FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

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April 2025

Working Paper 2023-35

https://doi.org/10.24148/wp2023-35

Suggested citation:

Mertens, Thomas M., and Tony Zhang. 2025. "A New Keynesian Model for Financial Markets." Federal Reserve Bank of San Francisco Working Paper 2023-35. https://doi.org/10.24148/wp2023-35

This paper was previously circulated under the title "A Financial New Keynesian Model."

A New Keynesian Model for Financial Markets*

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April 21, 2025

Abstract

Which levels of interest rates along the yield curve are consistent with stable economic activity? We use expectations at the daily frequency reflected in financial market prices to extract the state of the economy within a textbook New Keynesian model. We use these real-time estimates to derive neutral and optimal monetary policy rates at each horizon. The model identifies perceived demand and supply shocks on each day, along with their persistence and associated risk premiums. We find that financial markets started to predict the post-COVID surge in inflation by mid-2021 and inflation risk premiums turned positive soon thereafter. The resulting inflation forecasts from the model are at least as accurate as several leading alternatives.

Key Words: Risk-neutral expectations, inflation risk premium, term premium, natural rate, inflation forecasting

JEL Classifications: E43, E44, E45, G10

*We thank seminar participants at Arizona State University, the Federal Reserve Board, the Federal Reserve Bank of Chicago, UC Berkeley, UNC Chapel Hill, and the University of Virginia. We thank Regis Barnichon, Luigi Bocola, Mike Gallmeyer, Matteo Iacoviello, Emi Nakamura, and Jon Steinsson for helpful suggestions. The views expressed here are solely those of the authors and do not necessarily represent those of the Federal Reserve Bank of San Francisco, the Board of Governors of the Federal Reserve, or the Federal Reserve System.

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1 Introduction

Suppose the central bank could perfectly influence interest rates across the yield curve, which 10-year yield should it target to keep economic activity stable? A benchmark for monetary policy is the interest rate that neither slows nor stimulates economic activity. Such a benchmark rate is at the center of a classic literature in monetary economics (see, e.g., Wicksell (1898), Keynes (1930), Friedman (1968)).[1] Its estimation requires both data and a model. The data is necessary to extract the state of the economy, and the model is required because the benchmark interest rate is a counterfactual rate that keeps economic activity stable.

The existing literature is silent on the choice of maturity for the benchmark interest rate. For practical relevance, however, the time horizon plays a critical role. While many financial derivatives are based on short-term interest rates, longer-term rates, such as the 10-year Treasury yield, are arguably the most important interest rates for corporate borrowing and mortgage rates.

In this paper, we tackle the estimation of a benchmark interest rate in a novel way by taking advantage of innovations in both financial markets and monetary theory over the past decades. To this end, we solve the standard New Keynesian model in terms of risk-neutral expectations that have direct counterparts in financial market data. We then interpret the term structure of nominal yields and inflation compensation through the lens of the model and obtain market-based real-time estimates of the magnitude and persistence of shocks in the economy, as well as their associated risk premiums. We use these estimates along with the model to derive benchmark interest rates at various horizons. The estimates further facilitate decompositions of inflation and interest rates into demand-side and supply-side contributions, and provide inflation forecasts whose accuracy is on par with that of several leading survey-based alternatives.

We use a textbook New Keynesian model (as in Clarida, Gali and Gertler (1999) and Woodford (2003)) to cleanly illustrate how the standard three-equation New Keynesian model can be rewritten such that it has direct counterparts in financial market data. We solve for the non-linear optimality

[1] A popular concept is thereby the "natural rate of interest." The idea traces back to Böhm-Bawerk (1884) and Wicksell (1898) and appears prominently in Cassel (1928), Keynes (1930), Williams (1931), and Friedman (1968). A closely related literature measures the neutral real rate of interest (see, e.g., Laubach and Williams (2003)).

conditions for households and firms and rewrite them in terms of risk-neutral expectations. This reformulation does not require any approximation but rather takes the risk-pricing of households into account. For example, we show that the Euler equation of households directly implies a version of the Fisher equation that links the nominal interest rate to the risk-neutral expectation of inflation and the real interest rate.

We obtain a novel log-linearization of the New Keynesian model, which takes the form of the standard three-equation New Keynesian model with risk-neutral expectations taking the place of physical expectations. As a result, the solution of the model now contains additional terms that reflect the risk premiums associated with future demand and markup shocks. These demand and markup risk premiums produce first-order effects on inflation and output gaps, and show up as inflation risk premiums and term premiums in financial data.

The model directly links risk-neutral expectations of inflation and nominal interest rates to a subset of its parameters, specifically the shock realizations, the perceived persistence of the shocks, and their associated risk premiums. We observe the empirical counterparts of the risk-neutral expectations of inflation and nominal interest rates that appear in the model in financial market data. We therefore use inflation swaps as measures of inflation expectations and overnight indexed swap (OIS) rates written on the federal funds rate for nominal interest rates.

The standard three-equation New Keynesian model lacks the flexibility to match the various shapes of the yield curves and inflation compensation curves observed in the data. The model's ability to match risk-neutral expectations in the data is, however, critical for the estimation the model's parameters. One potential avenue would be to add features to the model that support an internal propagation of shocks. In this paper, we instead opt for specifying a more general shock process. Both demand and markup shocks are comprised of a short-run and a long-run component, each of which follows an autoregressive process of order one. With this specification, our model captures a wide variety of forward curves accurately.

We estimate the parameters of the model using financial data at the daily frequency. We use end-of-day OIS and inflation swap rates across all available maturities. We solve for the model's parameter values that minimize the sum-of-squared errors between the model-implied term structures and their empirical counterparts, both for inflation swap rates and nominal interest rates. Our estimation thus returns a time series of daily parameter values where each day's estimates are entirely independent of estimates on other days. We interpret these time series as the market's perceptions of the current state and outlook for the economy.

The model exhibits an excellent fit to the underlying data. The median average fitting error is just three basis points for a given maturity relative to the Nelson-Siegel curves for inflation and nominal interest rates.

Our estimates of the perceived state of the economy allow us to perform counterfactual analysis in the model and derive benchmark interest rates for monetary policy. We refer to the path for nominal rate consistent with stable economic activity as "natural rate". The nominal natural interest rate can be expressed in terms of the steady state neutral real interest rate, inflation compensation, and the shock realizations. We compute nominal natural rates at each horizon and convert them into real rates, which are the key variables for economic activity in the New Keynesian model.

Our analysis shows that comparing short-term real interest rates to measures of long-run real rates (e.g., r^*) can provide a misleading measure of the restrictiveness of monetary policy. On any single day, the long-term real natural rate converges to a level close to r^* only at the long end of the yield curve. However, at the short end, large discrepancies can arise due to perceived transition dynamics in the economy. Hence, a short-term policy rate that is much higher than the long-run real rate may actually be accommodative or neutral depending on the shocks affecting the economy.

Our time series of implied natural interest rates measure the stance of monetary and financial conditions while also taking into account the perceived transition dynamics in the economy and risk premiums. Monetary policy is restrictive at a given maturity only if the corresponding interest rate in the data exceeds the natural interest rate. We show realized policy rates towards the end of our sample period were close to the natural rates and therefore consistent with stable economic activity, even though the realized short-term real rates were much higher than r^*.

We further compute a benchmark rate associated with optimal policy under discretion, which we derive under the assumption that the central bank faces a quadratic loss function and solves for the optimal interest rate taking expectations and the state of the economy as given. The time series for nominal natural rates and optimal rates under discretion are closely aligned with observed interest rates prior to the COVID pandemic. Through the zero lower bound episode during and immediately after the pandemic, both the neutral and optimal rates are negative. Once inflation started to rise in 2021, benchmark interest rates increased sharply. As a result, restrictive policy then required a commensurate increase in the policy rate.

Longer-term yields display a similar picture, although with less variation. We compute the optimal 10-year yield under discretion to provide a benchmark for the stance of policy at the long end of the yield curve. Our estimate for the optimal discretionary rate is, on average, close to the observed long-term yield over the sample period. There are, however, discrepancies at various times. During the pandemic, financial conditions were tight relative to this benchmark while policy was constrained by the zero lower bound. By mid-2022, actual financial conditions again closely align with the implied optimal discretionary long-term rate while they run slightly below it during the disinflationary period.

Unlike in the standard New Keynesian model, the yields in our model are affected by an inflation risk premium. We estimate these risk premiums from the data. We highlight that these risk premium estimates are not the result of our specification of the stochastic discount factor. Instead, we only use the model to obtain the estimation equations that determine how risk premiums affect yields, and we let the data determine the magnitude of the risk premiums. The inflation risk premium at the 1-year horizon is on the order of about five basis points, while its counterpart at the 10-year horizon is about 10-15 basis points. Estimated long-term inflation risk premiums were negative over the late 2010s but turned negative when inflation surged in the aftermath of the pandemic. A decomposition of inflation into demand and markup shocks shows that this increase in inflation was largely driven by supply-side factors.

Due to the model's ability to obtain risk premiums, we extract an out-of-sample forecast for inflation under the physical measure at each point in time. We find that short-term inflation expectations picked up by mid-2021 and rose until mid-2022 when they peaked at slightly above 5%. The rise in inflation expectations was confined to the short-end, however. Even when observed inflation and short-term inflation expectations, peaked, long-term inflation expectations remained well-anchored.

The model-implied inflation forecast are surprisingly accurate, at least on par with leading survey-based alternatives. Their out-of-sample accuracy at the 1-year horizon compares favorably with corresponding estimates from the Survey of Professional Forecasters, the Michigan Survey, the New York Fed Survey of Consumer Expectations, as well as the ATSIX data set. They even slightly edge out the forecast accuracy of unadjusted inflation swaps.

We point out three main caveats to our analysis. First, we do not explicitly deal with the zero lower bound on interest rates. Since the lower bound affected the pricing of interest rates and inflation in the aftermath of the Great Financial Crisis (see Mertens and Williams (2021)), our estimation might be contaminated by the influence of the zero lower bound. We deal with this issue by filtering some of the resulting noise. Second, financial market prices are only available for inflation and interest rates, but not the output gap which also appears in the New Keynesian model. As in Bocola, Dovis, Jørgensen and Kirpalani (2024), the real side of the economy appears as a latent variable. Having said that, the implied output gaps in this model are broadly consistent with estimates from the Congressional Budget Office. Third, our estimation strategy can only reveal a subset of the parameters, as we show in our analysis. We keep all other parameters fixed. However, one could imagine combining our estimation technique of using financial market data with an estimation based on macroeconomic time series to jointly estimate all parameters via a mixed-frequency approach, similar to the one in Meyer-Gohde and Shabalina (2022). We view this estimation as outside the scope of this analysis.

This paper relates to several strands of the literature. First, a recent literature has emerged that uses and investigates the link between monetary policy and financial markets. Kiyotaki and Moore (1997), Bernanke et al. (1999), and Brunnermeier and Sannikov (2014) incorporate a financial sector into a macroeconomic framework. Dew-Becker (2014), Campbell, Pflueger and Viceira (2020), Caballero and Simsek (2020b), Caballero and Simsek (2020a), Bianchi, Lettau and Ludvigson (2022), and Bok, Mertens and Williams (2022) study the asset pricing implications from monetary policy and the real side of the economy. Closely related to this paper is Pflueger (2023) who measures risk of stagflation from nominal and real bonds. In this paper, we stay within the textbook New

Keynesian model which we re-write in terms of risk-neutral expectations so that it can be estimated directly from the data. We abstract from noise in financial market trading, as in Hassan and Mertens (2017).

Second, an emerging literature estimates perceived policy rules. Bauer, Pflueger and Sunderam (2022) and Bauer et al. (2024) use survey data to extract perceived coefficients in monetary policy rules. In closely related work, Bocola et al. (2024) study time-variation in the reaction function according to bond markets. Relative to these studies, we share the feature that our estimation is based off of expected future inflation and interest rates. We instead take a more structural approach by filtering these estimates through the textbook New Keynesian model.

Third, our estimation relates to the vast literature on the estimation of the components of the New Keynesian model. Following the seminal work of Laubach and Williams (2003), a series of papers, including Lubik and Matthes (2015), Holston, Laubach and Williams (2017), Johannsen and Mertens (2016), and Del Negro, Giannone, Giannoni and Tambalotti (2017), has presented measures of the natural real rate of interest, r^*. Closest to our measure is Christensen and Rudebusch (2017) who use TIPS markets as well but build a more elaborate model of risk premiums. The inflation risk premium is been estimated, among others, by d'Amico, Kim and Wei (2018), Chernov and Mueller (2012), Grishchenko and Huang (2013), Fleckenstein, Longstaff and Lustig (2017), and Andreasen, Christensen and Riddell (2021). A series of papers studies and analyzes inflation expectations under the physical measure, including Faust and Wright (2013), Coibion and Gorodnichenko (2015), Duffee (2018), and Aruoba (2020). Instead of estimating the various parts separately, we extract measures of expectations and risk premiums within the general framework of the New Keynesian model.

