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# THE WEAK JOB RECOVERY IN A MACRO MODEL OF SEARCH AND RECRUITING INTENSITY

SYLVAIN LEDUC AND ZHENG LIU

**ABSTRACT.** We show that cyclical fluctuations in search intensity and recruiting intensity are quantitatively important for explaining the weak job recovery from the Great Recession. We demonstrate this result using an estimated labor search model that features endogenous search and recruiting intensity. Since the textbook model with free entry implies constant recruiting intensity, we introduce a cost of vacancy creation, so that firms respond to aggregate shocks by adjusting both vacancies and recruiting intensity. Fluctuations in search and recruiting intensity driven by shocks to productivity and the discount factor help bridge the gap between the actual and model-predicted job filling rate.

## I. INTRODUCTION

The U.S. labor market has improved substantially since the Great Recession. The unemployment rate declined steadily from its peak of over 10 percent in 2009 to under 4 percent in 2018, accompanied by a steady increase in the job openings rate. However, the pace of recovery in hiring has been much more subdued.

These observations present a puzzle for the standard labor search model. In the standard model, hiring is related to unemployment and job vacancies through a matching function. The matching function implies that the job filling rate—defined as new hires per job vacancy—is inversely related to labor market tightness measured by the vacancy-unemployment ( $v-u$ ) ratio. It also implies that the job finding rate—defined as new job matches per unemployed worker—is positively related to labor market tightness. Thus,

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when the labor market tightens (i.e., when the v-u ratio rises), jobs are easier to find while openings are harder to fill, pushing the job finding rate up and the job filling rate down.

A model with the standard matching function fails to predict the deep labor market downturn and the subsequent weak job recovery. For example, Figure 1 shows that the implied path for the job filling rate from the standard matching function tracked the data fairly well up to early 2009. After that, however, it diverged from the data. This divergence reflects that the actual hiring rate fell more sharply during the Great Recession and rose more gradually during the recovery than the standard matching function predicted.<sup>1</sup>

To understand the forces behind this weak job recovery, we develop and estimate a dynamic stochastic general equilibrium (DSGE) framework that incorporates endogenous variations in two additional margins of labor market adjustment: search intensity and recruiting intensity. We examine the quantitative importance of cyclical fluctuations in these intensive margins for driving the weak job recovery in our estimated model.

Our model builds on the textbook model of recruiting intensity of Pissarides (2000) and extends it to incorporate costly vacancy creation. This simple extension allows our model to generate procyclical recruiting intensity. In good times, firms would have an incentive to raise recruiting intensity because the marginal cost of vacancy creation increases. In the textbook model, vacancy creation is free (i.e., there is free entry). When macroeconomic conditions change, firms vary the number of vacancies—which are costless to create or destroy—to meet new hiring needs and choose the level of recruiting intensity to minimize the cost of posting each vacancy. The free-entry assumption implies counterfactually that vacancies are a flow variable that can be adjusted continuously. Furthermore, as shown by Pissarides (2000), free entry also implies that recruiting intensity is independent of macroeconomic fluctuations. However, in our model with costly vacancy creation, vacancies become a slow-moving state variable, leading to persistent labor market dynamics. Importantly, firms adjust both the number of new vacancies and recruiting intensity in response to aggregate shocks. Thus, our model generates not only plausible vacancy dynamics but also business-cycle variations in recruiting intensity.<sup>2</sup>

We estimate our model using Bayesian methods to fit three monthly time series for the U.S. labor market: the unemployment rate, the job vacancy rate, and a measure of search

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<sup>1</sup>Since the job finding rate and the job filling rate are both functions of the v-u ratio, if the standard matching function correctly predicts one, it also correctly predicts the other. It is thus sufficient to show the predictions for one of these series. We focus on the job filling rate.

<sup>2</sup>Incorporating vacancy creation costs is important for understanding the shifts in the Beveridge curve (Elsby et al., 2015). It also helps explain sluggish responses of the v-u ratio to productivity shocks (Fujita and Ramey, 2007). See Coles and Moghaddasi Kelishomi (2011) for related discussions about the role of costly entry for understanding labor market dynamics.

intensity. The estimated model suggests that recruiting intensity is procyclical and positively correlated with aggregate hiring, with the magnitude of correlation comparable to the empirical estimates derived from micro data by Davis et al. (2013) despite the different empirical approaches. Our measure of search intensity based on Davis (2011) is also procyclical. Procyclical variations in search and recruiting intensities reinforce each other to amplify labor market fluctuations. With sharp declines in both search intensity and recruiting intensity during the Great Recession and with weak recoveries in these intensive margins following the recession, our model predicts a gradual path of recovery in the job filling rate. The predictions from our model are much more in line with the data than those from the standard matching function during the Great Recession and through the early part of the recovery. Importantly, this improvement does not come at the cost of a worsening performance in periods prior to the Great Recession.

The labor market dynamics in our model are driven primarily by a discount factor shock and a technology shock, which account for about 60 percent and 30 percent of the labor market fluctuations, respectively. In contrast, a job separation shock plays a relatively minor role. Our model shares the Shimer (2005) puzzle of the standard Diamond-Mortensen-Pissarides (DMP) model. Unless real wages are rigid, it is difficult to generate the observed large volatilities in unemployment and vacancies. With real wage rigidities assumed in our model, technology shocks are an important source of labor market fluctuations. In line with Hall (2017), we find that a discount factor shock is the most important driving force of labor market fluctuations. The shock has direct impacts on the present values of a job match and an open vacancy for firms and also on the employment surplus for job seekers. By driving changes in these present values, the discount factor shock contributes to a significant fraction of fluctuations in unemployment, vacancies, hiring, and search and recruiting intensity.<sup>3</sup>

In addition to the shocks, labor market dynamics are also driven by our model's internal propagation mechanism through search and recruiting intensity. To assess the importance of the model's internal mechanism relative to the shocks, we conduct alternative experiments in which we calibrate the parameters in the shock processes following Shimer (2005), and we re-estimate the model under this calibration. In these experiments, we find that our model still outperforms the standard matching function in predicting the deep recession and the weak job recovery, although the quantitative improvement is not as large as in our benchmark estimation.

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<sup>3</sup>Albuquerque et al. (2016) argue that discount factor shocks are important for asset pricing models because they give rise to a valuation risk that allows the model to account for the volatile asset price fluctuations and the weak correlations between stock returns and fundamentals.

## II. RELATED LITERATURE

Our paper contributes to the recent theoretical literature on cyclical variations in recruiting intensity. For example, Kaas and Kircher (2015) study a competitive search environment with heterogeneous firms facing a recruiting cost function that is convex in the number of open vacancies. In their model, since the marginal cost of recruiting increases with the number of vacancies, growing firms do not rely solely on vacancy postings to attract workers; they also rely on varying their posted wage offers. Gavazza et al. (2014) assume a recruiting cost function similar to that in Kaas and Kircher (2015) and study the importance of financial shocks for shifting the Beveridge curve through their impact on firms' recruiting intensity. We add to this literature by introducing an alternative departure from the textbook search model. In particular, we relax the free entry condition to allow for business cycle fluctuations in recruiting intensity. The resulting tractability of our framework has the added advantage of making it straightforward to estimate the model to fit time-series data using standard techniques.

Motivated by the observed patterns in labor adjustments at the establishment level, Cooper et al. (2007) estimate a labor search model with non-convexities in vacancy posting costs and firing costs using simulated methods of moments to match aggregate unemployment, vacancies, and hours. Our work is also motivated by micro-level observations about search intensity and recruiting intensity. We use some of these micro-level observations to discipline an aggregate DSGE model, and we estimate the model to understand aggregate fluctuations in the labor market.

Lubik (2009) estimates a macro model with the standard labor search friction, and he finds that the model relies heavily on exogenous shocks to matching efficiency to fit time series data of unemployment and vacancies. Our model enriches the standard model with search and recruiting intensity and thus relies on endogenous responses of search and recruiting intensity (instead of exogenous variations in matching efficiency) to explain the observed labor market dynamics.

Our paper is also related to recent work on screening, an implicit form of recruiting intensity. For instance, Ravenna and Walsh (2012) examine the effects of screening on the magnitude and persistence of unemployment following adverse technology shocks in a search model with heterogeneous workers and endogenous job destruction. Relatedly, Sedláček (2014) empirically studies the fluctuations in matching efficiency and proposes countercyclical changes in hiring standards as an underlying force.

By examining the interaction between search and recruiting intensity, our work also complements the analysis of Gomme and Lkhagvasuren (2015), who study how the addition of search intensity and directed search can amplify the responses of the unemployment and

vacancy rates following productivity shocks, although their model is not estimated to fit time-series data.

### III. A MODEL WITH SEARCH AND RECRUITING INTENSITY

This section presents a DSGE model that generalizes the standard DMP model to incorporate search intensity and recruiting intensity. The economy is populated by a continuum of infinitely lived and identical households and a continuum of firms. The representative household consists of working members and job seekers. A job seeker chooses the level of search effort (i.e., search intensity). Increasing search intensity raises search costs, but it also increases the probability of finding a job.

Creating new vacancies is costly. Posting existing vacancies also incurs a per-period fixed cost. Firms choose both the number of vacancies and the level of recruiting effort (i.e., recruiting intensity) for hiring. Increasing recruiting intensity raises the costs of advertising but it also raises the probability of filling a vacancy.

In the labor market, a matching technology transforms efficiency units of searching workers and vacancies into an employment relation. Real wages are determined by Nash bargaining between a searching worker and a hiring firm. The government finances transfer payments to unemployed workers by lump-sum taxes.

**III.1. The Labor Market.** In the beginning of period  $t$ , there are  $N_{t-1}$  employed workers. A fraction  $\delta_t$  of job matches are separated in each period. We assume that the job separation rate  $\delta_t$  is stochastic and follows the stationary process

$$\ln \delta_t = (1 - \rho_\delta) \ln \bar{\delta} + \rho_\delta \ln \delta_{t-1} + \varepsilon_{\delta t}. \quad (1)$$

In this shock process,  $\rho_\delta$  is the persistence parameter and the term  $\varepsilon_{\delta t}$  is an i.i.d. normal process with a mean of zero and a standard deviation of  $\sigma_\delta$ . The term  $\bar{\delta}$  denotes the steady-state rate of job separation.

We follow Fujita and Ramey (2007) and assume that a job position—filled or vacant—becomes obsolete at a constant rate of  $\rho^o$ . If a job match is separated or the position becomes obsolete, the worker becomes an unemployed job seeker. Under the assumption of full labor force participation, the number of job seekers at the beginning of period  $t$  is given by

$$u_t = 1 - (1 - \rho^o)(1 - \delta_t)N_{t-1}, \quad (2)$$

where we have normalized the size of the labor force to one.

After observing aggregate shocks, new vacancies are created at a cost. The stock of vacancies  $v_t$  evolves according to the law of motion

$$v_t = (1 - q_{t-1}^v)(1 - \rho^o)v_{t-1} + \delta_t(1 - \rho^o)N_{t-1} + n_t, \quad (3)$$

where  $q_t^v$  denotes the probability of filling a vacancy and  $n_t$  the number of newly created vacancies. The job filling probability is given by

$$q_t^v = \frac{m_t}{v_t}, \quad (4)$$

where  $m_t$  denotes the number of job matches.

