FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

Top Incomes, Rising Inequality, and Welfare

Kevin J. Lansing
Federal Reserve Bank of San Francisco

Agnieszka Markiewicz
Erasmus University Rotterdam

June 2016

Working Paper 2012-23
http://www.frbsf.org/publications/economics/papers/2012/wp12-23bk.pdf

Suggested citation:
Lansing J., Kevin and Agnieszka Markiewicz. 2016. "Top Incomes, Rising Inequality, and Welfare." Federal Reserve Bank of San Francisco Working Paper 2012-23.
http://www.frbsf.org/economicresearch/publications/working-papers/wp12-23bk.pdf

Top Incomes, Rising Inequality, and Welfare*

Kevin J. Lansing[†]
FRB San Francisco

Agnieszka Markiewicz
Erasmus University Rotterdam

June 15, 2016

Abstract

We introduce permanently-shifting income shares into a growth model with workers and capital owners. The model exactly replicates the U.S. time paths of the top quintile income share, capital's share of income, and key macroeconomic variables from 1970 to 2014. Welfare effects depend on changes in the time pattern of agents' consumption relative to a counterfactual scenario that holds income shares and the transfer-output ratio constant. Short-run declines in workers' consumption are only partially offset by longer-term gains from higher transfers and more capital per worker. The baseline simulation delivers large welfare gains for capital owners and significant welfare losses for workers.

Keywords: *Top Incomes, Inequality, Distribution shocks, Redistributive Transfer Payments, Welfare.*

JEL Classification: D31, E32, E44, H21, O33.

*Forthcoming in *Economic Journal*. For helpful comments and suggestions, we thank three anonymous referees, Baris Kaymak, John Komlos, Dirk Krueger, Morten Ravn, Kjetil Storesletten, Jacek Suda, and numerous seminar and conference participants. Part of this research was conducted while Lansing was a visiting economist at Norges Bank, whose hospitality is gratefully acknowledged. The online apppendix to this article contains all of the U.S. data employed and the computer programs used to produce the figures and quantitative results.

[†]Corresponding author. Research Department, Federal Reserve Bank of San Francisco, P.O. Box 7702, San Francisco, CA 94120-7702, kevin.j.lansing@sf.frb.org

Income inequality in the U.S. economy has increased markedly over the past several decades. Most of the increase can be traced to gains made by those near the top of the income distribution. As noted by Piketty (2014, p. 297), "if we consider the total growth of the U.S. economy in the thirty years prior to the crisis, that is, from 1977 to 2007, we find that the richest 10 percent appropriated three-quarters of the growth." Even if we restrict attention to college-educated workers, Lemieux (2006, p. 199), concludes that "changes in wage inequality are increasingly concentrated in the very top end of the wage distribution."

The top left panel of Figure 1 shows the dramatic upward shift in the share of before-tax income going to the top decile of U.S. households, as compiled by Piketty and Saez (2003, 2013a).[1] Using data from the U.S. Census Bureau, the top right panel shows that the before-tax income share of the top quintile of U.S. households increased by 8 percentage points, going from 43% in 1970 to 51% in 2014. Also using census data, the bottom left panel of Figure 1 shows that the growth in mean household income has significantly outpaced the growth in the median income since 1970. This pattern indicates a shift in the mass of income towards the upper tail of the distribution.[2]

The bottom right panel shows that capital's share of income increased from about 35% in 1970 to 43% in 2014.[3] Given that the distribution of financial wealth in the U.S. economy is highly skewed, the increase in capital's share of income would be expected to disproportionately benefit households near the top of the income distribution. According to a study by the U.S. Congressional Research Service (Hungerford 2011), changes in capital gains and dividend income were the two largest contributors to the increase in the Gini coefficient from 1996 to 2006. As a mitigating factor, transfer payments from the government to individuals increased from 7.5% of output in 1970 to 14.8% in 2014. These transfers would be expected to disproportionately benefit households outside the top quintile of the income distribution.[4]

[1] Updated annual data are available from The World Top Incomes Database.

[2] Census income is defined as income received on a regular basis (exclusive of capital gains) before payments for personal income taxes, social security, union dues, medicare deductions, food stamps, subsidized housing, etc. The data plotted in Figure 1 are from Tables H-2 and H-17 at www.census.gov/hhes/www/income/data/historical/household/.

[3] Following Lansing (2015), capital's share of income is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. Both series are from the Bureau of Economic Analysis (BEA), NIPA Table 1.14, lines 1 and 4. The increase in capital's share of income is not limited to the United States. Using data over the period 1975 to 2012, Karabarbounis and Neiman (2014) find that capital's share increased in 42 out of 59 countries with at least 15 years of data.

[4] Transfers include benefits from Old Age, Survivors, and Disability Insurance, Medicare and Medicaid benefits, Supplemental Security Income, Family Assistance, Food Stamps, and Unemployment Insurance Com-

Motivated by the above observations, this paper develops a quantitative growth model to assess the welfare consequences of rising U.S. income inequality over the period 1970 to 2014. The model includes two types of infinitely-lived agents: capital owners who represent the top income quintile of U.S. households and workers who represent the remainder. All agents supply labor inelastically to firms, consistent with the near-zero labor supply elasticity estimates obtained by most empirical studies (Blundell and McCurdy, 1999).[5] Our setup is similar to other concentrated capital ownership models that have been applied successfully to asset pricing.[6]

The top income quintile in our model owns 100% of the productive capital stock—a setup that roughly approximates the highly-skewed distribution of U.S. financial wealth. Using data from the Survey of Consumer Finances, Wolff (2010, p. 44) finds that the share of total financial wealth owned by the top quintile of U.S. households remained steady at around 92% from 1983 to 2007. Corporate stock is an important component of financial wealth. In 1995, the richest 25% of U.S. households sorted by wealth owned more than 90% of stocks.[7]

Income shares enter the model via stochastic exponents in a Cobb-Douglas aggregate production function, as in Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), and Lansing (2015). But in contrast to these papers, we assume that the exponent shifts are permanent rather than temporary. Our modeling strategy is similar to Goldin and Katz (2007) who allow for permanent shifts in the share parameters of a constant elasticity of substitution production function as a way of capturing technology-induced changes in the demand for skilled versus unskilled labor. Along these lines, a study by the OECD (2011) asserts that technological progress and a more integrated global economy have shifted production methods in favor of highly-skilled individuals. Here we remain agnostic about the underlying causes of the production function shifts and focus on the resulting consequences for welfare.

As inputs to the model, we incorporate the observed U.S. time paths of the top quintile income share and capital's share of total income, as plotted in Figure 1. Given these time paths from the data, we solve for the required time series of tax wedges, productivity shocks,

pensation.

[5]Alternatively, one may view our welfare results as applying to the vast majority of agents who remain fully-employed at all times. Along these lines, we find that business cycle-type fluctuations have very little impact on the welfare results.

[6]See, for example, Danthine, Donaldsen, and Siconolfi (2008), Guvenen (2009), and Lansing (2015).

[7]See Heaton and Lucas (2000, Figure 3, p. 224).

and capital accumulation shocks to make the model exactly replicate the observed trajectories of the following U.S. macroeconomic variables over the period 1970 to 2014: (1) real per capita output, (2) real per capita net stock of private nonresidential fixed assets, (3) real per capita aggregate consumption, (4) real per capita private nonresidential investment, (5) real per capita government consumption and investment, and (6) real per capita government transfer payments to individuals.[8] Figure 2 plots the last four of these variables as ratios relative to real output.[9]

Given time series for the income shares, tax wedges, and shocks, we use the model's decision rules to construct individual consumption paths for the capital owners and workers. Our procedure ensures that the individual consumption paths that we use to evaluate welfare are consistent with the evolution of the U.S. macroeconomic variables from 1970 to 2014.

Welfare effects are measured by the percentage change in consumption per annum that makes each type of agent indifferent between the baseline simulation and a counterfactual scenario in which income shares and the transfer-output ratio are held constant at year 1970 values. Both scenarios employ the same time series for the ratio of total government spending output and the same time series of productivity shocks, capital accumulation shocks, and investment tax wedges.

For the baseline simulation, the welfare gain for capital owners is 3.4% of their consumption per annum while workers suffer a welfare loss of 0.8% of their consumption per annum. These results reflect changes in the time pattern of consumption for each type of agent in both the short run and the long run. Due to discounting, the short-run changes in consumption are more important for welfare.

For capital owners, welfare gains derive in large measure from the post-2005 upward shift

[8] Our methodology is conceptually similar to that of Chari, McGrattan, and Kehoe (2007) who develop a quantitative model with four "wedges" that relate to labor, investment, productivity, and government consumption.

[9] Nominal personal consumption expenditures C_t are from the Bureau of Economic Analysis (BEA), NIPA Table 2.3.5. The corresponding price index is from Table 1.1.4. Nominal government consumption and investment G_t and the corresponding price index are from NIPA Tables 1.1.5 and 1.1.4. Nominal private nonresidential fixed investment I_t and the corresponding implicit price deflator are from the Federal Reserve Bank of St. Louis' FRED database. Nominal transfer payments to individuals T_t are also from FRED. Population data are from NIPA Table 2.1, line 40. We first define the nominal ratios C_t/Y_t, I_t/Y_t, G_t/Y_t and T_t/Y_t, where $Y_t \equiv C_t + I_t + G_t$. The nominal ratios capture shifts in relative prices. We then deflate Y_t by an output price index constructed as the weighted-average of the price indices for C_t, I_t, and G_t, where the weights are the nominal ratios relative to Y_t. Finally, we construct the per capita real series c_t, i_t, and g_t by applying the nominal ratios to the deflated output series and then dividing by population. In this way, the per capita real series reflect the same resource allocation ratios as the nominal series.

in their consumption path relative to the counterfactual. This pattern can be traced to the dramatic increase in capital's share of income starting around the year 2005. In the long run, the capital owners' consumption shifts up by 11.3% relative to the counterfactual path while investment shifts up by 12.7%. For workers, welfare losses are mitigated by the favorable period from 1971 to 1985 when the transfer-output ratio is rising faster than the top quintile income share, thus boosting their consumption relative to the counterfactual. Beyond 2014, the higher level of investment by capital owners contributes to more capital and more private-sector output per worker, allowing the worker's consumption to eventually surpass the counterfactual, achieving a permanent upward level shift of 1.3%. But these long-run consumption gains are heavily discounted in the welfare calculation.

As a validity check, we demonstrate that the model-predicted paths for a number of economic variables track reasonably well with the corresponding variables in U.S. data. These include: (i) the top quintile consumption share from the Consumer Expenditure Survey, (ii) the real S&P 500 stock market index, and (iii), an income-weighted average tax rate constructed using estimated U.S. tax rates on labor and capital incomes from Gomme, Ravikumar, and Rupert (2011, updated).

Experiments with the model show that the welfare results are sensitive to the precise time paths followed by the income shares and transfer payments during the early years of the simulation, which are lightly discounted. As a robustness check, we consider different evaluation dates for the welfare calculation. The evaluation date is the year in which the agent is presumed to be indifferent between the consumption path in baseline simulation and the consumption path in the counterfactual scenario. Regardless of whether the evaluation date is at the start, middle, or end of the U.S. data sample, the baseline simulation consistently delivers large welfare gains for capital owners and significant welfare losses for workers.

As a supplement to the positive analysis summarized above, we undertake two normative experiments. Given the paths of the U.S. before-tax income shares, we solve for a time series of transfers that equalizes agents' marginal utility of consumption each period from 1971 onwards. The new level of transfers is financed by adjusting the path of tax rates relative to those in the baseline simulation, but with other relevant time series unchanged. We find that the transfer-output ratio must rise to around 31% by the year 2014. Relative to the counterfactual (no change in income shares or the transfer-output ratio), capital owners suffer a welfare loss of 23% while workers enjoy a gain of 6.2%.

As a more realistic normative experiment, we compute a Pareto-improving time series of transfers that delivers small but equal welfare gains to capital owners and workers over a long simulation. In this case, the transfer-output ratio must rise to 18.6% by the year 2014—somewhat higher than the actual value of 14.8% observed in the data. The welfare gain for both types of agents is small, amounting to only 0.12% of consumption per annum. This result is due to the need for a higher average tax rate path to finance the higher level of transfers. Still, the experiment suggests that realistic policy actions could be effective in mitigating the negative impacts of rising income inequality.

It is important to note that our model assumes no mobility into or out of the top quintile of U.S. earners. Mazumder (2005) finds that intergenerational mobility is very low for U.S. households in the bottom three-quarters of the net worth distribution. His quantitative estimates imply that it would take many generations for a low or middle income family to make significant upward movement in the earnings distribution. While the returns to college have increased since 1980, intergenerational earnings mobility has declined substantially (Corak, 2013). This pattern suggests that the same forces which have contributed to rising U.S. income inequality may also be restricting intergenerational mobility. In support of this idea, Van der Weide and Milanovic (2014) find that the increase in inequality among the top 40% of U.S. earners is associated with lower real income growth for the bottom 40% of earners.

Our analysis examines the consequences of rising inequality that is driven by gains in top incomes, defined here as the highest 20% of earners. In contrast, the majority of previous research has focused on inequality that is driven by the rising wage premium of college-educated workers.[10] As an alternative to technological explanations for rising income inequality, Piketty, Saez, and Stantcheva (2014) argue that the dramatic rise in top incomes has been driven mainly by institutional changes which strengthened the bargaining power of top earners at the expense of lower earners. According to this "grabbing hand" theory, the shift in bargaining power has enabled rent-seeking top earners to successfully push their pay above their marginal product. While the grabbing-hand theory may have different implications for social welfare,

[10] See, for example, Attanasio and Davis (1996), Krussell, et al. (2000), Goldin and Katz (2007, 2008) and Heathcote, Storesletten, and Violante (2010, 2013).

the welfare consequences for each class of agents would still be linked to the resulting paths for their income and consumption, which our quantitative analysis explicitly takes into account. Kumhof, Rancière, and Winantet (2015) consider an endowment economy with rising income inequality, as measured by the income share of the top 5% of households. They do not consider welfare but instead focus on the links between rising inequality, increased household leverage, and the risk of a financial crisis.

1. Model

The model consists of workers, capital owners, competitive firms, and the government. There are n times more workers than capital owners, with the total number of capital owners normalized to one. Naturally, the firms are owned by the capital owners. Workers and capital owners both supply labor to the firms inelastically, but in different amounts.[11] The government levies distortionary taxes on both types of agents to finance public consumption expenditures and redistributive transfers.

1.1 Workers

The individual worker's decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log\left(c_t^w\right), \tag{1}$$

subject to the budget constraint

$$c_t^w \;=\; \left(1-\tau_t^w\right) w_t^w \ell^w \;+\; T_t/n, \tag{2}$$

where E_t represents the mathematical expectation operator, β is the subjective time discount factor, c_t^w is the individual worker's consumption, w_t^w is the worker's competitive market wage, ℓ^w is the constant supply of labor hours per worker, and τ_t^w is the worker's personal income tax rate. Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their resources each period, consisting of after-tax labor income $\left(1-\tau_t^w\right) w_t^w \ell^w$ and a per-worker transfer payment T_t/n received from the government.

1.2 Capital Owners

[11] The model setup is similar to a standard framework that is often used to study optimal redistributive capital taxation. See, for example, Judd (1985), Lansing (1999), and Krusell (2002). In these papers, however, capital owners do not supply labor.

Capital owners represent the top quintile of earners. Their decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log\left(c_t^c\right), \tag{3}$$

subject to the budget constraint

$$c_t^c \; + \; i_t \;\; = \;\; (1-\tau_t^c)\left(w_t^c\,\ell^c + r_t k_t\right) + \tau_t \phi_t i_t, \tag{4}$$

where c_t^c is the individual capital owner's consumption and ℓ^c is the constant supply of labor hours. The symbol i_t represents investment in physical capital k_t. For simplicity, we assume that the functional form of the utility function and the discount factor β are the same for both capital owners and workers. Capital owners derive income by supplying labor and capital services to firms. They earn a wage w_t^c for each unit of labor employed by the firm and receive the rental rate r_t for each unit of physical capital used in production. The capital owner's personal income tax rate is τ_t^c. Finally, the term $\tau_t \phi_t i_t$ captures the degree to which investment in physical capital can be "expensed," or immediately deducted from business taxable income, where τ_t is the effective business tax rate (which may differ from τ_t^c), and ϕ_t is an index number that captures elements of the tax code that encourage saving or investment.