The remainder of this paper is structured as follows. Section 2 lays out the model, derives the equilibrium and its log-linearized form, and discusses the theoretical implications of the model. Section 3 estimates the model. Section 4 discusses the quantitative results, and section 5 concludes.

2 Model

This section lays out and solves the textbook New Keynesian model with Rotemberg adjustment costs that underlies our analysis. The description of the economy results in an IS curve and a Phillips curve that we derive in terms of risk-neutral expectations. These expectations pin down asset prices in the model and are thus observable from financial market prices. Since the setup is standard, we give an incomplete description of the model in the main text and discuss remaining details in Appendix A.

2.1 Setup

Time is discrete and there exists a unit mass of households that live forever. In each period, households optimally consume, supply labor N_t, and save so as to maximize expected lifetime utility

$$\sum_{i=0}^{\infty} \mathbb{E}_{t+i} \left[\beta^{t+i} e^{\chi_{t+i}} \left(\frac{C_{t+1}^{1-\gamma} - 1}{1-\gamma} - \frac{N_{t+i}^{1-\phi} - 1}{1-\phi} \right) \right]. \tag{1}$$

Their preferences are pinned down by the coefficient of relative risk aversion γ, the time preference factor β, and the Frisch elasticity of labor supply determined by ϕ. $\mathbb{E}$ denotes the expectations operator under the physical probability measure, i.e., the $\mathbb{P}$-measure. Households are subject to a stochastic aggregate taste shock χ_t.

Consumption C_t is comprised of a CES aggregate of a unit mass of differentiated goods

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where i indexes varieties of differentiated goods and $\varepsilon > 1$ governs the elasticity of substitution. With P_t denoting the accompanying price index to the CES consumption aggregate, households face the budget constraint

$$P_t C_t + \frac{1}{I_t} B_{t+1} \leq B_t + W_t N_t + T_t, \tag{2}$$

where I_t denotes the gross return on risk-free assets, B_t bond payoffs, and W_t wages. T_t refers to transfers from the government that redistribute firm profits to households. We further allow households to trade a complete set of Arrow-Debreu securities. In our model, it is sufficient for them to trade the bond given the state contingent transfers T_t. Market completeness ensures that we have a unique stochastic discount factor that prices any asset.

The first-order condition for households with respect to bond holdings is given by

$$\mathbb{E}_t \left[M_{t+1} I_t \right] = 1, \tag{3}$$

where M_{t+1} denotes the nominal stochastic discount factor

$$M_{t+1} = \beta e^{\chi_{t+1} - \chi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}}. \tag{4}$$

A unit mass of monopolistically competitive firms produces the differentiated goods according to the production function $Y_t(i) = A_t N_t(i)$. Firms pay Rotemberg adjustment cost when updating their prices:

$$\frac{\eta}{2} \left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}} \right)^2 P_{t-1} Y_{t-1}.$$

With Ψ_t denoting the firm's real marginal cost of production, the firm's dynamic optimization problem leads to the first-order condition

$$(1 - \epsilon) + \epsilon \Psi_t + \eta \left(1 - \Pi_t\right) \frac{Y_{t-1}}{Y_t} = \eta \mathbb{E}_t \left[M_{t+1} \right] - \eta \mathbb{E}_t \left[M_{t+1} \Pi_{t+1} \right]. \tag{5}$$

The first-order conditions for households and firms share a common feature: Both equations contain expressions reflecting risk-neutral pricing. In other words, the stochastic discount factor appears in the expectations operator.

The common log-linearization separates expected stochastic discount factors from expectations about other variables. In the following section, we first rewrite the expectations in terms of the risk-neutral measure before log-linearizing.

2.2 Risk-neutral expectations

We follow the standard convention when defining the risk-neutral measure. We therefore start from the pricing equation $\mathbb{E}_t\left[M_{t+1}\widetilde{R}_{t+1}\right] = 1$ which holds for any nominal return $\widetilde{R}_{t+1}$. Dividing both sides by $\mathbb{E}_t[M_{t+1}]$ allows us to write expectations as

$$\hat{\mathbb{E}}_t[\widetilde{R}_{t+1}] \equiv \mathbb{E}_t\left[\frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]}\widetilde{R}_{t+1}\right] = \int \widetilde{R}_{t+1}(\omega)\frac{M_{t+1}(\omega)h_t(\omega)}{\int M_{t+1}(\omega)h_t(\omega)d\omega}d\omega = \frac{1}{\mathbb{E}_t[M_{t+1}]}, \tag{6}$$

where ω denotes the state of the economy and $h_t(\omega)$ the physical probability density function over those states. Note that, by definition, the fraction in the third part of equation (6) is non-negative and integrates to one. It is thus a probability measure: The risk-neutral probability measure or $\mathbb{Q}$-measure.

Here we show that these risk-neutral expectations are observable from financial market prices. We start by pricing a fixed-for-floating inflation swap. The rate on the fixed leg, $S_{t,t+1}$, is adjusted such that its fixed payoff equals the value on the floating inflation leg and no up-front payment is exchanged. We apply equation (6) to both the fixed payoff and inflation to obtain the (fixed leg) swap rate between time t and $t+1$

$$\mathbb{E}_t[M_{t+1}\Pi_{t+1}] = \mathbb{E}_t[M_{t+1}S_{t,t+1}] = S_{t,t+1}\mathbb{E}_t[M_{t+1}], \tag{7}$$

where the last step follows because the swap rate is known at time t. Dividing both sides by the expected discount factor leads to $S_{t,t+1} = \hat{\mathbb{E}}_t[\Pi_{t+1}]$, which shows that risk-neutral expectations about inflation are directly observable from the swap rate.

This same logic implies a version of the Fisher equation. To see this, plug the swap rate as the risk-neutral expectation of inflation back into the third term in (7) and observe that $\mathbb{E}_t[M_{t+1}]$ is the inverse of the nominal gross interest rate (as shown in equation (6)) and $\mathbb{E}_t[M_{t+1}\Pi_{t+1}]$ the inverse of the real gross interest rate to get

$$I_t = \hat{\mathbb{E}}_t\left[\Pi_{t+1}\right]R_t. \tag{8}$$

Taking logarithms on both sides of the equation shows that we can observe risk-neutral expectations of inflation from the difference between nominal and real interest rates.

With the conversion to risk-neutral expectations, the log-linearization of the Euler equation (3) turns into the IS curve

$$x_t = -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r^*\right) + \mathbb{E}_t[x_{t+1}] + g_t, \tag{9}$$

where x_t denotes the logarithm of the output gap and g_t is a demand shock that captures expected changes in taste shocks χ_t as in the textbook New Keynesian model.[2] Lower-case symbols thereby denote logarithms of the upper-case analogues. The IS curve takes the typical form of the textbook New Keynesian model except that physical inflation expectations are replaced by risk-neutral expectations. Expectations about the output gap remain under the physical measure.

Similarly to the optimality condition for households, the first-order condition for firms takes the standard form, including the markup shock u_t, when log-linearized

$$\pi_t = \lambda x_t + \beta \hat{\mathbb{E}}_t\left[\pi_{t+1}\right] + u_t. \tag{10}$$

Here again, the only difference to the standard model is the expectation operator.

While the changes to the textbook solution of the model may seem insignificant, they have at least two important consequences. First, as we showed above, the expectations in these equations have a direct counterpart in financial market data. Second, as we shall see in Section 2.4, the solution to the model contains additional terms reflecting risk premiums.

As is well known, this standard setup for the New Keynesian model described so far lacks the internal propagation to match the dynamics of the economy. For our purposes, it is essential that the model accurately captures the term structure of interest rates and inflation expectations. We accomplish this feature through the shock process rather than modifications to the model setup.

Therefore, we assume that the markup shock and the demand shock are each comprised of a long-term and a short-term component. Both components follow AR(1) processes where the persistence of the short-term component is smaller than the persistence of the long-term component. We

[2]Appendix A shows $g_t = \frac{1}{\gamma}(\chi_t - \mathbb{E}[\chi_{t+1}]) + \mathbb{E}[\bar{y}^f_{t+1}] - \bar{y}^f_t$, where $\bar{y}^f_t$ denotes log steady-state output in the flexible price economy against which the output gap is defined.

denote the long-term components of the shocks are denoted by ℓ superscript and the short-term components are denoted by s superscripts:

$$g_t = g_t^s + g_t^\ell, \text{ where } \begin{cases} g_{t+1}^s = \rho_g^s g_t^s + \varepsilon_{g,t+1}^s \\ g_{t+1}^\ell = \rho_g^\ell g_t^\ell + \varepsilon_{g,t+1}^\ell \end{cases} \text{ and } u_t = u_t^s + u_t^\ell, \text{ where } \begin{cases} u_{t+1}^s = \rho_u^s u_t^s + \varepsilon_{u,t+1}^s \\ u_{t+1}^\ell = \rho_u^\ell u_t^\ell + \varepsilon_{u,t+1}^\ell \end{cases} \quad (11)$$

The generalization of the shock process allows the model to match a variety of observed yield curves. While a standard AR(1) process only matches a mean-reverting curve, its generalization with short-term and long-term shocks can match hump-shaped curves. The long-term shock therefore matches the long end of the curve while a short-term shock of different sign can induce a hump.

2.3 Monetary policy

To close the model, we assume that financial market participants perceive the central bank to follow a Taylor rule (see Bauer, Pflueger and Sunderam (2022) and Bocola, Dovis, Jørgensen and Kirpalani (2024)). The central banks sets the nominal interest rate according to

$$i_t = \theta_0 + \theta_\pi \pi_t + \theta_x x_t. \quad (12)$$

We assume $\theta_\pi > 1$. To see the implications of interest rate policy on inflation and output gaps, we turn to the model's solution in the next section.

2.4 Model solution and inflation risk premiums

The previous sections derive a slight variant of the familiar three-equation New Keynesian model. The model is fully described by the IS curve (9), the Phillips curve (10), the interest rate rule (12), and the processes for the shocks (11).

Following the same steps as in the solution of the classic formulation of the New Keynesian model, we derive an expression for inflation as

$$\pi_t = \frac{\lambda + \beta(\theta_x + \gamma)}{\xi}\hat{\mathbb{E}}[\pi_{t+1}] + \frac{\lambda}{\xi}(\gamma\mathbb{E}[x_{t+1}] + r^* - \theta_0) + \frac{(\gamma + \theta_x)}{\xi}(u_t^l + u_t^s) + \frac{\gamma\lambda}{\xi}(g_t^l + g_t^s), \tag{13}$$

where $\xi = \gamma + \theta_x + \theta_\pi \lambda$ is a positive constant. Inflation thus depends on the realizations of short-term and long-term shocks, as well as expectations about future variables. The coefficients on the various terms thereby depend on the coefficients of the Taylor rule. In particular, inflation rises with risk-neutral expectations about future inflation.

Through this channel, risk premiums affect the current level of inflation in the economy through the risk-neutral expectation $\hat{\mathbb{E}}_t[\pi_{t+1}]$. They emerge in the model because both growth and markup shocks affect the output gap, which leads to comovement between the stochastic discount factor and inflation. To make the relationship between shocks, risk premiums and inflation explicit, we solve equation (13) in terms of shocks and their associated risk premiums. We denote risk premiums associated with innovations to the long-term and short-term components of the markup shocks by $\mu_{u,t}^{\ell}$ and $\mu_{u,t}^{s}$, respectively:

$$\hat{\mathbb{E}}_t[u_{t+1}] = \rho_u^{\ell} u_t^{\ell} + \hat{\mathbb{E}}_t[\epsilon_{u,t+1}^{\ell}] + \rho_u^s u_t^s + \hat{\mathbb{E}}_t[\epsilon_{u,t+1}^{s}] = \rho_u^{\ell} u_t^{\ell} + \mu_{u,t}^{\ell} + \rho_u^s u_t^s + \mu_{u,t}^{s}. \tag{14}$$

Analogously, we denote the risk premium associated with innovations to the long-term and short-term components of the growth shock by $\mu_{g,t}^{\ell}$ and $\mu_{g,t}^{s}$. In the estimation section, we discuss how we estimate perceived risk premiums and persistence parameters at each point in time. We thus allow these perceived parameters to vary over time.