New job matches are formed based on the matching function

$$m_t = \mu(s_t u_t)^\alpha (a_t v_t)^{1-\alpha}, \quad (5)$$

where  $s_t$  denotes search intensity,  $a_t$  denotes recruiting intensity (or advertising), the parameter  $\mu$  represents the scale of matching efficiency, and the parameter  $\alpha \in (0, 1)$  is the elasticity of job matches with respect to efficiency units of searching workers.

Newly formed matches add to the employment pool, whereas job separations and obsolescence subtract from it. Thus, aggregate employment evolves according to the law of motion

$$N_t = (1 - \rho^o)(1 - \delta_t)N_{t-1} + m_t. \quad (6)$$

At the end of period  $t$ , the searching workers who failed to find a job remain unemployed. Unemployment is given by

$$U_t = u_t - m_t = 1 - N_t. \quad (7)$$

For convenience, we define the job finding probability  $q_t^u$  as

$$q_t^u = \frac{m_t}{u_t}. \quad (8)$$

**III.2. The representative household.** The representative household has the utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \Theta_t (\ln C_t - \chi N_t), \quad (9)$$

where  $\mathbb{E}[\cdot]$  is an expectation operator,  $C_t$  denotes consumption, and  $N_t$  denotes the fraction of household members who are employed. The parameter  $\beta \in (0, 1)$  denotes the subjective discount factor, and the term  $\Theta_t$  denotes an exogenous shifter to the subjective discount factor.

The discount factor shock  $\theta_t \equiv \frac{\Theta_t}{\Theta_{t-1}}$  follows the stationary stochastic process

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}. \quad (10)$$

In this shock process,  $\rho_\theta$  is the persistence parameter and the term  $\varepsilon_{\theta t}$  is an i.i.d. normal process with a mean of zero and a standard deviation of  $\sigma_\theta$ . Here, we have implicitly assumed that the mean value of  $\theta$  is one.

The representative household chooses consumption  $C_t$ , savings  $B_t$ , and search intensity  $s_t$  to maximize the utility function in (9) subject to the sequence of budget constraints

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) - u_t h(s_t) + d_t - T_t, \quad \forall t \geq 0, \quad (11)$$

where  $B_t$  denotes the household's holdings of a risk-free bond,  $r_t$  denotes the gross real interest rate,  $w_t$  denotes the real wage rate,  $h(s_t)$  denotes the resource cost of search efforts,  $d_t$  denotes the household's share of firm profits, and  $T_t$  denotes lump-sum taxes. The parameter  $\phi$  measures the flow benefits of unemployment.

The search cost function  $h(s_i)$  for an individual unemployed worker  $i$  is increasing and convex. Raising search intensity, while costly, increases the job finding probability. For each efficiency unit of searching workers supplied, there will be  $m/(su)$  new matches formed. For a worker with search effort  $s_{it}$ , the probability of finding a job is

$$q^u(s_{it}) = \frac{s_{it}}{s_t u_t} m_t, \quad (12)$$

where  $s$  (without the subscript  $i$ ) denotes the average search intensity. The household takes the economy-wide variables  $s$ ,  $u$ , and  $m$  as given when choosing the level of search intensity  $s_i$ . A marginal effect of raising search intensity on the job finding probability is given by

$$\frac{\partial q^u(s)}{\partial s_i} = \frac{m_t}{s_t u_t} = \frac{q_t^u}{s_t}, \quad (13)$$

which depends only on aggregate economic conditions.

As we show in the Appendix A, the household's optimal search intensity decision (in a symmetric equilibrium) is given by

$$h'(s_t) = \frac{q_t^u}{s_t} \left[ w_t - \phi - \frac{\chi}{\Lambda_t} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \rho^o)(1 - \delta_{t+1})(1 - q_{t+1}^u) S_{t+1}^H \right], \quad (14)$$

where  $S_t^H$  is the employment surplus (i.e, the value of employment relative to unemployment). At the optimal level of search intensity, the marginal cost of searching equals the marginal benefit, which is the increased odds of finding a job multiplied by the net benefit of employment, including both the contemporaneous net flow benefits and the continuation value of employment.

The employment surplus  $S_t^H$  itself, as we show in the appendix, satisfies the Bellman equation

$$S_t^H = w_t - \phi - \frac{\chi}{\Lambda_t} + \frac{h(s_t)}{1 - q_t^u} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \rho^o)(1 - \delta_{t+1})(1 - q_{t+1}^u) S_{t+1}^H, \quad (15)$$

where  $\Lambda_t = \frac{1}{C_t}$  denotes the marginal utility of consumption.

The employment surplus has a straightforward economic interpretation. If the household adds a new worker in period  $t$ , then the current-period gain would be wage income net



of the opportunity costs of working, including unemployment benefit and the disutility of working. The contemporaneous benefit also includes saved search cost because it reduces the pool of job seekers, the measure of which is  $1 - q_t^u$  at the end of period  $t$ . In addition, the household also enjoys the continuation value of employment if the employment relation continues. Having an extra worker today adds to the employment pool tomorrow (if the employment relation survives job separation and obsolescence); however, adding a worker today would also reduce the pool of searching workers tomorrow, a fraction  $q_{t+1}^u$  of whom would be able to find jobs. Thus, the marginal effect of adding a new worker in period  $t$  on employment in period  $t + 1$  is given by  $(1 - \rho^o)(1 - \delta_{t+1})(1 - q_{t+1}^u)$ , resulting in the effective continuation value of employment shown in the last term of equation (15).

We also show in the appendix that the household's optimizing consumption/saving decision implies the intertemporal Euler equation

$$1 = \mathbb{E}_t \beta \theta_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} r_t. \quad (16)$$

**III.3. The firms.** A firm can produce the final consumption goods only if it successfully matches with a worker. The production function for firm  $j$  with one worker is given by

$$y_{jt} = Z_t,$$

where  $y_{jt}$  is output and  $Z_t$  is an aggregate technology shock. The technology shock follows the stochastic process

$$\ln Z_t = (1 - \rho_z) \ln \bar{Z} + \rho_z \ln Z_{t-1} + \varepsilon_{zt}. \quad (17)$$

The parameter  $\rho_z \in (-1, 1)$  measures the persistence of the technology shock. The term  $\varepsilon_{zt}$  is an i.i.d. normal process with a zero mean and a finite variance of  $\sigma_z^2$ . The term  $\bar{Z}$  is the steady-state level of the technology shock.<sup>4</sup>

If a firm  $j$  finds a match, it obtains a flow profit in the current period after paying the worker. In the next period, if the match survives (with probability  $(1 - \rho^o)(1 - \delta_{t+1})$ ), the firm continues; if the match breaks down, the firm posts a new job vacancy at a flow cost of  $\kappa_{jt}$ , with the vacancy value of  $J_{j,t+1}^V$ , provided that the job position does not become obsolete. The firm's match value therefore satisfies the Bellman equation

$$J_{jt}^F = Z_t - w_t + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \rho^o) \left\{ (1 - \delta_{t+1}) J_{j,t+1}^F + \delta_{t+1} J_{j,t+1}^V \right\}. \quad (18)$$

Here, the value function is discounted by the representative household's marginal utility because all firms are owned by the household.

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<sup>4</sup>The model can easily be extended to allow for trend growth. We do not present that version of the model to simplify presentation.

Following Coles and Moghaddasi Kelishomi (2011), we assume that vacancy creation incurs a non-negative entry cost of  $x$  drawn from an i.i.d. distribution  $F(\cdot)$ . A new vacancy is created if and only if  $x \leq J_t^V \equiv x_t^*$ , or equivalently, if and only if its net value is non-negative. Thus, the number of new vacancies  $n_t$  equal to  $F(J_t^V)$ —the cumulative density of entry costs evaluated at the vacancy value. This feature implies that the marginal cost of vacancy creation (i.e., the break-even entry cost  $x_t^*$ ) increases with the number of vacancies created and firms would have an incentive to raise recruiting intensity in good times.

With appropriate assumptions about the functional form of the distribution function  $F(\cdot)$ , the number of new vacancies created is related to the vacancy value through the equation

$$n_t = \eta(J_t^V)^\xi, \quad (19)$$

where  $\eta$  is a scale parameter and  $\xi$  measures the elasticity of new vacancies with respect to the value of the vacancy. The special case with  $\xi = \infty$  corresponds to the standard DMP model with free entry (i.e.,  $J_t^V = 0$ ). In general, a smaller value of  $\xi$  would imply a less elastic response of new vacancies to changes in aggregate conditions (through changes in the value of vacancies).

The flow cost of posting a vacancy is an increasing and convex function of the level of advertising. In particular, we follow Pissarides (2000) and assume that

$$\kappa_{jt} = \kappa(a_{jt}), \quad \kappa'(\cdot) > 0, \quad \kappa''(\cdot) \geq 0, \quad (20)$$

where  $a_{jt}$  is firm  $j$ 's level of advertising.

Increasing advertising efforts raises the probability of filling a vacancy. For each efficiency unit of vacancy supplied, there will be  $m/(av)$  new matches formed. Thus, for a firm that supplies  $a_{jt}$  units of advertising efforts, the probability of filling a vacancy is

$$q^v(a_{jt}) = \frac{a_{jt}}{a_t v_t} m_t, \quad (21)$$

where  $a_t$  is the average advertising effort by firms.

If the vacancy is filled (with probability  $q_{jt}^v$ ), the firm obtains the value of a match  $J_{jt}^F$ . If the vacancy remains unfilled, then the firm goes into the next period and obtains the continuation value of the vacancy, provided that the vacancy will not be obsolete. Thus, the value of an open vacancy is given by

$$J_{jt}^V = -\kappa(a_{jt}) + q^v(a_{jt})J_{jt}^F + (1 - \rho^o)(1 - q^v(a_{jt}))\mathbb{E}_t \frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t} J_{j,t+1}^V. \quad (22)$$

The firm chooses advertising efforts  $a_{jt}$  to maximize the value of vacancy  $J_{jt}^V$ . The optimal level of advertising is given by the first-order condition

$$\kappa'(a_{jt}) = \frac{\partial q^v(a_{jt})}{\partial a_{jt}} \left[ J_{jt}^F - (1 - \rho^o)\mathbb{E}_t \frac{\beta\theta_{t+1}\Lambda_{t+1}}{\Lambda_t} J_{j,t+1}^V \right], \quad (23)$$

where, from (21), we have

$$\frac{\partial q^v(a_{jt})}{\partial a_{jt}} = \frac{m_t}{a_t v_t} = \frac{q_t^v}{a_t}. \quad (24)$$

We concentrate on a symmetric equilibrium in which all firms make identical choices of the level of advertising. Thus, in equilibrium, we have  $a_{jt} = a_t$ . In such a symmetric equilibrium, the optimizing advertising decision (23) can be written as

$$\kappa'(a_t) = \frac{q_t^v}{a_t} \left[ J_t^F - (1 - \rho^o) \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} J_{t+1}^V \right]. \quad (25)$$

If the firm raises its advertising effort, it incurs the marginal cost of  $\kappa'(a_t)$ . The marginal benefit is that, by increasing the probability of forming a job match, the firm obtains the match value  $J_t^F$ , although it loses the continuation value of the vacancy, which represents the opportunity cost of filling the vacancy.<sup>5</sup>

In the special case with free entry, the value of vacancy would be zero. Thus, equation (22) reduces to

$$\kappa(a_t) = q_t^v J_t^F. \quad (26)$$

Furthermore, the optimal advertising choice (25) reduces to

$$\kappa'(a_t) = \frac{q_t^v}{a_t} J_t^F. \quad (27)$$

These two equations together imply that

$$\frac{\kappa'(a_t) a_t}{\kappa(a_t)} = 1. \quad (28)$$

In this case, the level of advertising is chosen such that the elasticity of the cost of advertising equals 1 and it is thus invariant to macroeconomic conditions, as in the textbook model of Pissarides (2000).

