FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

Market-Based Estimates of the Natural Real Rate: Evidence from Latin American Bond Markets

Luis Ceballos
University of San Diego

Jens H. E. Christensen
Federal Reserve Bank of San Francisco

Damian Romero
Banco Central de Chile

January 2024

Working Paper 2024-01

https://www.frbsf.org/economic-research/publications/working-papers/2024/01/

Suggested citation:

Ceballos, Luis, Jens H. E. Christensen, Damian Romero. 2024 "Market-Based Estimates of the Natural Real Rate: Evidence from Latin American Bond Markets," Federal Reserve Bank of San Francisco Working Paper 2024-01.
https://doi.org/10.24148/wp2024-01

Market-Based Estimates of the Natural Real Rate: Evidence from Latin American Bond Markets

Luis Ceballos

Jens H. E. Christensen

Damian Romero

Abstract

We provide market-based estimates of the natural real rate, that is, the steady-state short-term real interest rate, for Brazil, Chile, and Mexico. Our approach uses a dynamic term structure finance model estimated directly on the prices of individual inflation-indexed bonds with adjustments for bond-specific liquidity and real term premia. First, we find that inflation-indexed bond liquidity premia in all three countries are sizable with significant variation. Second, we find large differences in their estimated equilibrium real rates: Brazil's is large and volatile, Mexico's is stable but elevated, while Chile's is low and has fallen persistently. Although uncertain, our estimates could have important implications for the conduct of monetary policy in these three countries.

JEL Classification: C32, E43, E52, G12

Keywords: affine arbitrage-free term structure model, financial market frictions, monetary policy, rstar

---

Luis Ceballos (luisceballos@sandiego.edu), Jens H. E. Christensen (jens.christensen@sf.frb.org), Damian Romero (dromeroc@bcentral.cl). Corresponding author: Jens Christensen, Economic Research Department, Federal Reserve Bank of San Francisco, 101 Market Street (MS 1130), San Francisco, CA 94105. The views in this paper are solely the responsibility of the authors and do not necessarily reflect those of others at the Federal Reserve Bank of San Francisco, in the Federal Reserve System, or at the Central Bank of Chile.

This version: January 11, 2024.

1 Introduction

The equilibrium real rate is a key macroeconomic concept of general importance. For investors, the steady-state level of the real short-term rate serves as an anchor for projections of the future discount rates used in valuing assets (e.g., Clarida 2014). For policymakers and researchers, the equilibrium or natural rate of interest is a policy lodestar that provides a neutral benchmark to calibrate the stance of monetary policy: Monetary policy is expansionary if the short-term real interest rate lies below the natural rate and contractionary if it lies above. A good estimate of the equilibrium real rate is also necessary to operationalize popular monetary policy rules such as the Taylor rule.[1]

While most existing estimates of this key economic variable are drawn from *macroeconomic* models and data, we instead follow Christensen and Rudebusch (2019, henceforth CR) and use *financial* models. Specifically, we rely on inflation-indexed bond prices from three major Latin American countries, Brazil, Chile, and Mexico, for our analysis and therefore offer a unique emerging market perspective on recent trends in the natural real rate.[2]

We note that these securities have coupon and principal payments indexed to the consumer price index (CPI) in each country and hence provide compensation to investors for the erosion of purchasing power due to domestic price inflation.[3] Therefore, their bond prices can be expressed directly in terms of real yields. We assume that the longer-term expectations embedded in the bond prices reflect financial market participants' views about the steady state of each economy, including its natural rate of interest.

The bond data also offer additional advantages. First, all three countries have fairly liquid markets for inflation-indexed government debt. Second, with possible maturities of up to 40 years in the case of Brazil, these inflation-indexed bond markets contain significant forward-looking information and hence are likely to provide clear evidence for the issue at hand. Third, by relying on inflation-indexed bonds, we avoid any issues related to the zero lower bound that may apply to overnight rates and other nominal interest rates. Furthermore, as the underlying factors affecting long-term interest rates are likely global in nature—such as worldwide demographic shifts, changes in productivity trends, or persistent adjustments to global supply chains in the post-pandemic world—these three government bond markets may well be as informative as any other emerging sovereign bond market. Thus, we think of our collection of bond data as being representative for emerging bond markets more broadly.

Despite all these advantages, the use of inflation-indexed bonds for measuring the steady-

[1]For research on the role of the natural rate in monetary policy, see Rudebusch (2001), Orphanides and Williams (2002), Eggertsson et al. (2016), and Hamilton et al. (2016), among many others.

