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The Benefit of Inflation-Indexed Debt: Evidence from an Emerging Bond Market

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Abstract

Portfolio diversification is as important to debt management as it is to asset management. In this paper, we focus on diversification of sovereign debt issuance through greater reliance on inflation-indexed bonds for an emerging economy, Colombia. Using an arbitrage-free dynamic term structure model of fixed-coupon and inflation-indexed bond prices, we account for inflation and liquidity risk premia and calculate the forward-looking net benefit of issuing inflation-indexed bonds over nominal bonds. Our results suggest that the Colombian government could lower its funding costs by as much as 0.69 percent through increased issuance of inflation-indexed debt.

JEL Classification: D84, E31, E43, E44, G12

Keywords: term structure modeling, inflation risk, liquidity risk, financial market frictions, central bank credibility, debt management

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1 Introduction

Portfolio diversification is as important to debt management as it is to asset management. With respect to sovereign debt issuance, several studies have examined issues of time-varying maturity structure of debt as well as default resolution through maturity extension.[1] In light of the significant fiscal consequences arising from the COVID-19 pandemic and its aftermath, questions regarding the magnitudes of sovereign bond issuance and the optimization of its issuance terms are worthy of further review. This is particularly true for emerging economies whose debt capacity is widely perceived to be lower than that of more advanced economies that have a long tradition of deep and well-established financial markets. Furthermore, as global financial conditions have tightened while government deficits remain large, practical questions arise regarding how to finance new and maturing debt in a manner sustainable for both governments and their domestic financial markets.

In this paper, we examine a potentially relevant aspect of this sovereign debt management challenge; namely, whether the government might benefit from switching its debt issuance away from well-established nominal fixed-coupon bonds and towards inflation-indexed bonds.[2,3]

The choice between issuing conventional debt with a fixed notional amount and inflation-indexed debt, which maintains its real value, involves a tradeoff. When the government issues nominal debt, investors assume the inflation risk. Provided investors as a group are rational and forward-looking, they will demand a premium for taking on this risk, referred to as the inflation risk premium. If, instead, the government chooses to issue inflation-indexed debt, investors are protected against inflation risk as the cash flows of such financial claims (coupons and notional amount) are adjusted with the change in the economy-wide price index. As a consequence, the government can avoid paying the inflation risk premium and lower its debt servicing costs, assuming the government is less averse to inflation than investors.

One reason why the government may be less averse to inflation risk is that it controls the central bank and hence can directly affect inflation outcomes, unlike investors. Another reason is that taxes and many public benefits have structures with nominal rigidities such as fixed income brackets that are only adjusted with a lag, again with the government deciding when and what adjustments are made. This means that unexpected positive shocks to inflation, which are very costly to holders of nominal debt, tend to have a positive effect on the government's fiscal position, at least initially.[4] Moreover, positive inflation shocks in and of themselves erode the real value of the government's nominal debt and transfers wealth from bond holders to the government and taxpayers. In principle, this makes nominal debt

[1] See Arellano and Ramanarayanan (2012) and Michalache (2020), respectively.

[2] Ermolov(2021) studies this question for a large set of advanced economies, but fails to account for the liquidity risk premia of nominal bonds unlike the analysis in this paper.

[3] Christensen et al. (2024) examine whether the U.S. government could benefit from extending the maximum maturity of its debt issuance, which represents another way to diversify a government's debt portfolio. In our analysis, we stay within the prevailing maturity structure.

[4] See Neely (2022) for a recent discussion in the case of U.S. government debt.

preferable to the government in an unconditional sense, but investors are aware and demand the inflation risk premium in return, and the larger that premium is, the greater is the benefit and potential savings of inflation-indexed debt. These are the standard arguments put forward in favor of greater government reliance on inflation-indexed debt; see Price (1997) for an overview.[5]

Against this backdrop, one may ask why most government debt around the world remains nominal.[6] What can account for the low issuance of inflation-indexed debt despite this clear economic argument in its favor? The theory of revealed preferences suggests that some costs must be preventing governments from following this seemingly pareto-improving strategy.

One key factor is likely to be frictions in financial markets, which may impede the trading and liquidity of inflation-indexed debt in secondary markets and in turn lower the prices achieved in primary auctions. These frictions could materialize for several reasons. In the following, we identify one key friction that flows from the economic reasoning above and relates to the natural demand for inflation-indexed bonds.

If inflation shocks and changes to investors' outlook for inflation represent a major reason for investors to trade standard fixed-coupon securities, a security that is insulated from this risk should be traded less, all else being equal. This means that the trading environment for inflation-indexed bonds may be rather different and less active than the market for nominal securities.[7] In turn, this may have a determining effect on the types of investors who choose to hold this debt. First, the classic economic argument outlined earlier clearly has *domestic* holders in mind in that they are the ones exposed to the inflation risk protected by the inflation-indexed bonds. Hence, foreign investors may have little appetite for this type of debt.[8] Second, the lower level of trading may imply that the investor segment holding inflation-indexed bonds could be dominated by patient buy-and-hold investors defined as those who are less likely to be hit with liquidity shocks and forced to liquidate their holdings. Ultimately, these negative market dynamics can feed on themselves and become self-fulfilling—investors' fear of illiquidity cause the inflation-indexed bonds to become illiquid. This drives up the search frictions in the over-the-counter market for these bonds and leads to higher liquidity risk premia in the steady state of the market; see Duffie et al. (2005). As a consequence, the liquidity risk profile of inflation-indexed debt may be very different from that of nominal debt, and the

[5]We note that there are caveats to this basic reasoning that may make inflation-indexed debt less desirable from the point of view of the government. For example, if tax evasion for some reason is easier when inflation is high or taxes are based on nominal income and collected with a significant lag, the government's financial position could be negatively correlated with inflation. In that case, inflation-indexed debt could be risky for the government.

[6]In the United States, for example, Treasury Inflation-Protected Securities (TIPS) only account for about 8 percent of marketable government debt.

[7]Swanson and Williams (2014) find that positive surprises in inflation cause U.S. Treasury yields of all maturities to rise, while Beechey and Wright (2009) report no significant response of U.S. TIPS yields to such surprises.

[8]Beauregard et al. (2024) document for Mexico that only a very small fraction of its government's inflation-indexed debt is held by foreigners, while more than half of the nominal debt is foreign held.

premium investors demand for assuming the liquidity risk of these harder-to-sell securities could be quite sizable.[9] Finally, provided the trading in inflation-indexed debt securities is indeed concentrated among domestic investors, it follows as a corollary that their liquidity risk premia should be driven primarily by domestic factors and economic developments.[10]

In contrast, the inflation risk premium, which is compensation demanded by the holders of nominal bonds, is more likely to be determined by global factors and international developments. By the same logic, the liquidity risk premia of nominal debt securities are also likely to be heavily influenced by foreign factors.[11]

To summarize, there is a tradeoff to consider when a government decides whether to issue nominal or inflation-indexed debt. Although classic economic arguments seem to favor inflation-indexed debt, trading conditions and differences in investor preferences may cause the liquidity risk premia of inflation-indexed debt to be sufficiently large as to offset both the inflation risk premium and any liquidity risk premia in nominal debt. In that case, it may *not* be beneficial to issue inflation-indexed debt. Ultimately, it is an empirical question which of these factors dominates in any given government bond market.

For evidence of the potential benefits of the strategic switch in debt issuance away from nominal bonds and towards inflation-indexed debt, we choose to focus on Colombia, which is a country in which inflation and associated risk premia are likely to play a first-order role given its long history of high and fairly volatile inflation.[12] Additional motivations underlie this choice. First, Colombia has well-functioning markets for both standard nominal fixed-coupon government bonds, so-called bonos del tesoro, and real inflation-indexed government bonds, known as bonos del tesoro UVR (Unidad de Valor Real), with available data starting in 2005. Thus, the debt diversification question we consider is one of relevance to the Colombian government. Second, we think of Colombia as a representative emerging market economy for other economic and institutional reasons. Like many of its emerging market peers, Colombia has its own currency with a floating exchange rate. For monetary policy it has followed an inflation-targeting regime for many years, which is by now well established. Finally, it has open and internationally integrated financial markets.[13] For all these reasons we consider Colombia and its government bond markets to be a useful and informative case study for the wider set of large and medium-sized open emerging market economies in which both of these

[9] A large literature has documented sizable liquidity risk premia in the market for U.S. TIPS; see Andreasen et al. (2021), D'Amico et al. (2018), and Pflueger and Viceira (2016), among many others.

[10] Ceballos et al. (2025) estimate liquidity risk premia for Chilean inflation-indexed government bonds that are sizable and significantly influenced by the holdings of domestic pension funds.

[11] Christensen et al. (2021) find that the foreign share is a key determinant of liquidity risk premia in the Mexican bonos market, while Beauregard et al. (2024) find that Mexican inflation risk premia are determined by the foreign share of the Mexican bonos market in addition to global factors like oil prices, U.S. interest rates, and the VIX, a measure of risk aversion in the U.S. stock market.

[12] For our baseline 2005-2020 period, year-over-year inflation in Colombia as measured by the consumer price index averaged 4.13 percent with a standard deviation of 1.68 percent. For comparison, the respective corresponding statistics were 1.70 percent and 0.83 percent for Canada, 4.07 percent and 1.01 percent for Mexico, and 2.00 percent and 1.33 percent for the United States.

[13] See online Appendix A for further evidence of Colombia as a representative emerging market economy.

two types of debt securities are issued and being traded.[14]

The starting point for our empirical analysis is breakeven inflation (BEI)—the difference between yields on comparable-maturity nominal and real debt. This is a frequently used indicator of inflation expectations. However, as widely noted by most observers, BEI is a noisy measure of expected inflation because it contains both an inflation risk premium and differential liquidity risk premia. Thus, for our research question and to fully understand the variation in BEI, we need estimates of both the differential liquidity risk premia in nominal and real bond prices and investors' underlying inflation expectations in order to get an estimate of the inflation risk premium, which represents the main theoretical economic benefit of inflation-indexed debt.

The challenge in accounting for the differential liquidity risk premia in nominal and real bond prices is to distinguish them from the more fundamental factors such as inflation risk premia that would affect asset prices even in a world without any frictions to trading. To achieve this separation, we follow Beauregard et al. (2024, henceforth BCFZ), who introduce a flexible dynamic term structure model of nominal and real bond prices with separate liquidity risk factors for nominal and real bonds that we henceforth refer to as the BCFZ model. For each class of bonds, the identification of the liquidity risk factor comes from its unique loading, which mimics the idea that, over time, an increasing amount of the outstanding notional value of individual securities gets locked up in buy-and-hold investors' portfolios. This increases their sensitivity to variation in the marketwide liquidity risk captured by the corresponding liquidity risk factor. By observing prices for balanced panels of nominal and real bonds, their respective liquidity risk factors can be separately identified.

We estimate this novel model, as detailed in BCFZ, using our Colombian data. In terms of our empirical findings, we make a number of observations. First, our results indicate that the average liquidity risk premia embedded in both nominal and real Colombian bond yields exhibit notable time variation. For nominal yields, the estimated liquidity risk premia average 41 basis points with a standard deviation of 28 basis points. For real yields, the estimated liquidity risk premia average 244 basis points with a standard deviation of 27 basis points. Thus, as conjectured, the liquidity risk premia of Colombian inflation-indexed government bonds are significantly larger than those of standard Colombian nominal government bonds, as also reported by BCFZ for Mexico. Importantly, the nominal and real bond liquidity risk premia we estimate are mildly negatively correlated in levels, while their weekly changes are practically uncorrelated. These results suggest that inflation-indexed Colombian bonds indeed are less liquid and overall less desirable from an investment perspective than nominal Colombian bonds.

Second, the model's decomposition of liquidity-adjusted BEI rates indicates that investors' long-term inflation expectations in Colombia have been stable at a level close to the inflation

[14] This list includes Brazil, Chile, India, Israel, Korea, Mexico, Peru, and South Africa, among others.

target set by the Bank of the Republic (BR) with some mild fluctuations. This finding implies that most of the variation in the liquidity-adjusted BEI rates is driven by fluctuations in the inflation risk premium, which has trended lower since 2005 and fallen on net slightly more than 300 basis points during our sample period. Furthermore, we compare our estimated inflation risk premium series to estimates from Canada, Mexico, and the United States and find them to be weakly positively correlated, but larger and more volatile in Colombia, as anticipated. Still, for extended periods, inflation risk in Colombia only commands a premium slightly above the one estimated for Mexico, while it is markedly above those observed for Canada and the United States. These findings underscore that inflation risk is a significant source of risk for investors in Colombian nominal bonds.

As for the determinants of Colombian inflation risk premia, we perform regression analysis with a large battery of explanatory variables. The regressions have significant explanatory power with adjusted R^2s up to 0.74. Furthermore, focusing on our preferred regression specification, we note that increases in global interest rate risk as captured by the MOVE index, increases in global perceptions about credit risk as reflected in the Colombian five-year credit default swap (CDS) rate, and increases in U.S. long-term interest rates all tend to boost long-term inflation risk premia in Colombia. This is consistent with our conjecture that foreigners are likely to favor holding Colombian nominal bonds over inflation-indexed ones, which should make the inflation risk premium more sensitive to global risk factors. Furthermore, the results show that proxies for the liquidity risk in the markets for bonos del tesoro and bonos del tesoro UVR matter as well, even though we technically have adjusted for the liquidity risk premia within our model.

With estimates of liquidity and inflation risk premia in hand, we can turn our attention to our main research question about the net benefit to the Colombian government from increasing its issuance of inflation-indexed debt. As already noted, the liquidity risk premia embedded in the prices of bonos del tesoro UVR average significantly higher than those of bonos del tesoro. Thus, based on our results, issuance of bonos del tesoro UVR faces a liquidity disadvantage, as also conjectured earlier. However, this disadvantage is more than overcome by the large inflation risk premia demanded by investors to assume the inflation risk of nominal bonos del tesoro. To see this, we construct synthetic measures of the forward-looking net benefit to the Colombian government of issuing bonos del tesoro UVR over bonos del tesoro at constant maturities, where we focus on the 5-year, 10-year, and 15-year horizons to be consistent with the maturities of the securities actually issued in the bonos del tesoro and bonos del tesoro UVR markets. Our results show that the average net benefit of bonos del tesoro UVR issuance during our benchmark sample period from January 2005 to December 2020 was 37 basis points, 66 basis points, and 69 basis points, respectively. Thus, while issuance of 5-year securities can be viewed as fairly competitive, although still favoring inflation-indexed bonds, our results more clearly favor increased issuance of such bonds at the longer 10-year and 15-year maturities. Furthermore, a replication of our exercise for the Mexican bonos and udibonos market using an update of the data examined by BCFZ produces qualitatively similar results, again favoring inflation-indexed debt, in particular at longer maturities. Although qualitatively similar to the results reported by Ermolov (2021) for a large set of advanced economies, our estimates of the net benefit of inflation-indexed debt are generally larger. We interpret this as a sign that inflation risk is more material in emerging economies like Colombia and Mexico, in particular at longer horizons. These results also underscore the social benefits to society of the government in those economies issuing inflation-indexed debt with long maturities that affords domestic investors a class of securities through which they can protect their financial wealth against the inflation risk over the long term.

To examine whether our findings hold in the post-pandemic period, we repeat the cost-benefit analysis for updated data through December 2024. First, the results for the baseline 2005-2020 part of the updated samples are very similar to our findings detailed above confirming their robustness. Second, for the 2021-2024 period, the results are even more favorable to increased inflation-indexed bond issuance thanks to rising inflation risk premia. These results make us even more confident in our conclusion that increased issuance of inflation-indexed bonds, in particular at long maturities, would be beneficial to both the Colombian and Mexican government.

Although we consider Colombia to be a representative country among large and medium-sized emerging economies, we stress that our findings and results may not extend to others in that group. However, the general principles and approach we use can be applied to any country to examine the benefit of inflation-indexed debt as long as the requisite long samples of nominal and real prices are available.

The analysis in this paper relates to several strands of literature. First, our results regarding liquidity risk premia could be relevant for the financial market microstructure literature that aims to understand the factors that determine the size and dynamics of liquidity and financial frictions in government bond markets. Second, it speaks to the large literature focused on understanding the role and impact of foreign participation in emerging bond markets. Moreover, the paper contributes to the rapidly growing literature on the economic consequences of the COVID-19 pandemic and its aftermath. Finally, it has ties to the large literature on sovereign debt management and the benefits of portfolio diversification.

The remainder of the paper is structured as follows. Section 2 contains a description of our Colombian bond data along with institutional details about the markets for these securities. Section 3 details the BCFZ model and our estimation results, including a brief analysis of the estimated nominal and real bond liquidity risk premia and whether the two bond markets are segmented. Section 4 describes its BEI decomposition and scrutinizes the estimated inflation risk premia. Finally, Section 5 examines the benefits to the Colombian government of issuing bonos del tesoro UVR before Section 6 concludes the paper. An online appendix contains additional analysis, estimation results, and robustness exercises.

2 Colombian Government Bond Data

This section first describes the Colombian government bond bonos del tesoro and bonos del tesoro UVR data we use in the model estimation. We then briefly summarize the institutional structure of the public debt markets in Colombia before we proceed to a discussion of the holdings, the credit risk, and the bid-ask spreads in the markets for these bonds.

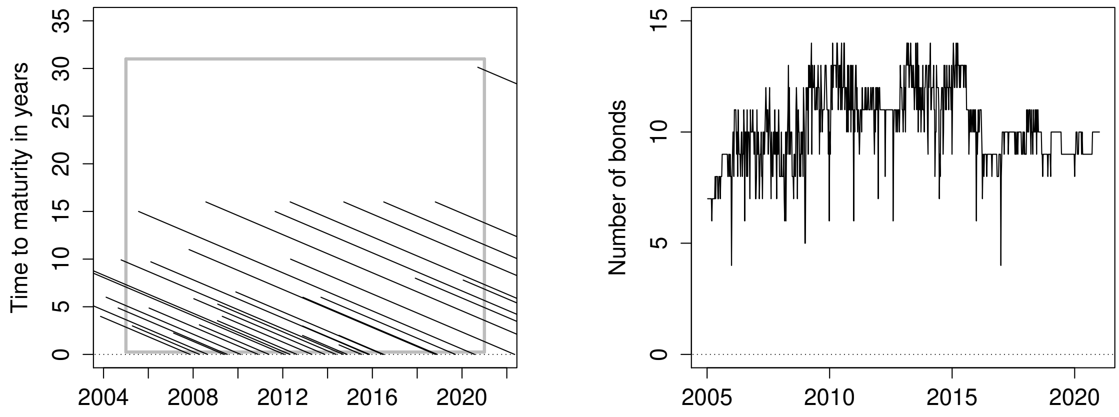
2.1 Bonos del Tesoro

The available universe of individual Colombian government fixed-coupon bonds, known as bonos del tesoro, is illustrated in Figure 1(a). Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in Colombian pesos that pay a fixed rate of interest annually. We note that we track the entire universe of bonos del tesoro issued since January 2005. In addition, we include a few bonds outstanding at the start of our sample period. In general, the Colombian government has been issuing 5-, 10-, and 15-year bonos del tesoro on a fairly regular basis during this period. In addition, it issued its first 30-year bonos in September 2020, which marked a significant milestone in its long-term debt strategy. As a result, there is a wide variety of bonds with different maturities and coupon rates in the data throughout our sample. This variation provides the foundation for the econometric identification of the factors in the yield curve model we use.

The contractual characteristics of all 35 bonos del tesoro in our sample are reported in Table 1. The number of weekly observations for each bond using three-month censoring before maturity is also reported in the table.

Figure 1(b) shows the distribution across time of the number of bonds included in the sample. We note that the number of bonds has fluctuated around 10 for most of our sample. Furthermore, the occasional sharp drops are days when some bonds have missing observations. Combined with the cross sectional dispersion in the maturity dimension observed in Figure 1(a), this implies that our panel of nominal bond prices is very well-balanced.

Figure 2 shows the time series of the yields to maturity implied by the observed Colombian bonos del tesoro prices. We note that the general yield level in Colombia trended down between 2005 and 2010, but has been fairly stable since then. Moreover, as in U.S. Treasury yield data, there is notable variation in the shape of the yield curve. At times, like in mid-2007, yields across maturities are relatively compressed. At other times, the yield curve is steep with long-term bonos del tesoro trading at yields that are 400-500 basis points above those of shorter-term securities like in 2010. Finally, the BR has never lowered its conventional policy



(a) Distribution of bonos

(b) Number of bonos

Figure 1: **Overview of the Colombian Bonos del Tesoro Data**
Panel (a) shows the maturity distribution of the Colombian government fixed-coupon bonos del tesoro considered in the paper. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on January 7, 2005, and end on December 30, 2020, and limited to bonos prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonos del tesoro at a given point in time.

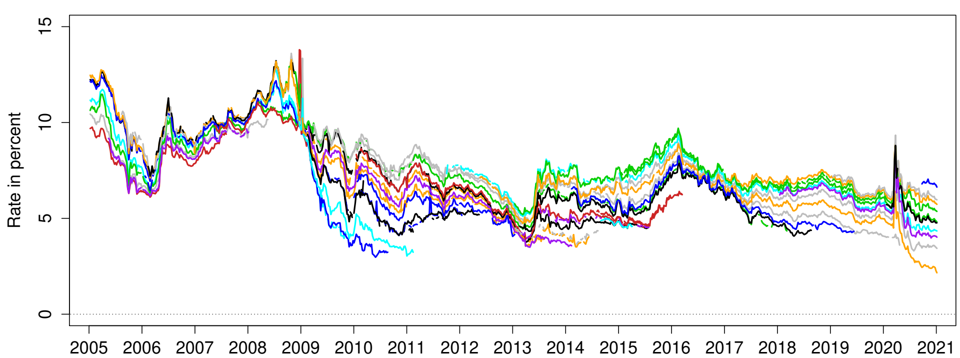


Figure 2: **Yield to Maturity of Colombian Bonos del Tesoro**
Illustration of the yields to maturity implied by the Colombian government fixed-coupon bonos del tesoro prices. The data are weekly covering the period from January 7, 2005, to December 30, 2020, and censors the last three months for each maturing bond. Each bond yield series is shown with its own colored line.

rate even close to zero. Thus, there is no need to account for any lower bounds to model these fixed-coupon bond prices, which motivates our focus on the Gaussian BCFZ model.

To support the choice to focus on the BCFZ model more formally based on the Colombian bonos del tesoro data, we note that researchers have typically found that three factors are

Fixed-coupon bonos	No. obs.	Issuance Date	Issuance Amount	Number of auctions	Total notional amount
(1) 15% 8/22/2008	174	8/22/2001	447.4	36	4,516.5
(2) 15% 1/25/2012	154	1/28/2002	289.4	4	1,522.3
(3) 15% 4/26/2012	259	4/24/2002	4.9	32	3,453.8
(4) 12% 11/9/2007	135	11/12/2003	107.2	30	4,904.3
(5) 13% 2/12/2010	220	2/11/2004	103.5	31	4,987.3
(6) 12.5% 7/10/2009	219	8/25/2004	94.0	34	5,440.2
(7) 13.5% 9/12/2014	427	10/12/2004	108.1	36	5,669.0
(8) 10% 4/11/2008	135	4/13/2005	92.9	42	4,461.3
(9) 11% 7/24/2020	748	7/24/2005	71.7	71	6,834.7
(10) 8% 10/28/2015	447	2/8/2006	174.0	53	6,067.4
(11) 7.5% 11/24/2010	215	1/11/2006	55.0	39	4,017.3
(12) 8.75% 5/14/2009	91	2/14/2007	37.4	26	4,399.9
(13) 11% 5/18/2011	141	4/9/2008	164.0	25	4,616.2
(14) 11.25% 10/24/2018	438	10/24/2007	171.6	75	10,385.7
(15) 10.25% 11/14/2013	275	1/9/2008	119.2	35	5,524.4
(16) 9.25% 5/14/2014	260	2/2/2009	96.2	43	8,671.8
(17) 10% 7/24/2024	613	7/24/2008	91.6	112	21,618.0
(18) 9.25% 8/15/2012	169	1/28/2009	191.8	30	6,066.8
(19) 6% 4/17/2013	156	4/17/2009	146.0	17	4,995.4
(20) 7.25% 6/15/2016	319	11/24/2009	142.5	69	11,753.3
(21) 7.5% 8/26/2026	449	8/26/2011	203.0	106	29,191.3
(22) 6% 4/28/2028	411	4/27/2012	470.5	146	32,479.1
(23) 7% 5/4/2022	442	5/4/2012	400.4	97	29,035.0
(24) 5.25% 11/11/2015	84	11/15/2012	237.7	26	3,076.7
(25) 5% 11/12/2014	50	11/15/2012	84.5	21	1,908.8
(26) 5% 11/21/2018	287	11/21/2012	485.1	28	5,289.4
(27) 7% 9/11/2019	279	9/11/2013	470.9	34	9,468.2
(28) 5.5% 7/1/2016	73	7/1/2014	50.2	21	4,924.2
(29) 5% 7/3/2015	33	7/3/2014	250.6	21	4,947.0
(30) 7.75% 9/18/2030	302	9/18/2014	478.0	70	22,399.6
(31) 7% 6/30/2032	207	6/30/2016	673.9	58	27,970.5
(32) 6.25% 11/26/2025	156	11/26/2017	693.5	55	18,089.5
(33) 7.25% 10/18/2034	104	10/18/2018	733.5	101	29,913.8
(34) 5.75% 11/3/2027	50	1/15/2020	711.9	52	20,101.9
(35) 7.25% 10/26/2050	16	9/9/2020	354.5	53	23,499.9

Table 1: **Sample of Colombian Bonos del Tesoro**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of Colombian pesos either at maturity or as of December 30, 2020, for the available universe of Colombian government fixed-coupon bonos del tesoro in the sample. Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2020.

sufficient to model the time variation in the cross section of U.S. Treasury yields (e.g., Litterman and Scheinkman 1991). To perform a similar analysis based on our sample of Colombian bonos del tesoro prices, we construct synthetic zero-coupon bond yields by fitting the flexible

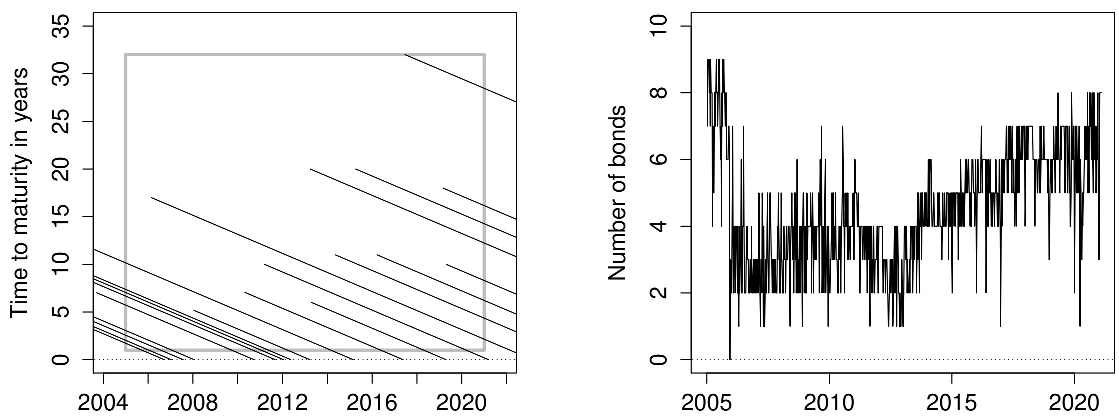
Maturity in months	First P.C.	Second P.C.	Third P.C.
12	0.34	0.74	-0.54
24	0.36	0.31	0.40
36	0.38	0.12	0.45
60	0.38	-0.09	0.26
84	0.37	-0.21	0.08
120	0.35	-0.29	-0.14
144	0.33	-0.32	-0.26
180	0.30	-0.32	-0.42
% explained	89.17	8.18	2.20

Table 2: **Factor Loadings of Colombian Bonos del Tesoro Yields**
The top rows show the eigenvectors corresponding to the first three principal components (PC). Put differently, they show how bond yields at various maturities load on the first three principal components. In the final row the proportion of all bond yield variability explained by each principal component is shown. The data are daily Colombian nominal zero-coupon government bond yields from January 3, 2005, to December 30, 2020, a total of 3,882 observations for each yield series.