This special case highlights the importance of incorporating costs of vacancy creation. Absent any vacancy creation cost, as in the textbook models, firms can freely adjust vacancies to respond to changes in macroeconomic conditions and choose the level of advertising to minimize the cost of each vacancy. In this case, the optimal level of advertising is independent of market variables. In contrast, if vacancy creation is costly, as we assume in our model, firms would rely on adjusting both the level of advertising and the number of vacancies to respond to changes in macroeconomic conditions.

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<sup>5</sup>The optimizing recruiting intensity (advertising) decision in Eq. (25) reveals that the cyclical properties of recruiting intensity are a priori ambiguous. In a recession, the job filling rate rises, and firms respond by exerting more recruiting efforts. However, in a recession, the match value  $J^F$  and the vacancy value  $J^V$  both decline, so that changes in the net value of filling a vacancy—the difference between  $J^F$  and  $J^V$ —are in general ambiguous. With empirically plausible parameters, recruiting intensity in our model is procyclical, as we show below.

**III.4. The Nash bargaining wage.** Firms and workers bargain over wages. The Nash bargaining problem is given by

$$\max_{w_t} (S_t^H)^b (J_t^F - J_t^V)^{1-b}, \quad (29)$$

where  $b \in (0, 1)$  represents the bargaining weight for workers. The first-order condition implies that

$$b (J_t^F - J_t^V) \frac{\partial S_t^H}{\partial w_t} + (1 - b) S_t^H \frac{\partial (J_t^F - J_t^V)}{\partial w_t} = 0, \quad (30)$$

where, from the household surplus equation (15), we have  $\frac{\partial S_t^H}{\partial w_t} = 1$ ; and from the firm's value function (18), we have  $\frac{\partial (J_t^F - J_t^V)}{\partial w_t} = -1$ .

Define the total surplus as

$$S_t = J_t^F - J_t^V + S_t^H. \quad (31)$$

Then the bargaining solution is given by

$$J_t^F - J_t^V = (1 - b)S_t, \quad S_t^H = bS_t. \quad (32)$$

The bargaining outcome implies that the firm's surplus is a constant fraction  $1 - b$  of the total surplus  $S_t$  and the household's surplus is a fraction  $b$  of the total surplus.

The bargaining solution (32) and the expression for household surplus in equation (15) together imply that the Nash bargaining wage  $w_t^N$  satisfies the Bellman equation

$$\begin{aligned} \frac{b}{1-b} (J_t^F - J_t^V) &= w_t^N - \phi - \frac{\chi_t}{\Lambda_t} + \frac{h(s_t)}{1 - q_t^u} \\ &+ \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} \left[ (1 - \rho^o)(1 - \delta_{t+1}) (1 - q_{t+1}^u) \frac{b}{1-b} (J_{t+1}^F - J_{t+1}^V) \right] \end{aligned} \quad (33)$$

**III.5. Wage rigidity.** Similar to the standard DMP models, real wage rigidities are important for our model to generate empirically plausible volatilities of vacancies and unemployment relative to the volatility of labor productivity.<sup>6</sup> We follow the literature and consider real wage rigidity (Hall, 2005a; Shimer, 2005). We assume that the real wage is a geometrically weighted average of the Nash bargaining wage and the realized wage rate in the previous period. That is,

$$w_t = w_{t-1}^\gamma (w_t^N)^{1-\gamma}, \quad (34)$$

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<sup>6</sup>The recent literature identifies several sources of real wage rigidities. For example, Christiano et al. (2015) report that an estimated DSGE model with wages determined by an alternating offer bargaining game in the spirit of Hall and Milgrom (2008) fits the data better than the standard model with Nash bargaining. Liu et al. (2016) show that, in an estimated DSGE model with labor search frictions and collateral constraints, endogenous real wage inertia can be obtained conditional on a housing demand shock even if wages are determined from the standard Nash bargaining game.

where  $\gamma \in (0, 1)$  represents the degree of real wage rigidity.<sup>7</sup>

**III.6. Government policy.** The government finances unemployment benefit payments  $\phi$  for unemployed workers through lump-sum taxes. We assume that the government balances the budget in each period so that

$$\phi(1 - N_t) = T_t. \quad (35)$$

**III.7. Search equilibrium.** In a search equilibrium, the markets for bonds and goods all clear. Since the aggregate bond supply is zero, the bond market-clearing condition implies that

$$B_t = 0. \quad (36)$$

Goods market clearing requires that consumption spending, search and recruiting costs, and vacancy creation costs add up to aggregate production. This requirement yields the aggregate resource constraint

$$C_t + h(s_t)u_t + \kappa(a_t)v_t + \int_0^{J_t^V} x dF(x) = Y_t, \quad (37)$$

where the last term on the left-hand side of the equation corresponds to the aggregate cost of creating job vacancies. Under our distribution assumption of the vacancy creation cost, the cumulative density function of  $x$  is given by  $F(x) = \eta x^\xi$ . Thus, the aggregate cost of vacancy creation is given by  $\int_0^{J_t^V} x dF(x) = \frac{\eta\xi}{1+\xi}(J_t^V)^{1+\xi}$ . Using the relation between the number of job vacancies and the value of an open vacancy in equation (19), the aggregate resource cost for vacancy creation can be written as  $\frac{\xi}{1+\xi}n_t J_t^V$ .

Aggregate output  $Y_t$  is related to employment through the aggregate production function

$$Y_t = Z_t N_t. \quad (38)$$

#### IV. EMPIRICAL STRATEGIES

We solve the DSGE model by log-linearizing the equilibrium conditions around the deterministic steady state.<sup>8</sup> We calibrate a subset of the parameters to match steady-state observations and estimate the remaining structural parameters and shock processes to fit the U.S. time-series data.

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<sup>7</sup>We have examined other wage rules as those in Blanchard and Galí (2010), and we find that our results do not depend on the particular form of the wage rule.

<sup>8</sup>Details of the equilibrium conditions, the steady state, and the log-linearized system are available in the online appendix at [http://www.frbsf.org/economic-research/files/wp2016-09\\_appendix.pdf](http://www.frbsf.org/economic-research/files/wp2016-09_appendix.pdf).

Consistent with the empirical evidence by Yashiv (2000) and Christensen et al. (2005), we assume that the vacancy cost function  $\kappa(a)$  and the search cost function  $h(s)$  take the quadratic forms

$$\kappa(a_t) = \kappa_0 + \kappa_1(a_t - \bar{a}) + \frac{\kappa_2}{2}(a_t - \bar{a})^2, \quad (39)$$

$$h(s_t) = h_1(s_t - \bar{s}) + \frac{h_2}{2}(s_t - \bar{s})^2, \quad (40)$$

where we normalize the steady-state levels of recruiting intensity and search intensity so that  $\bar{a} = 1$  and  $\bar{s} = 1$ . We also assume that the search cost is zero in the steady state.

We first calibrate a subset of model parameters using steady-state restrictions. These parameters include  $\beta$ , the subjective discount factor;  $\chi$ , the disutility of working;  $\alpha$ , the elasticity of matching with respect to searching workers;  $\mu$ , the matching efficiency;  $\bar{\delta}$ , the average job separation rate;  $\rho^o$ , the vacancy obsolescence rate;  $\phi$ , the unemployment benefits;  $b$ , the Nash bargaining weight;  $\kappa_0$  and  $\kappa_1$ , the intercept and the slope of the vacancy cost function;  $h_1$ , the slope parameter of the search cost function;  $\gamma$ , the parameter that measures real wage rigidities; and  $\xi$ , the elasticity parameter of vacancy creation.

We estimate the remaining structural and shock parameters using Bayesian methods to fit the time-series data of unemployment, vacancies, and search intensity. The structural parameters to be estimated include  $K \equiv \frac{1}{\eta}$ , the scale of the vacancy creation cost function;  $\kappa_2$ , the curvature of the vacancy posting cost function; and  $h_2$ , the curvature of the search cost function. The shock parameters include  $\rho_j$  and  $\sigma_j$ , the persistence and the standard deviation of the shock  $j \in \{z, \theta, \delta\}$ .

**IV.1. Calibration.** Table 1 shows the calibrated values of the model parameters.

We consider a monthly model. Thus, we set  $\beta = 0.9967$ , so that the model implies a steady-state annualized real interest rate of about 4 percent. We set  $\alpha = 0.5$  following the literature (Blanchard and Galí, 2010; Gertler and Trigari, 2009). In line with Hall and Milgrom (2008), we set  $b = 0.5$ . We also set  $\phi = 0.25$  so that the unemployment benefit is about 25 percent of normal earnings. In our baseline experiment, we set  $\xi = 1$ , as in Fujita and Ramey (2007) and Coles and Moghaddasi Kelishomi (2011).

In our calibration, we target a steady-state unemployment rate of  $U = 0.055$  and a steady-state job filling rate of  $q^v = 0.6415$ . This targeted job filling rate matches the empirical estimation in Davis et al. (2013) using establishment-level JOLTS data. In particular, Davis et al. (2013) estimate that the daily job filling rate averages about 0.05, which implies a

monthly job filling rate of  $q^v = 0.6415$ .<sup>9</sup> We set the average monthly job destruction rate to 0.034, as in the JOLTS data. Given the steady-state unemployment rate of  $U = 0.055$  and the job destruction rate of 0.034, we obtain the steady-state employment rate of  $N = 0.945$  and the hiring rate of  $m = 0.034$ . It follows that the steady-state number of job seekers is  $u = 0.0871$  and that the job vacancy rate is  $v = 0.0501$ . We calibrate the job obsolescence rate to  $\rho^o = 0.0196$ , implying an average ratio of newly created vacancies to employment of about 0.02, which is slightly lower than that estimated by Davis et al. (2013). The normal average job separation rate  $\bar{\delta}$  can then be inferred from the overall job destruction rate (0.034) and the job obsolescence rate. This leads to  $\bar{\delta} = 0.0147$ .

We calibrate the steady-state level of vacancy cost  $\kappa_0$  so that the total cost of posting vacancies is about 1 percent of gross output. We normalize the average level of total factor productivity (TFP) to  $Z = 1$ , so that the steady-state gross output is  $Y = 0.945$ . Given  $Y$  and  $v$ , we obtain  $\kappa_0 = 0.1887$ . We set  $\kappa_1 = 0.1902$  so that the steady-state recruiting intensity is  $\bar{a} = 1$ . We set  $h_1 = 0.1088$  so that the steady-state search intensity is  $\bar{s} = 1$ . Given the steady-state values of  $m$ ,  $u$ ,  $v$ ,  $s$ , and  $a$ , we use the matching function to obtain an average matching efficiency of  $\mu = 0.4864$ .

