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Top Incomes, Rising Inequality, and Welfare*

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November 23, 2015

Abstract

We introduce permanently-shifting income shares into a growth model with two types of agents. The model exactly replicates the U.S. time paths of the top quintile income share, capital's share of income, and key macroeconomic variables from 1970 to 2014. Welfare effects depend on changes in the time pattern of agents' consumption relative to a counterfactual scenario that holds income shares and the transfer-output ratio constant. Short-run declines in workers' consumption are only partially offset by longer-term gains from higher transfers and more capital per worker. The baseline simulation delivers large welfare gains for capital owners and nontrivial welfare losses for workers.

Keywords: *Top Incomes, Inequality, Distribution shocks, Redistributive Transfer Payments, Welfare.*

JEL Classification: D31, E32, E44, H21, O33.

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1 Introduction

Income inequality in the U.S. economy has increased markedly over the past several decades. Most of the increase can be traced to gains made by those near the top of the income distribution. As noted by Piketty (2014), p. 297 "if we consider the total growth of the U.S. economy in the thirty years prior to the crisis, that is, from 1977 to 2007, we find that the richest 10 percent appropriated three-quarters of the growth." Even if we restrict attention to college-educated workers, Lemieux (2006), p. 199 concludes that "changes in wage inequality are increasingly concentrated in the very top end of the wage distribution."

The top left panel of Figure 1 shows the dramatic upward shift in the share of pre-tax income going to the top decile of U.S. households, as compiled by Piketty and Saez (2003, 2013a).[1] Using data from the U.S. Census Bureau, the top right panel shows that the pre-tax income share of the top quintile of U.S. households increased by 8 percentage points, going from 43% in 1970 to 51% in 2014. Also using census data, the bottom left panel of Figure 1 shows that the growth in mean household income has significantly outpaced the growth in the median income since 1970. This pattern indicates a shift in the mass of income towards the upper tail of the distribution.[2]

The bottom right panel shows that capital's share of income increased from about 35% in 1970 to 43% in 2014.[3] Given that the distribution of financial wealth in the U.S. economy is highly skewed, the increase in capital's share of income would be expected to disproportionately benefit households near the top of the income distribution. According to a study by the U.S. Congressional Research Service (Hungerford 2011), changes in capital gains and dividend income were the two largest contributors to the increase in the Gini coefficient from 1996 to 2006. As a mitigating factor, transfer payments from the government to individuals increased from 7.5% of output in 1970 to 14.8% in 2014. These transfers would be expected to disproportionately benefit households outside the top quintile of the income distribution.[4]

Motivated by the above observations, this paper develops a quantitative growth model to assess the welfare consequences of rising U.S. income inequality over the period 1970 to

[1] Updated annual data are available from The World Top Incomes Database.

[2] Census income is defined as income received on a regular basis (exclusive of capital gains) before payments for personal income taxes, social security, union dues, medicare deductions, food stamps, subsidized housing, etc. The data plotted in Figure 1 are from Tables H-2 and H-17 at www.census.gov/hhes/www/income/data/historical/household/.

[3] Following Lansing (2015), capital's share of income is measured as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. Both series are from the Bureau of Economic Analysis (BEA), NIPA Table 1.14, lines 1 and 4. The increase in capital's share of income is not limited to the United States. Using data over the period 1975 to 2012, Karabarbounis and Neiman (2014) find that capital's share increased in 42 out of 59 countries with at least 15 years of data.

[4] Transfers include benefits from Old Age, Survivors, and Disability Insurance (OASDI), Medicare and Medicaid benefits, Supplemental Security Income, Family Assistance, Food Stamps, and Unemployment Insurance Compensation.

2014. The starting date for the simulation is when the top quintile income share in the data started rising in a sustained way. This is also when the mean household income started to pull away from the median. The model includes two types of infinitely-lived agents: capital owners who represent the top income quintile of U.S. households and workers who represent the remainder. All agents supply labor inelastically to firms, consistent with the near-zero labor supply elasticity estimates obtained by most empirical studies (Blundell and McCurdy, 1999).[5] Our setup is similar to other concentrated capital ownership models that have been applied successfully to asset pricing.[6]

The top income quintile in our model owns 100 percent of the productive capital stock—a setup that roughly approximates the highly-skewed distribution of U.S. financial wealth. Using data from the Survey of Consumer Finances, Wolff (2010), p.44, finds that the share of total financial wealth owned by the top quintile of U.S. households remained steady at around 92 percent from 1983 to 2007. Shares of corporate stock are an important component of financial wealth, representing claims to the tangible and intangible capital of firms. As recently as 1995, the lowest 75 percent of U.S. households sorted by wealth owned less than 10 percent of stocks.[7]

Our tractable growth model delivers approximate decision rules for consumption and investment that depend on income share variables, distortionary tax wedges, and the level of real output. The income share variables enter the model via stochastic exponents in a Cobb-Douglas aggregate production function, along the lines of Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), and Lansing (2015). But in contrast to these papers, we assume that the exponent shifts are permanent rather than temporary. Our modeling strategy is similar to Goldin and Katz (2007) who allow for permanent shifts in the share parameters of a constant elasticity of substitution production function as a way of capturing technology-induced changes in the demand for skilled versus unskilled labor. Along these lines, a study by the OECD (2011) asserts that technological progress and a more integrated global economy have brought profound changes in the ways that firms produce and distribute goods and services, and that these changes have shifted production technologies in favor of highly-skilled individuals. Here we remain agnostic about the underlying causes of the production function shifts and focus on the resulting consequences for welfare.

Tax wedges enter the model via the budget constraints of the agents and the government. We allow for a progressive personal income tax schedule a variable that governs the fraction of

[5] Alternatively, one may view our welfare results as applying to the vast majority of agents who remain fully-employed at all times. Along these lines, we find that business cycle fluctuations in agent's consumption paths have very little impact on the welfare results.

[6] See, for example, Danthine, Donaldsen, and Siconolfi (2008), Guvenen (2009), and Lansing (2015).

[7] See Heaton and Lucas (2000), Figure 3, p. 224.

business investment that can be "expensed," or immediately deducted from business taxable income. As inputs to the model, we incorporate the observed U.S. time paths of the top quintile income share and capital's share of total income, as plotted in Figure 1. Given these time paths from the data, we solve for the time series of tax wedges and productivity shocks such that the model exactly replicates the observed trajectories of the following U.S. macroeconomic variables over the period 1970 to 2014: (1) real per capital output, (2) real per capita aggregate consumption, (3) real per capita nonresidential private investment, (4) real per capita government consumption and investment, and (5) real per capita government transfer payments to individuals.[8] Figure 2 plots the latter four variables as ratios relative to real output.[9]

Given time series for the income shares, tax wedges, and productivity shocks, we use the model's decision rules to construct individual consumption paths for the capital owners and workers. Our procedure ensures that the individual consumption paths that we use to evaluate welfare are consistent with the evolution of the U.S. macro variables from 1970 to 2014.

Welfare effects are measured by the percentage change in per-period consumption that makes each type of agent indifferent between the baseline simulation and a counterfactual scenario in which income shares and the transfer-output ratio are held constant at year 1970 values. We assume that the resources saved by not increasing transfers are redirected to government consumption which does not affect production or utility. Both scenarios employ the same time series for the ratio of total government outlays to output and the same time series of labor-augmenting productivity shocks. An advantage of our quantitative modeling approach is that it allows us to construct a clean counterfactual scenario. In contrast, a purely empirical analysis based on ex post observed U.S. data cannot take into account how the economy would have evolved in the absence of shifting income shares and rising transfer payments.

For the baseline simulation, the welfare gain for capital owners is 3.3% of their per-period consumption while workers suffer a welfare loss of 0.5% of their per period consumption. These

[8]Our methodology is conceptullally similar to that of Chari, McGrattan, and Kehoe (2007) who develop a quantitative model with four "wedges" that relate to labor, investment, productivity, and government consumption.

[9]Nominal personal consumption expenditures C_t are from the Bureau of Economic Analysis (BEA), NIPA Table 2.3.5. The corresponding price index is from Table 1.1.4. Nominal government consumption and investment G_t and the corresponding price index are from NIPA Tables 1.1.5 and 1.1.4. Nominal private nonresidential fixed investment I_t and the corresponding implicit price deflator are from the Federal Reserve Bank of St. Louis' FRED database. Nominal transfer payments to individuals T_t are also from FRED. Population data are from NIPA Table 2.1, line 40. We first define the nominal ratios C_t/Y_t, I_t/Y_t, G_t/Y_t and T_t/Y_t, where $Y_t \equiv C_t + I_t + G_t$. The nominal ratios capture shifts in relative prices. We then deflate Y_t by an output price index constructed as the weighted-average of the price indices for C_t, I_t, and G_t, where the weights are the nominal ratios relative to Y_t. Finally, we construct the per capita real series c_t, i_t, and g_t by applying the nominal ratios to the deflated output series and then dividing by population. In this way, the per capita real series reflect the same resource allocation ratios as the nominal series.

results reflect changes in the time pattern of consumption for each type of agent in both the short-run and the long-run. Due to discounting, the short-run changes in consumption are more important for welfare.

For capital owners, welfare gains derive mainly from the post-2005 upward shift in their consumption path relative to the counterfactual. This pattern can be traced to the dramatic increase in capital's share of income starting around the year 2005. In the long-run, the capital owners' consumption shifts up by 12.1% relative to the counterfactual path while investment shifts up by 13.3%. For workers, welfare losses are mitigated by the favorable period from 1971 to 1985 when the transfer-output ratio is rising faster than the top quintile income share, thus boosting their consumption relative to the counterfactual. Beyond 2014, the higher level of investment by capital owners contributes to more capital and more private-sector output per worker, allowing the worker's consumption to eventually surpass the counterfactual, achieving a permanent upward level shift of 2.4%. But these long-run consumption gains are heavily discounted in the welfare calculation.

We examine the sensitivity of the welfare results to a wide variety of scenarios. In particular, we show that having a more-progressive tax schedule in place from 1970 to 2014 would have nearly eliminated the welfare loss for workers, while preserving a substantial welfare gain for capital owners.

As a validity check, we demonstrate that the model-predicted paths for a number of economic variables track reasonably well with the corresponding variables in U.S. data. These include: (i) the top quintile consumption share from the Consumer Expenditure Survey, (ii) the BEA's chain-type quantity index for the net stock of private nonresidential fixed assets, (iii) the real S&P 500 stock market index, and (iv), an income-weighted average tax rate constructed using estimated U.S. tax rates on labor and capital incomes from Gomme, Ravikumar, and Rupert (2011, updated).

Experiments with the model show that the welfare results are sensitive to the precise time paths followed by the income share variables and transfer payments during the early years of the simulation, which are lightly discounted. As a robustness check, we consider different evaluation dates for the welfare calculation. The evaluation date is the year in which the agent is presumed to be indifferent between the consumption path in baseline simulation and the consumption path in the counterfactual scenario. Regardless of whether the evaluation date is at the start-, middle-, or end-of-sample, the baseline simulation consistently delivers large welfare gains for capital owners and nontrivial welfare losses for workers.

As a supplement to the positive analysis summarized above, we undertake two normative experiments. Given the paths of the U.S. pre-tax income shares, we solve for a time series of transfers that equalizes agents' marginal utility of consumption each period from 1971 onwards.

The new level of transfers is financed by adjusting the path of tax rates relative to the baseline simulation, but with other relevant variables unchanged. We find that the transfer-output ratio must rise to around 32% by the year 2014. Relative to the counterfactual (no change in income shares or the transfer-output ratio), capital owners suffer a welfare loss of 24% while workers enjoy a gain of 8.2%.

As a more realistic normative experiment, we compute a Pareto-improving time series of transfers that delivers equal welfare gains to capital owners and workers over a long simulation. In this case, the transfer-output ratio must rise to 18.4% percent by the year 2014—somewhat higher than the actual value of 14.8% observed in the data. The welfare gain for both types of agents is modest at 0.43% of per-period consumption. This is due to the need for a higher average tax rate path to finance the higher level of transfers. Still, the experiment suggests that realistic policy actions could be effective in mitigating the negative impacts of rising income inequality.

It is important to note that our analysis abstracts from human capital accumulation and distributional mobility that could allow some workers to become capital owners (and vice versa) over the course of the model simulation. Hurst, Luoh, and Stafford (1998) document significant mobility within the midrange deciles of the U.S. wealth distribution, but much lower mobility for the top and bottom deciles. Our model assumes no mobility into or out of the top quintile. More recently, Mazumder (2005) finds that the intergenerational earnings elasticity (an inverse measure of mobility) is very high for U.S. households in the bottom three-quarters of the net worth distribution. His quantitative estimates imply that it would take many generations for a low or middle income family to make significant upward movement in the earnings distribution. Corak (2013) presents evidence that the returns to college and the intergenerational earnings elasticity both increased substantially since 1980. This pattern suggests that the same forces which have contributed to rising U.S. income inequality may also be restricting intergenerational mobility.

1.1 Related Literature

Our analysis examines the consequences of rising inequality that is driven by gains in top incomes, defined here as the highest 20% of earners. In contrast, the majority of previous research has focused on inequality that is driven by the rising wage skill premium of college-educated workers.[10] Our framework takes into account the simultaneous shifts in the distribution of both labor and capital incomes in U.S. data. According to Alvaredo, et al. (2013), the increased correlation between top labor incomes and top capital incomes is an important but

[10] See, for example, Attanasio and Davis (1996), Krussell, et al. (2000), Goldin and Katz (2007, 2008) and Heathcote, Storesletten, and Violante (2010, 2013).

often-overlooked factor contributing to the rise in U.S. income inequality.

As an alternative to technological explanations for rising income inequality (such as shifting production functions), Piketty, Saez, and Stantcheva (2014) argue that the dramatic rise in top incomes has been driven mainly by institutional changes which strengthened the bargaining power of top earners at the expense of lower earners. According to this "grabbing hand" theory, the shift in bargaining power has enabled rent-seeking top earners to successfully push their pay above their marginal product. While the grabbing-hand theory may have different implications for social welfare, the welfare consequences for each class of agents would still be linked to the resulting paths for their income and consumption, which our quantitative analysis explicitly takes into account. Kumhof, Rancière, and Winantet (2015) consider an endowment economy with rising income inequality, as measured by the income share of the top 5% of households. They do not consider welfare but instead focus on the links between rising inequality, increased household leverage, and the risk of a financial crisis.

2 Model

The model consists of workers, capital owners, competitive firms, and the government. There are n times more workers than capital owners, with the total number of capital owners normalized to one. Naturally, the firms are owned by the capital owners. Workers and capital owners both supply labor to the firms inelastically, but in different amounts.[11] The government levies distortionary taxes on both types of agents to finance public consumption expenditures and redistributive transfers.