Using these definitions, we obtain the solution for inflation by iterating equation (13) forward

$$\pi_t = \frac{\lambda(r^* - \theta_0)}{\Xi_1} + \sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_1\Xi_g^h}((\lambda + \beta\theta_x)\mu_{g,t}^h + g_t^h) + \frac{(\gamma(1-\rho_u^h) + \theta_x)}{\Xi_1\Xi_u^h}((\lambda + \beta\theta_x)\mu_{u,t}^h + u_t^h)\right], \tag{15}$$

where $\Xi_1 = (1-\beta)\theta_x + \lambda(\theta_\pi - 1)$ and $\Xi_j^h = \gamma(1-\rho_j^h)(1-\beta\rho_j^h) + \theta_x(1-\beta\rho_j^h) + \lambda(\theta_\pi - \rho_j^h)$ are both positive constants.

Equipped with the solution in equation (15), we solve for the inflation risk premium, which is commonly defined as the difference between the risk-neutral and the physical expectation of inflation:

$$\hat{\mathbb{E}}_t[\pi_{t+1}] - \mathbb{E}_t[\pi_{t+1}] = \sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_1\Xi_g^h}\mu_{g,t}^h + \frac{(\gamma(1-\rho_u^h)+\theta_x)}{\Xi_1\Xi_u^h}\mu_{u,t}^h\right] \tag{16}$$

Intuitively, the inflation risk premium increases with the markup shock and demand shock risk premiums, as well as with the persistence of the shocks.

3 Data, Identification, and Estimation Strategy

This section describes our estimation strategy and the data we use in the process. Our goal is to show how we can use the New Keynesian framework to extract the expected path of the economy from financial market data at any given point in time. Therefore, we describe the model parameters that can be identified from the cross-section of financial data alone, and we calibrate all other parameters that would require additional macroeconomic data to estimate.

3.1 Data

The estimation of the model requires financial market data on the nominal yield curve and risk-neutral expectations of inflation. To match risk-neutral expectations in the model with market prices, the financial instruments need to be written on the quantities that appear in the model. To this end, we rely on the innovations in financial markets over the past decades that have lead to a substantial expansion of the swap market.

As our measure of the nominal yield curve, we use overnight-indexed swap (OIS) rates written on the federal funds rate, the policy rate in the United States. We gather yields data at the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, and 30 year maturities from Bloomberg. To minimize the effects of market illiquidity on financial market prices at isolated maturities, we fit a Nelson-Siegel curve to OIS rates. We then evaluate the Nelson-Siegel-implied nominal yield curve at annual maturities between 1 and 20 years, and use these values in our estimation.

We follow the same procedure when gathering data on risk-neutral expectations of inflation, observable through inflation swap rates. We collect these data at the same maturities and use Nelson-Siegel implied swap rates in our estimation. We also require data on the current level of inflation at each date corresponding to π_t in the model. We use observed monthly CPI inflation rates for dates when the CPI is available, and we use the previous data print for dates without CPI releases.

Our sample period ranges from January 1, 2014 to November 8, 2024. We restrict the sample to post-2014 due to liquidity concerns in the inflation swap market. For example, Fleming and Sporn (2013) show that average daily inflation swap was growing but still modest through 2012. The overall inflation swap market size was around 350 million dollars per day in April 2012. For comparison, Fleming and Krishnan (2012) show trading activity in the TIPS market for 0 to 10 year maturity bonds averaged 512.8 million dollars per day between March 4, 2005 and March 27, 2008. Appendix B.1 contains the list of data sources.

3.2 Identification and Estimation Strategy

In this section, we discuss our estimation strategy. We show how we identify the magnitude and persistence of the shocks in the economy, and their associated risk premiums, from data on risk-neutral expectations.

Identification. The estimation strategy in this paper matches risk-neutral expectations in the model with those observed through financial market prices. Through this channel, we can infer the key model parameters from the data.

To gain intuition for how the method works, imagine an AR(1) process that mean-reverts to some long-run level. Expectations about future realizations inherit the mean-reverting structure from the underlying process. If one were to observe expectations at various horizons, one can infer key parameters of the process. From far-out expectations, one can infer the long-run level of the process. The rate of decay of expectations is directly linked to the persistence of the shock process. And from near-term expectations, one can infer the current level of the shock.

The estimation in this paper is more involved than the intuition above due to the presence of short-term and long-term shocks as well as risk premiums. But the basic logic still applies. The rate of decay at the long end is primarily driven by the persistence of the long-term shocks. The decay on the short end identifies the persistence of the short-term shocks, once one controls for the persistence of the long-term shocks. The gaps between observed risk-neutral expectations and the risk premium at short horizons identify the levels of the short-term and the long-term shocks, in line with the logic for the basic AR(1) process.

Where the intuition from the basic AR(1) process breaks down is with regards to the long-term level of the shocks. The expectations data does not separately identify long-term levels for individual processes in the sum of multiple standard AR(1) process. In the case of the model in Section 2, however, the level that the aggregate process gravitates towards changes with the time horizon. By iterating equation (15) forward, we can show the linear combination of risk-premiums that determine the level of the inflation swap curve varies at each tenor.[3] The change of this risk premium with the time horizon, in turn, depends on the persistence of the individual AR(1) shock processes. Exploiting this feature, we separately identify risk premiums associated with short-term and long-term components from the levels of the inflation swap curve and interest rate curve across the term structure.

In sum, we identify the shock realizations, along with their persistence and associated risk premiums, while all other parameters are not estimated. These remaining parameters are the discount rate, β, the coefficient of relative risk aversion, γ, the slope of the Phillips curve, λ, the parameters of the Taylor rule, and the long-run neutral real interest rate, r^*. We are therefore left with three options. We can calibrate these other parameters based on estimates in the literature, use the time-series dimension of the financial data, or we introduce additional macroeconomic data at lower frequencies to obtain estimates of these other parameters. We opt for the first and calibrate this remaining set of parameters. This choice is mainly driven by the desire to focus this paper on the high-frequency use of financial data for the New Keynesian model.

Table 1 presents our benchmark calibration. We calibrate the intertemporal elasticity of substi-

[3]The explicit formula for the inflation swap curve is presented in Appendix equation (29).

Table 1
Calibrated Invariant Parameters

Parameter	Description	Value
β	Discount factor	0.98
γ	Inverse IES	0.67
λ	Slope of Phillips curve	0.33
θ_0	Taylor rule constant	r^*
θ_π	Taylor rule coefficient on π_t	1.50
θ_x	Taylor rule coefficient on x_t	1.00

tution to be 1.5. We calibrate the slope of the Phillips curve to be consistent with Fitzgerald et al. (2024). We calibrate the constant of the Taylor rule coefficient to equal r^* as estimated by Laubach and Williams (2003), and the remaining coefficients of the Taylor rule according to Bernanke (2015).

The remaining set of parameters contains variables for which perceptions can change quickly. We denote this set of parameters by Θ_t and we attach t subscripts to each of them:

$$\Theta_t = \left\{\rho^s_{u,t}, \rho^\ell_{u,t}, \mu^s_{u,t}, \mu^\ell_{u,t}, u^s_t, u^\ell_t, \rho^s_{g,t}, \rho^\ell_{g,t}, \mu^s_{g,t}, \mu^\ell_{g,t}, g^s_t, g^\ell_t\right\}. \tag{17}$$

We obtain estimates of this vector of parameters on each day based on closing prices. While the parameters in the model in Section 2 are constant, the estimate of these parameters may vary day-by-day. We do not put any restrictions on these daily variations.

Estimation Strategy. In this section, we describe the procedure for estimating the vector of parameters in (17). We therefore exploit that the impact of short-term shocks fades more quickly than for long-term shocks. Depending on the persistence of this component, this may happen sooner or later. Assuming that short-term shocks have approximately died off after H_t periods, the first step in our estimation procedure is to pick a value for this horizon $H_t \in \{1, ..., 9\}$. In the second step, we estimate the parameter vector Θ_t by minimizing the sum of squared errors between model implied moments and the corresponding data moments. We describe these data moments and the parameters that they identify, below. We then iterate across H_t to find which cut-off for the persistence of short-term shocks minimizes the discrepancy between the model moments and the data moments.

We apply this process at the daily frequency to obtain a daily time-series of parameter estimates.

More specifically, we aim at matching two sets of moments when estimating the parameter vector Θ_t. The first set of moments targets the change in forward inflation rates and forward inflation rates over time, which are informative for identifying the persistence of shocks. The second set of moments targets the inflation swap curve and the nominal yield curve, which identify the markup shocks, the demand shocks, and their associated risk premiums. Appendix B.2 contains additional details about our estimation methodology, and includes the mathematical expressions for the model moments that we match with the data

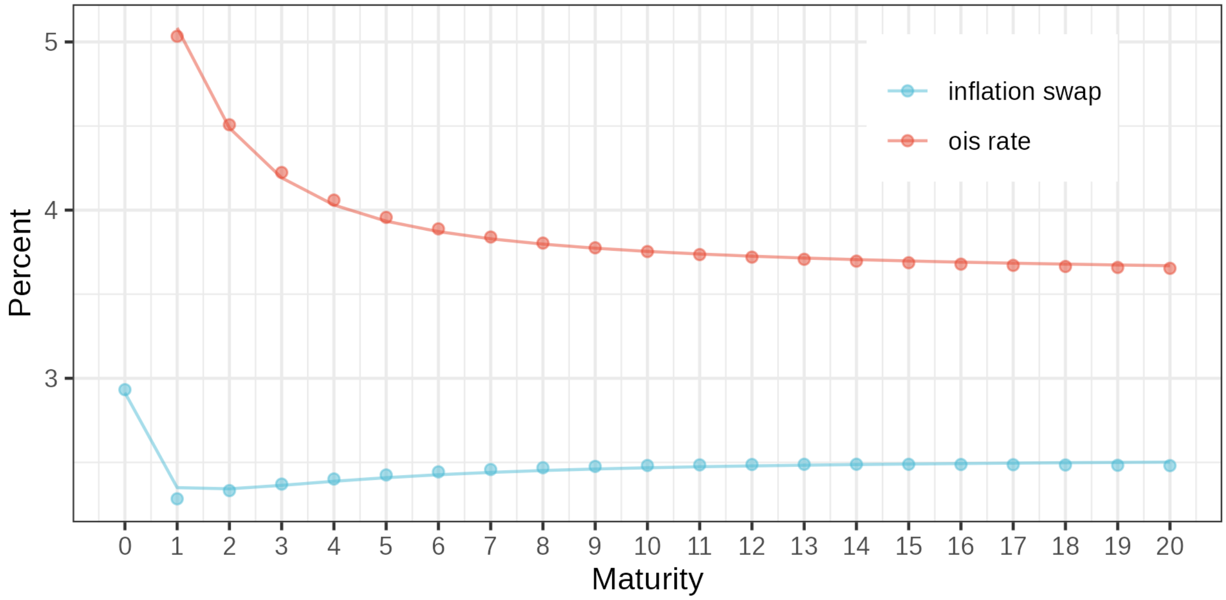
4 Empirical Results

This section presents the results of our estimation. We begin by discussing the model fit before presenting our estimates for benchmark interest rates for monetary policy. We therefore study the natural rate that keeps real activity constant in expectation as well as an optimal discretionary policy rate that assumes the central bank sets policy optimally under discretion each period. We further provide a broader discussion of the various estimated components of our model including estimates of inflation risk premiums and expectations for inflation.

Figure 1 shows the fit of our model for one particular day, June 18, 2024. The blue line plots the model-implied inflation swap curve against the data (plotted as dots), and the red line plots the model-implied nominal yield curve (ois rates) against the data (plotted as dots). The figure shows the model fits the data well.

The close fit of the model to the data is representative of the results throughout the sample period. In general, the model accurately captures both the inflation swap curve and the nominal yield curve with a median absolute value of the fitting error across all dates and tenors of only three basis points.

FIGURE 1
MODEL FIT ON JUNE 18, 2024



Notes: This figure presents the model fit on June 18, 2024. The blue line plots the model implied risk-neutral inflation expectations curve against the data (plotted as dots), and the red line plots the model implied nominal yield curve against the data (plotted as dots).