To obtain a value for  $\chi$ , we solve the steady-state system so that  $\chi$  is consistent with an unemployment rate of  $U = 0.055$ . The process results in  $\chi = 0.665$ . Finally, as in the standard DMP model, our model relies on real wage rigidities to generate the observed large fluctuations in labor market variables (Shimer, 2005). We set the wage rigidity parameter to  $\gamma = 0.95$ , which lies at the high end of the literature (Hall, 2005b).

**IV.2. Estimation.** We now describe our data and estimation approach.

**IV.2.1. Data and measurement.** We fit the DSGE model to three monthly time series for the U.S. labor market: the unemployment rate, the job vacancy rate, and a measure of search intensity. The sample covers the period from July 1967 to July 2017. Appendix B provides a detailed description of the data sources and measurements.

The unemployment rate in the data (denoted by  $U_t^{data}$ ) corresponds to the end-of-period unemployment rate in the model  $U_t$ . We demean the unemployment rate data (in log units) and relate it to our model variable according to

$$\ln(U_t^{data}) - \ln(U^{data}) = \hat{U}_t, \quad (41)$$

where  $U^{data}$  denotes the sample average of the unemployment rate in the data and  $\hat{U}_t$  denotes the log-deviations of the unemployment rate from its steady-state value in the model.

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<sup>9</sup>Assuming that one month consists of 20 business days, we can infer the monthly job filling rate  $q^v$  from the daily rate  $f = 0.05$  by using the relation  $q^v = f + f(1-f) + f(1-f)^2 + \dots + f(1-f)^{19} = 1 - (1-f)^{20} = 0.6415$ .

Similarly, we use demeaned vacancy rate data (also in log units) and relate it to the model variable according to

$$\ln(v_t^{data}) - \ln(\bar{v}^{data}) = \hat{v}_t, \quad (42)$$

where  $v_t^{data}$  denotes the sample average of the vacancy rate data and  $\hat{v}_t$  denotes the log-deviations of the vacancy rate from its steady-state value in the model.

We follow the approach of Davis (2011) to construct a measure of search intensity. He combines mean unemployment spells from the Current Population Survey (CPS) and regression results from Krueger and Mueller (2011), who find that search intensity declines as the duration of unemployment increases in high-frequency longitudinal data. In particular, Davis (2011) postulates that

$$s_t = A - Bd_t, \quad (43)$$

where  $s_t$  is search intensity and  $d_t$  is the mean unemployment duration (in weeks). He then constructs the search intensity index by setting  $A = 122.30$  and  $B = 0.90$  after adjusting for some potential biases in the regression results obtained by Krueger and Mueller (2011).<sup>10</sup> We follow the same methodology as Davis (2011) in constructing a search intensity series, with the exception that we use the median unemployment duration in weeks instead of the mean.<sup>11</sup>

Figure 2 displays this measure of aggregate search intensity. Clearly, search intensity is procyclical, rising in booms and falling in recessions. In the Great Recession and its aftermath, search intensity declined substantially, as the duration of unemployment lengthened.

*IV.2.2. Prior distributions and posterior estimates.* The prior and posterior distributions of the estimated parameters from our benchmark model are displayed in Table 2.

The priors for the structural parameters  $K$ ,  $\kappa_2$ , and  $h_2$  are drawn from the gamma distribution. We assume that the prior mean of each of these three parameters is 5, with a standard deviation of 1. The priors of the persistence parameter of each shock follow the beta distribution with a mean of 0.8 and a standard deviation of 0.1. The priors of the volatility parameter of each shock follow an inverse gamma distribution with a prior mean of 0.01 and a standard deviation of 0.1.

The posterior estimates and the 90 percent probability intervals for the posterior distributions are displayed in the last three columns of Table 2. The posterior mean estimate of

<sup>10</sup>See the discussions of this methodology in Davis (2011), p. 66.

<sup>11</sup>The Great Recession caused many workers to experience extremely long spells of unemployment, contributing to the sharp increase in the mean duration of unemployment for this period. We use the median unemployment duration to construct our search intensity series, since we believe this median measure better reflects the underlying factors that influence an individual job seeker's search efforts than does the mean. The time series of median unemployment durations from the BLS is available from July 1967 and on.



the vacancy creation cost parameter is  $K = 3.89$ , which implies a modest size of vacancy creation costs, at about 0.1 percent of aggregate output in the steady state. The posterior mean estimates of the curvature parameters for the vacancy cost function and the search cost function are  $\kappa_2 = 5.88$  and  $h_2 = 0.99$ , respectively. The 90 percent probability intervals suggest that the posterior estimates of these structural parameters are significantly different from their priors. Thus, the data are informative on these parameters.

The posterior estimation suggests that the technology shock and the discount factor shock are both highly persistent, with the posterior means of the AR(1) parameters at  $\rho_z = 0.995$  and  $\rho_\theta = 0.993$ , respectively. The job separation shock is less persistent, with the AR(1) parameter of  $\rho_\delta = 0.843$ . The standard deviations of the technology shock ( $\sigma_z = 0.019$ ) and the discount factor shock ( $\sigma_\theta = 0.030$ ) are both much smaller than that of the separation shock ( $\sigma_\delta = 0.187$ ).

## V. ECONOMIC IMPLICATIONS

We now examine the model's transmission mechanism and its quantitative performance for explaining the sharp labor market downturns during the Great Recession and the subsequent weak job recovery.

**V.1. The model's transmission mechanism.** The equilibrium dynamics in our model are driven by both the exogenous shocks and the model's internal propagation mechanism. To help understand the contributions of the shocks and the model's mechanism, we examine forecast error variance decompositions and impulse response functions.

*V.1.1. Forecast error variance decompositions.* Table 3 displays the mean (or unconditional) forecast error variance decompositions for several key labor market variables and aggregate output.<sup>12</sup>

The variance decompositions suggest that our model's labor market dynamics are primarily driven by the discount factor shock and the technology shock, while the job separation shock is less important.<sup>13</sup>

Consistent with the intuition provided by Shimer (2005), real wage rigidities in our model allow technology shocks to play an important role in driving labor market fluctuations. The

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<sup>12</sup>We have also computed the conditional forecast error variance decompositions with forecasting horizons between 12 months and 48 months and found that they deliver the same message as the unconditional variance decomposition.

<sup>13</sup>Historical decompositions show that discount factor shocks were the primary driving force of fluctuations in unemployment, vacancies, and search intensity in the Great Recession, and technology shocks played a more important role during the recover. To conserve space, we report the historical decomposition results in the online appendix.

variance decomposition results suggest that technology shocks account for 30-50 percent of the cyclical fluctuations in unemployment, vacancies, search intensity, recruiting intensity, hiring, and the job filling rate. Technology shocks are also the primary driving force of aggregate output, accounting for about 94 percent of its fluctuations.

Discount factor shocks can directly affect the present values of a job match and an open vacancy, and also the employment surplus for a job seeker. Thus, it is important for explaining the observed labor market fluctuations (Hall, 2017). Quantitatively, our variance decomposition shows that a discount factor shock contributes to a large fraction—about 40-70 percent—of fluctuations in labor market variables.

Job separation shocks do not play a big role in labor market fluctuations, except for hiring, for which they account for about 18 percent of the variance. As noted by Shimer (2005), job separation shocks generate a counterfactually positive correlation between unemployment and vacancies. Accordingly, in our estimated model, this shock plays a relatively minor role.

*V.1.2. Impulse responses.* Figure 3 shows the impulse responses of several key labor market variables to a one-standard-deviation negative technology shock, in two different models: our benchmark model (the black solid lines) and a standard model (the blue dashed line). The standard model here corresponds to a version our benchmark, where we impose free entry and constant search and recruiting intensity.

In the benchmark model, a decline in aggregate productivity reduces the value of new job matches. Firms respond by reducing hiring and vacancy postings. These responses lead to a drop in the job finding rate and an increase in the unemployment rate.

Unlike the standard model with free entry, our model with costly vacancy creation implies that the value of an unfilled vacancy is nonzero. Thus, as we have alluded to in the introduction, vacancies become a state variable that evolves slowly over time according to the law of motion in Equation (3). This gives rise to persistent dynamics in vacancies shown in the figure. The initial drop in the stock of vacancies is attributable to declines in newly created vacancies: since the shock reduces the value of an open vacancy ( $J_t^V$ ), firms have less incentive to create new vacancies.

In the benchmark model, the technology shock reduces both search intensity and recruiting intensity. The household's optimizing decisions for search intensity (Eq. (14)) show that search intensity increases with the job finding probability and the employment value, which is proportional to the match surplus from Nash bargaining. Since the technology shock lowers both the job finding rate and the match surplus, it reduces search intensity as well.

Recruiting intensity falls following the negative technology shock partly because the expected value of a job match declines. This can be seen from the optimizing decision for recruiting intensity in Eq. (25), which shows that recruiting intensity increases with both

the job filling probability and the value of a new job match ( $J^F$ ) relative to the value of an unfilled vacancy ( $J^V$ ). However, because the technology shock reduces both  $J^F$  and  $J^V$ , the net effect on recruiting intensity can be ambiguous. Under our estimated parameters, the net surplus falls following a contractionary technology shock. Thus, recruiting intensity falls as well.

Declines in search and recruiting intensity counteract the effects of the rise in unemployment on hiring and reinforce the effects of the drop in vacancies. The net effect leads to a fall in hiring. With both hiring and vacancies declining following the negative technology shock, the response of the job filling rate—which is the ratio of hires to vacancies—is a priori ambiguous. Under our estimation, the job filling rate rises following a decline in productivity, and this countercyclical behavior is in line with the data.

Since search and recruiting intensity both decline in the benchmark model, the technology shock leads to a decline in the measured matching efficiency and thus to an outward shift of the Beveridge curve. The measured matching efficiency here is defined as

$$\Omega_t = \mu s_t^\alpha a_t^{1-\alpha}. \quad (44)$$

Although there are no exogenous shifts in true matching efficiency (i.e.,  $\mu$  is constant), measured matching efficiency ( $\Omega$ ) in our model still fluctuates with endogenous variations in search and recruiting intensity.

Figure 3 also shows that the standard model implies less persistent responses of unemployment, vacancies, and hiring to a technology shock (the blue dashed lines) than those in the benchmark model. Free entry in the standard model implies that unfilled vacancies can be discarded and new vacancies can be created without costs in each period. Thus, the number of vacancies becomes a jump variable rather than a slow-moving state variable as in our benchmark model. Accordingly, the peak effect of the technology shock on vacancies occurs in the impact period, and the dynamic responses of the labor market variables are in general less persistent than in our benchmark model. Furthermore, the standard model does not allow for amplification through cyclical fluctuations in search and recruiting intensity.