2.1 Workers

The individual worker's decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log\left(c_t^w\right), \tag{1}$$

subject to the budget constraint

$$c_t^w \ = \ \left(1-\tau_t^w\right) w_t^w \ell_t^w \ + \ T_t/n, \tag{2}$$

where E_t represents the mathematical expectation operator, β is the subjective time discount factor, c_t^w is the individual worker's consumption, w_t^w is the worker's competitive market wage, $\ell_t^w = \ell^w$ is the constant supply of labor hours per worker, and τ_t^w is the worker's personal

[11] The model setup is similar to a standard framework that is often used to study optimal redistributive capital taxation. See, for example, Judd (1985), Lansing (1999), and Krusell (2002). In these examples, however, capital owners do not supply labor.

income tax rate. Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their resources each period, consisting of after-tax labor income $(1-\tau_t^w)\, w_t^w \ell_t^w$ and a per-worker transfer payment T_t/n received from the government.

2.2 Capital Owners

Capital owners represent the top quintile of earners. Their decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log\left(c_t^c\right), \tag{3}$$

subject to the budget constraint

$$c_t^c \;+\; i_t \;=\; (1-\tau_t^c)\left(w_t^c\, \ell_t^c + r_t k_t\right) + \tau_t \phi_t i_t, \tag{4}$$

where c_t^c is the individual capital owner's consumption and $\ell_t^c = \ell^c$ is the constant supply of labor hours. The symbol i_t represents investment in physical capital k_t. For simplicity, we assume that the functional form of the utility function and the discount factor β are the same for both capital owners and workers. Capital owners derive income by supplying labor and capital services to firms. They earn a wage w_t^c for each unit of labor employed by the firm and receive the rental rate r_t for each unit of physical capital used in production. The capital owner's personal income tax rate is τ_t^c. Finally, the term $\tau_t\phi_t i_t$ captures the degree to which business investment can be "expensed," or immediately deducted from business taxable income, where τ_t is the effective business tax rate (which may differ from τ_t^c), and ϕ_t is an index number that captures elements of the tax code that encourage saving or investment. In the quantitative analysis, we calibrate the average value of the term $\tau_t\phi_t i_t$ to reflect a standard depreciation allowance for physical capital.

Resources devoted to investment augment the stock of physical capital according to the law of motion

$$k_{t+1} \;=\; B\, k_t^{1-\lambda}\, i_t^{\lambda}, \tag{5}$$

with k_0 given. The parameter $\lambda \in (0,1]$ is the elasticity of new capital with respect to new investment. When $\lambda < 1$, equation (5) reflects the presence of capital adjustment costs.[12]

[12] This formulation is employed by Cassou and Lansing (2006) in a welfare analysis of tax reform. Since equation (5) can be written as $k_{t+1}/k_t = B\left(i_t/k_t\right)^{\lambda}$, our adjustment cost specification can be viewed as a log-linearized version of the following law of motion employed by Jermann (1998): $k_{t+1}/k_t = 1-\delta+\psi_0\left(i_t/k_t\right)^{\psi_1}$. For details, see Lansing (2012), p. 467.

2.3 Firms

Identical competitive firms are owned by the capital owners and produce output according to the technology

$$y_t = A\, k_t^{\theta_t} \left[\exp(z_t)\; (\ell_t^c)^{\alpha_t}\; (n\,\ell_t^w)^{1-\alpha_t}\right]^{1-\theta_t}, \qquad A>0, \tag{6}$$

$$z_t = z_{t-1} + \mu + \varepsilon_t, \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right) \tag{7}$$

$$s_t \equiv \frac{\theta_t}{\theta_t + \alpha_t\,(1-\theta_t)}, \tag{8}$$

$$s_t = (s_{t-1})^{\rho}\,(\widetilde{s}\,)^{1-\rho}\exp(u_t)\,, \qquad \begin{array}{l} \widetilde{s} \equiv \exp\{E[\log(s_t)]\}\,, \\ |\rho|<1, \\ u_t \sim NID\left(0,\sigma_u^2\right), \end{array} \tag{9}$$

with z_0 and s_0 given. In equation (6), z_t represents a labor-augmenting "productivity shock" that evolves as a random walk with drift. The drift parameter μ determines the trend growth rate of the economy. The shock innovation ε_t is normally and independently distributed (NID) with mean zero and variance σ_ε^2. Stochastic shifts in the the production function exponents θ_t and α_t represent "distribution shocks" along the lines of Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), and Lansing (2015). Given the Cobb-Douglas form of the production function, θ_t is capital's share of income, $\theta_t + \alpha_t\,(1-\theta_t)$ is the top quintile income share, $\alpha_t\,(1-\theta_t)$ is the labor income share of the capital owners, and $(1-\alpha_t)\,(1-\theta_t)$ is the income share of the workers, representing the bottom four quintiles.

Recall from Figure 1 that the U.S. income shares exhibit sustained upward trends over the period 1970 to 2014. To facilitate a solution of the model in terms of stationary variables, we define the variable s_t as the ratio of capital's share of income to the top quintile income share. Figure 3 shows that the empirical counterpart of s_t in the data appears to be stationary but persistent. To capture this feature, we postulate that s_t in the model evolves according to the law of motion (9) with persistence parameter ρ and innovation variance σ_u^2.

Profit maximization by firms yields the following factor prices

$$r_t = \theta_t\, y_t/k_t, \tag{10}$$

$$w_t^c = \alpha_t(1-\theta_t)\, y_t/\ell^c, \tag{11}$$

$$w_t^w = (1-\alpha_t)\,(1-\theta_t)\, y_t/\left(n\ell^w\right), \tag{12}$$

which reflect the constant labor supplies ℓ^c and ℓ^w.

2.4 Government

The government collects tax revenue to finance expenditures on public consumption and redistributive transfers. We assume that the government's budget constraint is balanced each period, as given by

$$g_t \ + \ T_t = n\,\tau_t^w \underbrace{w_t^w \ell^w}_{=y_t^w} + \tau_t^c \underbrace{(w_t^c \ell^c + r_t k_t)}_{=y_t^c} - \tau_t \phi_t i_t, \tag{13}$$

where g_t is public consumption, T_t is aggregate redistributive transfers, and y_t^i for $i = w$, c, is the pre-tax income for workers and capital owners, respectively. The balanced-budget constraint can be viewed as an approximation to the consolidated budgets of federal, state, and local governments. Public consumption does not provide direct utility to either capital owners or workers. Nevertheless, we include g_t in our analysis to obtain quantitatively realistic tax rates during the transition period from 1970 to 2014.

Following Guo and Lansing (1998) and Cassou and Lansing (2004), we introduce progressive income taxation via the formulation

$$\tau_t^i = 1 - (1 - \tau_t)\left(\frac{y_t^i}{\overline{y}_t}\right)^{-\kappa}, \tag{14}$$

where τ_t^i is the personal income tax rate of agent type i, y_t^i is the individual agent's pre-tax income, and $\overline{y}_t$ is the average per capita income level in the economy which the agent takes as given. The parameter $\kappa \geq 0$ governs the slope of the tax schedule while τ_t governs the level of the tax schedule. When $\kappa > 0$, the agent's personal tax rate is increasing in the agent's income, reflecting a progressive tax schedule. When $\kappa = 0$, the tax schedule is flat such that all agents face the same tax rate τ_t regardless of their income. For simplicity, we assume that τ_t also pins down the effective business tax rate which exhibits no progressivity.

The agent's marginal personal tax rate MTR_t^i is defined as the change in taxes paid divided by the change in income, that is, the rate applied to the last dollar earned. The expression for the agent's marginal personal tax rate is

$$MTR_t^i = \frac{\partial\left(\tau_t^i\, y_t^i\right)}{\partial y_t^i} = 1 - (1 - \kappa)\left(1 - \tau_t^i\right), \tag{15}$$

which implies $MTR_t^i > \tau_t^i$ when $\kappa > 0$.

The average per capita income level in the economy is given by $\overline{y}_t = y_t/\left(n + 1\right)$, where $n + 1$ is the total number of agents. Making use of the Cobb-Douglas production function (6) and the factor prices (10) through (12), the equilibrium personal income tax rates for each type of agent are given by:

$$\tau_t^w = 1 - (1-\tau_t)\left[(1-\alpha_t)(1-\theta_t)(n+1)\frac{1}{n}\right]^{-\kappa} \tag{16}$$

$$\tau_t^c = 1 - (1-\tau_t)\left\{[\theta_t + \alpha_t(1-\theta_t)](n+1)\right\}^{-\kappa}. \tag{17}$$

All else equal, higher values of θ_t or α_t will serve to increase the capital owner's tax rate, but decrease the worker's tax rate.

2.5 Decision Rules and Computation

Given that labor supply is inelastic, workers simply consume their after-tax income plus transfers each period according to their budget constraint (2). In equilibrium, the individual worker's ratio of consumption to total output is given by

$$\frac{c_t^w}{y_t} = \frac{1}{n}\left[(1-\tau_t^w)(1-\alpha_t)(1-\theta_t) + \frac{T_t}{y_t}\right], \tag{18}$$

where τ_t^w is given by equation (16) and we have substituted in the worker's equilibrium real wage (12).

For capital owners, we first use the capital law of motion (5) to eliminate i_t from the budget constraint (4). The capital owner's first-order condition with respect to k_{t+1} is given by

$$\underbrace{\frac{(1-\tau_t\phi_t)i_t}{\lambda}}_{p_t} = E_t\, M_{t+1}^c[\underbrace{(1-\kappa)\left(1-\tau_{t+1}^c\right)r_{t+1}k_{t+1} - \left(1-\tau_{t+1}\phi_{t+1}\right)i_{t+1}}_{d_{t+1}} + \underbrace{\frac{(1-\tau_{t+1}\phi_{t+1})i_{t+1}}{\lambda}}_{p_{t+1}}], \tag{19}$$

where $M_{t+1}^c \equiv \beta\left(c_{t+1}^c/c_t^c\right)^{-1}$ is the capital owner's stochastic discount factor and τ_{t+1}^c is given by equation (17) evaluated at time $t+1$.[13] In deciding how much to invest, the capital owner takes into account the slope of the personal tax schedule, as reflected by the term $(1-\kappa)$. The first-order condition takes the form of a standard asset pricing equation where $p_t = (1-\tau_t\phi_t)i_t/\lambda$ is the market value of the capital owner's equity shares in the firm. These equity shares entitle the capital owner to a perpetual stream of dividends d_{t+1} starting in period $t+1$. The model's adjustment cost specification (5) implies a direct link between equity values and investment. This feature is consistent with the observed low-frequency comovement between the real S&P 500 stock market index and real business investment in recent decades, as documented by Lansing (2012).

Capital owners must only decide the fraction of their after-tax income to be devoted to investment, with the remaining fraction devoted to consumption. As shown in Appendix

[13] After taking the derivitive of the capital owner's Lagrangian with respect to k_{t+1}, we have multiplied both sides of the resulting expression by the ratio k_{t+1}/c_t^c which is known at time t.

A, the capital owner's optimization problem can be formulated in terms of a single decision variable, namely, the tax-adjusted investment-consumption ratio given by $x_t \equiv (1-\tau_t\phi_t)i_t/c_t^c$, Our choice of functional forms (log utility and Cobb-Douglas forms for production and the capital law of motion) delivers a simple approximate decision rule for x_t in terms of the state variable s_t, where s_t evolves according to the law of motion (9). In Appendix A we show that x_t is increasing in θ_t. Hence, an increase in capital's share of income causes the capital owner to devote more resources to investment rather than consumption.

Given the decision rule $x_t = x(s_t)$, the equilibrium version of the capital owner's budget constraint (4) can be used to derive the following expressions for the capital owner's allocations:

$$\frac{c_t^c}{y_t} = \frac{1}{1+x(s_t)}(1-\tau_t^c)\left[\theta_t + \alpha_t(1-\theta_t)\right], \tag{20}$$

$$\frac{i_t}{y_t} = \frac{x(s_t)}{1+x(s_t)}\left(\frac{1-\tau_t^c}{1-\tau_t\phi_t}\right)\left[\theta_t + \alpha_t(1-\theta_t)\right]. \tag{21}$$

A convenient property of our setup is that we do not need to specify the laws of motion for the tax wedges in order to solve for the capital owner's allocations. This is because the income and substitution effects of changes in either τ_t or ϕ_t are offsetting. In equilibrium, τ_t^c depends only on τ_t and the income share variables θ_t and α_t. Given the observed paths for the income shares in the data, we solve for the time series of τ_t and ϕ_t that allow the model to exactly replicate the observed time paths of the four U.S. macroeconomic ratios plotted in Figure 2. Later, as a validity check, we compare the income-weighted average tax rate from the model simulation to a corresponding U.S. tax rate series constructed using estimated tax rates on labor and capital incomes from Gomme, Ravikumar, and Rupert (2011, updated).

We also solve for a time series of productivity shocks z_t that allow the model to exactly replicate the path of U.S. real per capita output over the period 1970 to 2014, where the level of real output in 1970 is normalized to 1.0. The time series for the state variable s_t is taken directly from U.S. data, as plotted in Figure 3. Afterwards, we use the laws of motion for the shocks (7) and (9) to recover the time series of innovations ε_t and u_t. For periods beyond 2014, we assume that all shock innovations are zero, while income shares, tax wedges, and the various macroeconomic ratios remain constant at year 2014 values. Details regarding the simulation procedure are contained in Appendix B.

3 Model Calibration

Table 1 summarizes the parameter values for the baseline simulation. Values are set to achieve targets based on observed U.S. variables within the sample period 1970 to 2014.

Table 1: Baseline Parameter Values

Parameter	Value	Description/Target
n	4	Capital owners = top income quintile.
θ_0	0.3500	Capital's share of income = 0.350 in 1970.
α_0	0.1277	Top quintile income share = 0.433 in 1970.
ℓ^c/ℓ^w	0.2928	Mean relative wage $w^c/w^w = 2$ in 1970.
μ	0.0201	Mean per capita consumption growth = 2.01%, 1970 to 2014.
β	0.9616	Mean log equity return = 5.92%, 1970 to 2014.
A	0.4367	$y_t = 1$ with $k_t/y_t = 1.51$ in 1970.
B	1.1269	Mean $i_t/k_t = 0.0803$, 1970 to 2014.
λ	0.0394	$\widetilde{x} \equiv \exp\{E[\log(x_t)]\} = 0.5326$, 1970 to 2014.
$\widetilde{s}$	0.7991	$\widetilde{s} \equiv \exp\{E[\log(s_t)]\} = 0.7991$, 1970 to 2014.
ρ	0.8607	Corr. $[\log(s_t),\ \log(s_{t-1})] = 0.8607$, 1970 to 2014.
σ_u	0.0250	Std. dev. $\log(s_t) = 0.0492$, 1970 to 2014.
σ_ε	0.0440	Std. dev. per capita output growth = 1.726%, 1970 to 2014.
κ	0.1254	Estimated tax schedule slope = 1.214, Cassou and Lansing (2004).
τ_0	0.3448	$g_t/y_t + T_t/y_t = 0.323$ in 1970.
ϕ_0	0.7562	$i_t/y_t = 0.121$ in 1970.

