all assets (equity and bonds) are priced by the capital owners. The labor supply of the capital owners is inelastic, consistent with the idea that asset prices are determined in securities markets by agents who remain fully-employed at all times. For simplicity, I also assume that the workers' labor supply is inelastic.<sup>6</sup>

I consider two types of shocks: (1) a standard labor-augmenting productivity shock that evolves as a random walk with drift, and (2) a temporary but persistent "distribution shock" that causes the income share of capital owners to fluctuate over time. Along similar lines, Young (2004) and Ríos-Rull and Santaeulàlia-Llopis (2010) introduce stochastic variation in capital's share of income in a representative-agent model to help account for various business cycle facts. However, they do not examine the asset pricing implications of this shock.<sup>7</sup>

I calibrate the volatility of the productivity shock innovation in the model to match the 2.2% standard deviation of U.S. real per capita aggregate consumption growth (nondurable goods and services) from 1930 to 2012.<sup>8</sup> There are several options for calibrating the volatility of the distribution shock. For example, it could be chosen to match the 5.6% standard deviation of the U.S. top decile income share (top left panel of Figure 1) or the 2% standard deviation of the U.S. capital income share (bottom left panel of Figure 1). Of these, fluctuations in the top decile income share would seem to be more indicative of fluctuations in stockholder consumption. Another option is to calibrate the distribution shock to match the observed volatility of the stockholder cash flows that the model seeks to price. Two candidates for such cash flows are real dividends for the S&P 500 stock index and so-called "macroeconnomic dividends" defined as capital-type income less investment. The latter measure has the advantage of mapping directly to the model's concept of dividends, where capital's share of income is included as part of the definition. Figure 2 shows that the growth rate of macroecononomic dividends exhibits a much stronger correlation with the historical equity premium than does

<sup>&</sup>lt;sup>6</sup>Allowing for elastic labor supply on the part of workers would not change the model's asset pricing results because workers do not participate in financial markets. Allowing for elastic labor supply on the part of capital owners would introduce an additional mechanism for these agents to smooth their consumption, making it more difficult for the model to achieve a sizeable equity premium.

<sup>&</sup>lt;sup>7</sup>Lansing and Markiewicz (2013) examine the welfare consequences of permanent shifts in the U.S. top decile income share.

<sup>&</sup>lt;sup>8</sup>Data on nominal consumption expenditures for nondurable goods and services are from the Bureau of Economic Analysis, NIPA Table 2.3.5, lines 8 and 13. The corresponding price indices are from Table 2.3.4, lines 8 and 13. Population data are from Table 2.1, line 40.

the growth rate of S&P 500 dividends. The correlation coefficient between the S&P 500 dividend growth and the equity premium is close to zero (top right panel of Figure 2), whereas the correlation coefficient between U.S. macroeconomic dividend growth and the equity premium is around 0.3 and statistically significant (bottom right panel of Figure 2).<sup>9</sup>

For the baseline calibration, I set the volatility of the distribution shock to match the 6.3%standard deviation of U.S. real per capita macroeconomic dividend growth for the period 1930 to 2012. With this choice, the model exhibits the property that dividend growth is about three times more volatile than aggregate consumption growth. The baseline calibration can be viewed as conservative given that S&P 500 dividend growth is over five times more volatile than U.S. aggregate consumption growth. In the sensitivity analysis, I show how different calibration targets for the volatility of the distribution shock influence the model's quantitative predictions for the mean equity premium and other statistics. The conservative calibration for the volatility of model dividend growth is helpful for matching another empirical observation, namely, the relative volatilities of consumption growth for stockholders versus non-stockholders. A study by Malloy, Moskowitz, and Vissing-Jørgensen (2009) finds that the volatility of consumption growth for U.S. stockholders is roughly twice that of non-stockholders over the period 1982 to 2004. This empirical target might also be viewed as conservative; a study by Aït-Sahalia, Parker, and Yogo (2004) suggests a much higher relative volatility of stockholder consumption growth based on retail sales data for luxury goods over the period 1961 to 2001.

With a steady state risk aversion coefficient around 4, the concentrated ownership model delivers an unlevered mean equity risk premium of 3.9% per year relative to short-term bonds and a premium of about 1.2% relative to long-term bonds. The corresponding mean risk premia in U.S. data for the period 1900 to 2012 are higher at 7% and 5% respectively, as documented by Dimson, Marsh, and Staunton (2002, updated). While higher risk aversion coefficients can raise the model's mean equity premium, such a calibration would cause the model to overpredict the 20% standard deviation of U.S. real equity returns. I show that an otherwise similar representative-agent version of the model delivers mean equity risk premia of only 0.4% and 0.25%, respectively.

Capital owners in the concentrated ownership model demand a high equity premium because their consumption is strongly linked to volatile dividends from equity. The volatility of

<sup>&</sup>lt;sup>9</sup>Data on real dividends for the S&P 500 index are from Robert Shiller's website. Macroeconomic dividends are defined as  $\theta_t y_t - i_t$ , where  $\theta_t$  is capital's share of income (footnote 4),  $i_t$  is real per capita private nonresidential fixed investment plus real per capita durable goods consumption, and  $y_t$  is constructed as the sum of  $i_t$ and real per capita consumption (footnote 8). Data on nominal private nonresidential fixed investment and the corresponding implicit price deflator are from the Federal Reserve Bank of St. Louis' FRED database. Data on nominal durable goods consumption and the corresponding price index are from NIPA Tables 2.3.5 (line 3) and 2.3.4 (line 3).

equity dividends derives primarily from the distribution shock. The capital owners' consumption growth is more volatile than aggregate consumption growth. This higher volatility serves to magnify the equity risk premium for any given level of risk aversion. In a representativeagent endowment economy with *iid* aggregate consumption growth, the equity risk premium relative to one-period bonds is given by the product of the coefficient of relative risk aversion and the variance of *aggregate* consumption growth.<sup>10</sup> In contrast, the concentrated ownership model links the equity risk premium to stockholders' consumption growth rather than aggregate consumption growth.

Along the lines of Rudebusch and Swanson (2008), a long-term bond is modeled as a decaying-coupon consol with a Macauly duration of 10 years. The model's underprediction of the equity risk premium relative to long-term bonds reflects the fact that long-term bonds in the model behave too much like equity—a result that is also typical of endowment economies.<sup>11</sup> The concentrated ownership model overpredicts the volatility of long-term bond returns, again because these bonds behave too much like equity. Nevertheless, the model is able to match the 20% standard deviation of U.S. real equity returns and delivers about one-third of the observed volatility in the price-dividend ratio for the S&P 500 index. The corresponding standard deviations in the representative agent model are substantially lower.

As part of the analysis, I investigate how some key model parameters influence the size of the mean equity premium. These include: (1) the standard deviation of the distribution shock innovation, (2) a curvature parameter in the law of motion for capital that governs the strength of the capital adjustment costs, (3) a utility curvature parameter that influences the degree of risk aversion, and (4) a utility habit parameter that allows for time-varying risk aversion. Since the distribution shock is an important source of consumption risk for capital owners, increasing its volatility raises the mean equity premium. All else equal, stronger capital adjustment costs would impair the capital owner's ability to smooth consumption, thereby raising the mean equity premium. Offsetting this effect, however, is the need to recalibrate both shock innovations to always match the volatilities of aggregate consumption growth and macroeconomic dividend growth in the data. The end result is that changes in the adjustment cost parameter have only a small effect on the mean equity premium in the calibrated model. The baseline value for the adjustment cost parameter is picked so that the model approximately matches the 12.7% standard deviation of real per capita investment growth in U.S. data over the period 1930 to 2012. Larger values for the utility curvature

<sup>&</sup>lt;sup>10</sup>Specifically, we have  $\log \left[ E\left(R_{t+1}^s\right) / E\left(R_{t+1}^b\right) \right] = \alpha \operatorname{Var}\left[ \log\left(c_{t+1}^a / c_t^a\right) \right]$ , where  $R_{t+1}^s$  is the gross return on equity,  $R_{t+1}^b$  is the gross return on a one-period discount bond (the risk free rate),  $\alpha$  is the coefficient of relative risk aversion, and  $c_t^a$  is real per capita aggregate consumption. For the derivation, see Abel (1994), p. 353.

 $<sup>^{11}\</sup>mathrm{See},$  for example, Abel (2008), Table 2.

parameter or the habit formation parameter both serve to raise the mean equity premium. However, if the values become too large, the model will overpredict the standard deviation of equity returns in the data.

On the quantity side, I show that the concentrated ownership model performs well in matching the business cycle moments of aggregate macro variables including the pro-cyclical behavior of capital's share of income in U.S. data. A positive innovation to the capital income share induced by the distribution shock causes an increase in real output. In simulations, the model delivers a contemporaneous correlation of 0.3 between capital's share of income and the growth rate of real output—consistent with U.S. data over the period 1930 to 2012.

Finally, I show that the equity premium generated by the concentrated ownership model is predictable using the preceding period's dividend yield (i.e., the inverse of the price-dividend ratio). The estimated coefficient in the predictability regression is similar in magnitude to that obtained using U.S. financial market data.

#### **1.2** Related Literature

The model developed here is most closely related to Danthine and Donaldson (2002) who also employ a setup with capital owners and workers.<sup>12</sup> The wage contract in their model smoothes workers' consumption against aggregate shocks, a mechanism they describe as "operational leverage." A persistent shock to the relative bargaining power of the two groups creates an additional source of risk that must be borne by the capital owners and contributes to a higher equity premium. When the bargaining power shocks are positively correlated with (temporary) productivity shocks, the model can produce an equity premium relative to one-period bonds close to 6%, but the result is accompanied by too much volatility in the one-period bond return, i.e., a standard deviation in excess of 10%.<sup>13</sup> Other counterfactual implications of their model are: (1) the consumption growth of stockholders is 10 times more volatile than aggregate consumption growth and, (2) the standard deviation of model-implied dividend growth is nearly 20%—about twice the volatility of S&P 500 dividend growth.<sup>14</sup> The model developed here avoids these counterfactual predictions while still delivering a sizeable equity premium.

Guvenen (2009) also develops a model with concentrated ownership of capital. Stockholders price equity while non-stockholders price one-period bonds. Stockholders must bear the risk of countercyclical interest payments to non-stockholders which amplifies the volatility

<sup>&</sup>lt;sup>12</sup>Further elaboration on the Danthine-Donaldson model can be found in Danthine, et al. (2008).

<sup>&</sup>lt;sup>13</sup>See Table 4, Panel B, p. 59 in Danthine and Donaldson (2002).

<sup>&</sup>lt;sup>14</sup>See Table 6, Panel A, p. 62 in Danthine and Donaldson (2002). They do not report the volatility of consumption growth for workers. To address the excessive volatility of dividend growth, they introduce an ad hoc mechanism for smoothing paid out dividends.

of the stockholders' consumption streams, thereby raising their required return on equity.<sup>15</sup> With a stockholder risk aversion coefficient of 6, Guvenen's baseline model delivers an equity premium relative to one-period bonds of about 5.5%, but he does not investigate the model's implications for long-term bonds. It is not clear how long-term bonds would be priced in his model, since it appears that both types of agents would be willing to buy these bonds.

De Graeve et al. (2010) develop a model that combines elements from both Danthine and Donaldson (2002) and Guvenen (2009). They allow for three types of agents, all with elastic labor supply: stockholders who price equity and long-term bonds, bondholders who price oneperiod bonds, and workers who do not save. The assumption that one-period bonds are priced by bondholders while long-term bonds are priced by stockholders seems hard to justify. An important limitation of all the foregoing models is that they abstract from long-run growth—a feature that affects the change in consumption from one period to the next. In contrast, the model developed here is calibrated to match both the mean and volatility of real per capita consumption growth in long-run U.S. data.