Figure 4 shows the impulse responses of the labor market variables in the two models following a one-standard-deviation negative discount factor shock. In the benchmark model (the black solid lines), the shock lowers the continuation value of a job match, leading to declines in hiring. Since the shock reduces the employment surplus, search intensity falls. Recruiting intensity also falls because the decline in the present value of a new job match outweighs the decline in the value of an open vacancy. Firms respond to the drop in vacancy value by reducing vacancy creation and postings. The decline in vacancies more than offsets that in hiring, leading to a rise in the job filling rate. This countercyclical response of

the job filling rate is in line with the data. Similar to the case with a technology shock, the responses of unemployment, vacancies, and hiring to the discount factor shock in the standard model (the blue dashed lines) are less persistent than in the benchmark model, and the intensive margins of adjustments are muted (by construction, given free entry). These results again highlight the importance of incorporating vacancy creation costs and endogenous adjustments in search and recruiting intensity, as we do in our benchmark model, for generating more persistent labor market dynamics.

Figure 5 shows the impulse responses following a positive shock to the job separation rate in the two models. In the benchmark model (the black solid lines), an increase in the rate of job separation leads to a rise in the unemployment rate. Since separated jobs add to the stock of vacancies, the number of vacancies rises along with unemployment. The equilibrium adjustments of hiring also depend on the responses of the intensive margins. The job separation shock reduces both the match value  $J^F$  and the vacancy value  $J^V$ , rendering the net effect on recruiting intensity ambiguous. Under our estimation, recruiting intensity edges down following the separation shock. On net, however, the increases in both  $u$  and  $v$  more than offset the decline in recruiting intensity, leading to an increase in hiring. Thus, the job finding rate increases. Although the match surplus declines, the increase in the job finding rate induces more search effort by the job seekers, so that search intensity rises. In the standard model (the blue dashed lines), an increase in the separation rate also raises  $u$  and  $v$ , as in the benchmark model. But the shock also raises the job filling rate on impact, unlike in the benchmark model. Free entry implies that  $J^V = 0$  and the job creation condition is given by  $\kappa = q^v J^F$ . An increase in job separation lowers the match value  $J^F$ . Thus, given the constant vacancy posting cost  $\kappa$ , the job filling rate rises. The increases in both  $u$  and  $v$  lead to an increase in hiring. The job finding rate declines on impact because the shock raises unemployment more than hiring. Overall, in both models, the job separation shock has relatively small effects on the labor market variables.

**V.2. The Great Recession and the weak job recovery.** We now examine the quantitative performance of our estimated model for explaining the sharp contraction in the labor market during the Great Recession and its subsequent weak recovery. We focus on the job filling rate. As Figure 1 shows, the job filling rate in the data (the blue solid line) surged in the recession and then recovered gradually to its pre-recession level.

The standard model without search and recruiting intensity has difficulties in replicating these observations. As Figure 1 shows (the red dashed lines), the predicted job filling rate from the standard matching function diverged from the data in early 2009, and it has stayed persistently above the data thereafter. These patterns reflect that the standard matching

function fails to replicate the sharp downturn in hiring in the recession and the subdued recovery.

Our model outperforms the standard matching function in predicting the job filling rate. As Figure 6 shows, our model's predicted job filling rate (the black dashed and dotted line) tracks the actual data much more closely than does the standard matching function in the recession and throughout the early part of the recovery up to 2015.

To get a quantitative sense of the goodness of fit of our model relative to the standard matching function, we compare the root mean squared errors (RMSE) of our benchmark model's predicted job filling rate with that implied by the standard matching function. We calculate the RMSEs for the sample period from December 2000 to July 2017, corresponding to the JOLTS sample period. Table 4 (Column (2)) shows that the RMSE from our benchmark model is about 42 percent of that implied by the standard matching function, representing a significant improvement in predicting the job filling rate.

One concern might be that the improvements in fit during the Great Recession and the subsequent recovery come at the cost of a worsening performance in the periods prior to the Great Recession. Figure 7 shows that this is not the case. While we focus on the post-2001 period in Figure 6 because of the availability of JOLTS data, we still estimate our model over a longer period (starting from July 1967) that captures several business cycles. This allows for a comparison of the job filling rate series from our benchmark model with that implied by the standard matching function for the periods prior to 2001. In contrast to the Great Recession and the subsequent recovery periods, Figure 7 shows that the two models' predicted job filling rates track each other more closely before the Great Recession. This result partly reflects the fact that the Great Recession led to a much more pronounced decline in our measure of search intensity than during previous downturns.<sup>14</sup>

However, it is important to note that the improved model performance is not mechanically attributable to the introduction of search intensity in the matching function. For instance, Table 4 (Column (3)) reports the RMSE of the standard matching function modified to include our measure of search intensity.<sup>15</sup> Augmenting the standard matching function with search intensity does improve the prediction for the job filling rate. The RMSE under the augmented matching function is about 85 percent of that under the standard matching function. However, this magnitude of improvement is much smaller than that obtained

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<sup>14</sup>Historical decompositions suggest that the sharp decline in search intensity during the Great Recession was mostly driven by discount factor shocks in our model, which capture financial factors that are not explicitly modeled here. See the online appendix for detailed discussions about the historical decompositions.

<sup>15</sup>Under the matching function augmented with search intensity, the job filling rate is given by  $\mu \left( \frac{u_t s_t}{v_t} \right)^\alpha$ , where the variables and the parameters are defined earlier.

from our general equilibrium model with cyclical fluctuations in both search and recruiting intensity. These findings suggest that incorporating both search intensity and recruiting intensity is important in accounting for the fluctuations in the job filling rate, especially for the post-2008 period.

Our model also implies that recruiting intensity is positively correlated with the hiring rate, as found by Davis et al. (2013), despite clear differences in empirical approaches. Davis et al. (2013) construct a measure of recruiting intensity based on establishment-level data. They show that recruiting intensity delivers a better-fitting Beveridge curve and accounts for a large share of fluctuations in aggregate hires. They further impute an aggregate relation between recruiting intensity and the hiring rate based on their estimated microeconomic relations. They show that this aggregate measure of recruiting intensity is highly correlated with the aggregate hiring rate, with a sample correlation of about 0.82.

The empirical measure of recruiting intensity obtained from our estimated macro model is also procyclical and highly correlated with the hiring rate in the model. In particular, the sample correlation between our model-based time series of recruiting intensity and the hiring rate is 0.83, which is remarkably similar to that reported by Davis et al. (2013). The cyclical behavior of recruiting intensity relies on the model's internal propagation mechanism, and the model is successful in generating procyclical recruiting intensity conditional on matching the time series of unemployment, vacancies, and search intensity. Our result lends support to the argument of Davis et al. (2013) that procyclical recruiting intensity plays an important role in explaining fluctuations in the labor market.

**V.3. Diagnosing the shocks.** We have used a Bayesian approach for estimating the stochastic processes of the shocks to technology, the discount factor, and the separation rate. The literature has typically followed a different approach, with the shock parameters calibrated instead of estimated. For instance, Shimer (2005) first estimates the stochastic processes for average labor productivity and the job separation rate using independent sources of information in the data, and then evaluates the business-cycle performance of a search and matching model simulated using these calibrated driving forces. Our estimated shock parameters turn out to differ somewhat from the calibrated values in the literature. In particular, the estimated standard deviation of the job separation shock is substantially larger than the typical calibrated value. To what extent, then, does our model's performance depend on the estimated value of the shock processes?

To address this question, we consider an alternative empirical strategy. First, we fix the parameters governing the stochastic processes for the technology shock and the job separation shock based on the approach in Shimer (2005). We then estimate the model to fit the time-series data of unemployment, vacancies, and search intensity.

As in Shimer (2005), we use average labor productivity to calibrate the technology shock parameters.<sup>16</sup> This calibration yields a monthly first-order autocorrelation of 0.9908 and a standard deviation of 0.005 for the technology shock. Similarly, we use the job separation rate as constructed by Shimer (2005) and monthly data from July 1967 to July 2017 to calibrate the stochastic process of the separation shock; we obtain an autocorrelation of 0.9806 and a standard deviation of 0.002.

The literature provides no direct evidence on the stochastic processes for the discount factor shock. Hall (2017) notes that calibrating the discount factor shock parameters is model dependent. In particular, it depends on the model's Euler equations, which are functions of expectations of future macroeconomic variables and the entire structure of the model. For this reason, we estimate the discount factor shock parameters along with other structural parameters.

Table 5 shows the estimation results under this alternative empirical approach, as well as the calibrated shock parameters. The posterior estimates of the structure parameters ( $K$ ,  $\kappa_2$ ,  $h_2$ ) are clearly different from those obtained in the benchmark estimation. The estimated discount factor shock is also different: it is highly persistent (with  $\rho_\theta = 0.9998$ ) and very volatile ( $\sigma_\theta = 0.21$ ).<sup>17</sup>

The model estimated with this alternative approach does not perform as well as the benchmark estimation in fitting the observed job filling rate. As shown in Table 4 (Column (4)), the RMSE under this alternative estimation is about 58 percent of that implied by the standard matching function.

To further examine the role of the shock processes in driving our model's dynamics, we estimated the model with all three shocks calibrated. In particular, we calibrate the technology and separation shocks following Shimer (2005), and we also calibrate the discount factor shock parameters to the values obtained under our benchmark estimation. As a result, the discount factor shock is not as volatile as in the previous exercise. In this version of the

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<sup>16</sup>The sample period covers 1967:Q3-2017:Q3. To stay consistent with the measurement in our model, we remove a linear trend from the average productivity series (instead of applying the HP filter). We convert the calibrated autocorrelation and standard deviation from quarterly values to monthly values.

<sup>17</sup>The data prefer the benchmark estimation to the alternative estimation, since the posterior data density for the benchmark estimation is much higher. To check whether the actual data lie in the tail of the model's posterior distribution when the technology shock and separation shock parameters are restricted to their calibrated values, we have performed a posterior predictive check following the approach described by An and Schorfheide (2007) and Faust and Gupta (2012). The posterior predictive analysis indicates that the calibrated parameters for the technology and separation shocks lie within the 90 percent probability intervals of the posterior distributions. See the online appendix for more details. We thank an anonymous referee for suggesting this approach.

model, we obtain an RMSE of 71 percent of that implied by the standard matching function, as shown in Table 4 (Column (5)).

Overall, the model's prediction errors (measured by the RMSE) with calibrated shocks are clearly greater than those under the benchmark estimation. Nonetheless, the improvement relative to the standard matching function is still economically important. Thus, our model's performance in predicting the actual job filling rate is not entirely driven by the estimated shocks, and endogenous cyclical fluctuations in search and recruiting intensity are quantitatively important for explaining the weak job recovery.

**V.4. The importance of using information from search intensity data.** In estimating our benchmark model, we have used three time series: the unemployment rate, the job vacancy rate, and a measure of search intensity. We followed Davis (2011) and constructed a time series of search intensity based on the median unemployment duration. The resulting search intensity series is procyclical, as shown in Figure 2. The procyclical behavior of search intensity is consistent with the textbook model (Pissarides, 2000).<sup>18</sup>

Yet, the empirical literature is not conclusive about whether search intensity is procyclical. For example, Shimer (2004) argues that search intensity is countercyclical based on evidence from cross-sectional data of the average number of search methods used by job seekers observed in the CPS. Mukoyama et al. (2014) combine information from the CPS data and the American Time Use Survey (ATUS) and obtain similar results.