4.1 A Natural Rate of Interest

As a first benchmark, we compute the "natural rate" that keeps the level of economic activity in the economy constant in expectation. To derive this natural rate, we return to equation (9) and set $x_t = \mathbb{E}_t[x_{t+1}]$ to keep the output gap constant in expectation. Solving for the interest rate i_t yields

$$i_t = r^* + \hat{\mathbb{E}}_t[\pi_{t+1}] + \gamma g_t.$$

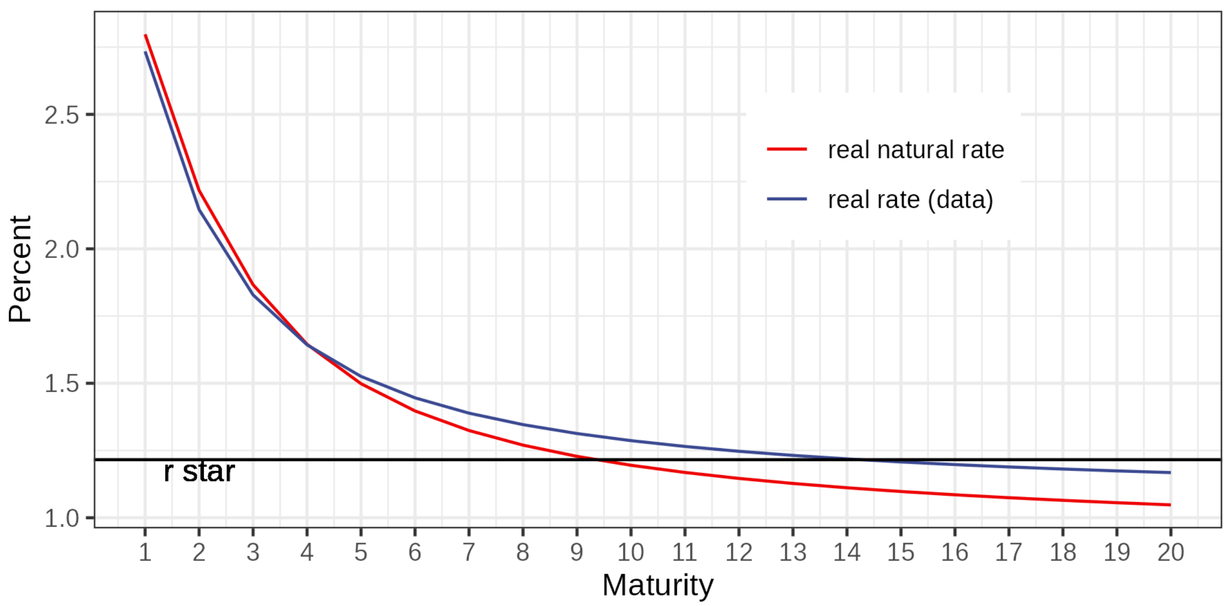
By iterating this equation forward and taking risk-neutral expectations, we arrive at the *forward* natural rate that keeps the output gap constant in expectation τ periods into the future

$$\hat{\mathbb{E}}_t\left[i_{t+\tau}\right] = r^* + \hat{\mathbb{E}}_t[\pi_{t+\tau+1}] + \gamma \sum_{i=0}^{\tau-1} \left(\rho_g^s\right)^i \mu_g^s + \gamma \sum_{i=0}^{\tau-1} \left(\rho_g^\ell\right)^i \mu_g^\ell + \gamma \left(\rho_g^s\right)^\tau g_t^s + \gamma \left(\rho_g^\ell\right)^\tau g_t^\ell. \tag{18}$$

According to the IS curve in equation (9), economic output in the New Keynesian model is determined by the level of real rates, which we derive by subtracting the inflation swap rate from equation (18).

Figure 2 presents the term structure of the resulting real natural rate, the observed real rate in the data, as well as the measure of the neutral real rate of interest r^* on June 18, 2024. These real rate curves represent real rates computed as averages of the forward real rates across maturities. At the long end, the real natural rates converge to a level close to the steady-state neutral real interest rate, r^*. The difference between long-run real natural rates and r^* thereby reflects risk premiums.[4] At the short end, the discrepancy between the real natural rate and r^* can be much larger due to perceived transition dynamics.

FIGURE 2
TERM STRUCTURE OF NEUTRAL REAL RATE ON JUNE 18, 2024



Notes: The above figure presents the estimated real natural rate against the real rate in the data on June 18, 2024. The value of the neutral real rate of interest, r^*, is depicted in black. At the long end, a discrepancy between the natural and neutral rates arises due to a risk premium.

Figure 2 illustrates that a simple comparison between a 1-year real interest rate and r^* is a misleading measure of the stance of policy. This simple metric does not take the current state and resulting transition dynamics into account and further ignores the effects of risk premiums. Instead,

[4]Specifically, the 1-period forward real natural rate converges to r^* plus compensation for growth risk:

$$\lim_{\tau \to \infty} \hat{\mathbb{E}}_t \left[i_{t+\tau} - \pi_{t+\tau+1} \right] = r^* + \gamma \frac{\mu_g^s}{1 - \rho_g^s} + \gamma \frac{\mu_g^\ell}{1 - \rho_g^\ell}.$$

the real natural rate curve provides a measure which varies by maturity and takes the state of the economy as well as risk-premiums into account. Using the curve on June 18, 2024 as an example, short-term real rates were close to the corresponding real natural rate and thus barely restrictive, despite far exceeding the natural real rate of interest.

Figure 2 also shows that the observed real rate was higher than the real natural rate starting at horizons of 3-years or longer. These distinctions are important for a central bank that has multiple policy tools and may be able to influence interest rates along the entire term structure.

How does the time series of the nominal interest rate compare to the nominal natural rate? Figure 3 presents the effective federal funds rate (EFFR) against the 1-year nominal natural rate derived from our estimates over the sample period.

The time series of the natural rate largely aligns with the federal funds rate prior to the COVID pandemic, suggesting that the Federal Reserve's policy rate was close to neutral. Following the onset of the pandemic, the federal funds rate hit the zero-lower-bound, while the estimated nominal natural rate was negative during a portion of this period. Thus, the zero lower bound then forced monetary policy to be restrictive during the post-pandemic period. When inflation rose in 2022, the nominal natural rate increased rapidly. At the end of the sample, the federal funds rate is very close to the nominal natural rate.

Despite having been the focus of an extensive literature, natural rates only focus on keeping prices or economic activity stable. To balance price objectives with the real side of the economy, we next compute benchmark rates as the solution to an optimization problem for the central bank.

4.2 Optimal policy

To compute an optimal policy rate, we assume that the central bank minimizes an expected loss function taking expectations as given

$$\min_{i_t} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\pi_s^2 + \alpha x_s^2 \right] ,$$

where α denotes the weight that the central bank puts on closing the output gap. The first-order condition for the central bank's optimization problem,

$$x_t = -\frac{\lambda}{\alpha}\pi_t, \tag{19}$$

establishes a direct link between the output gap and the rate of inflation. This first-order condition optimally addresses the trade-off between stabilizing inflation and minimizing the output gap.

Plugging the IS curve (9) and Phillips curve (10) into the central bank's first-order condition and solving for the interest rate results in

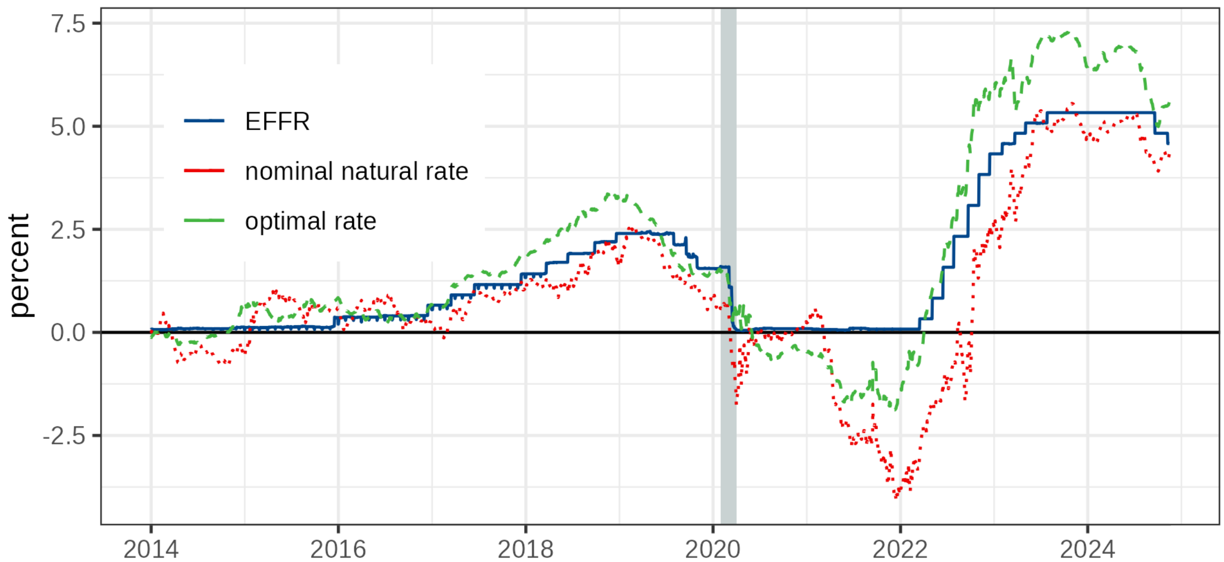
$$i_t = r_t^* + \left(1 + \frac{\beta\lambda\gamma}{\lambda^2 + \alpha}\right)\hat{\mathbb{E}}_t[\pi_{t+1}] + \gamma\mathbb{E}_t[x_{t+1}] + \frac{\lambda\gamma}{\lambda^2 + \alpha}u_t + \gamma g_t. \tag{20}$$

We can therefore obtain the time series of optimal discretionary interest rates by plugging in our time series of estimated shocks and by calibrating the one additional parameter α that governs the central bank's preferences. We calibrate $\alpha = 0.34$ to minimize the distance between optimal discretionary policy and the Taylor rule prior to January 2020.

Figure 3 presents the 1-year optimal discretionary rate along with the nominal natural rate and the observed 1-year OIS rate. The time series of the optimal discretionary rate differs from the natural rate along two key dimensions. First, the optimal rate starts to rise earlier in the aftermath of the COVID pandemic and increases more rapidly than the natural rate. Second, the optimal rate remains higher at the end of the sample period. Both of these features of the optimal rate reflect the fact that optimal policy places a greater weight on mitigating the surge in inflation after the COVID pandemic compared to the natural rate.

The timing of when the optimal rate rises above the zero lower bound closely matches the timing of the lift-off of the federal funds rate. However, the optimal rate rises higher than the actual federal funds rate after lift-off, and it remains at that higher level almost until the end of the sample period. These results suggest that given the financial market perceptions of the state of the economy in early 2022, the Federal Reserve did not start raising interest rates too late after inflation picked up

Figure 3
1-Year Natural and Optimal Discretionary Interest Rates



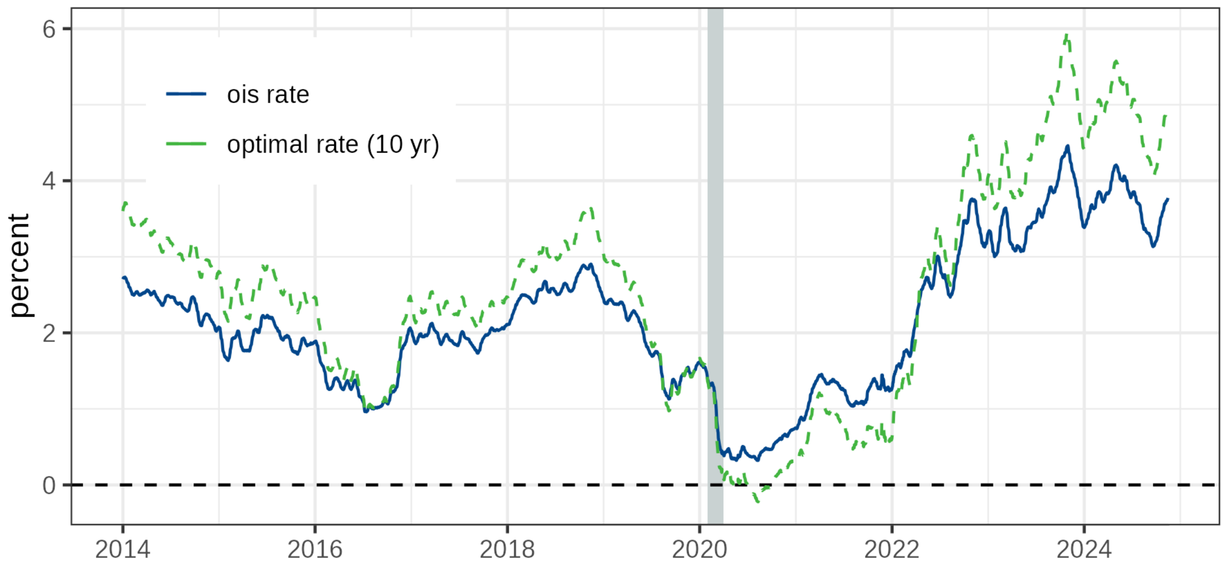
Notes: This figure presents the time series of the 1-year nominal natural rate (red, dotted), the effective federal funds rate (blue, solid), and the optimal discretionary rate (green, dashed) over the full sample period. Estimates are smoothed over a 10-day window. NBER recessions are shaded in gray.

in 2021. At the start of 2022, financial markets believed the output gap was negative and could therefore still justify maintaining low interest rates. However, once financial markets realized the severity of inflationary pressures, our estimates suggest that the Federal Reserve should have raised the short-term policy rate higher.