Nelson and Siegel (1987) yield curve to the set of bond prices observed for each observation date.[15] To have a yield panel representative of the underlying bonds in our sample, we include yields for eight constant maturities: 1, 2, 3, 5, 7, 10, 12, and 15 years. The data series are daily series, covering the period from January 3, 2005, to December 30, 2020.

The result of a principal component analysis of the yield panel is reported in Table 2. The top panel reports the eigenvectors that correspond to the first three principal components. The first principal component accounts for 89.2 percent of the variation in the bond yields, and its loading across maturities is uniformly positive. Thus, similar to a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 8.2 percent of the variation in these data and has sizable positive loadings for the shorter maturities and sizable negative loadings for the long maturities. Thus, similar to a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for 2.2 percent of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. These three factors combined account for 99.6 percent of the total variation. This supports the assumed factor structure for nominal bonds within the BCFZ model, which is rooted in the arbitrage-free Nelson and Siegel (AFNS) model derived in Christensen et al. (2011) with its level, slope, and curvature factors. However, to explain the remaining variation in the bonos del tesoro yield data not accounted for by these three fundamental factors, nominal bond prices within the BCFZ model are augmented with a liquidity risk factor structured as in Andreasen et al. (2021).

[15] Technically, we proceed as described in Andreasen et al. (2019).



(a) Distribution of bonos del tesoro UVR

(b) Number of bonos del tesoro UVR

Figure 3: **Overview of the Colombian Bonos del Tesoro UVR Data**
Panel (a) shows the maturity distribution of the Colombian government inflation-indexed bonos considered in the paper. These securities are also known as Colombian bonos del tesoro UVR. The solid gray rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on January 7, 2005, and limited to bonos prices with more than one year to maturity after issuance. Panel (b) reports the number of outstanding bonos del tesoro UVR at a given point in time.

2.2 Bonos del Tesoro UVR

The Colombian government also issues inflation-indexed bonds, which are bonos del tesoro denominated in consumption units known as UVR and therefore referred to as bonos del tesoro UVR. Unlike standard fixed-coupon bonds, interest and principal payments of bonos del tesoro UVR are adjusted for changes in the general price level as reflected in the Colombian consumer price index, abbreviated IPC in Spanish. Hence, UVR is used to convert the real return of bonos del tesoro UVR into the corresponding value measured in current Colombian pesos at any given point in time.[16]

The Colombian government launched its inflation-indexed bond program in the 1990s. However, the data provided to us by staff at the BR only start in January 2005 and end in December 2020, which determines our baseline sample period. The available universe of bonos del tesoro UVR and their maturity distribution across time is shown in Figure 3(a). It includes the entire universe of bonos del tesoro UVR issued since 2005 combined with the outstanding stock of bonos del tesoro UVR at the start of our sample. We note that the issuance is concentrated in the 10- and 20-year segment, while a single 30-year bonos del tesoro UVR was issued in 2017.

The contractual details of each bonos del tesoro UVR in our sample are reported in Table

[16]For technical reasons there is a modest lag of about two and a half months in the indexation of the cash flows of these bonds, which is similar to most other indexed bonds around the world, including U.S. TIPS.

Indexed bonos	No. obs.	Issuance Date	Issuance amount	Number of auctions	Total notional amount
(1) 8% 9/21/2006	34	9/23/1999	1.1	1	6.4
(2) 6% 1/12/2007	40	1/12/2000	1.2	3	9.5
(3) 8% 7/26/2007	56	9/21/2000	1.3	2	10.5
(4) 8% 1/22/2008	40	2/22/2001	0.4	1	6.0
(5) 8% 9/2/2011	50	8/30/2001	1.3	3	7.0
(6) 7% 1/17/2012	110	1/17/2002	1.3	2	14.8
(7) 7% 5/15/2012	89	5/3/2002	3.0	1	11.3
(8) 7% 2/25/2015	420	2/20/2003	7.0	57	51.1
(9) 7% 9/22/2010	234	9/16/2003	2.9	37	32.8
(10) 4.75% 2/23/2023	521	2/23/2006	0.5	132	100.4
(11) 5.25% 3/20/2013	182	1/16/2008	1.0	36	32.0
(12) 4.25% 5/17/2017	266	4/26/2010	0.7	54	63.4
(13) 3.5% 3/10/2021	364	3/10/2011	2.0	104	76.8
(14) 3% 3/25/2033	323	3/25/2013	1.4	90	44.4
(15) 3.5% 4/17/2019	216	4/17/2013	0.9	48	40.7
(16) 3.5% 5/7/2025	237	5/7/2014	1.5	147	61.6
(17) 4.75% 4/4/2035	224	4/4/2015	1.0	125	92.2
(18) 3.3% 3/17/2027	193	3/17/2016	1.4	176	80.1
(19) 3.75% 6/16/2049	43	6/16/2017	3.9	107	55.7
(20) 3.75% 2/25/2037	93	2/25/2019	1.4	127	83.9
(21) 2.25% 4/18/2029	33	4/18/2019	0.7	100	42.6

Table 3: **Sample of Colombian Bonos del Tesoro UVR**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of UVR either at maturity or as of December 30, 2020, for the available sample of Colombian government inflation-indexed bonos, also known as bonos del tesoro UVR. Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2020.

3. It also contains the number of weekly observations for each bond in our sample, with the last year before maturity censored to avoid erratic variation in their prices arising from seasonality in the inflation adjustment of their payoffs; see Andreasen et al. (2021).

The total number of bonos del tesoro UVR in our sample across time is shown in Figure 3(b). As with the regular bonos del tesoro, we stress that the sample of bonos del tesoro UVR we use is very well-balanced across maturities at all times, which underpins the econometric identification of the state variables in the term structure model we use.

Figure 4 shows the yields to maturity implied by the bonos del tesoro UVR prices. Similar to what we observe for the nominal bonos del tesoro yields, the yields of bonos del tesoro UVR have fluctuated around a fairly stable level since 2010, but with some variation in the steepness of the bonos del tesoro UVR yield curve. Ideally, we therefore would like to model the bonos del tesoro UVR yields with a level, slope, and curvature factor structure similar to what we do for the nominal bonos del tesoro yields. Unfortunately, for extended periods, the number of available bonos del tesoro UVR prices is insufficient to allow this as can be seen in

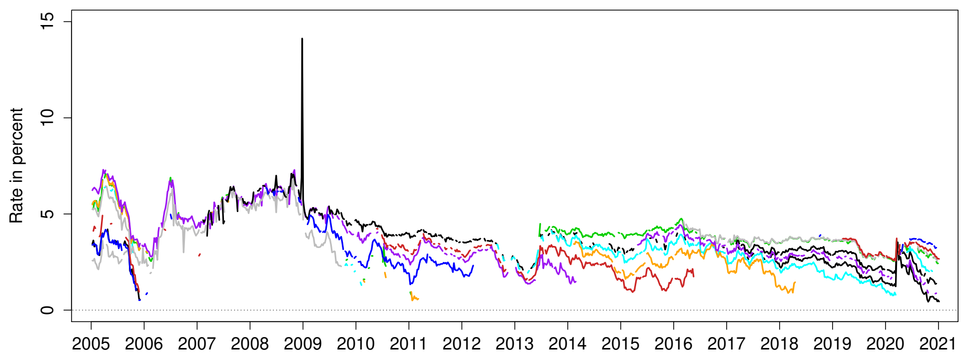


Figure 4: **Yield to Maturity of Colombian Bonos del Tesoro UVR**
Illustration of the yield to maturity implied by the Colombian government inflation-indexed bonos del tesoro UVR prices considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 7, 2005, to December 30, 2020; (2) censoring of a bond's price when it has less than one year to maturity. Each bond yield series is shown with its own colored line.

Figure 3(b). Hence, we are limited to only include a fundamental level and slope factor as in the analysis by BCFZ, who faced a similar data availability constraint in modeling Mexican udibonos prices.[17] Moreover, as in BCFZ, the real side of the model is augmented with a separate liquidity risk factor to account for the liquidity risk in the bonos del tesoro UVR market, which is again structured as in Andreasen et al. (2021).

2.3 The Institutional Structure of the Colombian Public Debt Markets

In this section, we describe the main institutional details characterizing the Colombian public debt markets and explain why they represent a compelling case study in emerging market financial development. Their evolution from a nascent, institutionally fragmented system to modern, liquid fixed-income markets reflects a combination of deliberate policy choices and strategic reforms. In the following, we outline the key institutional foundations, market mechanisms, and investor dynamics that characterize Colombia's Treasury bond markets, highlighting their current position as an attractive destination for fixed-income allocations among both domestic and foreign investors.

2.3.1 Institutional Foundations and Financial Market Development

The key institutional foundations of today's Colombian public debt markets were introduced in the early 1990s through a series of reforms that redefined the state's role in the financial system. The current constitution of Colombia adopted in 1991 granted independence to the central bank and explicitly prohibited it from financing the government directly. Simultaneously,

[17] Finlay and Wende (2012) examine prices from a limited number of Australian inflation-indexed bonds.

financial sector liberalization increased competition among financial intermediaries, and a reformed social security system created a stable demand base for fixed-income securities by fostering long-term institutional investors. Collectively, these measures created the necessary conditions for the development of a modern domestic government bond market.

In this initial phase, though, central government financing still relied predominantly on external borrowing. Domestic debt placements remained largely obligatory and were directed toward public entities, such as the Social Security Institute (ISS). A critical turning point took place in 1996 with the consolidation of domestic bond issuance through three key institutional innovations: (i) the adoption of a stable auction calendar for Class B Treasury Bonds, known as TES B; (ii) the designation of specialized market-makers with defined privileges and obligations; and (iii) the establishment by the BR of a comprehensive electronic platform for the auction, administration, registration, and settlement of dematerialized bonds. These innovations marked the beginnings of a systematic and deliberate market-building process.

2.3.2 Public Debt Market Structure and Trading Mechanisms

Colombian bonos del tesoro are issued in the primary market through auctions organized by the Ministry of Finance and Public Credit, known as MHCP in Spanish, and are subsequently traded on the secondary market. The main trading platforms are: (i) the Sistema Electrónico de Negociación (SEN), administered by the BR, which serves as the official wholesale interbank trading system; and (ii) the Master Trader, managed by the Colombian Stock Exchange (BVC), which allows for broader participation, including institutional and retail investors.[18] Clearing and settlement of trades are centralized through the Depósito Central de Valores (DCV), the national central securities depository operated by the BR.

The BR has played a central and multifaceted role in shaping the structure and functioning of the Colombian government bond markets. As part of its monetary policy operations, the central bank relies exclusively on bonos del tesoro for open market interventions. Initially, TES B securities were used primarily for temporary monetary expansions, but over time they also became instruments for permanent liquidity management. In addition to outright purchases and sales, the BR actively conducts repo operations, which serve as a key instrument for short-term liquidity provision and absorption. The central bank has also intervened in the government bond markets during episodes of pronounced imbalances between money market supply and demand; when such pressures could not be addressed through temporary operations, it resorted to the bonos del tesoro market to restore orderly market conditions. These interventions not only served monetary policy objectives but also supported market development by providing liquidity and stabilizing prices during short-term spells of market stress.[19]

[18] In 2024, approximately 69 percent of the trading volume in government securities was conducted through the SEN, underscoring its dominant role in supporting market liquidity.

[19] The bonos del tesoro market has become a cornerstone of Colombia's money markets, with significant vol-

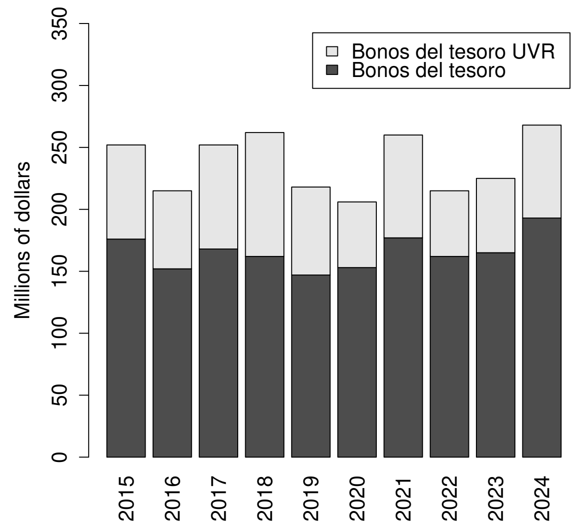


Figure 5: **Average Weekly Primary Issuance**

Bonos del tesoro are issued in Colombian pesos, with a face value of COP 1,000,000 per bond. As described in Section 2.2, the Colombian government also issues inflation-indexed bonds denominated in UVR, which adjust daily according to the CPI. This dual structure—comprising of nominal and inflation-indexed bonds—facilitates portfolio diversification and provides the government with access to different investor bases.

To further support this market structure, the BR has operated a primary dealer program since 2003. These institutions are known as Creadores de Mercado. The program is designed to enhance demand in primary auctions and support liquidity in the secondary market. It currently includes approximately 14 financial institutions, primarily large domestic banks and a few foreign banks with significant local operations. To preserve their designation, these institutions are required to (i) participate actively in primary auctions by submitting competitive bids that help stabilize demand, and (ii) provide secondary market liquidity by quoting firm two-way prices in benchmark bonds on the SEN platform. Thus, their role combines obligations for primary auction participation with market-making duties, a structure consistent with practices in other emerging bond markets.[20]

Since 2015 the primary market has been characterized by regular auctions with average weekly issuances equivalent to about USD 237 million of which about two thirds represent standard bonos del tesoro and about one third bonos del tesoro UVR as shown in Figure 5. Since 2021, the average bid-to-cover ratio in the primary bonos del tesoro market has

umes traded through repos and reverse repos, where government securities are the dominant form of collateral.

[20] As noted in the OECD (2024) survey on secondary market liquidity, one of the main obligations of primary dealers is to ensure the liquidity of the government debt securities market by promoting secondary market trading and the effective pricing of securities.

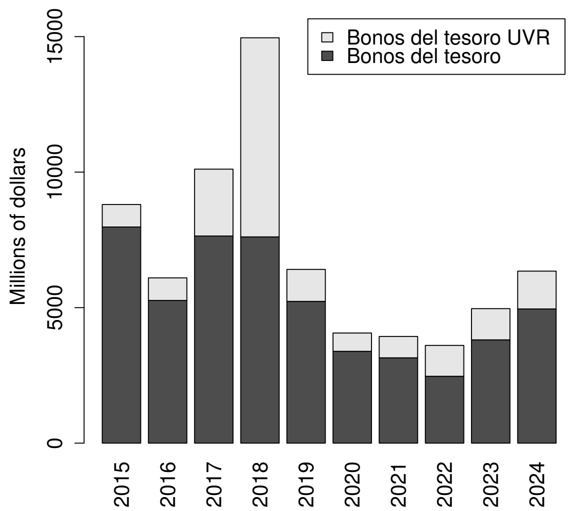


Figure 6: **Average Weekly Secondary Market Trading Volumes**

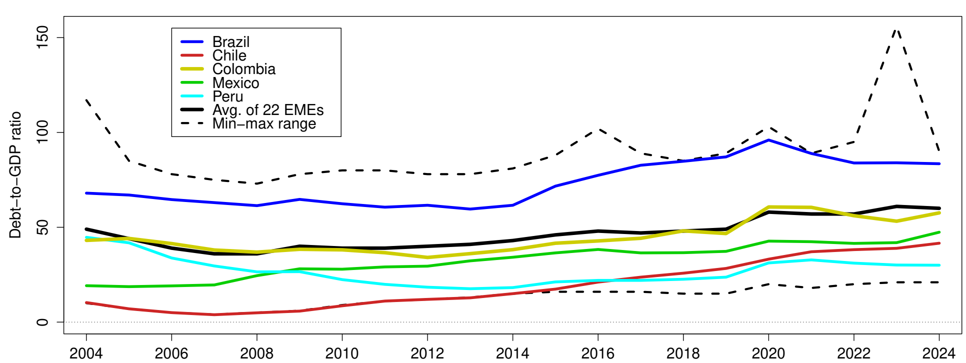


Figure 7: **Sovereign Debt-to-GDP Ratios**

been 3.66, compared with 3.0 for bonos del tesoro UVR. A ratio above 1 is particularly important, as it signals that investor demand exceeds the amount offered, reflecting healthy market participation and competitive auction dynamics.

As shown in Figure 6, weekly trading volumes in the secondary market have averaged the equivalent of about USD 6.9 billion, with 77% concentrated in the bonos del tesoro market. By contrast, the trading of bonos del tesoro UVR has consistently been smaller in trade sizes and trading volumes, pointing to higher liquidity risk premia in that market—a conjecture we explicitly examine in our analysis.

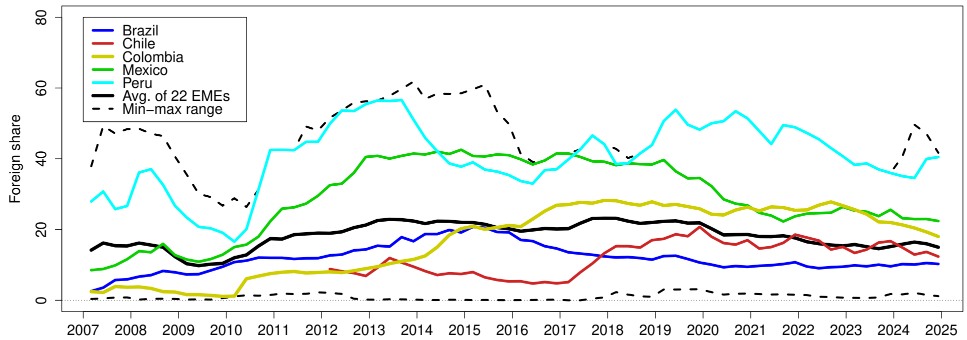


Figure 8: **Share of Foreign Investors in Sovereign Bond Markets**

2.3.3 Sovereign Debt Profile and Investor Base

Regarding the total outstanding amount of government debt, Colombia's public-debt ratio stood at 58 percent of nominal GDP as of 2024, which is near the emerging-market average as shown in Figure 7.[21] Its debt ratio remains well below Brazil's, but above that of regional peers like Chile and Mexico. This moderate level of indebtedness, combined with Colombia's modern-era record of servicing its sovereign obligations without a default, makes its domestic government bond markets an attractive destination for emerging-market fixed-income allocations.

This attractiveness has been amplified by sound debt management and Colombia's growing prominence in global emerging bond market benchmarks. A key catalyst was J.P. Morgan's March 2014 announcement of a significant increase in Colombia's weight within its GBI-EM index, which rose from 3.9 percent to 8.0 percent. This change triggered a surge in nonresident demand: foreign investors' share of local bonos del tesoro holdings climbed from roughly 3 percent before 2014 to above one quarter by 2018. By the end of 2024, this share stood at 20.4 percent—well above pre-2014 levels and in line with the average among emerging market economies as shown in Figure 8.

This broader and more diverse investor base confers significant advantages. In the academic literature, such investor diversity has been linked to improved international risk sharing (Sill, 2011; Stulz, 1999; Tesar, 1995); deeper secondary-market liquidity (World Bank & IMF, 2001); and more accurate sovereign bond pricing (Ejsing et al., 2012). Thus, Colombia's government bond market is characterized by healthy diversification. This is further evidenced by a cross-sector Herfindahl-Hirschman Index (HHI) of approximately 2,726, which is similar to Brazil's (2,796), but well below the more concentrated markets of Mexico (4,961) or China

[21] The included countries are Argentina, Brazil, Bulgaria, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, and Uruguay.

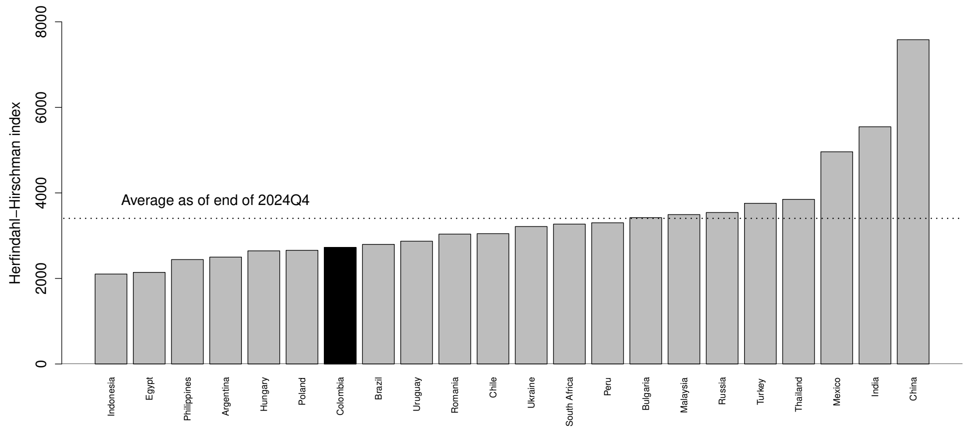


Figure 9: **Concentration of Investors in EM General Government Debt Markets**

(7,581) as shown in Figure 9. This relatively low concentration would point to somewhat healthier liquidity conditions in the Colombian sovereign debt markets compared to several of its emerging market peers.

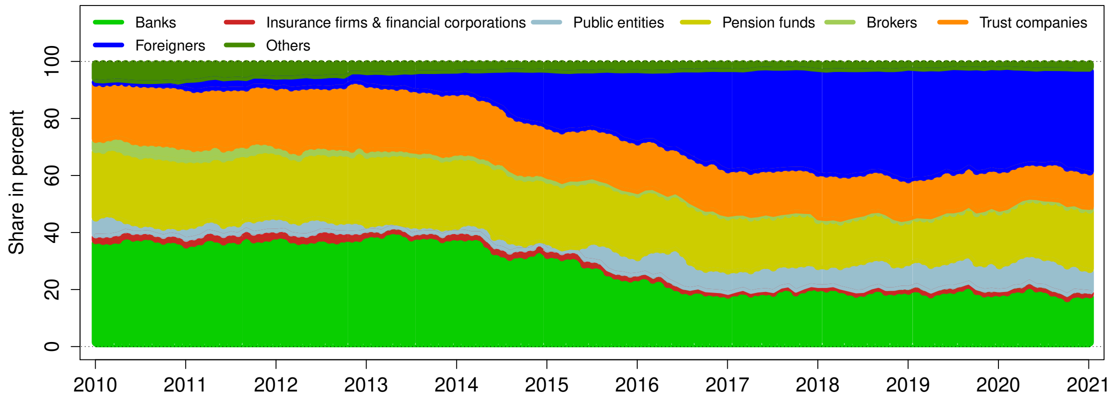
To summarize, the history of Colombia's bonos del tesoro market shows how institutional reforms, sound debt management, and strategic integration into global financial bond index benchmarks can transform a domestic sovereign debt market into an attractive destination for international investors. The country's moderate debt level, diverse investor base, and commitment to honoring its obligations position it favorably among emerging markets, offering both stability and liquidity for fixed-income investors.

2.4 Colombian Government Bond Holdings

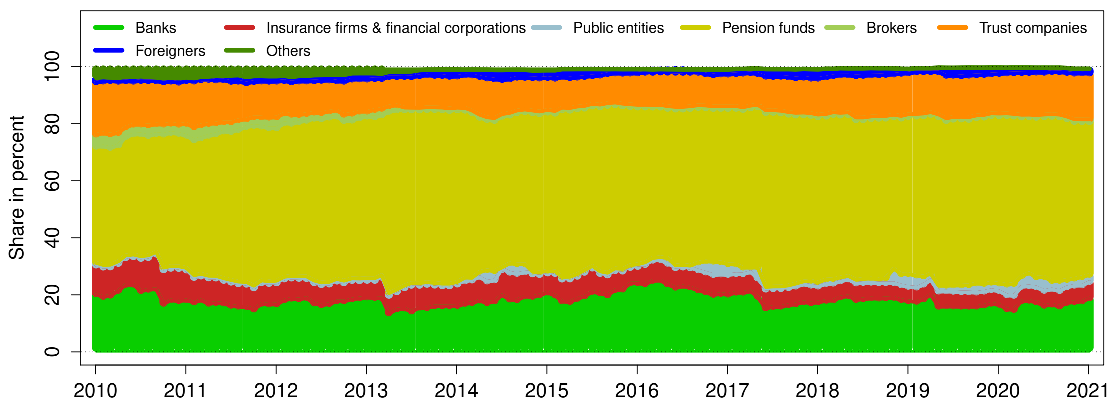
In this section, we provide further detailed data on the investor groups holding Colombian government bonds. The data we use have been collected by the BR since 2010 to track market activity in the Colombian sovereign bond markets. Importantly, the data break down investor holdings into multiple groups, which include banks, insurance companies, pension funds, and foreigners, among others.

Figure 10(a) shows the share held by each of these groups of investors in the regular bonos del tesoro market. Note that there has been a significant increase in the foreign-held share since 2010. As a result, foreigners have become the single largest investor group with about one-third of the market by the end of our baseline sample that runs through December 2020.[22] This expansion of the foreign role has come at the expense of the participation of domestic banks, while the holdings of the other domestic investor groups have changed little on net since 2010.