Resources devoted to investment augment the stock of physical capital according to the law of motion

$$k_{t+1} \;\; = \;\; B \exp\left(v_t\right)\, k_t^{1-\lambda}\, i_t^{\lambda}, \qquad B > 0, \qquad \lambda \in (0,1], \tag{5}$$

$$v_t \;\; = \;\; \rho_v v_{t-1} \; + \; \eta_t, \qquad |\rho_v| < 1, \qquad \eta_t \sim NID\left(0, \sigma_\eta^2\right), \tag{6}$$

with k_0 and v_0 given. The parameter λ is the elasticity of new capital with respect to new investment. When $\lambda < 1$, equation (5) reflects the presence of capital adjustment costs.[12] Following Cassou and Lansing (1997), we allow for a "capital accumulation shock" v_t that evolves as a stationary AR(1) process with persistence parameter ρ_v. The shock innovation η_t is normally and independently distributed (NID) with mean zero and variance σ_η^2. The capital accumulation shock can be viewed as capturing stochastic variation in capital depreciation rates or shifts in the marginal efficiency of investment, along the lines of Justiniano, Primiceri,

[12] This formulation is employed by Cassou and Lansing (2006) in a welfare analysis of tax reform. Equation (5) can be viewed as a log-linearized version of the capital law of motion employed by Jermann (1998). For details, see Lansing (2012, p. 467).

and Tambalotti (2010). More generally, shocks that appear in the capital accumulation equation can be interpreted as a reduced-form way of capturing financial frictions that impact the supply of new capital.

1.3 Firms

Identical competitive firms are owned by the capital owners and produce output according to the technology

$$y_t = A\, k_t^{\theta_t} \left[\exp(z_t)\, (\ell^c)^{\alpha_t}\, (n\,\ell^w)^{1-\alpha_t}\right]^{1-\theta_t}, \qquad A > 0, \tag{7}$$

$$z_t = z_{t-1} + \mu + \varepsilon_t, \qquad \mu > 0, \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \tag{8}$$

$$s_t \equiv \frac{\theta_t}{\theta_t + \alpha_t\,(1-\theta_t)}, \tag{9}$$

$$s_t = (s_{t-1})^{\rho_s}\,(\widetilde{s}\,)^{1-\rho_s} \exp(u_t), \qquad \begin{array}{l} \widetilde{s} \equiv \exp\{E[\log(s_t)]\}, \\ |\rho_s| < 1, \quad u_t \sim NID\left(0, \sigma_u^2\right), \end{array} \tag{10}$$

with z_0 and s_0 given. In equation (7), z_t represents a labor-augmenting "productivity shock" that evolves as a random walk with drift. The drift parameter μ determines the trend growth rate of the economy. Stochastic shifts in the production function exponents θ_t and α_t represent "distribution shocks" along the lines of Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), and Lansing (2015). Given the Cobb-Douglas form of the production function, θ_t is capital's share of income, $\theta_t + \alpha_t\,(1-\theta_t)$ is the top quintile income share, $\alpha_t\,(1-\theta_t)$ is the labor income share of the capital owners, and $(1-\alpha_t)\,(1-\theta_t)$ is the income share of the workers, representing the bottom four quintiles.

Recall from Figure 1 that the U.S. income shares exhibit sustained upward trends over the period 1970 to 2014. To facilitate a solution of the model in terms of stationary variables, we define the variable s_t as the ratio of capital's share of income to the top quintile income share. Figure 3 shows that the empirical counterpart of s_t in the data appears to be stationary but persistent. To capture this feature, we postulate that s_t in the model evolves according to the law of motion (10) with persistence parameter ρ_s and innovation variance σ_u^2.

Profit maximization by firms yields the following factor prices

$$r_t = \theta_t\, y_t / k_t, \tag{11}$$

$$w_t^c = \alpha_t (1-\theta_t)\, y_t / \ell^c, \tag{12}$$

$$w_t^w = (1-\alpha_t)\,(1-\theta_t)\, y_t / \left(n \ell^w\right). \tag{13}$$

1.4 Government

The government collects tax revenue to finance expenditures on public consumption and redistributive transfers. We assume that the government's budget constraint is balanced each period, as given by

$$g_t \;+\; T_t = n\,\tau_t^w \underbrace{w_t^w \ell^w}_{=y_t^w} + \tau_t^c \underbrace{\left(w_t^c \ell^c + r_t k_t\right)}_{=y_t^c} - \tau_t \phi_t i_t, \tag{14}$$

where g_t is public consumption, T_t is aggregate redistributive transfers, and y_t^i for $i = w,\, c$, is the before-tax income for workers and capital owners, respectively. The balanced-budget constraint can be viewed as an approximation to the consolidated budgets of federal, state, and local governments. Public consumption does not provide direct utility to either capital owners or workers. Nevertheless, we include g_t in our analysis to obtain quantitatively realistic tax rates during the transition period from 1970 to 2014.

Following Guo and Lansing (1998) and Cassou and Lansing (2004), we introduce progressive income taxation via the formulation

$$\tau_t^i = 1 - (1 - \tau_t)\left(\frac{y_t^i}{\overline{y}_t}\right)^{-\kappa}, \tag{15}$$

where τ_t^i is the personal income tax rate of agent type i, y_t^i is the individual agent's before-tax income, and $\overline{y}_t$ is the average per capita income level in the economy which the agent takes as given. The parameter $\kappa \geq 0$ governs the slope of the tax schedule while τ_t governs the level of the tax schedule. When $\kappa > 0$, the agent's personal tax rate is increasing in the agent's income, reflecting a progressive tax schedule. When $\kappa = 0$, the tax schedule is flat such that all agents face the same tax rate τ_t regardless of their income level. For simplicity, we assume that τ_t also pins down the effective business tax rate which exhibits no progressivity.

The agent's marginal personal tax rate MTR_t^i is defined as the change in taxes paid divided by the change in income, that is, the tax rate applied to the last dollar earned. The expression for the agent's marginal personal tax rate is

$$MTR_t^i = \frac{\partial\left(\tau_t^i\, y_t^i\right)}{\partial y_t^i} = 1 - (1-\kappa)\left(1 - \tau_t^i\right), \tag{16}$$

which implies $MTR_t^i > \tau_t^i$ when $\kappa > 0$.

The average per capita income level in the economy is given by $\overline{y}_t = y_t / (n+1)$, where $n + 1$ is the total number of agents. Making use of the Cobb-Douglas production function (7) and the factor prices (11) through (13), the equilibrium personal income tax rates for each type of agent are given by:

$$\tau_t^w = 1 - (1-\tau_t)\left[(1-\theta_t/s_t)(n+1)/n\right]^{-\kappa}, \tag{17}$$

$$\tau_t^c = 1 - (1-\tau_t)\left[(\theta_t/s_t)(n+1)\right]^{-\kappa}, \tag{18}$$

where $\theta_t/s_t = \theta_t + \alpha_t(1-\theta_t)$ is the top quintile income share. All else equal, higher values of θ_t or α_t will increase the capital owner's tax rate, but decrease the worker's tax rate.

1.5 Decision Rules and Computation

Given that workers neither save or borrow, they simply consume their after-tax wage income plus transfers each period according to their budget constraint (2). In equilibrium, the individual worker's consumption is given by

$$c_t^w = \frac{1}{n}\left[(1-\tau_t^w)(1-\theta_t/s_t)\,y_t \ + \ T_t\right], \tag{19}$$

where τ_t^w is given by equation (17) and we have substituted in the worker's equilibrium real wage (13).

For capital owners, we first use the capital law of motion (5) to eliminate i_t from the budget constraint (4). The capital owner's first-order condition with respect to k_{t+1} is given by

$$\underbrace{\frac{(1-\tau_t\phi_t)i_t}{\lambda}}_{p_t} = E_t\, M_{t+1}^c[\underbrace{(1-\kappa)\left(1-\tau_{t+1}^c\right)r_{t+1}k_{t+1} - \left(1-\tau_{t+1}\phi_{t+1}\right)i_{t+1}}_{d_{t+1}} + \underbrace{\frac{(1-\tau_{t+1}\phi_{t+1})i_{t+1}}{\lambda}}_{p_{t+1}}], \tag{20}$$

where $M_{t+1}^c \equiv \beta\left(c_{t+1}^c/c_t^c\right)^{-1}$ is the capital owner's stochastic discount factor and τ_{t+1}^c is given by equation (18) evaluated at time $t+1$.[13] In deciding how much to invest, the capital owner takes into account the slope of the personal tax schedule, as reflected by the term $(1-\kappa)$. The first-order condition takes the form of a standard asset pricing equation where $p_t = (1-\tau_t\phi_t)i_t/\lambda$ is the market value of the capital owner's equity shares in the firm. These equity shares entitle the capital owner to a perpetual stream of dividends d_{t+1} starting in period $t+1$. The model's adjustment cost specification (5) implies a direct link between equity values and investment. This feature is consistent with the observed low-frequency comovement

[13] After taking the derivitive of the capital owner's Lagrangian with respect to k_{t+1}, we have multiplied both sides of the resulting expression by k_{t+1} and by c_t^c, which are both known at time t.

between the real S&P 500 stock market index and real business investment in recent decades, as documented by Lansing (2012, p. 466).

Since labor supplies are fixed, the observed values of z_t, θ_t, and α_t, together with the existing capital stock k_t, uniquely determine the amount of total income according to the production technology (7). Each period, capital owners must only decide the fraction of their after-tax income to be devoted to investment, with the remaining fraction devoted to consumption. As shown in Appendix A, the capital owner's optimization problem can be formulated in terms of a single decision variable, namely, the tax-adjusted investment-consumption ratio given by $x_t \equiv (1 - \tau_t \phi_t) i_t / c_t^c$. Our choice of functional forms (log utility and Cobb-Douglas specifications for production and the capital law of motion) delivers a simple approximate decision rule for x_t in terms of the state variable s_t, where s_t evolves according to the law of motion (10). In Appendix A, we show that x_t is increasing in θ_t. Hence, an increase in capital's share of income causes the capital owner to devote more resources to investment rather than consumption. If capital owners supply no labor $(\alpha_t = 0)$, then $s_t = 1$ and x_t is constant for all t.

Given the decision rule $x_t = x(s_t)$, the equilibrium version of the capital owner's budget constraint (4) can be used to derive the following expressions for the capital owner's allocations:

$$c_t^c = \frac{1}{1 + x(s_t)} \left(1 - \tau_t^c\right) \left(\theta_t / s_t\right) y_t, \tag{21}$$

$$i_t = \frac{x(s_t)}{1 + x(s_t)} \left(\frac{1 - \tau_t^c}{1 - \tau_t \phi_t}\right) \left(\theta_t / s_t\right) y_t, \tag{22}$$

where θ_t / s_t is the top quintile income share and $y_t = y(z_t, \theta_t, \alpha_t, k_t)$.

A convenient property of our setup is that we do not need to specify the laws of motion for the tax wedges in order to solve for the capital owner's allocations. This is because the income and substitution effects of changes in either τ_t or ϕ_t are offsetting. While the rational expectation solution for x_t depends on s_t, it does not depend on the tax wedges; the tax wedges are subsumed within the definition of x_t.

In equilibrium, τ_t^c depends only on τ_t and the top quintile income share θ_t / s_t. Given the observed paths for the income shares in the data, we solve for the time series of τ_t and ϕ_t that allow the model to exactly replicate the observed time paths of the four U.S. macroeconomic ratios plotted in Figure 2. Later, as a validity check, we compare the income-weighted average tax rate from the model simulation to a corresponding U.S. tax rate series constructed using estimated tax rates on labor and capital incomes from Gomme, Ravikumar, and Rupert (2011, updated).

The time series for the state variable s_t is taken directly from U.S. data, as plotted in Figure 3. We solve for the time series of productivity shocks z_t and capital accumulation shocks v_t that cause the model to exactly replicate the observed paths of (1) U.S. real per capita output, and (2) U.S. real per capita private nonresidential fixed assets from 1970 to 2014. Afterwards, we use the laws of motion for the shocks to recover the time series of innovations u_t, ε_t, and η_t. For periods beyond 2014, we assume that all shock innovations are zero, while income shares, tax wedges, and the various macroeconomic ratios remain constant at year 2014 values. Details regarding the simulation procedure are contained in Appendix B.

2. Model Calibration

Table 1 summarizes the parameter values for the baseline simulation. Values are set to achieve targets based on observed U.S. variables within the sample period 1970 to 2014. The time period in the model is one year. The number of workers per capital owner is $n = 4$ so that capital owners represent the top quintile of earners. In the model, capital owners possess 100% of the physical capital wealth—a reasonable approximation to the U.S. financial wealth distribution in which the ownership share of the top quintile of earners is around 92% (Wolff 2010, Table 2, p. 44).

Table 1

Baseline Parameter Values

Parameter	Value	Description/Target
n	4	Capital owners = top income quintile.
θ_0	0.3500	Capital's share of income = 0.350 in 1970.
α_0	0.1277	Top quintile income share = 0.433 in 1970.
ℓ^c/ℓ^w	0.2928	Mean relative wage $w^c/w^w = 2$ in 1970.
μ	0.0201	Mean per capita consumption growth = 2.01%, 1970 to 2014.
β	0.9634	Mean log equity return $\simeq$ 6%, 1970 to 2014.
A	0.4225	$y_t = 1$ with $k_t = 1.661$ in 1970.
B	1.1392	Estimated from U.S. data on k_t and i_t, 1970 to 2014.
λ	0.0441	Estimated from U.S. data on k_t and i_t, 1970 to 2014.
$\widetilde{s}$	0.7991	$\widetilde{s} \equiv \exp\{E[\log(s_t)]\} = 0.7991$, 1970 to 2014.
ρ_s	0.8607	Corr. $[\log(s_t),\ \log(s_{t-1})] = 0.8607$, 1970 to 2014.
σ_u	0.0250	Std. dev. $\log(s_t) = 0.0492$, 1970 to 2014.
σ_ε	0.0418	Std. dev. real output growth = 1.726%, 1970 to 2014.
ρ_v	0.7682	Corr. $(v_t,\ v_{t-1}) = 0.7682$, 1970 to 2014.
σ_η	0.0058	Std. dev. real fixed asset growth = 0.969%, 1970 to 2014.
κ	0.1204	Estimated tax schedule slope = 1.214, Cassou and Lansing (2004).
τ_0	0.3354	$g_t/y_t + T_t/y_t = 0.323$ in 1970.
ϕ_0	0.5421	$i_t/y_t = 0.121$ in 1970.

The initial capital income share θ_0 is set to match the 1970 observed value of 0.35, as shown in Figure 1. The initial production elasticity of the capital owner's labor supply α_0 is set to achieve an initial top quintile income share of $\theta_0 + (1 - \theta_0)\,\alpha_0 = 0.433$, corresponding to the 1970 observed value as shown in Figure 1. Given these values, the labor supply ratio ℓ^c/ℓ^w is set so that the initial wage ratio in 1970 is $w^c/w^w = 2$ with ℓ^w normalized to 1. For comparison, Heathcote, Storesletten, and Violante (2010, p. 686) report a male college wage premium of about 1.5 in 1970, whereas Gottschalk and Danziger (2005, p. 238) report a male wage ratio of 4.1 in 1979 when comparing the top decile to the bottom decile. The wage ratio w^c/w^w in our model compares the top quintile to the remainder of households, so one would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, Storesletten, and Violante (2010). The quantitative results exhibit little sensitivity to the value of the initial wage ratio.

The value $\mu = 0.0201$ matches the average growth rate of real per capita aggregate consumption over the period 1970 to 2014, where the consumption series is constructed as described in footnote 9. Given μ, we choose β to achieve a mean log equity return of about 6%, coinciding with the real return delivered by the S&P 500 stock index over the period 1970 to

2014.[14]

As described in Appendix B, we use the BEA's chain-type quantity index for the net stock of private nonresidential fixed assets to construct a normalized path for the real per capita U.S. capital stock from 1970 to 2014.[15] We calibrate the value of A in the production function (7) to yield $y_t = 1$ in 1970 when $k_t = 1.661$, corresponding to the normalized capital stock value in 1970. Our normalization procedure delivers the sample means of $i_t/k_t = 0.078$ and $k_t/y_t = 1.683$ from 1970 to 2014. Given the U.S. data for k_t and i_t, we run a regression of $\log(k_{t+1}/k_t)$ on a constant and $\log(i_t/k_t)$ to estimate the values of the parameters B and λ that appear in the capital law of motion (5). The estimates are $B = \exp(0.1303)$, s.e. $=$ 0.0488 and $\lambda = 0.0441$, s.e. $= 0.0191$. These parameters are re-estimated for the alternative simulations that begin at $t_0 = 1975$ or $t_0 = 1980$.

Recall that s_t represents the ratio of capital's share of income to the top quintile income share (Figure 3). We choose the parameters $\widetilde{s}$, ρ, and σ_u in the law of motion (10) to match the mean, persistence, and volatility of $\log(s_t)$ in U.S. data from 1970 to 2014.

As described in Appendix B, we compute the time series of productivity shocks z_t and capital accumulation shocks v_t that cause the model to exactly replicate the U.S. data paths for y_t and k_t. Given the time series of shocks, we use the laws of motion (6) and (8) to recover the implied sequence of innovations, where $\rho_v = 0.7682$ is the autocorrelation of the identified capital accumulation shocks from 1970 to 2014. The standard deviations of the implied shock innovations are $\sigma_\varepsilon = 0.0418$ and $\sigma_\eta = 0.0058$.