Polkovnichenko (2004) and Walentin (2010) show that a permanent increase in the share of dividend income in stockholders' total income serves to increase the equity premium in endowment economies. A similar mechanism is at work here, except that the distribution shock delivers temporary but persistent fluctuations in the share of dividend income in stockholders' total income (which consists of dividends and wage income).

As a caveat, it should be noted that model comparisons with the U.S. equity return data pertain only to publically-traded firms. A study by Davis, et al. (2006), p. 119 finds that privately-held firms account for more than two-thirds of total private business employment. The inclusion of privately-held firms in the equity return data would provide a broader measure of the equity risk premium. Moskowitz and Vissing-Jørgensen (2002), p. 765 find that while average equity returns for public and private firms are similar, private equity returns exhibit a lower standard deviation relative to public-firms' market equity returns. According to these measures, the inclusion of private equity return data would increase the magnitude of the Sharpe ratio in the data that any model would seek to explain.

### 2 Model

The model consists of workers, capital owners, and competitive firms. There are n times more workers than capital owners, with the total number of capital owners normalized to one. Naturally, the firms are owned by the capital owners. Workers and capital owners both supply

 $<sup>^{15}</sup>$ Guo (2004) develops a similar mechanism in the context of an endowment economy.

labor to the firms inelastically, but in different amounts.<sup>16</sup>

#### 2.1 Workers

Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, workers simply consume their labor income each period such that

$$c_t^w = w_t^w \, \ell_t^w,$$

where  $c_t^w$  is the individual worker's consumption,  $w_t^w$  is the worker's competitive market wage, and  $\ell_t^w = \ell^w$  is the constant supply of labor hours per worker.

### 2.2 Capital Owners

The capital owner's decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{c_t}{H_t} - \kappa \frac{C_{t-1}}{H_{t-1}}\right)^{1-\alpha} - 1}{1-\alpha},$$
(1)

subject to the budget constraint

$$c_t + p_t^s q_{t+1}^s + p_t^b q_{t+1}^b + p_t^c q_{t+1}^c = (p_t^s + d_t) q_t^s + q_t^b + (1 + \delta^c p_t^c) q_t^c + w_t^c \ell_t^c, \quad (2)$$

where  $E_t$  represents the mathematical expectation operator,  $\beta$  is the subjective time discount factor,  $c_t$  is the individual capital owner's consumption, and  $\alpha \geq 0$  is a curvature parameter that influences the coefficient of relative risk aversion. Along the lines of Abel (1999), an individual capital owner derives utility from consumption relative to an exogenously-growing living standard index  $H_t = \exp(\mu t)$ , where  $\mu$  is the economy's trend growth rate. This setup implies that capital owners today are not substantially "happier" (as measured in utility terms) than they were a hundred years ago because individual consumption is measured relative to an ever-improving living standard. The net effect of  $H_t$  is to change the effective time discount factor which turns out to be useful in the calibration procedure.<sup>17</sup> To allow for time-varying risk aversion, I assume that an individual capital owner's felicity is also measured relative to the lagged per capita consumption basket  $C_{t-1}/H_{t-1}$ , which the agent views as outside of his control.<sup>18</sup> The parameter  $\kappa \geq 0$  governs the importance of the external habit stock. When  $\alpha = 1$ , the within-period utility function can be written as  $\log (c_t/H_t - \kappa C_{t-1}/H_{t-1})$ .

<sup>&</sup>lt;sup>16</sup>The model setup is similar to a standard framework that is often used to study optimal redistributive capital taxation. See, for example, Judd (1985), Lansing (1999), and Krusell (2002). In these examples, however, capital owners do not supply labor.

<sup>&</sup>lt;sup>17</sup>The value of  $\beta$  is chosen to match the mean price-dividend ratio in long-run U.S. data. The presence of  $H_t$  in (1) allows the calibration target to be achieved with  $\beta < 1$ , even if steady-state risk aversion is high.

<sup>&</sup>lt;sup>18</sup>Maurer and Meier (2008) find strong empirical evidence for "peer-group effects" on individual consumption decisions using panel data on U.S. household expenditures.

Capital owners derive labor income in the amount  $w_t^c \ell_t^c$ , where  $\ell_t^c = \ell^c$  is the constant supply of labor hours per person. Capital owners may purchase the firm's equity shares in the amount  $q_{t+1}^s$  at the ex-dividend price  $p_t^s$ . Shares purchased in the previous period yield a dividend  $d_t$ . One-period discount bonds purchased in the amount  $q_{t+1}^b$  at the price  $p_t^b$  yield a single payoff in the following period of one consumption unit per bond. Capital owners may also purchase long-term bonds (consols) in the amount  $q_{t+1}^c$  at the ex-coupon price  $p_t^c$ . A long-term bond purchased in period t yields the following stream of decaying coupon payments (measured in consumption units) starting in period t + 1: 1,  $\delta^c$ ,  $(\delta^c)^2$ ,  $(\delta^c)^3$  ..., where  $\delta^c$  is the decay parameter that governs the Macauly duration of the bond, i.e., the present-value weighted average maturity of the bond's cash flows.<sup>19</sup> When  $\delta^c = 0$ , the long-term bond collapses to a one-period bond. Equity shares are assumed to exist in unit net supply while both types of bonds exist in zero net-supply. Market clearing therefore implies  $q_t^s = 1$  and  $q_t^b = q_t^c = 0$  for all t.

The capital owner's first-order conditions with respect to  $q_{t+1}^s$ ,  $q_{t+1}^b$ , and  $q_{t+1}^c$  are as follows:

$$p_{t}^{s} = E_{t} \beta \exp(-\phi\mu) \left[ \frac{c_{t+1} - \kappa \exp(\mu) c_{t}}{c_{t} - \kappa \exp(\mu) c_{t-1}} \right]^{-\alpha} \left( p_{t+1}^{s} + d_{t+1} \right),$$
(3)

$$p_t^b = E_t \beta \exp\left(-\phi\mu\right) \left[\frac{c_{t+1} - \kappa \exp\left(\mu\right) c_t}{c_t - \kappa \exp\left(\mu\right) c_{t-1}}\right]^{-\alpha},\tag{4}$$

$$p_t^c = E_t \beta \exp\left(-\phi\mu\right) \left[\frac{c_{t+1} - \kappa \exp\left(\mu\right) c_t}{c_t - \kappa \exp\left(\mu\right) c_{t-1}}\right]^{-\alpha} \left(1 + \delta^c p_{t+1}^c\right),\tag{5}$$

where  $\phi \equiv 1 - \alpha$  and I have made the substitutions  $(H_{t+1}/H_t)^{-(1-\alpha)} = \exp(-\phi\mu)$  and  $c_t = C_t$ for all t. In equilibrium, the capital owner's budget constraint becomes  $c_t = d_t + w_t^c \ell^c$ , which shows that the capital owner's consumption is funded from dividends and wage income.

<sup>&</sup>lt;sup>19</sup>Rudebusch and Swanson (2008) employ a similar setup except that a long-term bond purchased in period t yields a declining coupon stream of 1,  $\delta^c$ ,  $(\delta^c)^2$ ... starting in period t rather than in period t + 1.

#### 2.3 Firms

The firm's output is produced according to the technology

$$y_t = k_t^{\theta_t} \left[ \exp(z_t) \left( \ell_t^c \right)^a \left( n \, \ell_t^w \right)^{1-a} \right]^{1-\theta_t}, \qquad a \in (0,1)$$
(6)

$$z_t = z_{t-1} + \mu + \varepsilon_t, \qquad \varepsilon_t \sim NID\left(0, \sigma_{\varepsilon}^2\right), \tag{7}$$

$$\theta_t = \theta \exp(v_t), \quad \theta \in (0,1),$$
(8)

$$v_t = \rho v_{t-1} + u_t, \quad |\rho| < 1, \quad u_t \sim NID(0, \sigma_u^2),$$
(9)

with  $z_0$  and  $v_0$  given. The symbol  $k_t$  is the firm's stock of physical capital and  $z_t$  is a laboraugmenting "productivity shock" that evolves as a random walk with drift. The drift parameter  $\mu$  determines the trend growth rate of output. The shock innovation  $\varepsilon_t$  is normally and independently distributed (*NID*) with mean zero and variance  $\sigma_{\varepsilon}^2$ . The parameter *a* governs the relative productivity of the two types of labor inputs. Along the lines of Young (2004) and Ríos-Rull and Santaeulàlia-Llopis (2010), the capital income share  $\theta_t$  can fluctuate over time in response to a "distribution shock"  $v_t$  which evolves as a stationary AR(1) process with persistence parameter  $\rho$  and innovation variance  $\sigma_u^2$ . Allowing the share parameter *a* to similarly fluctuate in response to the distribution shock does not substantially alter the quantitative results.<sup>20</sup>

Resources devoted to investment augment the firm's stock of physical capital according to the law of motion

$$k_{t+1} = B \left\{ (1-\lambda) \left[ k_t \left( 1-\delta \right) \right]^{\psi_k} + \lambda i_t^{\psi_k} \right\}^{\frac{1}{\psi_k}}, \quad \begin{array}{l} B > 0, \quad \lambda \in (0,1), \quad \delta \in [0,1) \\ \psi_k \equiv (\sigma_k - 1) / \sigma_k \\ \sigma_k \in (0,\infty) \end{array}$$
(10)

with  $k_0$  given. The parameter  $\delta$  is the capital depreciation rate. The parameter  $\psi_k$  depends on the elasticity of substitution  $\sigma_k$  between the two inputs that are used to produce new capital, namely, existing capital net of depreciation  $k_t (1 - \delta)$  and new investment  $i_t$ . As  $\sigma_k \to 0$  (or  $\psi_k \to -\infty$ ), new investment and existing capital become more complimentary (i.e., more tightly coupled) which raises the implicit cost of adjusting the capital stock from one period to the next. Kim (2003) shows that the intertemporal adjustment cost specification (10) can also be interpreted as a multisectoral adjustment cost that imposes a nonlinear transformation between consumption and investment in the national income identity. A convenient feature of

<sup>&</sup>lt;sup>20</sup>I experimented with versions of the model where  $a_t = a \exp(\gamma v_t)$  and  $\gamma > 0$ .

the above specification is that it nests the standard linear law of motion with no adjustment costs as a special case. The standard linear law of motion can be recovered by imposing the following parameter settings:  $\sigma_k = \infty$  (or  $\psi_k = 1$ ), B = 2, and  $\lambda = 1/2$ .

Under the assumption that the labor market is perfectly competitive, firms take  $w_t^c$  and  $w_t^w$  as given and choose sequences of  $\ell_{t+j}^c$ ,  $\ell_{t+j}^w$ , and  $k_{t+1+j}$ , to maximize the following discounted stream of expected dividends:

$$E_0 \sum_{j=0}^{\infty} M_{t+j} \underbrace{\left[ y_{t+j} - w_{t+j}^c \,\ell_{t+j}^c - n \, w_{t+j}^w \,\ell_{t+j}^w - i_{t+j} \right]}_{d_{t+j}},\tag{11}$$

subject to the production function (6) and the capital law of motion (10). Firms act in the best interests of their owners such that dividends in period t + j are discounted using the capital owner's stochastic discount factor  $M_{t+j}$  which is given by

$$M_{t+j} = \beta^{j} \exp(-\phi \mu j) \left[ \frac{c_{t+j} - \kappa \exp(\mu) \ c_{t+j-1}}{c_t - \kappa \exp(\mu) \ c_{t-1}} \right]^{-\alpha}.$$
 (12)

The firm's first-order conditions are:

$$w_t^c = \frac{(1-\theta_t) a y_t}{\ell^c},\tag{13}$$

$$w_t^w = \frac{(1 - \theta_t) (1 - a) y_t}{n \,\ell^w},\tag{14}$$

$$i_t g (k_{t+1}/k_t) = E_t M_{t+1} [\theta_{t+1} y_{t+1} - i_{t+1} + i_{t+1} g (k_{t+2}/k_{t+1})], \qquad (15)$$

where 
$$g(k_{t+1}/k_t) \equiv 1 + \frac{1-\lambda}{\left[\frac{k_{t+1}/k_t}{B(1-\delta)}\right]^{\psi_k} - (1-\lambda)}$$

which reflect the constant labor supplies  $\ell^c$  and  $\ell^w$ . Equations (13) and (14) show that each type of labor is paid its marginal product. The share of total income going to the top decile (i.e., capital owners) is  $s_t^c = \theta_t + (1 - \theta_t) a$ , while the share of total income going to workers is  $s_t^w = (1 - \theta_t) (1 - a)$ . Comparing the first-order condition (15) to the equity pricing equation (3), we see that the ex-dividend price of an equity share is given by  $p_t^s = i_t g (k_{t+1}/k_t) \cdot 2^{11}$  The equity share is a claim to a perpetual stream of dividends  $d_{t+1} = \theta_{t+1} y_{t+1} - i_{t+1}$  starting in period t + 1. In the version of the model with no capital adjustment costs ( $\psi_k = 1, B = 2$ ,  $\lambda = 1/2$ ), we have  $p_t^s = k_{t+1}$ . When  $\sigma_k = 1$  such that  $\psi_k = 0$ , the capital law of motion

<sup>&</sup>lt;sup>21</sup>After taking the derivitive of the profit function (11) with respect to  $k_{t+1}$ , I have multiplied both sides of the resulting first-order condition by  $k_{t+1}$ , which is known at time t.