The time period in the model is one year. The number of workers per capital owner is $n = 4$ so that capital owners represent the top quintile of earners. In the model, capital owners possess 100% of the physical capital wealth—a reasonable approximation to the U.S. financial wealth distribution in which the ownership share of the top quintile of earners is around 92%.[14]

The initial capital income share θ_0 is set to match the 1970 observed value of 0.35, as shown in Figure 1. The initial production elasticity of the capital owner's labor supply α_0 is set to achieve an initial top quintile income share of $\theta_0 + (1-\theta_0)\,\alpha_0 = 0.433$, corresponding to 1970 observed value as shown in Figure 1. Given these values, the labor supply ratio ℓ^c/ℓ^w is set so that the initial wage ratio in 1970 is $w^c/w^w = 2$ with ℓ^w normalized to 1. For comparison, Heathcote, Storesletten, and Violante (2010), p. 686 report a male college wage premium of about 1.5 in 1970, whereas Gottschalk and Danziger (2005), p. 238 report a male wage ratio of 4.1 in 1979 when comparing the top decile to the bottom decile. The wage ratio w^c/w^w in our model compares the top quintile to the remainder of households, so one would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, Storesletten, and Violante (2010). The quantitative results exhibit little sensitivity to the value of the initial wage ratio.

The value $\mu = 0.0201$ matches the average growth rate of real per capita aggregate consumption over the period 1970 to 2014, where the consumption series is constructed as described in footnote 9. Given μ, we choose β to achieve a mean log equity return of 5.92%,

[14] See Wolff (2010), Table 2, p. 44.

coinciding with the corresponding real return delivered by the S&P 500 stock index over the period 1970 to 2014.[15]

The level of real per capita output in the U.S. data is normalized to 1.0 in the year 1970. We calibrate the value of A in the production function (6) to yield $y_t = 1$ at $t_0 = 1970$. The calibration assumes $k_t/y_t = 1.51$ in 1970 which is obtained by combining the observed U.S. value of $i_t/y_t = 0.121$ in 1970 with a reasonable value for the investment-capital ratio i_t/k_t. For example, in a model with no capital adjustment costs, we have $i_t/k_t = k_{t+1}/k_t - 1 + \delta$, where δ is the annual depreciation rate of physical capital. For the calibration, we employ the value $\delta = 0.06$ which in turn yields the target value $i_t/k_t = \exp(0.0201) - 1 + 0.06 = 0.0803$. Given the calibrated value for λ (described below), we set the parameter B in the capital law of motion (5) such that $B = (k_{t+1}/k_t)\,(i_t/k_t)^{-\lambda}$, where $k_{t+1}/k_t = \exp(0.0201)$ and $i_t/k_t = 0.0803$.

The parameter λ governs the strength of capital adjustment costs and depreciation. The value of λ influences the coefficients in the capital owner's decision rule $x_t = x\,(s_t)$, where $x_t \equiv (1 - \phi_t \tau_t) i_t / c_t^c$. We choose the value of λ to a achieve the target value $\widetilde{x} \equiv \exp\{E\,[\log\,(x_t)]\} = 0.5326$. This target value is computed using the 1970 to 2014 average values for the U.S. income shares and the U.S. macroeconomic ratios plotted in Figure 2. Details of the calibration procedure for λ are contained in Appendix A.

Recall that s_t represents the ratio of capital's share of income to the top quintile income share (Figure 3). We choose the parameters $\widetilde{s}$, ρ, and σ_u in the law of motion (9) to match the mean, persistence, and volatility of $\log\,(s_t)$ in U.S. data from 1970 to 2014.

After computing the time series of productivity shocks z_t that cause the model to exactly replicate the path of U.S. real per capita output, we use the law of motion (7) to recover the implied sequence of innovations ε_t, with $z_t = 0$ at $t_0 = 1970$. The standard deviation of the implied shock innovations turns out to be $\sigma_\varepsilon = 0.0440$. The corresponding standard deviation of output growth in both the model and the data is 1.73% from 1970 to 2014.

The slope parameter for the progressive tax schedule is set to $\kappa = 0.125$ so that the hypothetical average-income agent in the model with $y_t^i/\overline{y}_t = 1$ faces a tax schedule slope of $MTR_t^i/\tau_t^i = 1.124$ when the top quintile income share and the macroeconomic ratios i_t/y_t, g_t/y_t, and T_t/y_t take on their average values from 1970 to 2014. The target slope corresponds to the value estimated by Cassou and Lansing (2004) using the 1994 U.S. tax schedule for married taxpayers with no children, filing IRS form 1040 jointly.[16] Given the many significant changes to the U.S. tax code that have taken place since 1970, we examine the sensitivity of

[15] The mean log equity return in the model is given by $E\,[\log\,(R_{t+1}^s)] = -\log\,(\beta) + \mu$. Data on real log equity returns for the U.S. are from Dimson, Marsh, and Staunton (2002), updated through 2014.

[16] The tax schedule, taken from Mulligan (1997), Table 5-2, displays twelve different tax brackets that derive from the combined effects of the federal individual income tax, the earned income tax credit, and employee and employer contributions to Social Security and Medicare.

our results to different values for κ.[17] Table 2 shows the personal income tax rates faced by each type of agent for the baseline calibration with $\kappa = 0.125$ and an alternative calibration with $\kappa = 0.224$. The alternative calibration implies a more progressive tax schedule such that the average-income agent now faces a steeper slope of $MTR_t^i/\tau_t^i = 1.4$. The income ratios $y_t^i/\overline{y}_t$ that determine the personal income tax rates from equations (14) and (15) are based on the average top quintile income share from 1970 to 2014. For both calibrations, the tax rates faced by capital owners are higher than those for workers.

Table 2: Model Tax Rates Implied by 1970 to 2014 Average Values

	Average-income Agent $y_t^i/\overline{y}_t = 1$		Capital Owner $y_t^i/\overline{y}_t = 2.368$		Worker $y_t^i/\overline{y}_t = 0.658$	
	$\kappa = 0.125$	$\kappa = 0.224$	$\kappa = 0.125$	$\kappa = 0.224$	$\kappa = 0.125$	$\kappa = 0.224$
τ_t^i	36.9	35.9	43.4	47.2	33.5	29.6
MTR_t^i	44.8	50.2	50.5	59.0	41.8	45.4
MTR_t^i/τ_t^i	1.214	1.400	1.164	1.251	1.249	1.533

Notes: All tax rates in percent. τ_t^i = personal income tax rate. MTR_t^i = marginal personal income tax rate. Values for the tax rates and $y_t^i/\overline{y}_t$ are based on the 1970 to 2014 average values for the U.S. top quintile income share and the U.S. macroeconomic ratios i_t/y_t, g_t/y_t, and T_t/y_t.

Given values for κ, λ, $\widetilde{x}$, and $\widetilde{s}$, and the capital owner's decision rules for x_t and i_t, we solve for ϕ_0 and τ_0 such that the model delivers the observed U.S. values $i_t/y_t = 0.121$ and $g_t/y_t + T_t/y_t = 0.323$ at $t_0 = 1970$. A similar procedure is used to solve for ϕ_t and τ_t for each $t > t_0$, as described in Appendix B.

4 Intuition for the Results

Before moving to the quantitative analysis, this section examines the basic mechanism that determines how a permanently shifting income share impacts capital owners versus workers. Let us consider a stripped-down version of the model with no labor supply for capital owners ($\alpha_t = 0$), unit labor supply for workers $(\ell^w = 1)$, no growth $(z_t = 0)$, equal number of capital owners and workers $(n = 1)$, no taxes $(\tau_t = 0,\ \kappa = 0)$, and no capital adjustment costs such that $k_{t+1} = (1 - \delta)k_t + i_t$, where δ is the capital depreciation rate. With these simplifying assumptions, output is given by $y_t = Ak_t^{\theta_t}$. The incomes of the capital owners and workers are $\theta_t y_t$ and $(1 - \theta_t)\, y_t$, respectively.

In response to a one-time increase in θ_t, the capital stock cannot respond immediately so the short-run response of output is muted relative to the long-run response. In the short-run,

[17] Significant tax code changes were enacted by the Economic Recovery Tax Act of 1981 (ERTA81) and the Tax Reform Act of 1986 (TRA86). ERTA81 imposed a 23 percent across-the-board cut in all marginal tax rates and reduced the top marginal rate for individual income from 70 to 50 percent. TRA86 further lowered marginal rates for individuals and corporations, dramatically reduced the number of tax brackets, and eliminated or reduced many tax breaks. For additional details, see Guo and Lansing (1997).

the income of capital owners will rise while the income of workers will fall. These short-run effects will have a large influence on welfare because they are not discounted much in calculating lifetime utility.

But the increase in θ_t will also stimulate an increase in i_t, thus raising k_{t+1} and y_{t+1}.[18] As time goes by, the income (and consumption) of workers will be boosted by the rising level of private-sector output. In the long-run steady state, private-sector output is given by

$$y = A^{\frac{1}{1-\theta}} \left[\frac{\beta\theta}{1-\beta\left(1-\delta\right)}\right]^{\frac{\theta}{1-\theta}}, \tag{22}$$

which shows that an increase in θ leads to an increase in y. It is straightforward to show that for reasonable parameterizations, an increase in θ also leads to an increase in $(1-\theta)\, y$, which determines the steady state level of workers' consumption. In other words, an increase in capital's share of income can also boost the long-run level of workers' consumption. But since this event takes place in the very long-run, the resulting impact on workers' welfare is small due to discounting.

While an increase in θ unambiguously benefits the welfare of capital owners, the welfare impact for workers will depend on how fast capital and output converge to the new steady state. Short-term negative impacts must be balanced against long-term gains.[19] In the quantitative analysis that follows, we show that the welfare impact also depends on the time path followed by θ_t during the transition and the time path followed by total government outlays (including redistributive transfers) which must be financed with distortionary taxes.

5 Quantitative Analysis

We first consider a baseline simulation that exactly replicates the observed U.S. time paths of the top quintile income share, capital's share of income, and key macroeconomic variables from 1970 to 2014. The baseline simulation is compared to a counterfactual scenario in which income shares and the transfer-output ratio T_t/y_t are held constant at year 1970 values, while the ratio of total government outlays to output $g_t/y_t + T_t/y_t$ is identical to that in the baseline simulation. Details of the computation procedure are contained in Appendix A.

As a validity check, we compare model-predicted paths for a number of variables to the corresponding variables in U.S. data. Finally, we undertake two normative experiments that construct alternative paths for redistributive transfers and income tax rates relative to those in the baseline simulation.

[18] The closed-form investment decision rule for capital owners is: $i_t = \beta\theta_t y_t - (1-\beta)(1-\delta)\, k_t$.

[19] We thank an anonymous referee for suggesting this simple intuition.

5.1 Baseline Simulation vs. Counterfactual Scenario

Figure 4 plots the simulated trajectories of four model variables: aggregate consumption $c_t^c + nc_t^w$, aggregate investment i_t, the capital owner's consumption c_t^c, and the worker's consumption c_t^w. For each variable, we compare the baseline simulation to the counterfactual scenario described above. By holding T_t/y_t constant in the counterfactual scenario (with $g_t/y_t + T_t/y_t$ identical to the baseline), we adopt the view that the upward trend of T_t/y_t observed in the data was a deliberate government policy response to the trend of rising pre-tax income inequality. In other words, the upward trend in T_t/y_t would not have been needed if pre-tax income inequality had remained low. The resources thus saved could have been used to increase g_t/y_t. Consistent with this view, a recent study by Ostry, Berg, and Tsangarides (2014) finds that countries with higher pre-tax income inequality tend to undertake more redistribution than countries with lower pre-tax income inequality. Later, in the welfare analysis, we will consider an alternative counterfactual in which we hold T_t/y_t constant, but then allow for shifting income shares as in the data.[20]

The top panels of Figure 4 show that aggregate consumption and investment in the baseline simulation can fluctuate below the counterfactual path during portions of the transition period from 1970 to 2014. The sum of these two variables represents private-sector output. The increase in capital's share of income θ_t in the baseline simulation shrinks the output contribution coming from the model's growth engine, namely, labor-augmenting technological progress as given by $\exp\left[(1-\theta_t)\, z_t\right]$. This effect can produce a temporary slowdown in the growth rate of private-sector output. Along these lines, Hornstein and Krusell (1996) and Greenwood and Yörükoğlu (1997) develop models in which a biased technology change initially leads to a measured slowdown in total factor productivity.

It takes a long time for the model transition dynamics to fully play out. The increase in the marginal product of capital, as measured by θ_t, stimulates an increase in investment i_t relative to the counterfactual scenario (top right panel of Figure 4). Once θ_t stops increasing and all of the transition dynamics have died out, there is a permanent upward level shift of 13.3% in investment relative to the counterfactual. The higher investment level leads to a permanent upward level shift of 6.2% in private-sector output relative to the counterfactual. These permanent shifts derive from the unit root in the law of motion for z_t.

The lower two panels in Figure 4 show the paths for the capital owner's consumption c_t^c and

[20] Figure 2 shows that T_t/y in the data rose from 7.5% in 1970 to 12% in 2005. It remained approximately constant at around 12% through 2007. Then, over the next three years, the ratio increased rapidly, peaking at 15.6% in 2010. The ratio has since come down a bit to 14.8% in in 2014. While some of the run-up in T_t/y_t in recent years appears to have been triggered by the government's response to the financial crisis of 2007-09, it is also true that the top quintile income share continued to trend upward over this same period. Moreover, the value of T_t/y_t in 2014 is only slightly below the peak value achieved in 2010, suggesting that much of the recent run-up may be permanent rather than temporary.

the worker's consumption c_t^w. Relative to the counterfactual scenario, consumption growth for capital owners exhibits a higher mean (2.2% versus 2.0%) and a lower volatility (2.8% versus 3.5%) from 1970 to 2014. Beyond 2014, the capital owner's consumption pulls further away from the counterfactual path. In the long-run (i.e., at the end of a 3000 period simulation), the capital owner's consumption experiences a permanent upward level shift of 12.1% relative to the counterfactual.

The worker's consumption (lower right panel of Figure 4) falls below the counterfactual path during a substantial portion of the transition period from 1970 to 2014. But after 45 years, the level of the worker's consumption is only slightly lower than in the counterfactual. This result is due mainly to the rising transfer-output ratio in the baseline simulation. The volatility of the worker's consumption growth is substantially lower in the baseline simulation (1.8% versus 3.4%). The lower volatility stems from the countercyclical behavior of government transfers. In the baseline simulation (and in the U.S. data), the correlation coefficient between the growth rate of real transfer payments and the growth rate of real output is -0.46 from 1970 to 2014. The consumption-smoothing effect of these transfers is taken into account by our welfare analysis, as described further below. Beyond 2014, the worker's consumption starts to surpass the counterfactual path around the year 2021. This effect is driven by the higher long-run level of investment in the baseline simulation which contributes to more capital accumulation and more private-sector output per worker. At the end of the 3000 period simulation, the worker's consumption experiences a permanent upward level shift of 2.4% relative to the counterfactual.