[22]Although we treat foreigners as a single group, we note that there is significant heterogeneity among foreign investors in Colombian bond markets in terms of investment strategies and horizons; see Gamboa-Estrada and Sanchez-Jabba (2024).



(a) Bonos del tesoro



(b) Bonos del tesoro UVR

Figure 10: **Holdings of Colombian Government Bonds**

Figure 10(b) shows the corresponding breakdown of holdings in the bonos del tesoro UVR market, where the shares are distributed very differently across groups. Most importantly, foreigners are nearly absent in this market. Instead, the holdings are concentrated among domestic investors, such as pension funds, insurance companies, and banks, that all tend to pursue a more stable buy-and-hold strategy for their fixed-income investments.

These patterns are similar to the holdings statistics reported by BCFZ for the Mexican government bond markets, where foreigners own more than half of the regular fixed-coupon bond market and less than 10 percent of the inflation-indexed bond market.[23]

Overall, this represents tangible evidence in favor of our conjectures laid out in the introduction whereby inflation-indexed bonds mainly meet the needs of patient domestic buy-and-

[23]Ceballos et al. (2025) report a similar high concentration among domestic patient institutional investors for the Chilean inflation-indexed government bond market.

hold investors exposed to the fluctuations in the Colombian CPI. As a consequence, foreigners will tend to gravitate towards the more liquid nominal bond market.

In the public debt market in Colombia, the main changes that could have had an impact on the behavior of portfolio investment flows occurred between 2010 and 2013. During that period the country's credit rating reached investment-grade level.[24] After those changes and with global financial markets characterized by benign liquidity conditions driven in part by unconventional monetary policies pursued by major central banks in advanced economies, J.P. Morgan announced that it would increase Colombia's weight in three of its main emerging market bond indices in 2014. As a result, the share of foreign investors in the local public debt market, i.e. the bonos del tesoro market, rose from approximately 5 percent in mid-2013 to 19 percent at the end of 2014. The foreign participation further increased to levels around 35 percent by 2017, where it has remained since then.[25] In contrast, domestic pension funds and insurance companies continue to participate heavily in the bonos del tesoro UVR market to hedge their liabilities. This is particularly true for pension funds as their liabilities tend to have a large share of claims indexed to inflation.

Ultimately, these differences in investor concentrations have consequences for the size and dynamics of inflation and liquidity risk premia in each of the two bond markets that we aim to quantify in our empirical analysis.

2.5 The Credit Risk of Colombian Government Bonds

Following the analysis in BCFZ, we gauge whether there are any material credit risk issues to consider in modeling Colombian government bond prices by examining rates on CDS contracts. They reflect the annual rate investors are willing to pay to buy protection against default-related losses on these bonds over a fixed period of time stipulated in the contract. These derivatives have been used to price the credit risk of many countries, including Colombia, since the early 2000s.

In Figure 11, we plot the series for the one- and five-year Colombian CDS rate since 2005 with solid gray and black lines, respectively. Also shown with a solid red line is the spread between these two CDS rates. We note that the five-year CDS rate has fluctuated in a fairly narrow range between 100 and 200 basis points, except for a few brief episodes including the Global Financial Crisis in 2008-2009, when Colombian CDS rates temporarily spiked above 300 basis points, and the early stages of the COVID-19 pandemic. This is a level of credit risk on par with most investment-grade firms in the United States, and its variation is mostly very gradual. This suggests that credit risk-related components are unlikely to be the driver of the results we present later on.

[24] Granted in March 2011 by Standard & Poor's, in May 2011 by Moody's, and in June 2011 by Fitch Ratings.

[25] See Romero et al. (2021) for an analysis of how the increased foreign participation in the Colombian government bond market helped ease domestic financial conditions by lowering interest rates and increase

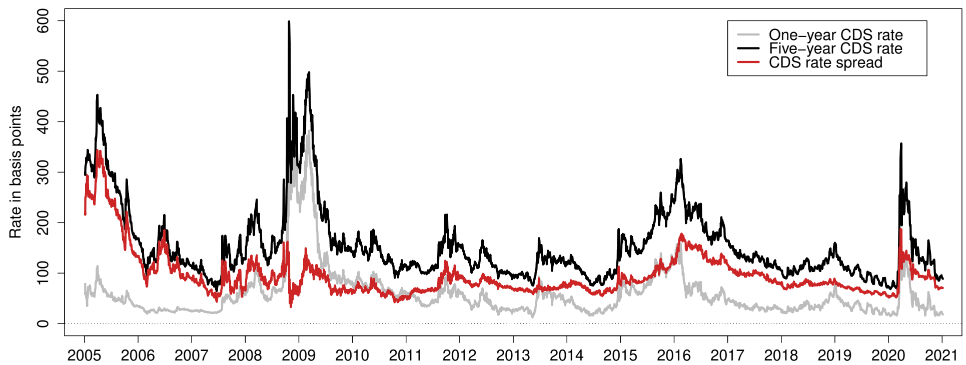


Figure 11: **Colombian CDS Rates**

More importantly, on a practical note, there are no differences in the credit risk of nominal and real bonds in the sense that they will receive the same treatment in case the Colombian government stops servicing its debt. Thus, using arguments similar to those made by Fleckenstein et al. (2014) for U.S. Treasuries and TIPS, there is no reason to believe that there are any differentials in the pricing of bonos del tesoro and bonos del tesoro UVR tied to credit risk. By implication, our measure and decomposition of Colombian BEI are unaffected by variation in the credit risk premia of Colombian government debt as they cancel out in the calculation of BEI.[26]

2.6 Bid-Ask Spreads of Colombian Government Bonds

In this section, to shed light on the trading frictions in the markets for bonos del tesoro and bonos del tesoro UVR, we compare the median bid-ask spread of the bonos del tesoro in our sample with the median of the bid-ask spread of the bonos del tesoro UVR in our sample. These series are available starting in 2013 and shown in Figure 12.[27]

Note that, outside of sharp, short-lived spikes in 2013 and again in 2020, these series have fluctuated near a fairly stable and similar level. The main takeaway is that, with a level of bid-ask spreads around 5 basis points, the trading of these securities is indeed associated with some amount of liquidity risk. Consistent with this evidence, our model assumes that both nominal and real bond prices contain liquidity risk premia that investors demand to assume their liquidity risk.

credit extension.

[26]We note that this view of equal treatment of nominal and real government debt in bankruptcy is not universally accepted. For an example, see Dittmar et al. (2024) for evidence and arguments in favor of an asymmetric impact on nominal and real sovereign debt as the government approaches the default threshold.

[27]The shown bid-ask spread series are calculated using the daily median of the difference between the best bid and the best ask for each government bond. This is done separately for bonos del tesoro and bonos del tesoro UVR.

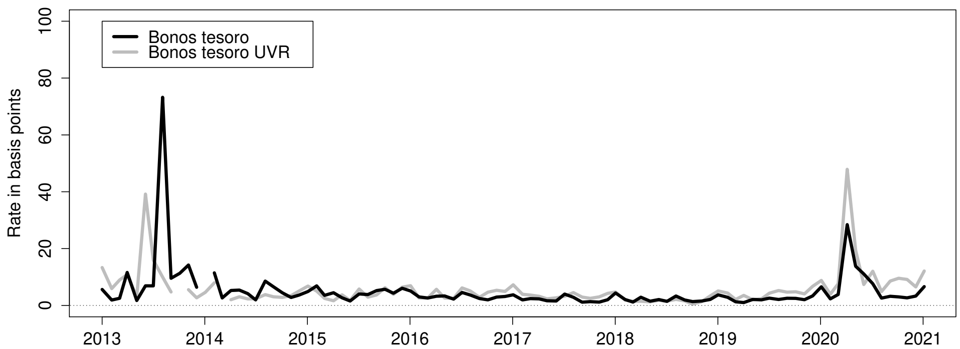


Figure 12: **Bid-Ask Spreads of Colombian Government Bonds**

The purpose of the remainder of the paper is to quantify the relative importance of these bond risk premia in the pricing of bonos del tesoro and bonos del tesoro UVR and what adjustments for them may imply about bond investors' underlying inflation expectations and associated inflation risk premia using the BCFZ model.

3 Model and Estimation Results

In this section, we first provide a brief summary description of the BCFZ model we rely on for our empirical analysis. We then describe the results we obtain when estimating the BCFZ model using the Colombian bond price data before we proceed to a short analysis of the estimated nominal and real bond liquidity risk premia. We end the section with an examination of the connectedness of the Colombian nominal and real bond markets using U.S. unconventional monetary policy announcements as unexpected exogenous shocks.

3.1 The BCFZ Model

Although we direct interested readers to consult BCFZ for the full details of their model, we briefly summarize its structure and discuss some of its main features and how they relate to our Colombian data.

To begin, let $X_t = (L_t^N, S_t^N, C_t^N, X_t^N, L_t^R, S_t^R, X_t^R)$ denote the state vector of the seven-factor BCFZ model. At its core and consistent with the principal component analysis in Section 2.1, the model has a nominal yield curve with a level, a slope, and a curvature factor in the style of the arbitrage-free Nelson-Siegel (AFNS) models derived in Christensen et al. (2011), while its real yield curve is simpler and only contains a level and a slope factor for the reasons laid out in Section 2.2, but also modeled in the style of the AFNS model. This core model structure was first used by Carriero et al. (2018) to analyze U.K. nominal and real yield curves. The novelty of the analysis in BCFZ is to augment this five-factor model with a liquidity risk factor for the nominal bond market and another separate liquidity risk factor for the real bond market, both in the style described in Andreasen et al. (2021).

The above interpretation of the latent state variables is achieved by imposing a unique structure on the risk-free short rates and the risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing. Specifically, the instantaneous nominal and real risk-free rates must be defined as

$$r_t^N = L_t^N + S_t^N, \tag{1}$$

$$r_t^R = L_t^R + S_t^R, \tag{2}$$

while the risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing must be given by

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dC_t^N \\ dX_t^N \\ dL_t^R \\ dS_t^R \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & -\lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda^N & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_N^{\mathbb{Q}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_R^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_N^{\mathbb{Q}} \\ 0 \\ 0 \\ \theta_R^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t^N \\ C_t^N \\ X_t^N \\ L_t^R \\ S_t^R \\ X_t^R \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{C^N,\mathbb{Q}} \\ dW_t^{X^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \\ dW_t^{X^R,\mathbb{Q}} \end{pmatrix},$$

where Σ is assumed to be a diagonal matrix as per Christensen et al. (2011).

Here, we note that the two level factors, L_t^N and L_t^R, are unit-root processes under the $\mathbb{Q}$-measure. As demonstrated by Christensen et al. (2011), this implies that the mean parameters under the $\mathbb{Q}$-measure for the standard level, slope, and curvature factor are not econometrically identified within the AFNS models and instead must be fixed at zero at no loss of generality. We add that this result extends to the simplified version of those models with only a level and a slope factor. In contrast, as noted by Andreasen et al. (2021), this does not apply to the liquidity risk factors, X_t^N and X_t^R, which can be allowed to have their own mean parameter under the $\mathbb{Q}$-measure. As a consequence, $\theta_N^{\mathbb{Q}}$ and $\theta_R^{\mathbb{Q}}$ are free to be determined by the data.

Due to the liquidity risk in the markets for nominal and real bonds, their yields are sensitive to liquidity pressures consistent with the sizable bid-ask spread series described in Section 2.6. As a consequence, the pricing of nominal and real bonds is not performed with the frictionless short rates in equations (1) and (2), but rather with discount functions that account for the liquidity risk as in Andreasen et al. (2021):

$$\bar{r}^{N,i}(t,t_0^i) = r_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N = L_t^N + S_t^N + \beta^{N,i}(1 - e^{-\delta^{N,i}(t-t_0^i)})X_t^N, \tag{3}$$

$$\bar{r}^{R,j}(t,t_0^j) = r_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R = L_t^R + S_t^R + \beta^{R,j}(1 - e^{-\delta^{R,j}(t-t_0^j)})X_t^R, \tag{4}$$

where t_0^i and t_0^j denote the dates of issuance of the specific nominal and real bonds, respectively, and $\beta^{N,i}$ and $\beta^{R,j}$ are their sensitivities to the variation in their respective liquidity risk factors. Furthermore, the decay parameters $\delta^{N,i}$ and $\delta^{R,j}$ are assumed to vary across securities.

Now, consider the whole value of the nominal bond i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C^i. Its price is given by[28]

$$\begin{aligned}\overline{P}_t^{N,i}(t_0^i,\tau^i,C^i) &= C^i(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{N,i}(s,t_0^i)ds}\Big]+\sum_{k=2}^{n}C^iE^{\mathbb{Q}}\Big[e^{-\int_t^{t_k}\overline{r}^{N,i}(s,t_0^i)ds}\Big]\\ &\quad +E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\overline{r}^{N,i}(s,t_0^i)ds}\Big].\end{aligned}$$

Next, consider the whole value of the real bond j issued at time t_0^j with maturity at $t+\tau^j$ that pays an annual coupon C^j. Its clean price is given by[29]

$$\begin{aligned}\overline{P}_t^{R,j}(t_0^j,\tau^j,C^j) &= C^j(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{R,j}(s,t_0^j)ds}\Big]+\sum_{k=2}^{n}C^jE^{\mathbb{Q}}\Big[e^{-\int_t^{t_k}\overline{r}^{R,j}(s,t_0^j)ds}\Big]\\ &\quad +E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^j}\overline{r}^{R,j}(s,t_0^j)ds}\Big].\end{aligned}$$

The only minor omission in the real bond price formula above is that we do not account for the lag in the inflation indexation of the real bond payoff, but the potential error should be modest in most cases; see Grishchenko and Huang (2013) and D'Amico et al. (2018) for evidence in the case of the U.S. TIPS market.

To complete the BCFZ model description, we need to specify the risk premia that connect the factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the objective $\mathbb{P}$-measure, where we follow BCFZ and use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t=\gamma^0+\gamma^1X_t,$$

where $\gamma^0\in\mathbf{R}^7$ and $\gamma^1\in\mathbf{R}^{7\times7}$ contain unrestricted parameters. Thus, the resulting unrestricted BCFZ model has $\mathbb{P}$-dynamics given by

$$dX_t=K^{\mathbb{P}}(\theta^{\mathbb{P}}-X_t)+\Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 7×7 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 7×1 vector of mean levels, and Σ is a 7×7 diagonal volatility matrix. This is the transition equation in the extended Kalman filter estimation of the model.

Finally, similar to BCFZ, we also incorporate long-term forecasts of inflation from the

[28]This is the clean price that does not account for any accrued interest and maps to our observed bond prices.
[29]Unlike U.S. TIPS, Colombian bonos del tesoro UVR have no embedded deflation protection option, which makes their pricing straightforward.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$K^{\mathbb{P}}_{\cdot,6}$	$K^{\mathbb{P}}_{\cdot,7}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	9.5776	12.0497	6.9333	9.7760	0.5315	-24.0237	1.3179	0.1831	σ_{11}	0.0093
	(0.2731)	(0.2730)	(0.1816)	(0.3915)	(0.4321)	(0.3834)	(0.1122)	(0.0462)		(0.0005)
$K^{\mathbb{P}}_{2,\cdot}$	-17.8577	-23.4151	-16.6144	-14.6790	-9.5236	55.5653	-3.4163	-0.1163	σ_{22}	0.0001
	(0.3700)	(0.4275)	(0.3717)	(0.4708)	(0.5299)	(0.4885)	(0.2543)	(0.0329)		(0.5854)
$K^{\mathbb{P}}_{3,\cdot}$	22.8974	27.7585	22.2970	15.6342	15.5499	-69.5200	5.0490	-0.0350	σ_{33}	0.0409
	(0.4641)	(0.4655)	(0.4255)	(0.5331)	(0.4978)	(0.5939)	(0.3370)	(0.0430)		(0.0017)
$K^{\mathbb{P}}_{4,\cdot}$	11.3909	12.7440	9.2231	9.0429	1.9937	-29.9800	1.9031	0.0197	σ_{44}	0.0148
	(0.3519)	(0.2770)	(0.2407)	(0.3888)	(0.5493)	(0.3893)	(0.1642)	(0.0091)		(0.0011)
$K^{\mathbb{P}}_{5,\cdot}$	3.2554	7.1137	3.8373	6.3634	6.3335	-13.7058	0.9282	0.0799	σ_{55}	0.0089
	(0.2838)	(0.2882)	(0.1949)	(0.3694)	(0.4684)	(0.4622)	(0.0911)	(0.0211)		(0.0006)
$K^{\mathbb{P}}_{6,\cdot}$	-8.2703	-11.7627	-6.2892	-8.7168	2.7181	22.8101	-0.5065	-0.0603	σ_{66}	0.0201
	(0.3662)	(0.3555)	(0.2554)	(0.3993)	(0.4828)	(0.4868)	(0.1286)	(0.0173)		(0.0007)
$K^{\mathbb{P}}_{7,\cdot}$	1.3189	3.0441	-2.0631	-3.1821	4.4762	-5.3026	0.8968	-0.3207	σ_{77}	0.1923
	(0.5541)	(0.4771)	(0.5121)	(0.5350)	(0.5349)	(0.5196)	(0.3303)	(0.1481)		(0.0143)

Table 4: **Estimated Dynamic Parameters of the BCFZ Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the BCFZ model. The estimated value of λ^N is 0.5195 (0.0095), while $\lambda^R = 0.2660$ (0.0264), $\kappa^{\mathbb{Q}}_N = 2.9608$ (0.1737), $\theta^{\mathbb{Q}}_N = 0.0035$ (0.0003), $\kappa^{\mathbb{Q}}_R = 10.0312$ (0.5200), and $\theta^{\mathbb{Q}}_R = 0.0238$ (0.0025). The maximum log likelihood value is 61,203.72. The numbers in parentheses are the estimated parameter standard deviations.

Consensus Forecasts survey for Latin America in our model estimation. These include monthly data on inflation forecasts for the following full calendar year and semiannual data on 5-year, 10-year, and so-called 5yr5yr inflation forecasts, which represent long-term inflation forecasts covering a 5-year period starting 5 years ahead.

The full details of the model estimation can be found in online Appendix B.

3.2 Estimation Results

In this section, we briefly summarize the main estimation results, while details about the remaining results can be found in online Appendix C.

For the nominal bonos del tesoro, the root mean-squared error (RMSE) for all bonds combined is 10.83 basis points, while the corresponding statistics for the real bonos del tesoro UVR is 15.85 basis points. Thus, the BCFZ model provides a good fit to both sets of bond prices, similar to what BCFZ report for their Mexican data.

As for the monthly data on inflation forecasts for the following full calendar year and the semiannual data on 5-year, 10-year, and so-called 5yr5yr inflation forecasts, the RMSEs are 34.31 basis points, 31.52 basis points, 26.03 basis points, and 28.19 basis points, respectively, which are all well below the 75 basis points assumed in the model estimation. Thus, the model is also able to simultaneously deliver a reasonably accurate fit to the full term structure of available survey inflation forecasts.

The estimated dynamic parameters in the BCFZ model are reported in Table 4. We note that the estimated mean and volatility parameters for the seven state variables are generally similar to those reported by BCFZ using Mexican data. Hence, the Colombian and Mexican government bond markets seem to share some fundamental characteristics.

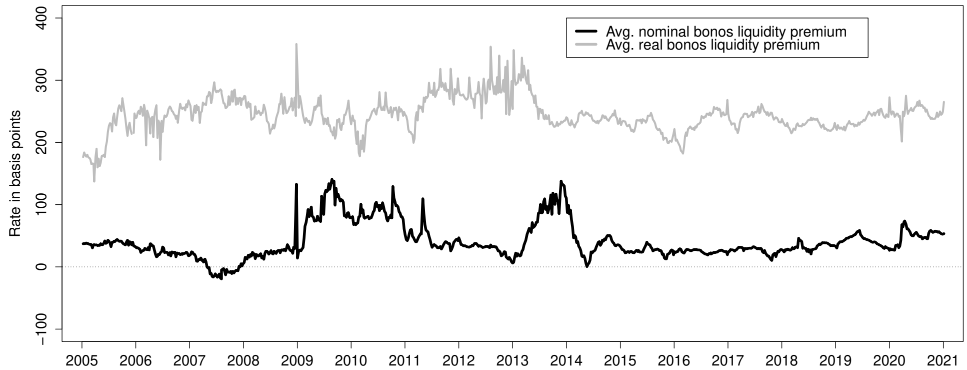


Figure 13: **Average Estimated Liquidity Risk Premia of Colombian Government Bonds**
Illustration of the average estimated liquidity risk premium of Colombian bonos del tesoro and bonos del tesoro UVR for each observation date implied by the BCFZ model. The liquidity risk premia are measured as the estimated yield difference between the fitted yield to maturity of individual bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from January 7, 2005, to December 30, 2020.

3.3 The Estimated Liquidity Risk Premia

We now use the estimated BCFZ model to extract the liquidity risk premia in the bonos del tesoro and bonos del tesoro UVR prices. To compute these premia, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^{T}$ to calculate the fitted bond prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$.

To obtain the corresponding yields with correction for liquidity risk, we compute a new set of model-implied bond prices from the estimated BCFZ model but using only its frictionless part. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$. They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity risk premium for the ith bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{5}$$

This can be calculated for bonos del tesoro and bonos del tesoro UVR separately.

Figure 13 shows the average bonos del tesoro and bonos del tesoro UVR liquidity risk premium series, denoted $\bar{\Psi}_t^N$ and $\bar{\Psi}_t^R$, across the outstanding set of each type of bond at each point in time. The average liquidity risk premium of bonos del tesoro has a mean equal to 40.72 basis points with a standard deviation of 28.38 basis points, while the average bonos del tesoro UVR liquidity risk premium has a significantly higher mean equal to 244.42 basis points with a standard deviation of 27.08 basis points. Hence, according to our model, the liquidity risk in the bonos del tesoro UVR market is an order of magnitude above that in the standard bonos del tesoro market. Furthermore, their correlation in levels is -15 percent, while it is 1 percent in first differences. Thus, the liquidity risk in the two markets is hardly correlated, which is similar to what BCFZ report for the Mexican government bond market.

As documented in Section 2.4, the trading of nominal and inflation-indexed debt is concentrated among very different investor groups in each market. We take the very low correlation in the estimated liquidity risk premium series from the two markets to be a sign that the liquidity needs and demands across these diverse investor groups are rather different as well.

This leaves the question whether the two bond markets are segmented, which would invalidate a key assumption underlying the BCFZ model, namely that the marginal investor is the same across the two markets, and the same across bonds in each market. Regarding the bonds in each market, the small fitted errors within the BCFZ model represent tangible evidence that the bonds in each market are priced consistently relative to each other most of the time. As for the consistency across markets, we aim to provide evidence that the two markets are connected and not segmented by examining their joint response to a series of unexpected exogenous foreign monetary policy shocks. This is the focus of the next section.

3.4 International Spillovers of U.S. Unconventional Monetary Policy

In accordance with the established literature on arbitrage-free dynamic term structure models, a key assumption underlying the BCFZ model is that the marginal investor is the same across the nominal and indexed bond markets.

To validate this key modeling assumption, we examine the Colombian bond market response to eight U.S. unconventional monetary policy announcements in the 2008-2009 period. These announcements listed in Table 5 have also been analyzed in previous papers such as Gagnon et al. (2011) and Christensen and Rudebusch (2012), among many others. That large literature argues that, to a large extent, these announcements came as surprises to financial market participants. Thus, we adopt that position in the following. Moreover, given that these decisions were clearly made without regard for economic developments in Colombia at the time, the resulting monetary policy shocks can safely be assumed to be exogenous to the Colombian economy. As a result, the Colombian bond market reaction to these events can serve as a natural experiment to shed light on how the nominal and inflation-indexed bond markets respond to such unexpected exogenous shocks.[30]

Table 6 reports the two-day changes in Colombian nominal and real yields as well as BEI rates across four maturities in response to the eight U.S. QE events. In general, both bond markets responded to these events, but to varying degrees depending on the specific

[30] A potential caveat regarding drawing general conclusions based on this evidence is that some of these announcements were made in extraordinary times with high volatility in financial markets. Hence, for this exercise to be able to speak to the bond market behavior in general, we are essentially assuming that the bond markets will respond in a similar way during normal times.

No.	Date	Event	Description
I	Nov. 25, 2008	Initial LSAP announcement	Fed announces purchases of $100 billion in GSE debt and up to $500 billion in MBS.
II	Dec. 1, 2008	Bernanke speech	Chairman Bernanke indicates that the Fed could purchase long-term Treasury securities.
III	Dec. 16, 2008	FOMC statement	The first FOMC statement that mentions possible purchases of long-term Treasuries.
IV	Jan. 28, 2009	FOMC statement	FOMC states that it is ready to expand agency debt and MBS purchases and to purchase long-term Treasuries.
V	Mar. 18, 2009	FOMC statement	Fed will purchase an additional $750 billion in agency MBS and $100 billion in agency debt. Also, it will purchase $300 billion in long-term Treasury securities.
VI	Aug. 12, 2009	FOMC statement	Fed is set to slow the pace of the LSAP. The final purchases of Treasury securities will be in the end of October instead of mid-September.
VII	Sep. 23, 2009	FOMC statement	Fed's purchases of agency debt and MBS will end in the first quarter of 2010, while its Treasury purchases will end as planned in October.
VIII	Nov. 4, 2009	FOMC statement	Amount of agency debt capped at $175 billion instead of the $200 billion previously announced.

Table 5: **Key U.S. Federal Reserve QE Announcements**

announcement. As a result, BEI rates changed as well in response to these announcements.

Importantly, We note the following regarding these yield changes. First, with few exceptions, Colombian nominal yields fell in response to these announcements, and they fell more notably around the most forceful announcements (events I, III and V). This seems reasonable given that these announcements were made by the Fed to provide additional monetary stimulus after its key overnight federal funds policy rate had reached its effective zero lower bound. Second, the real yields mostly move in the same direction as the nominal yields (events I-II and IV-VII), but events III and VIII are notable exceptions to this general pattern. These varying responses of the real yields imply that the responses of BEI rates can go in either direction. For three of the events (I, III, and VIII) BEI rates fell notably, for the second event they barely changed, while they increased in response to the four remaining events.

Overall, these findings suggest that the two bond markets are connected and that there are investors ensuring a coherent response to unexpected shocks like the ones examined here. Furthermore, these results provide evidence of significant spillover effects of U.S. monetary policy shocks onto Colombian financial markets, but we leave it for future research to explore that relationship further.