On the other side of the debate, Tumen (2014) emphasizes that the cross-sectional measures based on CPS data are likely to suffer from a composition bias if a job seeker with stronger labor market attachment also uses more search methods. Since the share of job seekers with stronger labor-market attachment increases during a recession, the measured search intensity appears to be countercyclical. When the composition bias is corrected, Tumen (2014) finds that search intensity is procyclical. Gomme and Lkhagvasuren (2015) make a similar argument about the composition bias. They use merged data from the ATUS and the CPS to study cyclical variations in search intensity. They find that, when the composition bias is corrected, the evidence suggests procyclical search intensity.<sup>19</sup>

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<sup>18</sup>Our measure of search intensity is constructed based on estimated parameters using longitudinal data that track unemployed workers' amount of time spent for job searching as well as the number of weeks they have been unemployed (see Davis (2011)). A drawback of this method is that it is based on answers from interviews conducted over a 24-week period during the fall of 2009 and winter of 2010, so the estimation uses a relatively short time-series dimension.

<sup>19</sup>Mueller (2017) shows that the pool of unemployed shifts to high-wage workers in recessions. If high-wage workers search more intensely, this can lead to a substantial composition bias. Faberman and Kudlyak (2016) also discuss the implications of composition bias in measuring search intensity. They report that



Given this debate, we assess the robustness of our findings by fitting our model to the observed unemployment rate and the vacancy rate only. Under this alternative estimation, we do not use information of search intensity in the data.<sup>20</sup>

When we estimate the model to fit unemployment and vacancies data without using information from search intensity data, we obtain an RMSE for the job filling rate of about 58 percent relative to that implied by the standard matching function. This prediction error is clearly larger than that under our benchmark estimation. But the improvement relative to the standard matching function is substantial.

In addition, information from search intensity in the data also has implications for the estimated cyclical behaviors of recruiting intensity. When we do not use information from search intensity to estimate the model, the correlation between recruiting intensity and hiring becomes smaller than that in the benchmark estimation (0.46 vs. 0.83).

Overall, these exercises suggest that there are important general equilibrium interactions between search and recruiting intensity that amplify the impact of the shocks on labor market variables. Procyclical fluctuations in search intensity and recruiting intensity help bridge the gap between the model's predicted job filling rate and that in the data.

## VI. CONCLUSION

The sharp contraction in the labor market during the Great Recession and the subsequent weak recovery present a challenge for the standard model of labor search and matching. We have developed and estimated a DSGE model that generalizes the standard model to incorporate cyclical fluctuations of search and recruiting intensity. We find that these intensive margins of labor market adjustments are quantitatively important. In the depth of the recession and during the early part of the recovery, the job filling rate predicted from our estimated model is much closer to the actual time-series data than those implied by the standard matching function. Our model suggests that procyclical fluctuations in search and recruiting intensity played an important role in explaining the deep recession and the weak recovery.

To allow for aggregate fluctuations in recruiting intensity, we modified the standard model by assuming that creating a new job vacancy is costly. This simple modification facilitates tractability and makes it straightforward to estimate the model to fit time-series data using standard techniques. Our macro emphasis nonetheless yields predictions of the cyclical movements in recruiting intensity that are very much in line with those obtained by Davis

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long-term unemployed job seekers tend to exert more efforts throughout their search process, reflecting that long-term unemployed individuals have stronger labor market attachment.

<sup>20</sup>To conserve space, we report the priors and posterior estimation results in the online appendix.

et al. (2013) based on establishment-level data. In particular, both approaches highlight that recruiting intensity is procyclical. In addition, our empirical findings also point to an important interaction between search and recruiting intensity that helps account for the observed behavior of the job filling rate since the onset of the Great Recession.

To better highlight our model's mechanism, we focus on three particular sources of business cycle fluctuations: technology shocks, discount factor shocks, and job separation shocks. All of these shocks are arguably reduced-form representations of some microeconomic frictions or policy distortions that are not considered in our model. For example, the discount factor shock in our model that represents stochastic changes in households' intertemporal preferences may reflect factors outside of the model that drive asset market fluctuations (Hall, 2017). Our model also assumes that job separations vary exogenously, while in reality, job separations occur endogenously in response to the state of the economy.

Our model also restricts the labor force participation rate to be constant. Relaxing this assumption can have important implications for labor market dynamics. For example, Diamond (2013) argues that incorporating flows into and out of the labor force helps better understand the shifts of the Beveridge curve after the Great Recession. Kudlyak and Schwartzman (2012) show that persistent declines in labor force participation contributed to the large increases in unemployment during the Great Recession and to the subsequent slow decline in unemployment. Future research should extend the framework in this paper to incorporate endogenous job separations and labor force participation to study their potential interactions with search and recruiting intensity. This more general framework would help further improve our understanding of labor market fluctuations and policy design.

TABLE 1. Calibrated parameters

Parameter	Description	value
$\beta$	Subjective discount factor	0.9967
$\phi$	Unemployment benefit	0.25
$\alpha$	Elasticity of matching function	0.50
$\mu$	Matching efficiency	0.4864
$\bar{\delta}$	Job separation rate	0.0147
$\rho^o$	Vacancy obsolescence rate	0.0196
$\kappa_0$	Steady-state advertising cost	0.1887
$\kappa_1$	Slope of vacancy posting cost	0.1902
$h_1$	Slope of search cost	0.1088
$b$	Nash bargaining weight	0.50
$\gamma$	Real wage rigidity	0.95
$\xi$	Elasticity of vacancy creation	1
$\chi$	Mean value of preference shock	0.6650
$\bar{Z}$	Mean value of technology shock	1

TABLE 2. Estimated parameters

Parameter description	Priors		Posterior		
	Type	[mean, std]	Mean	5%	95%
$K$ vacancy creation cost	G	[5, 1]	3.8888	3.5572	4.2272
$\kappa_2$ vacancy posting cost	G	[5, 1]	5.8751	5.6821	6.0537
$h_2$ search cost function	G	[5, 1]	0.9928	0.9778	1.0116
$\rho_z$ AR(1) of tech shock	B	[0.8, 0.1]	0.9953	0.9952	0.9953
$\sigma_z$ std of tech shock	IG	[0.01, 0.1]	0.0194	0.0171	0.0217
$\rho_\theta$ AR(1) of dis. factor shock	B	[0.8, 0.1]	0.9932	0.9928	0.9937
$\sigma_\theta$ std of dis. factor shock	IG	[0.01, 0.1]	0.0302	0.0290	0.0313
$\rho_\delta$ AR(1) of sep shock	B	[0.8, 0.1]	0.8429	0.8370	0.8492
$\sigma_\delta$ std of sep shock	IG	[0.01, 0.1]	0.1868	0.1829	0.1909

*Note:* This table shows our benchmark estimation results. For the prior distribution types, we use G to denote the gamma distribution, B the beta distribution, and IG the inverse gamma distribution.

TABLE 3. Forecasting Error Variance Decomposition

Variables	Technology shock	Discount factor shock	Job separation shock
Unemployment	32.10	67.27	0.63
Vacancy	38.93	58.85	2.21
Search intensity	26.83	73.13	0.04
Recruiting intensity	51.95	47.93	0.12
Hiring	48.11	33.29	18.60
Job filling	35.17	64.81	0.03
Aggregate output	94.36	5.59	0.05

*Note:* The numbers reported are the posterior mean contributions (in percentage terms) of each of the three shocks in the benchmark estimation to the forecast error variances of the variables listed in the rows.

TABLE 4. Prediction errors (RMSE) relative to the standard matching function for the job filling rate

(1)	(2)	(3)	(4)	(5)	(6)
Standard match function	Benchmark estimation	Standard match function with search intensity	Estimation with calibrated $z$ and $\delta$ shocks	Estimation with all shocks calibrated	Estimation without search intensity data
1	0.419	0.856	0.581	0.707	0.579

*Note: This table shows the prediction errors of alternative models (or estimation methods) for the job filling rate, which is defined as the ratio of hires to the end-of-period job vacancies. We measure the prediction errors by the root mean squared error (RMSE), and we normalize the RMSE implied by the standard matching function to one (Column (1)). The rest of the columns show the ratios of the RMSE in each case to that implied by the standard matching function. Column (2) shows the relative RMSE of the benchmark estimation of our DSGE model, Column (3) shows the standard matching function augmented with search intensity, Column (4) shows the estimation of the DSGE model with calibrated parameters for the technology shock and the job separation shock, Column (5) shows the estimation of the DSGE model with calibrated parameters for all three shocks, and Column (6) shows the estimation without using search intensity data. The sample period for computing the RMSEs ranges from December 2000 to July 2017, corresponding to the JOLTS sample.*

TABLE 5. Alternative estimation

Estimated parameters						
Parameter description	Priors		Posterior			
	Type	[mean, std]	Mean	5%	95%	
$K$ vacancy creation cost	G	[5, 1]	0.9779	0.9771	0.9780	
$\kappa_2$ vacancy posting cost	G	[5, 1]	0.9775	0.9772	0.9776	
$h_2$ search cost function	G	[5, 1]	1.6037	1.6017	1.6090	
$\rho_\theta$ AR(1) of dis. factor shock	B	[0.8, 0.1]	0.9998	0.9997	0.9999	
$\sigma_\theta$ std of dis. factor shock	IG	[0.01, 0.1]	0.2149	0.2142	0.2159	
Calibrated shock parameters						
Parameter description	Calibrated value					
$\rho_z$ AR(1) of tech shock	0.9908					
$\sigma_z$ std of tech shock	0.0050					
$\rho_\delta$ AR(1) of sep shock	0.9806					
$\sigma_\delta$ std of sep shock	0.0020					

*Note:* This table shows the estimation results when we fix the parameters in the technology and separation shocks to their calibrated values following the approach in Shimer (2005). The technology shock parameters are calibrated based on the quarterly time series of average labor productivity from 1967Q3 to 2017Q3, with a linear trend removed. We convert the autocorrelation and standard deviation parameters in the technology shock from quarterly frequency to monthly frequency. The job separation shock parameters are calibrated using monthly data from July 1967 to July 2017, following Shimer's (2005) approach.