Figure 5 plots the time series of the two tax wedge innovations ($\Delta\tau_t$ and $\Delta\phi_t$) and the two stochastic shock innovations (ε_t and u_t) that are needed to make the baseline simulation exactly replicate the paths of U.S. macroeconomic variables from 1970 to 2014. By construction, the innovations are zero at $t_0 = 1970$ and for $t > 2014$. The mean values of $\Delta\tau_t$, $\Delta\phi_t$, ε_t and u_t are all close to zero over the period 1970 to 2014. In the lower left panel of Figure 5, the identified productivity shock innovation ε_t is negative during the U.S. recession years of 1974-75, 1980-82, 1990-91, and 2008-09.[21]

Figure 6 plots the ratios of macroeconomic variables to output generated by the model. In the top two panels, the baseline simulation exactly replicates the 1970 to 2014 observed U.S. time paths for the ratios c_t/y_t and i_t/y_t, as plotted earlier in Figure 2.[22] We use the model decision rules to construct paths for c_t^c/y_t, and c_t^w/y_t which, when aggregated, are consistent with the evolution of the ratio c_t/y_t in the U.S. data.

In the baseline simulation, the capital owner's consumption increases faster than output

[21] The U.S. recession years are from www.nber.org/cycles.html.

[22] Although not shown, the baseline simulation also replicates the observed U.S. time paths for the ratios g_t/y_t and T_t/y_t.

such that c_t^c/y_t goes from 16.8% in 1970 to 19.6% in 2014 (bottom left panel of Figure 6). In contrast, c_t^w/y_t increases only slightly from 11.6% in 1970 to 12.3% in 2014 (bottom right panel of Figure 6). The small increase in c_t^w/y_t is due to the rising transfer-output ratio in the baseline simulation which offsets the workers' shrinking income share. In the absence of a rising transfer-output ratio, the shifting income shares would cause the worker's consumption ratio to drop to 10.5% by 2014. In the counterfactual scenario, the consumption-output ratios for both types of agents can fluctuate in response to changes in tax wedges and productivity shocks, but the ratios experience very little net change after 45 years.

5.2 Model vs. Data: Income and Consumption Inequality

Figure 7 shows that the rise in consumption inequality in the model is far less-pronounced than the rise in pre-tax income inequality. The model's top quintile income share before taxes and transfers (solid blue line) rises by 8 percentage points, from 43% to 51%, exactly replicating the census data plotted in Figure 1. In contrast, the after tax and transfer income share (dashed red line) and the consumption share (dashed-dot green line) both rise by only about 2 percentage points. The smaller rise in consumption inequality in the model is due to two factors: (1) the progressive nature of the tax schedule which extracts proportionally more tax revenue from capital owners as their income share rises, and (2) the rising transfer-output ratio which helps to mitigate the workers' shrinking income share.

The sustained increase in U.S. pre-tax income inequality has prompted suggestions for increasing the marginal tax rate on top incomes.[23] Our model allows us to assess the degree to which a more progressive tax schedule from 1970 to 2014 could have mitigated the rise of inequality, as measured after taxes and transfers. In Table 3, we show results for three different values of the tax schedule slope parameter κ. In each case, we follow the computation methodology of the baseline simulation. Higher values of κ serve to increase the capital owners' marginal tax rate MTR_t^c. Since the simulated paths for g_t/y_t and T_t/y_t are the same in each case, a higher marginal tax rate on capital owners serves to lessen the proportional tax burden on workers. Consequently, a more progressive tax schedule helps to reduce the top quintile share of after tax income and consumption by the end of the simulation in 2014. In section 5.5, we show that imposing a more progressive tax schedule from 1970 to 2014 (with $\kappa = 0.224$) would nearly eliminate the welfare losses for workers while preserving a substantial welfare gain for capital owners.

[23] See, for example, Piketty (2014), Chapter 14, Piketty, Saez, and Stantcheva (2014), and Kindermann and Krueger (2014).

Table 3: Effects of Tax Schedule Progressivity on Income and Consumption Shares

Tax Schedule Slope Parameter	Capital Owner MTR_t^c in 2014	Model Top Quintile Share in 2014: Income Before Taxes & Transfers	Income After Taxes & Transfers	Consumption
$\kappa = 0$	35.6	51.2	43.4	32.7
$\kappa = 0.125$	48.5	51.2	39.8	28.4
$\kappa = 0.224$	57.7	51.2	37.0	25.1

Note: Tax rates and shares in percent.

For comparison with U.S. data, Figure 7 plots the consumption share of high-income households (those in the 80th through 95th percentiles) using data from the Consumer Expenditure Survey (CES) for the period 1980 and 2010. The consumption of high-income households is computed using two methods: (1) reported after-tax income minus saving, and (2) reported expenditures. The consumption share from the first method is noticeably higher than that from the second method. This gap is similarly evident in the data reported by Aguiar and Bils (2011), Table 1, p. 30. A later version of the same paper (Aguiar and Bils, 2015) highlights the growing discrepancy between the CES expenditure data and the aggregate consumption data from the National Income and Product Accounts (NIPA). This discrepancy affects the comparison in Figure 7 because our model exactly replicates the path of the NIPA aggregate consumption data from 1970 to 2014.[24]

Notwithstanding the data issues noted above, the model's prediction for the capital owners' consumption share tracks reasonably well with the consumption share of high-income households computed from the CES data (grey lines). From 1980 to 2010, the net increase in the CES consumption share is 3.1 percentage points using the income minus saving data and 1.9 percentage points using the reported expenditure data. For the same 1980 to 2010 time period, the model predicts an increase of about 1.4 percentage points in the capital owners' consumption share. The results in Table 3 show that a less-progressive tax schedule (lower value for κ) would allow the model to deliver a higher net increase in the capital owners' consumption share during the simulation.

There is disagreement in the literature regarding the extent to which U.S. consumption inequality has increased. Studies by Krueger and Perri (2006) and Meyer and Sullivan (2013) find that consumption inequality has risen by much less than income inequality. Both studies measure consumption inequality using reported expenditures from the CES. However, Aguiar and Bils (2015) argue that the reported expenditure data for high-income households is subject to under-measurement error which has been growing over time. After designing a correction

[24] The CES data and associated stata codes are the same as those used by Aguiar and Bils (2015) and are available from Mark Aguiar's website. The data excludes the top and bottom five percent of households sorted by before tax income. For comparison with the model, we treat households in the 80th through 95th percentiles as the top quintile and households in the 5th through 80th percentiles as the remainder.

for the measurement error, they conclude that the rise in consumption inequality is close to the rise in income inequality.

5.3 Model vs. Data: Capital Stock and Real Equity Value

The top left panel of Figure 8 compares the model-predicted path for the stock of physical capital k_t to the BEA's chain-type quantity index for the net stock of private nonresidential fixed assets, where each series is indexed to 1 in 1970.[25] The top right panel shows that the yearly growth rates of the two series move together, exhibiting a correlation coefficient of 0.62, which is statistically significant. Recall that by construction, the baseline simulation for model investment i_t matches the BEA series for private nonresidential fixed investment (footnote 9). The BEA fixed asset data is constructed by cumulating investment flows and then adjusting for depreciation and relative price changes. By matching the time path of investment flows in the data, the model's capital accumulation equation (5) delivers a reasonable approximation for the time path of fixed assets in the data.

A recent empirical study by Greenwald, Lettau, and Ludvigson (2014) finds that highly persistent "factor share shocks" which redistribute income between stockholders and non-stockholders are an important driver of U.S. stock prices over the period 1952 to 2012. Along these lines, Lansing (2015) develops a concentrated capital ownership model (similar to the one used here) in which persistent shocks to capital's share of income serve to substantially magnify the equity premium relative to a otherwise similar representative agent model.

While asset pricing is not our focus here, it is interesting to examine the model's prediction for the path of real equity values from 1970 to 2014. Recall from equation (19) that the market value of the capital owner's equity shares is $p_t = (1-\tau_t\phi_t)i_t/\lambda$. The bottom left panel of Figure 8 plots p_t from the baseline simulation versus the real per capita market capitalization of the firms in S&P 500 stock market index, where each series is indexed to 1 in 1970.[26]

The bottom right panel of Figure 8 shows that the S&P 500 market cap is far more volatile than p_t in the model. Moreover, at the end of the data sample in 2014, the S&P market cap is 34% higher than the endpoint predicted by the model. These differences are perhaps not surprising given that our fully-rational model excludes the possibility of "bubbles" or "excess volatility," both which are the subject of a large literature.[27]

The bottom right panel of Figure 8 plots *changes* in model equity values versus those in the

[25] The BEA fixed asset data are from NIPA Table 4.2, line 1.

[26] Data on the nominal S&P 500 market capitalization in $ billions are from Haver Analytics. We convert to real per capita values using the output price index described in foonote 9 and the U.S. population data from NIPA Table 2.1.

[27] Lansing and LeRoy (2014) provide a recent update on the excess volatility literature. The model fit could potentially be improved for both of the data series plotted in Figure 8 by allowing for stochastic variation in the parameters B and λ that appear in the capital law of motion (5).

data, where each series is scaled by its sample standard deviation. The correlation coefficient between the two series is 0.31 and statistically significant. These results lend support to the idea that there is important link between shifting U.S. income shares and movements in equity values.

5.4 Normative Transfer Experiments

Figure 9 plots the results of two normative experiments in which the time series of government transfers and tax rates depart from those in the baseline simulation. In the first experiment, we solve for the time series of transfers T_t^* that equates agents' marginal utility of consumption (MUC) each period such that $1/c_t^w = 1/c_t^c$ for $t > t_0$. Equating agents' marginal utility of consumption would be the goal of a social planner who seeks to maximize the weighted-sum of agents' lifetime utilities in an economy without distortions, where the weights correspond to the population share of each agent-type.

In the second experiment, we solve for a Pareto-improving time series of transfers T_t^p that achieves the less ambitious goal of $1/c_t^w = 1/\left(\psi c_t^c\right)$ where $0 < \psi < 1$. We set $\psi = 0.69025$ to achieve equal per-period welfare gains for capital owners and workers over a long simulation of the model. For each experiment, we solve for the time path of τ_t that is needed to satisfy the government budget constraint (13) each period, where other relevant variables take on the same values as those in the baseline simulation. Details of the computation procedure are contained in Appendix C.

The left panel of Figure 9 shows that T_t^*/y_t jumps from 7.5% in 1970 (the starting value in the data) to 22% in 1971. The ratio then trends upwards to 32% by the year 2014, after which it remains constant because income inequality in the model stops rising by assumption. Interestingly, the correlation coefficient between the growth rate of T_t^* and the growth rate of y_t in the model experiment is -0.44 from 1970 to 2014. This is nearly identical to the observed correlation of -0.46 in the data, suggesting that U.S. government transfers exhibit about the right amount of countercyclicality.[28]

The right panel of Figure 9 plots the income-weighted average tax rate that is needed to finance each of the normative transfer experiments. The income-weighted average tax rate in the model is given by

$$\overline{ATR}_t = \left(1 - q_t\right) \tau_t^w + q_t \left[\tau_t^c y_t^c - \tau_t \phi_t i_t\right] / y_t^c, \tag{23}$$

where $q_t \equiv \theta_t + \alpha_t \left(1 - \theta_t\right)$ is the top quintile income share and y_t^c is the capital owner's pre-tax income. In the case of MUC-equalizing transfers, the income-weighted average tax rate

[28] Our exploration of normative redistribution policies is necessarily brief here. For a more comprehensive treatment, see Piketty and Saez (2013b).

jumps from 32% in 1970 to 46.5% in 1971, and then trends upward to 50% by the year 2014. While fiscal policy shifts of this magnitude are not realistic, they illustrate the severity of the actions that would have been needed to achieve equality of marginal utility (and equality of consumption) given the historical pattern of rising U.S. income inequality.

The second normative experiment shows that much milder policy actions would have sufficed to achieve welfare gains for everyone, according to the model. In this case, there is no need for an immediate jump in either transfers or tax rates. The ratio T_t^p/y_t rises from 7.5% in 1970 to 18.4% in 2014. The ending value is not much higher than the actual value of 14.8% observed in the data. In the data, the average annual growth rate of transfer payments is 3.59% per year from 1970 to 2014. The Pareto-improving policy calls for T_t^p to grow at an average annual growth rate of 4.05% per year. There is a jump in T_t^p/y_t (and T_t^*/y_t) that occurs in the mid-1990s. This feature can be traced to the jump in the U.S. top quintile income share that occurred at the same time (Figure 1). The income-weighted average tax rate that is needed to finance the Pareto-improving transfers goes from 32% in 1970 to 37% in 2014. The ending value is near the low end of the range of average tax rates observed in OECD countries.[29]

Figure 9 also plots the time series for T_t/y_t and $\overline{ATR}_t$ from the baseline simulation. Recall that the baseline series for T_t/y_t exactly replicates the U.S. data (Figure 2). The baseline series for $\overline{ATR}_t$ ranges from a low of 29% to a high of 38%. These values are realistic in comparison to tax rates that have been estimated directly for the U.S. economy. Gomme, Ravikumar, and Rupert (2011, updated) construct average U.S. tax rates on labor and capital incomes for the period 1954 to 2013.[30] Starting with their estimates, we compute an income-weighted average tax rate by weighting their labor and capital income tax rates by $1 - \theta_t$ and θ_t, respectively, where θ_t is capital's share of income in the data, as plotted in Figure 1. Figure 9 shows that the $\overline{ATR}_t$ series from the baseline simulation is close to the income-weighted average tax rate series computed from the Gomme-Ravikumar-Rupert estimates.

5.5 Welfare Analysis

Table 4 summarizes the effects of rising income inequality for various model specifications. As detailed in Appendix D, welfare effects are calculated as the constant percentage amount by which each agent's consumption in the counterfactual scenario must be adjusted upward or downward each period to make lifetime utility equal to that in the baseline (or other) simulation. Table 4 also shows the long-run percentage shifts in consumption and investment for each type of agent, measured relative to the counterfactual scenario.

[29] According to Piketty and Saez (2013b), p. 141, the ratio of tax revenue to national income in OECD countries ranges from 35% to 50%.

[30] The updated tax rate series are available from Paul Gomme's website.