Longer-term rates are arguably the most relevant interest rates for corporate borrowing and mortgage rates. We can use equation (20) to construct a term structure of optimal rates. We therefore calculate risk-neutral expectations of the one-year optimal rate at different maturities, and we convert those into yields by averaging them across maturities to get both short- and long-term rates. We compare the optimal 10-year interest rate and compare it to the observed 10-year interest rate in Figure 4.

Prior to the COVID-pandemic, the observed 10-year OIS rate largely tracked the 10-year optimal rate. However, this tight relationship between observed and optimal 10-year rates weakened in the post-pandemic period. Starting after the onset of the COVID pandemic, the 10-year optimal rate fell below the observed 10-year rate. During the rise in post-pandemic inflation, the optimal 10-year

Figure 4
10-Year Optimal Discretionary Interest Rate



Notes: This figure presents the time series of the 10-year OIS rate (blue, solid), and the optimal discretionary 10-year interest rate (green, dashed) over the full sample period. Estimates are smoothed over a 10-day window. NBER recessions are shaded in gray.

rate rapidly rose above the observed 10-year rate and remained there until the end of the sample period.

The yield curves of benchmark rates serve as reference rates for the stance of policy. Central banks have tools through which they can affect different parts of the yield curve. The policy rate is typically a very short-term interest rate that anchors the yield curve at short maturities.

We show here that the short end of the yield curve deserves a separate benchmark from the long end due to the influence of transition dynamics. Central banks can use forward guidance to influence financial conditions at the near- to medium-term. For the longer end, many central banks have engaged in balance sheet policies. And a few central banks have discussed, or even implemented, yield curve control. The framework in this paper serves as a guidepost for these policy tools.

4.3 Variation in the Taylor Rule

While early studies of policy rules used fixed coefficients, there has been recent work documenting time variation in (at least the perceived) coefficients of the Taylor rule. Bauer et al. (2022) document substantial time series variation in market perceptions of the Federal Reserve's reaction function using professional forecasts of interest rates and macroeconomic conditions. And Bocola et al. (2024) show that the Federal Reserve policy function was perceived to be less reactive to inflation during the post-pandemic period.

How does variation in the perceived Taylor rule impact our estimates of benchmark interest rates? We study how a less reactive Taylor rule impacts our benchmark policy rates by estimating our model under the assumption that the Taylor rule coefficient on π_t declined from 1.5 to 1 starting in 2021.

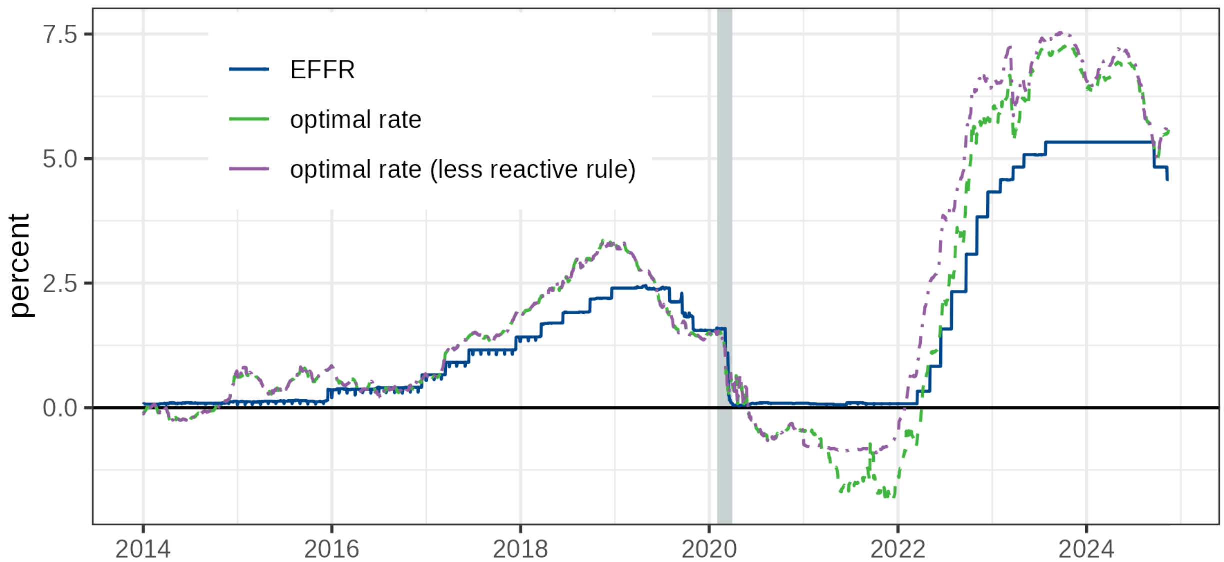
Figure 5 presents the 1-year optimal discretionary policy rate computed with parameter estimates under the Taylor rule that is less reactive to inflation. For reference, we also include the original optimal policy rate and the observed effective federal funds rate from Figure 3.

The figure shows that the optimal policy rate under a less reactive Taylor rule exceeds the original optimal discretionary rate. Starting in 2021, the optimal rate under a less reactive Taylor rule does not drop as low as the original optimal rate, it rises more quickly than the original optimal rate, and it peaks at a higher level in 2023. However, the two optimal rate series converge towards the end of our sample.

Figure 5 also shows that after accounting for the financial market's perception that the Federal Reserve was acting under a less reactive Taylor rule, then the timing and pace of the Federal Reserve policy rate hikes post-pandemic were too slow relative to optimal policy. The intuition behind this result is that if the market believes Federal Reserve policy is less reactive towards inflation, then the market will also infer that the current output gap, x_t, is less negative than it would have been if the policy rule was more responsive to inflation. Hence, the optimal policy rate under a less reactive rule is shifted up, because there is less incentive to boost output through lower interest rates.

Even though optimal policy under the less reactive rule rises faster than observed policy in-

FIGURE 5
OPTIMAL DISCRETIONARY INTEREST RATE WITH LESS REACTIVE TAYLOR RULE

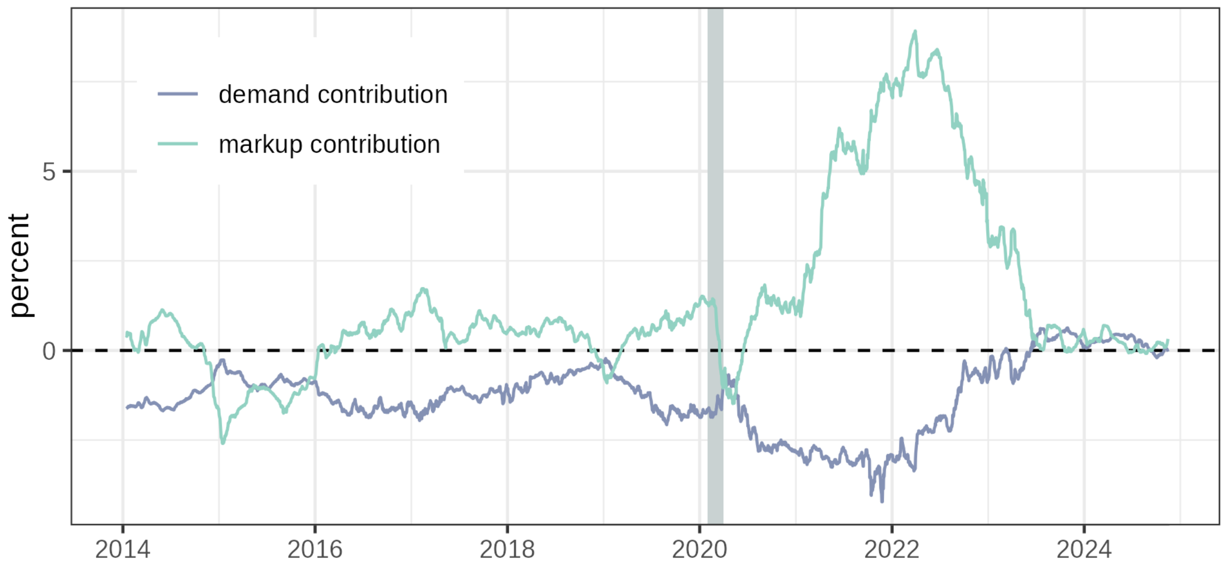


Notes: This figure presents the time series of the effective federal funds rate (blue, solid), and the original optimal discretionary 1-year interest rate (green, dashed), and the optimal discretionary 1-year rate under a less reactive rule (purple, dot-dashed) over the full sample period. Estimates are smoothed over a 10-day window. NBER recessions are shaded in gray.

creases, both benchmarks for optimal policy rates in Figure 5 show that interest rates should have risen to a higher level and remained more restrictive for longer during the post-pandemic period of high inflation.

The exercise in this section shows our estimation procedure facilitates changes in the perception of Taylor rules over time, because the estimates on one day does not depend on the estimate on another day. It is therefore possible to use the benchmark optimal discretionary rate in Figure 5 for certain time periods and the estimated optimal discretionary rate under a less reactive rule at other times. Overall, our results suggest that the Taylor rule is an important element when estimating the New Keynesian model. Hence, providing more accurate measurements of the market's perceptions of Federal Reserve's policy is a useful avenue for future research.

Figure 6
Impact of Demand Shocks and Markup Shocks on Current Inflation



Notes: This figure presents estimates of effect of demand (light purple) and markup (light green) shocks on the current inflation rate, π_t. Estimates are smoothed over a 10-day window. Grey shading indicates NBER recessions.

4.4 Impact of Demand and Markup Shocks on Inflation

The estimation gives rise to a decomposition of inflation rates and expectations due to demand and supply factors. Figure 6 shows the time series of how the demand shocks and the markup shocks impact the current inflation rate, π_t.

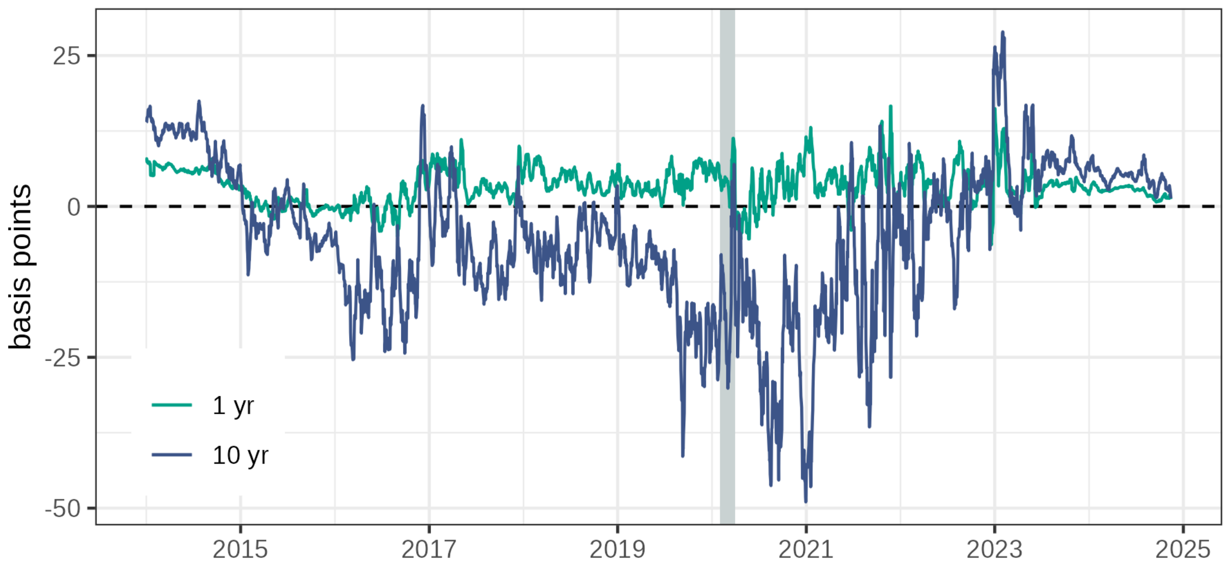
The estimated markup contributions appear close to zero over most of the sample period. Note that this result is not imposed by the estimation since there is no restriction that the estimated shocks have to be zero on average. A notable exception is the post-COVID spike in inflation when markup-shocks are the main source of inflation.