[2]Our analysis differs from Blake et al. (2015), who analyze *nominal* term premium estimates from multiple Latin American countries, including the three countries considered here, in that they rely on fitted synthetic zero-coupon bond yields for their model estimations.

[3]The CPI is also the price index targeted by the three respective central banks for monetary policy purposes.

state short-term real interest rate faces its own empirical challenges. One problem is that inflation-indexed bond prices include a real term premium. Given the generally upward slope of the inflation-indexed bond yield curves in our data, the real term premium is presumably usually positive. However, little is known with certainty about its size or variability. In addition, despite the fairly large notional amount of outstanding inflation-indexed bonds in the three countries under analysis, these securities inherently face appreciable liquidity risk for structural reasons, as argued by Cardozo and Christensen (2023). First, since they provide a hedge against inflation risk, they are likely to be much less traded than nominal bonds. Second, as this hedge argument only applies to domestic investors whose consumption expenditures track the local CPI, foreigners will not benefit from holding these securities. As a consequence, the trading of inflation-indexed bonds ends up being concentrated among patient domestic buy-and-hold investors like pension funds and insurance companies. Presumably, investors require a premium for bearing the liquidity risk associated with holding inflation-indexed bonds, but the extent and time variation of this liquidity premium deserve further examination. Finally, we note that we perform our analysis under the standard finance assumption that the government bonds are default free. Hence, we do not account for credit risk. While this omission may bias our results to some extent, we note in support of this model choice that the public debt to nominal GDP ratio in 2022 was 72.9%, 37.6%, and 48.3% for Brazil, Chile, and Mexico, respectively,[4] which are low values compared to most advanced economies, including the United States.

To estimate the natural real rate of interest in the presence of liquidity and real term premia, we use an arbitrage-free dynamic term structure model of real yields augmented with a liquidity risk factor. The identification of the liquidity risk factor comes from its unique loading for each individual security as in Andreasen et al. (2021, henceforth ACR). Our analysis uses prices of individual bonds rather than the more usual input of yields from fitted synthetic curves. The underlying mechanism assumes that, over time, an increasing proportion of the outstanding inventory is locked up in buy-and-hold portfolios. Given forward-looking investor behavior, this lockup effect means that a particular bond's sensitivity to the market-wide liquidity factor will vary depending on how seasoned the bond is and how close to maturity it is. By observing a cross section of bond prices over time—each with a different time-since-issuance and time-to-maturity—we can identify the overall liquidity factor and each bond's loading on that factor. This technique is particularly useful for analyzing inflation-indexed debt when only a limited sample of bonds may be available as in the early years of our samples.[5]

The theoretical arbitrage-free formulation of the model also provides identification of a

[4]See https://focus-economics.com.

[5]Finlay and Wende (2012) examine prices from a limited number of Australian inflation-indexed bonds but do not account for liquidity premia.

time-varying real term premium in the pricing of inflation-indexed bonds. Identifying the liquidity premium and real term premium allows us to estimate the underlying frictionless real rate term structure and the natural real rate of interest, which we measure as the average expected real short rate over a five-year period starting five years ahead, as in CR.

We summarize our findings as follows. First, there are sizable and time-varying liquidity risk premia in all three markets, consistent with the arguments laid out in Cardozo and Christensen (2023). This also underscores the importance of accounting for these premia in our analysis. Second, there is large dispersion in the estimates of the natural real rate across the three considered economies. This likely reflects the large differences in their economic fundamentals. We leave it for future research to examine whether this applies to other emerging economies in Latin America and beyond. As a final exercise, we use our estimates of the natural real rate to construct a measure of the stance of monetary policy in each country. These measures suggest that, while accommodative during the pandemic period, monetary policy had reached a highly restrictive stance in all three economies by the end of our sample period. However, we stress that our natural real rate estimates are associated with significant uncertainty, which is worth keeping in mind if used to draw implications for the conduct of monetary policy.

The remainder of the paper is organized as follows. Section 2 contains a description of the inflation-indexed bond data, while Section 3 details the no-arbitrage term structure models we use and presents the empirical results. Section 4 briefly examines the estimated real bond-specific liquidity premia, while Section 5 analyzes our bond-based estimates of the natural rate. Finally, Section 6 concludes. Appendices contain details of our Brazilian data and a summary of the model selection procedure for each country.