Table 4: Effects of Rising U.S. Income Inequality

Model Specification	Welfare Change Capital Owners	Welfare Change Workers	Long-run Consumption Shift Capital Owners	Long-run Consumption Shift Workers	Long-run Investment Shift Capital Owners
Baseline simulation	3.32	−0.52	12.12	2.35	13.34
No productivity shocks	3.26	−0.58	12.12	2.35	13.64
Flat tax schedule, $\kappa = 0$	3.70	−1.18	14.00	1.74	15.64
Steeper tax schedule, $\kappa = 0.224$	2.86	−0.06	10.25	2.74	11.42
Constant $\theta_t = \theta_0$	−0.57	3.66	−0.92	10.46	−1.83
Constant $T_t/y_t = T_0/y_0$	3.32	−8.22	12.12	−12.83	13.34
MUC-equalizing $T_t/y_t = T_t^*/y_t$	−24.22	8.23	−29.52	2.21	−19.13
Pareto-improving $T_t/y_t = T_t^p/y_t$	0.43	0.43	2.83	2.93	6.68
Less patience, $\beta = 0.9516$	2.78	−0.06	12.12	2.44	13.34
Linear transition path for $\theta_t,\ \alpha_t$	4.38	0.47	12.12	2.35	13.34
Start date $t_0 = 1975$	−1.15	−10.17	5.68	−9.32	4.68
Start date $t_0 = 1980$	2.82	−5.67	10.77	−1.67	12.99

Notes: Welfare effects are measured by the percentage change in per-period consumption to make each agent indifferent between the baseline (or other) simulation and the counterfactual scenario which holds income shares and the transfer-output ratio constant at year 1970 values, while maintaining the baseline path for $g_t/y_t + T_t/y_t$. The long-run consumption and investment shifts are the percent changes relative to the counterfactual scenario, computed at the end of a long simulation.

For the baseline simulation, capital owners achieve a welfare gain of 3.3% of their per-period consumption while workers suffer a welfare loss of 0.5% of their per period consumption. The welfare effects are determined by changes in the time pattern of consumption for each type of agent in both the short-run and the long-run. The changes in consumption patterns can be seen in the bottom two panels of Figure 4. Changes that take place in the short-run, i.e., closer to $t_0 = 1970$, have more influence on welfare due to light discounting.

For capital owners, welfare gains derive mainly from the post-2005 upward shift in their consumption path relative to the counterfactual. This pattern can be traced to movements in capital's share of income θ_t. From Figure 1, we see that capital's share of income in the data experienced a dramatic increase starting around the year 2005. In the long-run, the capital owners' consumption shifts up by 12.1% relative to the counterfactual path. Given the permanently higher marginal product of capital, investment expenditures shift up by 13.3% in the long-run.

The time pattern of the workers' consumption is more complicated. From 1971 to 1985, the baseline path is above the counterfactual. This 15-year period has a strong positive influence on the worker's welfare because of light discounting. During this time, the transfer-output ratio is rising faster than the top quintile income share, thus boosting the worker's consumption relative to the counterfactual. From 1985 to 2014, the upward trend in capital's share of income θ_t shrinks the worker's income share and the output contribution coming from labor-augmenting technological progress. This effect pushes down the worker's consumption relative to the counterfactual. Beyond 2014, the higher level of investment in the baseline economy (due to a higher θ_t) contributes to more capital accumulation and more private-sector output per worker, allowing the worker's consumption to eventually surpass the counterfactual, achieving a permanent upward level shift of 2.4%. But these long-run consumption gains are heavily discounted.

The second row of Table 4 shows that shutting off the productivity shock innovations has only small effects on welfare relative to the baseline simulation. The model delivers the standard result that business cycles are not very important for welfare (Lucas, 1987). Interestingly, the historical pattern of productivity shock innovations that we identify in Figure 5 for the period 1971 to 2014 serves to slightly improve the welfare outcomes for both types of agents relative to the simulation with no productivity shocks.

Changes in the progressivity of the tax schedule have the expected effects. A flat tax schedule with $\kappa = 0$ makes capital owners better off but leaves workers worse off relative to baseline simulation with $\kappa = 0.125$. A steeper tax schedule with $\kappa = 0.224$ is successful in reducing the welfare loss for workers to only 0.06% while preserving a substantial welfare gain of 2.9% for capital owners. These results lend support to the idea that an increase in the marginal tax rate on top incomes could be an effective policy tool for addressing the rise in U.S. income inequality.

When the capital income share θ_t is held constant at the year 1970 value of 35%, capital owners experience a welfare loss 0.6% while workers enjoy a welfare gain of 3.7%. In this experiment, capital owners must now help pay for the rising time path of T_t/y_t (which directly benefits workers) without the help of a rising capital income stream.

The sixth row of Table 4 holds T_t/y_t constant at its year 1970 value of 7.5% (with $g_t/y_t + T_t/y_t$ identical to the baseline) while allowing the income shares to shift as in the data. In this case, workers suffer a much larger welfare loss of 8.2% versus 0.5% in the baseline simulation. The welfare gain for capital owners is unaffected at 3.3% because the time paths for $g_t/y_t + T_t/y_t$, τ_t^c, τ_t, ϕ_t, and z_t in this experiment are identical to those in the baseline simulation. This result suggests that the historical pattern of U.S. transfer payments has helped to mitigate the negative impacts of rising income inequality on households who fall outside the top quintile of the income distribution.[31]

The transfer policy that achieves MUC equality from 1971 onwards produces a substantial welfare gain of 8.2% for workers. But for capital owners, the higher tax rates needed to finance the higher level of transfers produces a enormous welfare loss of 24.2%. Moreover, the

[31] But as a caveat, it should be noted that our model implies that there are no negative welfare consequences for capital owners when resources are shifted away from government consumption g_t for the purpose of increasing transfers T_t while holding $g_t/y_t + T_t/y_t$ constant.

economy with MUC-equalizing transfers suffers a permanent downward level shift of 19.1% in investment relative to the counterfactual. Private-sector output shifts down by 8.1% in the long-run.

The Pareto-improving transfer policy achieves a modest welfare gain of 0.43% for both capital owners and workers. Still, this outcome is a significant improvement for workers relative to the 0.52% welfare loss suffered in the baseline simulation. Moreover, the economy experiences a permanent upward level shift of 6.7% in investment relative to the counterfactual. Private-sector output shifts up by 3.5% in the long-run. The Pareto-improving experiment suggests that realistic policy movements in the direction of more redistribution could be successful in combating the negative effects of rising income inequality without sacrificing long-run economic performance.

The welfare gain for capital owners' shrinks to 2.8% if agents are less patient such that $\beta = 0.9516$. Recall that baseline simulation has $\beta = 0.9616$ to match the mean log equity return for the S&P 500 stock index. A lower value of β reduces the lifetime utility benefit of the capital owners' long-run upward consumption shift. When workers are less patient, the favorable 1971 to 1985 period for their consumption relative to the counterfactual takes on added-importance for welfare, thus generating a smaller loss of only 0.06%.

The above discussion highlights the importance of accurately modeling the historical paths of the U.S. income shares because these affect the time pattern of agents' consumption and hence welfare. For example, implementing a linear transition path for the income shares over the period 1970 to 2014 (while preserving the endpoints) improves the welfare outcomes for both types of agents relative to the baseline simulation. Capital owners now achieve a larger gain of 4.4% versus 3.3% in the baseline simulation. Workers now enjoy welfare gain of 0.47% versus a welfare loss of 0.52% in the baseline simulation. For capital owners, a linear transition causes θ_t to be higher than the baseline path during the early years of the simulation. For workers, a linear transition causes their income share $(1 - \alpha_t)(1 - \theta_t)$ to be higher than the baseline path from 1991 to 2006—an unfavorable period when their baseline consumption path falls below the counterfactual.

To further examine the sensitivity of the welfare results to the time paths of the variables, we repeat the baseline simulation using different starting dates, specifically $t_0 = 1975$ and $t_0 = 1980$. When $t_0 = 1975$, capital's share of income θ_t and the transfer-output ratio T_t/y_t both undergo net declines during the first five years of the simulation, replicating the patterns observed in the U.S. data (Figures 1 and 2). Since the first five years of the simulation are lightly discounted, the initial declines in θ_t and T_t/y_t have large negative welfare consequences for both types of agents. While previously enjoying a welfare gain of 3.3%, capital owners now suffer a welfare loss of 1.2% when $t_0 = 1975$. The welfare loss for workers now increases considerably to 10.2%. However, if we start the baseline simulation at $t_0 = 1980$, the capital owners' welfare gain is restored, but to the slightly smaller value of 2.8%. The welfare loss for workers remains sizable at 5.7%. Again, these results are driven by the different time paths for θ_t and T_t/y_t during the early years of the simulation.

One way of addressing the sensitivity of the welfare results to the starting date of the simulation is to consider different evaluation dates for the welfare calculation. The evaluation date is year in which the agent is presumed to be indifferent between the consumption path in baseline simulation and the consumption path in the counterfactual scenario. For a given pair of consumption paths, the resulting utility streams will differ depending on the date when the agent is asked to make a welfare comparison between the two paths. An evaluation date that occurs later in the sample will diminish the influence of the starting date in the welfare calculation. The baseline simulation in Table 4 use 1970 as the evaluation date, with zero weight placed on pre-1970 consumption. In Table 5, we show the results for two alternate evaluation dates: 1992 and 2014. For these dates, we assume that agents discount the utility of past consumption going back to 1970 using the same discount factor β.[32] Details of the calculation are contained in Appendix D.

Table 5 shows that, regardless of the evaluation date, the baseline simulation consistently delivers large welfare gains for capital owners and nontrivial welfare losses for workers. These results lend support to the view that the pattern of rising U.S. income inequality has been detrimental to a substantial fraction of the population.

Table 5: Welfare Effects of Baseline Simulation at Different Evaluation Dates

Specification	Evaluation Date	Welfare Change Capital Owners	Welfare Change Workers
Start of sample	1970	3.32	−0.52
Middle of sample	1992	4.16	−2.49
End of sample	2014	7.03	−1.58

Notes: Welfare effects are measured by the percentage change in per-period consumption to make the agent indifferent between the baseline simulation and the counterfactual scenario. The evaluation date is the year in which the agent is indifferent. The baseline simulation in Table 4 uses 1970 as the evaluation date with zero weight on pre-1970 consumption. Here, we also consider evaluation dates of 1992 or 2014, with the utility of past consumption to 1970 discounted using the same discount factor β.

6 Conclusion

The increase in U.S. income inequality over the past 45 years can be traced to gains made by those near the top of the income distribution where financial wealth and corporate stock

[32] Caplin and Leahy (2004) show that discounting the utility of past consumption is a necessary condition to ensure retrospective time consistency of an agent's chosen consumption path.

ownership is highly concentrated. The economic and political implications of increasingly-skewed income distributions in the United States and other countries have risen to the forefront of current policy debates.

Our contribution is to try to assess the welfare consequences of rising U.S. income inequality using a standard growth model with two types of agents and concentrated-ownership of physical capital. The model is designed to exactly replicate the observed time paths of numerous U.S. macroeconomic variables from 1970 to 2014. The welfare consequences of rising income inequality depend crucially on changes in agents' consumption paths relative to a plausible counterfactual scenario. Our methodology ensures that agents' consumption paths are consistent with the evolution of U.S. macroeconomic variables over the same period. Our quantitative modeling approach has the additional advantage of providing us with full knowledge of the counterfactual—something which is not possible using purely empirical methods.

According to our analysis, the increase in income inequality since 1970 has delivered large welfare gains to the top income quintile of U.S. households. For households outside this exclusive group, the welfare losses appear to have been sizeable, albeit mitigated by the doubling of the share of U.S. output devoted to redistributive transfers since 1970. Our simulations also suggest that a more-progressive U.S. tax schedule from 1970 to 2014 could have helped to reduce welfare losses for agents outside the top quintile.

Our analysis of a transfer policy that equalizes agents' marginal utility of consumption within the model suggests that U.S. transfer payments exhibit about the right amount of countercyclicality. In addition, we showed that a relatively modest increase in the historical growth rate of U.S. transfer payments (from 3.59% to 4.05%) could have achieved modest welfare gains for all households while continuing to deliver significant upward shifts in long-run consumption and investment relative to the counterfactual scenario. Overall, our results suggest that there is room for policy actions to address the negative consequences of rising U.S. income inequality.

A Appendix: Capital Owner Decision Rule and λ Calibration

By combining equations (4), (10), and (11), and then dividing both sides of the expression by c_t^c, we obtain the following transformed version of the capital owner's budget constraint:

$$1 + x_t = (1 - \tau_t^c) \left[\theta_t + \alpha_t (1 - \theta_t)\right] y_t / c_t^c, \tag{A.1}$$

where $x_t \equiv (1 - \tau_t \phi_t) i_t / c_t^c$. Solving the above equation for c_t^c / y_t yields equation (20) in the text. Equation (21) in the text follows directly from the definition of x_t.

The capital owner's first-order condition (19) can be re-written as follows

$$\begin{aligned} x_t &= E_t \, \beta \left[\frac{\lambda (1-\kappa) \left(1 - \tau_{t+1}^c\right) \theta_{t+1} y_{t+1}}{c_{t+1}^c} + (1-\lambda) \; x_{t+1} \right], \\ &= E_t \, \beta \left[\lambda (1-\kappa) \; s_{t+1} (1 + x_{t+1}) + (1-\lambda) \; x_{t+1}\right] \end{aligned} \tag{A.2}$$

where $s_{t+1} \equiv \theta_{t+1} / \left[\theta_{t+1} + \alpha_{t+1} (1 - \theta_{t+1})\right]$ and we have eliminated $\left(1 - \tau_{t+1}^c\right) y_{t+1} / c_{t+1}^c$ using equation (A.1). Notice that the rational expectation solution for x_t will depend on the state variable s_t but not on the tax wedges. The tax wedges are subsumed within the definition of x_t.

To solve for the approximate decision rule $x_t = x(s_t)$, we first log linearize the right-side of equation (A.2) to obtain

$$x_t = E_t \, \mathrm{a}_0 \left[\frac{x_{t+1}}{\widetilde{x}}\right]^{\mathrm{a}_1} \left[\frac{s_{t+1}}{\widetilde{s}}\right]^{\mathrm{a}_2}, \tag{A.3}$$

where a_0, a_1, and a_2 are Taylor-series coefficients. The expressions for the Taylor-series coefficients are

$$\mathrm{a}_0 = \beta \left[\lambda (1-\kappa) \; \widetilde{s} (1 + \widetilde{x}) + (1-\lambda) \; \widetilde{x}\right], \tag{A.4}$$

$$\mathrm{a}_1 = \frac{\left[\lambda (1-\kappa) \; \widetilde{s} + (1-\lambda)\right] \widetilde{x}}{\lambda (1-\kappa) \; \widetilde{s} (1 + \widetilde{x}) + (1-\lambda) \; \widetilde{x}}, \tag{A.5}$$

$$\mathrm{a}_2 = \frac{\lambda (1-\kappa) \; \widetilde{s} \; (1 + \widetilde{x})}{\lambda (1-\kappa) \; \widetilde{s} (1 + \widetilde{x}) + (1-\lambda) \; \widetilde{x}}, \tag{A.6}$$

where the approximation is taken around the ergodic mean such that $\widetilde{x} \equiv \exp\{E\left[\log(x_t)\right]\}$ and $\widetilde{s} \equiv \exp\{E\left[\log(s_t)\right]\}$.