In summary, although many investors in the indexed bond market can be characterized

Event		Two-day yield changes			
		3-year	5-year	7-year	10-year
I	Nominal yield	-29.41	-31.14	-29.78	-25.81
	Real yield	-6.13	-8.42	-8.52	-7.33
	BEI	-23.28	-22.73	-21.25	-18.49
II	Nominal yield	-3.22	-3.71	-2.64	-0.15
	Real yield	-4.24	-3.26	-2.87	-2.75
	BEI	1.02	-0.45	0.23	2.60
III	Nominal yield	-10.24	-15.04	-17.54	-19.17
	Real yield	114.26	69.06	44.66	26.52
	BEI	-124.49	-84.10	-62.20	-45.69
IV	Nominal yield	2.26	1.13	-1.27	-5.29
	Real yield	-15.15	-19.13	-19.52	-17.85
	BEI	17.41	20.26	18.25	12.56
V	Nominal yield	-8.01	-5.48	-3.39	-0.97
	Real yield	-9.69	-7.23	-5.48	-3.71
	BEI	1.69	1.75	2.09	2.74
VI	Nominal yield	-2.53	-1.86	-1.39	-0.92
	Real yield	-14.63	-10.78	-8.39	-6.26
	BEI	12.10	8.92	7.00	5.34
VII	Nominal yield	1.30	-0.90	-2.32	-3.59
	Real yield	-12.40	-10.72	-9.21	-7.41
	BEI	13.70	9.81	6.89	3.82
VIII	Nominal yield	-6.51	-6.88	-6.57	-5.67
	Real yield	1.32	4.08	4.59	3.87
	BEI	-7.83	-10.96	-11.16	-9.54

Table 6: **Colombian Yield Responses to U.S. QE Announcements**
The table reports the two-day response of Colombian government bond yields at four different maturities around eight U.S. QE announcements. All numbers are measured in basis points.

as passive buy-and-hold investors as noted in Section 2.4, there are investors and institutions that appear to be trading across the two markets taking on the classic textbook role of acting as arbitrageurs. As a consequence, we do not think of these markets as segmented. More importantly, this evidence justifies the key assumption underlying the BCFZ model structure, namely that there exists a single marginal investor that simultaneously trades all bonds in both the nominal and indexed bond markets.

Crucially, this evidence also makes us comfortable in relying on the joint bond price data to estimate the inflation risk premium investors demand for holding the Colombian nominal bonds. The task we now turn to.

4 Empirical BEI Decomposition

In this section, we first briefly describe how we decompose BEI into the model-implied expected inflation and the associated inflation risk premium that investors in nominal bonos del tesoro demand to assume their inflation risk. We then examine the properties of the BEI decomposition implied by the BCFZ model with a particular emphasis on both the model-implied expected inflation and the associated inflation risk premium. We end the section with an analysis of the estimated inflation risk premia and their determinants, including an international comparison.

4.1 Decomposing BEI

Christensen et al. (2010) show that the price of a nominal zero-coupon bond with maturity in τ years can be written as

$$P_t^N(\tau) = P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right),$$

where $P_t^R(\tau)$ is the price of a real zero-coupon bond that pays one consumption unit in τ years, M_t^R is the real stochastic discount factor, and Π_t is the price level.[31]

By taking logarithms, this can be converted into

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real *frictionless* zero-coupon yields adjusted for the embedded liquidity risk premia as described in BCFZ, while the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right] \quad (6)$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right). \quad (7)$$

Equation (7) demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0. \quad (8)$$

[31] The full details of the decomposition can be found in BCFZ.

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2012).

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

4.1.1 Identification of Expected Inflation and the Inflation Risk Premium

In this section, we briefly elaborate on the identification of expected inflation and the inflation risk premium within the BCFZ model.

In principle, it is feasible to rely on a yields-only approach for the identification of expected inflation, as in for example the Canadian study by Christensen et al. (2025). However, in general, this requires relatively long time-series samples in order for the estimated model dynamics to adequately reflect the persistence of the state variables. Or put differently, if the sample length is too short, the estimated persistence of the state variables may be plagued by the finite-sample bias issues discussed at length in Bauer et al. (2012). Moreover, the more latent state variables there are in the model, the harder it is for the estimator to assign the right persistence to each one. As a consequence, our Colombian model with seven state variables and only 16 years of data in our benchmark 2005-2020 sample falls in the grey zone where it may in principle appear feasible to estimate the model using a yields-only approach, but in reality this is very challenging and not really doable.[32]

To improve identification, one can add a mix of backward-looking realized inflation, which may be available for a longer history, and forward-looking survey information as done in for example the U.S. TIPS study by D'Amico et al. (2018). This represents the other extreme.

Here, we follow BCFZ and only add survey information. This provides the cleanest way to leverage the superiority of the survey inflation forecasts as documented by Ang et al. (2008) based on U.S. data. Crucially, as demonstrated by Kim and Orphanides (2012), the inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics and hence overcome the finite-sample bias challenge highlighted in Bauer et al. (2012).

Overall, the purpose of including the survey information is to have the model-implied long-term inflation expectations be well behaved and realistic. In turn, this helps ensure that the residual inflation risk premia are also reasonably well behaved.

[32] In online Appendix E, we report results for the BCFZ model estimated using yields only.

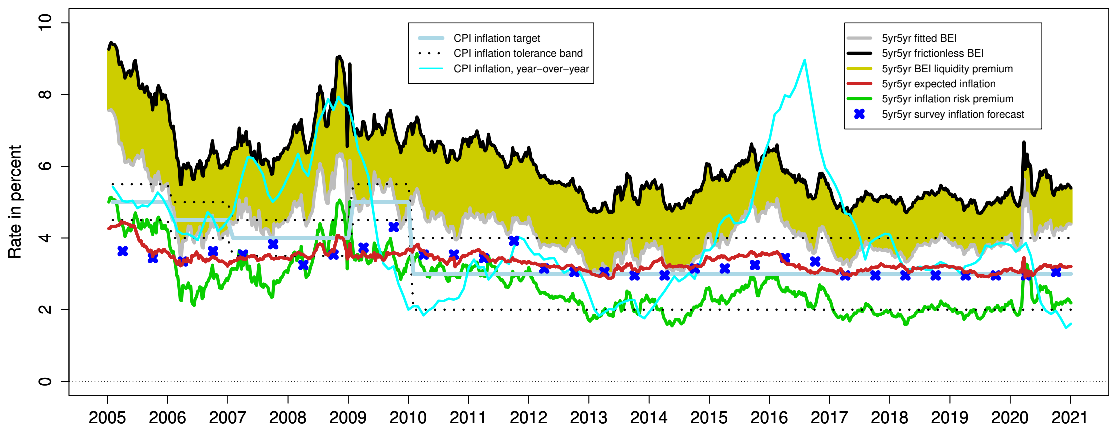


Figure 14: **Decomposition of 5yr5yr BEI**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Colombian bonos and bonos UVR prices separately and its decomposition based on the BCFZ model estimated with an unrestricted specification of $K^{\mathbb{P}}$ and a diagonal specification of Σ into: (1) the estimated frictionless BEI, (2) expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from January 7, 2005, to December 30, 2020.

4.2 BEI Decomposition

The formulas for decomposing BEI provided in the previous section are valid for any maturity τ. However, to be consistent with the existing literature, we focus on a horizon long enough into the future that most transitory shocks to the economy can be expected to have vanished. At the same time, the horizon must be practically relevant and covered by the available maturities in the underlying bond data. Balancing these considerations, we limit our analysis to the five-year forward BEI rate that starts five years ahead, denoted 5yr5yr BEI.

The result of decomposing 5yr5yr BEI based on the BCFZ model is shown in Figure 14. The solid gray line shows the fitted 5yr5yr BEI obtained by estimating a standard three-factor arbitrage-free Nelson-Siegel (AFNS) model using nominal bonos del tesoro and real bonos del tesoro UVR prices separately. This can be compared to the estimated 5yr5yr frictionless BEI implied by the BCFZ model, which is shown with a solid black line in Figure 14. The difference between these two measures of 5yr5yr BEI represents the net liquidity premium or distortion of the observed BEI series due to bond-specific liquidity risk premia in both bonos del tesoro and bonos del tesoro UVR prices. The fact that the 5yr5yr frictionless BEI is entirely above the 5yr5yr fitted BEI implies that the distortions due to liquidity risk are systematically larger in the real yields compared to those in the nominal yields at the 5yr5yr horizon, consistent with the evidence in Figure 13.

Due to its theoretical consistency, the BCFZ model allows us to break down the 5yr5yr frictionless BEI into an expected inflation component, shown with a solid red line in Figure 14, and the residual inflation risk premium, shown with a solid green line. Also shown in Figure 14 with a solid light blue line is the inflation target of the BR, which has varied over time but has been stable at 3 percent since 2010. Also shown with dotted black lines is the ± 1 percentage point tolerance band around the target rate. For comparison, Figure 14 also shows in blue crosses the 5yr5yr expected CPI inflation reported semiannually in the Consensus Forecasts surveys. Although these survey inflation forecasts are included in the model estimation, the model-implied 5yr5yr expected inflation rate does deviate from them at times thanks to the assumed standard deviation of 75 basis points for the associated measurement errors. Still, the closeness of the model's expected inflation to all the considered survey forecasts reported earlier underscores its ability to appropriately capture the term structure of inflation expectations among investors in the Colombian bonos del tesoro and bonos del tesoro UVR markets. Finally, Figure 14 also shows the year-over-year change in the Colombian CPI with a solid cyan line to provide a measure of the actual inflation outcomes during this 16-year period. Since 2010, CPI inflation has mostly remained within the tolerance band except for a brief period in 2015-2016. We take this evidence to imply that both the survey inflation forecasts and the model-implied inflation expectations can be viewed as anchored at a level consistent with the central bank's inflation target.

4.2.1 Comparison with an Alternative BEI Decomposition

To validate the BEI decomposition implied by the BCFZ model, we compare it to the BEI decomposition from an existing model of Colombian nominal and real government bond yields described in Espinosa-Torres et al. (2017, henceforth EMM). They use nominal and real yields with maturities ranging from one to eight years and estimate a six-factor dynamic term structure model based on the approach of Adrian et al. (2013). Furthermore, their model makes adjustments for liquidity risk premia in both bond markets using an approach similar to Abrahams et al. (2016). Finally, no CPI or survey information is used in the model estimation.

Figure 15 compares the estimated eight-year expected inflation from the two models. For reference we include the semiannual ten-year inflation forecasts from the Consensus Forecasts surveys shown with blue crosses. Although clearly positively correlated, the EMM estimate is uniformly lower. This is explained by its lower estimate of the net liquidity premium in Colombian BEI rates. Crucially, the EMM estimate implies that long-term inflation expectations in Colombia have frequently been below the target of the central bank. This appears to be somewhat at odds with the fact that realized CPI inflation has mostly come in above the target during our sample period, and it also deviates notably from the long-term inflation expectations reflected in surveys of professional forecasters.

Figure 16 compares the estimated eight-year inflation risk premium from the two models.

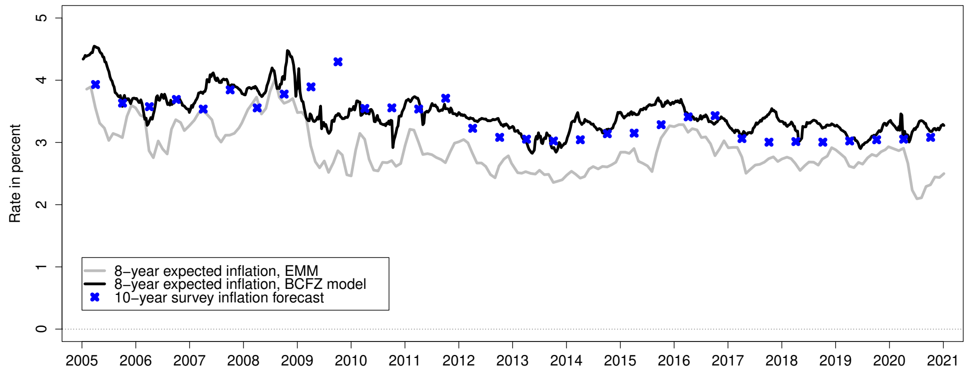


Figure 15: **Comparison of Measures of Market-Based Expected Inflation**

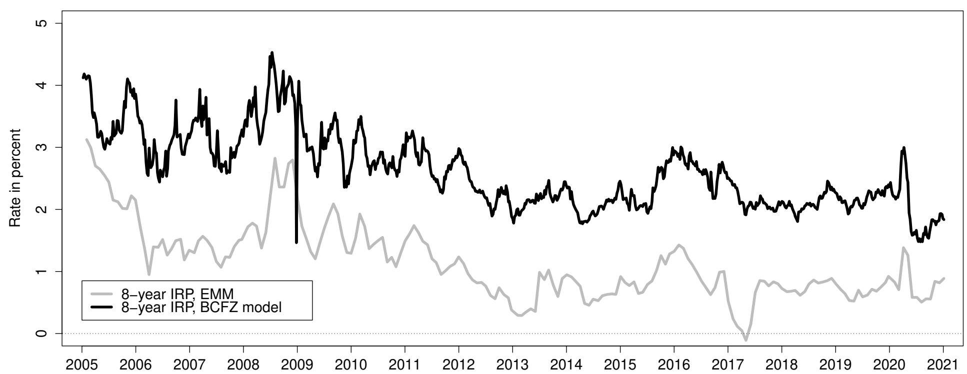


Figure 16: **Comparison of Market-Based Inflation Risk Premia**

The two series are also highly positively correlated, but with a sizable and persistent difference in magnitude that again can be traced back to differences in the estimated liquidity risk premia. Furthermore, the low level of the inflation risk premium implied by the EMM model seems to be somewhat counterintuitive given the high level of both inflation and its volatility in Colombia.

Overall, by more accurately accounting for the liquidity risk premia in both bonos del tesoro and bonos del tesoro UVR prices, the BCFZ model is able to produce what appears to be more realistic estimates of both investors' inflation expectations and the premia they demand for being exposed to inflation risk. This is instrumental to our assessment later on of the net benefit to the Colombian government of issuing inflation-indexed debt.

4.3 Analysis of the Model-Implied Inflation Expectations

In this section, we examine the properties of the inflation expectations implied by the BCFZ model in greater detail. First, we evaluate its ability to forecast inflation for the coming calendar year by comparing its performance with that of the Consensus Forecasts survey. Second, we assess how anchored inflation expectations appear to be in Colombia using a statistical measure from the literature.

4.3.1 Performance Comparison with Consensus Forecasts

In this section, we explore whether the desirable properties of the BCFZ model-implied long-term inflation expectations documented so far allow it to also generate realistic short-term inflation dynamics.

We structure the forecast exercise to match the monthly Consensus Forecasts survey. At the start of each month, the professional forecasters are asked about their expectations for the change in the CPI for both the current and the following calendar year. To have a series of pure forecasts that are not distorted by incoming information on realized inflation outcomes, we focus on the monthly survey forecasts of CPI inflation over the next calendar year. We then use the estimated BCFZ model to generate the matching model-implied CPI inflation forecasts. This has the advantage that the model-implied forecasts reflect information available at the end of each month and therefore lag the official survey dates by between one and two weeks. Thus, this exercise is by design conservative, although we stress the model forecasts are based on the full-sample estimates, unlike the survey forecasts, which are real-time forecasts by construction. Finally, our sample of monthly Consensus Forecasts starts in January 2009. Hence, to align the exercise with the available survey forecasts, we start the sample of corresponding model output in December 2008 and end it in November 2020, a total of 144 forecasts.

As benchmarks, we include two additional forecasting methods. The first is the classic random walk assumption of no change for which the one-year inflation forecast each month equals the past 12-month change in the Colombian CPI. Hence, the fact that the forecast does not start until the beginning of the next calendar year is without importance. The second is constructed from the observed BEI rates and equals the one-year forward BEI rate that starts at the beginning of the next calendar year and hence align exactly with the forecast horizon in the Consensus Forecasts surveys.[33]

The summary statistics of the monthly forecast errors from the four forecast methods are reported in Table 7. First, we note that observed BEI rates outperform the random walk assumption. This suggests that the bond yield data are informative about inflation dynamics. However, as an inflation forecasting tool, observed BEI rates are inferior to both the survey

[33]Similar to Figure 14, the BEI rates are obtained by estimating a standard three-factor AFNS model to nominal bonos del tesoro and real bonos del tesoro UVR prices separately.

Model	Mean	RMSE	MAE
Random walk	9.38	222.99	186.97
BEI	58.43	181.15	141.53
Consensus Forecasts	35.80	166.13	128.11
BCFZ model	38.13	167.77	130.83

Table 7: **Summary Statistics of CPI Inflation Forecast Errors**
This table reports the mean forecasting errors (Mean), the root mean-squared forecasting errors (RMSE), and the mean absolute forecasting errors (MAE). The BCFZ model forecasts are computed from the full sample estimation results. The forecast errors are reported as the true value minus the model-implied prediction, and all numbers are reported in annual basis points.

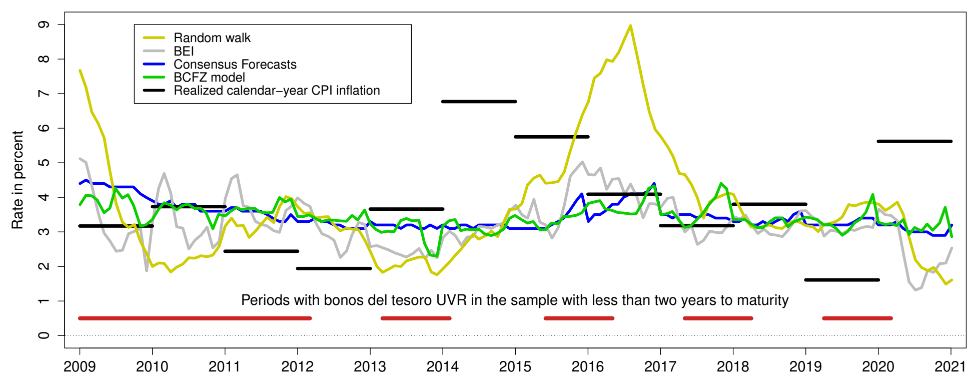


Figure 17: **CPI Inflation Forecasts and Realizations**

forecasts and the BCFZ model-implied forecasts because of the noise added by both the inflation risk premium and differential liquidity risk premia. This explains its higher RMSE forecast errors and mean absolute forecast errors. Importantly, this also underscores the value of adjusting for these risk premia within the BCFZ model. Finally, in terms of the direct comparison with the survey forecasts, we note that the BCFZ model produces slightly higher forecast errors as measured by all three reported statistics. Given the flexible structure of the BCFZ model and its high number of parameters and state variables, we consider this an encouraging outcome.

In comparing the forecast series, Figure 17 shows that the survey forecasts are very stable, even at the short calendar-year-ahead horizon examined here, another sign that inflation expectations in Colombia appear to be well anchored. In contrast, BEI rates and the BCFZ model-implied forecasts are slightly more volatile. Furthermore, Figure 17 also includes the subsequent CPI calendar-year inflation realizations shown with solid black lines. Lastly, the random walk forecasts are the most volatile as they span the full swings in realized one-year inflation by construction.

To better understand the periodic deviations between the survey and BCFZ model-implied forecasts, we note that the deviations are positively correlated with the periods during which there are bonos del tesoro UVR with less than two years to maturity in our sample, highlighted with solid red lines in Figure 17. Given that the latter are periods when the bonos del tesoro UVR data may be particularly informative about investors' near-term inflation expectations, it seems reasonable that these would also be times when the model-implied inflation expectations are more likely to differ from those reported in the surveys.

Overall, these results and findings lead us to conclude that the BCFZ model is able to generate realistic inflation dynamics with properties that match those of the actual CPI series, even though we stress that no inflation data is included in the model estimation.

4.3.2 A Statistical Measure of Inflation Anchoring

For an inflation-targeting central bank like the BR, an important policy question is to what extent inflation expectations in Colombia appear to be anchored at a level consistent with the announced inflation target. In this section, to focus more squarely on that question, we consider a statistical measure of inflation anchoring inspired by Grishchenko et al. (2019).

This measure is centered around the conditional probability of our chosen anchoring measure—the 5yr5r expected inflation rate, here denoted $\pi^e_{t+\tau}(5yr5yr)$—being within the (2 percent, 4 percent) tolerance band used by the BR since 2010. That is, we are interested in the following conditional probability:

$$P\Big(\pi^e_{t+\tau}(5yr5yr) \leq 0.04 \Big| X_t\Big) - P\Big(\pi^e_{t+\tau}(5yr5yr) \leq 0.02 \Big| X_t\Big),$$

where τ is the considered horizon. Hence, this measure emphasizes whether the crucial 5yr5yr expected inflation among bond investors and other financial market participants is likely to remain within the tolerance band.

Since $\pi^e_t(5yr5yr)$ is affine in the state variables,

$$\pi^e_t(5yr5yr) = A^\pi + \Big(B^\pi\Big)' X_t,$$

it follows from the Gaussian dynamics of our model that

$$\pi^e_{t+\tau}(5yr5yr) \sim N\Big(A^\pi + \Big(B^\pi\Big)' E^{\mathbb{P}}_t[X_{t+\tau}], \Big(B^\pi\Big)' V^{\mathbb{P}}_t[X_{t+\tau}] B^\pi\Big).$$

Thus, the involved probabilities are easily calculated given that the first and second moments of X_t within the BCFZ model follow well-known formulas.

Figure 18 shows these probabilities based on our estimated model at the one- and three-year horizon starting in 2010 when the BR adopted its current inflation target of 3 percent. As noted in Figure 14, the estimated value of $\pi^e_{t+\tau}(5yr5yr)$ has tended to be within the tolerance

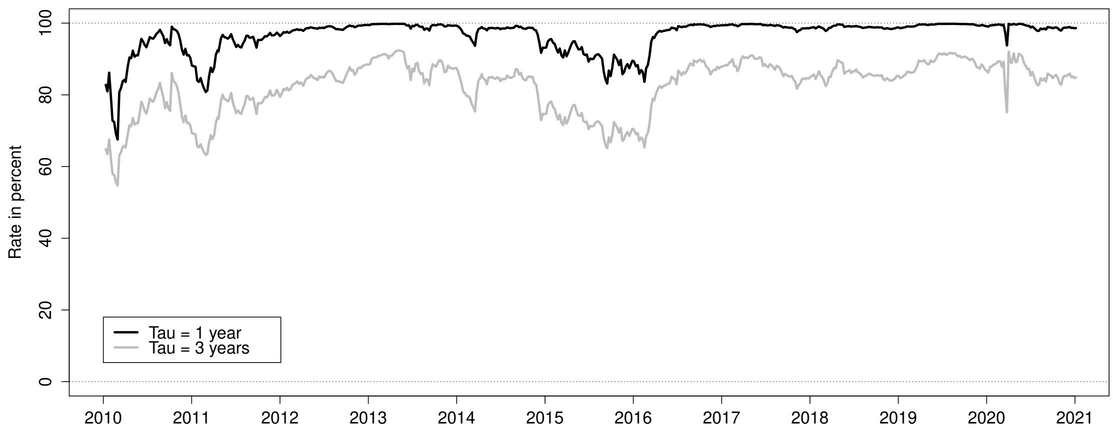


Figure 18: **Probability of 5yr5yr Expected Inflation Remaining Anchored**

band. As a consequence, it is not surprising that the probability of it remaining within the band one year ahead has fluctuated close to 100 percent since 2010. However, as we increase the considered horizon to three years, the probability declines uniformly to a level between 75 percent and 85 percent. This is thanks to the increase in the uncertainty in the underlying projections as we lengthen the forecast horizon.

Mapping to the results reported for the United States and the euro area in Grishchenko et al. (2019), we note that the probabilities we obtain for Colombia are remarkably close to theirs and much higher than those reported by BCFZ for Mexico. This shows that, although inflation overall is more volatile in an emerging market economy like Colombia, this need not translate into materially more uncertain or less well-anchored long-term inflation expectations.[34]

4.3.3 Summary

In this section, we have performed a careful examination of the BCFZ model-implied inflation dynamics. First and most importantly, we find that long-term inflation expectations in Colombia appear to be stable at a level slightly above 3 percent. This makes us draw the conclusion that inflation expectations in Colombia appear to be well anchored close to the 3 percent inflation target of the BR.[35] Furthermore, based on the high probabilities of statistical measures of inflation anchoring, which are comparable to those reported for advanced economies, we feel extra confident drawing that conclusion. Lastly, the documented reasonableness of the model's estimated inflation dynamics also gives us confidence in its estimated inflation risk premia, which we analyze next.

[34] Garcia and Gimeno (2024) report estimates of trend inflation for Colombia that are fully consistent with the BR's inflation target.

[35] De Pooter et al. (2014) reached a similar conclusion for Brazil, Chile, and Mexico.

4.4 Analysis of the Estimated Inflation Risk Premia

In this section, we first explore what determines the size of and variation in Colombian inflation risk premia using regression analysis. We then examine the term structure of unconditional inflation risk premia in Colombia. This is followed by an international comparison to Canadian, Mexican, and U.S. inflation risk premia.

4.4.1 Determinants of Inflation Risk Premia

While the long-term inflation expectations in Colombia are largely determined by the inflation target of the BR, it is less clear which factors would matter for the size of Colombian long-term inflation risk premia. To explain the variation of the 5yr5yr Colombian inflation risk premium series, we therefore run a battery of standard regressions with it as the dependent variable and a wide set of explanatory variables that are thought to play a role for inflation risk premia as explained in the following.

To begin, we are interested in the role of factors that are believed to matter for bonos del tesoro and bonos del tesoro UVR market liquidity specifically or bond market liquidity more broadly as they could matter for the estimated inflation risk premia, even though we have explicitly accounted for bonos del tesoro and bonos del tesoro UVR liquidity risk premia in the model estimation. First, we include the average bonos del tesoro age and the one-month realized volatility of the 10-year bonos del tesoro yield as proxies for bonos del tesoro liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et al. (2013), we also include a noise measure of bonos del tesoro prices to control for variation in the amount of arbitrage capital available in this market. Third, we include the average bonos del tesoro UVR age to proxy for liquidity risk in the market for those bonds. Finally, we add a Colombian overnight interbank rate known as TIB to proxy for the opportunity cost of holding money and the associated liquidity premia of Colombian government bonds, as explained in Nagel (2016). Combining these five explanatory variables tied to market liquidity and functioning produces the results reported in regression (1) in Table 8. We note a high adjusted R^2 of 0.58. The average bonos del tesoro age has a highly significant negative coefficient. This implies that an increase in the liquidity risk of bonos del tesoro is associated with lower inflation risk premia. The other measures of financial frictions tend to push up the Colombian inflation risk premia.