## Job Filling Rate

3-month moving average, logged

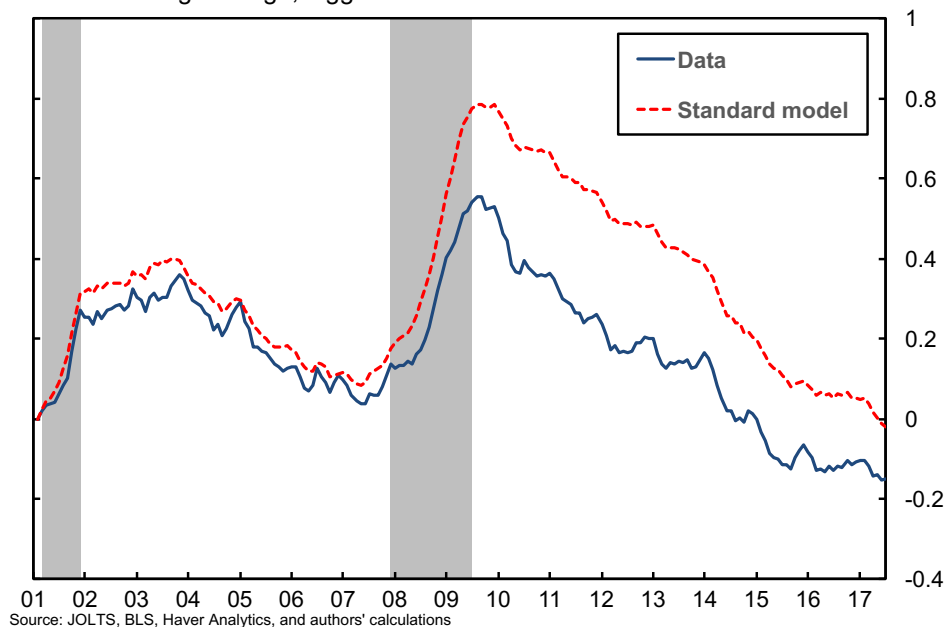


FIGURE 1. Job filling rate: Data vs. standard matching function. The shaded areas indicate the NBER recession dates. The job filling rate in the data is defined as the ratio of hiring to the end-of-period job vacancies. The job filling rate implied by the standard matching function is calculated based on the matching function  $m_t = \mu u_t^\alpha v_t^{1-\alpha}$ , where  $m_t$  denotes new job matches,  $u_t$  and  $v_t$  denote unemployment and job vacancies, respectively,  $\alpha$  measures the elasticity of matching with respect to unemployment, and  $\mu$  is a scale parameter. The job filling rate is given by  $q_t^v \equiv \frac{m_t}{v_t} = \mu \left(\frac{v_t}{u_t}\right)^{-\alpha}$  and the job finding rate is given by  $q_t^u \equiv \frac{m_t}{u_t} = \mu \left(\frac{v_t}{u_t}\right)^{1-\alpha}$ . Since these two implied series are perfectly (and negatively) correlated, we choose to display only the job filling rate in the figure. In calculating the job filling rate implied by the standard matching function, we follow Davis et al. (2013) and use the calibrated parameter  $\alpha = 0.5$ , the observed job openings from the JOLTS, and the unemployment rate from the BLS. (The qualitative results are similar for  $\alpha = 0.4$  or  $\alpha = 0.6$ .) All series are in log terms and normalized relative to the February 2001 observation, corresponding to the starting point of the 3-month moving averages of the JOLTS data. With this normalization, the scale parameter  $\mu$  in the matching function becomes irrelevant.



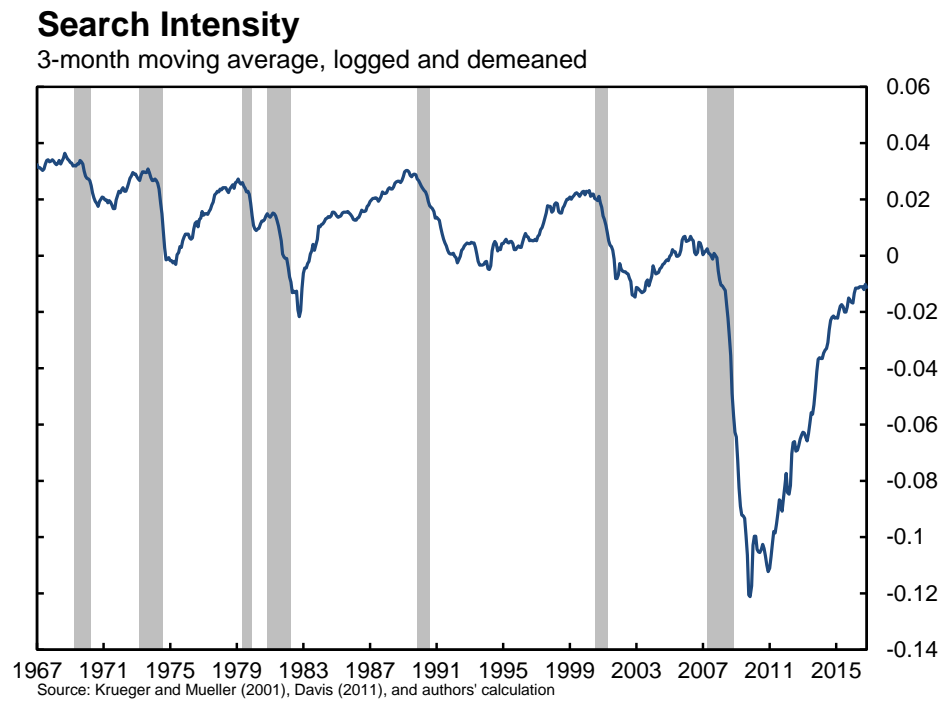


FIGURE 2. Time series of search intensity. The shaded areas indicate recession dates. The search intensity series is imputed from the median duration of unemployment (weeks) based on the regression analysis of Davis (2011).

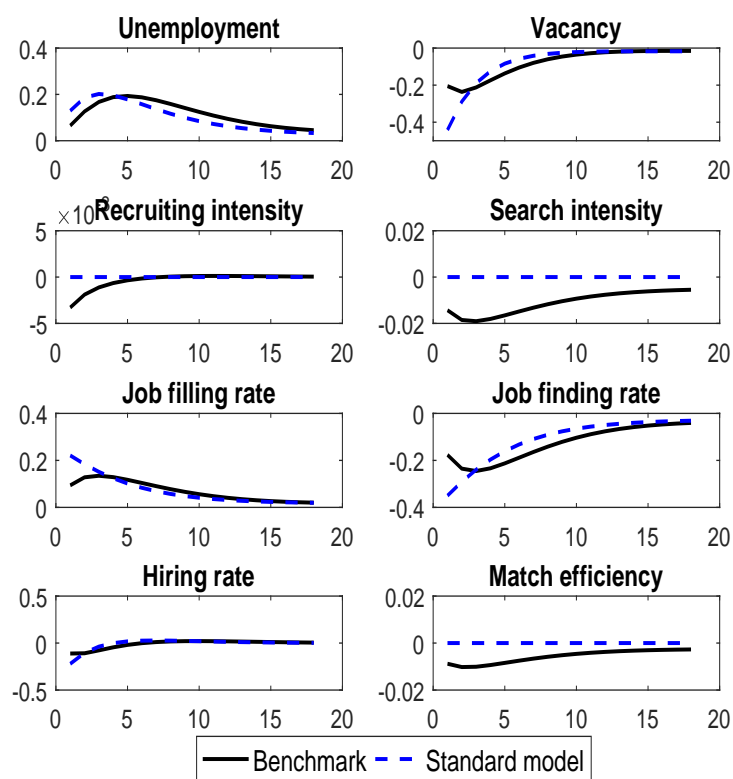


FIGURE 3. Impulse responses to a negative technology shock. The solid lines are impulse responses from the benchmark model. The dashed lines are from the standard model, which assumes free entry and constant search and recruiting intensity.

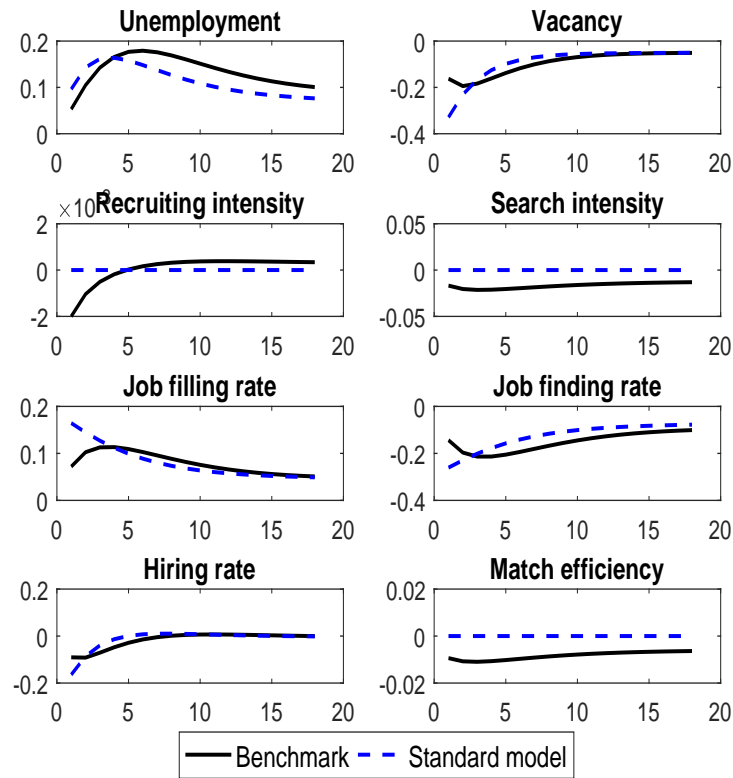


FIGURE 4. Impulse responses to a negative discount factor shock. The solid lines are impulse responses from the benchmark model. The dashed lines are from the standard model, which assumes free entry and constant search and recruiting intensity.

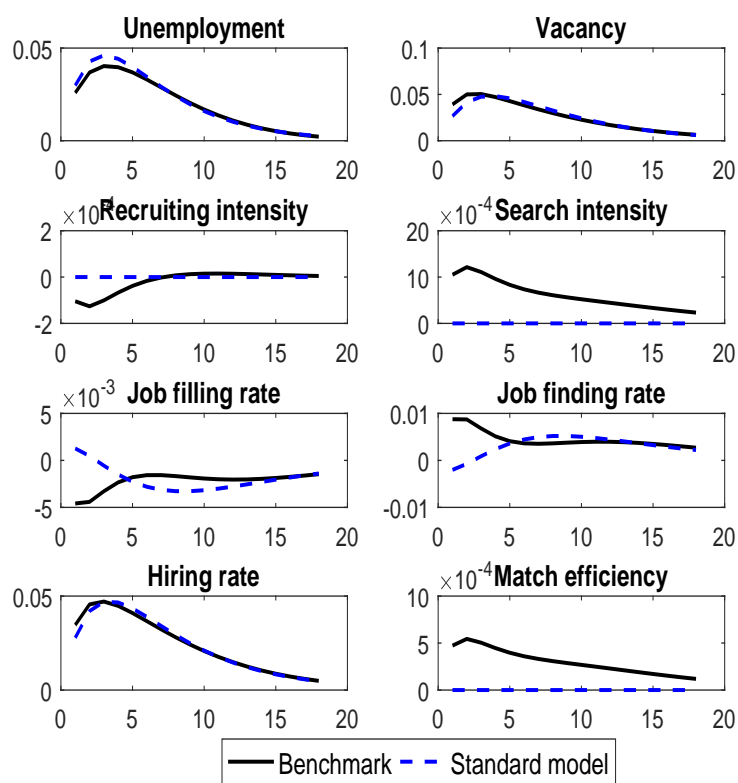


FIGURE 5. Impulse responses to a positive job separation shock. The dashed lines are from the standard model, which assumes free entry and constant search and recruiting intensity.