We conjecture that the decision rule for x_t takes the form $x_t = \widetilde{x} \left[s_t / \widetilde{s}\right]^{\gamma}$. The conjectured solution is iterated ahead one period and then substituted into the right-side of equation (A.3) together with the law of motion for s_{t+1} from equation (9). After evaluating the conditional expectation and then collecting terms, we have

$$x_t = \underbrace{\mathrm{a}_0 \exp\left[\tfrac{1}{2} \left(\mathrm{a}_2 + \gamma \mathrm{a}_1\right)^2 \sigma_u^2\right]}_{=\widetilde{x}} \times \left[\frac{s_t}{\widetilde{s}}\right]^{\overbrace{\rho \left(\mathrm{a}_2 + \gamma \mathrm{a}_1\right)}^{=\gamma}} \tag{A.7}$$

which yields two equations in the two unknown solution coefficients $\widetilde{x}$ and γ.

Using the decision rule for x_t and the definition of s_t from equation (8), we have

$$\frac{\partial x_t}{\partial \theta_t} \;=\; \frac{\partial x_t}{\partial s_t}\frac{\partial s_t}{\partial \theta_t} \;=\; \frac{\gamma x_t}{\theta_t + \alpha_t\left(1-\theta_t\right)} \;>\; 0, \tag{A.8}$$

which shows that an increase in θ_t causes the capital owner to devote more resources to investment instead of consumption.

From equations (A.4) through (A.7), we see that the value of λ will influence the value of the Taylor-series coefficients and hence the value of $\widetilde{x}$. To calibrate the value of λ, we first use the investment decision rule (21) to eliminate the term $\tau_t \phi_t i_t$ from the government budget constraint (13). Next, we substitute in the equilibrium expressions for τ_t^w and τ_t^c from equations (16) and (17) and then solve the resulting expression for $1-\tau_t$ to obtain

$$1-\tau_t \;=\; \frac{1-\left(g_t/y_t + T_t/y_t + i_t/y_t\right)}{q_t^{1-\kappa}\left(n+1\right)^{-\kappa}\left(1+x_t\right)^{-1} + \left(1-q_t\right)^{1-\kappa}\left[\left(n+1\right)/n\right]^{-\kappa}}, \tag{A.9}$$

where $q_t \equiv \theta_t + \alpha_t\left(1-\theta_t\right)$ is the top quintile income share. Using equation (A.9), we construct an expression for the term $1-\tau_t\phi_t$ which appears in the investment decision rule (21). Substituting for $1-\tau_t^c$ and $1-\tau_t\phi_t$ in the investment decision rule and then solving for x_t yields a complicated expression for the target value of x_t in terms of the top quintile income share q_t, the macroeconomic ratios g_t/y_t, T_t/y_t, and i_t/y_t, and the investment tax wedge ϕ_t.

We choose a target value for ϕ_t that is based on a standard depreciation allowance with a depreciation rate of $\delta = 0.06$ and an investment-capital ratio of $i_t/k_t = 0.0803$. Specifically, we choose $\phi_t i_t = \delta k_t$ such that $\phi_t = \delta\left(k_t/i_t\right) = 0.06/\left(0.0803\right) = 0.747$. Given this target value for ϕ_t together with the 1970 to 2014 average top quintile income share of $q_t = 0.4737$ and the 1970 to 2014 average values for the U.S. macroeconomic ratios g_t/y_t, T_t/y_t, and i_t/y_t, we solve for the corresponding target value $x_t = 0.5326$. Using equation (A.7), we then solve for the value of $\lambda = 0.0394$ to achieve the ergodic mean target value $\widetilde{x} = 0.5326$. Also using equation (A.7), we obtain the decision rule coefficient $\gamma = 0.3602$ for the baseline calibration.

B Appendix: Numerical Simulation Procedure

B.1 Baseline Simulation

Given the agents' decision rules (18), (20), and (21), together with the government budget constraint (13) and the equilibrium personal income tax rates (16) and (17), we solve for the time series of tax wedges τ_t and ϕ_t so that the model exactly replicates the observed time paths of the four U.S. macroeconomic ratios plotted in Figure 2. The resulting changes in τ_t and ϕ_t feed through to bring about changes in the personal income tax rates τ_t^c and τ_t^w levied on capital owners and workers.

Given the observed U.S. time series for s_t from Figure 3, we use the decision rule (A.8) to compute $x_t = x\left(s_t\right)$ for each period from 1970 to 2014. The time series for τ_t is computed using equation (A.10), where q_t, g_t/y_t, T_t/y_t, and i_t/y_t are the observed U.S. values. Given

x_t, q_t, i_t/y_t, and τ_t, we use the investment decision rule (21) to compute the time series for the investment tax wedge ϕ_t.

The aggregate resource constraint for the model economy implies $c_t/y_t = 1 - g_t/y_t - i_t/y_t$. The computed time series for τ_t and ϕ_t ensure that we exactly replicate the observed U.S. time paths for g_t/y_t and i_t/y_t. Since we define y_t in the data as $c_t + i_t + g_t$ (footnote 9), our procedure ensures that we also replicate the observed U.S. time path for c_t/y_t, as plotted in Figure 2.

The final step is to compute a time series of productivity shocks z_t that cause the model to exactly replicate the path of U.S. real per capita output from 1970 to 2014. The level of real output in the data is normalized to 1.0 in the year 1970. We calibrate the value of A in the production function (6) to yield $y_t = 1$ at $t_0 = 1970$. The calibration assumes $k_t/y_t = 1.51$ in 1970 which is obtained by combining the observed U.S. value of $i_t/y_t = 0.1214$ in 1970 with the calibration target of $i_t/k_t = 0.0803$. Given the computed time series for τ_t and ϕ_t described above, we conjecture a time series for z_t from 1970 to 2014 with $z_0 = 0$. Using the agents' decision rules, we then simulate the model. After each simulation, we compute a new time series for z_t as follows

$$z_t = \frac{\log(y_t) - \log\left\{A\, k_t^{\theta_t} \left[(\ell^c)^{\alpha_t}\, (n\,\ell^w)^{1-\alpha_t}\right]^{1-\theta_t}\right\}}{1-\theta_t}, \tag{B.1}$$

where y_t is given by the normalized real output series from the U.S. data, θ_t and α_t are pinned down by the income share data, and k_t is the model capital stock series implied by the law of motion (5) with i_t determined by the capital owner's decision rule (21). We repeat this procedure until the computed time series for z_t does not change from one simulation to the next. In practice, convergence is achieved after about 12 simulations. For $t > 2014$, we assume that the shock innovations ε_t and u_t are zero each period while θ_t, α_t, τ_t, and ϕ_t are held constant at year 2014 values. As a result, the macroeconomic ratios c_t^c/y_t, c_t^w/y_t, i_t/y_t, g_t/y_t, and T_t/y_t all remain constant at year 2014 values.

B.2 Counterfactual Scenario

The counterfactual scenario holds the variables θ_t, α_t, and T_t/y_t constant at year 1970 values. The time series for g_t is constructed so that the ratio of total government outlays to output $g_t/y_t + T_t/y_t$ is identical to that in the baseline simulation. Specifically, the time series for g_t evolves according to $g_t = y_t \left(g_t^b/y_t^b + T_t^b/y_t^b\right) - T_0/y_0$, where g_t^b/y_t^b and T_t^b/y_t^b are the values from the baseline simulation and $T_0/y_0 = 0.075$ is the year 1970 value. We assume that the time series for the investment tax wedge ϕ_t is identical to that in the baseline simulation. We then solve for the time series of tax rates τ_t that satisfies equation (A.10) for each $t > t_0$, where i_t/y_t is now pinned down by the capital owner's investment decision rule (21). The resulting equation that determines τ_t each period is quadratic. We choose the solution that lies on the upward-sloping portion of the Laffer curve.

C Appendix: Normative Transfer Experiments

This appendix outlines our procedure for computing the MUC-equalizing and Pareto-improving transfers plotted in Figure 9. The MUC-equalizing level of transfers achieves the condition $1/c_t^w = 1/c_t^c$, or equivalently, $c_t^w = c_t^c$, for each $t > t_0$. The Pareto-improving level of transfers achieves the condition $1/c_t^w = 1/\left(\psi c_t^c\right)$, or equivalently $c_t^w = \psi c_t^c$, where $0 < \psi < 1$. Substituting the consumption decision rules (18) and (20) into the condition $c_t^w = \psi c_t^c$ and then solving for the required transfer-output ratio yields

$$T_t^p/y_t \;=\; \left(1-\tau_t\right)\left(n+1\right)^{-\kappa}\left\{\frac{n\psi q_t^{1-\kappa}}{1+x\left(s_t\right)} - \left(1-q_t\right)^{1-\kappa} n^{\kappa}\right\}, \tag{C.1}$$

where T_t^p is the Pareto-improving level of transfers and $q_t \equiv \theta_t + \alpha_t\left(1-\theta_t\right)$ is the top quintile income share. When $\psi = 1$, we recover the MUC-equalizing level of transfers T_t^*.

For the computation, we assume that the time series for θ_t, α_t, g_t/y_t and ϕ_t are identical to those in the baseline simulation. We then solve for the required time series of tax rates τ_t that satisfies equation (A.10) for each $t > t_0$, where T_t/y_t is now pinned down by equation (C.1) and i_t/y_t is pinned down by the capital owner's investment decision rule (21). The resulting equation that determines τ_t each period is quadratic. We choose the solution that lies on the upward-sloping portion of the Laffer curve. Through repeated simulations of the model, we guess and verify that the value $\psi = 0.69024746$ achieves the result $\Delta^w = \Delta^c = 0.0042586882$, where Δ^w and Δ^c are the per-period welfare effects described in Appendix D.

D Appendix: Welfare Calculation

The welfare effects in Table 4 are calculated as the constant percentage amount by which each agent's consumption in the counterfactual scenario must be adjusted upward or downward each period to make lifetime utility equal to that in the baseline transition simulation. Specifically, we find Δ^w and Δ^c that solve the following two equations

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left(c_t^w\right) \;=\; \sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left[\overline{c}_t^w\,\left(1+\Delta^w\right)\right], \tag{D.1}$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left(c_t^c\right) \;=\; \sum_{t=t_0}^{\infty}\beta^{t-t_0}\log\left[\overline{c}_t^c\,\left(1+\Delta^c\right)\right], \tag{D.2}$$

where c_t^w and c_t^c are the consumption outcomes in the baseline simulation and $\overline{c}_t^w$ and $\overline{c}_t^c$ are the consumption outcomes in the counterfactual scenario. The infinite sums in (D.1) and (D.2) are approximated by sums over a 3000 period simulation, after which the results are not changed. The initial conditions correspond to year $t_0 = 1970$ values for all variables.

The evaluation date for the welfare calculation is the year in which the agent is presumed to be indifferent between the two consumption paths being compared. In Table 4, the evaluation date is $t_0 = 1970$ (start of sample) with zero weight placed on pre-1970 consumption. In Table 5, we also consider evaluation dates of 1992 (middle of sample) or 2014 (end of sample) with past consumption going back to 1970 discounted using the discount factor β. For example, when 2014 is the evaluation date, the welfare effects for the worker are computed as the value of Δ^w that solves the following equation:

$$\begin{aligned}\sum_{t=1970}^{2013} \beta^{2014-t} \log\left(c_t^w\right) + \sum_{t=2014}^{\infty} \beta^{t-2014} \log\left(c_t^w\right) \quad = \quad & \sum_{t=1970}^{2013} \beta^{2014-t} \log\left[\bar{c}_t^w \ \left(1+\Delta^w\right)\right] \\ & + \sum_{t=2014}^{\infty} \beta^{t-2014} \log\left[\bar{c}_t^w \ \left(1+\Delta^w\right)\right].\end{aligned} \tag{24}$$

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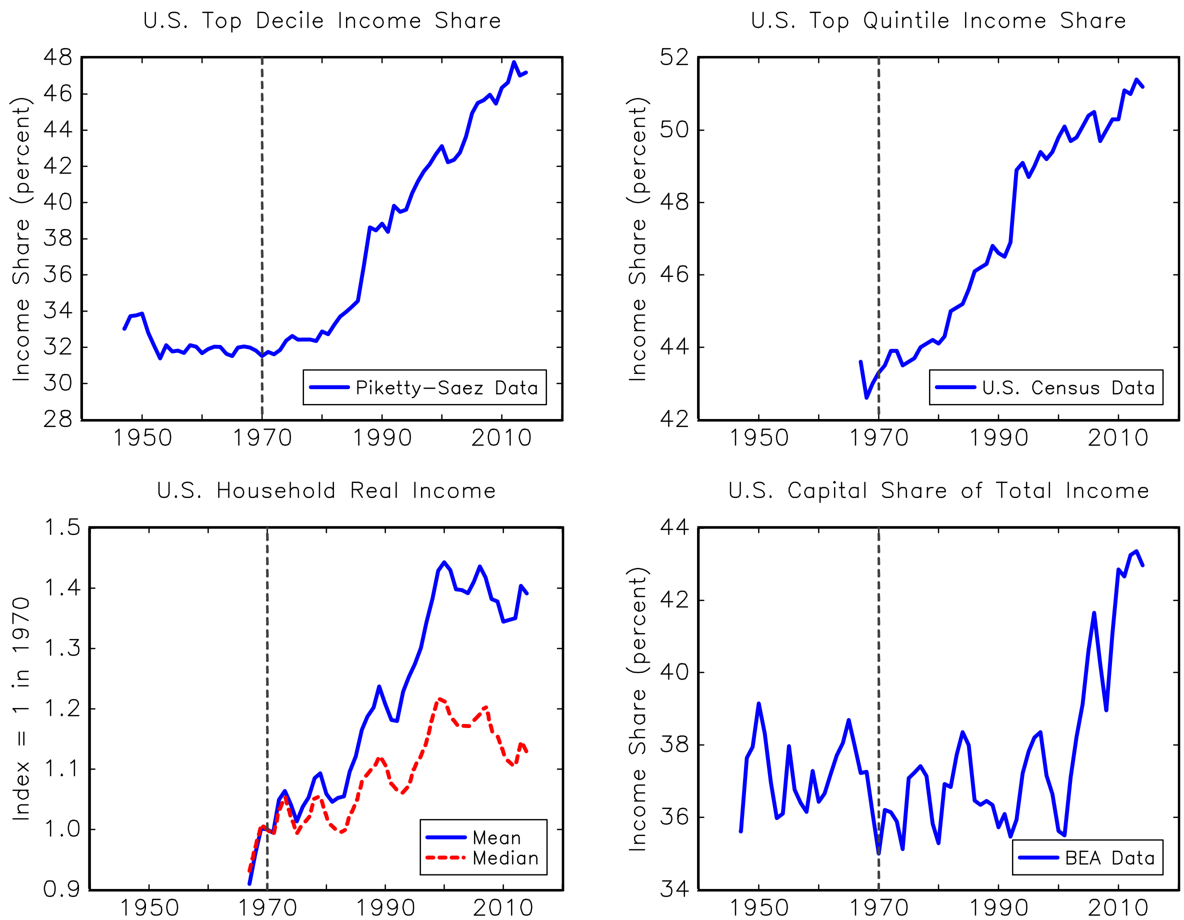

Figure 1: The increase in U.S. income inequality over the past 45 years can be traced to gains made by those near the top of the income distribution where financial wealth and corporate stock ownership is highly concentrated.