(10) takes a Cobb-Douglas form and we have  $p_t^s = i_t/\lambda$ . These examples demonstrate that the degree of curvature in the capital law of motion can influence the volatility of the equity price and hence the volatility of the equity return.

### 3 Model Calibration

A time period in the model is taken to be one year. The baseline parameters are chosen simultaneously to match various empirical targets, as summarized in Table 1. In addition to the concentrated ownership model, I similarly calibrate a representative-agent version of the same model. Analytical moment formulas derived from the log-linear approximate solution of both models are used in the calibration procedure.

	Concentrated	Representative	
Parameter	Ownership	Agent	Description/Target
$\overline{n}$	9	_	Capital owners $=$ top income decile.
heta	0.37	0.37	Mean value of capital's share of income.
a	0.048	_	Mean top decile income share $= 0.40$ .
$\ell^c/\ell^w$	0.225	_	Mean relative wage $w^c/w^w = 2$ .
$\delta$	0.07	0.07	Annual capital depreciation rate = $7\%$ .
$\mu$	0.0186	0.0186	Mean per capita consumption growth = $1.86\%$ .
B	1.135	1.135	Mean $i_t / y_t = 0.16$ .
$\lambda$	0.0076	0.0076	Mean $i_t/k_t = \exp(\mu) + 1 - \delta = 0.089.$
$\sigma_k$	0.65	0.65	Std. dev. investment growth $\simeq 12.7\%$ .
$\sigma_{arepsilon}$	0.0331	0.0312	Std. dev. aggregate consumption growth = $2.2\%$ .
$\sigma_u$	0.0614	0.0320	Std. dev. macroeconomic dividend growth = $6.3\%$ .
ho	0.8	0.8	AR(1) for distribution shock.
$\beta$	0.9640	0.9648	Mean $p_t^s/d_t \simeq 29$ .
$\delta^c$	0.9512	0.9504	Consol duration $= 10$ years.
$\alpha$	3.3	3.3	Std. dev. equity return $\simeq 20\%$ .
$\kappa$	0.2	0.2	Implies some predictability of excess returns.

Table 1: Baseline Parameter Values

The number of workers per capital owner is set to n = 9 so that capital owners represent the top income decile of households in the concentrated ownership model. The steady state capital income share  $\theta$  is set to match the sample mean of 0.37, as plotted in the bottom left panel of Figure 1. The production elasticity of the capital owner's labor supply is set to a = 0.048. This value implies a top decile income share in steady state of  $s^c = \theta + (1 - \theta) a = 0.40$ , corresponding to the U.S. sample mean, as plotted in the top left panel of Figure 1. Given these values, the labor supply ratio  $\ell^c/\ell^w$  is set so that the steady state wage ratio is  $w^c/w^w = 2$ . For comparison, Heathcote, Perri, and Violante (2010), p. 24 report a male college wage premium of about 1.4 in 1980, whereas Gottschalk and Danziger (2005), p. 238 report a male wage ratio of about 4 when comparing the top decile to the bottom decile. The wage ratio  $w^c/w^w$ in this model compares the top decile to the remainder of households, so one would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, Perri, and Violante (2010). The quantitative results are not sensitive to the value of this wage ratio.

The capital depreciation rate is set to  $\delta = 0.07$ , a typical value. The drift parameter  $\mu$  of the random walk productivity process (7) is set to achieve a trend growth rate of 1.86%, corresponding to the sample mean of U.S. real per capita aggregate consumption growth (footnote 8). The capital law of motion parameters B and  $\lambda$  are chosen to deliver realistic target values for the steady-state investment-output ratio and the steady-state investment-capital ratio. The steady-state target for  $i_t/y_t$  corresponds to the sample mean in U.S. data over the period 1930 to 2012 (footnote 9). The steady-state target for  $i_t/k_t$  corresponds to the value implied by a model with no capital adjustment costs. The values for B and  $\lambda$  depend on the chosen value for the curvature parameter  $\sigma_k$ . Each time  $\sigma_k$  is changed, the values of B and  $\lambda$  are adjusted to maintain the same steady-state ratios  $i_t/y_t$  and  $i_t/k_t$  as before. In this way, changes in  $\sigma_k$  identify a family of CES production functions that are distinguished only by the elasticity parameter, and not by the steady-state ratios  $i_t/y_t$  and  $i_t/k_t$ .<sup>22</sup> All else equal, smaller values for  $\sigma_k$  (implying stronger capital adjustment costs) result in a lower standard deviation of investment growth. Given the other parameters, the value  $\sigma_k = 0.65$  delivers a standard deviation for investment growth around 12.7% in the concentrated ownership model.

As discussed in the introduction, the standard deviations for the two shock innovations  $\sigma_{\varepsilon}$  and  $\sigma_u$  are chosen to match the 2.2% volatility of U.S. real per capita aggregate consumption growth and the 6.3% volatility of U.S. real per capita macroeconomic dividend growth. Macroeconomic dividends are constructed from the data as  $d_t = \theta_t y_t - i_t$ , where  $\theta_t$  is capital's share of income. The concentrated ownership model requires a higher value of  $\sigma_u$  because the investment decision of the top-decile agents has a smaller proportional impact on the volatility of model dividends versus the investment decision of a representative agent. In the sensitivity analysis, I investigate the effects of changing the relative volatility of the two shock innovations, as measured by the ratio  $\sigma_u/\sigma_{\varepsilon}$ . Different values for this ratio can be interpreted as reflecting alternative calibration targets for the distribution shock.

Using quarterly data, Greenwald, Lettau, and Ludvigson (2014) identify a "factor share shock" that is highly persistent—close to a random walk. The autocorrelation of capital's share of income in *annual* U.S. data from 1930 to 2012 is 0.8. Given that the capital income

<sup>&</sup>lt;sup>22</sup>This methodology follows the standard normalization procedure that is used when comparing CES production models with different parameterizations. See Klump and Saam (2008).

share  $\theta_t$  appears directly in the definition of macroeconomic dividends, I choose  $\rho = 0.8$  for the baseline calibration. A more-persistent distribution shock would impose less consumption risk on the capital owner, thereby shrinking the mean equity premium predicted by the model.

The discount factor  $\beta$  is chosen to achieve a mean price-dividend ratio of about 29, consistent with the long-run average for the S&P 500 stock index. Dimson, Marsh, and Staunton (2002) pp. 74-75 indicate that their return measure for long-term bonds is based on a portfolio of U.S. government bonds with maturities ranging from 5 to 20 years. Following Rudebusch and Swanson (2008), I calibrate the long-term bond in the model to have a Macaulay duration of 10 years. The Macaulay duration is the present-value-weighted average maturity of the bond's cash flows. The consol coupon decay parameter  $\delta^c$  is set so that the Macaulay duration of the model consol is D = 10, computed as follows:

$$D = \frac{\sum_{t=0}^{\infty} \left(\delta^{c} \widetilde{M}\right)^{t} (t+1)}{\sum_{t=0}^{\infty} \left(\delta^{c} \widetilde{M}\right)^{t}} = \frac{1}{1 - \delta^{c} \widetilde{M}},$$
(16)

where  $\widetilde{M} \equiv \exp\left[E\log\left(M_{t+1}\right)\right] = \beta \exp\left(-\mu\right)$  from equation (12).

The capital owner's time-varying coefficient of relative risk aversion  $(CRRA_t)$  is given by

$$CRRA_t \equiv -\frac{c_t U_{cc}}{U_c} = \frac{\alpha}{1 - \kappa (c_{t-1}/c_t) \exp(\mu)},$$
(17)

which collapses to  $\alpha/(1-\kappa)$  in steady state. The baseline values  $\alpha = 3.3$  and  $\kappa = 0.2$ imply a steady-state risk aversion coefficient of 4.125. These values deliver a sizeable equity risk premium in the concentrated ownership model without overpredicting the volatility of equity returns in the data. When  $\kappa > 0$ , excess returns on equity in the model exhibit some predictability—a well-documented feature of U.S. return data (Cochrane, 2008). I also examine the effects of employing different combinations for the values of  $\alpha$  and  $\kappa$ .

Table 2 compares the model distribution for income and wealth to the corresponding distribution in the U.S. economy. The U.S. financial wealth distribution data are from Wolff (2006), covering the period 1983 to 2001. The Gini coefficient data for income are from Heathcote, Perri, and Violante (2010) using the Current Population Survey for the period 1967 to 2005.

Statistic	U.S. Data	Concentrated Ownership Model		
Top decile share	400%a	40%		
Income	4070	4070		
Top decile share	800%b	100%		
Financial wealth	8070	10070		
Gini coefficient	0.22 0.42°	0.20		
Income	0.32 - 0.42	0.50		
Gini coefficient	0.80 0.03 <sup>b</sup>	0.00		
Financial wealth	0.09 - 0.93	0.30		

Table 2: Income and Wealth Distribution: Data versus Model

Sources: a = Piketty and Saez (2003, 2013), b = Wolff (2006), c = Heathcote, Perri, and Violante (2010).

### 4 Quantitative Results

#### 4.1 Impulse Response Functions

Details regarding the model solution are contained in the appendix. The capital growth rate  $x_t \equiv k_{t+1}/k_t$  is the only decision variable. There are four state variables: (1) the normalized capital stock  $k_{n,t} \equiv k_t/[\exp(z_t) (\ell^c)^a (n \ell^w)^{1-a}]$  which subsumes the productivity shock  $z_t$ , (2) the distribution shock  $v_t$ , (3) the lagged consumption-capital ratio  $c_{t-1}/k_{t-1}$ , and (4) the lagged decision variable  $x_{t-1}$ . The last two state variables summarize the influence of the external habit stock. An approximate log-linear solution is used as a starting value for an alternative solution method that preserves the model's nonlinear equilibrium conditions. The alternative solution employs a version of the parameterized expectation algorithm (PEA) described by Den Haan and Marcet (1990). The results obtained using the PEA solution are not much different from those generated by the log-linear solution.

Figure 3 plots the concentrated ownership model's response to a one standard deviation innovation of the distribution shock (solid blue line) and the productivity shock (dashed red line). The vertical axes measure the percentage deviation of each variable from the no-shock trend. The effects of the distribution shock are temporary but highly persistent—lasting in excess of 20 years. The effects of the productivity shock are permanent due to the unit root in the law of motion (7). With the exception of the workers' consumption, both shocks move the variables in the same direction. A positive distribution shock raises the capital owners' consumption but lowers the workers' consumption, resulting in a small increase in aggregate consumption. In this way, the model is able to match the low volatility of aggregate consumption growth in U.S. data while still delivering a sizeable equity premium.