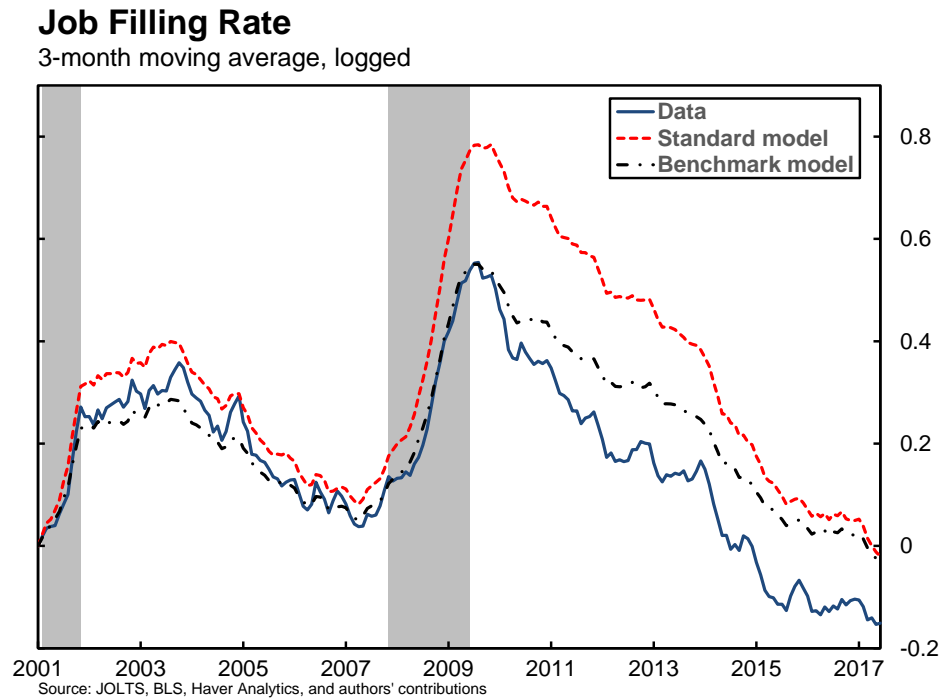


FIGURE 6. Job filling rate: Data, standard matching function, and benchmark DSGE model. The shaded areas indicate the NBER recession dates. For explanations of the variable constructions in the data and the standard model, see Figure 1. The job filling rate in the benchmark DSGE model is the smoothed series from the estimated DSGE model.

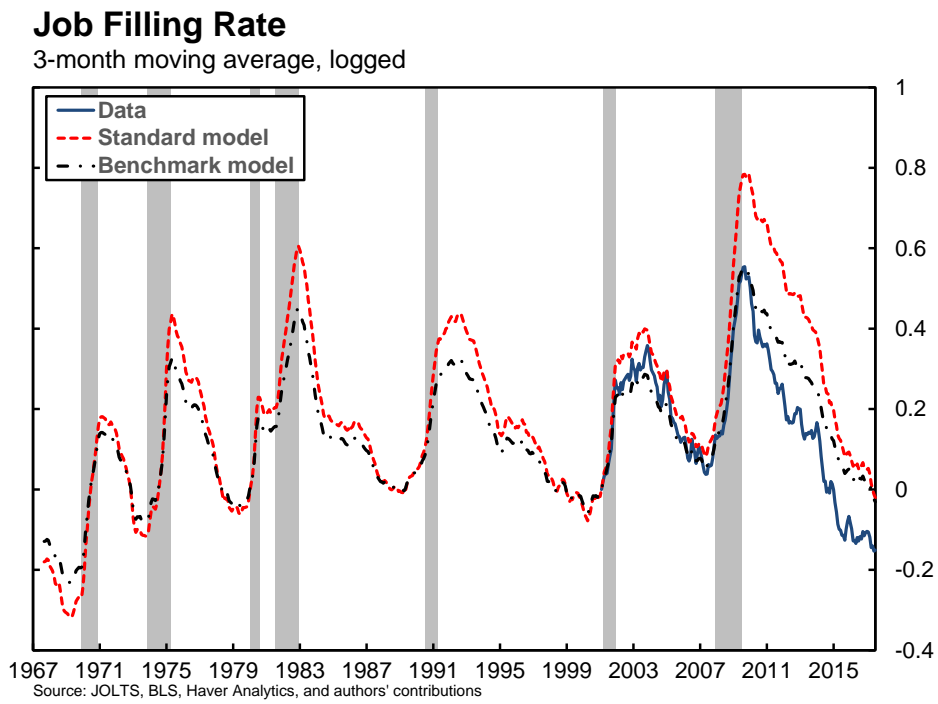


FIGURE 7. Job filling rate: Data, standard matching function, and benchmark DSGE model in the full sample. The shaded areas indicate the NBER recession dates. For explanations of the variable constructions, see Figure 6.

## APPENDIX A. DERIVATIONS OF HOUSEHOLD'S OPTIMIZING CONDITIONS

Our approach to incorporating search intensity in the DSGE model builds on the textbook treatment by Pissarides (2000). The basic idea is that the representative household can choose the effort level that is devoted to searching for those members who are unemployed. Increasing search effort incurs some resource costs, but it also creates the benefits of increasing the individual searching worker's job finding rate.

We now derive the optimal search intensity decision from the first principle. To economize notations, we do not carry around the individual index  $i$  in describing the household's optimizing problem. Keep in mind that, in choosing the individual search intensity and employment, the household takes the economy-wide variables as given. In a symmetric equilibrium, the individual optimal choices coincide with the aggregate optimal choices.

Denote by  $V_t(B_{t-1}, N_{t-1})$  the value function for the representative household. The household's optimizing problem can be written in the recursive form

$$V_t(B_{t-1}, N_{t-1}) \equiv \max \ln C_t - \chi N_t + \beta \mathbb{E}_t \theta_{t+1} V_{t+1}(B_t, N_t). \quad (\text{A1})$$

The household's utility-maximizing decision is subject to the budget constraint

$$C_t + \frac{B_t}{r_t} = B_{t-1} + w_t N_t + \phi(1 - N_t) - u_t h(s_t) + d_t - T_t, \quad (\text{A2})$$

and the law of motion for employment

$$N_t = (1 - \rho^o)(1 - \delta_t)N_{t-1} + q^u(s_t)u_t, \quad (\text{A3})$$

where the measure of job seekers is given by

$$u_t = 1 - (1 - \rho^o)(1 - \delta_t)N_{t-1}. \quad (\text{A4})$$

The household chooses  $C_t$ ,  $B_t$ ,  $N_t$ , and  $s_t$ , taking prices and the average job finding rate as given.

Denote by  $\Lambda_t$  the Lagrangian multiplier for the budget constraint (A2). The first-order condition with respect to consumption implies that

$$\Lambda_t = \frac{1}{C_t}. \quad (\text{A5})$$

The optimizing decision for  $B_t$  implies that

$$\frac{\Lambda_t}{r_t} = \beta \mathbb{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial B_t}, \quad (\text{A6})$$

where  $\theta_t \equiv \frac{\Theta_t}{\Theta_{t-1}}$  denotes the discount factor shock. Combining equation (A6) with the envelope condition with respect to  $B_{t-1}$ , we obtain the intertemporal Euler equation

$$1 = \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} r_t, \quad (\text{A7})$$

which is equation (16) in the text.

Optimal choice of search intensity  $s_t$  implies that

$$h'(s_t) = \frac{q_t^u}{s_t} \left[ w_t - \phi - \frac{\chi_t}{\Lambda_t} + \beta \mathbb{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t} \frac{1}{\Lambda_t} \right], \quad (\text{A8})$$

where we have used equation (13) to replace the term  $\frac{\partial q^u(s_t)}{\partial s_t}$  by  $\frac{q_t^u}{s_t}$ . The envelope condition implies that

$$\frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} = \left[ \Lambda_t (w_t - \phi) - \chi_t + \beta \mathbb{E}_t \theta_{t+1} \frac{\partial V_{t+1}(B_t, N_t)}{\partial N_t} \right] \frac{\partial N_t}{\partial N_{t-1}} - \Lambda_t h_t \frac{\partial u_t}{\partial N_{t-1}} \quad (\text{A9})$$

Equations (A3) and (A4) imply that

$$\frac{\partial N_t}{\partial N_{t-1}} = (1 - \rho^o)(1 - \delta_t)(1 - q^u(s_t)) \quad (\text{A10})$$

and that

$$\frac{\partial u_t}{\partial N_{t-1}} = -(1 - \rho^o)(1 - \delta_t). \quad (\text{A11})$$

Define the employment surplus (i.e., the value of employment relative to unemployment) as

$$\begin{aligned} S_t^H &= \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_t} = \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} \frac{\partial N_{t-1}}{\partial N_t} \\ &= \frac{1}{\Lambda_t} \frac{\partial V_t(B_{t-1}, N_{t-1})}{\partial N_{t-1}} \frac{1}{(1 - \rho^o)(1 - \delta_t)(1 - q^u(s_t))}. \end{aligned} \quad (\text{A12})$$

Thus,  $S_t^H$  is the value for the household to send an additional worker to work in period  $t$ . Then the envelope condition (A9) implies that

$$S_t^H = w_t - \phi - \frac{\chi}{\Lambda_t} + \frac{h_t}{1 - q_t^u} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \rho^o)(1 - \delta_{t+1})(1 - q_{t+1}^u) S_{t+1}^H. \quad (\text{A13})$$

The employment surplus  $S_t^H$  derived here corresponds to equation (15) in the text and it is the relevant surplus for the household in the Nash bargaining problem.

Given the definition of employment surplus in equation (A12), the optimal search intensity decision (A8) can be rewritten as

$$h'(s_t) = \frac{q_t^u}{s_t} \left[ w_t - \phi - \frac{\chi}{\Lambda_t} + \mathbb{E}_t \frac{\beta \theta_{t+1} \Lambda_{t+1}}{\Lambda_t} (1 - \rho^o)(1 - \delta_{t+1})(1 - q_{t+1}^u) S_{t+1}^H \right]. \quad (\text{A14})$$

Thus, at the optimum, the marginal cost of search intensity equals the marginal benefit, where the benefit derives from the increased job finding rate and the net value of employment. This last equation corresponds to equation (14) in the text.



## APPENDIX B. DATA

We fit the DSGE model to three monthly time series for the U.S. labor market: the unemployment rate, job vacancies, and a measure of search intensity. We also use monthly time-series data of hires to construct our measure of the job filling rate.

- (1) Unemployment: Civilian unemployment rate (16 years and over) from the Bureau of Labor Statistics (LR@USECON in Haver), seasonally adjusted monthly series.
- (2) Job vacancies: For the period from December 2000 and on, we use the seasonally adjusted job opening rate series from JOLTS (LJJTPA@USECON in Haver). For the period prior to December 2000, we use the measure of the job vacancy rate constructed by Barnichon (2010) based on the Help Wanted Index.
- (3) Search intensity: constructed based on the approach in Davis (2011), using the following empirical relation between search intensity ( $s_t$ ) and the duration of unemployment ( $d_t$ )

$$s_t = 122.30 - 0.90d_t, \tag{A15}$$

where  $d_t$  is measured by the seasonally adjusted monthly series of the median duration of unemployment (in weeks) reported in the Current Population Survey (WAMED@EMPL in Haver).

- (4) Hires: Total hires rate from JOLTS (LJHTPA@USECON in Haver), seasonally adjusted monthly series.
- (5) The job filling rate: the ratio of hires to job vacancies.

The sample range for the unemployment rate, the job vacancy rate, and the measure of search intensity covers the period from July 1967 to July 2017. The sample for the hiring rate from the JOLTS ranges from December 2000 to July 2017.

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