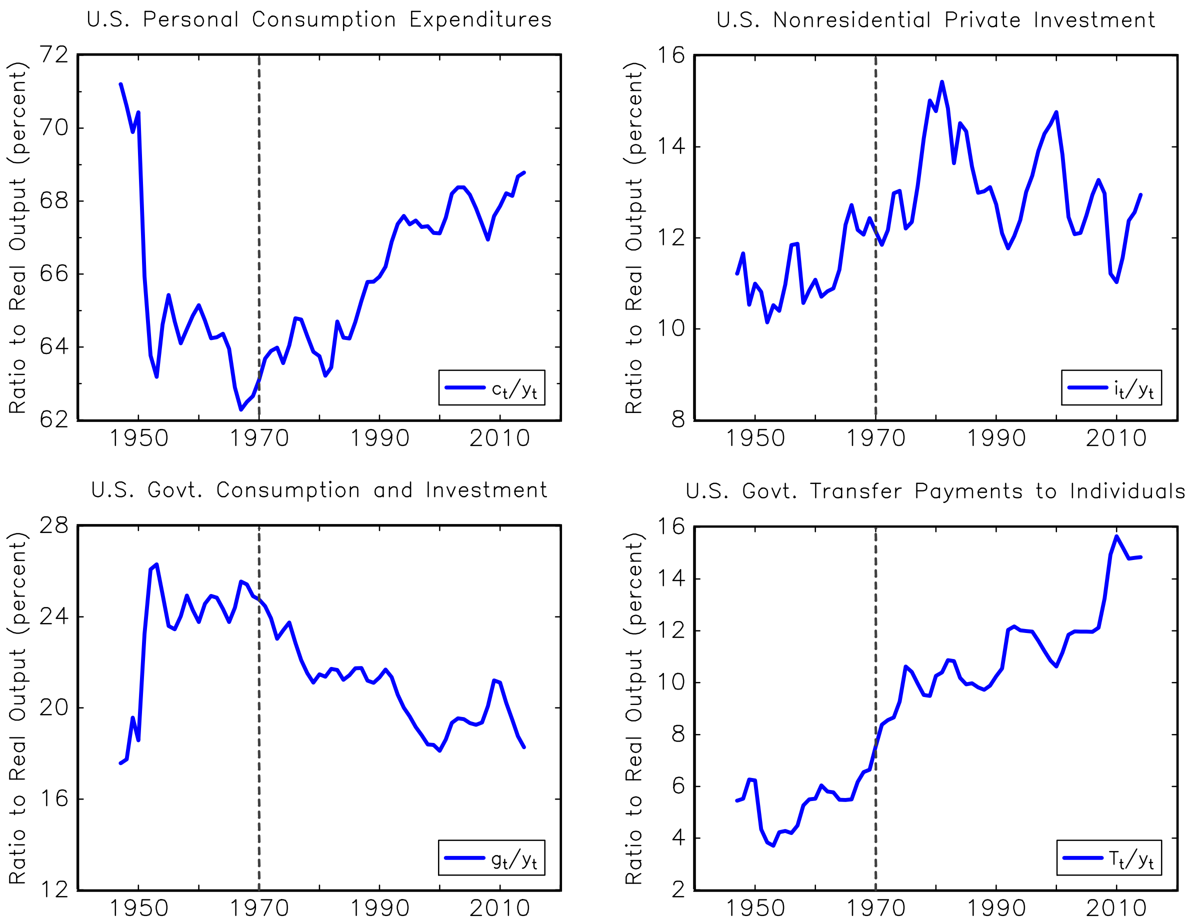

Figure 2: The baseline simulation exactly replicates the observed U.S. time paths for the ratios c_t/y_t, i_t/y_t, g_t/y_t, and T_t/y_t from 1970 to 2014. The vertical dashed line marks $t_0 = 1970$. Data series are constructed as described in footnote 9.

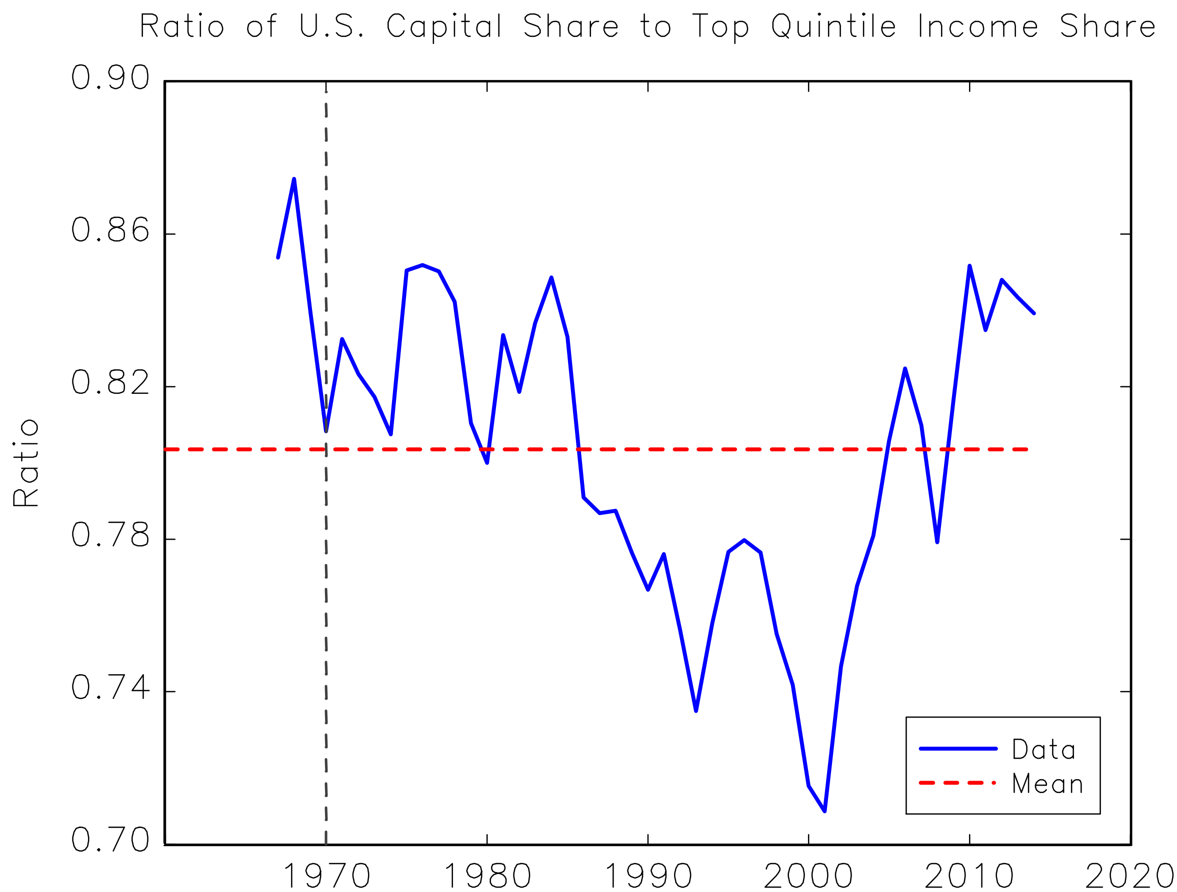

Figure 3: The ratio of capital's share of income to the top quintile income share in U.S. data appears stationary but persistent. In the model, this ratio is a state variable that pins down the capital owner's tax-adjusted income-consumption ratio.

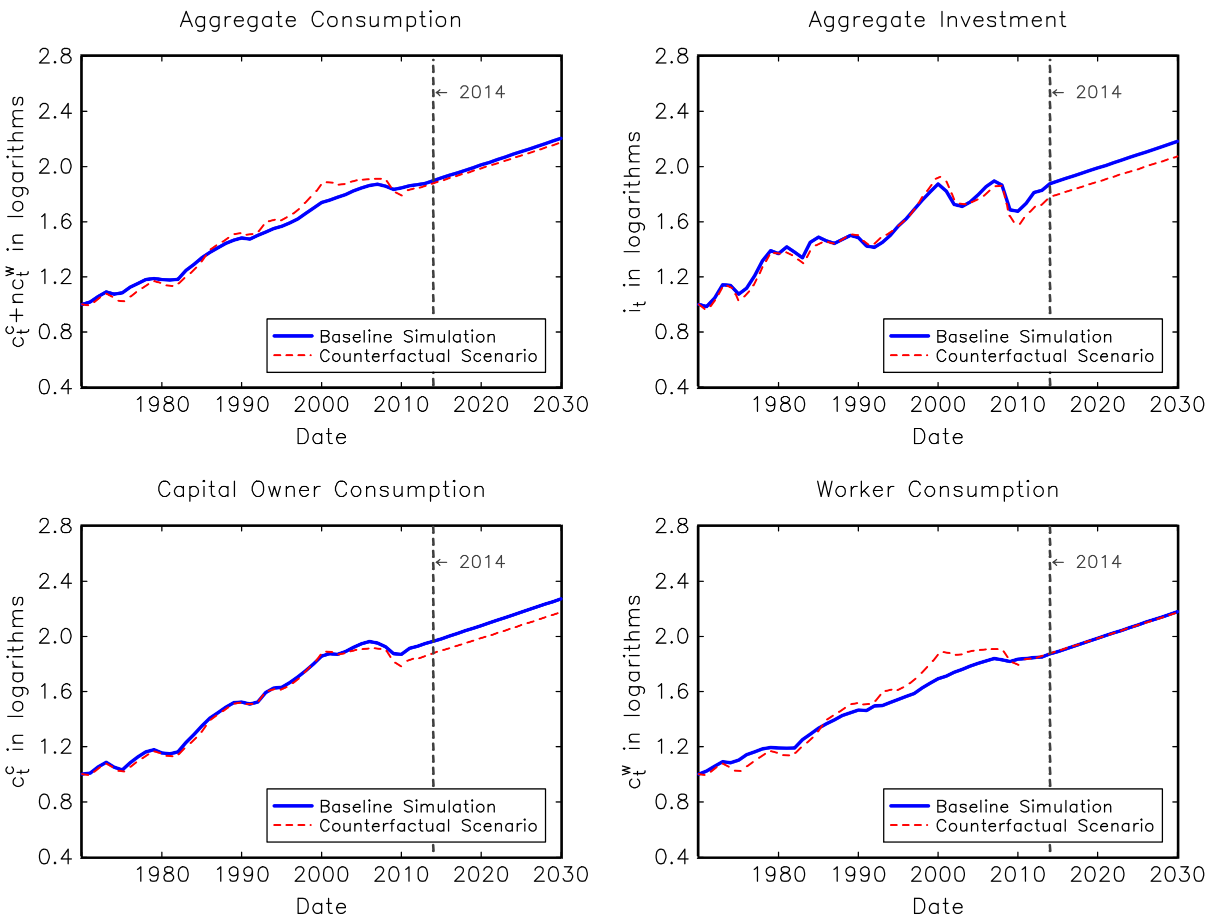

Figure 4: The figure plots the paths of model variables in the baseline simulation versus a counterfactual scenario in which the income shares and the transfer-output ratio T_t/y_t are held constant at year 1970 values, while the ratio of total government outlays to output $g_t/y_t + T_t/y_t$ is identical to that in the baseline simulation. Capital owners and workers both achieve long-run upward level shifts in consumption relative to the counterfactual scenario. But the short-run consumption paths are more important for welfare.

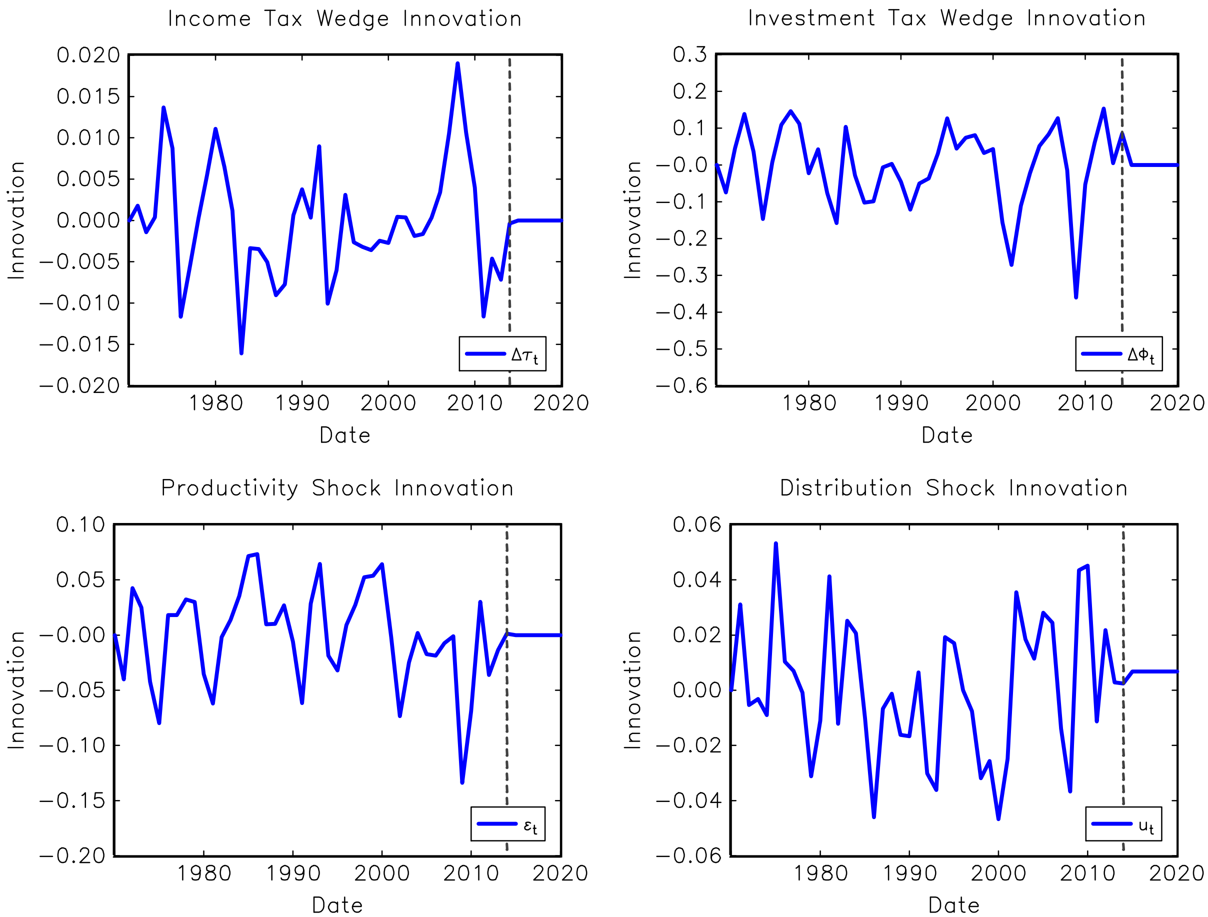

Figure 5: The figure plots the time series of tax wedge innovations and stochastic shock innovations that are needed to make the baseline simuation exactly replicate the paths of U.S. macroeconomic variables from 1970 to 2014. By construction, the innovations are zero at $t_0 = 1970$ and for $t > 2014$. The vertical dashed line marks $t = 2014$.