2 The Inflation-Indexed Bond Data

In this section, we briefly describe the inflation-indexed bond data we use in our model estimations.

Our sample of Brazilian inflation-indexed bonds is downloaded from Bloomberg and contains prices for 31 bonds starting in November 2005 and ending in September 2023. Appendix A provides more details of our sample of Brazilian bonds.[6]

The Chilean government issued its first inflation-indexed bonds back in the 1960s. These bonds are known as bonos tesoreria UF (BTUs) and bonos central UF (BCUs). Our sample from Riskamerica (www.riskamerica.com) covers the period from August 2003 to July 2023 and contains prices for a total of 72 bonds. It was first analyzed in Ceballos et al. (2024, henceforth CCR). Thus, we refer interested readers to that paper for further details on the

[6] De Pooter et al. (2014) use these Brazilian real bond prices, but not at our level of detail.

Chilean data.[7]

Mexican inflation-indexed bonds are known as udibonos. Our sample of udibonos prices is downloaded from Bloomberg and covers the period from May 2009 to September 2023 with prices for a total of 20 bonds. It was first analyzed by Beauregard et al. (2023). Thus, we refer interested readers to that paper for additional details on our Mexican data.

Importantly, as noted by Gürkaynak et al. (2010) and ACR, prices of inflation-indexed bonds near their maturity tend to be somewhat erratic because of the indexation lag in their payoffs. Therefore, to facilitate model estimation, we drop inflation-indexed bonds from all three samples when they have less than one year to maturity.

Figure 1 shows the yield-to-maturity series for all inflation-indexed bonds in our sample for each of the three countries. All three yield samples are characterized by pronounced business cycle variation. Moreover, real yields in all three countries fell sharply at the peak of the COVID-19 pandemic. However, outside that period, the commonalities are less obvious. Furthermore, they generally operate at different levels, with Brazil offering the highest real rates, while Chile tends to have the lowest real rates closely followed by Mexico.[8]

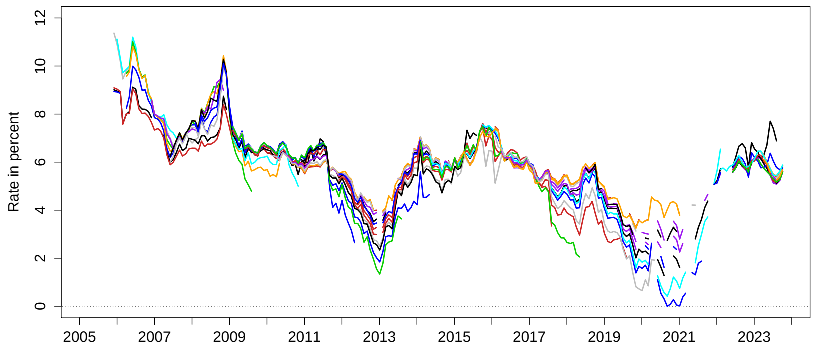
One key thing to note is that both the number of bonds and their maturity distribution vary across the three samples. Thus, these yields are not directly comparable. Fortunately, our term structure models allow us to accurately account for these sample differences in addition to providing adjustments to the observed real rates for both bond-specific liquidity risk premia and general term premia. This way we get readings on investors' expectations for the underlying real short-term rates in each of these three important inflation-indexed bond markets. The rest of the paper is dedicated to this task.

3 Model Estimation and Results

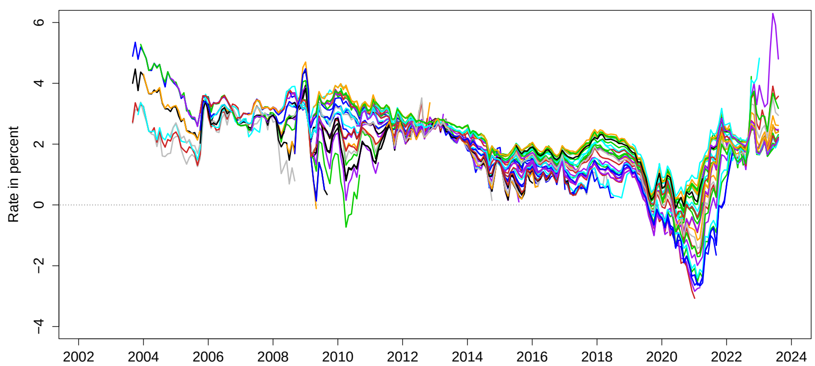
In this section, we first describe how we model yields in a world without any frictions to trading. This model of frictionless dynamics is fundamental to our analysis. We then detail the augmented model that accounts for the liquidity premia in the inflation-indexed bond yields. This is followed by a description of the restrictions imposed to achieve econometric identification of this model and its estimation. We end the section with a brief summary of our model selection strategy and associated estimation results.