The estimated demand shocks are mostly negative. In particular, demand shocks exerted downwards pressure on currency inflation during the onset of the COVID pandemic. These negative demand shocks are reflected in an upward sloping yield curve conditional on the contribution for markup shocks, since a negative demand shock allows interest rates to slowly increase as the shock dissipates at longer horizons. Positive demand shocks appear only during the post-COVID inflationary episode, which coincides with a period in which the yield curve was inverted.

4.5 Inflation Risk Premiums

The estimation procedure uses the structure of the model to extract risk premiums from financial market data. Figure 7 presents the 1-year inflation risk premium in green and the average 10-year inflation risk-premium in blue.

FIGURE 7
INFLATION RISK PREMIUMS



Notes: The figure above presents estimates of the 1-year (green), and the 10-year inflation risk premiums (blue). Estimates are smoothed over a 10-day window. Grey shading indicates NBER recessions.

The estimated inflation risk premiums are small at the 1-year horizon. The median 1-year inflation risk premium is 3 basis points and the standard deviation is 3 basis points after smoothing over 10-day windows. It is important to remember that these risk premium estimates are not the result of our specification of the stochastic discount factor. In fact, estimated risk premiums might not coincide with the risk premium in the model which arises from the covariance between the stochastic discount factor and shocks. We view this as a strength of the procedure: Risk pricing in macroeconomic models with low risk aversion is known to be too small to explain financial market data. Instead, we only used the model to obtain the estimation equations, and we let the data determine the size of the risk premiums associated with each of the shocks.

The 10-year inflation risk premiums are larger in magnitude than the 1-year inflation risk premiums. The median 10-year inflation risk premium is -6 basis points, its standard deviation is 12 basis points. Figure 7 shows the 10-year inflation risk premiums are also noticeably more volatile than the 1-year risk premiums.

There is some comovement between the 1-year and 10-year risk premiums. Due to the larger magnitude of longer-term risk premiums, the trends stand out more clearly. In the mid- to late-2010s, inflation risk premiums were largely negative. This remained true during the COVID pandemic where spikes became somewhat more pronounced, likely in part due to liquidity problems. With the higher inflation rates, inflation risk premiums turned positive from 2022 on.

4.6 Inflation Forecasts and Forecast Performance

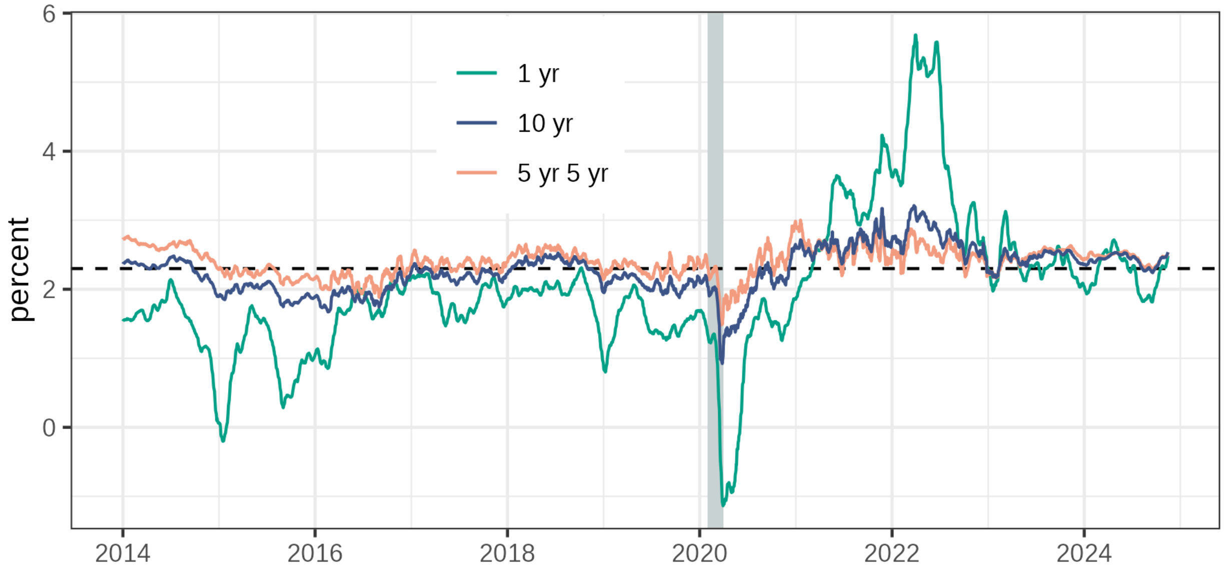
From inflation swap rates and our estimates of inflation risk premiums, we construct physical expectations of inflation. Figure 8 presents these inflation expectations at various horizons. The five-year inflation rate five years ahead, which we interpret as long-term expectations of inflation, is shown in orange. We plot the average inflation rate over 10-years in purple and estimates of the 1-year inflation forecast in green.

These estimates should be interpreted as physical expectations of inflation according to the Consumer Price Index (CPI). CPI inflation has tended to run about 30 to 40 bps above Personal Consumption Expenditure (PCE) price inflation which the Federal Reserve uses to measure its inflation goals. Taking this discrepancy into account, CPI inflation expectations at levels of 2.3% or 2.4% are thus consistent with the Federal Reserve's target level of 2% inflation. We therefore plot a horizontal dashed line at 2.3 percent to represent the Federal Reserve target.

Short-term inflation expectations, as measured by the 1-year inflation rate, undershot the target rate before the pandemic. They increased rapidly when the rate of inflation rose, peaking at above 5% in early 2022 before sharply dropping of an returning to target.

Figure 8 also shows that throughout our sample period long-term inflation expectations, as measured by five-year five-year forward expectations, remained anchored around the Federal Reserve inflation target. Even as annual inflation rates increased to 9 percent in aftermath of the COVID

Figure 8
Inflation Expectations



Notes: This figure presents the physical expectation of the 1-year inflation rate (green), the average 5-year inflation rate 5 years ahead (orange), and the average 10-year inflation rate (purple). Estimates are smoothed over a 10-day window. Grey shading indicates NBER recessions.

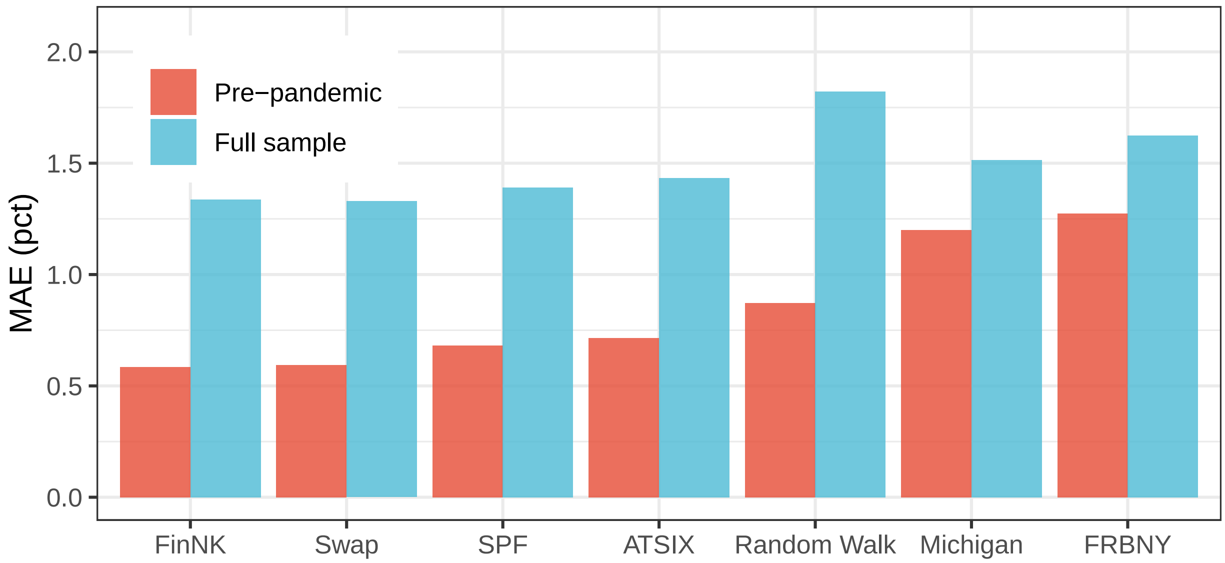
pandemic, long-term inflation expectations never rose above 3 percent. Financial markets thus expected inflationary pressures to fade or be dealt with over time and inflation to return to a level close to the Federal Reserve's target in the long run throughout the post-pandemic period.

Note that these forecasts are truly out of sample since the estimation is conducted from separate data at each point in time. Since financial market prices are forward-looking and adjust rapidly, the forecasts have the potential to capture information about future inflation.

But how accurate is this measure of inflation expectations? We compare the accuracy of our model's 1-year inflation forecasts to various leading inflation forecasts. These forecasts include the Michigan Survey, the Survey of Professional Forecasters, and the Aruoba Term Structure of Inflation Expectations (ATSIX). We also include the raw inflation swap rates as an additional forecast for inflation, even though this series represents risk-neutral expectations and, as such, contains risk premiums.

Due to the different frequencies at which surveys are conducted, we first aggregate all of our forecasts to the quarterly level by averaging forecasts within the quarter. We then compute the mean absolute value of the distance between each forecast and the average realized inflation over a horizon of 1 year. Our sample period therefore stops in 2024 — 1 year before the end of the full sample period to be able to compute the ex-post realized rate of inflation.

FIGURE 9
ACCURACY OF SHORT-RUN INFLATION FORECASTS



Notes: This figure compares the accuracy of 1-year inflation forecasts from our model (FinNK) against the accuracy other inflation forecasts. Each bar represents the mean absolute distance between 1-year inflation forecasts and realized average inflation over a 1-year horizon. The data are aggregated to a quarterly level to account for the fact that surveys are conducted at different frequencies.

Figure 9 summarizes the accuracy of the various forecast methods. We separate out the performance of the forecasts in into a pre-pandemic period (ending in 2019), and a full sample period. Short-run inflation forecasts generated by our model were the most accurate in the pre-pandemic period. Over the full sample period, the forecast from our our model outperforms all non-financial forecasts, and it only performs marginally worse than inflation swaps.

Survey evidence from ATSIX and the Survey of Professional Forecasters also perform relatively well. The Michigan Survey and the FRBNY consumer surveys have provided the least accurate forecasts during our sample period. This results mainly arises because the average level of these consumer-based forecasts exceeds the true rate of inflation.

Figure 9 also shows that risk-neutral expectations for inflation provide reasonable forecasts for inflation, even without adjusting for risk premiums. The fact that inflation swap rates have performed better at forecasting inflation than survey-based forecasts is documented in Diercks et al. (2023). Adjusting for the estimated risk premiums in this paper marginally improves on the forecasting power of inflation swaps during the pre-pandemic period. However, during the post-pandemic period inflation risk premiums were positive and inflation swap measures underpredicting realized inflation. Hence, adjusting inflation swaps for inflation risk premiums resulted in worse forecast performance post-pandemic.

5 Conclusion

This paper provides benchmarks for monetary policy and the restrictiveness of financial market conditions across the entire yield curve. To this end, we solve the canonical New Keynesian model in terms of risk-neutral expectations. This formulation allows us to extract the state of the economy at daily frequencies from the inflation swap curve and nominal yield curve.

The estimation results give rise to two types of benchmark interest rates, natural interest rates and optimal discretionary rates, which can be used to guide policy rates and forward guidance. Our framework further provides measures of the inflation risk premiums, short-term and long-term inflation expectations, and a decomposition of inflation rates into the contribution of demand and markup shocks.

While we take a strong position by relying on financial market data at a snapshot in time, future research can augment the use of financial data with macroeconomic data to jointly inform the stance of monetary policy.

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Appendix

A The New Keynesian model

A.1 Households

Time is discrete and there exists a unit mass of households that live forever. In each period, households consume, they supply labor, and they save. The household objective is to maximize lifetime utility given by equation (1).

A.2 IS Curve: Details

In the following section, we derive the log-linear IS relationship. We deviate from the canonical derivation by substituting in the definition of the inflation swap rate prior to log-linearizing the equilibrium.

For any real rate of return R_{t+1} realized at time $t+1$, the household's Euler equation reads:

$$\mathbb{E}_t \left[\beta e^{\chi_{t+1}-\chi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] = 1, \tag{21}$$

where $R_{t+1} = I_t/\Pi_{t+1}$, and I_t is a nominal rate of return known at time t. We substitute this relationship on the left-hand side, and apply equation (6) with $\tilde{R}_{t+1} = \Pi_{t+1}$ to show:

$$\frac{I_t}{S_{t,t+1}} \mathbb{E}_t \left[\beta e^{\chi_{t+1}-\chi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] = 1, \tag{22}$$

where $S_{t,t+1} = \hat{\mathbb{E}}_t[\Pi_{t+1}]$ is the risk-neutral expectation of inflation (alternatively the inflation swap rate). The first-order approximation of the above expression shows:

$$i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r^* + \mathbb{E}[\chi_t] - \chi_t - \gamma \left(\mathbb{E}[\bar{c}_{t+1}] - \bar{c}_t \right) = 0, \tag{23}$$

where bars denote deviations from steady state and r^* denotes the steady state real interest rate.