After having explored the role of liquidity factors, we examine the effects of factors reflecting risk sentiment domestically and globally on the inflation risk premia. This set of variables includes the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The set also contains the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same 10-year maturity. This on-the-run (OTR) premium is a fre-

Explanatory variables	(1)	(2)	(3)
Avg. bonos del tesoro age	-31.06***		-25.01***
	(6.547)		(5.571)
One-month vol. of 10yr bonos del tesoro yield	0.320		-0.157
	(0.597)		(0.465)
Bonos del tesoro noise measure	2.044**		0.481
	(0.999)		(0.618)
Avg. bonos del tesoro UVR age	4.551		7.580**
	(3.627)		(2.892)
TIB	8.159**		-0.019
	(4.037)		(4.799)
VIX		0.702	0.299
		(0.734)	(0.682)
OTR premium (bps)		-0.364	-0.570
		(1.206)	(1.178)
MOVE Index		0.587	0.539*
		(0.371)	(0.285)
TED spread (bps)		-0.250**	-0.059
		(0.117)	(0.156)
5yr CDS rate (bps)		0.528***	0.396***
		(0.142)	(0.114)
10yr US Treasury yield (%)		32.172***	14.397*
		(10.853)	(7.563)
WTI		0.141	-0.390
		(0.373)	(0.448)
Constant	304.381***	36.113	217.747***
	(51.308)	(33.850)	(71.155)
N	835	835	835
Adjusted R^2	0.58	0.68	0.74

Table 8: **Regression Results for the 5yr5yr Inflation Risk Premium**
The table reports the results of regressions with the estimated 5yr5yr inflation risk premium as the dependent variable and 12 explanatory variables. Standard errors computed by the Newey-West estimator (with 13 lags) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

quently used measure of financial frictions in the U.S. Treasury market. To control for factors related to the uncertainty about the interest rate environment, we include the MOVE index. The fourth variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate. This spread represents a measure of the perceived general credit risk in global financial markets. As an additional indicator of credit risk and credit risk sentiment, we use the five-year CDS rate for Colombia shown in Figure 11. The next variable in the set is the 10-year U.S. Treasury yield from the Federal Reserve's H.15 database, which is included to control for reach-for-yield effects in advanced economies. This may be particularly relevant for our sample during the period between December 2008 and December 2015 and again in 2020 when U.S. short-term interest rates were constrained by the zero lower bound. Finally, we include the West Texas Intermediate (WTI) Cushing crude oil price to proxy for energy prices, which represent a significant risk to the inflation outlook in many countries around the world, including Colombia. The results of the regression with these seven explanatory variables is reported in regression (2) in Table 8. This produces a notable adjusted R^2 of 0.68. Furthermore, the MOVE index, the five-year CDS rate, and the 10-year U.S. Treasury yield are among the most significant variables and with the expected positive sign. This contrasts with the TED spread, which has a negative coefficient.

To assess the robustness of the results from these two regressions, we include all 12 explanatory variables with the results reported in column (3) in the table. Although this joint regression produces a high adjusted R^2 of 0.74, we do see a few variables with switches in the signs of their estimated coefficients.

Given the mixed results from the large regression models, we use informed priors to identify a simple preferred regression model to explain the variation in the 5yr5yr inflation risk premium series. First, we do want to account for liquidity risk in both bonos del tesoro and bonos del tesoro UVR. This makes us include the average bonos del tesoro and bonos del tesoro UVR age. Second, the MOVE index is widely used as a measure of interest rate uncertainty that matters for risk premia in both bond and stock markets. Third, perceptions about credit risk as measured by CDS rates appear to matter. As a consequence, we include the five-year CDS rate as well. Finally, as a small open economy, the Colombian government bond market is significantly affected by the interest rate level prevailing in the U.S. Treasury market, which we proxy with the 10-year U.S. Treasury yield. Thus, we run a second set of regressions with these five variables individually and combined. This allows us to identify a final preferred regression model for the Colombian 5yr5yr inflation risk premium series.

The results are reported in Table 9. Regression (6) with all five representative variables combined delivers an adjusted R^2 of 0.73. This is very close to the value obtained when we include all 12 variables. Hence, this supports our selection of this particular set of representative variables. These results also underscore that our five representative variables are responsible

Explanatory variables	(1)	(2)	(3)	(4)	(5)	(6)
Avg. bonos del tesoro age	-39.65***					-22.75***
	(6.141)					(4.990)
Avg. bonos del tesoro UVR age		-22.40***				7.38***
		(5.775)				(2.580)
MOVE Index			1.47***			0.35***
			(0.131)			(0.209)
5yr CDS rate (bps)				0.70***		0.46***
				(0.111)		(0.112)
10yr US Treasury yield (%)					40.08***	12.46*
					(16.959)	(6.727)
Constant	433.83***	375.61***	151.19***	161.64***	150.89***	191.62***
	(31.65)	(34.34)	(20.36)	(23.14)	(46.77)	(39.07)
N	835	835	835	835	835	835
Adjusted R^2	0.51	0.11	0.35	0.43	0.34	0.73

Table 9: **Preferred Regression Results for the 5yr5yr Inflation Risk Premium**
The table reports the results of regressions with the estimated 5yr5yr inflation risk premium as the dependent variable and the five representative explanatory variables identified in the initial round of regressions. Standard errors computed by the Newey-West estimator (with 13 lags) are reported in parentheses. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

for essentially all of the significant explanatory power. Furthermore, all five variables are statistically significant. Most importantly, their regression coefficients have consistent and sensible signs. As a consequence, we consider regression (6) to be our preferred explanatory regression model for the Colombian 5yr5yr inflation risk premium series.

As for the involved magnitudes, we note that a one-year increase in the average bonos del tesoro age lowers the 5yr5yr inflation risk premium by 23 basis points. Thus, the persistent increase in the average bonos del tesoro age during our sample from 1.67 years to 5.65 years has been a key factor in the persistent decline in the Colombian 5yr5yr inflation risk premium since 2005. On the other hand, increases in the average bonos del tesoro UVR age, the MOVE index, and the CDS rate tend to put significant upward pressure on Colombian inflation risk premia. Lastly, a one-percentage-point increase in the 10-year U.S. Treasury yield will tend to boost the Colombian 5yr5yr inflation risk premium by about 12 basis points. For comparison, BCFZ also report an estimate of 12 basis points for the effect of the 10-year U.S. Treasury yield on Mexican inflation risk premia. Hence, the spillover effects from U.S. interest rates onto the inflation risk premia in Colombia and Mexico appear to be quite similar.

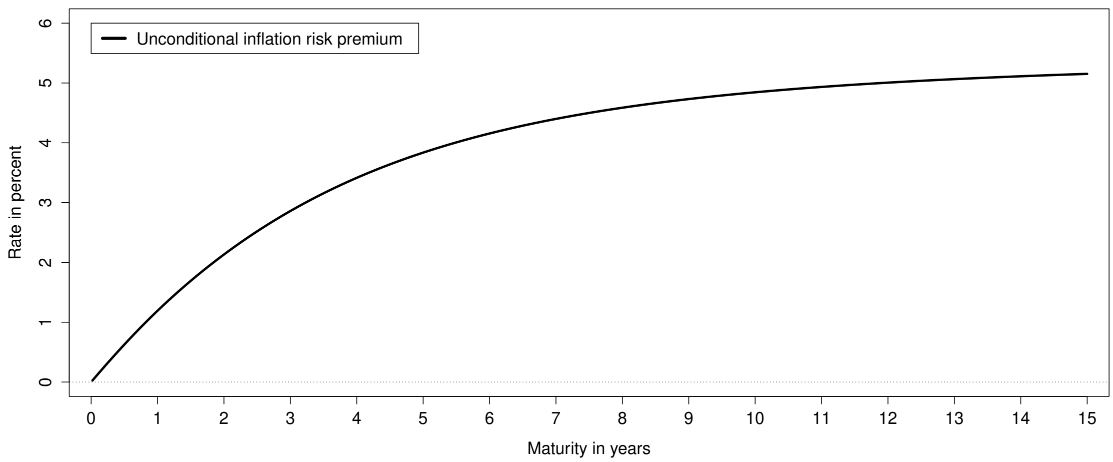


Figure 19: **Term Structure of Unconditional Inflation Risk Premia**

4.4.2 Unconditional Inflation Risk Premia

Figure 19 shows the term structure of unconditional inflation risk premia implied by the estimated BCFZ model. It is obtained by evaluating the function for the inflation risk premium at the unconditional mean of the state variables. Consistent with economic intuition, inflation risk premia are near zero at short horizons where the involved uncertainty is inherently small, even in an emerging bond market like the one in Colombia exposed to the high level of domestic inflation. In contrast—and also consistent with economic intuition—Colombian inflation risk premia at long horizons approach 5 percent as investors in nominal bonds face considerable inflation risk over the long term given the high and volatile level of inflation in Colombia. As a consequence, the term structure of unconditional inflation risk premia in Colombia is steeply upward sloping in the 1- to 15-year maturity range. We note that this could serve as an important stylized fact to match for macrofinance models of emerging bond markets. Moreover, it underscores the advantage of working with a theoretically consistent arbitrage-free dynamic term structure model of nominal and real yields jointly that allows for the calculation of key outputs such as the full term structure of the unconditional first moment of inflation risk premia discussed here.

4.4.3 International Comparison of Inflation Risk Premia

To go beyond the regression analysis in Section 4.4.1, we compare the estimated 5yr5yr inflation risk premium from Colombia with matching estimates from Canadian, Mexican, and U.S. nominal and real yields.[36] Figure 20 shows all four series for the available overlapping

[36]The Canadian estimate is taken from Christensen et al. (2025), the Mexican estimate comes from an update of the analysis by BCFZ, while the U.S. estimate represents an update of the model described in Andreasen et al. (2021) using all available U.S. TIPS.

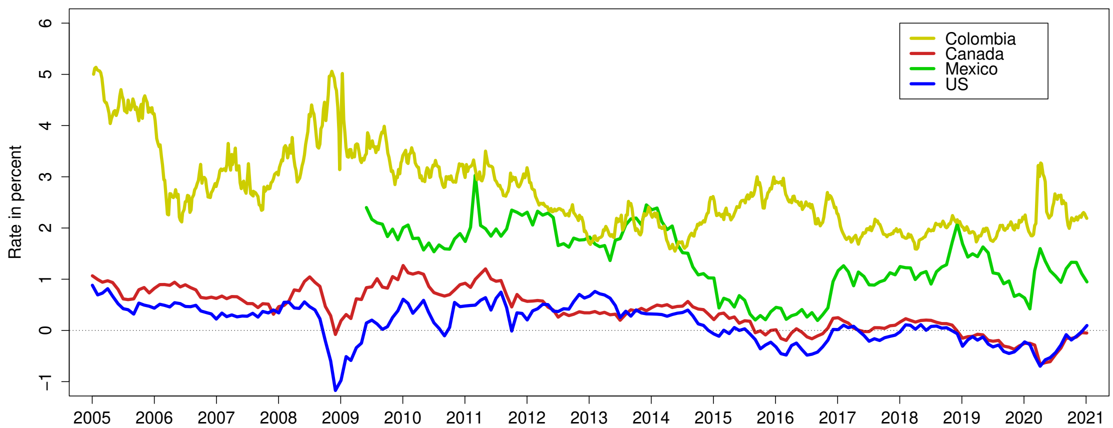


Figure 20: **International Panel of 5yr5yr Inflation Risk Premia**
Illustration of the estimated 5yr5yr inflation risk premium series from Mexican, Canadian, and U.S. nominal and real bond prices as described in the text. The shown data for Colombia are weekly and cover the period from January 7, 2005, to December 30, 2020, while the shown data for Canada and the United States are monthly and cover the period from January 31, 2005, to December 31, 2020. Finally, the shown data for Mexico are monthly and cover the period from May 31, 2009, to December 30, 2020.

sample period.

The Canadian and U.S. inflation risk premia are highly positively correlated (77 percent). The Mexican inflation risk premium series is also positively correlated with each for the overlapping period, 64 percent and 69 percent, respectively. Thus, both in terms of size and time variation, Mexican inflation risk premia share similarities with those observed in Canadian and U.S. bond markets.

The estimated 5yr5yr inflation risk premium for Colombia has a much lower correlation with the Canadian, U.S., and Mexican estimates of 54 percent, 16 percent, and 27 percent, respectively. Furthermore, Colombian inflation risk premia are more volatile with a standard deviation of 78 basis points compared with 42 basis points, 37 basis points, and 63 basis points for the Canadian, U.S., and Mexican series, respectively.

The mostly positive and small inflation risk premia in Canada and the United States are consistent with the findings from simple macro-finance representative agent models; see Hördahl and Tristani (2012). For the United States, D'Amico et al. (2018) also report empirical estimates of inflation risk premia, which are mostly positive and relatively small. In turn, to observe larger and more volatile inflation risk premia in an emerging market economy such as Colombia would seem like a reasonable result given the higher and more volatile CPI inflation in Colombia compared with Canada and the United States. Given that CPI inflation has averaged a notch higher in Colombia compared to Mexico in addition to being notably more

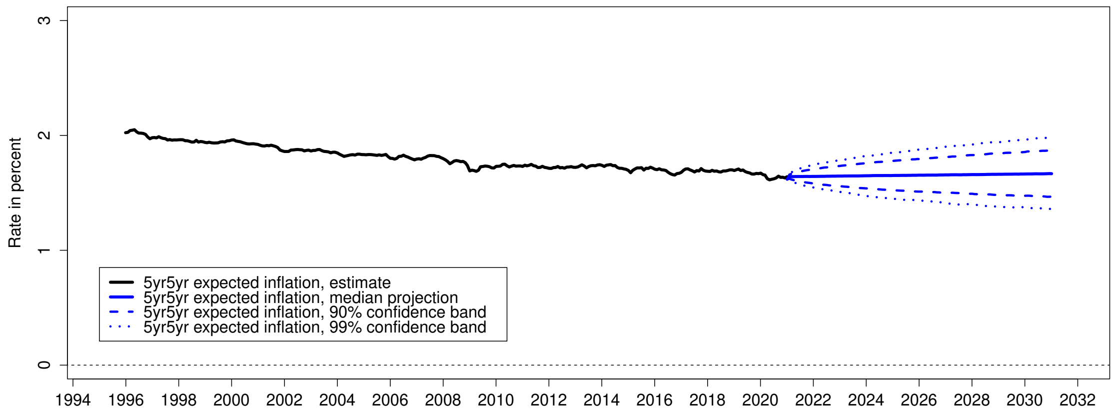


Figure 21: **Ten-Year Projections of 5yr5yr Expected Inflation in Canada**

volatile, it also seems reasonable that Colombian inflation risk premia are higher and more volatile than those observed in Mexico.

In an attempt to understand why inflation risk premia are so much higher in Colombia, we leverage the BCFZ model and its estimated dynamic structure to examine the full distribution of potential changes to investors' 5yr5yr expected inflation. To do so, we follow the approach of Christensen et al. (2015) and simulate 10,000 factor paths over a ten-year horizon, conditioned on the shapes of the nominal and real yield curves and investors' embedded forward-looking expectations as of the end of December 2020 (i.e., using the estimated state variables and factor dynamics as of December 30, 2020). The simulated factor paths are then converted into forecasts of 5yr5yr expected inflation, a key economic fundamental when it comes to inflation uncertainty in the long run.

To create the sharpest contrast, we repeat this exercise for the Canadian model described in Christensen et al. (2025) since Canada has the lowest and most stable CPI inflation among our panel of countries.[37]

The Canadian simulation results are reported in Figure 21, which shows the median projection and the 0.51th, 5th, 95th, and 99.5th percentile values for the simulated 5yr5yr expected inflation over the ten-year forecast horizon.[38] We note that the median projection of 5yr5yr expected inflation in Canada is essentially a flat line close to the 2 percent inflation target set by the Bank of Canada. Moreover, the simulated paths suggest that 5yr5yr expected inflation is mostly to remain in vicinity of their estimated level as of December 30, 2020. More importantly, for the question about inflation risk, the width of the 90 percent confidence five years

[37]For the 2005-2020 period, CPI inflation in Colombia averaged 4.13 percent with a standard deviation of 1.68 percent. In Canada, it averaged 1.70 percent with a standard deviation of 0.83 percent. Thus, both the level and standard deviation in Colombia are more than twice the size of those in Canada.

[38]Note that the lines do not represent paths from a single simulation run over the forecast horizon; instead, they delineate the distribution of all simulation outcomes at a given point in time.

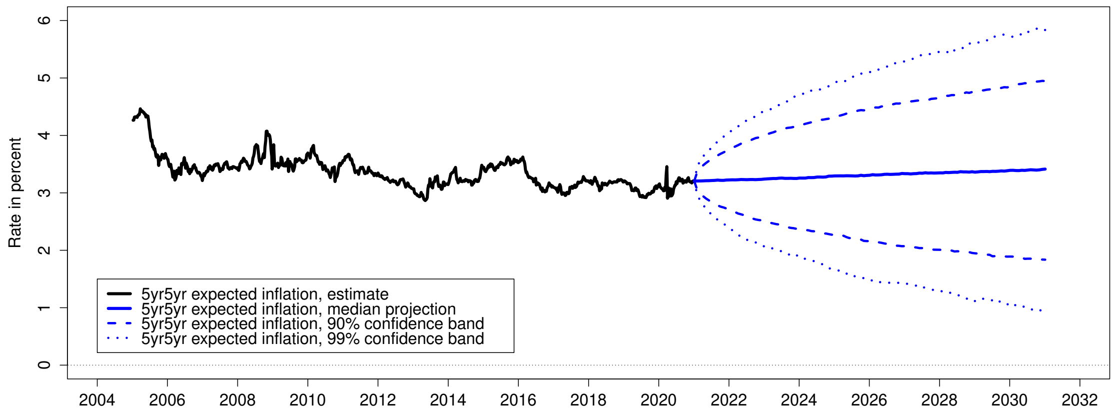


Figure 22: **Ten-Year Projections of 5yr5yr Expected Inflation in Colombia**

out is only 0.28 percent, while it increases modestly to 0.4 percent at the ten-year horizon. The corresponding statistics for the width of the 99 percent confidence band are 0.41 percent and 0.63 percent, respectively. Thus, from the point of view of the model, there is only a tiny chance of any major changes to the 5yr5yr expected inflation in Canada. This low risk to long-term inflation expectations in Canada is a key factor behind its very low 5yr5yr inflation risk premium the last many years.

This contrasts with the corresponding BCFZ model-based simulation results for Colombia shown in Figure 22. In Colombia, five years out, there is a 5 percent chance that 5yr5yr expected inflation will be 4.46 percent and, at the ten-year horizon, there is a 5 percent chance that it will touch 4.95 percent—notably above the 3.20 percent estimate as of December 30, 2020. Moreover, further out the tail and five years out, there is a 0.5 percent chance that 5yr5yr expected inflation will be as high as 5.10 percent and as high as 5.84 percent at the ten-year horizon. These represent scenarios with a likely deanchoring of inflation expectations that presumably the inflation risk premium serves as the compensation against.

Thus, based on the BCFZ model, there is a meaningful chance of some fairly severe tail risks when it comes to investors' long-term inflation expectations in Colombia, even though our analysis suggests that they were well anchored as of December 2020. Furthermore, this exercise highlights the advantage of using a full dynamic term structure model that allows us to attach probabilities to specific outcomes. This is crucial for risk management purposes, as also stressed by Christensen et al. (2015).

To conclude, although both survey information and model-based measures of anchoring could be interpreted to suggest that investors' long-term inflation expectations in Colombia appear to be about as well anchored as they are in an advanced economy like Canada, a more detailed simulation-based analysis of the risk of tail outcomes for investors' long-term inflation expectations reveals a sharp contrast. In Canada, the chance of investors' long-term inflation expectations changing persistently from their current level by any material margin is practically nonexistent. In Colombia, in contrast, our BCFZ model suggests that there is more than a 5 percent chance of a large and persistent upward movement in investors' long-term inflation expectations within the next five to ten years. We believe this is a significant factor behind why inflation risk premia are so much larger in Colombia compared with advanced economies like Canada or the United States.

4.4.4 Summary

In this section, we examined the properties of the Colombian 5yr5yr inflation risk premium. First, we explored its determinants. Our preferred regression model showed that inflation risk premia in Colombia are significantly affected by international factors, namely the global interest rate level proxied by the U.S. 10-year yield, the uncertainty surrounding the global interest rate level as captured by the MOVE index, and global perceptions about credit risk as reflected in the Colombian five-year CDS rate. In addition, the liquidity risk of domestic government bond markets also plays a role as proxied by the average age of the bonos del tesoro and bonos del tesoro UVR in our sample. Overall, these results largely conform to our conjecture laid out in the introduction that, provided the bonos del tesoro market is dominated by foreign investors, the inflation risk premium demanded by investors in nominal bonos del tesoro should indeed be sensitive to global risk factors.

Second, we showed that the term structure of unconditional inflation risk premia implied by the BCFZ model is strongly upward sloping. This could be an important stylized fact to match for theoretical macrofinance models of emerging bond markets.

This was followed by an international comparison, which revealed that, although positively correlated with estimates from other countries, inflation risk premia in Colombia are unique in addition to being larger and more volatile in an international context. These characteristics seem reasonable in light of the fact that CPI inflation in Colombia has been higher and more volatile over the past 15 years than in the countries included in the comparison. Moreover, this finding is consistent with our conjecture spelled out in the introduction that inflation risk is likely to be country-specific in nature outside of global spells of inflation like the one experienced in the aftermath of the COVID-19 pandemic.

Equipped with an estimate of Colombian inflation risk premia along with estimates of the liquidity risk premia of bonos del tesoro and bonos del tesoro UVR, we are ready to tackle our main research question about the benefit to the Colombian government of issuing inflation-indexed debt. This is the focus of the next section.

5 Cost-Benefit Analysis of Inflation-Indexed Debt

In this section, we assess the relative costs and benefits between issuing bonos del tesoro and bonos del tesoro UVR to the Colombian government, which constitutes an important policy application of our estimation results.

Given that the Colombian government issues bonds with fairly fixed maturities (5-year, 10-year, 15-year), we limit our focus to measuring both liquidity and inflation risk premia at those fixed constant maturities. To that end, we first calculate fitted nominal and real zero-coupon yields for all relevant maturities.[39] Note that these yields embed the liquidity risk premia of the nominal and real bonds. We then use the estimated frictionless factor dynamics within the BCFZ model to calculate the corresponding frictionless nominal and real zero-coupon yields that do not contain any liquidity risk premia. The difference between the fitted and frictionless nominal zero-coupon yields then becomes an alternative synthetic estimate of the bonos del tesoro liquidity risk premia at constant maturities. These are shown in Figure 23(a) for three maturities: 5-year, 10-year, and 15-year. Repeating this for the real yields produces the synthetic estimates of bonos del tesoro UVR liquidity risk premia at the same three maturities shown in Figure 23(b).

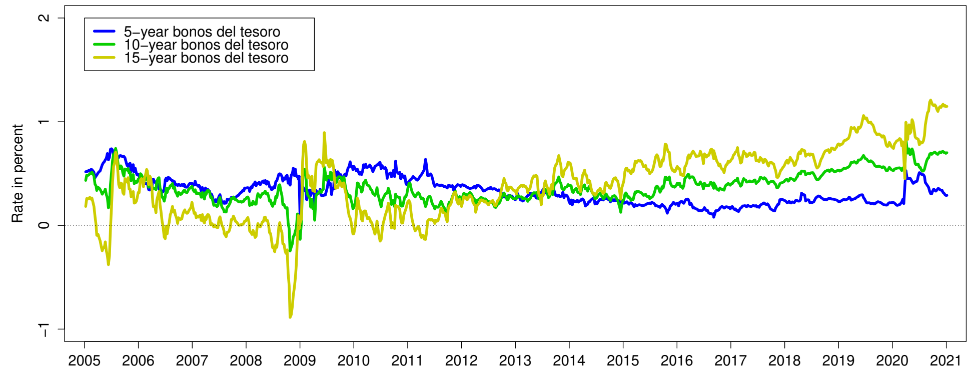
In the final step, we use the frictionless nominal and real zero-coupon yields implied by the BCFZ model to construct the corresponding frictionless BEI at the same three fixed maturities and deduct the associated model-implied expected inflation to obtain the corresponding three inflation risk premium series shown in Figure 23(c). For reference, we also plot the 5yr5yr inflation risk premium used in the regression analysis in the previous section.

Note the diverse shape of the term structure for each of these three types of risk premia. While the term structures of the inflation and bonos del tesoro liquidity risk premia both vary between flat, upward sloping, and downward sloping, the slope of the bonos del tesoro UVR liquidity risk premium curve is mostly downward sloping. This diverse pattern underscores the importance of combining the full term structure of bond prices for both bonos del tesoro and bonos del tesoro UVR with a full joint term structure model of both nominal and real yields.

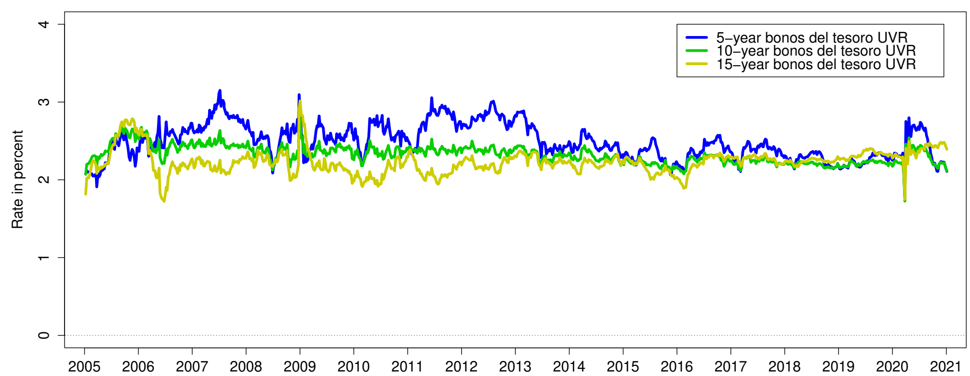
Now, for each fixed maturity, adding the inflation risk premium and the bonos del tesoro liquidity risk premium before deducting the bonos del tesoro UVR liquidity risk premium produces a measure of the net benefit of issuing bonos del tesoro UVR over bonos del tesoro to the Colombian government.[40] The resulting three net benefit series at the 5-year, 10-year, and 15-year maturities are shown in Figure 24.

[39] Similar to Figure 14, the required nominal and real zero-coupon yields are obtained by estimating a standard three-factor AFNS model to nominal bonos del tesoro and real bonos del tesoro UVR prices separately.