Investment and dividends both exhibit strong positive responses to the distribution shock. Relative to the no-shock trend, investment increases by 12% on impact while dividends increase by nearly 6%. Recall that dividends are given by  $d_t = \theta_t y_t - i_t$ . A positive distribution shock raises the productivity of physical capital as measured by  $\theta_t$ . The capital owner reacts by devoting more resources to capital investment. But even with more resources devoted to investment, the higher value of  $\theta_t$  combined with the resulting increase in aggregate output  $y_t$ still allows for a nearly 6% increase in dividends relative to trend. The increase in dividends combined with a larger value of the capital owners's stochastic discount factor (explained further below) deliver an 18% increase in the equity price, in accordance with equation (3). The fact that a temporary distribution shock can induce a large move in the equity price allows the model to match the 20% standard deviation of real equity returns in U.S. data. However, as we shall see in the simulations, the volatility of the model price-dividend ratio is still below that observed in the financial market data.

For both shocks, the responses of investment and the equity price look qualitatively similar. The two variables are linked by the equilibrium relationship  $p_t^s = i_t g(k_{t+1}/k_t)$ , where movements in  $i_t$  and the nonlinear function  $g(k_{t+1}/k_t)$  are both influenced by the curvature parameter  $\sigma_k$  which governs the strength of capital adjustment costs. The small degree of overshooting in investment (and the equity price) that occurs in response to the permanent productivity shock can be traced to the influence of the external habit stock which introduces the lagged variables  $c_{t-1}/k_{t-1}$  and  $x_{t-1} = k_t/k_{t-1}$  as additional state variables in the model solution. Intuitively, the overshooting in investment helps to smooth the capital owners' felicity which depends on the lagged consumption basket  $C_{t-1}/H_{t-1}$ .

Both shocks cause the 1-year bond price to increase so as to satisfy the risk adjusted no-arbitrage condition across the different asset classes. An increase in the 1-year bond price implies an increase in the capital owners's stochastic discount factor via the equilibrium condition (4). Although not shown, the consol bond price also increases in response to both shocks, but with a magnitude that lies in between the responses of the equity price and the 1-year bond price.

#### 4.2 Sensitivity of Mean Equity Premium to Key Parameters

Figure 4 plots the mean equity premium relative to short-term bonds  $E(R_{t+1}^s - R_{t+1}^b)$  as some key parameters are varied in the concentrated ownership model (solid blue line) and the representative agent model (dashed red line).

I examine the effects of: (1) the relative volatility of the distribution shock innovation, as measured by the ratio  $\sigma_u/\sigma_{\varepsilon}$ , (2) the capital-investment substitution elasticity  $\sigma_k$  which governs the strength of capital adjustment costs, (3) the utility curvature parameter  $\alpha$ , and (4) the utility habit parameter  $\kappa$ . The vertical line in each panel marks the baseline calibration in the concentrated ownership model.<sup>23</sup> The return moments are computed analytically using the approximate log-linear solution of the model. When either  $\sigma_k$ ,  $\alpha$ , or  $\kappa$  is changed, the remaining non-curvature parameters are adjusted to maintain the same empirical targets shown in Table 1. To vary the ratio  $\sigma_u/\sigma_{\varepsilon}$ , I choose  $\sigma_u$  to be a fixed multiple of  $\sigma_{\varepsilon}$  while the latter continues to be chosen in each model to match the 2.2% standard deviation of U.S. aggregate consumption growth. Hence, for the plot that varies  $\sigma_u/\sigma_{\varepsilon}$ , the models do not match the 6.3% standard deviation of U.S. macroeconomic dividend growth, except at their respective baseline values for the ratio  $\sigma_u/\sigma_{\varepsilon}$ .

The top left panel of Figure 4 shows the effect of changing the relative volatility of the distribution shock innovation. Higher values of the ratio  $\sigma_u/\sigma_{\varepsilon}$  raise the equity premium in both models, but the gradient is very small in the representative agent model. The equity premium in the concentrated ownership model is more sensitive to the parameter shift because the ratio  $\sigma_u/\sigma_{\varepsilon}$  strongly impacts the volatility of dividends and hence the volatility of the capital owners' consumption growth. In contrast, an increase in  $\sigma_u/\sigma_{\varepsilon}$  has less impact on the volatility of the representative agent's consumption growth.

At the baseline calibration with  $\sigma_u/\sigma_{\varepsilon} = 1.855$ , the mean equity premium in the concentrated ownership model is 3.9% versus 0.5% in the representative agent model. When  $\sigma_u/\sigma_{\varepsilon} \simeq 2.5$ , the equity premium in the concentrated ownership model is nearly 7%, which is close to the U.S. average over the period 1900 to 2012. However, such a calibration would cause the model to overpredict the standard deviations of other variables, including investment growth and the equity return.

Table 3 shows how changes in the ratio  $\sigma_u/\sigma_{\varepsilon}$  affect the standard deviations of selected variables in the concentrated ownership model. Given the other parameter settings, values of  $\sigma_u/\sigma_{\varepsilon}$  that exceed the baseline ratio 1.855 cause the concentrated ownership model to start significantly overpredicting the 6.3% standard deviation of U.S. macroeconomic dividend growth and the 20% standard deviation of U.S. equity returns. The table also provides insight into how different calibration targets for the volatility of the distribution shock would influence the model's quantitative predictions. For example, calibrating the distribution shock to match the 5.6% standard deviation of the U.S. top decile income share  $s_t^c$  would imply  $\sigma_u/\sigma_{\varepsilon} \simeq 3$  and deliver a mean equity premium around 9%. In contrast, calibrating the distribution shock to match the 2% standard deviation of the U.S. capital income share  $\theta_t$  would imply  $\sigma_u/\sigma_{\varepsilon} \simeq 1$ and deliver a mean equity premium of only 1.4%. In this case, however, the model would significantly underpredict the volatility of U.S. macroeconomic dividend growth, yielding a standard deviation for  $\Delta \log (d_t)$  of only 3.9% versus 6.3% in the data. Since capital's share

<sup>&</sup>lt;sup>23</sup>The baseline values for  $\sigma_k$ ,  $\alpha$ , and  $\kappa$  are the same for both models. However, the representative agent model requires a lower baseline ratio  $\sigma_u/\sigma_{\varepsilon} = 1.024$  in order to match the calibration targets shown in Table 1.

of income is included as part of macroeconomic dividends, i.e.,  $d_t = \theta_t y_t - i_t$ , it would seem more appropriate to match the volatility of the relevant cash flows to be priced, as opposed to matching the volatility of  $\theta_t$  in isolation. In any case, Table 3 shows the results that would obtain under different calibration targets for the volatility of the distribution shock. Ideally, one would wish to calibrate the distribution shock to match the volatility of U.S. stockholder consumption growth, but reliable long-run data on this object is not available.

Standard Deviations						Mean			
$\sigma_u/\sigma_{\varepsilon}$	$\Delta \log \left( d_t \right)$	$s_t^c$	$\theta_t$	$\Delta \log \left( c_t^a \right)$	$\Delta \log (c_t)$	$\Delta \log \left( c_t^w \right)$	$\Delta \log (i_t)$	$R_{t+1}^s$	$R_{t+1}^s - R_{t+1}^b$
0	2.3%	0%	0%	2.2%	2.3%	2.1%	2.0%	3.1%	0.3%
1	3.9%	2.0%	2.1%	2.2%	3.5%	2.3%	7.0%	11.3%	1.4%
1.5	5.3%	2.9%	3.1%	2.2%	4.6%	2.5%	10.2%	16.6%	2.7%
1.855	6.3%	3.6%	3.8%	2.2%	5.4%	2.7%	12.4%	20.5%	3.9%
2	6.7%	3.9%	4.1%	2.2%	5.8%	2.7%	13.4%	22.1%	4.4%
2.5	8.2%	4.9%	5.1%	2.2%	7.0%	3.0%	16.6%	27.7%	6.7%
3	9.6%	5.8%	6.1%	2.2%	8.1%	3.3%	19.7%	33.5%	9.3%
U.S. Data	6.3%	5.6%	2.0%	2.2%	_	_	12.7%	20.1%	7.3%

Table 3: Effect of Distribution Shock in Concentrated Ownership Model

Notes: Moments of concentrated ownership model are computed analytically from the log-linear solution. The baseline calibration from Table 1 implies  $\sigma_u/\sigma_{\varepsilon} = 1.855$ . For the U.S. data,  $d_t$  is macroeconomic dividends, given by  $\theta_t y_t - i_t$ .  $s_t^c =$  top decile income share.

The top right panel of Figure 4 shows the effect of changing  $\sigma_k$ . Smaller values of  $\sigma_k$  imply more curvature in the capital law of motion (10). More curvature implies that new investment is more complimentary (i.e., more tightly coupled) to existing capital, thereby increasing the cost of adjusting next period's capital stock via changes in new investment. All else equal, stronger capital adjustment costs impair the capital owner's ability to smooth consumption, thereby raising the mean equity premium. In the figure, however, all else is not equal. Whenever the value of  $\sigma_k$  is changed, the standard deviations of the two shock innovations must be recalibrated to match the volatilities of U.S. aggregate consumption growth and U.S. macroeconomic dividend growth. For both models, smaller values of  $\sigma_k$  necessitate a lower ratio  $\sigma_u/\sigma_{\varepsilon}$  to achieve the calibration targets. The end result is that smaller values of  $\sigma_k$  serve only to mildly raise the mean equity premium in the calibrated models. The baseline value for  $\sigma_k$  is picked so that the concentrated ownership model approximately matches the 12.7% standard deviation of investment growth in U.S. data from 1930 to 2012.

Figure 5 depicts how smaller values of  $\sigma_k$  imply more curvature in the relationship between the investment-capital ratio  $i_t/k_t$  and gross capital growth  $k_{t+1}/k_t$ . When  $\sigma_k = 1$ , capital law of motion takes the form of a Cobb-Douglas function. As  $\sigma_k \to \infty$ , the capital adjustment costs become vanishingly small. During model simulations, the realized adjustment costs are generally small but can occasionally be large if realizations of  $i_t/k_t$  fall outside a two standard deviation range surrounding the mean.

The bottom two panels in Figure 4 show that higher values for either  $\alpha$  or  $\kappa$  lead to a higher mean equity premium in both models. This is not surprising given that an increase in either parameter contributes to a higher coefficient of relative risk aversion, as shown by equation (17). For any given level of risk aversion, the high volatility of the capital owner's consumption growth serves to magnify the equity risk premium in the concentrated ownership model relative to the representative agent model. The concentrated ownership model can deliver a mean equity premium near 7% if  $\alpha$  is increased to around 4.5 or if  $\kappa$  is increased to around 0.4. However, as with increasing the ratio  $\sigma_u/\sigma_{\varepsilon}$ , increasing either  $\alpha$  or  $\kappa$  will cause the model to start significantly overpredicting the volatility of the equity return in U.S. data. At the baseline values of  $\alpha = 3.3$ ,  $\kappa = 0.2$ ,  $\sigma_k = 0.65$  and  $\sigma_u/\sigma_{\varepsilon} = 1.855$ , the concentrated ownership model comes very close to matching the 20.1% standard deviation of the equity return in the data.