The slope parameter for the progressive tax schedule is set to $\kappa = 0.1204$ so that the hypothetical average-income agent in the model with $y_t^i/\overline{y}_t = 1$ faces a tax schedule slope of $MTR_t^i/\tau_t^i = 1.214$ when the top quintile income share and the macroeconomic ratios i_t/y_t, g_t/y_t, and T_t/y_t take on their average values from 1970 to 2014. The target slope corresponds to the value estimated by Cassou and Lansing (2004) using the 1994 U.S. tax schedule for married taxpayers with no children, filing IRS form 1040 jointly.[16]

Given the many significant changes to the U.S. tax code that have taken place since 1970,

[14] Data on real log equity returns for the U.S. are from Welch and Goyal (2008), updated through 2014 using data available from www.hec.unil.ch/agoyal/.

[15] The BEA fixed asset data are from NIPA Table 4.2, line 1.

[16] The tax schedule, taken from Mulligan (1997, Table 5-2), displays twelve different tax brackets that derive from the combined effects of the federal individual income tax, the earned income tax credit, and employee and employer contributions to Social Security and Medicare.

we examine the sensitivity of our results to different values for κ.[17] Table 2 shows the personal income tax rates faced by each type of agent for the baseline calibration with $\kappa = 0.1204$ and an alternative calibration with $\kappa = 0.1730$. The alternative calibration implies a more progressive tax schedule such that the average-income agent now faces a steeper slope of $MTR_t^i/\tau_t^i = 1.3$. The income ratios $y_t^i/\overline{y}_t$ that determine the personal income tax rates from equations (15) and (16) are based on the average top quintile income share from 1970 to 2014. For both calibrations, the tax rates faced by capital owners are higher than those for workers.

Table 2
Model Tax Rates Implied by 1970 to 2014 Average Values

	Average-income Agent		Capital Owner		Worker	
	$y_t^i/\overline{y}_t = 1$		$y_t^i/\overline{y}_t = 2.368$		$y_t^i/\overline{y}_t = 0.658$	
	$\kappa = 0.1204$	$\kappa = 0.1730$	$\kappa = 0.1204$	$\kappa = 0.1730$	$\kappa = 0.1204$	$\kappa = 0.1730$
τ_t^i	36.0	36.6	42.3	45.4	32.6	31.8
MTR_t^i	43.7	47.5	49.2	54.8	40.8	43.6
MTR_t^i/τ_t^i	1.214	1.300	1.164	1.208	1.248	1.371

Notes: All tax rates in %. τ_t^i = personal income tax rate. MTR_t^i = marginal personal income tax rate. Values for the tax rates and $y_t^i/\overline{y}_t$ are based on the 1970 to 2014 average values for the U.S. top quintile income share and the U.S. macroeconomic ratios i_t/y_t, g_t/y_t, and T_t/y_t.

As described in Appendix B, we use the capital owner's decision rules to solve for ϕ_0 and τ_0 such that the model delivers the observed U.S. values $i_t/y_t = 0.121$ and $g_t/y_t + T_t/y_t = 0.323$ at $t_0 = 1970$. A similar procedure is used to solve for ϕ_t and τ_t for each $t > t_0$.

3. Intuition for the Results

Before moving to the quantitative analysis, this section examines the basic mechanism that determines how a permanently shifting income share impacts capital owners versus workers. Let us consider a stripped-down version of the model with no labor supply for capital owners ($\alpha_t = 0$), unit labor supply for workers ($\ell^w = 1$), no growth ($z_t = 0$), equal number of capital owners and workers ($n = 1$), no taxes ($\tau_t = 0,\ \kappa = 0$), no capital accumulation shocks ($v_t = 0$), and no capital adjustment costs such that $k_{t+1} = (1-\delta)k_t + i_t$, where δ is the capital depreciation rate. With these simplifying assumptions, output is given by $y_t = Ak_t^{\theta_t}$. The incomes of the capital owners and workers are $\theta_t y_t$ and $(1-\theta_t)\, y_t$, respectively.

[17] Significant tax code changes were enacted by the Economic Recovery Tax Act of 1981 (ERTA81) and the Tax Reform Act of 1986 (TRA86). ERTA81 imposed a 23% across-the-board cut in all marginal tax rates and reduced the top marginal rate for individual income from 70% to 50%. TRA86 further lowered marginal rates for individuals and corporations, dramatically reduced the number of tax brackets, and eliminated or reduced many tax breaks. For additional details, see Guo and Lansing (1997).

In response to a one-time increase in θ_t, the capital stock cannot respond immediately so the short-run response of output is muted relative to the long-run response. In the short-run, the income of capital owners will rise while the income of workers will fall. These short-run effects will have a large influence on welfare because they are not discounted much in calculating lifetime utility.

But the increase in θ_t will also stimulate an increase in i_t, thus raising k_{t+1} and y_{t+1}.[18] As time goes by, the workers' income and consumption will be boosted by the rising level of private-sector output. In the long-run steady state, private-sector output is given by

$$y = A^{\frac{1}{1-\theta}} \left[\frac{\beta\theta}{1 - \beta(1-\delta)} \right]^{\frac{\theta}{1-\theta}}, \tag{23}$$

which shows that an increase in θ leads to an increase in y. It is straightforward to show that for reasonable parameterizations, an increase in θ also leads to an increase in $(1-\theta)\,y$, which determines the steady state level of workers' consumption. In other words, an increase in capital's share of income can also boost the long-run level of workers' consumption. But since this event takes place in the very long run, the resulting impact on workers' welfare is small due to discounting.

While an increase in θ unambiguously benefits the welfare of capital owners, the welfare impact for workers will depend on how fast capital and output converge to the new steady state. Short-term negative impacts must be balanced against long-term gains.[19] In the quantitative analysis that follows, we show that the welfare impact also depends on the time path followed by θ_t during the transition and the time path followed by total government spending (including redistributive transfers) which must be financed with distortionary taxes.

4. Quantitative Analysis

We first consider a baseline simulation that exactly replicates the observed U.S. time paths of the top quintile income share, capital's share of income, and key macroeconomic variables from 1970 to 2014. The baseline simulation is compared to a counterfactual scenario in which the income shares and the transfer-output ratio T_t/y_t are held constant at year 1970 values, while maintaining the baseline time series for z_t, v_t, ϕ_t, and $g_t/y_t + T_t/y_t$. For the welfare analysis, we also consider an alternative counterfactual scenario that allows a different

[18] The closed-form investment decision rule for capital owners is: $i_t = \beta\theta_t y_t - (1-\beta)(1-\delta)\,k_t$.

[19] We thank an anonymous referee for suggesting this simple intuition.

trajectory for $g_t/y_t + T_t/y_t$ for $t > 1970$. We also examine the sensitivity of the welfare results to a wide variety of alternative simulations. As a validity check, we compare model-predicted paths for a number of variables to the corresponding variables in U.S. data.

4.1 Baseline Simulation versus Counterfactual Scenario

Figure 4 plots the simulated trajectories of four model variables: aggregate consumption $c_t^c + nc_t^w$, aggregate investment i_t, the capital owner's consumption c_t^c, and the worker's consumption c_t^w. For each variable, we compare the baseline simulation to the counterfactual scenario described above. By holding T_t/y_t constant in the counterfactual scenario (with $g_t/y_t + T_t/y_t$ identical to the baseline), we adopt the view that the upward trend of T_t/y_t observed in the data was a deliberate government policy response to the trend of rising before-tax income inequality. In other words, the upward trend in T_t/y_t would not have been needed if before-tax income inequality had remained low. The resources thus saved could have been used to increase g_t/y_t. Consistent with this view, a study by Ostry, Berg, and Tsangarides (2014) finds that countries with higher before-tax income inequality tend to undertake more redistribution than countries with lower before-tax income inequality.[20]

The top panels of Figure 4 show that aggregate consumption and investment in the baseline simulation can fluctuate below the counterfactual path during portions of the transition period from 1970 to 2014. The sum of these two variables represents private-sector output. The increase in capital's share of income θ_t in the baseline simulation shrinks the output contribution coming from the model's growth engine, namely, labor-augmenting technological progress as given by $\exp\left[\left(1-\theta_t\right) z_t\right]$. This effect can produce a temporary slowdown in the growth rate of private-sector output. Along these lines, Hornstein and Krusell (1996) and Greenwood and Yörükoğlu (1997) develop models in which a biased technology change initially leads to a measured slowdown in total factor productivity.

It takes a long time for the model transition dynamics to fully play out. The increase in the marginal product of capital, as measured by θ_t, stimulates an increase in investment

[20] Figure 2 shows that T_t/y in the data rose from 7.5% in 1970 to 12% in 2005. It remained approximately constant at around 12% through 2007. Then, over the next three years, the ratio increased rapidly, peaking at 15.6% in 2010. The ratio has since come down a bit to 14.8% in 2014. While some of the run-up in T_t/y_t in recent years appears to have been triggered by the government's response to the financial crisis of 2007-09, it is also true that the top quintile income share continued to trend upward over this same period. Moreover, the value of T_t/y_t in 2014 is only slightly below the peak value achieved in 2010, suggesting that much of the recent run-up may be permanent rather than temporary.

relative to the counterfactual scenario (top right panel of Figure 4). Once θ_t stops increasing and all of the transition dynamics have died out, there is a permanent upward level shift of 12.7% in investment relative to the counterfactual. The higher investment level leads to a permanent upward level shift of 5.2% in private-sector output relative to the counterfactual. These permanent shifts derive from the permanent movements in the income share variables.

The lower two panels in Figure 4 show the paths for the capital owner's consumption c_t^c and the worker's consumption c_t^w. Relative to the counterfactual scenario, consumption growth for capital owners exhibits a higher mean (2.2% versus 2.0%) and a lower volatility (2.9% versus 3.5%) from 1970 to 2014. Beyond 2014, the capital owner's consumption pulls further away from the counterfactual path. In the long run (i.e., at the end of a 3000 period simulation), the capital owner's consumption experiences a permanent upward level shift of 11.3% relative to the counterfactual.

The worker's consumption falls below the counterfactual path during a substantial portion of the transition period from 1970 to 2014. But after 45 years, the level of the worker's consumption is only slightly below the counterfactual. This result is due mainly to the rising transfer-output ratio in the baseline simulation which supports worker consumption in the face of a shrinking income share. The volatility of the worker's consumption growth is substantially lower in the baseline simulation (1.8% versus 3.4%). The lower volatility stems from the countercyclical behavior of government transfers. In the baseline simulation (and in the U.S. data), the correlation coefficient between the transfer-output ratio and the growth rate of real output is -0.4 through 2014. The consumption-smoothing effect of these transfers is taken into account by our welfare analysis, as described further below. Beyond 2014, the worker's consumption starts to surpass the counterfactual path around the year 2050. This effect is driven by the higher long-run level of investment in the baseline simulation which contributes to more capital accumulation and more private-sector output per worker. At the end of the 3000 period simulation, the worker's consumption experiences a permanent upward level shift of 1.3% relative to the counterfactual.

Figure 5 plots the time series of the two tax wedge innovations ($\Delta\tau_t$ and $\Delta\phi_t$) and the two stochastic shock innovations (ε_t and η_t) that are needed to make the baseline simulation exactly replicate the paths of U.S. macroeconomic variables from 1970 to 2014. By construction, the innovations are zero at $t_0 = 1970$ and for $t > 2014$. The mean values of $\Delta\tau_t$, $\Delta\phi_t$, ε_t, and η_t are all close to zero over the period 1970 to 2014. The mean values of τ_t and ϕ_t are 0.330 and 0.498, respectively. In the top right panel of Figure 5, the identified productivity shock innovation ε_t is negative during the U.S. recession years of 1974-75, 1980-81, 1990-91, and 2007-09. The correlation between ε_t and η_t is close to zero for the period 1970 to 2014.

Figure 6 plots the ratios of macroeconomic variables to output generated by the model. In the top two panels, the baseline simulation exactly replicates the 1970 to 2014 observed U.S. time paths for the ratios c_t/y_t and i_t/y_t, as plotted earlier in Figure 2.[21] We use the model decision rules to construct paths for c_t^c/y_t, and c_t^w/y_t which, when aggregated, are consistent with the evolution of the ratio c_t/y_t in the U.S. data.

In the baseline simulation, the capital owner's consumption increases faster than output such that c_t^c/y_t goes from 16.3% in 1970 to 19.0% in 2014 (bottom left panel of Figure 6). In contrast, c_t^w/y_t increases only slightly from 11.7% in 1970 to 12.4% in 2014 (bottom right panel of Figure 6). The small increase in c_t^w/y_t is due to the rising transfer-output ratio in the baseline simulation which offsets the workers' shrinking income share. In the absence of a rising transfer-output ratio, the shifting income shares would cause the worker's consumption ratio to drop to 10.6% by 2014. In the counterfactual scenario, the consumption-output ratios for both types of agents can fluctuate in response to changes in tax wedges and stochastic shocks, but the ratios experience very little net change after 45 years.

4.2 Model versus Data: Income and Consumption Inequality

Figure 7 shows that the rise in consumption inequality in the model is far less-pronounced than the rise in before-tax income inequality. The model's top quintile income share before taxes and transfers (solid blue line) rises by 8 percentage points, from 43% to 51%, exactly replicating the census data plotted in Figure 1. In contrast, the after tax and transfer income share (dashed red line) rises by about 1.4 percentage points while the top quintile consumption share (dashed-dot green line) rises by about 1.9 percentage points. The modest rise in consumption inequality in the model is due to two factors: (1) the progressive nature of the tax schedule which extracts proportionally more tax revenue from capital owners as their income share rises, and (2) the rising transfer-output ratio which helps to mitigate the workers'

[21] Although not shown, the baseline simulation also replicates the observed U.S. time paths for the ratios g_t/y_t and T_t/y_t.

shrinking income share.

The sustained increase in U.S. before tax income inequality has prompted suggestions for increasing the marginal tax rate on top incomes.[22] Our model allows us to assess the degree to which having a more progressive tax schedule in place from 1970 onwards could have mitigated the rise of inequality, as measured after taxes and transfers. In Table 3, we show simulation results for three different values of the tax schedule slope parameter κ.[23] Higher values of κ serve to increase the capital owners' marginal tax rate MTR_t^c. Since the simulated paths for g_t/y_t and T_t/y_t are the same in each case, a higher marginal tax rate on capital owners serves to lessen the proportional tax burden on workers. Consequently, a more progressive tax schedule helps to reduce the capital owners' share of after tax income and consumption by the year 2014. However, as we show in section 4.5, a more progressive tax schedule with $\kappa = 0.1730$ ends up reducing welfare for both types of agents relative to the baseline simulation with $\kappa = 0.1204$.

Table 3

Effects of Tax Schedule Progressivity on Income and Consumption Shares

Tax Schedule Slope Parameter	MTR_t^c in 2014 Capital Owner	2014 Capital Owner Share: Income Before Taxes & Transfers	2014 Capital Owner Share: Income After Taxes & Transfers	2014 Capital Owner Share: Consumption
$\kappa = 0$	34.7	51.2	42.8	28.8
$\kappa = 0.1204$	47.1	51.2	39.1	27.7
$\kappa = 0.1730$	52.1	51.2	37.5	27.1

Notes: Tax rates and shares in %. Results for $\kappa = 0.1204$ are from the baseline simulation. For other values of κ, the simulation employs the baseline time series for θ_t, α_t, z_t, v_t, ϕ_t, g_t/y_t, and T_t/y_t but we compute a new time series for τ_t to satisfy the government budget constraint (14) each period.

For comparison with U.S. data, Figure 7 plots the consumption share of high-income households (those in the 80th through 95th percentiles) using data from the Consumer Expenditure Survey (CES) for the period 1980 and 2010. The consumption of high-income households is computed using two methods: (1) reported after-tax income minus saving, and (2) reported expenditures. The consumption share from the first method is noticeably higher than that

[22] See, for example, Piketty (2014, Chapter 14), Piketty, Saez, and Stantcheva (2014), and Kindermann and Krueger (2014).

[23] Results for $\kappa = 0.1204$ are from the baseline simulation that replicates the path of U.S. macroeconomic variables from 1970 to 2014. For other values of κ, the simulation employs the baseline time series for θ_t, α_t, z_t, v_t, ϕ_t, g_t/y_t, and T_t/y_t. We then solve for the required time series of tax rates τ_t to satisfy the government budget constraint (14) each period, with i_t pinned down by the capital owner investment decision rule (22).

from the second method. This gap is similarly evident in the data reported by Aguiar and Bils (2011, Table 1, p. 30). A later version of the same paper (Aguiar and Bils, 2015) highlights the growing discrepancy between the CES expenditure data and the aggregate consumption data from the National Income and Product Accounts (NIPA). This discrepancy affects the comparison in Figure 7 because our model exactly replicates the path of the NIPA aggregate consumption data from 1970 to 2014.[24]

Notwithstanding the data issues noted above, the model's prediction for the capital owners' consumption share tracks reasonably well with the consumption share of high-income households computed from the CES data (grey lines). From 1980 to 2010, the net increase in the CES consumption share is 3.1 percentage points using the income minus saving data and 1.9 percentage points using the reported expenditure data. For the same 1980 to 2010 time period, our model predicts an increase of 1.5 percentage points in the capital owners' consumption share. The results in Table 3 show that a less-progressive tax schedule (lower value for κ) would allow the model to deliver a higher net increase in the capital owners' consumption share during the simulation. Alternatively, the model could deliver a larger increase in consumption inequality if a fraction of government transfer payments were distributed to capital owners rather than being wholly distributed to workers.