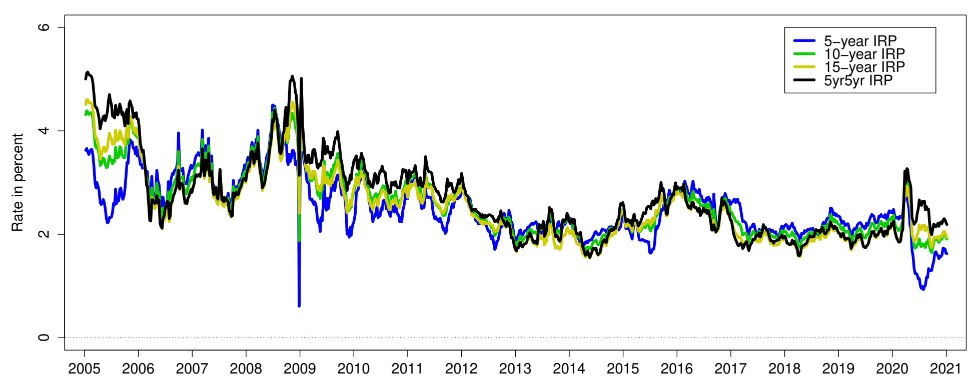
[40] An alternative way of measuring the net benefit of issuing inflation-indexed debt is to compare the ex post realized costs of issued inflation-indexed bonds to those of matching issued nominal bonds. See Dudley et al. (2009) for a discussion in the context of U.S. TIPS and arguments why ex ante measures of benefits like ours are to be preferred over ex post cost calculations that are overly influenced by the realized inflation surprises over the life of the considered bonds.



(a) Bonos del tesoro liquidity risk premia



(b) Bonos del tesoro UVR liquidity risk premia



(c) Inflation risk premia

Figure 23: **Term Structure of Bond Risk Premia**

The average net benefit of issuing bonos del tesoro UVR at the 5-, 10-, and 15-year maturity is estimated at 0.37 percent, 0.66 percent, and 0.69 percent, respectively. Thus, while bonos del tesoro issuance can be considered to be competitive, at least periodically, at the liquid

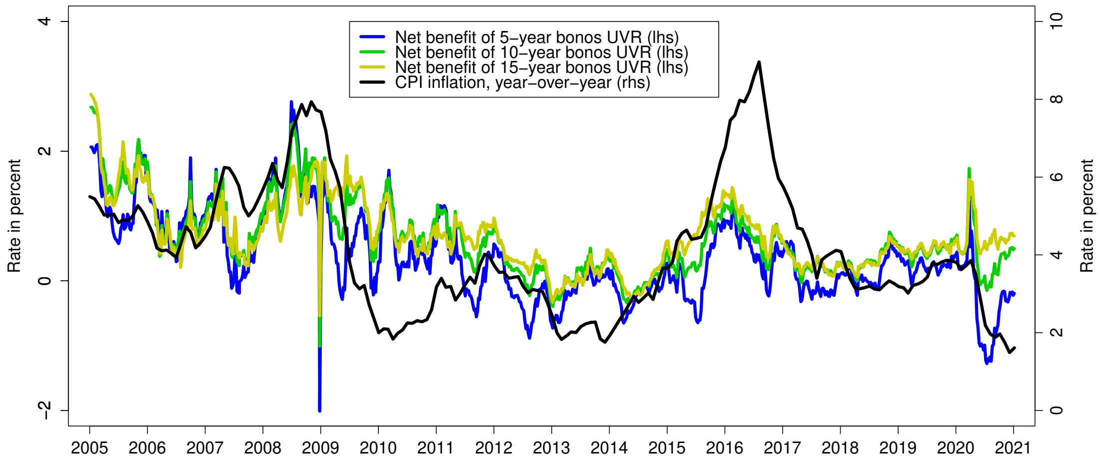


Figure 24: **Term Structure of Net Benefit of Bonos UVR Issuance and CPI Inflation**

5-year maturity, our results are unfavorable regarding issuance of 10-year and 15-year bonos del tesoro. Based on our estimates, it is recommendable that the Colombian government tilt its issuance of such long-term bonds towards the bonos del tesoro UVR market. This result shows that the sizable inflation risk premia in Colombia dominate over the relative liquidity disadvantage of bonos del tesoro UVR compared to regular bonos del tesoro, in particular at longer maturities.[41]

Figure 24 also shows the year-over-year CPI inflation on the right hand scale where we note its positive correlation with the net benefit measures of 50 percent, 48 percent, and 49 percent at the 5-, 10-, and 15-year maturity, respectively. Hence, the data suggest that the benefit of increased inflation-indexed debt issuance is particularly pronounced when inflation is elevated. An examination of the three components that go into the construction of our net benefit measure reveals that a positive correlation between inflation risk premia and realized CPI inflation seems to be the main driver behind this finding. The economic intuition behind this dynamic relationship appear to be straightforward. When inflation spikes, investors become increasingly concerned about future inflation, which has the potential to erode the real value of nominal debt. In response, they demand higher compensation for assuming the inflation risk. As a consequence, inflation risk premia are pushed up, which makes issuance of nominal debt relatively more costly compared to issuance of a matching amount of inflation-indexed debt with the same maturity. Based on these observations we conjecture that inflation-indexed debt will be even more beneficial relative to nominal debt in economies with inflation above the

[41] One minor caveat to our calculation is that it is based on secondary market prices, while the interest paid by the government is determined in the primary auctions. However, as we document in Tables 1 and 3, most auctions in the Colombian government bond market are reopenings of existing bonds with simultaneously observable secondary market prices. Thus, this distinction should matter little for our results.

levels observed in Colombia, but we leave it for future research to explore that idea further.[42]

In light of these empirical results, we feel compelled to stress that the size and sign of the inflation risk premium should not be determined by the level of inflation, but rather by its covariance with the real stochastic discount factor as made clear by equation (7).

Finally, although the unconditional inflation risk premia have an upward sloping term structure as demonstrated in Figure 19, we note in Figure 23(c) that during our sample period the term structure of inflation risk premia can be upward sloping (2005 and 2020); flat (2007); downward sloping (2017-2019); and even hump shaped (2010). This suggests a more complex reality than that implied by workhorse theoretical macrofinance models such as the habit model of Wachter (2006), the rare disasters model of Gabaix (2012), or the long-run risk model of Bansal and Shaliastovich (2013), which all imply that the inflation risk premium should be increasing with maturity because the inflation process is persistent and on average negatively correlated with consumption growth.[43] Still, despite the significant variation in the level and slope of the term structure of conditional inflation risk premia, it is the case that we find an average net benefit of issuing inflation-indexed debt at all three examined maturities.

5.1 Evidence from Mexico

As a point of reference, we repeat the analysis for Mexico by estimating the BCFZ model using an update of the monthly Mexican data examined in BCFZ, which starts in May 2009. Furthermore, we note that inflation-indexed bonds in Mexico are known as udibonos, which is the term used in the following.

Given that the Mexican government also issues bonds with fairly fixed maturities (5-year, 10-year, 20-year), we calculate the net benefit measure of issuing udibonos for those three maturities with the resulting series shown in Figure 25. In this case, we see a fairly consistent upward sloping pattern with the net benefit of issuing 5-year udibonos being lower than the benefits of issuing longer-term udibonos. Still, all three series are positive on average: 0.15 percent, 0.28 percent, and 0.49 percent, respectively, at the 5-, 10-, and 20-year horizon. Hence, these results lead to a similar conclusion, namely issuance at the 5-year maturity appear to be competitive for both types of debt with a mild advantage towards inflation-indexed issuance, while at the long-term maturities the results point to a more material benefit from increased issuance of inflation-indexed bonds.

5.2 Update through 2024

In this section, we provide an update of our analysis based on data through the end of December 2024. We consider this an important out-of-sample robustness check of the results

[42]Brazil would be a prime candidate given that CPI inflation there is higher and more volatile than in Colombia.

[43]We leave it for future research to explore more formally the relationship between the inflation process and consumption growth in Colombia.

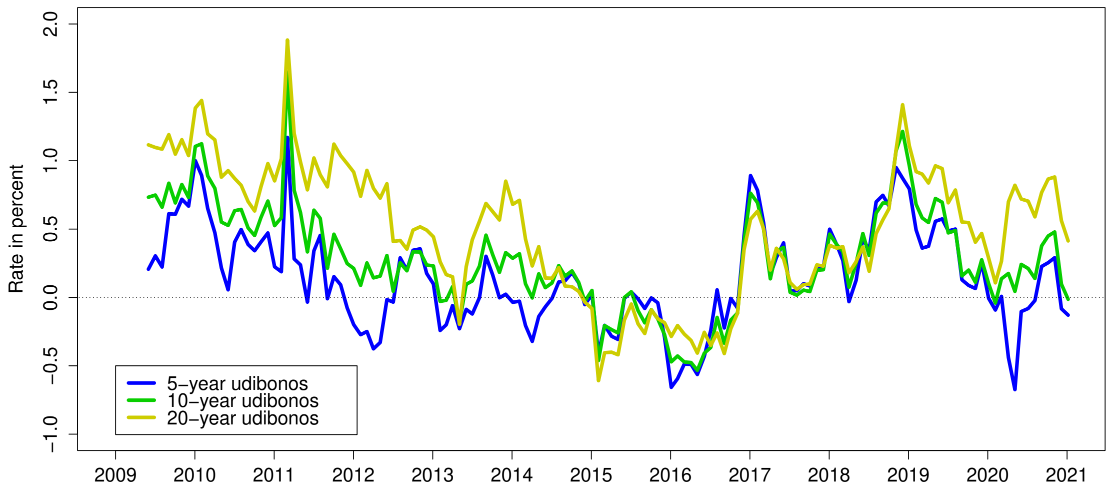


Figure 25: **Term Structure of Net Benefit of Mexican Udibonos Issuance**

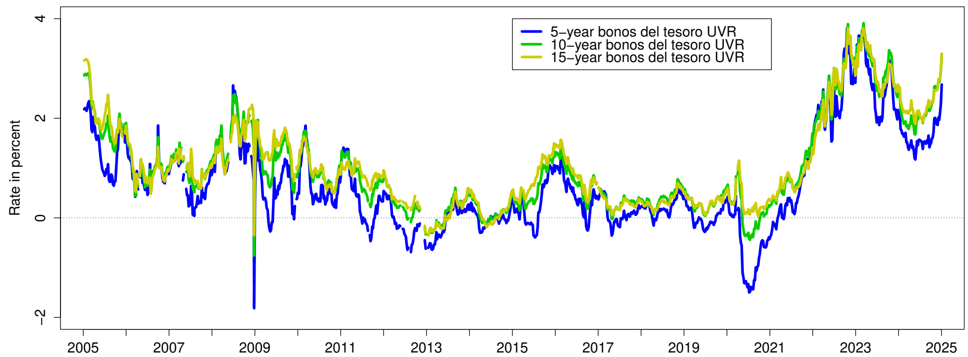
reported based on our benchmark 2005-2020 sample in light of the elevated levels of inflation since then prevailing around the world, including in Colombia and Mexico.

The updated estimates of the net benefits of increased inflation-indexed bond issuance to the Colombian and Mexican government are shown in Figure 26. First, our Colombian results for the benchmark 2005-2020 period are very similar to those shown in Figure 26(a) for the overlapping period, and the same holds for our updated Mexican results. Thus, our original results for both Colombia and Mexico are robust to an update.

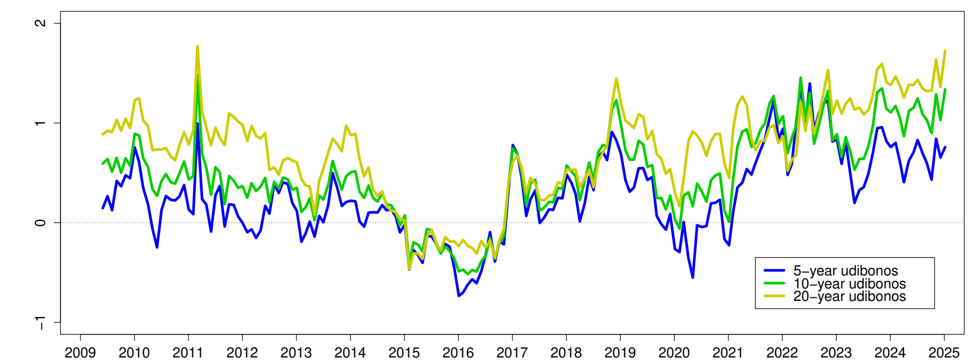
Second, our conjecture from Section 5 is confirmed in that when inflation spikes as it did in the 2021-2022 period, the benefit of issuing inflation-indexed bonds increases, partly driven by higher inflation risk premia, but also by somewhat lower liquidity risk premia in inflation-indexed bond prices as they become more desirable securities to hold under those circumstances.[44] The same pattern is observed in the updated results for Mexico.

Finally, the clear upward sloping term structure of the net benefit of issuing inflation-indexed bonds remains evident in the 2021-2024 period. Hence, our recommendation to increase issuance of inflation-indexed bonds at long maturities is even stronger based on the updated data. Specifically, for Colombia, the average net benefit is 0.69 percent, 1.02 percent, and 1.06 percent at the 5-, 10-, and 15-year maturities, respectively. For Mexico, the average estimated net benefit is 0.27 percent, 0.48 percent, and 0.71 percent at the 5-, 10-, and 20-year maturities, respectively. Given that these numbers reflect the annual saved interest rate expenses for long-term debt contracts, the total savings end up being quite material sums.

[44] Updated Colombian liquidity and inflation risk premium estimates are reported in online Appendix F.



(a) Colombia



(b) Mexico

Figure 26: **Term Structure of Net Benefit of Inflation-Indexed Bond Issuance**

5.3 Evidence from Advanced Economies

Ermolov (2021) examines the benefit of inflation-indexed over nominal debt at various maturities calculated simply as the difference between BEI and expected inflation, measured either from surveys of professional forecasters or from a statistical model, for 9 advanced economies. He finds that nominal bonds are favorable at the 5-year maturity, while it is mostly a tie between issuing nominal and real debt at the 10-year maturity. Thus, it is only at maturities from 15 years and above that inflation-indexed debt becomes cost effective based on his data. Importantly, unlike our analysis, his simplified approach does not speak to the underlying source behind this pattern across maturities as the inflation and liquidity risk premia are not separately identified. However, our international comparison of inflation risk premia in Section 4.4.3 suggests that a key factor behind his results may be the lower level of inflation risk premia in advanced economies as compared to the their higher level in the two emerging economies examined in this section.

For his sample of advanced economies, Ermolov (2021) also reports that the benefit of inflation-indexed debt increases as the relative size of the inflation-indexed bond market grows.

Moreover, he finds that the costs of inflation-indexed debt are lower in countries with more countercyclical inflation. In our more detailed analysis for Colombia, we can tie the latter pattern to the positive correlation between CPI inflation and inflation risk premia in Colombia.

Overall, despite differences in data and approaches, the results of our analysis and those of Ermolov (2021) paint a similar picture, namely that inflation-indexed debt is beneficial at longer-term maturities as the inflation risk premium demanded by investors for holding nominal debt start to become material. The main difference across the two studies is that the breakeven point for the benefit of inflation-indexed debt is reached at a shorter maturity in emerging economies due to their higher level of inflation risk.

5.4 Summary

To summarize, without other strategic benefits from nominal debt, a strict actuarial cost-benefit analysis would recommend an increased issuance of inflation-indexed debt in both Colombia and Mexico, in particular at longer maturities and even more forcefully so based on updated data through December 2024. Furthermore, these governments could introduce regulatory rules for financial institutions and institutional investors such as pension funds and life insurance companies with some form of beneficial treatment of holdings of long-term inflation-indexed debt. The goal would be to provide additional incentives to gain exposure to this particular market and thereby aim to reduce the high liquidity risk premia in the prices of the inflation-indexed bonds.

6 Conclusion

In this paper, we estimate a novel flexible joint model of nominal and real yields taken from BCFZ on the complete sample of nominal and real government bond prices from Colombia. The novel feature of the BCFZ model is that it accounts for liquidity risk premia in both nominal and real bond prices. As a consequence, it provides us with estimates of the liquidity-adjusted frictionless BEI along with its decomposition into investors' underlying inflation expectations and associated inflation risk premia. In particular, our work significantly improves upon the joint estimation of liquidity and inflation risk premia in nominal and real bond prices, going well beyond the existing model of Colombian government bond yields described in Espinosa-Torres et al. (2017).

Our examination of the BCFZ model-implied inflation dynamics reveals stable long-term inflation expectations in Colombia located slightly above 3 percent. This stability provides a strong basis for us to confidently conclude that these expectations are well anchored near the 3 percent target set by the BR. Despite the seemingly stable long-term inflation expectations, we find that Colombian inflation risk premia are larger and more volatile than corresponding estimates from Canada, Mexico, and the United States. Moreover, model-based simulations indicate that tail risks to the stability of long-term inflation expectations are orders of magnitude larger in Colombia as compared with Canada and may account for its elevated inflation risk premia. Both findings seem reasonable as CPI inflation in Colombia is generally higher and more volatile than in the countries included in the comparison. As a result, inflation uncertainty represents a material risk for investors in the Colombian nominal bonos del tesoro market.

A comprehensive analysis of the determinants of long-term inflation risk premia in Colombia identifies five variables of particular importance, namely the MOVE index, the five-year CDS rate, and the 10-year U.S. Treasury yield in addition to proxies of the liquidity risk in the markets for bonos del tesoro and bonos el tesoro UVR. While the last two variables can be considered domestic, the first three variables are international and have positive coefficients, meaning that increases in global interest rate risk, global perceptions about credit risk, and U.S. long-term interest rates tend to boost long-term inflation risk premia in Colombia. The finding that foreign factors strongly influence Colombian inflation risk premia is consistent with our conjectures detailed in the introduction that foreigners would concentrate their trading in the nominal bonos del tesoro market. Thus, to maintain the credibility of its monetary policy target, the BR will have to navigate these global influences on its domestic bond markets carefully.

With estimates of bonos del tesoro and bonos del tesoro UVR liquidity risk premia and general inflation risk premia in hand, we are able to assess the net benefit of bonos del tesoro UVR issuance to the Colombian government. Our results show a clear advantage of bonos del tesoro UVR over comparable bonos del tesoro in the relevant 5-year to 15-year maturity range that is particularly pronounced for long-term bonos del tesoro UVR. Besides, the data suggest that the benefit of increased inflation-indexed debt issuance is more pronounced when inflation is elevated, and updated results through December 2024 confirm this finding. Thus, barring other strategic motives behind the issuance of long-term bonds, our results suggest that a tilt towards greater issuance of long-term bonos del tesoro UVR would be cost-effective to the Colombian government. Moreover, we note that our results for Mexico are qualitatively similar, leading to a similar recommendation for the Mexican government.

Finally, while it is worth emphasizing that our findings and results for Colombia and Mexico may not extend to other large and medium-sized emerging economies, we feel compelled to stress that our model framework can be applied to other emerging market economies with established nominal and real bond markets such as Brazil, Chile, and Peru, among many others. However, we leave those applications for future research.

References

Abrahams, Michael, Tobias Adrian, Richard K. Crump, Emanuel Moench, and Rui Yu, 2016, "Decomposing Real and Nominal Yield Curves," *Journal of Monetary Economics*, Vol. 84, 182-200.

Adrian, Tobias, Richard K. Crump, and Emanuel Moench, 2013, "Pricing the Term Structure with Linear Regressions," *Journal of Financial Economics*, Vol. 110, 110-138.

Andreasen, Martin M., Jens H. E. Christensen, and Simon Riddell, 2021, "The TIPS Liquidity Premium," *Review of Finance*, Vol. 25, No. 6, 1639-1675.

Andreasen, Martin M., Jens H. E. Christensen, and Glenn D. Rudebusch, 2019, "Term Structure Analysis with Big Data: One-Step Estimation Using Bond Prices," *Journal of Econometrics*, Vol. 212, 26-46.

Arellano, Christina and Ananth Ramanarayanan, 2012, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, Vol. 120, No. 2, 187-232.

Bansal, Ravi and Ivan Shaliastovich, 2013, "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets," *Review of Financial Studies*, Vol. 26, No. 1, 1-33.

Bauer, Michael D., Glenn D. Rudebusch, and Jing (Cynthia) Wu, 2012, "Correcting Estimation Bias in Dynamic Term Structure Models," *Journal of Business and Economic Statistics*, Vol. 30, No. 3, 454-467.

Beauregard, Remy, Jens H. E. Christensen, Eric Fischer, and Simon Zhu, 2024, "Inflation Expectations and Risk Premia in Emerging Bond Markets: Evidence from Mexico," *Journal of International Economics*, Vol. 151, 103961.

Beechey, Meredith J. and Jonathan H. Wright, 2009, "The High-Frequency Impact of News on Long-Term Yields and Forward Rates: Is it Real?," *Journal of Monetary Economics*, Vol. 56, No. 4, 535-544.

Carriero, Andrea, Sarah Mouabbi, and Elisabetta Vangelista, 2018, "U.K. Term Structure Decompositions at the Zero Lower Bound," *Journal of Applied Econometrics*, Vol. 33, No. 5, 643-661.

Ceballos, Luis, Jens H. E. Christensen, and Damian Romero, 2025, "A Post-Pandemic New Normal for Interest Rates in Emerging Bond Markets? Evidence from Chile," *Journal of International Money and Finance*, Vol. 150, 103234.

Christensen, Jens H. E., Francis X. Diebold, and Glenn D. Rudebusch, 2011, "The Affine Arbitrage-Free Class of Nelson-Siegel Term Structure Models," *Journal of Econometrics*, Vol. 164, No. 1, 4-20.

Christensen, Jens H. E., Eric Fischer, and Patrick J. Shultz, 2021, "Bond Flows and Liquidity: Do Foreigners Matter?," *Journal of International Money and Finance*, Vol. 117-102397, 1-20.

Christensen, Jens H. E., Jose A. Lopez, and Paul L. Mussche, 2024, "International Evidence on Extending Sovereign Debt Maturities," *Journal of International Money and Finance*, Vol. 141-103009, 1-18.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2010, "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," *Journal of Money, Credit and Banking*, Supplement to Vol. 42, No. 6, 143-178.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2019, "A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt," *Review of Economics and Statistics*, Vol. 101, No. 5, 933-949.

Christensen, Jens H. E., Glenn D. Rudebusch, and Patrick J. Shultz, 2025, "Accounting for Changes in Long-Term Interest Rates: Evidence from Canada," *Journal of Financial Econometrics*, Vol. 23, No. 2, nbaf006.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2018, "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," *Journal of Financial and Quantitative Analysis*, Vol. 53, No. 1, 243-268.

De Pooter, Michiel, Patrice Robitaille, Ian Walker, and Michael Zdinak, 2014, "Are Long-Term Inflation Expectations Well Anchored in Brazil, Chile, and Mexico?," *International Journal of Central Banking*, Vol. 10, No. 2, 337-400.

Dittmar, Robert F., Alex Hsu, Guillaume Roussellet, and Peter Simasek, 2024, "Default Risk and the Pricing of U.S. Sovereign Bonds," forthcoming *Journal of Finance*.

Dudley, William C., Jennifer Roush, and Michelle Steinberg Ezer, 2009, "The Case for TIPS: An Examination of the Costs and Benefits," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 15, No. 1, 1-17.

Duffee, Gregory R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, Vol. 57, No. 1, 405-443.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, 2005, "Over-the-Counter Markets," *Econometrica*, Vol. 73, No. 6, 1815-1847.

Ejsing, J., M. Grothe, and O. Grothe, 2012, “Liquidity and Credit Risk Premia in Government Bond Yields,” *ECB Working Paper Series*, No 1440, June 2012.

Ermolov, Andrey, 2021, “When and Where Is It Cheaper to Issue Inflation-Linked Debt?,” *Review of Asset Pricing Studies*, Vol. 11, No. 3, 610-653.

Espinosa-Torres, Juan Andrés, Luis Fernando Melo-Velandía, and José Fernando Moreno-Gutiérrez, 2017, “Expectativas de Inflación, Prima de Riesgo Inflationario y Prima de Liquidez: Una Descomposición del Break-Even Inflation para los Bonos del Gobierno Colombiano,” Desarrollo y Sociedad, Vol. 78, No. 1, 315-365.

Finlay, Richard and Sebastian Wende, 2012, “Estimating Inflation Expectations with a Limited Number of Inflation-Indexed Bonds,” *International Journal of Central Banking*, Vol. 8, No. 2, 111-142.

Fleckenstein, Mathias, Francis A. Longstaff, and Hanno Lustig, 2014, “The TIPS-Treasury Bond Puzzle,” *Journal of Finance*, Vol. 69, No. 5, 2151-2197.

Gabaix, Xavier, 2012, “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance,” *Quarterly Journal of Economics*, Vol. 127, No. 2, 645-700.

Gamboa-Estrada, Fredy and José Vicente Romero, 2024, “Modelling CDS Volatility at Different Tenures: An Application for Latin-American Countries,” *Borsa Istanbul Review*, Vol. 24, No. 4, 772-786.

Gamboa-Estrada, Fredy and Andres Sanchez-Jabba, 2024, “The Effects of Foreign Investor Composition on Colombia’s Sovereign Debt Flows,” *International Economics*, Vol. 178, 100507.

Garcia, Juan Angel and Ricardo Gimeno, 2024, “Navigating High Inflation: A Joint Analysis of Inflation Dynamics and Long-Term Inflation Expectations in Latin America,” *Latin American Journal of Central Banking*, Vol. 5, 100133.

Grishchenko, Olesya V. and Jing-Zhi Huang, 2013, “Inflation Risk Premium: Evidence from the TIPS Market,” *Journal of Fixed Income*, Vol. 22, No. 4, 5-30.

Grishchenko, Olesya V., Sarah Mouabbi, and Jean-Paul Renne, 2019, “Measuring Inflation Anchoring and Uncertainty: A U.S. and Euro Area Comparison, ” *Journal of Money, Credit and Banking*, Vol. 51, No. 5, 1053-1096.

Holston, Kathryn, Thomas Laubach, and John C. Williams, 2017, “Measuring the Natural Rate of Interest: International Trends and Determinants,” *Journal of International Economics*, Vol. 108, 559-575.

Hördahl, Peter and Oreste Tristani, 2012, "Inflation Risk Premia in the Term Structure of Interest rates," *Journal of the European Economic Association*, Vol. 10, No. 3, 634-657.

Houweling, Patrick, Albert Mentink, and Ton Vorst, 2005, "Comparing Possible Proxies of Corporate Bond Liquidity," *Journal of Banking and Finance*, Vol. 29, No. 6, 1331-1358.

Hu, Grace Xing, Jun Pan, and Jiang Wang, 2013, "Noise as Information for Illiquidity," *Journal of Finance*, Vol. 68, No. 6, 2341-2382.

Kim, Don H. and Athanasios Orphanides, 2012, "Term Structure Estimation with Survey Data on Interest Rate Forecasts," *Journal of Financial and Quantitative Analysis*, Vol. 47, No. 1, 241-272.