#### 4.3 Model Simulations

Table 4 provides some direct evidence in support of the model's main mechanism, namely an empirical link between distribution risk, as measured by movements in two separate income share variables, and the contemporaneous equity risk premium. I regress the equity premium in the data on the change in the top decile income share  $\Delta s_t^c$  and the change in capital's share of income  $\Delta \theta_t$ . The U.S. data regressions employ the same equity premium and income share data plotted earlier in Figure 1, where the variables in each regression have been scaled by their sample standard deviations. The first U.S. data regression shows a positive estimated coefficient b = 0.308, which is statistically significant with a t-statistic of 0.308/0.099 = 3.11. The second U.S. data regression also shows a positive estimated coefficient b = 0.280 with a t-statistic of 0.280/0.107 = 2.62. These results complement the empirical findings of Greenwald, Lettau, and Ludvigson (2014) who employ a vector autoregression analysis on quarterly U.S. data over the period 1952.Q2 to 2012.Q4. Their study identifies a statistically significant impact of "factor share shocks" on detrended stock market wealth and detrended stock prices.

The regressions on model-generated data in Table 4 show that an increase in either income share variable serves to raise the contemporaneous equity premium  $R_t^s - R_t^{b.24}$  The statistical correlation between  $\Delta \theta_t$  and the equity premium is stronger in the concentrated ownership model than in the representative agent model, consistent with the sensitivity results plotted

<sup>&</sup>lt;sup>24</sup>Regressions involving  $\Delta s_t^c$  are not reported for the representative agent model because  $s_t^c = 1$  for all t in this version of the model.

earlier in the top left panel of Figure 4.

The bottom row of Table 4 shows that there is a positive and statistically significant correlation between the U.S. equity premium and the growth rate of macroeconomic dividends with b = 0.289 and a t-statistic of 0.289/0.106 = 2.73. In contrast, the correlation between the U.S. equity premium and the growth rate of S&P 500 dividends is close to zero. This evidence suggests that U.S. macroeconomic dividends can be viewed as a relevant cash flow for equity pricing. In simulations, both versions of the model exhibit a positive and strongly significant correlation between the equity premium and the growth rate of model dividends  $\Delta \log(d_t)$ .

Table 4: Slope Coefficient in Equity Premium Regressions						
Concentrated Representative						
Regression	Dates	U.S. Data	Ownership	Agent		
$R_t^s - R_t^b =$	1010 2012	0.308	0.952	_		
$a + b\Delta s_t^c + \eta_{t+1}$	1919-2012	(0.099)	(0.002)	_		
$R_t^s - R_t^b =$	1020 9019	0.280	0.952	0.738		
$a + b\Delta\theta_t + \eta_{t+1}$	1950-2012	(0.107)	(0.002)	(0.005)		
	1020 0010	0.289				
$R_t^s - R_t^b =$	1950-2012	(0.106)	0.945	0.879		
$a + b\Delta\log\left(d_t\right) + \eta_{t+1}$	1000 0010	-0.008	(0.002)	(0.003)		
	1900-2012	(0.095)				

Notes: Standard errors in parentheses. In all regressions, the equity premium and right-side variables are scaled by their sample standard deviations. Model regressions are based on data from a 20,000 period simulation. For the U.S. data, the top regression results for  $\Delta \log (d_t)$  are based on macroeconomic dividends given by  $\theta_t y_t - i_t$ , while the bottom regression results are based on S&P 500 dividends.  $s_t^c = top$  decile income share. Regressions involving  $\Delta s_t^c$  are not reported for the representative agent model because  $s_t^c = 1$  for all t.

Table 5 presents unconditional moments of asset pricing variables computed from model simulations using the baseline parameter values shown in Table 1. The table also shows the corresponding statistics from U.S. data.<sup>25</sup> Figures 6, 7, and 8 provide a visual comparison between the U.S. data and the model-generated data for selected variables.

 $<sup>^{25}</sup>$ Data on the price-dividend ratio are from Robert Shiller's website. The price-dividend ratio in year t is defined as the value of the S&P 500 stock index at the beginning of year t + 1, divided by the accumulated S&P 500 dividends over year t. The U.S. real return statistics shown in Table 5 are for equity, short term bills, and long-term bonds from Dimson, et al. (2002), updated through 2012.

			0	
		U.S. Data	Concentrated	Representative
Variable	Statistic	1900-2012	Ownership	Agent
$p_t^s/d_t$	Mean	29.0	29.3	28.8
	Std. Dev.	15.3	5.30	1.74
	Corr. Lag 1	0.93	0.72	0.88
$R_{t+1}^s - 1$	Mean	8.2%	7.4%	5.6%
·	Std. Dev.	20.1%	20.4%	5.4%
	Corr. Lag 1	-0.01	-0.16	-0.24
$R_{t+1}^b - 1$	Mean	0.95%	3.6%	5.2%
·	Std. Dev.	4.7%	7.1%	1.7%
	Corr. Lag 1	0.62	0.66	0.35
$R_{t+1}^c - 1$	Mean	2.6%	6.2%	5.4%
	Std. Dev.	10.2%	15.3%	3.5%
	Corr. Lag 1	0.05	-0.15	-0.27
Sharpe	$E(R_{t+1}^{s} - R_{t+1}^{b})$	0.264	0.152	0.051
Ratio	$\overline{SD\left(R_{t+1}^{s}-R_{t+1}^{b}\right)}$	0.004	0.100	0.001
Sharpe	$E\left(R_{t+1}^s - R_{t+1}^c\right)$	0.970	0.049	0.092
Ratio	$\overline{SD\left(R_{t+1}^s - R_{t+1}^c\right)}$	0.279	0.040	0.025

Table 5: Unconditional Asset Pricing Moments

Note: Model results are computed from a 20,000 period simulation. For the U.S. data, statistics for  $p_t^s/d_t$  are based on the S&P 500 stock index and S&P 500 dividends.

Table 5 and the top panels of Figure 7 show that the concentrated ownership model underpredicts the volatility of the price-dividend ratio in the data but delivers about three times more volatility than the representative agent model. The standard deviation of the S&P 500 price-dividend ratio is 15.3 versus 5.3 in the concentrated ownership model and 1.7 in the representative agent model. The volatility of the S&P 500 price-dividend ratio is influenced by the dramatic bubble-like run-up starting in the mid-1990s that is partially retraced by the end of the data sample in 2012. A large literature finds evidence that real-world stock prices exhibit "excess volatility" when compared to the discounted stream of ex post realized dividends.<sup>26</sup> If findings of excess volatility in the data are genuine, then one would not expect a fully rational model like this one to be able to match the volatility of the price-dividend ratio in the data. An extension of the present model that allows for boundedly-rational expectations on the part of capital owners could potentially magnify the volatility of the price-dividend ratio, providing a better match with the data.<sup>27</sup>

Despite underpredicting the volatility of the price-dividend ratio, the concentrated ownership model provides a decent match with mean and volatility of the U.S. equity return, which are around 8% and 20%, respectively. The concentrated ownership model overpredicts

<sup>&</sup>lt;sup>26</sup>Lansing and LeRoy (2014) provide a recent update on this literature.

<sup>&</sup>lt;sup>27</sup>For examples along these lines, see Bansal and Shaliastovich (2010), Fuster, Hebert, and Laibson (2012), and Lansing (2010, 2012), among others.

the mean and volatility of the U.S. short-term bond return, although it should be noted that the return data constructed by Dimson, Marsh, and Staunton (2002, updated) pertain to a 3-month "bill" whereas the short-term bond in the model has a one-year maturity. The prediction of too much volatility in the short term bond return is a typical shortcoming of models with habit formation (Jermann 1998 and Abel 2008). It is possible, however, to reverseengineer more complicated laws of motion for the stochastic discount factor (12) so that the *expected* stochastic discount factor  $E_t M_{t+1}$  exhibits little or no volatility, thereby reducing or even eliminating the volatility in the short term bond return. The reverse-engineering approach has the unfortunate side effect of magnifying the degree of steady-state risk aversion that is needed to generate a sizeable equity premium (Campbell and Cochrane 1999).

As noted in the introduction, the concentrated ownership model's long-term bond behaves too much like equity such that the mean and volatility of the consol are too high relative to the mean and volatility of the U.S. long-term bond return. This deficiency in the model is well-summarized by the Sharpe ratio comparison at the bottom of Table 5. The concentrated ownership model does capture the fact that returns on equity and long-terms bonds exhibit near-zero autocorrelation in the data while returns on short-term bonds exhibit strong positive autocorrelation. Overall, the concentrated ownership model substantially outperforms the representative agent model in matching the majority of the U.S. data statistics in Table 5.

The bottom panel of Figure 7 shows that the equity premium  $R_{t+1}^s - R_{t+1}^b$  in the concentrated ownership model exhibits a correlation coefficient with aggregate consumption growth of 0.25 versus a value of 0.17 in the data. The correlation coefficient in the representative agent model is much higher at 0.86. Movements in the equity premium are determined in part by movements in the stochastic discount factor. In the concentrated ownership model, the stochastic discount factor (12) depends on the capital owner's consumption which responds differently to shocks than does aggregate consumption (see Figure 3). In contrast, the stochastic discount factor in the representative agent model depends on aggregate consumption which helps to explain the representative agent model's counterfactual prediction of a strong positive correlation between aggregate consumption growth and the equity premium. Croce (2014) develops a representative agent production economy with long-run risk that exhibits a sizeable equity premium (when applying a leverage multiplier of 2) and a low correlation between the equity premium and aggregate consumption growth. However, his model significantly underpredicts the volatility of U.S. equity returns.

Figure 8 shows that asset returns in the data and the concentrated ownership model exhibit time-varying means and volatilities. The time-varying behavior in the data suggests the presence of nonlinearities. The time-varying behavior in the concentrated ownership model is wholly endogenous, owing to the nonlinear nature of the model's equilibrium conditions combined with a calibration that necessitates a more-volatile distribution shock in comparison to the representative agent model. In contrast, Bansal and Yaron (2004) introduce *exogenous* time-varying volatility via the stochastic process for consumption growth within an endowment economy.

			Concentrated	Representative
Variable	Dates	U.S. Data	Ownership	Agent
$\Delta \log \left( y_t \right)$	1930-2012	3.04%	3.05%	2.73%
$\Delta \log \left( c_t^a  ight)$	1930-2012	2.16%	2.19%	2.16%
$\Delta \log \left( c_t \right)$	_	_	5.40%	_
$\Delta \log \left( c_t^w \right)$	_	_	2.77%	_
$\Delta \log (i_t)$	1930-2012	12.7%	12.4%	3.35%
$\Delta \log \left( d_t \right)$	1930-2012	6.31%	6.30%	6.31%
$\Delta \log \left( p_t^s \right)$	1930-2012	19.0%	19.0%	5.12%
$s_t^c$	1930-2012	5.64%	3.60%	_
$ heta_t$	1930-2012	1.95%	3.78%	1.96%
$\Delta s_t^c$	1930-2012	1.55%	2.29%	_
$\Delta \theta_t$	1930-2012	1.20%	2.40%	1.24%
$\frac{SD \left[\Delta \log(c_t)\right]}{SD \left[\Delta \log(c_t^w)\right]}$	1982 - 2004	1.63	1.95	_

Table 6: Standard Deviation of Macro Variables

Note: Model results computed from a 20,000 period simulation. For the U.S. data,  $d_t$  is macroeconomic dividends given by  $\theta_t y_t - i_t$ .  $s_t^c = \text{top decile income share.}$ 

Tables 6 and 7 show that the concentrated ownership model performs well in matching the business cycle moments of aggregate macro variables. By design, the model matches the standard deviations of U.S. aggregate consumption growth  $\Delta \log (c_t^a)$ , U.S. macroeconomic dividend growth  $\Delta \log (d_t)$ , and U.S. investment growth  $\Delta \log (i_t)$ . The model also matches the standard deviation of U.S. output growth  $\Delta \log (y_t)$  because data on real output are constructed as the sum of aggregate consumption and investment, consistent with the model. Given that the model does a good of matching the volatility of U.S. real equity returns, it also does a good job of matching the 19% standard deviation of real equity price changes  $\Delta \log (p_t^s)$ .