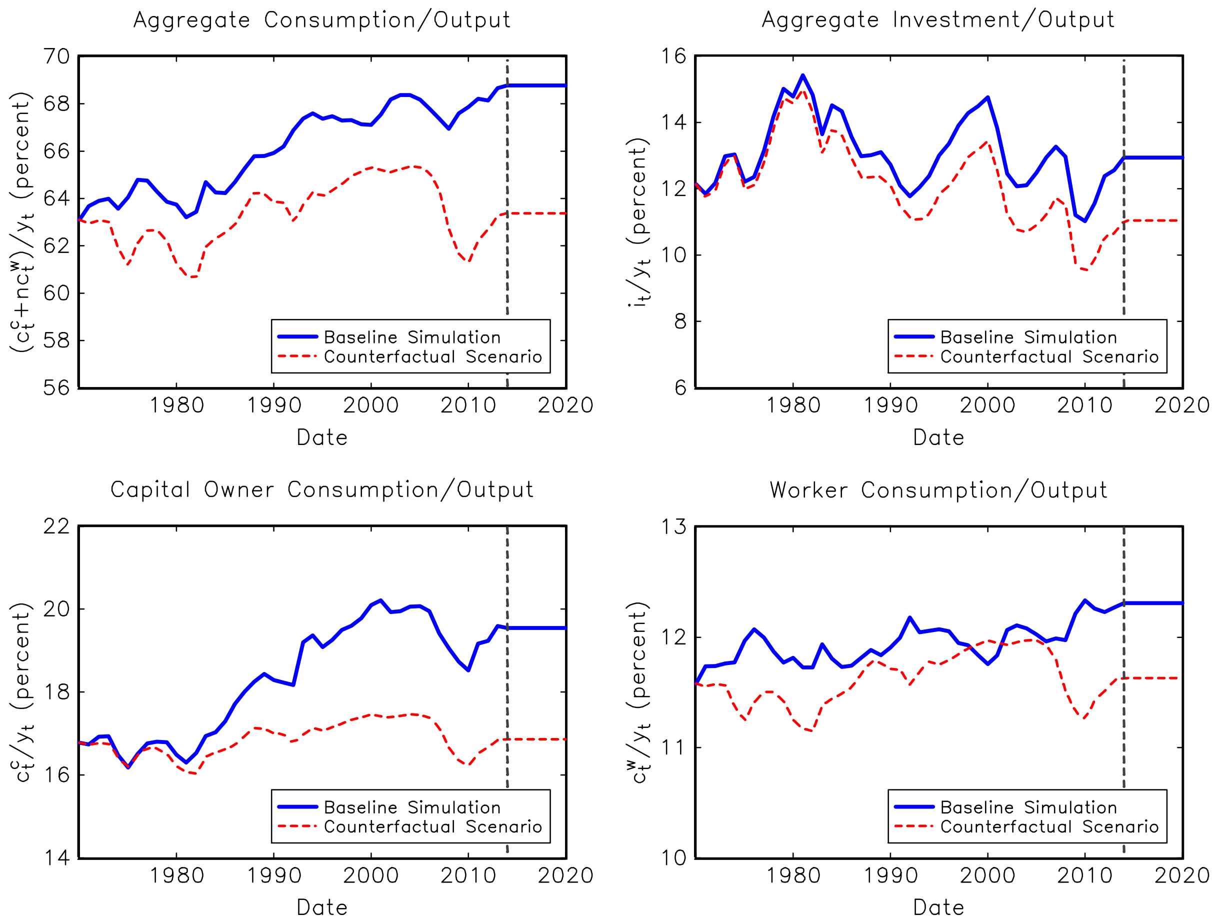

Figure 6: The baseline simulation exactly replicates the observed paths of aggregate c_t/y_t and aggregate i_t/y_t in U.S. data from 1970 to 2014 (Figure 2). We use the model decision rules (18) and (20) to construct individual consumption paths for the two types of agents. The vertical dashed line marks $t = 2014$.

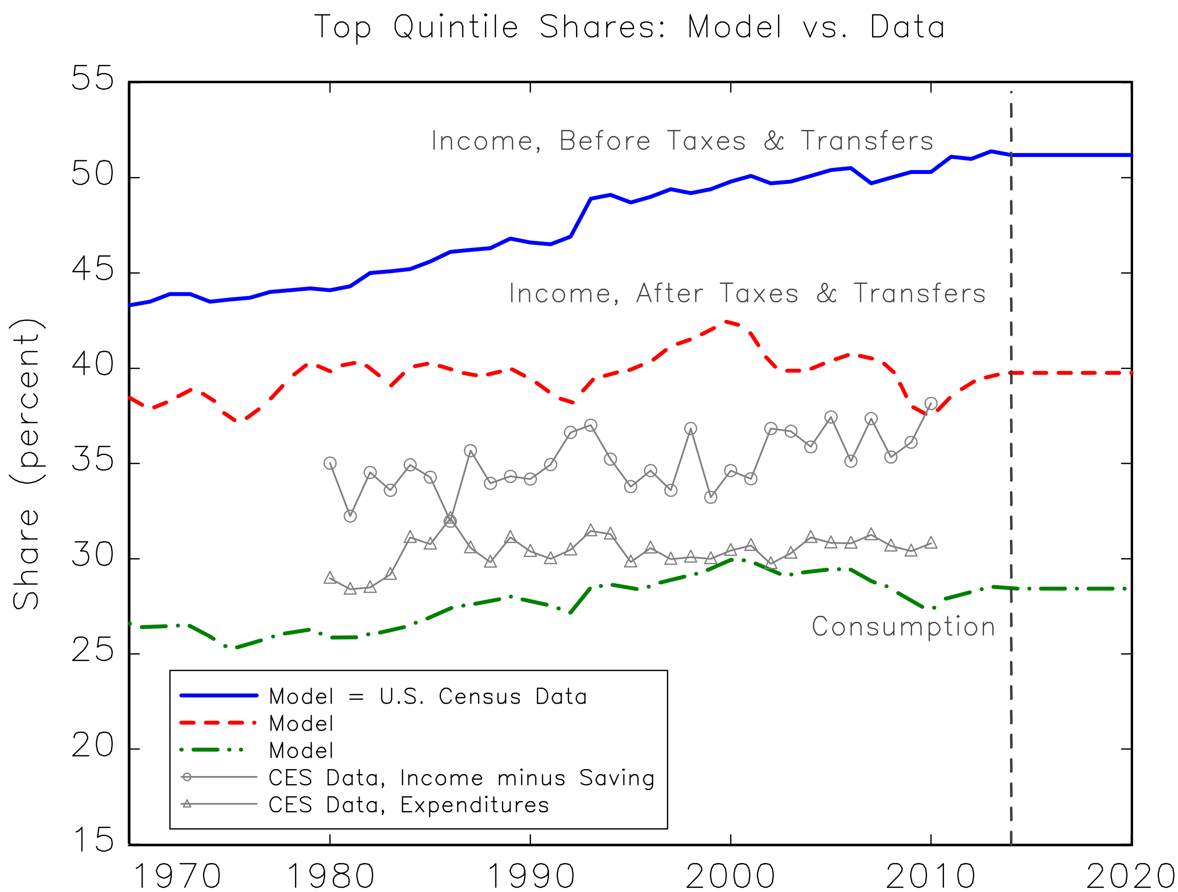

Figure 7: In the baseline simulation, the consumption share of the top quintile (capital owners) rises by much less than the before tax and transfer income share. The top quintile consumption share in the model tracks reasonably well with data from the Consumer Expenditure Survey (CES) for the period 1980 to 2010. The vertical dashed line marks $t = 2014$.

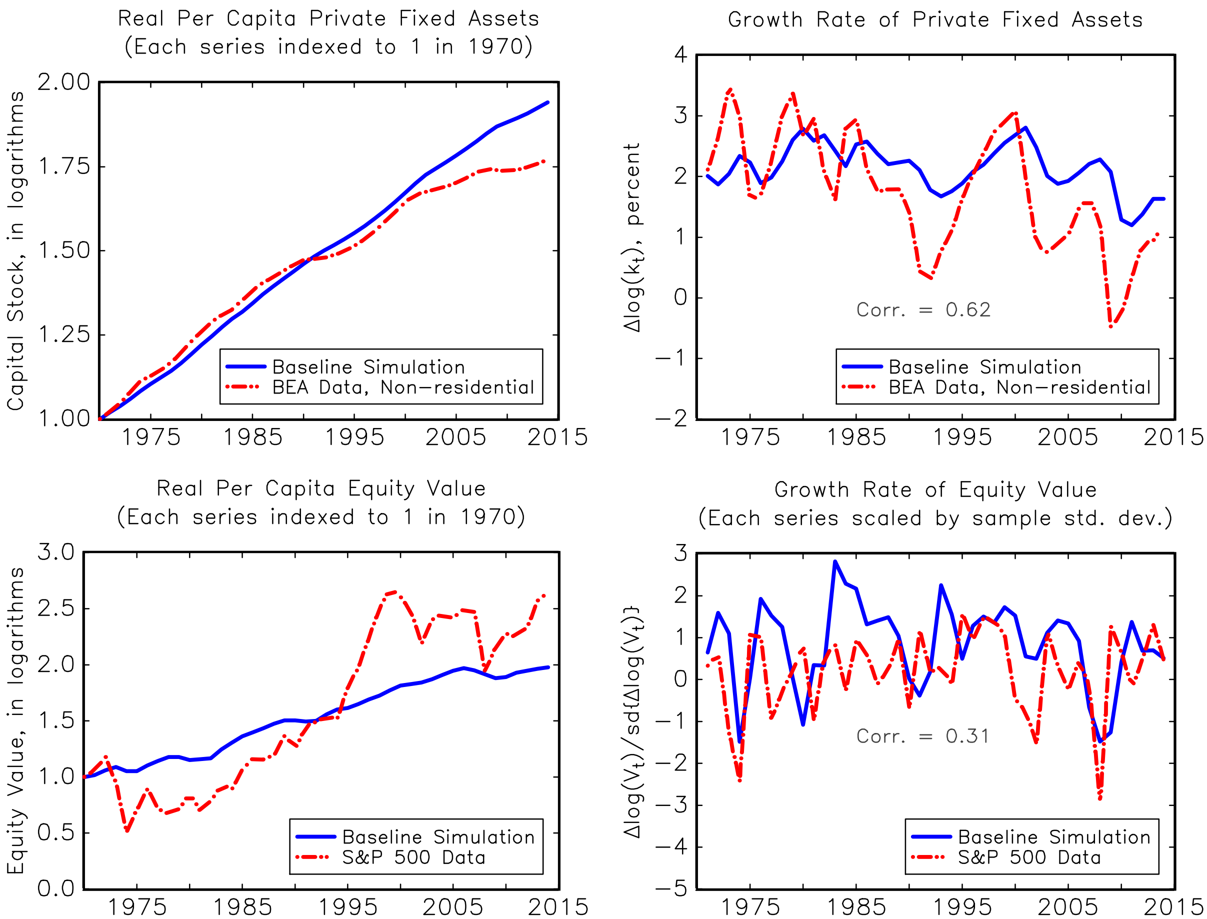

Figure 8: The simulated capital stock series from the model tracks reasonably well with the BEA's chain-type quantity index for the net stock of U.S. private nonresidential fixed assets. The correlation coefficient between the growth rates of the two series is 0.62. The real market value of the S&P 500 is far more volatile than the model equity value. Nevertheless, the correlation coefficient between the growth rates of the two series is 0.31.

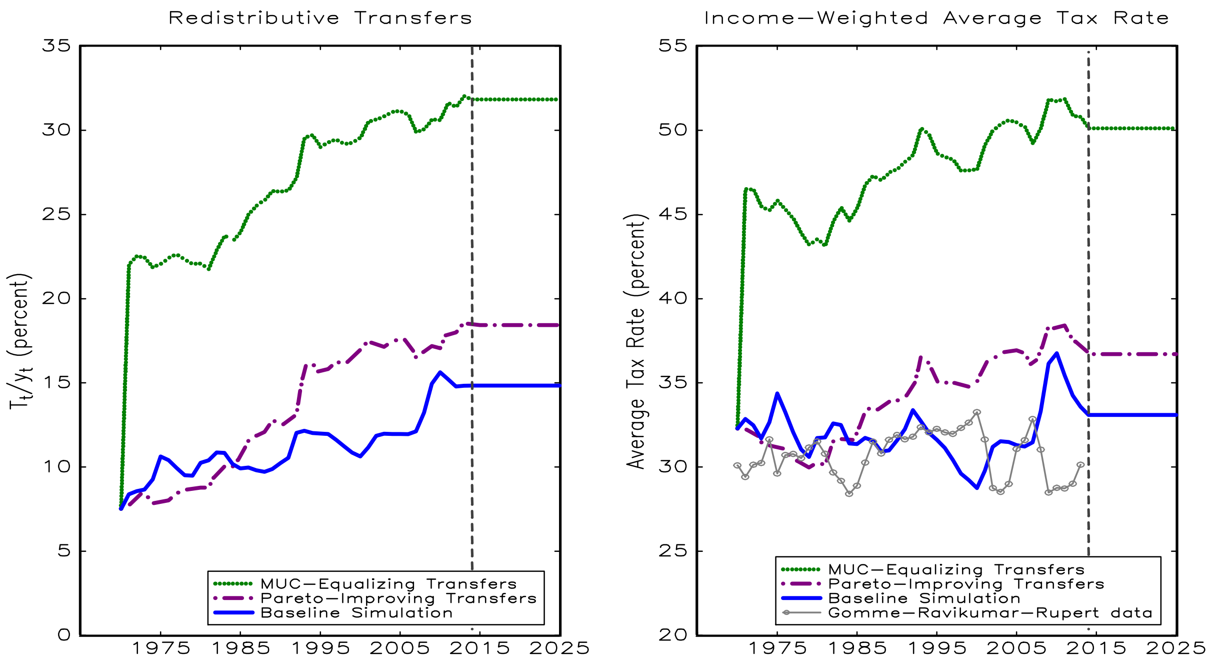

Figure 9: The MUC-equalizing transfers achieve the condition $1/c_t^w = 1/c_t^c$ for all $t > 1970$. The Pareto-improving transfers deliver equal welfare gains to capital owners and workers over a long simulation. The income-weighted average tax rates from the baseline simulation are close to those estimated by Gomme, Ravikumar, and Rupert (2011, updated). The vertical dashed line marks $t = 2014$.