There is disagreement in the literature regarding the extent to which U.S. consumption inequality has increased.[25] Studies by Krueger and Perri (2006) and Meyer and Sullivan (2013) find that consumption inequality has risen by much less than income inequality. Both studies measure consumption inequality using reported expenditures from the CES. However, Aguiar and Bils (2015) argue that the reported expenditure data for high-income households is subject to under-measurement error which has been growing over time. After designing a correction for the measurement error, they conclude that the rise in consumption inequality is close to the rise in income inequality. Auerbach, Kotlikoff, and Koehler (2016) perform a simulation study using data from the Federal Reserve's Survey of Consumer Finances. They find that the distribution of "spending power," defined as the present value of expected lifetime spending,

[24] The CES data and associated Stata codes are the same as those used by Aguiar and Bils (2015) and are available from Mark Aguiar's website. The data excludes the top and bottom 5% of households sorted by before tax income. For comparison with the model, we treat households in the 80th through 95th percentiles as the top quintile and households in the 5th through 80th percentiles as the remainder.

[25] Attansio and Pistaferri (2016) provide an overview of the research that seeks to compare trends in income inequality with trends in consumption inequality.

exhibits much less inequality than the distribution of net income. In particular, they find that the progressivity of the U.S. fiscal system plays an important role in reducing spending inequality.

4.3 Model versus Data: Capital Stock and Real Equity Value

The top left panel of Figure 8 plots the model-predicted path for the capital stock k_t which exactly replicates the BEA's chain-type quantity index for the net stock of private nonresidential fixed assets. The top right panel plots the yearly growth rate of the capital stock. The BEA fixed asset data is constructed by cumulating investment flows and then adjusting for depreciation and relative price changes. Recall that the baseline simulation for model investment exactly replicates the BEA series for private nonresidential fixed investment (footnote 9). The parameters B and λ in the capital law of motion (5) are estimated using the BEA fixed asset and investment data. Any remaining difference between the model-predicted path for k_t and the BEA data is thus attributed to the capital accumulation shock v_t. All else equal, when we shut off the identified capital accumulation shocks, the model-predicted path for k_t ends up 6.2% above the 2014 BEA fixed asset value.

A recent empirical study by Greenwald, Lettau, and Ludvigson (2014) finds that highly persistent "factor share shocks" which redistribute income between stockholders and non-stockholders are an important driver of U.S. stock prices over the period 1952 to 2012. Along these lines, Lansing (2015) develops a concentrated capital ownership model (similar to the one used here) in which persistent shocks to capital's share of income serve to substantially magnify the equity premium relative to a otherwise similar representative agent model.

While asset pricing is not our focus here, it is interesting to examine the model's prediction for the path of real equity values from 1970 to 2014. Recall from equation (20) that the market value of the capital owner's equity shares is $p_t = (1 - \tau_t \phi_t) i_t / \lambda$. The bottom left panel of Figure 8 plots p_t from the baseline simulation versus the real per capita market value of the firms in S&P 500 stock market index, where each series is indexed to 1 in 1970.[26]

The bottom left panel of Figure 8 shows that the S&P 500 market value is far more volatile than p_t in the model. Moreover, while the two series are approximately equal in 1993 and

[26] Data on the nominal S&P 500 market capitalization in $ billions are from Haver Analytics. We convert to real per capita values using the output price index described in foonote 9 and the U.S. population data from NIPA Table 2.1.

2008, the S&P 500 market value in 2014 is substantially higher than the endpoint predicted by the model. These differences are perhaps not surprising given that our fully-rational model excludes the possibility of "bubbles" or "excess volatility," both which are the subject of a large literature.[27]

The bottom right panel of Figure 8 compares the yearly *growth rates* of equity value in the model and in the data, where each series is scaled by its sample standard deviation. The correlation coefficient between the two growth rate series is 0.31 and statistically significant. These results lend support to the notion of a link between shifting U.S. income shares and movements in equity values.

4.4 Normative Transfer Experiments

Figure 9 plots the results of two normative experiments in which the time series of government transfers and tax rates depart from those in the baseline simulation. In the first experiment, we solve for the time series of transfers T_t^* that equates agents' marginal utility of consumption (MUC) each period such that $1/c_t^w = 1/c_t^c$ for $t > 1970$. Equating agents' marginal utility of consumption would be the goal of a social planner who seeks to maximize the weighted-sum of agents' lifetime utilities in an economy without distortions, where the weights correspond to the population share of each agent-type.

In the second experiment, we solve for a Pareto-improving time series of transfers T_t^p that achieves the less ambitious goal of $1/c_t^w = 1/(\psi c_t^c)$ where $0 < \psi < 1$. We set $\psi = 0.71836466$ to achieve small but equal welfare gains for capital owners and workers over a long simulation of the model. The welfare results are shown in Table 4. For each of the two experiments, we solve for the required time path of τ_t from 1971 onwards to satisfy the government budget constraint (14) each period, where other relevant time series (for θ_t, α_t, z_t, v_t, g_t/y_t, and ϕ_t) are identical to those in the baseline simulation. Details of the computation procedure are contained in Appendix C.

The left panel of Figure 9 shows that T_t^*/y_t jumps from 7.5% in 1970 (the starting value in the data) to about 21% in 1971. The ratio then trends upwards to about 31% by the year 2014, after which it remains constant because income inequality in the model stops rising by

[27] Lansing and LeRoy (2014) provide a recent update on the excess volatility literature. The model fit for equity values could potentially be improved by allowing for stochastic variation in the parameter λ that appears in the capital law of motion (5).

assumption. The correlation coefficient between T_t^*/y_t and the growth rate of y_t in the model experiment is -0.2 from 1972 to 2014. The corresponding correlation is -0.4 in the data, suggesting that U.S. government transfers exhibit a reasonable degree of countercyclicality.[28]

The right panel of Figure 9 plots the income-weighted average tax rate that is needed to finance each of the normative transfer experiments. The income-weighted average tax rate in the model is given by

$$ATR_t = (1 - \theta_t/s_t)\, \tau_t^w + (\theta_t/s_t)\, [\tau_t^c y_t^c - \tau_t \phi_t i_t] \,/ y_t^c, \tag{24}$$

where θ_t/s_t is the top quintile income share and y_t^c is the capital owner's before-tax income. In the case of MUC-equalizing transfers, the income-weighted average tax rate jumps from 32.3% in 1970 to 45.3% in 1971, and then trends upward to 49.2% in the year 2014. While fiscal policy shifts of this magnitude are obviously not realistic, the experiment illustrates the severity of the actions that would have been needed to achieve equality of marginal utility (and equality of consumption) given the historical pattern of rising U.S. income inequality.

The second normative experiment shows that much milder policy actions would have sufficed to achieve small but equal welfare gains for everyone, given our model calibration. Because capital owners are immediately enriched by the shifting production technology, it is possible to increase the growth rate of redistributive transfers to workers while leaving both types of agents better off from the perspective of $t_0 = 1970$. While alternative model calibrations might not allow for mutual welfare gains, it always possible to solve for a value of ψ that delivers equal welfare *changes* for both types of agents.[29]

In Figure 9, the ratio T_t^p/y_t rises from 7.5% in 1970 to 18.6% in 2014. The ending value is not much higher than the actual value of 14.8% observed in the data. In the data, the average growth rate of transfer payments is 3.59% per year from 1970 to 2014. The Pareto-improving policy calls for T_t^p to grow at an average growth rate of 4.06% per year. There is a jump in T_t^p/y_t (and T_t^*/y_t) that occurs in the mid-1990s. This feature can be traced to the jump in the U.S. top quintile income share that occurred at the same time (Figure 1). The income-weighted average tax rate that is needed to finance the Pareto-improving transfers goes from

[28] Our exploration of normative redistribution policies is necessarily brief here. For a more comprehensive treatment, see Piketty and Saez (2013b).

[29] For example, higher calibrated values for the tax schedule slope parameter κ can preclude the achievement of mutual welfare gains.

32.3% in 1970 to 36.9% in 2014. The ending value is near the low end of the range of average tax rates observed in OECD countries.[30]

Figure 9 also plots the time series for T_t/y_t and ATR_t from the baseline simulation. Recall that the baseline series for T_t/y_t exactly replicates the U.S. data (Figure 2). The baseline series for ATR_t ranges from a low of 28.8% to a high of 36.8%. These values are realistic in comparison to tax rates that have been estimated directly for the U.S. economy. Gomme, Ravikumar, and Rupert (2011, updated) construct average U.S. tax rates on labor and capital incomes for the period 1954 to 2013.[31] Starting with their estimates, we compute an income-weighted average tax rate by weighting their labor and capital income tax rates by $1 - \theta_t$ and θ_t, respectively, where θ_t is capital's share of income in the data, as plotted in Figure 1. Figure 9 shows that the ATR_t series implied by the baseline simulation is reasonable in comparison to the income-weighted average tax rate series computed from the Gomme-Ravikumar-Rupert estimates.

4.5 Welfare Analysis

Table 4 summarizes the effects of rising income inequality for various model specifications. As detailed in Appendix D, welfare effects are calculated as the constant percentage amount by which each agent's annual consumption in the counterfactual scenario must be adjusted upward or downward each year in perpetuity to make lifetime utility equal to that in the baseline (or other) simulation. Table 4 also shows the long-run percentage shifts in consumption and investment for each type of agent, each measured relative to the counterfactual scenario.

For the baseline simulation, capital owners achieve a welfare gain of 3.4% of their consumption per annum while workers suffer a welfare loss of 0.8% of their consumption per annum. The welfare effects are determined by changes in the time pattern of consumption for each type of agent in both the short-run and the long run. The changes in consumption patterns can be seen in the bottom two panels of Figure 4. Changes that take place in the short-run, i.e., closer to $t_0 = 1970$, have more influence on welfare due to light discounting.

[30] According to Piketty and Saez (2013b, p. 141), the ratio of tax revenue to national income in OECD countries ranges from 35% to 50%.

[31] The updated tax rate series are available from Paul Gomme's website.

Table 4
Effects of Rising U.S. Income Inequality

Model Specification	Welfare Change Capital Owners	Welfare Change Workers	Long-run Consumption Shift Capital Owners	Long-run Consumption Shift Workers	Long-run Investment Shift Capital Owners
Baseline simulation	3.44	−0.77	11.31	1.28	12.69
Baseline, alternative counterfactual	−3.20	−6.30	−4.46	−11.63	−1.03
Baseline, $\beta = 0.9538$	2.67	−0.29	10.58	1.17	11.95
Linear transition paths for $\theta_t,\ \alpha_t$	4.64	0.39	11.31	1.28	12.69
Start date $t_0 = 1975$	−1.35	−10.60	4.23	−10.51	3.24
Start date $t_0 = 1980$	2.85	−5.62	9.91	−2.32	12.28
No productivity shocks	3.55	−0.67	11.31	1.28	12.69
No capital accumulation shocks	3.45	−0.76	11.31	1.28	12.69
Constant capital share, $\theta_t = \theta_0$	−0.54	3.63	−0.82	10.33	−1.75
Constant $T_t/y_t = T_0/y_0$	3.44	−8.55	11.31	−13.59	12.69
Steeper tax schedule, $\kappa = 0.1730$	2.75	−0.98	8.11	−0.69	9.35
Flat tax schedule, $\kappa = 0$	4.87	−0.34	18.08	5.37	19.88
MUC-equalizing $T_t/y_t = T_t^*/y_t$	−22.79	6.18	−29.93	−2.37	−25.22
Pareto-improving $T_t/y_t = T_t^p/y_t$	0.12	0.12	0.96	1.05	3.47

Notes: Welfare effects are measured by the percentage change in consumption per annum to make each agent indifferent between the baseline (or other) simulation and the counterfactual scenario which holds income shares and T_t/y_t constant at year 1970 values, while maintaining the baseline time series for z_t, v_t, ϕ_t, and $g_t/y_t + T_t/y_t$. The alternative counterfactual scenario holds income shares and T_t/y_t constant at year 1970 values, while maintaining the baseline time series for z_t, v_t, ϕ_t, and g_t/y_t. The long-run consumption and investment shifts are the % changes relative to the counterfactual scenario, computed at the end of a 3000 period simulation.

For capital owners, welfare gains derive in large measure from the post-2005 upward shift in their consumption path relative to the counterfactual. This pattern can be traced to movements in capital's share of income θ_t. From Figure 1, we see that capital's share of income in the data experienced a dramatic increase starting around the year 2005. In the long run, the capital owners' consumption shifts up by 11.3% relative to the counterfactual path. Given the permanently higher marginal product of capital, investment expenditures shift up by 12.7% in the long run.

The time pattern of the workers' consumption is more complicated. From 1971 to 1985, the baseline path is above the counterfactual. This 15-year period has a strong positive influence on the worker's welfare because of light discounting. During this time, the transfer-output ratio is rising faster than the top quintile income share, thus boosting the worker's consumption relative to the counterfactual. From 1985 to 2014, the upward trend in capital's share of income θ_t shrinks the worker's income share and the output contribution coming from labor-augmenting technological progress. This effect pushes down the worker's consumption relative to the counterfactual. Beyond 2014, the higher level of investment in the baseline economy (due to a higher θ_t) contributes to more capital accumulation and more private-sector output per worker, allowing the worker's consumption to eventually surpass the counterfactual around the year 2050, achieving a permanent upward level shift of 1.3%. But these long-run consumption gains are heavily discounted.

Recall that our counterfactual scenario assumes that the resources saved by not increasing T_t/y_t are devoted to increasing g_t/y_t such that the size of government, as measured by $g_t/y_t + T_t/y_t$, follows the same path as in the baseline simulation. This setup is similar to the "revenue neutrality" assumption that is typically employed in the analysis of proposed tax reforms. The second row of Table 4 considers an alternative counterfactual that relaxes the revenue neutrality assumption. Specifically, the alternative counterfactual assumes that g_t/y_t follows the same downward path that is observed in the baseline simulation (and in the U.S. data). Like the original counterfactual, the alternative counterfactual holds T_t/y_t constant at the year 1970 value. Hence, the alternative counterfactual allows the size of government to shrink relative to that in the baseline simulation. In 2014, the size of government is 25.8% in the alternative counterfactual versus 33.1% in the baseline simulation. The smaller size of government allows for lower tax rates on both types of agents, making the alternative counterfactual look much better in terms of welfare. Consequently, the baseline simulation now delivers large welfare losses to both types of agents. The welfare loss for workers (6.3%) is about twice the loss for capital owners (3.2%). In our view, it seems somewhat implausible that the size of U.S. government would be so much smaller in 2014 if income inequality had not increased. Nevertheless, regardless of the counterfactual scenario, it remains true that the welfare outcome for capital owners is much better than the welfare outcome for workers. The remaining rows of Table 4 revert to the original counterfactual scenario.

The welfare gain for capital owners shrinks to 2.7% if we repeat the baseline simulation for a calibration where agents are less patient such that $\beta = 0.9538$, implying a mean log equity return of about 7%. Recall that baseline simulation has $\beta = 0.9634$ to match the real return of 6% for the S&P 500 stock index. A lower value of β reduces the lifetime utility benefit of the capital owners' long-run upward consumption shift. When workers are less patient, the favorable 1971 to 1985 period for their consumption relative to the counterfactual takes on added-importance for welfare, thus generating a smaller welfare loss of 0.3%.

The above discussion highlights the importance of accurately modeling the historical paths of the U.S. income shares because these affect the time pattern of agents' consumption and hence welfare. For example, implementing a linear transition path for the income shares over the period 1970 to 2014 (while preserving the endpoints) improves the welfare outcomes for both types of agents relative to the baseline simulation.[32] Capital owners now achieve a larger gain of 4.6% versus 3.4% in the baseline simulation. Workers now enjoy welfare gain of 0.4% versus a welfare loss of 0.8% in the baseline simulation. For capital owners, a linear transition causes θ_t to be higher than the baseline path during the early years of the simulation. For workers, a linear transition causes their income share $(1-\alpha_t)(1-\theta_t)$ to be higher than the baseline path from 1991 to 2006—an unfavorable period when their baseline consumption path falls below the counterfactual.

To further examine the sensitivity of the welfare results to the time paths of the variables, we repeat the methodology of the baseline simulation, but now use different starting dates, specifically $t_0 = 1975$ and $t_0 = 1980$.[33] When $t_0 = 1975$, capital's share of income θ_t and the transfer-output ratio T_t/y_t both undergo net declines during the first five years of the simulation, replicating the patterns observed in the U.S. data (Figures 1 and 2). Since the first five years of the simulation are lightly discounted, the initial declines in θ_t and T_t/y_t have large negative welfare consequences for both types of agents. While previously enjoying a welfare gain of 3.4%, capital owners suffer a welfare loss of 1.4% when $t_0 = 1975$. The welfare loss for workers increases considerably to 10.6% when $t_0 = 1975$. However, if we start the baseline simulation at $t_0 = 1980$, the capital owners' welfare gain is restored, but to the slightly smaller value of 2.9%. The welfare loss for workers remains sizable at 5.6%. Again, these results are driven by the different time paths for θ_t and T_t/y_t during the early years of the simulation.