Using the same calibration targets for the stochastic shocks and the same value for the capital adjustment cost parameter  $\sigma_k$ , the representative agent model can only match the standard deviations of  $\Delta \log (c_t^a)$  and  $\Delta \log (d_t)$  in the data, but not the standard deviations of  $\Delta \log (y_t)$ . All else equal, the temporary distribution shock has less impact on investment growth volatility in the representative agent model because the agent's consumption-investment decision pertains to aggregate consumption which is a much larger base than the capital owner's consumption in the concentrated ownership model. Moreover, recall from Ta-

ble 1 that the representative agent model employs a baseline calibration with  $\sigma_u/\sigma_{\varepsilon} = 1.024$ whereas the concentrated ownership model requires  $\sigma_u/\sigma_{\varepsilon} = 1.855$  to match the same empirical targets.

In the middle section of Table 6, we see that the concentrated ownership model underpredicts the volatility of the U.S. top decile income share  $s_t^c$  but overpredicts the volatility of the U.S. capital income share  $\theta_t$ . This is a consequence of the calibration whereby the value of  $\sigma_u$ is chosen to match the volatility of U.S. macroeconomic dividend growth. Recall that  $\theta_t$  in the data is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector (footnote 4). This measure could underestimate the volatility of U.S. stockholders' capital income share, which is not directly observable. While the representative agent model can match the volatility of  $\theta_t$  in the data, it makes no predictions regarding the volatility of the top decile income share  $s_t^c$ .

The bottom row of Table 6 shows that the capital owner's consumption growth  $\Delta \log (c_t)$  is about two times more volatile than the worker's consumption growth  $\Delta \log (c_t^w)$ . The source of the extra volatility for capital owners is their heavy reliance on volatile dividends to fund their consumption whereas workers' consumption is funded solely from labor income. From the impulse response functions in Figure 3, we see that a positive distribution shock that raises capital's share of income  $\theta_t$  also causes output to rise, implying that labor's share of income  $(1 - \theta_t)$  is countercyclical. The countercyclical nature of labor's share helps to smooth the workers' consumption relative to that of capital owners.

Malloy, Moskowitz, and Vissing-Jørgensen (2009) study consumption growth data for stockholders versus non-stockholders over the period 1982 to 2004.<sup>28</sup> Using their data, the consumption growth volatility ratio for the two groups is 1.63, as shown Table 6. The corresponding volatility ratio in the model is a bit higher at 1.95. Expanding the Malloy, Moskowitz, and Vissing-Jørgensen (2009) sample period to include the Great Depression and other volatile stock market episodes would likely magnify the volatility of stockholders' consumption growth relative to that of non-stockholders. Aït-Sahalia, Parker, and Yogo (2004) argue that luxury goods sales provide a better proxy for the consumption of U.S. stockholders than does aggregate consumption. They find (p. 2974) that luxury retail sales growth is about 4 times more volatile than aggregate consumption growth and that sales of luxury goods covary positively with excess stock market returns over the period 1961 to 2001. By comparison, Table 6 shows that the capital owner's consumption growth in the concentrated ownership model is only 2.5 times more volatile than aggregate consumption growth.

In the model of Guvenen (2009), the source of extra volatility for stockholders is the bond

<sup>&</sup>lt;sup>28</sup>The data are available from <http://faculty.haas.berkeley.edu/vissing/>.

market; stockholders make interest payments to bondholders which smooths the bondholders' consumption but magnifies the volatility of stockholders' consumption. Guvenen's model delivers a consumption growth volatility ratio for stockholders relative to non-stockholders of 2.4. In the model of Danthine and Donaldson (2002), the source of extra volatility for capital owners is the wage contract which smoothes workers' consumption at the expense of larger fluctuations in capital owners' consumption. In the version of their model that delivers an equity premium approaching 6%, the capital owners' consumption growth is 10 times more volatile than aggregate consumption growth.<sup>29</sup>

The top section of Table 7 shows that macro variables in the concentrated ownership model exhibit mostly strong correlations with output growth—a typical feature of productivity-shock driven real business cycle models. The sole exception is the workers' consumption growth  $\Delta \log (c_t^w)$  which exhibits a correlation coefficient with  $\Delta \log (y_t)$  of only 0.11. This result is due to the distribution shock which causes the workers' consumption to move opposite to output, as shown earlier in the impulse response functions (Figure 3).

In the middle section of Table 7, we see that model dividend growth  $\Delta \log (d_t)$  and the model asset pricing variables  $\Delta \log (p_t^s)$ ,  $R_{t+1}^s - R_{t+1}^b$ , and  $R_{t+1}^s - R_{t+1}^c$  all exhibit strong correlations with output growth, but the corresponding correlations in the U.S. data are very weak. This observation suggests the presence of additional fundamental or non-fundamental factors that induce movements in real-world dividends and asset prices, but are missing from the model. Along these lines, Greenwald, Lettau, and Ludvigson (2014) argue that "risk aversion shocks" which are unrelated to either aggregate consumption or aggregate labor income are a significant driver of short-term movements in U.S. stock prices. Such a shock could be introduced into the present model by allowing for stochastic variation in the capital owner's utility curvature parameter  $\alpha$  which appears in the expression for the risk aversion coefficient (17). Another possibility would be to allow for stochastic variation in another preference parameter, such as the stockholder's subjective discount factor  $\beta$ .

The bottom section of Table 7 shows that both models capture the procyclical movement of capital's share of income  $\theta_t$ . As shown earlier in Figure 3, a positive distribution shock that raises  $\theta_t$  also induces a temporary but persistent increase in real output. In simulations, the concentrated ownership model delivers a correlation coefficient of 0.28 between  $\theta_t$  and the growth rate of real output  $\Delta \log (y_t)$ . This result matches the correlation coefficient in the data over the period 1930 to 2012.<sup>30</sup> The top decile income share  $s_t^c$  exhibits a negative correlation

<sup>&</sup>lt;sup>29</sup>See Table 6, Panel A, p. 62 in Danthine and Donaldson (2002). They do not report the volatility of workers' consumption growth.

<sup>&</sup>lt;sup>30</sup>Young (2004) and Ríos-Rull and Santaeulàlia-Llopis (2010) report very similar correlation statistics using post-WWII data on the labor income share and real output, both detrended using the Hodrick-Prescott filter.

of -0.13 with output growth over the period 1930 to 2012. In contrast, the concentrated ownership model delivers a positive correlation of 0.28. Some portion of the observed movements in  $s_t^c$  in the data may reflect a permanent trend whereas the model implies that all movements in  $s_t^c$  are temporary. Looking at the correlation between  $\Delta s_t^c$  and  $\Delta \log (y_t)$  eliminates the influence of any permanent trend. In this case, the data and the model both exhibit a positive correlation, but the correlation coefficient in the model (0.71) is higher than that in the data (0.24).

Table 7: Correlations with Output Growth						
	U.S. Data Concentrated Representative					
Variable	1930-2012	Ownership	Agent			
$\Delta \log \left( y_t \right)$	1.00	1.00	1.00			
$\Delta \log \left( c_t^a \right)$	0.94	0.76	0.99			
$\Delta \log (c_t)$	_	0.92	_			
$\Delta \log \left( c_t^w  ight)$	_	0.11	_			
$\Delta \log (i_t)$	0.87	0.80	0.90			
$\Delta \log \left( d_t  ight)$	-0.05	0.90	0.74			
$\Delta \log \left( p_t^s \right)$	0.07	0.80	0.90			
$R_{t+1}^s - R_{t+1}^b$	0.13	0.79	0.92			
$R_{t+1}^s - R_{t+1}^c$	0.14	0.92	0.85			
$s_t^c$	-0.13	0.28	_			
$ heta_t$	0.28	0.28	0.17			
$\Delta s_t^c$	0.24	0.71	_			
$\Delta \theta_t$	0.15	0.71	0.50			

Notes: Model results computed from a 20,000 period simulation. For the U.S. data,  $d_t$  is macroeconomic dividends given by  $\theta_t y_t - i_t$ .  $s_t^c = \text{top decile income share.}$ 

Table 8 provides an alternative way of comparing the properties of U.S. macroeconomic dividends to those of model dividends. In each case, dividends are normalized by output to create the stationary ratio  $d_t/y_t = \theta_t - i_t/y_t$ . By construction, both models match the mean of  $d_t/y_t$  in the data because the model parameters are chosen to match the means of  $\theta_t$  and  $i_t/y_t$ in the data. Both models underpredict the standard deviation of  $d_t/y_t$  in the U.S. data, again suggesting that the baseline calibration for the distribution shock volatility can be viewed as conservative. Both models come reasonably close to matching the standard deviation of  $\Delta (d_t/y_t)$  in the data, as well as the other U.S. statistics shown in Table 8. The table shows that  $d_t/y_t$  in the data exhibits a near-zero correlation with output growth over the period 1930 to 2012. The corresponding correlations in the models are positive but weak.

Table 8: Properties of Macroeconomic Dividend-Output Ratio					
	U.S. Data	Concentrated	Representative		
Statistic	1930-2012	Ownership	Agent		
$E\left(d_t/y_t\right)$	0.214	0.210	0.211		
$SD  \left( d_t / y_t  ight)$	0.047	0.016	0.018		
$SD \left[\Delta \left(d_t/y_t\right)\right]$	0.014	0.008	0.010		
$Max \ (d_t/y_t)$	0.322	0.278	0.288		
$Min \ (d_t/y_t)$	0.113	0.156	0.148		
Corr $(d_t/y_t, d_{t-1}/y_{t-1})$	0.95	0.87	0.84		
Corr $(d_t/y_t, \Delta \log(y_t))$	0.03	0.21	0.16		

Notes: Model results computed from a 20,000 period simulation. For the U.S. data,  $d_t$  is macroeconomic dividends, given by  $\theta_t y_t - i_t$ .

Finally, Table 9 shows the results of forecasting regressions of the type that commonly appear in the finance literature (e.g., Cochrane, 2008). The regressions seek to predict either the excess return on equity relative to short-term bonds  $R_{t+1}^s - R_{t+1}^b$  or the gross dividend growth rate  $d_{t+1}/d_t$  using the prior year's value of the dividend yield  $d_t/p_t$  (i.e., the inverse of the price-dividend ratio). For comparability with the finance literature, I use S&P 500 dividends to represent  $d_t$  in the data, not macroeconomic dividends. For the model regressions, the table shows results for the baseline calibration with  $\alpha = 3.3$  and  $\kappa = 0.2$  and an alternative calibration with  $\alpha = 2.0625$  and  $\kappa = 0.5$ . The alternative calibration has the same steady-state coefficient of relative risk aversion as the baseline, but the higher value of the habit parameter  $\kappa$  implies more time-variation in the risk aversion coefficient in response to shocks, as governed by equation (17). The signs and magnitudes of the model-generated regression coefficients are influenced by the values of  $\alpha$  and  $\kappa$ .

Both U.S. data regressions imply that a higher dividend yield predicts higher excess returns on equity. Put another way: when equity prices are temporarily low relative to dividends (i.e., a high value for  $d_t/p_t$ ) future equity prices will tend to rise faster than dividends, pushing the dividend yield back down towards its long-run mean and in so doing, delivering a higher return on equity relative to bonds. The U.S. data results for the excess return regression in Table 9 are in the range of those reported by Cochrane (2008), p. 1534. He estimates a statistically significant value of b = 3.83 (standard error = 1.47) using U.S. stock market data for the period 1929 to 2004.

Like the data, the concentrated ownership model delivers a positive estimated slope coefficient in the excess return regression, with b = 3.96 for the baseline calibration and b = 8.70 for the alternative calibration. The higher value of the habit parameter  $\kappa$  in the alternative calibration delivers more time variation in the capital owner's risk aversion coefficient and hence more time variation in the excess return which compensates the capital owner for undertaking the risk of holding equity. The representative agent model delivers a negative estimated slope coefficient b = -0.62 for the baseline calibration but a positive estimated slope coefficient b = 3.08 for the alternative calibration, where the latter result is much closer to the data. Since the representative agent's stochastic discount factor is driven by aggregate consumption, excess returns and the dividend yield are less volatile relative to the concentrated ownership model, thus influencing the value of the slope coefficient in the regressions.