Kim, Don H. and Kenneth J. Singleton, 2012, "Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields," *Journal of Econometrics*, Vol. 170, No. 1, 32-49.

Litterman, R. and J. A. Scheinkman, 1991, "Common Factors Affecting Bond Returns," *Journal of Fixed Income*, Vol. 1, No. 1, 62-74.

Michalache, Gabriel, 2020, "Sovereign Default Resolution through Maturity Extension," *Journal of International Economics*, Vol. 125, https://doi.org/10.1016/j.jinteco.2020.103326.

Nagel, Stefan, 2016, "The Liquidity Premium of Near-Money Assets," *Quarterly Journal of Economics*, Vol. 131, No. 4, 1927-1971.

Neely, Christopher J., 2022, "Inflation and the Real Value of Debt: A Double-edged Sword," *On the Economy Blog*, Federal Reserve Bank of St. Louis (August 1), https://www.stlouisfed.org/on-the-economy/2022/aug/inflation-real-value-debt-double-edged-sword#

Nelson, Charles R. and Andrew F. Siegel, 1987, "Parsimonious Modeling of Yield Curves," *Journal of Business*, Vol. 60, No. 4, 473-489.

Ocampo, José Antonio, Mauricio Villamizar-Villegas, Germán Orbegozo-Rodríguez, Nicolás Fajardo-Baquero, Oscar Botero-Ramírez, and Camilo Orozco-Vanegas, "The Role of Investor Participation in Sovereign Debt Markets: Evidence from an Emerging Economy," *Emerging Markets Review*, Vol. 66, 101284.

Organisation for Economic Co-operation and Development, 2024, "Global Debt Report 2024: Bond Markets in a High-Debt Environment," *OECD Publishing*, Paris.

Pflueger, Carolin E. and Luis M. Viceira, 2016, "Return Predictability in the Treasury Market: Real Rates, Inflation, and Liquidity," in Handbook of Fixed-Income Securities, edited by P. Veronesi, Wiley, Hoboken, N.J., pp. 191-209 (Chapter 10).

Price, Robert, 1997, "The Rationale and Design of Inflation-Indexed Bonds," IMF Working Paper 97/12 (Washington: International Monetary Fund).

Romero, José Vicente, Hernando Vargas, Pamela Cardozo, and Andrés Murcia, 2021, "How Foreign Participation in the Colombian Local Public Debt Market has Influenced Domestic Financial Conditions," *Latin American Journal of Central Banking*, Vol. 2, 100043.

Rudebusch, Glenn D. and Eric Swanson, 2012, "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journals: Macroeconomics*, Vol. 4, No. 1, 105-143.

Sack, Brian and Robert Elsasser, 2004, "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 10, No. 1, 47-63.

Sill, K., 2001, "The Gains from International Risk-Sharing," *Federal Reserve Bank of Philadelphia Business Review*, Third Quarter 2001.

Stulz, R. M., 1999, "International Portfolio Flows and Security Markets," *in Martin Feldstein, ed. International Capital Flows*, Chicago: University of Chicago Press.

Swanson, Eric T. and John C. Williams, 2014, "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates," *American Economic Review*, Vol. 104, No. 10, 3154-3185.

Tesar, L., 1995, "Evaluating the Gains from International Risksharing," *Carnegie-Rochester Conference Series on Public Policy*, 42, 95–143.

Wachter, Jessica, 2006, "A Consumption-Based Model of the Term Structure of Interest Rates," *Journal of Financial Economics*, Vol. 79, No. 2, 365-399.

World Bank and International Monetary Fund, 2021, "Developing Government Bond Markets: A Handbook," *World Bank Publications (Washington: International Monetary Fund).*

Online Appendix

“The Benefit of Inflation-Indexed Debt: Evidence from an Emerging Bond Market”

Cristhian Hernando Ruiz Cardozo†

&

Jens Henrik Eggert Christensen‡

The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Bank of the Republic of Colombia, the Federal Reserve Bank of San Francisco, or the Federal Reserve System.

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Contents

A On the Representativeness of Colombia as an EME

In this appendix, we provide further evidence of the sense in which Colombia is a representative member of the wider group of large and medium-sized emerging market economies.

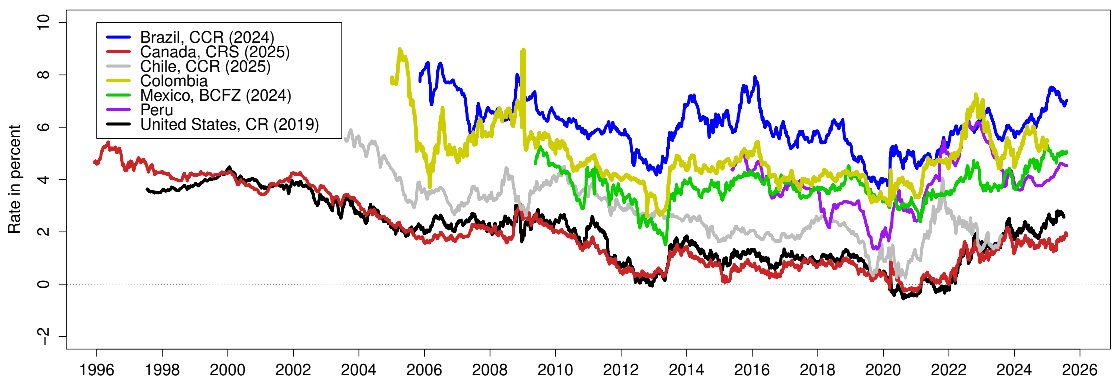


Figure 1: **International Panel of 5yr5yr Real Yields**

As for real rates, an important component in our analysis, we note that long-term real yields vary across time and across countries as shown in Figure 1 for a total of seven countries.

When it comes to Colombia, we think of it as a representative member of the set of large and medium-sized emerging market economies, while being clearly different from advanced economies in this regard.[1] Of particular note, Colombia does not have either the highest or the lowest real yields, but falls in the middle among the group of comparable emerging economies shown in Figure 1.

Next, we aim to examine to what extent Colombia has more or less counter-cyclical inflation than other emerging economies. To address that important question, Figure 2 shows year-over-year consumer price index (CPI) inflation for Brazil, Chile, Colombia, Mexico, and the United States since January 2000. To the naked eye, Colombian CPI inflation looks very similar to that of its Latin American peers. Moreover, all these economies have been in sync the last 10 years: They were all doing well in the years ahead of the COVID-19 pandemic, in part thanks to the longest U.S. expansion on record; the pandemic hit them all at the same

[1]The Brazilian real yields come from an update of Ceballos et al. (2024, CCR), while the Chilean real yields are taken from Ceballos et al. (2025, CCR). The Canadian real yields come from an update of Christensen et al. (2025, CRS). The Mexican real yields come from an update of Beauregard et al. (2024, BCFZ). The Peruvian real yields we construct using an arbitrage-free Nelson-Siegel (AFNS) model from Christensen et al. (2011) to Peruvian inflation-indexed bond prices downloaded from Bloomberg with available daily data starting May 15, 2015. Finally, the U.S. real yields come from an update of Christensen amd Rudebusch (2019, CR).

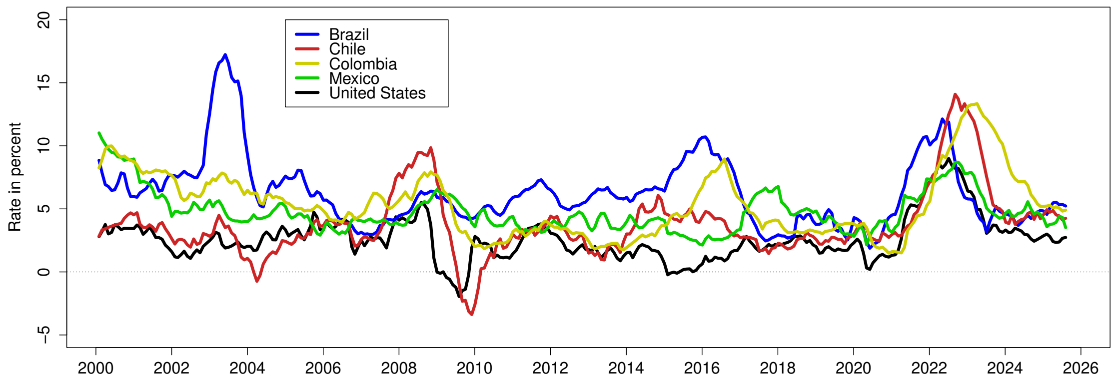


Figure 2: **International Panel of CPI Inflation**

time; and they all reopened after the pandemic around the same time. Lastly, they all had to deal with the fallout from Russia's invasion of Ukraine in March 2022. Thus, as far as where the world economy has been the last 10 years, it is not clear that CPI inflation in Colombia is more counter-cyclical than in its peers. Thus, when it comes to CPI inflation, its general level and variation in Colombia seem broadly similar to those of its EME peers. Hence, Colombia can also be characterized as representative in this regard.

B The BCFZ Model Estimation

In this appendix, we briefly summarize the estimation of the BCFZ model we use in the paper.

Due to the nonlinearity of the coupon bond pricing formulas detailed in the paper, the BCFZ model cannot be estimated with the standard Kalman filter, even though it is a standard affine Gaussian dynamic term structure model. Instead, we use the extended Kalman filter as in Kim and Singleton (2012); see Christensen and Rudebusch (2019) for details. To make the fitted errors comparable across bonds of various maturities, we follow BCFZ and scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form:

$$\frac{\overline{P}_t^j(\tau^i)}{D_t^j(\tau^i)} = \frac{\widehat{P}_t^j(\tau^i)}{D_t^j(\tau^i)} + \varepsilon_t^{j,i}, \quad j \in \{N, R\},$$

where $\widehat{P}_t^j(\tau^i)$ is the model-implied price of bond i and $D_t^j(\tau^i)$ is its duration, which is fixed and calculated before estimation. See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equations.

Since the liquidity risk factors are latent factors that we do not observe, their levels are not identified without additional restrictions. As a consequence, we let the first 15-year bonos del tesoro issued after the start of our sample window have a unit loading on the nominal liquidity risk factor, that is, bonos del tesoro number (9) in our sample issued on July 24, 2005, with maturity on July 24, 2020, and a coupon rate of 11 percent. As for the real liquidity risk factor, we let the bonos del tesoro UVR (Unidad de Valor Real) number (10) issued on February 23, 2006, with maturity on February 23, 2023, and a coupon rate of 4.75 percent have a unit loading.

In addition, we assume that all nominal bond price measurement equations have *i.i.d.* fitted errors with zero mean and standard deviation σ_{ε}^{N}. Similarly, all real bond price measurement equations have fitted errors that are assumed to be *i.i.d.* with zero mean and standard deviation σ_{ε}^{R}.

The measurement equation for the survey expectations takes the form

$$\pi_t^{CF}(\tau) = \pi_t^e(\tau) + \varepsilon_t^{CF},$$

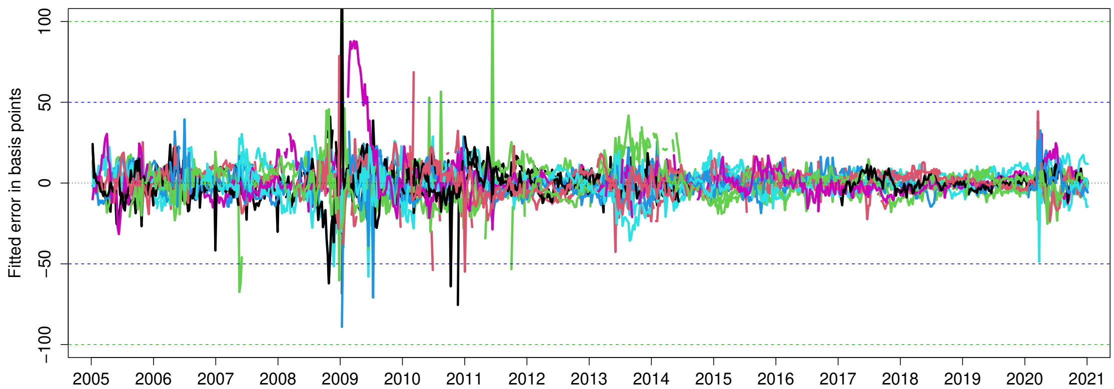
where τ is the forecast horizon. The required expected inflation is calculated using equation (6) in the paper, which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\left(0, (\sigma_{\varepsilon}^{CF})^2\right)$. As for the value of $\sigma_{\varepsilon}^{CF}$, we follow D'Amico et al. (2018, DKW) and fix it at 75 basis points in order to not overly influence the estimation results by including the survey forecasts. Alternatively, this approach can be interpreted as treating the survey forecasts as relatively noisy and infrequent measures of bond investors' inflation expectations.

Finally, we assume that the state variables are stationary, which is standard in the finance literature. As a consequence, we start the Kalman filter at the unconditional mean and covariance matrix.

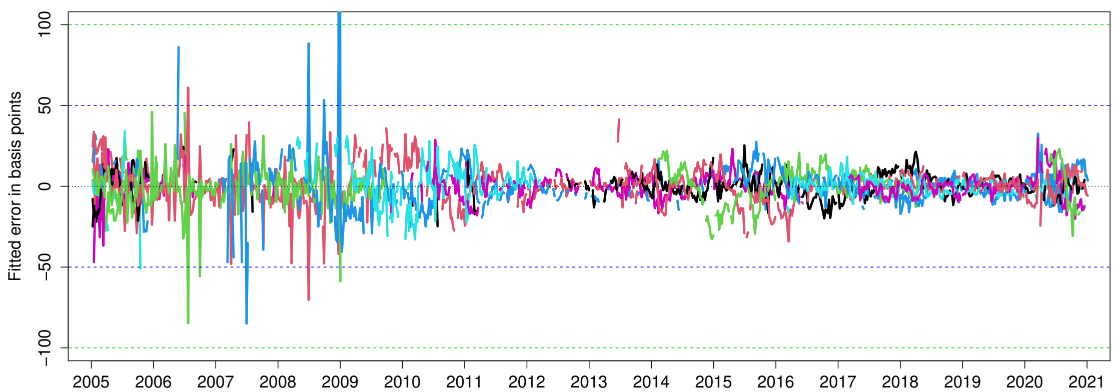
C Additional BCFZ Model Estimation Results

In this appendix, we summarize additional estimation results for the BCFZ model not reported in the paper.

To examine the model fit, pricing errors are computed based on the implied yield on each coupon bond to make these errors comparable across securities. That is, for the price on the



(a) Bonos del tesoro



(b) Bonos del tesoro UVR

Figure 3: **Fitted Errors of Colombian Bonos del Tesoro and Bonos del Tesoro UVR**

ith coupon bond $P_t^i(\tau, C^i)$, we find the value of $y_t^{i,c}$ that solves

$$P_t^i(\tau^i, C^i) = C^i(t_1 - t)\exp\left\{-y_t^{i,c}(t_1 - t)\right\} + \sum_{j=2}^{N} C^i \exp\left\{-y_t^{i,c}(t_j - t)\right\} + \exp\left\{-y_t^{i,c}(t_N - t)\right\}. \quad (1)$$

For the model-implied estimate of this bond price, denoted $\hat{P}_t^i(\tau, C^i)$, we find the corresponding implied yield $\hat{y}_t^{i,c}$ and report the pricing error as $y_t^{i,c} - \hat{y}_t^{i,c}$. Figure 3 shows the fitted error series for each bond price calculated this way. The top panel shows the results for the 35 bonos del tesoro in our sample, while the bottom panel shows the results for the 21 bonos del tesoro UVR in the sample. For the nominal bonos del tesoro, the root mean-squared error (RMSE) for all bonds combined is 10.83 basis points, while the corresponding statistics for the real bonos del tesoro UVR is 15.85 basis points. Thus, the $G^{X^N,X^R}(7)$ model provides a

Bonos del tesoro	Pricing errors		Estimated parameters			
	Mean	RMSE	$\beta^{N,i}$	SE	$\delta^{N,i}$	SE
(1) 15% 8/22/2008	-2.19	8.79	7.1007	0.5969	0.0162	0.0022
(2) 15% 1/25/2012	0.25	29.59	1.8942	0.1196	9.9276	3.2936
(3) 15% 4/26/2012	1.20	14.70	1.4760	0.0958	9.7409	3.2921
(4) 12% 11/9/2007	0.86	8.47	0.4768	0.0566	9.9999	0.4640
(5) 13% 2/12/2010	4.68	11.53	65.4073	0.5610	0.0043	0.0003
(6) 12.5% 7/10/2009	1.27	11.01	50.6204	0.5671	0.0052	0.0004
(7) 13.5% 9/12/2014	0.33	10.63	3.4853	0.4217	0.0807	0.0146
(8) 10% 4/11/2008	1.91	8.52	0.4773	0.0603	9.9931	0.5804
(9) 11% 7/24/2020	-0.29	12.09	1	n.a.	0.1698	0.0438
(10) 8% 10/28/2015	-0.08	9.90	1.2459	0.0848	9.9477	0.4713
(11) 7.5% 11/24/2010	-3.61	14.33	1.2541	0.0939	0.9205	0.3195
(12) 8.75% 5/14/2009	-2.26	5.40	163.7278	0.6871	0.0186	0.0014
(13) 11% 5/18/2011	-1.15	19.25	1.4205	0.0905	1.4634	0.1959
(14) 11.25% 10/24/2018	-0.50	10.64	1.0598	0.0818	2.6574	0.6016
(15) 10.25% 11/14/2013	0.15	9.76	7.4125	0.5671	0.0465	0.0052
(16) 9.25% 5/14/2014	-0.83	8.19	1.9847	0.1664	0.3744	0.0624
(17) 10% 7/24/2024	0.46	8.50	0.5932	0.0993	9.9983	0.8315
(18) 9.25% 8/15/2012	8.03	24.77	1.0279	0.0780	2.1244	0.1043
(19) 6% 4/17/2013	0.96	8.25	1.4019	0.1114	1.4018	0.4656
(20) 7.25% 6/15/2016	0.02	7.40	1.6220	0.5042	0.2559	0.1525
(21) 7.5% 8/26/2026	-0.36	10.00	1.1111	0.1266	2.7793	1.6852
(22) 6% 4/28/2028	0.53	6.59	1.2449	0.1512	5.3486	1.6066
(23) 7% 5/4/2022	-0.01	7.43	0.7363	0.0884	9.9999	1.7603
(24) 5.25% 11/11/2015	0.17	6.56	1.7873	0.2049	1.8272	0.7990
(25) 5% 11/12/2014	-0.01	7.36	1.7965	0.1163	6.1491	1.4514
(26) 5% 11/21/2018	1.25	6.11	55.6376	1.8275	0.0017	0.0004
(27) 7% 9/11/2019	-0.19	6.17	2.7158	1.6431	0.0646	0.0481
(28) 5.5% 7/1/2016	0.53	4.17	1.6323	0.1850	9.9954	1.6223
(29) 5% 7/3/2015	-1.73	4.28	1.8856	0.2848	9.9999	1.8282
(30) 7.75% 9/18/2030	0.30	7.03	1.5457	0.1814	9.1149	1.8102
(31) 7% 6/30/2032	0.54	5.39	1.9276	0.2315	9.9985	1.7616
(32) 6.25% 11/26/2025	0.28	6.60	0.8873	0.1421	8.5479	1.8754
(33) 7.25% 10/18/2034	1.53	5.88	2.1240	0.2449	9.9992	1.8900
(34) 5.75% 11/3/2027	1.16	8.03	1.3091	0.2301	9.9971	2.0323
(35) 7.25% 10/26/2050	-0.97	2.53	26.1083	3.2655	0.0172	0.0038
All bonos del tesoro	0.30	10.83	-	-	-	-

Table 1: **Pricing Errors and Estimated Liquidity Risk Parameters of Bonos del Tesoro**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of bonos del tesoro in the $G^{X^N,X^R}(7)$ model. The errors are computed as the difference between the bonos market price expressed as yield to maturity and the corresponding model-implied yield. All errors are reported in basis points. Standard errors (SE) are not available (n.a.) for the normalized value of $\beta^{N,9}$.

Bonos del tesoro UVR	Pricing errors		Estimated parameters			
	Mean	RMSE	$\beta^{R,j}$	SE	$\delta^{R,j}$	SE
(1) 8% 9/21/2006	1.64	11.19	1.8825	0.5190	0.1171	0.0502
(2) 6% 1/12/2007	3.89	13.09	8.1206	0.5106	0.0249	0.0030
(3) 8% 7/26/2007	-1.76	20.65	0.9354	0.0752	9.5170	3.2131
(4) 8% 1/22/2008	1.28	21.73	1.0725	0.0707	7.7152	3.2117
(5) 8% 9/2/2011	-1.37	14.75	2.5291	0.4367	0.0867	0.0215
(6) 7% 1/17/2012	0.03	12.26	2.3417	0.3561	0.1042	0.0243
(7) 7% 5/15/2012	-0.08	13.55	1.1859	0.0570	0.6063	0.2713
(8) 7% 2/25/2015	0.40	13.48	1.4754	0.1219	0.1906	0.0333
(9) 7% 9/22/2010	-0.25	18.14	6.0312	0.5035	0.0424	0.0046
(10) 4.75% 2/23/2023	0.31	28.95	1	n.a.	7.4382	0.5488
(11) 5.25% 3/20/2013	0.74	26.86	1.3207	0.0706	0.6660	0.0919
(12) 4.25% 5/17/2017	0.10	6.61	1.1467	0.0424	1.2378	0.3923
(13) 3.5% 3/10/2021	0.31	6.95	1.0536	0.0195	1.1710	0.8777
(14) 3% 3/25/2033	-0.23	9.26	0.9483	0.0441	9.9999	1.6606
(15) 3.5% 4/17/2019	0.18	10.63	1.0590	0.0255	9.9993	1.7250
(16) 3.5% 5/7/2025	-0.08	7.94	0.9174	0.0218	9.9984	1.7213
(17) 4.75% 4/4/2035	-0.06	4.68	0.9718	0.0491	9.4250	1.5668
(18) 3.3% 3/17/2027	-0.34	7.33	0.9107	0.0280	9.8410	1.9378
(19) 3.75% 6/16/2049	0.30	6.53	2.1579	1.8171	0.0602	0.0873
(20) 3.75% 2/25/2037	-0.13	6.64	0.9781	0.0628	9.9953	1.8286
(21) 2.25% 4/18/2029	1.70	15.03	0.9205	0.0396	9.6838	1.6013
All bonos del tesoro UVR	0.15	15.85	-	-	-	-

Table 2: **Pricing Errors and Estimated Liquidity Risk Parameters of Bonos del Tesoro UVR**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of bonos del tesoro UVR in the $G^{X^N,X^R}(7)$ model. The errors are computed as the difference between the bonos del tesoro UVR market price expressed as yield to maturity and the corresponding model-implied yield. All errors are reported in basis points. Standard errors (SE) are not available (n.a.) for the normalized value of $\beta^{R,10}$.

good fit to both sets of bond prices. Moreover, the better fit to the bonos del tesoro prices come from the fact that the BCFZ model is fitting those with four factors, while the bonos del tesoro UVR prices are only fitted using three factors.

Table 1 reports the pricing error statistics for individual bonos del tesoro along with their bond-specific liquidity risk parameters ($\beta^{N,i}, \delta^{N,i}$). Note that all the $\beta^{N,i}$ estimates are located in the admissible range from 0 to 250, while many of the $\delta^{N,i}$ estimates are at or close to the upper limit of 10 imposed to ensure numerical stability during the likelihood optimization. This constraint is without practical consequences. To see this, note that the involved term, $e^{-\delta^{N,i}(t-t_0^i)}$, becomes really tiny for values of $\delta^{N,i}$ above 10. As a consequence, allowing $\delta^{N,i}$ to be even larger would matter little for our reported results; see Christensen et al. (2021) for evidence.

The results for individual bonos del tesoro UVR are reported in Table 2. Here, it is also the case that the $\beta^{R,j}$ estimates remain within the admissible range from 0 to 250, while it is again the case that some of the $\delta^{R,j}$ estimates are at or close to the upper limit of 10. However, we reiterate that this constraint is without practical implications for our reported results.

D Inflation Forecast Survey Sensitivity Analysis

In this appendix, we examine the sensitivity of our estimation results to two key assumptions in the model estimation. In the first set of robustness exercises, we change the assumption about the standard deviation of the measurement errors for the model-implied inflation forecasts, while in the other set of exercises we vary the horizon and number of survey inflation forecasts used in the model estimation. To map the exercise to the analysis in the paper, we only show results for long-term inflation expectations covering a five-year period starting five years ahead, labeled 5yr5yr expected inflation. Moreover, we stress up front that the benchmark model implementation described in the paper uses all available survey inflation forecasts in the model estimation.

Figure 4 shows the 5yr5yr model-implied expected inflation from changing the assumption made about the standard deviation of the measurement error of the survey inflation forecasts used in our benchmark model implementation. We vary this value from 0.0025 to 0.01 in 0.0025 increments, with 0.0075 being our benchmark as recommended by DKW. In addition, in a separate estimation, we leave it as a free parameter to be determined by the data, which yields a value of 0.0023. We note that the model-implied 5yr5yr inflation expectations have little sensitivity to this choice. Since the model-implied frictionless breakeven inflation (BEI) rates are entirely unaffected by the value of σ_ε^{CF} as they are determined by the model's $\mathbb{Q}$-dynamics, it follows that the residual 5yr5yr inflation risk premium is also very insensitive to alternative choices of σ_ε^{CF}. Based on these observations, we choose to proceed with our benchmark fixed value of 0.0075 for the standard deviation of the survey forecast measurement errors, σ_ε^{CF}, throughout the paper and in this analysis.