We use the fact that $c_t = y_t$ and we add and subtract the deviation of output from steady state in a flexible price economy to write the log-linear Euler equation in terms of the output gap. This simplifies to equation (9):

$$x_t = -\frac{1}{\gamma}\left(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r^*\right) + \mathbb{E}_t[x_{t+1}] + \underbrace{\frac{1}{\gamma}\left(\chi_t - \mathbb{E}[\chi_{t+1}]\right) + \mathbb{E}[\bar{y}^f_{t+1}] - \bar{y}^f_t}_{g_t}, \tag{24}$$

where $x_t = \bar{y}_t - \bar{y}^f_t$ is the output gap, f superscripts denote allocations in a flexible price economy, and g_t is a demand shock.

A.3 Firms

Assume there exists a unit mass of monopolisticly competitive firms with the production function $Y_t(i) = A_t N_t(i)$. Furthermore, assume firms pay a nominal adjustment cost in order to update their prices following Rotemberg (1982). This adjustment cost is:

$$\frac{\eta}{2}\left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}}\right)^2 P_{t-1}Y_{t-1}.$$

Substitute the production function and household demand into the firm's objective to write the firm's real value function as:

$$V(P_{t-1}(i)) = \max_{P_t(i)} \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} Y_t - \Psi_t \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\eta}{2}\left(\frac{P_t(i) - P_{t-1}(i)}{P_{t-1}}\right)^2 \frac{P_{t-1}}{P_t} Y_{t-1} + \mathbb{E}\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} V(P_t(i))\right]$$

where

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

and

$$\Psi_t = \frac{W_t}{P_t A_t}$$

is the firm's real marginal cost. We assume the firm is owned by the households, and therefore discount the future using the household's stochastic discount factor. The first order condition with respect to $P_t(i)$ yields

$$(1-\epsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t}+\epsilon\Psi_t\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t(i)}-\eta\left(\frac{P_t(i)-P_{t-1}(i)}{P_{t-1}}\right)\frac{Y_{t-1}}{P_t}=-\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}V'(P_t(i))\right] \quad (25)$$

Iterating forward the envelope condition yields:

$$V'(P_t(i))=\eta\left(\frac{P_{t+1}(i)-P_t(i)}{P_t}\right)\frac{Y_t}{P_{t+1}} \quad (26)$$

Combining equations (25) and (26), and dividing through by Y_t shows:

$$\begin{aligned}&(1-\epsilon)\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{1}{P_t}+\epsilon\Psi_t\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{1}{P_t(i)}-\eta\left(\frac{P_t(i)-P_{t-1}(i)}{P_{t-1}}\right)\frac{1}{P_t}\frac{Y_{t-1}}{Y_t}=\\&-\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\eta\left(\frac{P_{t+1}(i)-P_t(i)}{P_t}\right)\frac{1}{P_{t+1}}\right]\end{aligned}$$

Imposing symmetry across price-setters, multiplying through by P_t, and plugging in the equilibrium consumption of households yields:

$$\begin{aligned}(1-\epsilon)+\epsilon\Psi_t+\eta\left(1-\Pi_t\right)\frac{Y_{t-1}}{Y_t}&=\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\eta\left(1-\Pi_{t+1}\right)\frac{1}{\Pi_{t+1}}\right]\\&=\eta\mathbb{E}_t\left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\frac{1}{\Pi_{t+1}}\right]-\eta\mathbb{E}_t\left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\right]\\&=\eta\mathbb{E}_t\left[M_{t+1}\right]-\eta\mathbb{E}_t\left[M_{t+1}\Pi_{t+1}\right]\\&=\eta\mathbb{E}_t\left[M_{t+1}\right]\left(1-S_{t,t+1}\right).\end{aligned}$$

The log-linear approximation of the equation above is:

$$\varepsilon(\psi_t-\overline{\psi}_t)-\eta(y_t-y_{t-1})-\eta\pi_t+\eta(y_t-y_{t-1})=-\eta\beta s_{t,t+1} \quad (27)$$

In the textbook New Keynesian model, the real marginal cost ψ_t is proportional to the output gap x_t. We add in an ad-hoc cost-push shock u_t, and simplify the previous equation to recover (10).

A.4 Central Bank

We assume that the central bank sets the nominal interest rate i_t according to a Taylor rule,

$$i_t = \theta_0 + \theta_\pi \pi_t + \theta_x x_t,$$

where $\theta_\pi > 1$.

A.5 Equilibrium

In this section, we derive the equilibrium inflation process, the inflation risk premiums, and the nominal yield curve. The equilibrium is described by the following set of equations:

$$\begin{aligned} x_t &= g_t - \frac{1}{\gamma}(i_t - \hat{\mathbb{E}}_t[\pi_{t+1}] - r_t^*) + \mathbb{E}_t[x_{t+1}] && \text{(IS Curve)} \\ \pi_t &= u_t + \lambda x_t + \beta \hat{\mathbb{E}}_t[\pi_{t+1}] && \text{(Phillips Curve)} \\ i_t &= \theta_0 + \theta_\pi \pi_t + \theta_x x_t, && \text{(Taylor rule)} \end{aligned}$$

along with the shock processes defined by Equation (11). $\mathbb{E}[\varepsilon^j] = \mathbb{E}[\varepsilon_u^j] = 0$ for $j \in \{s, \ell\}$. In risk-neutral expectations, $\hat{\mathbb{E}}[\varepsilon_g] = 0$ and $\hat{\mathbb{E}}[\varepsilon_u^j] = \mu_u^j$ for $j \in \{s, \ell\}$.

Before deriving the solutions to the endogenous variables in the model, it is useful to define a few constants:

Definition 1 *Let*

$$\Xi_1 = (1-\beta)\theta_x + \lambda(\theta_\pi - 1) > 0,$$

and

$$\Xi_j^h = \gamma(1-\rho_j^h)(1-\beta\rho_j^h) + \theta_x(1-\beta\rho_j^h) + \lambda(\theta_\pi - \rho_j^h) > 0,$$

for shock horizon $h \in \{s, \ell\}$ and shock type $j \in \{u, g\}$.

The following lemma describes the inflation process.

Lemma 1 (Inflation) *The inflation process is given by*

$$\begin{aligned}
\pi_t &= \frac{\lambda(r^* - \theta_0) + (\gamma + \theta_x)u_t + \gamma\lambda(\mathbb{E}_t[x_{t+1}] + g_t) + (\beta(\gamma + \theta_x) + \lambda)\hat{\mathbb{E}}_t[\pi_{t+1}]}{\gamma + \theta_x + \theta_\pi \lambda} \\
&= \frac{\lambda(r^* - \theta_0)}{\Xi_1} + \sum_{h=s,\ell} \left[\frac{\gamma\lambda}{\Xi_1 \Xi_g^h}((\lambda + \beta\theta_x)\mu_g^h + g_t^h) + \frac{(\gamma(1 - \rho_u^h) + \theta_x)}{\Xi_1 \Xi_u^h}((\lambda + \beta\theta_x)\mu_u^h + u_t^h) \right],
\end{aligned}$$

where the constants Ξ_1 and Ξ_j^h are defined in Definition 1. The risk-neutral expectation of inflation one period ahead is:

$$\hat{\mathbb{E}}_t[\pi_{t+1}] = \frac{\lambda(r^* - \theta_0)}{\Xi_1} + \sum_{h=s,\ell} \left[\frac{\gamma\lambda}{\Xi_g^h} \left(\frac{(\theta_x + \theta_\pi\lambda)}{\Xi_1}\mu_g^h + \rho_g^h g_t^h \right) + \frac{(\gamma(1 - \rho_u^h) + \theta_x)}{\Xi_u^h} \left(\frac{(\theta_x + \theta_\pi\lambda)}{\Xi_1}\mu_u^h + \rho_u^h u_t^h \right) \right],$$

and the physical expectation of the output gap one period ahead is:

$$\begin{aligned}
\mathbb{E}_t[x_{t+1}] = \frac{(1 - \beta)(r^* - \theta_0)}{\Xi_1} + \sum_{h=s,\ell} \Bigg[&\frac{\gamma}{\Xi_g^h} \left(\frac{(\lambda(1 - \beta\theta_\pi))}{\Xi_1}\mu_g^h + \rho_g^h(1 - \beta\rho_g^h)g_t^h \right) + \\
&\frac{1}{\Xi_u^h} \left(\frac{(\theta_x + \gamma(1 - \rho_u^h))(1 - \beta\theta_\pi)}{\Xi_1}\mu_u^h + \rho_u^h(\rho_u^h - \theta_\pi)u_t^h \right) \Bigg],
\end{aligned}$$

Proof: We use a guess and verify strategy to prove the lemma. Conjecture:

$$\begin{aligned}
\pi_t &= \kappa_0^\pi + \kappa_u^{\pi,s}u_t^s + \kappa_u^{\pi,\ell}u_t^\ell + \kappa_g^{\pi,s}g_t^s + \kappa_g^{\pi,\ell}g_t^\ell \\
x_t &= \kappa_0^x + \kappa_u^{x,s}u_t^s + \kappa_u^{x,\ell}u_t^\ell + \kappa_g^{x,s}g_t^s + \kappa_g^{x,\ell}g_t^\ell.
\end{aligned}$$

Take the risk-neutral and the physical expectations of the expressions above to arrive at conjectors for inflation expectations and output gap expectations:

$$
\begin{aligned}
\mathbb{E}_t[\pi_{t+1}] &= \kappa_0^{\pi} + \kappa_u^{\pi,s}\rho_u^s u_t^s + \kappa_u^{\pi,\ell}\rho_u^{\ell} u_t^{\ell} + \kappa_g^{\pi,s}\rho_g^s g_t^s + \kappa_g^{\pi,\ell}\rho_g^{\ell} g_t^{\ell} \\
\mathbb{E}_t[x_{t+1}] &= \kappa_0^{x} + \kappa_u^{x,s}\rho_u^s u_t^s + \kappa_u^{x,\ell}\rho_u^{\ell} u_t^{\ell} + \kappa_g^{x,s}\rho_g^s g_t^s + \kappa_g^{x,\ell}\rho_g^{\ell} g_t^{\ell}, \\
\hat{\mathbb{E}}_t[\pi_{t+1}] &= \kappa_0^{\pi} + \kappa_u^{\pi,s}(\mu_u^s + \rho_u^s u_t^s) + \kappa_u^{\pi,\ell}(\mu_u^{\ell} + \rho_u^{\ell} u_t^{\ell}) + \kappa_g^{\pi,s}(\mu_g^s + \rho_g^s g_t^s) + \kappa_g^{\pi,\ell}(\mu_g^{\ell} + \rho_g^{\ell} g_t^{\ell}) \\
\hat{\mathbb{E}}_t[x_{t+1}] &= \kappa_0^{x} + \kappa_u^{x,s}(\mu_u^s + \rho_u^s u_t^s) + \kappa_u^{x,\ell}(\mu_u^{\ell} + \rho_u^{\ell} u_t^{\ell}) + \kappa_g^{x,s}(\mu_g^s + \rho_g^s g_t^s) + \kappa_g^{x,\ell}(\mu_g^{\ell} + \rho_g^{\ell} g_t^{\ell}).
\end{aligned}
$$

Next, we plug these conjectures and the Taylor rule into the Phillips curve and the IS curve. We match coefficients:

$$
\begin{aligned}
\kappa_0^{\pi} &= \frac{\lambda(r^* - \theta_0)}{\Xi_1} + \frac{(\beta\theta_x + \lambda)}{\Xi_1}\sum_{h=s,\ell}\left(\frac{\gamma\lambda\mu_g^h}{\Xi_g^h} + \frac{(\theta_x + \gamma(1-\rho_u^h))\mu_u^h}{\Xi_u^h}\right) \\
\kappa_u^{\pi,s} &= \frac{\theta_x + \gamma(1-\rho_u^s)}{\Xi_u^s} \\
\kappa_u^{\pi,\ell} &= \frac{\theta_x + \gamma(1-\rho_u^{\ell})}{\Xi_u^{\ell}} \\
\kappa_g^{\pi,s} &= \frac{\gamma\lambda}{\Xi_g^s} \\
\kappa_g^{\pi,\ell} &= \frac{\gamma\lambda}{\Xi_g^{\ell}} \\
\kappa_0^{x} &= \frac{(1-\beta)(r^* - \theta_0)}{\Xi_1} + \frac{(1-\beta\theta_\pi)}{\Xi_1}\sum_{h=s,\ell}\left(\frac{\gamma\lambda\mu_g^h}{\Xi_g^h} + \frac{(\theta_x + \gamma(1-\rho_u^h))\mu_u^h}{\Xi_u^h}\right) \\
\kappa_u^{x,s} &= \frac{\rho_u^s - \theta_\pi}{\Xi_u^s} \\
\kappa_u^{x,\ell} &= \frac{\rho_u^{\ell} - \theta_\pi}{\Xi_u^{\ell}} \\
\kappa_g^{x,s} &= \frac{\gamma(1-\beta\rho_g^s)}{\Xi_g^s} \\
\kappa_g^{x,\ell} &= \frac{\gamma(1-\beta\rho_g^{\ell})}{\Xi_g^{\ell}}
\end{aligned}
$$