For the dividend growth regression, Table 9 shows that the data yield a negative estimated slope coefficient b = -2.60 for sample period from 1900 to 2012. This result implies that when equity prices are temporarily low relative to dividends (i.e., a high value for  $d_t/p_t$ ) future dividend growth rates will tend to be lower on average, thus helping to justify the current state of low equity prices relative to dividends. However, in the more recent sample period from 1948 to 2012, the estimated slope coefficient in the dividend growth regression is positive but not statistically significant, i.e., b = 0.12 (standard error = 0.60). Cochrane (2008) estimates b = 0.07 (standard error = 1.17) using data for the period 1929 to 2004. Hence, the more recent data imply that a low dividend yield is not predictive of higher future dividend growth.

Table 9 shows that the concentrated ownership model can deliver either a positive or negative value of the slope coefficient in the dividend growth regression, depending on the calibration. The baseline calibration yields b = 2.29 while the alternative calibration yields b = -0.12, but latter estimate is not statistically significant. These results can be traced to the more-volatile stochastic discount factor in the alternative calibration which causes movements in the equilibrium dividend yield to be driven almost entirely by movements in expected future returns as opposed to movements in expected future dividend growth rates. Overall, Table 9 shows that the concentrated ownership model can produce regression results that are broadly similar to those obtained using U.S. financial market data.

Table 9: Stope Coefficient in Forecasting Regressions					
		Concentrated	Representative		
	U.S. Data	Ownership	Agent		
Domogram		$\alpha = 3.3  \alpha = 2.0625$	$\alpha = 3.3  \alpha = 2.0625$		
Regression	1900-2012 1948-2012	$\kappa = 0.2$ $\kappa = 0.5$	$\kappa = 0.2$ $\kappa = 0.5$		
$R_{t+1}^s - R_{t+1}^b =$	1.72 3.41	3.96 8.70	-0.62 3.08		
$a+b \ (d_t/p_t^s)+\eta_{t+1}$	(1.10) $(1.50)$	(0.28) $(0.37)$	(0.23) $(0.29)$		
$d_{t+1}/d_t =$	-2.60 0.12	2.29 - 0.12	-6.50 -8.16		
$a + b (d_t/p_t^s) + \eta_{t+1}$	(0.60) $(0.60)$	(0.07) $(0.08)$	(0.21) $(0.16)$		

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Notes: Standard errors in parentheses. Model regressions based on data from a 20,000 period simulation. For the U.S. data,  $d_t$  is S&P 500 dividends and  $p_t^s$  is the S&P 500 stock price index. For the models, both calibrations imply a steady-state coefficient of relative risk aversion equal to 4.125.

### 5 Conclusion

A long history of research since Mehra and Prescott (1985) has sought to develop models that can account for the high mean and high volatility of observed equity returns relative to bond returns. One branch of this research has focused on investigating modifications to agents' preferences that govern attitudes towards risk or intertemporal substitution. Another branch has focused on investigating changes to the structure of the cash flows that are priced by agents in the model. This paper falls mainly into the second category. The basic intuition for the results is that capital owners demand a high equity premium to compensate for the risk of linking their consumption to a volatile dividend stream. Since ownership of stock market wealth in the U.S. economy is highly concentrated at the top end of the income distribution, the owners of this wealth must bear a disproportionate share of the risk from shocks that cause their dividend-type income to fluctuate.

In the model, volatility derives from two sources: (1) a random walk productivity shock and (2) a temporary but persistent distribution shock that shifts income between capital owners (stockholders) and workers (non-stockholders). As the volatility of the distribution shock increases, the mean equity premium rises and the return on equity becomes more volatile (Table 3). With reasonable levels of risk aversion (coefficients of relative risk aversion in steady state around 4), the concentrated ownership model delivers an unlevered mean equity premium relative to short-term bonds of nearly 4%. The model can match other quantitative features of U.S. data under the assumption of fully-rational expectations, but it notably underpredicts the volatility of the price-dividend ratio in the financial market data. This underprediction could potentially be addressed by a richer model that allows for than two fundamental shocks or non-fundamental elements that give rise to excess volatility.

### A Appendix: First-Order Condition in Stationary Variables

To facilitate a solution for the equilibrium allocations, the first-order condition (15) must be rewritten in terms of stationary variables. Given that labor supply is inelastic, the combined entity of the firm and capital owner must only decide the fraction of available resources to be devoted to investment, with the remaining fraction devoted to consumption. The investmentcapital ratio  $i_t/k_t$  is uniquely pinned down by the growth rate of capital. Hence, I employ  $x_t \equiv k_{t+1}/k_t$  as the capital owner's single decision variable. There are four stationary state variables: (1) the normalized capital stock defined as  $k_{n,t} \equiv k_t/[\exp(z_t) (\ell^c)^a (n \ell^w)^{1-a}]$ , (2) the distribution shock  $v_t$ , (3) the lagged consumption-capital ratio  $c_{t-1}/k_{t-1}$ , and (4) the lagged decision variable  $x_{t-1}$ . The last two state variables summarize the influence of the external habit stock. From the definition of  $k_{n,t}$  and the production function (6), it follows that  $y_t/k_t = (k_{n,t})^{\theta_t-1}$ .

Dividing both sides of the firm's first-order condition (15) by  $k_{t+1}$  and then employing the definitions of  $x_t$  and  $k_{n,t}$  yields:

$$(i_t/k_t) g(x_t) x_t^{-1} = E_t M_{t+1} \left\{ \theta_{t+1} (k_{n,t+1})^{\theta_{t+1}-1} - (i_{t+1}/k_{t+1}) [1 - g(x_{t+1})] \right\},$$

(A1)

where

$$i_t/k_t = (1-\delta) \left\{ \frac{1}{\lambda} \left[ \frac{x_t}{B(1-\delta)} \right]^{\psi_k} - \frac{(1-\lambda)}{\lambda} \right\}^{1/\psi_k} \text{ for all } t,$$

$$g(x_t) \equiv 1 + \frac{1-\lambda}{\left[ \frac{x_t}{B(1-\delta)} \right]^{\psi_k} - (1-\lambda)}, \text{ for all } t,$$

$$M_{t+1} \equiv \beta \exp\left(-\phi\mu\right) \left[ \frac{c_{t+1}/k_{t+1} - \kappa \exp\left(\mu\right) \left(c_t/k_t\right) x_t^{-1}}{c_t/k_t - \kappa \exp\left(\mu\right) \left(c_{t-1}/k_{t-1}\right) x_{t-1}^{-1}} \right]^{-\alpha} x_t^{-\alpha},$$

$$c_t/k_t = [\theta_t + a(1-\theta_t)] (k_{n,t})^{\theta_t - 1} - i_t/k_t, \text{ for all } t.$$

Using the definitions of  $k_{n,t}$  and  $x_t$ , the law of motion for the normalized capital stock is

$$k_{n,t+1} \equiv \frac{k_{t+1}}{\exp(z_{t+1}) (\ell^c)^a (n \,\ell^w)^{1-a}} = \frac{k_t \, x_t \exp(z_t - z_{t+1})}{\exp(z_t) (\ell^c)^a (n \,\ell^w)^{1-a}}$$
$$= k_{n,t} \, x_t \exp(-\mu - \varepsilon_{t+1}), \qquad (A2)$$

which is conveniently log-linear before undertaking any approximation.

An expression for the capital owner's consumption growth in terms of stationary variables is given by

$$c_{t+1}/c_t = x_t \left\{ \frac{\left[\theta_{t+1} + a\left(1 - \theta_{t+1}\right)\right] \left(k_{n,t+1}\right)^{\theta_{t+1}-1} - i_{t+1}/k_{t+1}}{\left[\theta_t + a\left(1 - \theta_t\right)\right] \left(k_{n,t}\right)^{\theta_{t-1}} - i_t/k_t} \right\},$$
(A3)

where  $i_{t+1}/k_{t+1}$  and  $i_t/k_t$  depend on the decision variables  $x_{t+1}$  and  $x_t$ , respectively, as shown in (A1). It is straightforward to derive analogous expressions for dividend growth  $d_{t+1}/d_t$ , output growth  $y_{t+1}/y_t$ , aggregate consumption growth  $c_{t+1}^a/c_t^a$ , and the worker's consumption growth  $c_{t+1}^w/c_t^w$ .

#### A.1 Asset Pricing Variables

Given the equilibrium relationships  $p_t^s = i_t g(x_t)$  and  $d_t = \theta_t y_t - i_t$ , it is straightforward to derive the following expressions for the equity price-dividend ratio and the gross equity return in terms of stationary variables:

$$p_t^s/d_t = \frac{(i_t/k_t) g(x_t)}{\theta_t (k_{n,t})^{\theta_t - 1} - i_t/k_t},$$
(A4)

$$R_{t+1}^{s} = \frac{p_{t+1}^{s} + d_{t+1}}{p_{t}^{s}}$$
$$= \frac{\left(p_{t+1}^{s}/d_{t+1} + 1\right) \left[\theta_{t+1} \left(k_{n,t+1}\right)^{\theta_{t+1}-1} - \frac{i_{t+1}}{k_{t+1}}\right]}{\left(p_{t}^{s}/d_{t}\right) x_{t}^{-1} \left[\theta_{t} \left(k_{n,t}\right)^{\theta_{t}-1} - \frac{i_{t}}{k_{t}}\right]}, \quad (A5)$$

where  $i_{t+1}/k_{t+1}$  and  $i_t/k_t$  depend on the decision variables  $x_{t+1}$  and  $x_t$ , respectively, as shown in (A1). The above expressions show that the distribution shock (which drives fluctuations in  $\theta_t$ ) has a direct impact on the volatility of the equity return.

The remaining asset pricing variables are the one-period bond return  $R_{t+1}^b$  (the risk free rate) and the long-term bond return  $R_{t+1}^c$  which are defined as follows:

$$R_{t+1}^b = \frac{1}{p_t^b} = \frac{1}{E_t M_{t+1}},$$
(A6)

$$R_{t+1}^{c} = \frac{1+\delta^{c} p_{t+1}^{c}}{p_{t}^{c}} = \frac{1+\delta^{c} p_{t+1}^{c}}{E_{t} \left[M_{t+1} \left(1+\delta^{c} p_{t+1}^{c}\right)\right]},$$
(A7)

where  $M_{t+1}$  is shown in (A1). Approximate solutions for the stationary bond prices  $p_t^b$  and  $p_t^c$  take the form of log-linear decision rules as a function of the four state variables  $k_{n,t}$ ,  $v_t$ ,  $c_{t-1}/k_{t-1}$ , and  $x_{t-1}$ . The approximate solutions are used as a starting values for the PEA solution described in Appendix C.