Next, we examine the sensitivity of the model-implied long-term inflation expectations to the specific survey inflation forecasts used in the model estimation. We assess this by studying output from four different model estimations. The first one is our benchmark estimation using all available inflation survey forecasts, while the three other results only use one of the inflation

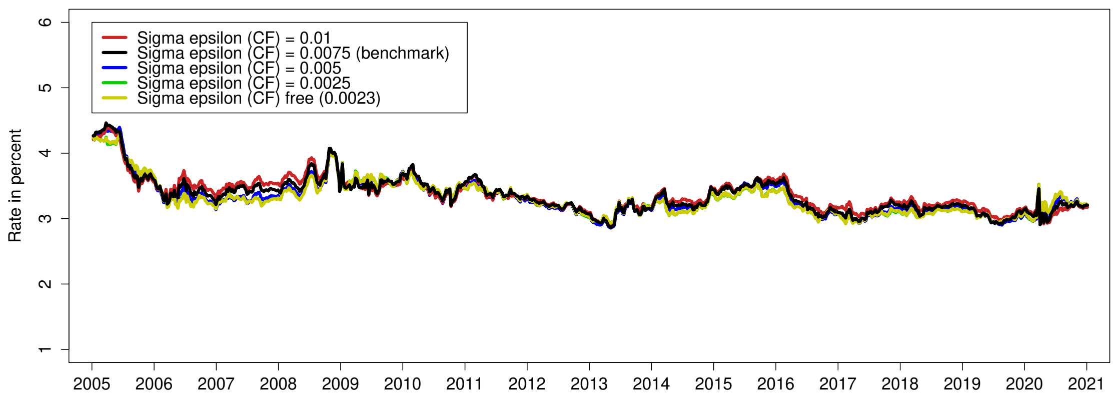


Figure 4: **Sensitivity of 5yr5yr Model-Implied Expected Inflation to σ_ε^{CF}**
Illustration of the 5yr5yr expected inflation implied by the $G^{X^N,X^R}(7)$ model estimated with five different assumptions about the standard deviation of the measurement error of the observed survey inflation forecasts: (1) fixed 0.01; (2) fixed 0.0075, which is our benchmark model implementation; (3) fixed 0.005; (4) fixed 0.0025; (5) leave σ_ε^{CF} as a free parameter to be determined by the model estimation, which produces a value of 0.0023. The shown data cover the period from January 7, 2005, to December 30, 2020.

forecast survey series in the model estimation, namely monthly data on inflation forecasts for the following full calendar, semiannual data on five-year inflation forecasts, and semiannual data on 5yr5yr inflation forecasts, respectively. Figure 5 shows the 5yr5yr model-implied expected inflation from these four different estimations. We note that using the monthly inflation forecasts for the coming calendar year produces somewhat elevated model-implied long-term inflation expectations with a pronounced downward trend. This seems to mimic the general decline in Colombian CPI inflation during the 2005-2020 period. Hence, using solely short-term inflation forecasts for the coming calendar year makes the model-implied long-term inflation expectations inherit many of the properties of short-term inflation forecasts, which makes them appear less realistic. In contrast, using semiannual 5yr5yr inflation forecasts or all survey inflation forecasts as in our benchmark implementation produces very similar and more stable model-implied long-term inflation expectations. Overall, we consider our benchmark implementation using all available survey inflation forecasts to strike a reasonable balance between, on one side, the very stable estimate for the 5yr5yr expected inflation when we only use the 5yr5yr inflation forecast from the surveys and, on the other side, the somewhat volatile and lower-trending estimates obtained when we only use the short-term survey inflation forecasts in the model estimation.

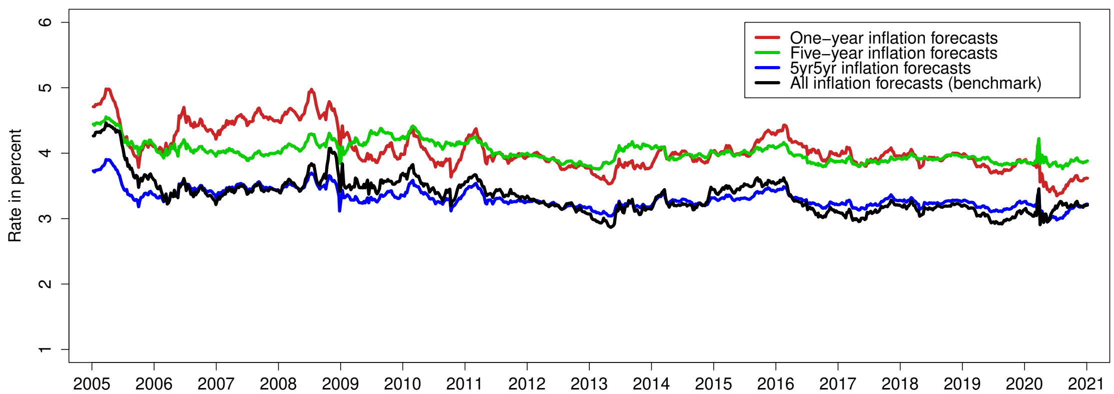


Figure 5: **Sensitivity of 5yr5yr Model-Implied Expected Inflation to Survey Inflation Forecasts**
Illustration of the 5yr5yr expected inflation implied by the $G^{X^N,X^R}(7)$ model estimated with four types of survey inflation forecasts: (1) the following calendar year; (2) the next five years; (3) five-year forecasts five years ahead; (4) all four survey forecasts, which is our benchmark implementation. The shown data cover the period from January 7, 2005, to December 30, 2020.

Furthermore, unreported results show that the estimated state variables are essentially indistinguishable across all four model estimations. In addition, their estimated $\mathbb{Q}$-dynamics are also practically identical. Thus, the source of the differences in the model-implied inflation expectations can be traced back to differences in the estimated real-world $\mathbb{P}$-dynamics.

In summary, the provided evidence suggests that the model-implied inflation expectations are not overly sensitive to the assumption made about the distribution of the fitted errors of the inflation forecasts. In contrast, greater dispersion is observed depending on the specific inflation forecast horizon used. Given that we have no reason to put more weight on any particular forecast horizon we simply include all available inflation forecasts in our benchmark implementation.

Technically, the results are sensitive to the survey forecast used because the model-implied BEI decompositions are unconvincing when the BCFZ model is estimated without any survey information based on a “yields-only” approach as we demonstrate in online Appendix E. Thus, the estimation is taking a signal from the survey information. As a consequence, it is not all that surprising that the exact survey information used in the estimation matters for the results.

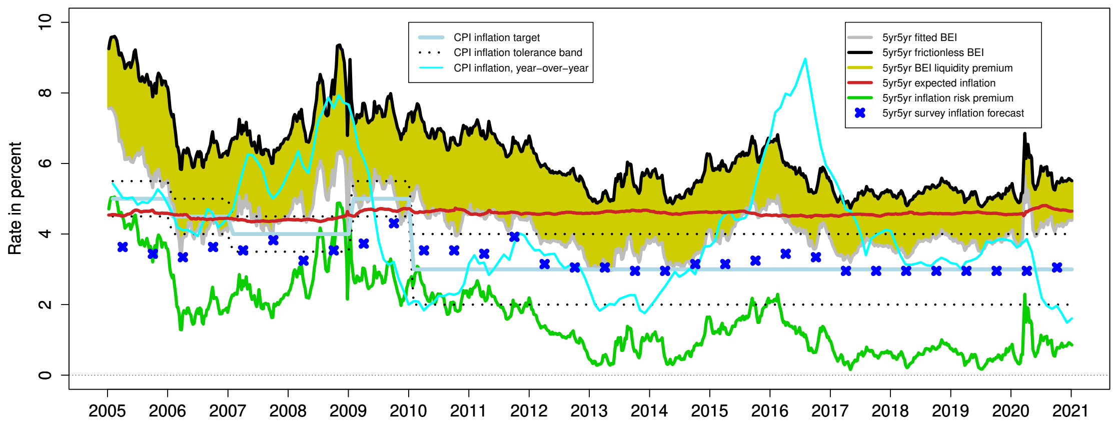
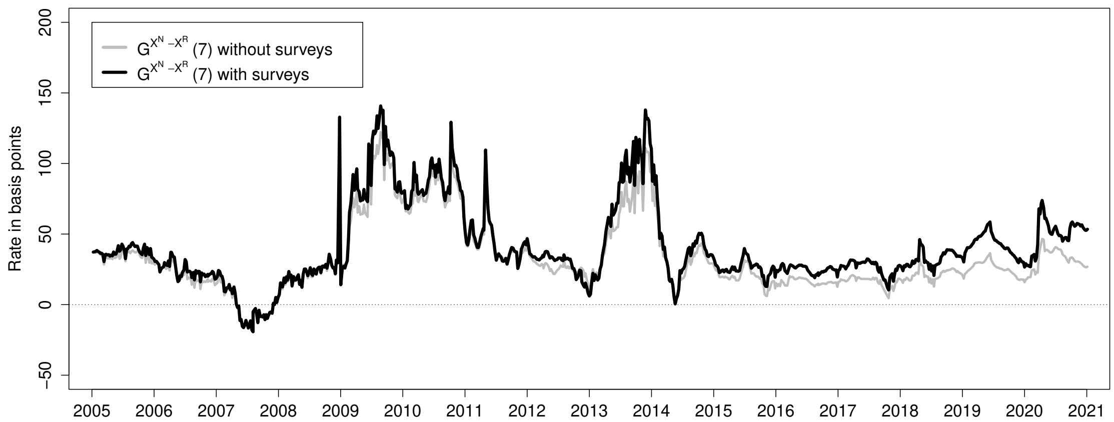


Figure 6: **Decomposition of 5yr5yr BEI: No Survey Information**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Colombian bonos del tesoro and bonos del tesoro UVR prices separately and its decomposition based on the BCFZ model estimated with an unrestricted specification of $K^{\mathbb{P}}$ and a diagonal specification of Σ and *without* any survey information into: (1) the estimated frictionless BEI, (2) expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from January 7, 2005, to December 30, 2020.

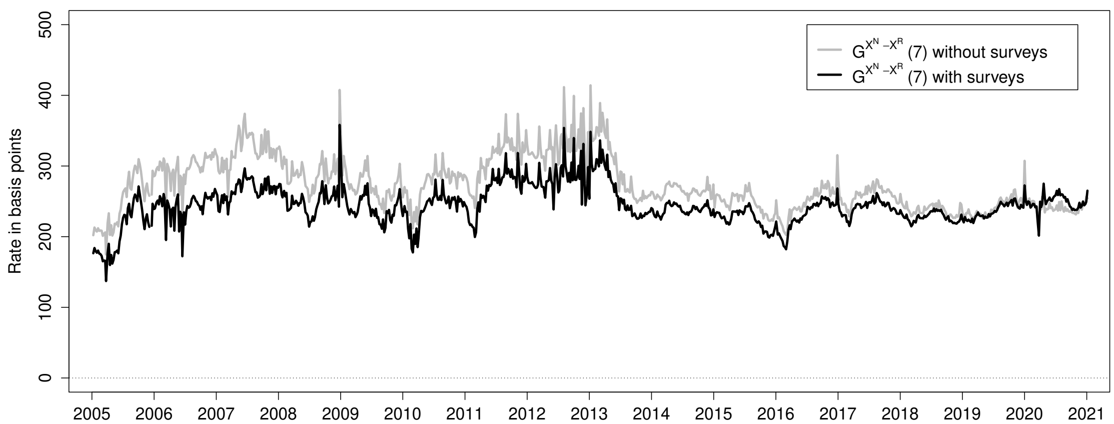
E BCFZ Model Results without Survey Information

In this appendix, we assess the sensitivity of our estimation results to the exclusion of the survey inflation forecasts in the BCFZ model estimation.

Figure 6 shows the decomposition of 5yr5yr BEI implied by the BCFZ model when estimated without any inflation expectations from the Consensus Forecasts surveys of professional forecasters. As established in online Appendix D, with survey information included, the BCFZ model is able to provide a close fit to the survey inflation forecasts. However, it is also clear that the model-implied long-term inflation expectations are sensitive to the specific survey forecasts used. Therefore, it is not all that surprising that the model results change quite notably when we exclude the survey information from the model estimation. Specifically, the model-implied inflation expectations appear to be unreasonably high reaching almost 5 percent. We note that the model-implied inflation expectations are well above those reported in the Consensus Forecasts surveys. Moreover, they are outside the 2 percent to 4 percent tolerance band for inflation adopted by the Bank of Republic in 2010. Thus, to be true, these model-implied inflation expectations require inflation to be above the tolerance band for a full five-year period. At the same time and as a direct consequence of the high model-implied



(a) Bonos del tesoro



(b) Bonos del tesoro UVR

Figure 7: **Average Estimated Liquidity Risk Premia without Survey Information**

inflation expectations, the residual 5yr5yr inflation risk premium series seems overly low.

Overall, we take these findings to support our choice in the paper to focus on the BCFZ model estimated using the survey information.

Figure 7 shows the updated average estimated liquidity risk premium series in the bonos del tesoro and bonos del tesoro UVR market based on the BCFZ model estimated without any survey information including a comparison with the corresponding results from our benchmark BCFZ model implementation with the survey information. Note the closeness of the estimated liquidity risk premium series in each market across the two model implementations. This explains why the frictionless BEI is also very similar across the two models.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$K^{\mathbb{P}}_{\cdot,6}$	$K^{\mathbb{P}}_{\cdot,7}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	8.0231	10.0991	6.5012	11.2658	4.3714	-22.9945	1.6164	0.1654	σ_{11}	0.0096
	(0.2887)	(0.2400)	(0.2100)	(0.2906)	(0.4691)	(0.4465)	(0.1167)	(0.1268)		(0.0004)
$K^{\mathbb{P}}_{2,\cdot}$	-18.9503	-21.3204	-15.478	-22.8152	-7.3330	52.1059	-3.6408	-0.1149	σ_{22}	0.0104
	(0.4807)	(0.3701)	(0.3819)	(0.4666)	(0.5232)	(0.5063)	(0.2595)	(0.0806)		(0.0010)
$K^{\mathbb{P}}_{3,\cdot}$	24.2155	27.7450	20.2813	28.7923	13.4931	-66.5752	5.0747	-0.0656	σ_{33}	0.0422
	(0.5852)	(0.4803)	(0.4340)	(0.4591)	(0.6538)	(0.5429)	(0.3587)	(0.0857)		(0.0017)
$K^{\mathbb{P}}_{4,\cdot}$	11.4863	13.5726	9.2425	16.8029	3.8246	-32.8714	2.1744	0.0099	σ_{44}	0.0314
	(0.4796)	(0.3899)	(0.3424)	(0.5446)	(0.6119)	(0.5582)	(0.1984)	(0.0120)		(0.0017)
$K^{\mathbb{P}}_{5,\cdot}$	4.3758	6.3240	4.1381	7.2893	4.6749	-14.3282	1.0653	0.0775	σ_{55}	0.0076
	(0.2634)	(0.2138)	(0.1676)	(0.2562)	(0.4850)	(0.4035)	(0.0942)	(0.0579)		(0.0005)
$K^{\mathbb{P}}_{6,\cdot}$	-5.0570	-6.4215	-3.9481	-5.7283	1.9061	15.2311	-0.3535	-0.0699	σ_{66}	0.0191
	(0.3560)	(0.2564)	(0.1911)	(0.3334)	(0.5616)	(0.4928)	(0.0829)	(0.0307)		(0.0006)
$K^{\mathbb{P}}_{7,\cdot}$	-1.6202	-2.04226	-6.9506	-7.7370	1.5864	13.0585	0.5289	-0.2427	σ_{77}	0.1838
	(0.5907)	(0.5321)	(0.5370)	(0.5784)	(0.5133)	(0.5670)	(0.3567)	(0.3850)		(0.0123)

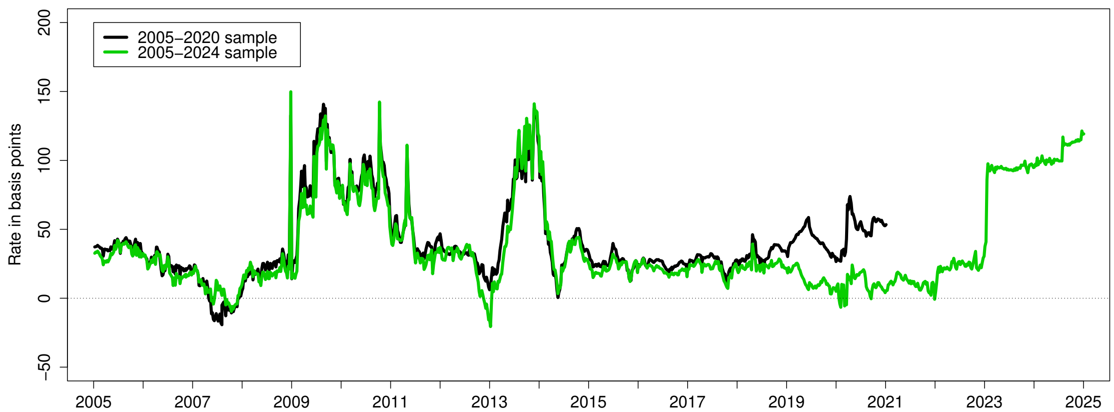
Table 3: **Updated Estimated Dynamic Parameters of the BCFZ Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the BCFZ model. The estimated value of λ^N is 0.5904 (0.0087), while $\lambda^R = 0.1919$ (0.0125), $\kappa_N^{\mathbb{Q}} = 2.3740$ (0.1263), $\theta_N^{\mathbb{Q}} = 0.0029$ (0.0002), $\kappa_R^{\mathbb{Q}} = 9.0951$ (0.5065), and $\theta_R^{\mathbb{Q}} = 0.0233$ (0.0017). The maximum log likelihood value is 79,747.24. The numbers in parentheses are the estimated parameter standard deviations.

F Updated BCFZ Model Results

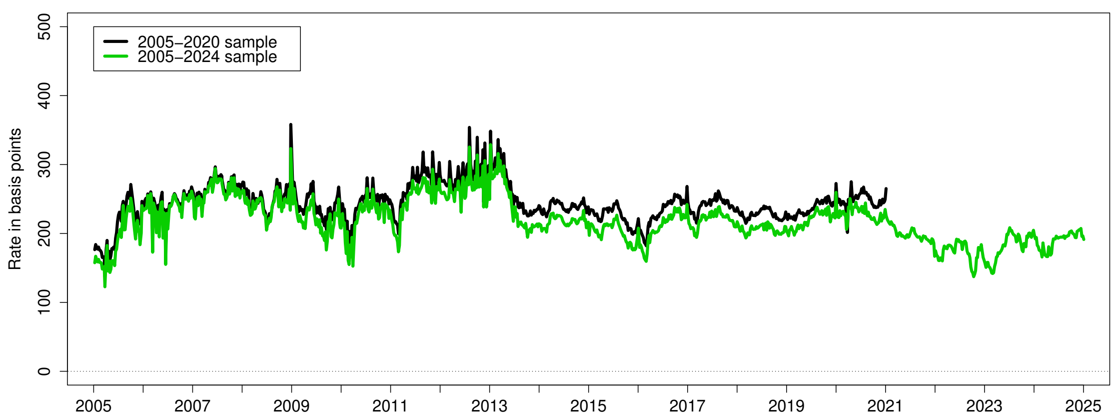
In this appendix, we report updated estimation results for the BCFZ model based on weekly data covering the period from January 7, 2005, to December 30, 2024, a total of 1,043 weekly observations.

Table 3 contains the estimated dynamic parameters of the BCFZ model when estimated on the updated weekly sample through December 30, 2024.

Figure 8 shows the updated average estimated liquidity risk premium series from the bonos del tesoro and bonos del tesoro UVR market including a comparison with the corresponding results from our benchmark 2005-2020 sample. First, we note that there are mostly only minor changes between the original and updated series for the overlapping period. Thus, our reported results in the paper for our benchmark 2005-2020 period are robust to updates. Second, there is a notable and seemingly lasting increase in the liquidity risk premia of the nominal bonos del tesoro, including a sharp spike in early 2023. Anecdotally, this may be related to a decline in the foreign share during this period. However, we leave it for future research to explore that conjecture more formally. At the same time, there has been a meaningful decline in the liquidity risk premia of real bonos del tesoro UVR. As a consequence, the net liquidity premium across the two markets that has historically favored the nominal bonos del tesoro has turned favorable to real bonos del tesoro UVR at the 5yr5yr maturity since 2024 as can be seen in Figure 9.



(a) Bonos del tesoro



(b) Bonos del tesoro UVR

Figure 8: **Average Estimated Liquidity Risk Premia**

Figure 9 shows the updated 5yr5yr BEI decomposition based on data through the end of December 2024. Of particular importance for our net benefit calculation, the net liquidity premium in 5yr5yr BEI switched sign in 2024 and is net favorable to bonos del tesoro UVR by the end of the updated sample, a phenomenon not observed during the first 23 years of the sample, so this is a historic development. Notably, the path to this outcome runs through both markets, there is a marked deterioration on the nominal side combined with a meaningful improvement on the real side. Moreover, Figure 9 also includes a comparison of the updated 5yr5yr inflation risk premium series with the estimate examined in the paper based on our 2005-2020 benchmark sample. Note that the two series are barely distinguishable for the overlapping period. As a result, the reported inflation risk premium estimates in the paper

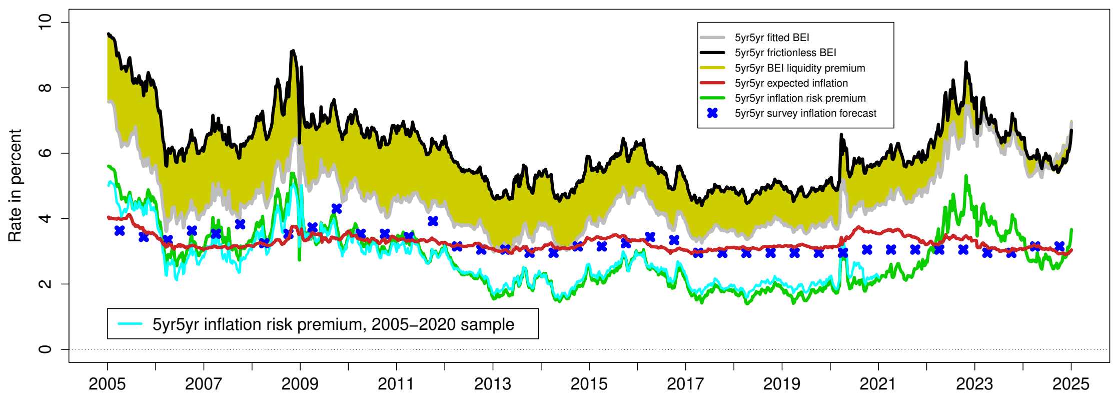


Figure 9: **Decomposition of 5yr5yr BEI**
Illustration of the fitted 5yr5yr BEI obtained by fitting an AFNS model to Colombian bonos and bonos UVR prices separately and its decomposition based on the BCFZ model estimated with an unrestricted specification of $K^{\mathbb{P}}$ and a diagonal specification of Σ into: (1) the estimated frictionless BEI, (2) expected inflation, and (3) the residual inflation risk premium. The difference between the fitted and frictionless 5yr5yr BEI is highlighted in yellow and represents the net liquidity premium of the observed 5yr5yr BEI. The shown data cover the period from January 7, 2005, to December 30, 2024.

are also robust to sample updates.

G Canadian Model Results

In this appendix, we reported the estimated parameters for the Canadian model we use in Section 4.4.3 in the paper.

The Canadian model we use is the preferred CLR-L model from Christensen et al. (2025) estimated using their data but ending in December 30, 2020. The estimated dynamic parameters of the preferred specification are reported in Table 4. The estimated $\mathbb{Q}$-dynamics used for pricing and determined by (Σ, λ, α^R, $\kappa^{\mathbb{Q}}_{liq}$, $\theta^{\mathbb{Q}}_{liq}$) are close to those reported Christensen et al. (2025). This implies that both model fit and the estimated RRB liquidity premia are similar to theirs and therefore not shown here. Furthermore, the estimated objective $\mathbb{P}$-dynamics in terms of $\theta^{\mathbb{P}}$ and Σ are also qualitatively similar to those reported in Christensen et al. (2025). Hence, the overall dynamic properties of the model are preserved, which makes us comfortable in relying on this model for our model-based simulation exercise in Section 4.4.3 of the paper.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	0.5743	0	0	-0.6111	0	0.0834	σ_{11}	0.0055
	(0.0527)			(0.0575)		(0.0035)		(0.0001)
$K^{\mathbb{P}}_{2,\cdot}$	0	0.6118	0	0	0.0512	-0.0193	σ_{22}	0.0118
		(0.0654)			(0.0165)	(0.0036)		(0.0003)
$K^{\mathbb{P}}_{3,\cdot}$	0	0	0.8314	0	0	-0.0203	σ_{33}	0.0221
			(0.0682)			(0.0048)		(0.0009)
$K^{\mathbb{P}}_{4,\cdot}$	-0.6796	-0.2578	0	0.7384	0	0.0574	σ_{44}	0.0037
	(0.0552)	(0.0424)		(0.0572)		(0.0037)		(0.0001)
$K^{\mathbb{P}}_{5,\cdot}$	0	0	0	0	0.6350	0.0765	σ_{55}	0.2583
					(0.0743)	(0.0548)		(0.0148)

Table 4: **Estimated Dynamic Parameters of the Preferred CLR-L Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the preferred CLR-L model according to the BIC. The estimated value of λ is 0.3888 (0.0049), while $\alpha^R =$ 0.7094 (0.0141), $\kappa^{\mathbb{Q}}_{liq} = 13.6780$ (0.0737), and $\theta^{\mathbb{Q}}_{liq} =$ -0.0010 (0.0001). The maximum log likelihood value is 28,104.06. The numbers in parentheses are the estimated parameter standard deviations.

References

Andreasen, Martin M., Jens H. E. Christensen, and Glenn D. Rudebusch, 2019, "Term Structure Analysis with Big Data: One-Step Estimation Using Bond Prices," *Journal of Econometrics*, Vol. 212, 26-46.

Beauregard, Remy, Jens H. E. Christensen, Eric Fischer, and Simon Zhu, 2024, "Inflation Expectations and Risk Premia in Emerging Bond Markets: Evidence from Mexico," *Journal of International Economics*, Vol. 151, 103961.

Ceballos, Luis, Jens H. E. Christensen, and Damian Romero, 2024, "Market-Based Estimates of the Natural Real Rate: Evidence from Latin American Bond Markets," Working Paper 2014-03, Federal Reserve Bank of San Francisco.

Ceballos, Luis, Jens H. E. Christensen, and Damian Romero, 2025, "A Post-Pandemic New Normal for Interest Rates in Emerging Bond Markets? Evidence from Chile," *Journal of International Money and Finance*, Vol. 150, 103234.

Christensen, Jens H. E., Eric Fischer, and Patrick J. Shultz, 2021, "Bond Flows and Liquidity: Do Foreigners Matter?," *Journal of International Money and Finance*, Vol. 117-102397, 1-20.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2019, "A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt," *Review of Economics and Statistics*, Vol. 101, No. 5, 933-949.

Christensen, Jens H. E., Glenn D. Rudebusch, and Patrick J. Shultz, 2025, "Accounting for Changes in Long-Term Interest Rates: Evidence from Canada," *Journal of Financial Econometrics*, Vol. 23, No. 2, nbaf006.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2018, "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," *Journal of Financial and Quantitative Analysis*, Vol. 53, No. 1, 243-268.

Kim, Don H. and Kenneth J. Singleton, 2012, "Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields," *Journal of Econometrics*, Vol. 170, No. 1, 32-49.