These coefficients determine the solution for inflation and the output gap as a function of shocks, risk premiums, and other model parameters. □

Plugging the solutions for inflation and the output gap into the Taylor rule allows us to derive the nominal interest rate in terms of shocks and risk premiums:

$$
\begin{aligned}
i_t = & \frac{r_t^*(\theta_x(1-\beta)+\theta_\pi\lambda)-\theta_0\lambda}{\Xi_1} + \sum_{h=s,\ell}\left[\frac{\gamma}{\Xi_g^h}\left(\frac{\lambda(\theta_x+\theta_\pi\lambda)}{\Xi_1}\mu_g^h + (\theta_x(1-\beta\rho_g^h)+\theta_\pi\lambda)g_t^h\right)+\right. \\
& \left.\frac{1}{\Xi_u^h}\left(\frac{(\theta_x+\gamma(1-\rho_u^h))(\theta_x+\theta_\pi\lambda)}{\Xi_1}\mu_u^h + (\theta_x\rho_u^h+\gamma\theta_\pi(1-\rho_u^h))u_t^h\right)\right],
\end{aligned}
\tag{28}
$$

We iterate forward our expressions for 1-period inflation and nominal interest rates, and then we take risk-neutral expectations to derive 1-period forward rates for inflation and interest rates. The risk-neutral expectation of inflation from period $t+\tau-1$ to $t+\tau$ is:

$$
\begin{aligned}
\hat{\mathbb{E}}_t[\pi_{t+\tau+1}] = & \underbrace{\frac{(r^*-\theta_0)\lambda}{\Xi_1} + \frac{\beta+\theta_x\lambda}{\Xi_1}\sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_g^h}\mu_g^h + \frac{\theta_x+\gamma(1-\rho_u^h)}{\Xi_u^h}\mu_u^h\right]}_{\text{constant across horizons}} + \\
& \underbrace{\sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_g^h}\left(\left(\sum_{k=0}^{\tau-1}\left(\rho_g^h\right)^k\right)\mu_g^h + \left(\rho_g^h\right)^\tau g_t^h\right) + \frac{\theta_x+\gamma(1-\rho_u^h)}{\Xi_u^h}\left(\left(\sum_{k=0}^{\tau-1}\left(\rho_u^h\right)^k\right)\mu_u^h + \left(\rho_u^h\right)^\tau u_t^h\right)\right]}_{\text{incremental contributions over horizons}},
\end{aligned}
\tag{29}
$$

The risk-neutral expectation of the interest rate from $t+\tau$ to $t+\tau+1$ is:

$$
\begin{aligned}
\hat{\mathbb{E}}_t[i_{t+\tau}] = & \underbrace{\frac{\theta_0\lambda + r^*(\theta_x(1-\beta)+\theta_\pi\lambda)}{\Xi_1} + \frac{\theta_x+\theta_\pi\lambda}{\Xi_1}\sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_g^h}\mu_g^h + \frac{\theta_x+\gamma(1-\rho_u^h)}{\Xi_u^h}\mu_u^h\right]}_{\text{constant across horizons}} + \\
& \underbrace{\sum_{h=s,\ell}\left[\frac{\gamma(\theta_x(1-\beta\rho_g^h)+\theta_\pi\lambda)}{\Xi_g^h}\left(\left(\sum_{k=0}^{\tau-1}\left(\rho_g^h\right)^k\right)\mu_g^h + \left(\rho_g^h\right)^\tau g_t^h\right) + \frac{\theta_x\rho_u^h+\gamma\theta_\pi(1-\rho_u^h)}{\Xi_u^h}\left(\left(\sum_{k=0}^{\tau-1}\left(\rho_u^h\right)^k\right)\mu_u^h + \left(\rho_u^h\right)^\tau u_t^h\right)\right]}_{\text{incremental contributions over horizons}},
\end{aligned}
\tag{30}
$$

for $\tau \geq 1$. When $\tau = 0$,

$$
\begin{aligned}
\hat{\mathbb{E}}_t[i_{t+\tau}] = & \frac{\theta_0\lambda + r^*(\theta_x(1-\beta)+\theta_\pi\lambda)}{\Xi_1} + \frac{\theta_x+\theta_\pi\lambda}{\Xi_1}\sum_{h=s,\ell}\left[\frac{\gamma\lambda}{\Xi_g^h}\mu_g^h + \frac{\theta_x+\gamma(1-\rho_u^h)}{\Xi_u^h}\mu_u^h\right] + \\
& \sum_{h=s,\ell}\left[\frac{\gamma(\theta_x(1-\beta\rho_g^h)+\theta_\pi\lambda)}{\Xi_g^h}g_t^h + \frac{\theta_x\rho_u^h+\gamma\theta_\pi(1-\rho_u^h)}{\Xi_u^h}u_t^h\right].
\end{aligned}
$$

Taking averages of equations (29) and (30) over horizons produces the inflation swap curve and the nominal yield curve.

B Estimation

B.1 Data Sources

Inflation Swap Rates. Inflation swap rates are found in Bloomberg under the ticker USSWITX Curncy, where X is the maturity of the swap rate. We use swap rates of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25 and 30 year maturities in our paper.

OIS Rates written on the Federal Funds Rate. OIS rates written on the Federal Funds Rate are found in Bloomberg under the ticker USSOX Curncy, where X is the maturity of the OIS rate. We use OIS rates of 1, 2, 3, 4, 5, 7, 10 and 20 year maturities in our paper.

B.2 Estimate the Parameter Vector

In the following appendix, we provide additional details about the estimation of the parameters. As discussed in section 3, we estimate parameters using a multistep step procedure that attempts to minimize the volatlity induced by liquidity issues. We begin by describing how we estimate the persistence of long-term shocks and then short-term shocks. After estimating the persistence of shocks, we then estimate the shocks themselves and the associated risk premiums.

Step 0: Assume a long-term horizon We start by choosing a horizon H after which we assume the short-term shock has died out. Specifically, we assume $\left(\rho_g^s\right)^h = (\rho_u^s)^h = 0 \; \forall \; h \geq H$ for the purposes of estimation. We will estimate all parameters governing risk-neutral expectations of inflation for all horizons H between 2 to 20 years. Afterwards, we will choose the long-term horizon H and its associated parameter estimates that minimizes the squared error between the model implied inflation swap curve and the data.

Step 1: Estimate the persistence of long-run shocks $\left(\rho_u^\ell, \rho_g^\ell\right)$. We estimate the persistence parameters ρ_u^ℓ and ρ_g^ℓ by minimizing the sum-of-squared errors between the model implied forward inflation swap rates and forward interest rates and the data. At each date t, the model implied difference between two consecutive forward inflation swap rates at horizon $h > H$ is:

$$\hat{\mathbb{E}}_t[\pi_{t+h+1}] - \hat{\mathbb{E}}_t[\pi_{t+h}] = \zeta_g^\ell \left(\rho_g^\ell\right)^h + \zeta_u^\ell \left(\rho_u^\ell\right)^h, \tag{31}$$

and the model implied difference between two consecutive forward interest rates at horizon $h > H$ is:

$$\hat{\mathbb{E}}_t[i_{t+h+1}] - \hat{\mathbb{E}}_t[i_{t+h}] = \zeta_g^\ell \left(\frac{\theta_x(1-\beta\rho_g^\ell) + \theta_\pi\lambda}{\lambda}\right)\left(\rho_g^\ell\right)^h + \zeta_u^\ell \left(\frac{\theta\rho_u^\ell + \gamma\theta_\pi(1-\rho_u^\ell)}{\gamma(1-\rho_u^\ell)+\theta_x}\right)\left(\rho_u^\ell\right)^h, \tag{32}$$

where ζ_u^ℓ and ζ_g^ℓ are constant across horizons and are functions of the other variables in the model. However, because ζ_u^ℓ and ζ_g^ℓ are constant across horizon, we can treat them as constants in this part of the estimation.

After plugging in the parameters listed in Table 1, equations (31) and (32) are functions of $\rho_u^\ell, \rho_g^\ell, \zeta_u^\ell$ and ζ_g^ℓ. We compute values of these variables that minimizes the sum of squared difference between the model-implied differences in forward inflation swap rates and forward interest rates and their data counterparts at all values of $h \geq H$.

Step 2: Estimate the persistence of short-term shocks $\left(\rho_u^s, \rho_g^s\right)$ After estimating the parameters governing the decay of the long-term shock, we estimate the persistence of the short-term shock. We subtract off the effect of the long-term shock on forward inflation swap rates and forward interest rates, and we solve for the persistence of the short-term shock that best matches the gap between the data and the impact of the long-term mark-up shock on the inflation swap rate.

Specifically, at each date t, the model implied difference between two consecutive forward inflation swap rates at horizon h is:

$$\hat{\mathbb{E}}_t[\pi_{t+h+1}] - \hat{\mathbb{E}}_t[\pi_{t+h}] = \zeta_g^\ell \left(\rho_g^\ell\right)^h + \zeta_u^\ell \left(\rho_u^\ell\right)^h + \zeta_g^s \left(\rho_g^s\right)^h + \zeta_u^s \left(\rho_u^s\right)^h \tag{33}$$

and the model implied difference between two consecutive forward interest rates at horizon h is:

$$\begin{aligned}\hat{\mathbb{E}}_t[i_{t+h+1}] - \hat{\mathbb{E}}_t[i_{t+h}] =& \zeta_g^\ell \left(\frac{\theta_x(1-\beta\rho_g^\ell) + \theta_\pi\lambda}{\lambda}\right)\left(\rho_g^\ell\right)^h + \zeta_u^\ell \left(\frac{\theta\rho_u^\ell + \gamma\theta_\pi(1-\rho_u^\ell)}{\gamma(1-\rho_u^\ell)+\theta_x}\right)\left(\rho_u^\ell\right)^h + \\ & \zeta_g^s \left(\frac{\theta_x(1-\beta\rho_g^s) + \theta_\pi\lambda}{\lambda}\right)\left(\rho_g^s\right)^h + \zeta_u^s \left(\frac{\theta\rho_u^s + \gamma\theta_\pi(1-\rho_u^s)}{\gamma(1-\rho_u^s)+\theta_x}\right)\left(\rho_u^s\right)^h ,\end{aligned} \tag{34}$$

where ζ_u^s and ζ_g^s are also constant across horizons and are functions of the other variables in the model. However, because ζ_u^s and ζ_g^s are constant across horizon, we can treat them as constants in this part of the estimation.

After plugging in the parameters listed in Table 1 and the previously estimated values of $\rho_u^\ell, \rho_g^\ell, \zeta_u^\ell$ and ζ_g^ℓ, equations (33) and (34) are functions of $\rho_u^s, \rho_g^s, \zeta_u^s$ and ζ_g^s. We compute values of these variables that minimizes the sum of squared difference between the model-implied differences in forward inflation swap rates and forward interest rates and their data counterparts at all horizons.

Step 3: Estimate shocks and risk premiums After estimating the persistence of shocks, equations (29) and (30) describe the model implied inflation swap curve and the nominal yield curve as functions of shocks, risk premiums and other parameters that are either calibrated or have been previously estimated. Therefore, we estimate the remaining parameters by minimizing the sum-of-squared difference between the model implied inflation swap curve and the model implied nominal yield curve and their data counterparts. These remaining parameters are: $u_t^\ell, u_t^s, g_t^\ell, g_t^s, \mu_{g,t}^\ell, \mu_{u,t}^\ell, \mu_{g,t}^s, \mu_{u,t}^s$.

Step 4: Iterate across horizons H . As a final step, we iterate across the threshold H and find the value that minimizes the sum-of-squared fitting error between the model implied inflaiton swap curve and nominal yield curve and their data counterparts, as computed in Step 3. We perform this estimation on each data using only the cross-section of asset prices on that date.