### **B** Appendix: Approximate Log-linear Solution

An approximate solution to the transformed first-order condition (A.1) takes the form of the following log-linear decision rule for  $x_t$  as a function of the four state variables

$$x_t = \widetilde{x} \left[ \frac{k_{n,t}}{\widetilde{k}_n} \right]^{\mathbf{s}_1} \exp\left(\mathbf{s}_2 v_t\right) \left[ \frac{c_{t-1}/k_{t-1}}{\widetilde{c/k}} \right]^{\mathbf{s}_3} \left[ \frac{x_{t-1}}{\widetilde{x}} \right]^{\mathbf{s}_4}, \tag{B1}$$

where s<sub>1</sub> through s<sub>4</sub> are solution coefficients. The Taylor-series approximation is taken around the ergodic mean such that  $\tilde{x} \equiv \exp \{E [\log (x_t)]\} = \exp (\mu), \ \tilde{k}_n \equiv \exp \{E [\log (k_{n,t})]\}, \text{ and} \\ \widetilde{c/k} \equiv \exp \{E [\log (c_t/k_t)]\}.$  After substituting in the various laws of motion, including (A2), into the transformed firstorder condition (A1), I take logarithms and apply a first-order Taylor series approximation to each side to obtain the following expression

$$a_{0} \left[\frac{x_{t}}{\widetilde{x}}\right]^{\mathbf{a}_{1}} \exp\left[\mathbf{a}_{2} v_{t}\right] \left[\frac{k_{n,t}}{\widetilde{k}_{n}}\right]^{\mathbf{a}_{3}} \left[\frac{c_{t-1}/k_{t-1}}{\widetilde{c/k}}\right]^{\mathbf{a}_{4}} \left[\frac{x_{t-1}}{\widetilde{x}}\right]^{\mathbf{a}_{5}}$$
$$= E_{t} \operatorname{b}_{0} \left[\frac{x_{t}}{\widetilde{x}}\right]^{\mathbf{b}_{1}} \exp\left[\operatorname{b}_{2} v_{t}\right] \left[\frac{k_{n,t}}{\widetilde{k}_{n}}\right]^{\mathbf{b}_{3}} \exp\left(\operatorname{b}_{4} v_{t+1} + \operatorname{b}_{5} \varepsilon_{t+1}\right) \left[\frac{x_{t+1}}{\widetilde{x}}\right]^{\mathbf{b}_{6}}, \quad (B2)$$

where  $a_i$  and  $b_i$  for i = 0, 1, 2, ... are Taylor series coefficients. The Taylor-series coefficients are functions of the ergodic-mean approximation points  $\tilde{x}$ ,  $\tilde{k}_n$ , and  $\tilde{c/k}$ . Similarly, the laws of motion governing the evolution of the endogenous state variables  $k_{n,t}$  and  $c_{t-1}/k_{t-1}$  are approximated as

$$\frac{k_{n,t+1}}{\widetilde{k}_n} = \left[\frac{x_t}{\widetilde{x}}\right] \left[\frac{k_{n,t}}{\widetilde{k}_n}\right] \exp\left(-\varepsilon_{t+1}\right),\tag{B3}$$

$$\frac{c_t/k_t}{\widetilde{c/k}} = \left[\frac{x_t}{\widetilde{x}}\right]^{f_1} \exp\left[f_2 v_t\right] \left[\frac{k_{n,t}}{\widetilde{k}_n}\right]^{f_3}, \tag{B4}$$

where (B3) follows directly from (A2) with  $\tilde{x} = \exp(\mu)$ .

The conjectured form of the solution (B1) is iterated ahead one period and then substituted into the right-side of equation (B2) together with the approximate laws of motion (B3) and (B4) and the law of motion for the distribution shock (9). After evaluating the conditional expectation and then collecting terms, we have

$$x_{t} = \underbrace{\tilde{x} \left[\frac{b_{0}}{a_{0}}\right]^{\frac{1}{a_{1}-b_{1}-b_{6}(s_{1}+f_{1}s_{3}+s_{4})}}_{\tilde{x}} \exp\left[\frac{\frac{1}{2}\left(b_{4}+b_{6}s_{2}\right)^{2}\sigma_{u}^{2}+\frac{1}{2}\left(b_{5}-b_{6}s_{1}\right)^{2}\sigma_{\varepsilon}^{2}}{a_{1}-b_{1}-b_{6}\left(s_{1}+f_{1}s_{3}+s_{4}\right)}\right]}_{\tilde{x}}$$

$$\times \left[\frac{k_{n,t}}{\tilde{k}_{n}}\right]^{\frac{b_{3}-a_{3}+b_{6}\left(s_{1}+f_{3}s_{3}\right)}{s_{1}}}_{s_{1}}\exp\left[\frac{b_{2}-a_{2}+b_{6}f_{2}s_{3}+\rho\left(b_{4}+b_{6}s_{2}\right)}{a_{1}-b_{1}-b_{6}\left(s_{1}+f_{1}s_{3}+s_{4}\right)}\right]}_{s_{2}}v_{t}\right]$$

$$\times \left[\frac{k_{n,t}}{\tilde{k}_{n}}\right]^{\frac{-a_{4}}{s_{1}}}_{s_{3}}\left[\frac{a_{1}-b_{1}-b_{6}\left(s_{1}+f_{1}s_{3}+s_{4}\right)}{s_{3}}\right]}{\left[\frac{x_{t-1}}{\tilde{x}}\right]} \underbrace{\frac{-a_{5}}{a_{1}-b_{1}-b_{6}\left(s_{1}+f_{1}s_{3}+s_{4}\right)}_{s_{4}}}_{(B5)}$$

which yields four equations in the four solution coefficients  $s_1$  through  $s_4$ .

From the transformed first-order condition (A1), the Taylor-series coefficients  $a_0$  and  $b_0$ 

are given by

$$\mathbf{a}_0 = i/k \operatorname{g}(\widetilde{x}) / \widetilde{x}, \tag{B6}$$

$$\mathbf{b}_{0} = \widetilde{M} \left\{ \theta \left( \widetilde{k}_{n} \right)^{\theta - 1} - \widetilde{i/k} \left[ 1 - \mathbf{g} \left( \widetilde{x} \right) \right] \right\}, \tag{B7}$$

where  $\widetilde{x} = \exp(\mu)$ ,  $\widetilde{i/k} = \operatorname{func}(\widetilde{x})$ ,  $\widetilde{M} = \operatorname{func}(\widetilde{c/k}, \widetilde{x})$ , and  $\widetilde{c/k} = \operatorname{func}(\widetilde{k}_n, \widetilde{x})$ . Given these relationships, the constant term in (B5) yields a fifth equation that pins down the approximation point  $\widetilde{k}_n$  which depends on the values for  $\sigma_u^2$  and  $\sigma_{\varepsilon}^2$ .

### C Appendix: Nonlinear Model Solution

The impulse response functions in Figure 3 and the model simulation results are generated using the solution method outlined below that preserves the model's nonlinear equilibrium conditions. The method employs a version of the parameterized expectation algorithm (PEA) described by Den Haan and Marcet (1990).

After substituting in the various laws of motion, the transformed first-order condition (A1) can be represented as:

$$f(x_t, k_{n,t}, v_t, c_{t-1}/k_{t-1}, x_{t-1}) = E_t h(x_t, k_{n,t}, v_t, x_{t+1}, u_{t+1}, \varepsilon_{t+1}),$$
(C1)

where  $h(\cdot)$  is the nonlinear object to be forecasted. For purposes of constructing the conditional expectation, the function  $h(\cdot)$  is approximated as

$$h(\cdot) \simeq d_0 [k_{n,t}]^{d_1} \exp[d_2 v_t] [c_{t-1}/k_{t-1}]^{d_3} [x_{t-1}]^{d_4} \exp[d_5 u_{t+1} + d_6 \varepsilon_{t+1}], \qquad (C2)$$

where  $d_0$  through  $d_6$  are regression coefficients that are obtained by projecting the true nonlinear function  $h(\cdot)$  onto the form (C2) during repeated simulations of the model, as described below. The initial guesses for  $d_0$  through  $d_6$  are computed using the approximate log-linear solution from Appendix B.

Given a set of initial guesses for  $d_0$  through  $d_6$ , a simulation is run where the conditional expectation on the right side of (B1) is constructed each period as

$$E_t h(\cdot) = d_0 [k_{n,t}]^{d_1} \exp[d_2 v_t] [c_{t-1}/k_{t-1}]^{d_3} [x_{t-1}]^{d_4} \exp\left[\frac{1}{2} (d_5 \sigma_u)^2 + \frac{1}{2} (d_6 \sigma_\varepsilon)^2\right].$$
(C3)

Given the forecast  $E_t h(\cdot)$ , the nonlinear function (C1) is solved each period for the decision variable  $x_t$  using a nonlinear equation solver. The endogenous state variables  $k_{n,t}$  and  $c_{t-1}/k_{t-1}$ evolve according to their exact nonlinear laws of motion. The endogenous state variable  $x_{t-1}$ is simply the lagged decision variable. During the simulation, realized values of the nonlinear function  $h(\cdot)$  are constructed. At the end of the simulation, the realized values of  $h(\cdot)$  are projected onto the form (C2) to obtain new guesses for  $d_0$  through  $d_6$ . The simulation is then repeated using the new guesses for  $d_0$  through  $d_6$  with the same sequence of draws for the shock innovations  $u_{t+1}$  and  $\varepsilon_{t+1}$ . The procedure is stopped when the guesses for  $d_0$  through  $d_6$  do not change from one simulation to the next. In practice, convergence to five decimal places occurs after about 160 simulations. An analogous procedure is used to construct the conditional expectations in the bond pricing equations (4) and (5) to solve for  $p_t^b$  and  $p_t^c$  each period. Specifically, the nonlinear objects to be forecasted are approximated by power functions of the state variables and shock innovations as follows:

$$p_{t}^{b} = E_{t} M_{t+1},$$
where  $M_{t+1} \simeq m_{0} [k_{n,t}]^{m_{1}} \exp[m_{2} v_{t}] [c_{t-1}/k_{t-1}]^{m_{3}} [x_{t-1}]^{m_{4}} \exp[m_{5} u_{t+1} + m_{6} \varepsilon_{t+1}],$ 
(C4)
$$p_{t}^{c} = E_{t} M_{t+1} + E_{t} \delta^{c} M_{t+1} p_{t+1}^{c}$$
where  $\delta^{c} M_{t+1} p_{t+1}^{c} \simeq n_{0} [k_{n,t}]^{n_{1}} \exp[n_{2} v_{t}] [c_{t-1}/k_{t-1}]^{n_{3}} [x_{t-1}]^{n_{4}} \exp[n_{5} u_{t+1} + n_{6} \varepsilon_{t+1}].$ 
(C5)

The initial guesses for the regression coefficients  $m_0$  through  $m_6$  and  $n_0$  through  $n_6$  are computed using the approximate log-linear solution of the model. After each simulation, new guesses for the regression coefficients are obtained by projecting the realized values of the nonlinear functions  $M_{t+1}$  and  $\delta^c M_{t+1} p_{t+1}^c$  onto the forms shown in (C4) and (C5) until convergence is achieved.

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Figure 1: The U.S. top decile income share and capital's share of total income both exhibit significant volatility, motivating a model with distribution shocks. Horizontal dashed lines show the sample means. The equity premium (return on stocks minus the return on short-term bonds) is positively correlated with the *change* in both income share variables.



Figure 2: The growth rate of S&P 500 dividends is much more volatile than the growth rate of macroeconomic dividends, defined as real capital income less real investment. Horizontal dashed lines show the sample means. The equity premium (real return on stocks minus the real return on short-term bonds) is positively correlated with the growth rate of macroeconomic dividends but not the growth rate of S&P 500 dividends.



Figure 3: Concentrated ownership model: Impulse responses to one standard deviation innovation of the temporary distribution shock (solid blue line) or the permanent productivity shock (dashed red line).



Figure 4: Sensitivity of the mean equity premium  $E(R_{t+1}^s - R_{t+1}^b)$  to parameter values in the concentrated ownership model (solid blue line) and the representative agent model (dashed red line). Vertical lines mark the baseline calibration in the concentrated ownership model.



Figure 5: The figure shows how different values for the curvature parameter  $\sigma_k$  influence capital adjustment costs in the concentrated ownership model. Smaller values of  $\sigma_k$  imply more complementarity between new investment and existing capital, which raises adjustment costs.



Figure 6: Simulated variables in the concentrated ownership model (solid blue line) are considerably more volatile than those in the representative agent model (dashed red line).



Figure 7: The concentrated ownership model underpredicts the volatility of the price-dividend ratio in the data but delivers three times more volatility than the representative agent model. Similar to the data, the concentrated ownership model exhibits a low correlation between the equity premium and aggregate consumption growth.



Figure 8: Asset returns in the concentrated ownership model exhibit time-varying means and volatilities, similar to that observed in the data.