One way of addressing the sensitivity of the welfare results to the starting date of the simulation is to consider different evaluation dates for the welfare calculation. The evaluation

[32] Given the linear transition paths for θ_t and α_t, we recompute the time series of tax wedges and shocks to match the observed paths of the U.S. macroeconomic variables from 1970 to 2014.

[33] Given the new starting dates, we recompute the time series of tax wedges and shocks to match the observed paths of the U.S. macroeconomic variables from t_0 to 2014.

date is year in which the agent is presumed to be indifferent between the consumption path in the baseline simulation and the consumption path in the counterfactual scenario. For a given pair of consumption paths, the resulting utility streams will differ depending on the date when the agent is asked to make a welfare comparison between the two paths. An evaluation date that occurs later in the sample will diminish the influence of the starting date in the welfare calculation.

The baseline simulation in Table 4 uses 1970 as the evaluation date, with zero weight placed on pre-1970 consumption. In Table 5, we show the results for two alternative evaluation dates: 1992 and 2014. For these dates, we assume that agents discount the utility of past consumption going back to 1970 using the same discount factor β.[34] In this way, each of the three evaluation dates employ the same data on agents' consumption from 1970 until the end of the long simulation; the only difference involves the utility weight assigned to consumption at each date. Details of the calculation are contained in Appendix D.

Table 5 shows that, regardless of the evaluation date, the baseline simulation consistently delivers large welfare gains for capital owners and significant welfare losses for workers. These results lend support to the view that the pattern of rising U.S. income inequality has been detrimental to a substantial fraction of the population.

Table 5

Welfare Effects of Baseline Simulation at Different Evaluation Dates

Specification	Evaluation Date	Welfare Change Capital Owners	Welfare Change Workers
Start of sample	1970	3.44	−0.77
Middle of sample	1992	4.14	−2.78
End of sample	2014	6.42	−2.40

Notes: Welfare effects are measured by the % change in consumption per annum to make the agent indifferent between the baseline simulation and the counterfactual scenario. The evaluation date is the year in which the agent is indifferent. The baseline simulation in Table 4 uses 1970 as the evaluation date, with zero weight on pre-1970 consumption. Here, we also consider evaluation dates of 1992 or 2014, with the utility of past consumption to 1970 discounted using the same discount factor β.

The seventh and eighth rows of Table 4 show that the stochastic shock innovations to productivity or capital accumulation have only small effects on welfare. All else equal, shutting off the shock innovations in both the transition economy and the counterfactual scenario

[34] Caplin and Leahy (2004) show that discounting the utility of past consumption is a necessary condition to ensure retrospective time consistency of an agent's chosen consumption path.

delivers results that are very close to those in the baseline simulation. The model delivers the standard result that business cycle-type fluctuations in agents' consumption are not very important for welfare (Lucas, 1987). The long-run consumption and investment shifts are not affected because all simulations set the shock innovations to zero for $t > 2014$.

When the capital income share θ_t is held constant at the year 1970 value of 35%, capital owners experience a welfare loss 0.5% while workers enjoy a welfare gain of 3.6%.[35] In this experiment, capital owners must now help pay for the rising time path of T_t/y_t (which directly benefits workers) without the help of a rising capital income stream. Workers see their income share $(1-\alpha_t)(1-\theta_t)$ shrink by less than in the baseline simulation.

The tenth row of Table 4 holds the transfer-output ratio T_t/y_t constant at its year 1970 value of 7.5% while allowing the income shares to shift as in the data and maintaining the baseline time series for z_t, v_t, ϕ_t, τ_t, and $g_t/y_t + T_t/y_t$. Relative to the baseline simulation, this experiment simply reallocates government spending in the direction of increasing g_t/y_t rather than increasing T_t/y_t, with no impact on agents' tax rates. In this case, workers suffer a much larger welfare loss of 8.6% versus 0.8% in the baseline simulation. The welfare gain for capital owners is unaffected at 3.4% because the time paths for the income shares, stochastic shocks, and tax wedges, and are identical to those in the baseline simulation. This result suggests that the historical pattern of U.S. transfer payments has helped to mitigate the negative impacts of rising income inequality on households who fall outside the top quintile of the income distribution.[36]

Toward the bottom of Table 4, we show the results for simulations that employ different values for the tax schedule slope parameter κ. These simulations were discussed earlier in relation to Table 3 which shows that a higher value for κ reduces the consumption share of capital owners by the year 2014. But reducing the consumption share of capital owners does not translate into higher welfare for workers. All else equal, a steeper tax schedule with $\kappa = 0.1730$ leaves both types of agents worse-off relative to the baseline simulation which has $\kappa = 0.1204$. The quantitative impact for workers is small; their welfare loss is now 1.0% versus

[35] The simulation employs the baseline time series for α_t, z_t, v_t, ϕ_t, g_t/y_t, and T_t/y_t, but we compute a new time series for τ_t to satisfy the government budget constraint (14) each period, with i_t pinned down by the capital owner investment decision rule (22).

[36] But as a caveat, it should be noted that our model implies that there are no negative welfare consequences for capital owners when resources are shifted away from government consumption g_t for the purpose of increasing transfers T_t while holding the size of government constant, as measured by $g_t/y_t + T_t/y_t$.

0.8% in the baseline simulation. The welfare gain for capital owners shrinks to 2.8% from 3.4% in the baseline simulation. In contrast, a flat tax schedule with $\kappa = 0$ leaves both types of agents better-off relative to the baseline simulation. The welfare gain for capital owners shoots up to 4.9% while the welfare loss for workers shrinks to 0.3%. These experiments show that workers can actually benefit from a less-progressive tax schedule because it encourages more capital accumulation, leading to higher real wages and a higher long-run level of their consumption.

The transfer policy that achieves MUC equality from 1971 onwards produces a substantial welfare gain of 6.2% for workers. But for capital owners, the higher tax rates needed to finance the higher level of transfers produces a enormous welfare loss of 22.8%. Moreover, the economy with MUC-equalizing transfers suffers a permanent downward level shift of 25.2% in investment relative to the counterfactual. Private-sector output shifts down by 11.8% in the long run.

The Pareto-improving transfer policy achieves small but equal welfare gains of 0.12% for both capital owners and workers. Still, this outcome is a significant improvement for workers relative to the 0.8% welfare loss suffered in the baseline simulation. Moreover, the economy experiences a permanent upward level shift of 3.5% in investment relative to the counterfactual. Private-sector output shifts up by 1.4% in the long run. The Pareto-improving experiment suggests that realistic policy movements in the direction of more redistribution could be successful in combating the negative effects of rising income inequality without sacrificing long-run economic performance.

5. Concluding Remarks

The increase in U.S. income inequality over the past half-century can be traced to gains made by those near the top of the income distribution—where financial wealth and corporate stock ownership is highly concentrated. The economic and political implications of increasingly-skewed income distributions in the United States and other countries have risen to the forefront of current policy debates.

Our contribution is to try to assess the welfare consequences of rising U.S. income inequality. The starting point is a standard growth model with two types of agents and concentrated-ownership of physical capital. The model is designed to exactly replicate the observed time paths of numerous U.S. macroeconomic variables from 1970 to 2014. The welfare consequences of rising income inequality depend crucially on changes in agents' consumption paths relative to a plausible counterfactual scenario. Our methodology ensures that agents' consumption paths are consistent with the evolution of U.S. macroeconomic variables over the same period. Our approach has the additional advantage of providing us with full knowledge of the counterfactual scenario—something that is not possible using purely empirical methods.

According to our analysis, the increase in income inequality since 1970 has delivered large welfare gains to the top income quintile of U.S. households. For households outside this exclusive group, the welfare losses appear to have been significant, albeit substantially mitigated by the doubling of the share of U.S. output devoted to redistributive transfers since 1970. Our model simulations also suggest that, all else equal, having a more-progressive U.S. tax schedule in place from 1970 to 2014 would not have helped to reduce the welfare losses for agents outside the top quintile.

Our analysis of a transfer policy that equalizes agents' marginal utility of consumption within the model suggests that U.S. transfer payments exhibit a reasonable degree of countercyclicality. In addition, we find that a relatively modest increase in the historical growth rate of U.S. transfer payments (from 3.6% to 4.1%) could have achieved small but equal welfare gains for all households while continuing to deliver nontrivial upward shifts in long-run consumption and investment relative to the counterfactual scenario. Overall, our results suggest that there is room for policy actions that could address the negative consequences of rising income inequality.

Appendix A. Capital Owner's Decision Rule

By combining equations (4), (11), and (12), and then dividing both sides of the expression by c_t^c, we obtain the following transformed version of the capital owner's budget constraint:

$$1 + x_t \; = \; (1 - \tau_t^c)\left[\theta_t + \alpha_t\left(1 - \theta_t\right)\right] y_t / c_t^c, \tag{A.1}$$

where $x_t \equiv (1 - \tau_t \phi_t) i_t / c_t^c$. Solving the above equation for c_t^c yields equation (21) in the text. Equation (22) in the text follows directly from the definition of x_t.

The capital owner's first-order condition (20) can be re-written as follows

$$\begin{aligned} x_t \; &= \; E_t \, \beta \left[\frac{\lambda\left(1-\kappa\right)\left(1-\tau_{t+1}^c\right)\theta_{t+1} y_{t+1}}{c_{t+1}^c} + \left(1-\lambda\right) \, x_{t+1} \right], \\ &= \; E_t \, \beta \left[\lambda\left(1-\kappa\right) \, s_{t+1}\left(1 + x_{t+1}\right) + \left(1-\lambda\right) \, x_{t+1}\right], \end{aligned} \tag{A.2}$$

where $s_{t+1} \equiv \theta_{t+1} / \left[\theta_{t+1} + \alpha_{t+1}\left(1 - \theta_{t+1}\right)\right]$ and we have eliminated $\left(1 - \tau_{t+1}^c\right) y_{t+1} / c_{t+1}^c$ using equation (A.1). Notice that the rational expectation solution for x_t will depend on the state variable s_t but not on the tax wedges. The tax wedges are subsumed within the definition of x_t.

To solve for the approximate decision rule $x_t = x\left(s_t\right)$, we first log linearize the right-side of equation (A.2) to obtain

$$x_t \; = \; E_t \, \mathrm{a}_0 \left[\frac{x_{t+1}}{\widetilde{x}}\right]^{\mathrm{a}_1} \left[\frac{s_{t+1}}{\widetilde{s}}\right]^{\mathrm{a}_2}, \tag{A.3}$$

where a_0, a_1, and a_2 are Taylor-series coefficients. The expressions for the Taylor-series coefficients are

$$\mathrm{a}_0 \; = \; \beta\left[\lambda\left(1-\kappa\right) \, \widetilde{s}\left(1+\widetilde{x}\right) + \left(1-\lambda\right) \, \widetilde{x}\right], \tag{A.4}$$

$$\mathrm{a}_1 \; = \; \frac{\left[\lambda\left(1-\kappa\right) \, \widetilde{s} + \left(1-\lambda\right)\right] \, \widetilde{x}}{\lambda\left(1-\kappa\right) \, \widetilde{s}\left(1+\widetilde{x}\right) + \left(1-\lambda\right) \, \widetilde{x}}, \tag{A.5}$$

$$\mathrm{a}_2 \; = \; \frac{\lambda\left(1-\kappa\right) \, \widetilde{s} \, \left(1+\widetilde{x}\right)}{\lambda\left(1-\kappa\right) \, \widetilde{s}\left(1+\widetilde{x}\right) + \left(1-\lambda\right) \, \widetilde{x}}, \tag{A.6}$$

where the approximation is taken around the ergodic mean such that $\widetilde{x} \equiv \exp\left\{E\left[\log\left(x_t\right)\right]\right\}$ and $\widetilde{s} \equiv \exp\left\{E\left[\log\left(s_t\right)\right]\right\}$.

We conjecture that the decision rule for x_t takes the form $x_t = \widetilde{x} \, \left[s_t / \widetilde{s}\right]^{\gamma}$. The conjectured solution is iterated ahead one period and then substituted into the right-side of equation (A.3) together with the law of motion for s_{t+1} from equation (10). After evaluating the conditional expectation and then collecting terms, we have

$$x_t \; = \; \underbrace{\mathrm{a}_0 \exp\left[\tfrac{1}{2}\left(\mathrm{a}_2 + \gamma \mathrm{a}_1\right)^2 \sigma_u^2\right]}_{=\widetilde{x}} \times \left[\frac{s_t}{\widetilde{s}}\right]^{\overbrace{\rho_s\left(\mathrm{a}_2 + \gamma \mathrm{a}_1\right)}^{=\gamma}} \tag{A.7}$$

which yields two equations in the two unknown solution coefficients $\widetilde{x}$ and γ. For the baseline calibration, we obtain $\widetilde{x} = 0.6070$ and $\gamma = 0.3745$.[37]

Using the decision rule for x_t and the definition of s_t from equation (9), we have

$$\frac{\partial x_t}{\partial \theta_t} = \frac{\partial x_t}{\partial s_t}\frac{\partial s_t}{\partial \theta_t} = \frac{\gamma x_t}{\theta_t + \alpha_t\,(1-\theta_t)} > 0, \tag{A.8}$$

which shows that an increase in θ_t causes the capital owner to devote more resources to investment instead of consumption.

Appendix B. Numerical Simulation Procedure

B.1 Baseline Simulation

For the baseline simulation, we must compute a time series of tax rates that satisfy the government budget constraint (14) each period, conditional on the observed time paths of the U.S. macro variables from 1970 to 2014. A useful expression for this purpose is

$$1-\tau_t = \frac{1-(g_t/y_t + T_t/y_t + i_t/y_t)}{(\theta_t/s_t)^{1-\kappa}\,(n+1)^{-\kappa}\,(1+x_t)^{-1} + (1-\theta_t/s_t)^{1-\kappa}\,[(n+1)\,/n]^{-\kappa}}, \tag{B.1}$$

where $\theta_t/s_t = \theta_t + \alpha_t\,(1-\theta_t)$ is the top quintile income share. To derive equation (B.1), we first use the capital owner investment decision rule (22) to eliminate the term $\tau_t\phi_t i_t$ from the government budget constraint (14). Next, we substitute in the equilibrium expressions for τ_t^w and τ_t^c from equations (17) and (18) and then solve the resulting expression for $1-\tau_t$.

Given the observed U.S. time series for s_t from Figure 3, we use the decision rule (A.7) to compute $x_t = x\,(s_t)$ for each period from 1970 to 2014. The time series for τ_t is computed using equation (B.1), where θ_t/s_t, g_t/y_t, T_t/y_t, and i_t/y_t are the observed U.S. values. Given x_t, θ_t/s_t, i_t/y_t, and τ_t, we use the investment decision rule (22) to compute the required time series for the investment tax wedge ϕ_t. The resulting values for τ_t and ϕ_t feed through to determine the personal income tax rates τ_t^c and τ_t^w from equations (17) and (18). The aggregate resource constraint for the model economy implies $c_t/y_t = 1 - g_t/y_t - i_t/y_t$. The computed time series for τ_t and ϕ_t ensure that we exactly replicate the observed U.S. time paths for g_t/y_t and i_t/y_t. Since we define y_t in the data as $c_t + i_t + g_t$ (footnote 9), our procedure ensures that we also replicate the observed U.S. time path for c_t/y_t, as plotted in Figure 2.

The final step is to compute the time series of productivity shocks z_t and capital accumulation shocks v_t that cause the model to exactly replicate: (1) the path of U.S. real per capita output, and (2) the path of U.S. real per capita private nonresidential fixed assets, as measured by the BEA's chain-type quantity index. The level of real output in the data is normalized to 1.0 in the year 1970. The level of private nonresidential fixed assets is normalized to deliver a mean value of $i_t/k_t = 0.0777$ from 1970 to 2014. The target mean is the steady state value implied by a model with no capital adjustment costs, such that $i_t/k_t = k_{t+1}/k_t - 1 + \delta$, where $\delta = 0.06$ is the target depreciation rate of physical capital. For the normalization, we employ

[37] Alternatively, we could have specified separate laws of motion for θ_t and α_t such that the equilibrium decision rule takes the form $x_t = x\,(\theta_t,\ \alpha_t)$. Our procedure simplifies the capital owner's decision problem and exploits the fact that s_t appears stationary in the data (Figure 3), whereas θ_t and α_t are both trending up in the data.

the mean growth rate of the per capita BEA quantity index such that $k_{t+1}/k_t = \exp(0.0175)$. We calibrate the value of A in the production function (7) to yield $y_t = 1$ in 1970 when k_t is equal to the normalized capital stock in 1970. Our procedure delivers a sample mean of $k_t/y_t = 1.683$ from 1970 to 2014.

Given the computed time series for τ_t and ϕ_t described above, we conjecture a pair of time series for z_t and v_t from 1970 to 2014 with $z_0 = 0$. Using the agents' decision rules, we then simulate the model. After each simulation, we compute a new pair of time series for z_t and v_t as follows

$$z_t = \frac{\log\left(y_t^{\mathrm{us}}\right) - \log\left\{A\, k_t^{\theta_t} \left[\left(\ell^c\right)^{\alpha_t} \left(n\,\ell^w\right)^{1-\alpha_t}\right]^{1-\theta_t}\right\}}{1-\theta_t}, \tag{B.2}$$

$$v_t = \log\left(k_{t+1}^{\mathrm{us}}\right) - \log\left(B\, k_t^{1-\lambda}\, i_t^{\lambda}\right), \tag{B.3}$$

where y_t^{us} and k_{t+1}^{us} are the normalized series from the U.S. data, θ_t and α_t are pinned down by the U.S. income share data, and k_t is the model capital stock series implied by the law of motion (5) with i_t determined by the capital owner decision rule (22). We repeat this procedure until the computed time series for z_t and v_t do not change from one simulation to the next. For $t > 2014$, we assume that the shock innovations ε_t and η_t are zero each period while θ_t, α_t, τ_t, and ϕ_t are held constant at year 2014 values. As a result, the macroeconomic ratios c_t^c/y_t, c_t^w/y_t, i_t/y_t, g_t/y_t, and T_t/y_t all remain constant at year 2014 values.[38]

B.2 Counterfactual Scenario

The initial conditions at t_0 for all variables in the counterfactual scenario are the same as those in the baseline simulation. The counterfactual scenario holds the variables θ_t, α_t, and T_t/y_t constant at year 1970 values. The time series for g_t is constructed so that the ratio of total government spending to output $g_t/y_t + T_t/y_t$ is identical to that in the baseline simulation. Specifically, the time series for g_t evolves according to $g_t = y_t\left(g_t^b/y_t^b + T_t^b/y_t^b\right) - T_0/y_0$, where g_t^b/y_t^b and T_t^b/y_t^b are the values from the baseline simulation and $T_0/y_0 = 0.075$ is the year 1970 value. We assume that the time series for the shocks z_t and v_t and the investment tax wedge ϕ_t are identical to those in the baseline simulation. We then solve for the time series of tax rates τ_t that satisfies equation (B.1) for each $t > t_0$, where i_t/y_t is now pinned down by the capital owner investment decision rule (22). The resulting equation that determines τ_t each period is quadratic. We choose the solution that lies on the upward-sloping portion of the Laffer curve.

The second row of Table 4 employs an alternative counterfactual scenario that holds θ_t, α_t, and T_t/y_t constant at year 1970 values. The time series for g_t is constructed so that the ratio g_t/y_t is identical to that in the baseline simulation. We solve for the required time series of τ_t using equation (B.1) with the time series for z_t, v_t, and ϕ_t identical to those in the baseline simulation.

[38] For $t > 2014$, the ratio k_t/y_t converges to a constant as the mean-reverting capital accumulation shock v_t converges to zero.

Appendix C. Normative Transfer Experiments

This appendix outlines our procedure for computing the MUC-equalizing and Pareto-improving transfers plotted in Figure 9. The MUC-equalizing level of transfers achieves the condition $1/c_t^w = 1/c_t^c$, or equivalently, $c_t^w = c_t^c$, for each $t > t_0$. The Pareto-improving level of transfers achieves the condition $1/c_t^w = 1/\left(\psi c_t^c\right)$, or equivalently $c_t^w = \psi c_t^c$, where $0 < \psi < 1$. Substituting the consumption decision rules (19) and (21) into the condition $c_t^w = \psi c_t^c$ and then solving for the required transfer-output ratio yields

$$T_t^p/y_t \;=\; (1-\tau_t)\,(n+1)^{-\kappa}\left\{\frac{n\psi\,(\theta_t/s_t)^{1-\kappa}}{1+x\,(s_t)} - (1-\theta_t/s_t)^{1-\kappa}\,n^{\kappa}\right\}, \tag{C.1}$$

where T_t^p is the Pareto-improving level of transfers and $\theta_t/s_t = \theta_t + \alpha_t\,(1-\theta_t)$ is the top quintile income share. When $\psi = 1$, we recover the MUC-equalizing level of transfers T_t^*.

For the computation, we employ the baseline time series for θ_t, α_t, z_t, v_t, ϕ_t, and g_t/y_t. We then solve for the required time series of tax rates τ_t that satisfies equation (B.1) for each $t > t_0$, where T_t/y_t is now pinned down by equation (C.1) and i_t/y_t is pinned down by the capital owner investment decision rule (22). The resulting equation that determines τ_t each period is quadratic. We choose the solution that lies on the upward-sloping portion of the Laffer curve. Through repeated simulations of the model, we guess and verify that the value $\psi = 0.71836465669$ achieves the result $\Delta^w = \Delta^c = 0.0012160615$, where Δ^w and Δ^c are the per annum welfare effects described below in Appendix D.

Appendix D. Welfare Calculation

The welfare effects in Table 4 are calculated as the constant percentage amount by which each agent's consumption in the counterfactual scenario must be adjusted upward or downward each year in perpetuity to make lifetime utility equal to that in the baseline simulation. Specifically, we find Δ^w and Δ^c that solve the following two equations

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left(c_t^w\right) \quad = \quad \sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left[\bar{c}_t^w\,\left(1+\Delta^w\right)\right], \tag{D.1}$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left(c_t^c\right) \quad = \quad \sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left[\bar{c}_t^c\,\left(1+\Delta^c\right)\right], \tag{D.2}$$

where c_t^w and c_t^c are the consumption outcomes in the baseline simulation and $\bar{c}_t^w$ and $\bar{c}_t^c$ are the consumption outcomes in the counterfactual scenario. The infinite sums in (D.1) and (D.2) are approximated by sums over a 3000 period simulation, after which the results are not changed. The initial conditions correspond to year $t_0 = 1970$ values for all variables.

The evaluation date for the welfare calculation is the year in which the agent is presumed to be indifferent between the two consumption paths being compared. In Table 4, the evaluation date is $t_0 = 1970$ (start of sample) with zero weight placed on pre-1970 consumption. In Table 5, we also consider evaluation dates of 1992 (middle of sample) or 2014 (end of sample) with past consumption going back to 1970 discounted using the discount factor β. For example, when 2014 is the evaluation date, the welfare effects for the worker are computed as the value of Δ^w that solves the following equation:

$$\sum_{t=1970}^{2013} \beta^{2014-t} \log\left(c_t^w\right) + \sum_{t=2014}^{\infty} \beta^{t-2014} \log\left(c_t^w\right) \quad = \quad \sum_{t=1970}^{2013} \beta^{2014-t} \log\left[\bar{c}_t^w \ (1+\Delta^w)\right] + \sum_{t=2014}^{\infty} \beta^{t-2014} \log\left[\bar{c}_t^w \ (1+\Delta^w)\right]. \tag{D.3}$$

Federal Reserve Bank of San Francisco
Erasmus University, Rotterdam

References.

Aguiar, M. and Bils, M. (2011). 'Has Consumption Inequality Mirrored Income Inequality?', NBER Working Paper No. 16807.

Aguiar, M. and Bils, M. (2015). 'Has Consumption Inequality Mirrored Income Inequality?', *American Economic Review,* vol. 105(9), pp. 2725-2756.

Attanasio, O. and Davis, S. (1996). 'Relative Wage Movements and the Distribution of Consumption', *Journal of Political Economy,* vol. 104(6), pp. 1227-1262.

Attanasio, O., and Pistaferri, L. (2016). 'Consumption Inequality', *Journal of Economic Perspectives,* vol. 30(2), pp. 3–28.

Auerbach, A.J., Kotlikoff, L.J. and Koehler, D.R. (2016). 'U.S. Inequality, Fiscal Progressivity, and Work Disincentives: An Intragenerational Accounting', NBER Working Paper No. 22032.

Blundell, R. and MaCurdy, T. (1999). 'Labor Supply: A Review of Alternative Approaches', in (O. Ashenfelter and D. Card, eds.), *Handbook of Labor Economics,* pp. 1559-1695, Amsterdam: Elsevier.

Caplin, A. and Leahy, J. (2004). 'The Social Discount Rate', *Journal of Political Economy,* vol. 112(6), pp. 257-1268.

Cassou, S.P. and Lansing, K.J. (1997). 'Welfare, Stabilization, or Growth: A Comparison of Different Fiscal Objectives, in (J. Hairault, P. Hénin, and F. Portier, eds.), *Business Cycles and Macroeconomic Stability: Should We Rebuild Built-in Stabilizers?,* pp. 55-78, Boston: Kluwer Academic Publishers.

Cassou, S.P. and Lansing, K.J. (2004). 'Growth Effects of Shifting from a Graduated-rate Tax System to a Flat Tax', *Economic Inquiry,* vol. 42(2), pp. 194-213.

Cassou, S.P. and Lansing, K.J. (2006). 'Tax Reform with Useful Public Expenditures', *Journal of Public Economic Theory,* vol. 8(4), pp. 631-676.

Chari, V.V., Kehoe, P.J. and McGrattan, E.R. (2007). 'Business Cycle Accounting', *Econometrica,* vol. 75(3), pp. 781-836.

Corak, M. (2013) 'Income Inequality, Equality of Opportunity, and Intergenerational Mobility', *Journal of Economic Perspectives,* vol. 27 (3), pp. 79-102.

Danthine, J.-P., Donaldson, J.B. and Siconolfi, P. (2008). 'Distribution Risk and Equity Returns', in (R. Mehra, eds.), *Handbook of the Equity Risk Premium*, pp. 415-462, Amsterdam: Elsevier.

Goldin, C. and Katz, L.F. (2007). 'The Race between Education and Technology: The Evolution of U.S. Educational Wage Differentials, 1890 to 2005', NBER Working Paper No. 12984.

Goldin, C. and Katz, L.F. (2008). *The Race between Education and Technology,* Cambridge MA: Harvard University Press.

Gomme, P., Ravikumar, B. and Rupert, P. (2011). 'The Return to Capital and the Business Cycle', *Review of Economic Dynamics,* vol. 14(2), pp. 262-278.

Gottschalk, P. and Danziger, S. (2005). 'Inequality of Wage Rates, Earnings and Family Income in the United States, 1975-2002', *Review of Income and Wealth,* vol. 51(2), pp. 231-254.

Greenwald, D.L., Lettau, M. and Ludvigson, S.C. (2014). 'The Origins of Stock Market Fluctuations', NBER Working Paper No. 19818.

Greenwood, J. and Yörükoğlu, M. (1997). '1974', *Carnegie-Rochester Conference Series on Public Policy,* vol. 46(1), pp. 49-96.

Guo, J.-T. and Lansing, K.J. (1997). 'Tax Structure and Welfare in a Model of Optimal Fiscal Policy', Federal Reserve Bank of Cleveland, *Economic Review* vol 30, pp. 2-14.

Guo, J.-T. and Lansing, K.J. (1998). 'Indeterminacy and Stabilization Policy', *Journal of Economic Theory,* vol. 82(2), pp. 481-490.

Guvenen, F., (2009) 'A Parsimonious Macroeconomic Model for Asset Pricing', *Econometrica,* vol. 77(6), pp. 1711-1750.

Heathcote, J., Storesletten, K. and Violante, G.L. (2010). 'The Macroeconomic Implications of Rising Wage Inequality in the United States', *Journal of Political Economy,* vol. 118(4), pp. 681-722.

Heathcote, J., Storesletten, K. and Violante, G.L. (2013). 'From Wages to Welfare: Decomposing Gains and Losses from Rising Inequality', in (D. Acemoglu, M. Arellano and E. Dekel, eds.), *Advances in Economics and Econometrics: Theory and Applications, vol. 2,* pp. 235-282, Cambridge: Cambridge University Press.

Heaton, J. and Lucas, D. (2000). 'Stock Prices and Fundamentals', in (B.S. Bernanke and J.J. Rotemberg, eds.), *NBER Macroeconomics Annual 1999,* pp. 213-264, Cambridge MA: MIT Press.

Hornstein, A. and Krusell, P. (1996). 'Can Technology Improvements Cause Productivity Slowdowns?' *NBER Macroeconomics Annual 11,* pp. 209-259, Cambridge MA: MIT Press.

Hungerford, T.L. (2011). 'Changes in the Distribution of Income among Tax Filers Between 1996 and 2006: The Role of Labor Income, Capital Income, and Tax Policy', Congressional Research Service, Washington D.C.

Jermann, U.J. (1998). 'Asset Pricing in Production Economies', *Journal of Monetary Economics,* vol. 41, pp. 257-275.

Judd, K.L. (1985). 'Redistributive Taxation in a Simple Perfect Foresight Model', *Journal of Public Economics,* vol. 28(1), pp. 59-83.

Justiniano, A., Primiceri, G.E., and Tambalotti, A. (2010). 'Investment Shocks and Business Cycles', *Journal of Monetary Economics,* vol. 57(2), pp. 132-145.

Karabarbounis, L. and Neiman, B. (2014). 'The Global Decline of the Labor Share', *Quarterly Journal of Economics,* vol. 129(1), pp. 61-103.

Kindermann, F. and Krueger, D. (2014). 'High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk', NBER Working Paper No. 20601.

Krueger, D. and Perri, F. (2006). 'Does Income Inequality Lead to Consumption Inequality? Evidence and Theory', *Review of Economic Studies,* vol. 73(1), pp. 163-193.

Krusell, P., Ohanian, L., Ríos-Rull, J.V. and Violante, G.L. (2000). 'Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis', *Econometrica,* vol. 68(5), pp. 1029-1054.

Krusell, P. (2002). 'Time-Consistent Redistribution', *European Economic Review,* vol. 46(4-5), pp. 755-769.

Kumhof, M., Rancière, R. and Winant, P. (2015). 'Inequality, Leverage and Crises: The Case of Endogenous Default', *American Economic Review,* vol. 105(3), pp. 1217-1245.

Lansing, K.J. (1999). 'Optimal Redistributive Capital Taxation in a Neoclassical Growth Model', *Journal of Public Economics,* vol. 73(3), pp. 423-453.

Lansing, K.J. (2012). 'Speculative Growth, Overreaction, and the Welfare Cost of Technology-driven Bubbles', *Journal of Economic Behavior and Organization,* vol. 83(3), pp. 461-483.

Lansing, K.J. (2015). 'Asset Pricing with Concentrated Ownership of Capital and Distribution Shocks', *American Economic Journal-Macroeconomics, vol.* 7(4), pp. 67-103.

Lansing, K.J. and LeRoy, S.F. (2014). 'Risk Aversion, Investor Information and Stock Market Volatility', *European Economic Review,* vol. 70, pp. 88–107.

Lemieux, T. (2006) 'Postsecondary Education and Increasing Wage Inequality', *American Economic Review Papers and Proceedings,* vol. 96(2), pp. 195-199.

Lucas, R. E. Jr., (1987). *Models of Business Cycles*, Oxford: Basil Blackwell.

Mazumder, B. (2005). 'Fortunate Sons: New Estimates of Intergenerational Mobility in the United States Using Social Security Earnings Data', *Review of Economics and Statistics,* vol. 87(2), pp. 235-255.

Meyer, B.D. and Sullivan, J.X. (2013). 'Consumption and income inequality in the U.S. since the 1960s', Working paper, University of Chicago.

Mulligan, C.B. (1997). *Parental Priorities and Economic Inequality*, Chicago: University of Chicago Press.

OECD (2011). *Divided We Stand: Why Inequality Keeps Rising,* Paris: OECD Publishing.

Ostry, J.D., Berg, A. and Tsangarides, C.G., (2014). 'Redistribution, Inequality, and Growth', International Monetary Fund Staff Discussion Note SDN/14/02.

Piketty, T. (2014). *Capital in the Twenty-First Century*, Cambridge, MA: Harvard University Press.

Piketty, T. and Saez, E. (2003). 'Income Inequality in the United States, 1913-1998', *Quarterly Journal of Economics,* vol. 118(1), pp. 1-39.

Piketty, T. and Saez, E. (2013a). 'Top Incomes and the Great Recession: Recent Evolutions and Policy Implications', *IMF Economic Review,* vol. 61(3), pp. 456-478.

Piketty, T. and Saez, E. (2013b). 'Optimal Labor Income Taxation', in (A.J. Auerbach, R. Chetty, M. Feldstein, and E. Saez, eds.), *Handbook of Public Economics vol. 5,* pp. 391-474, Amsterdam: Elsevier.

Piketty, T., Saez, E. and Stantcheva, S. (2014). 'Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities', *American Economic Journal: Economic Policy,* vol. 6(1), pp. 230-271.

Ríos-Rull, J.V. and Santaeulàlia-Llopis, R. (2010). 'Redistribution Shocks and Productivity Shocks', *Journal of Monetary Economics,* vol. 57(8), pp. 931–948.

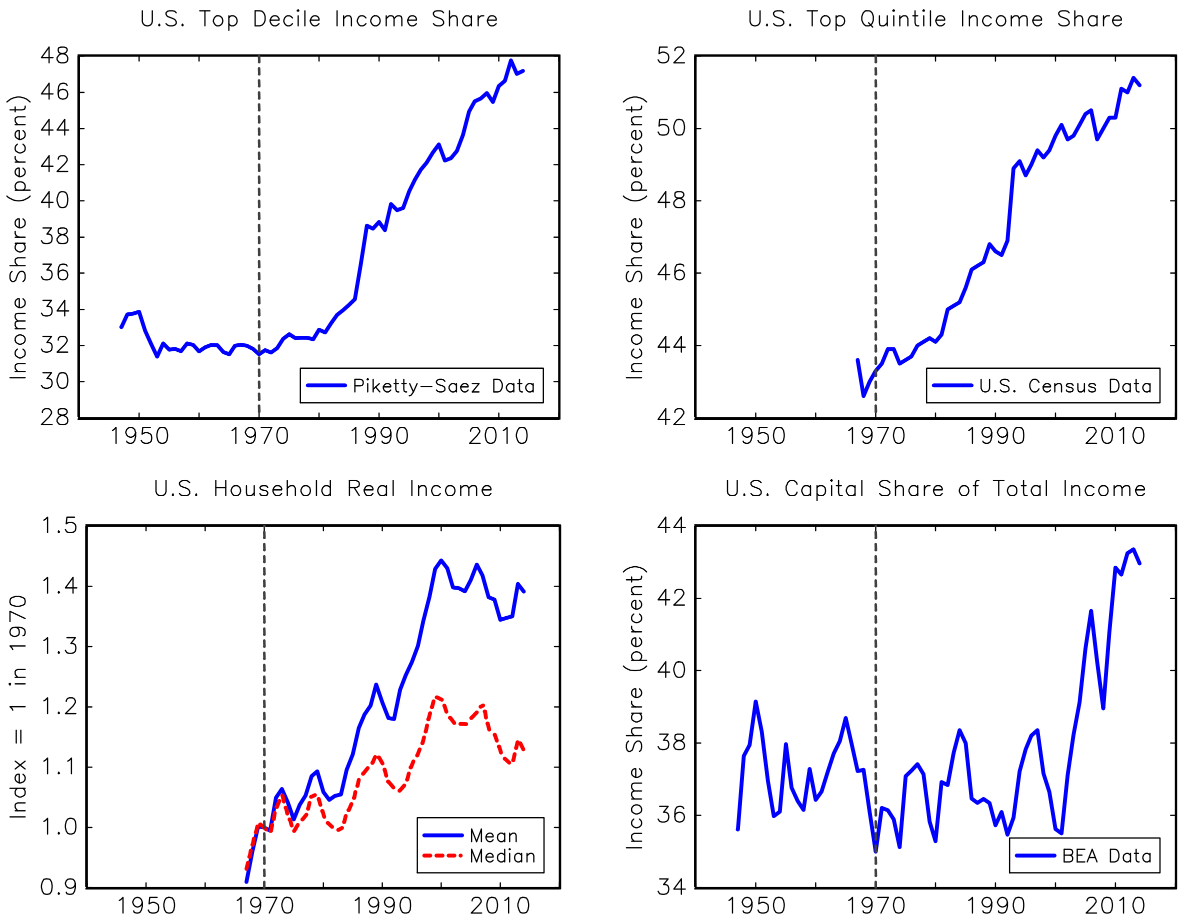
Van der Weide, R. and Milanovic, B. (2014). 'Inequality Is Bad for Growth of the Poor, But Not for That of the Rich', World Bank Policy Research Working Paper 6963.

Welch, I. and Goyal, A. (2008). 'A Comprehensive Look at the Empirical Performance of Equity Premium Prediction', *Review of Financial Studies,* vol. 21(4), pp. 1455-1508.

Wolff, E.N. (2010). 'Recent Trends in Household Wealth in the United States: Rising Debt and the Middle-Class Squeeze—an Update to 2007', Levy Economics Institute Working Paper 589.

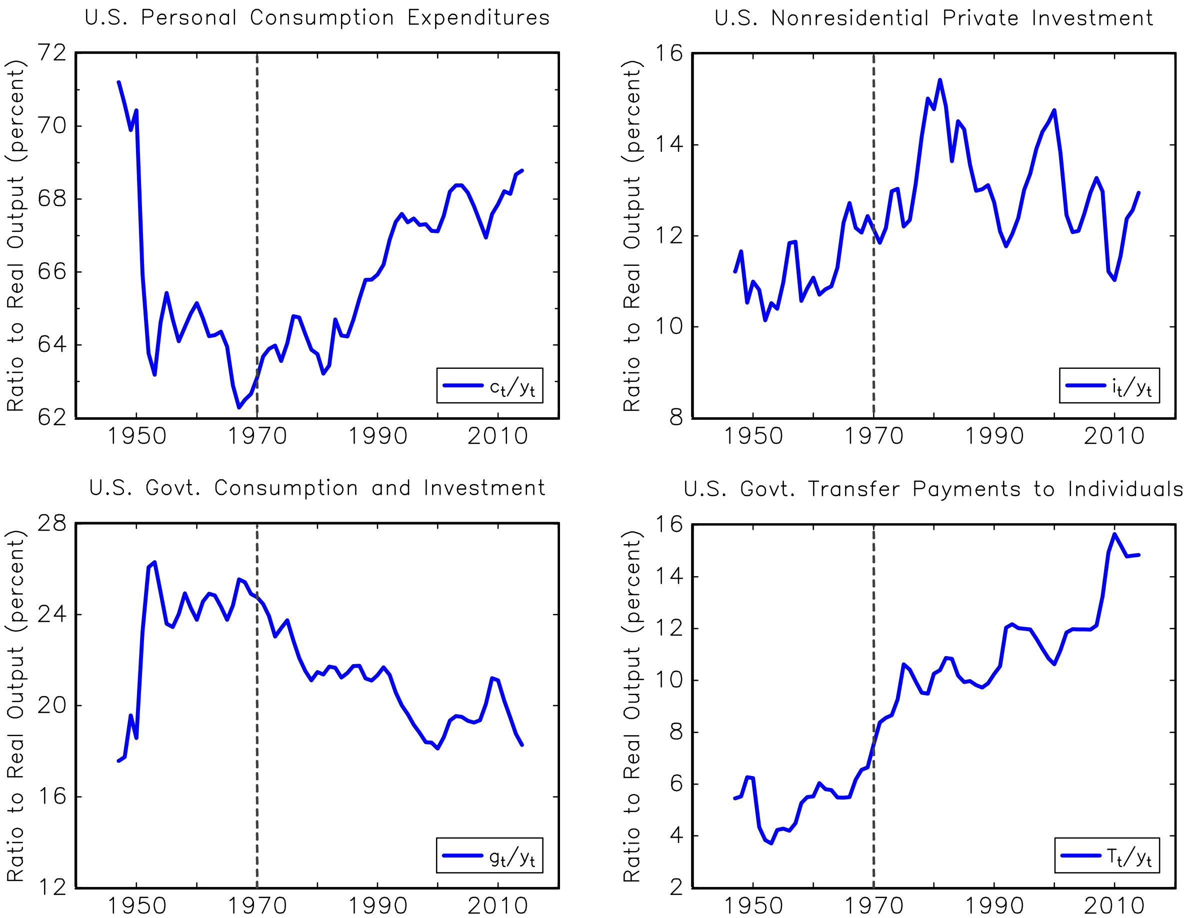
Young, A.T. (2004). 'Labor's Share Fluctuations, Biased Technical Change, and the Business Cycle', *Review of Economic Dynamics,* vol. 7(4), pp. 916–931.

Figure 1: *Tracking the Rise in U.S. Income Inequality*



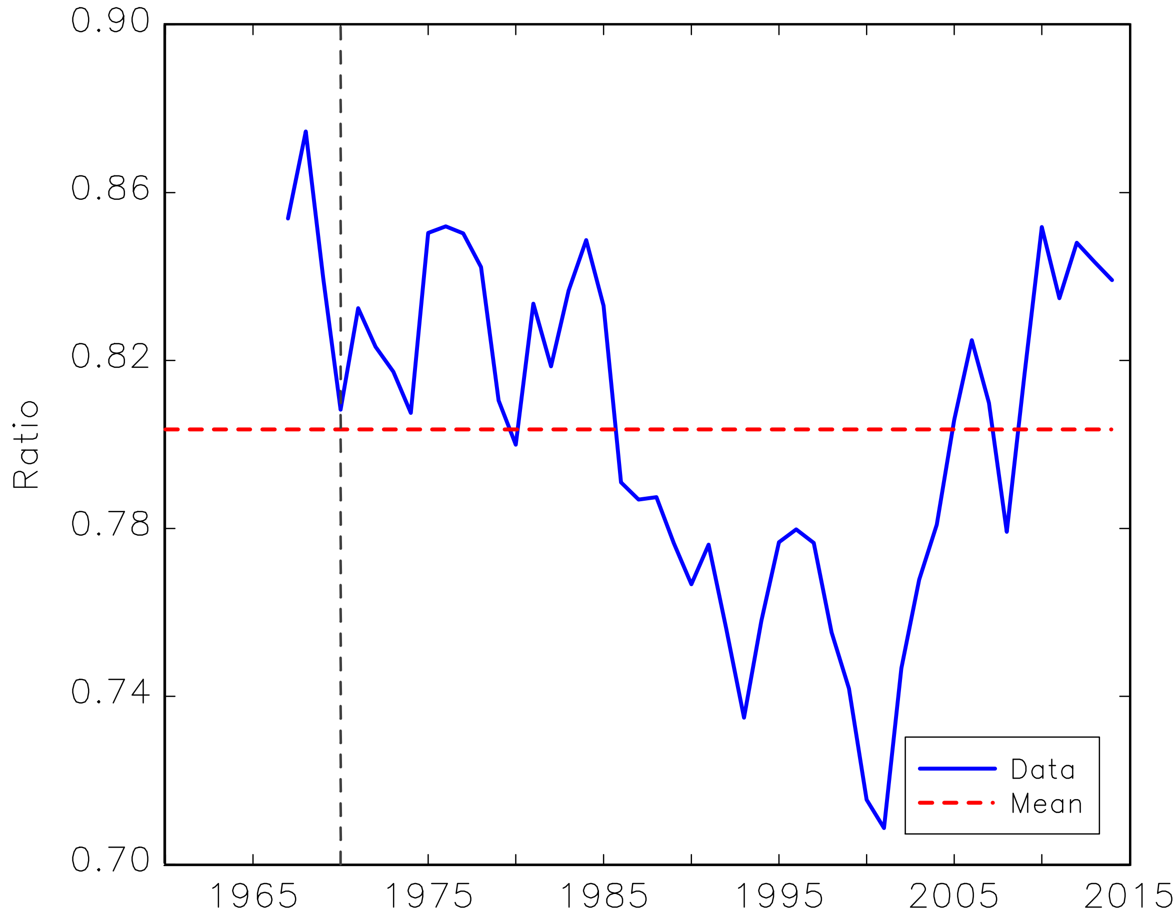
The increase in U.S. income inequality over the past 45 years can be traced to gains made by those near the top of the income distribution where financial wealth and corporate stock ownership is highly concentrated.

Figure 2: *U.S. Macroeconomic Ratios*



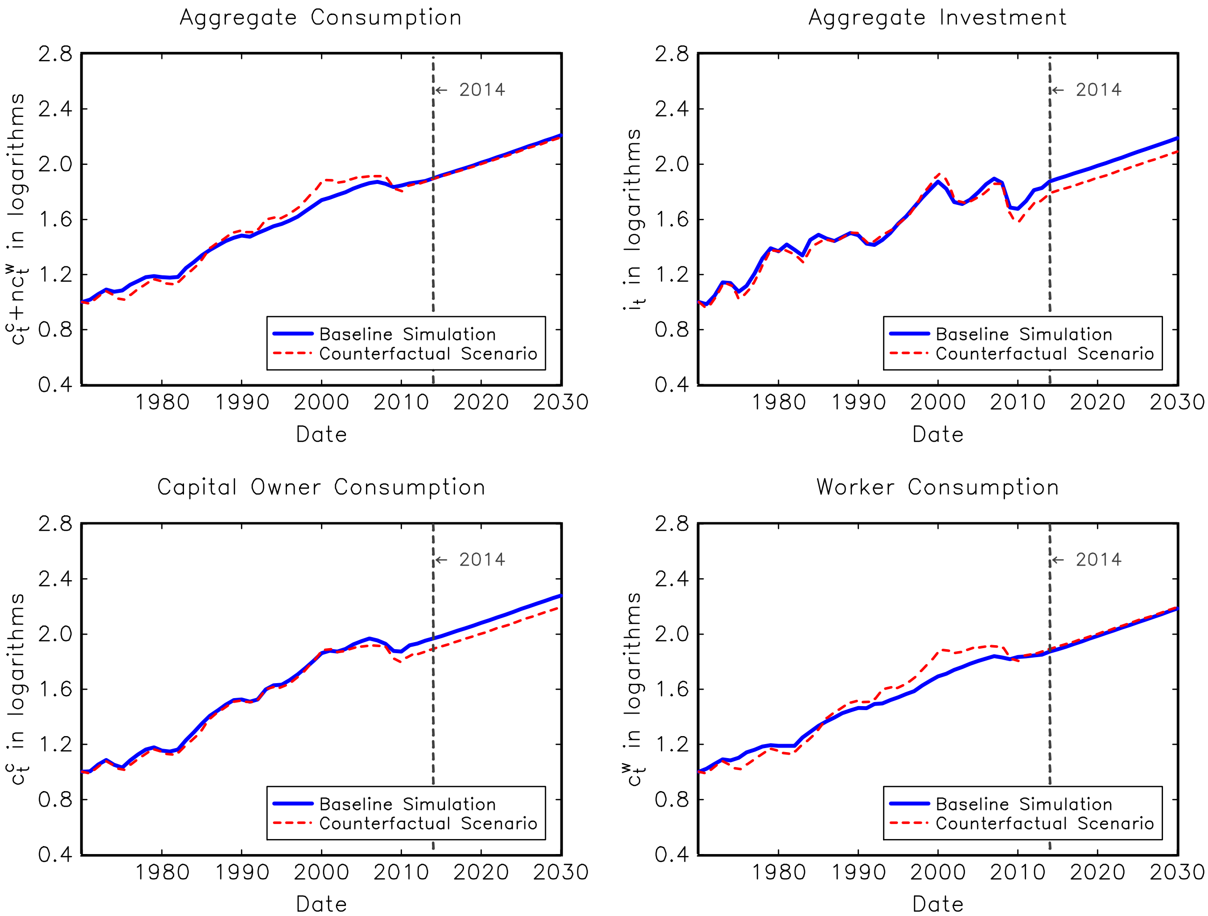
The baseline simulation exactly replicates the observed U.S. time paths for the ratios c_t/y_t, i_t/y_t, g_t/y_t, and T_t/y_t from 1970 to 2014. The vertical dashed line marks $t_0 = 1970$. Data series are constructed as described in footnote 9.

Figure 3: *Ratio of U.S. Capital Income Share to Top Quintile Income Share*



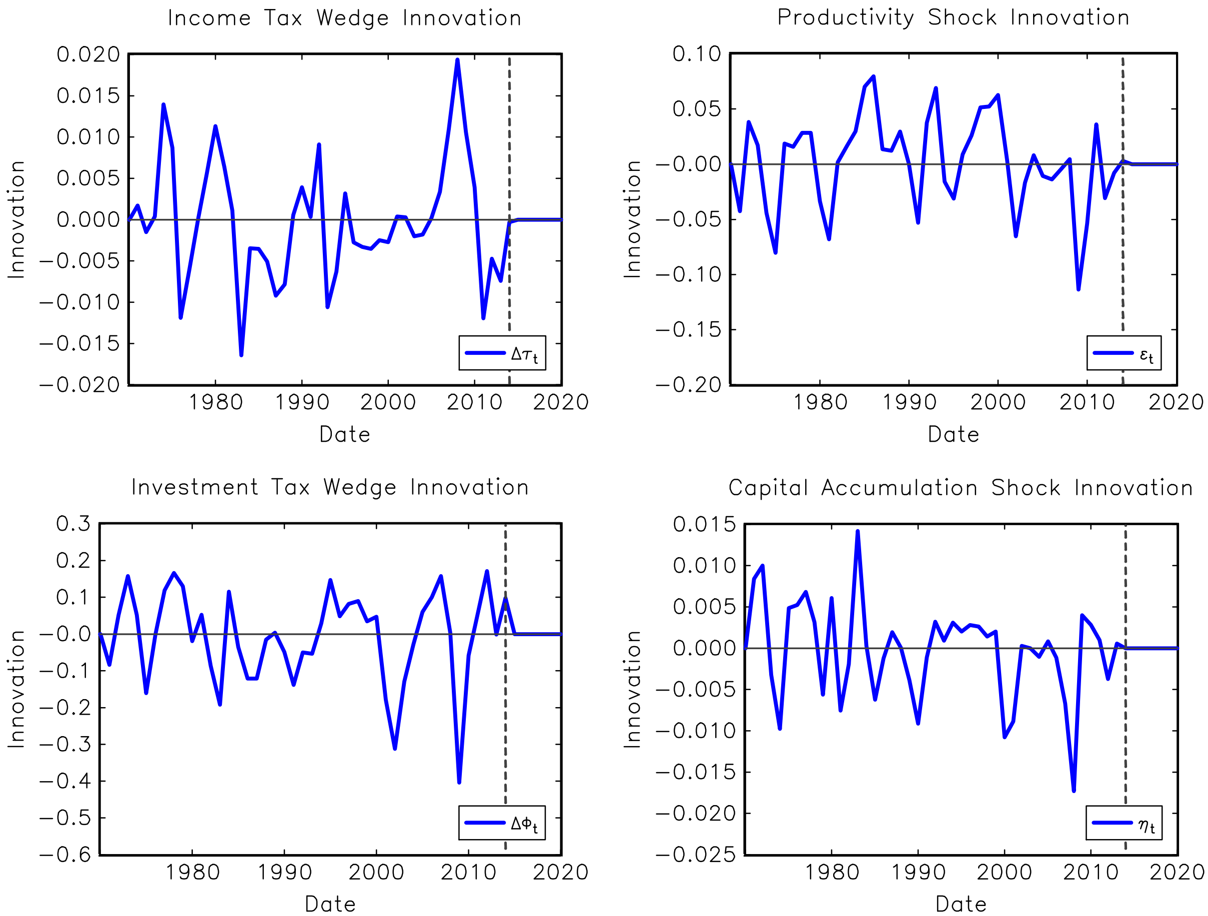
The ratio of capital's share of income to the top quintile income share in U.S. data appears stationary but persistent. In the model, this ratio is a state variable that pins down the capital owner's tax-adjusted income-consumption ratio.

Figure 4: *Simulated Consumption and Investment Paths*



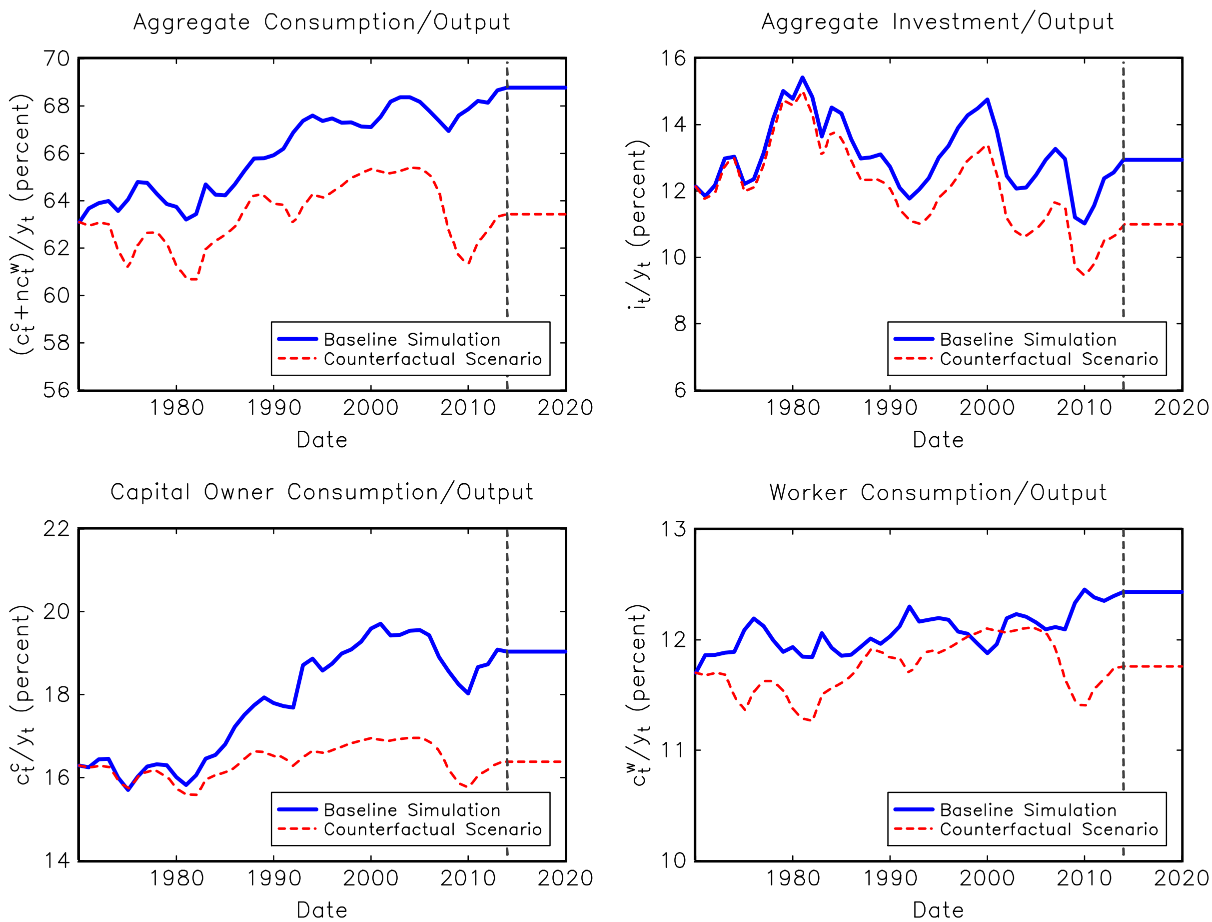
The figure plots the paths of model variables in the baseline simulation versus a counterfactual scenario in which the income shares and the transfer-output ratio T_t/y_t are held constant at year 1970 values, while maintaining the baseline time series for z_t, v_t, ϕ_t, and $g_t/y_t + T_t/y_t$. Each series is indexed to 1 in 1970. Capital owners and workers both achieve long-run upward level shifts in consumption relative to the counterfactual scenario. But the short-run consumption paths are more important for welfare.

Figure 5: *Innovations to Tax Wedges and Stochastic Shocks*



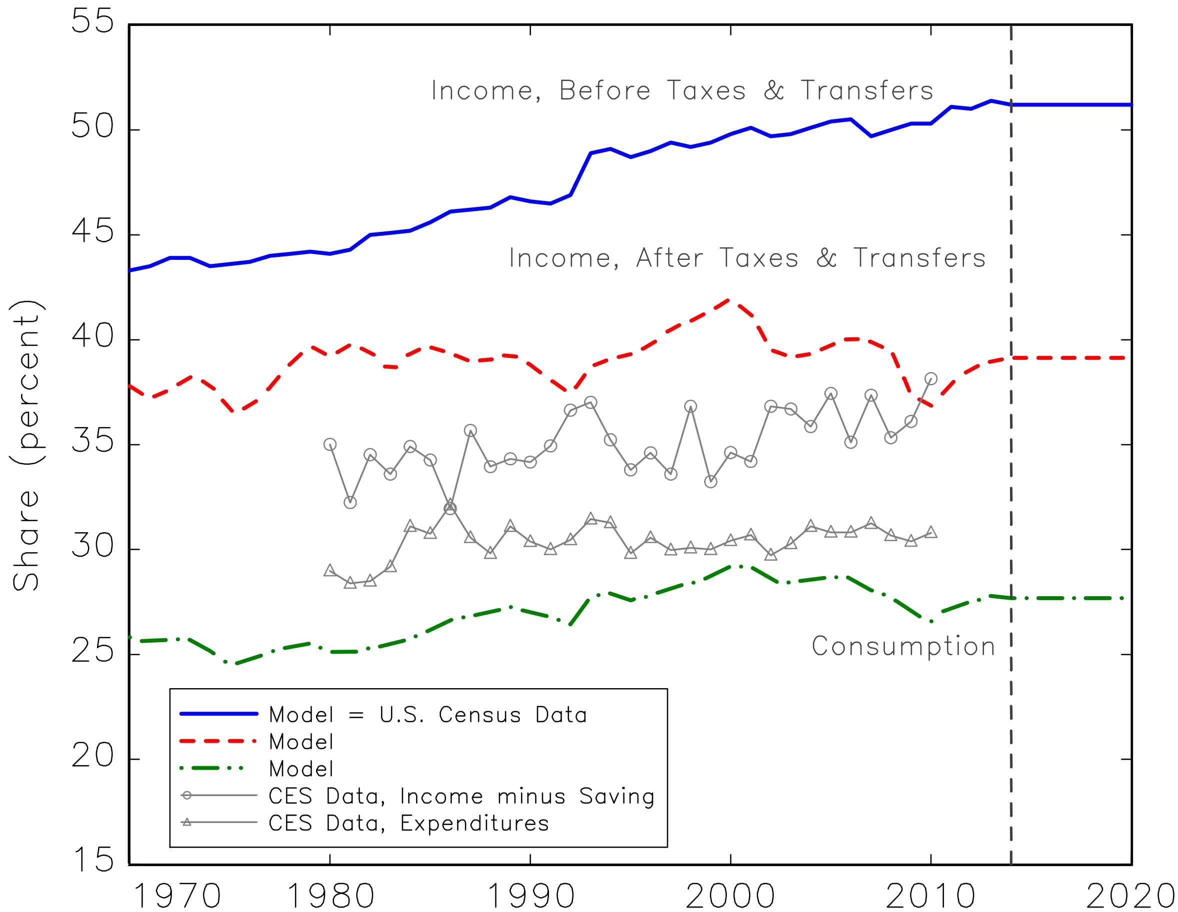
The figure plots the time series of tax wedge innovations and stochastic shock innovations that are needed to make the baseline simulation exactly replicate the paths of U.S. macroeconomic variables from 1970 to 2014. By construction, the innovations are zero at $t_0 = 1970$ and for $t > 2014$. The vertical dashed line marks $t = 2014$.

Figure 6: *Simulated Consumption and Investment Ratios*



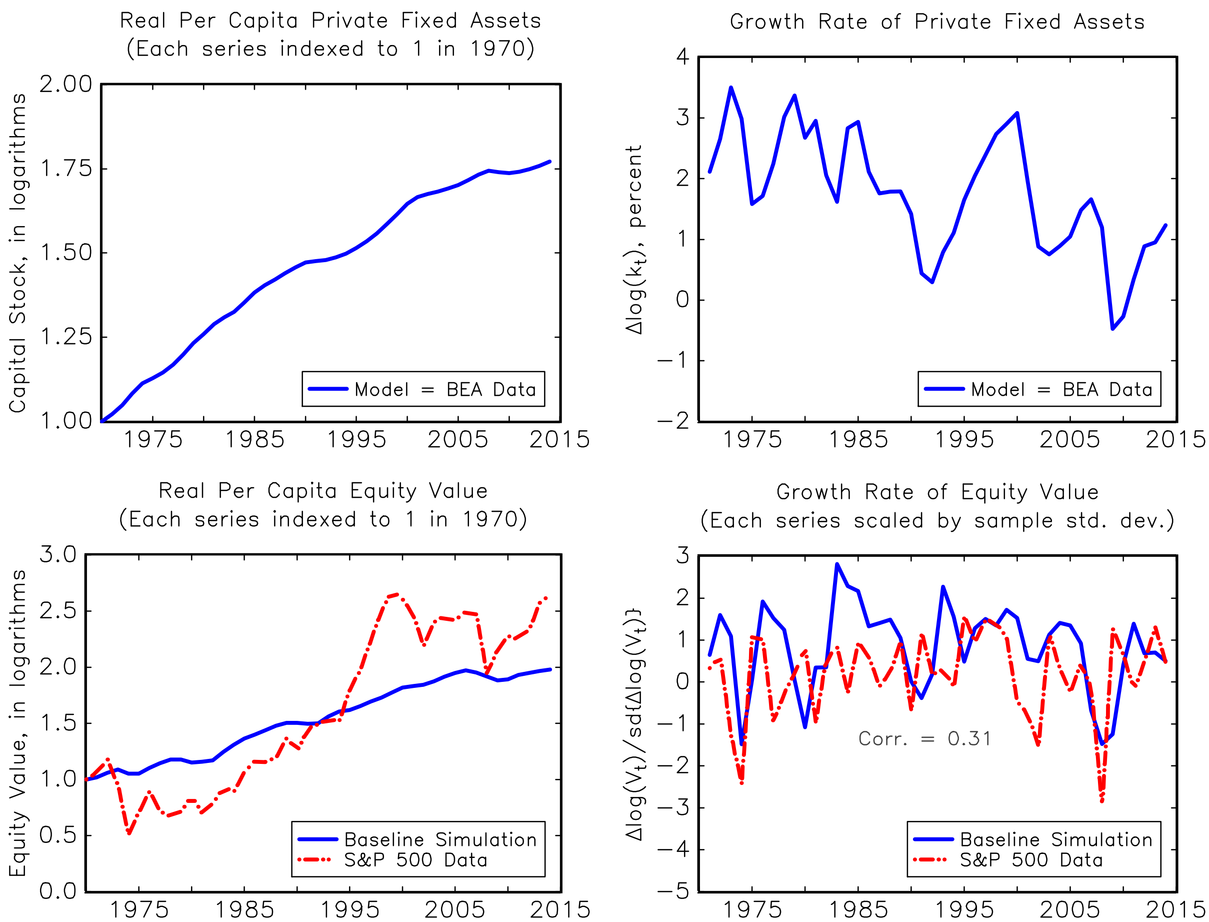
The baseline simulation exactly replicates the observed paths of aggregate c_t/y_t and aggregate i_t/y_t in U.S. data from 1970 to 2014 (Figure 2). We use the model decision rules (19) and (21) to construct individual consumption paths for the two types of agents. The vertical dashed line marks $t = 2014$.

Figure 7: *Top Quintile Shares: Model versus Data*



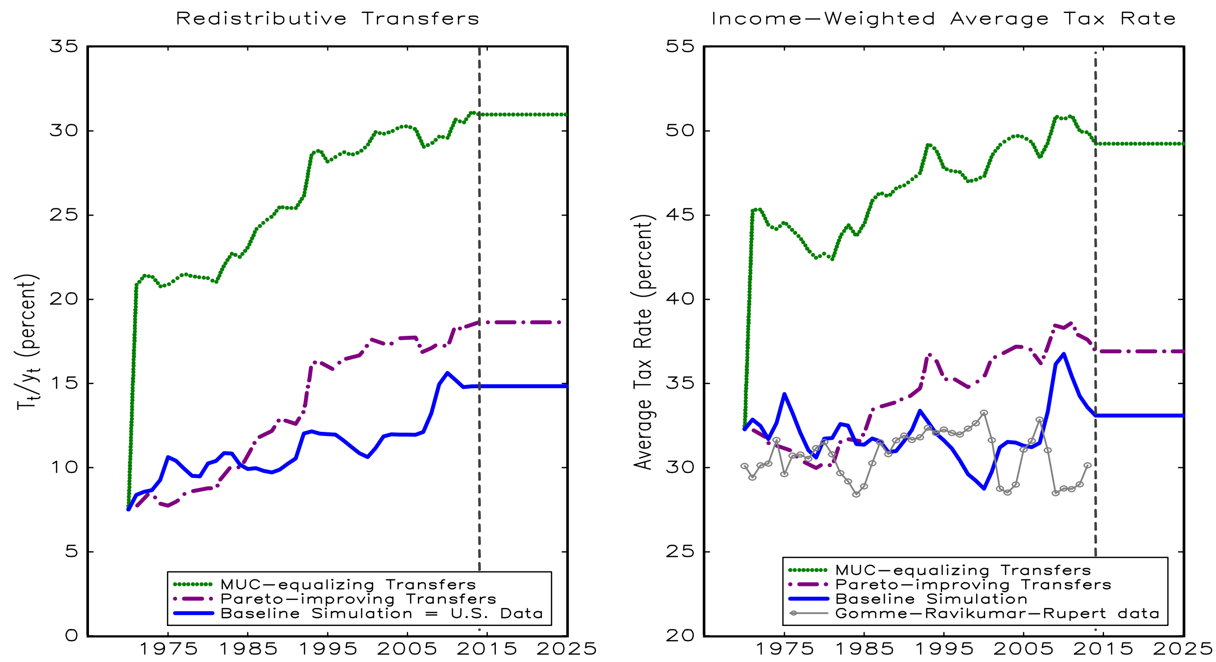
In the baseline simulation, the consumption share of the top quintile (capital owners) rises by much less than their income share before taxes and transfers. The top quintile consumption share in the model tracks reasonably well with data from the Consumer Expenditure Survey (CES) for the period 1980 to 2010. The vertical dashed line marks $t = 2014$.

Figure 8: *Capital Stock and Equity Value: Model versus Data*



The simulated capital stock series from the model exactly replicates the BEA's chain-type quantity index for the net stock of private nonresidential fixed assets. The model equity value is much less volatile than the real per capita market value of firms in the S&P 500 stock index. Nevertheless, the correlation coefficient between the growth rates of the two series is 0.31.

Figure 9: *Redistributive Transfers and Tax Rates: Model versus Data*



The MUC-equalizing transfers achieve the condition $1/c_t^w = 1/c_t^c$ for all $t > 1970$. The Pareto-improving transfers deliver small but equal welfare gains to capital owners and workers over a long simulation. The income-weighted average tax rates from the baseline simulation are close to those estimated by Gomme, Ravikumar, and Rupert (2011, updated). The vertical dashed line marks $t = 2014$.