[7]See Ceballos et al. (2016) for an analysis of Chilean *nominal* yields and term premia.

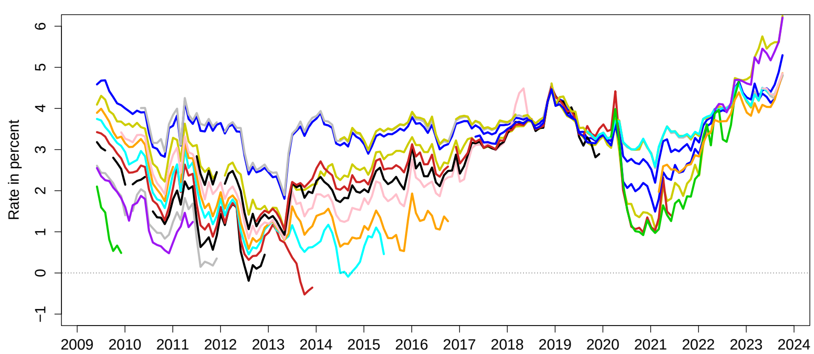
[8]The high level of interest rates in Brazil has not gone unnoticed in the academic literature. Arida et al. (2015), Gonçalves et al. (2007), and Bacha et al. (2009) examine the role of jurisdictional uncertainty for Brazilian long-term borrowing contracts, while Barbosa (2006) stresses the role of the high government debt level in Brazil. Finally, Segura-Ubiergo (2012) emphasizes Brazil's low level of domestic savings.



(a) Brazil



(b) Chile



(c) Mexico

Figure 1: **Yield to Maturity of Inflation-Indexed Bonds**

3.1 A Frictionless Arbitrage-Free Model of Real Yields

To capture the fundamental or frictionless factors operating the real yield curves in our data, we choose to focus on the tractable affine dynamic term structure model introduced in

Christensen et al. (2011).[9]

In this arbitrage-free Nelson-Siegel (AFNS) model, the state vector is denoted by $X_t = (L_t, S_t, C_t)$, where L_t is a level factor, S_t is a slope factor, and C_t is a curvature factor. The instantaneous risk-free real rate is defined as

$$r_t = L_t + S_t. \tag{1}$$

The risk-neutral (or $\mathbb{Q}$-) dynamics of the state variables are given by the stochastic differential equations[10]

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \end{pmatrix}, \tag{2}$$

where Σ is the constant covariance (or volatility) matrix. Based on this specification of the $\mathbb{Q}$-dynamics, zero-coupon real bond yields preserve the Nelson-Siegel factor loading structure as

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \tag{3}$$

where $\frac{A(\tau)}{\tau}$ is a convexity term that adjusts the functional form in Nelson and Siegel (1987) to ensure absence of arbitrage (see Christensen et al. (2011)).

To complete the description of the model and to implement it empirically, we will need to specify the risk premia that connect these factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the real-world (or physical) $\mathbb{P}$-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical $\mathbb{P}$-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^3$ and $\gamma^1 \in \mathbf{R}^{3\times 3}$ contain unrestricted parameters.

[9] Although the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the canonical Gaussian model, see Christensen et al. (2011) for details.

[10] As discussed in Christensen et al. (2011), with a unit root in the level factor, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

Thus, the resulting unrestricted three-factor AFNS model has $\mathbb{P}$-dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

3.2 An Arbitrage-Free Model of Real Yields with Liquidity Risk

In this section, we augment the frictionless model introduced above to account for the liquidity premium of the inflation-indexed bond prices we use in the empirical analysis. To do so, let $X_t = (L_t, S_t, C_t, X_t^{liq})$ denote the state vector of the four-factor model with liquidity risk premium adjustment, denoted the AFNS-L model. As in the non-augmented model, we let the frictionless instantaneous real risk-free rate be defined by equation (1), while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \kappa_{liq}^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_{liq}^{Q} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^{liq} \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{liq,\mathbb{Q}} \end{pmatrix},$$

where Σ continues to be a diagonal matrix.

In the augmented model, inflation-indexed bond yields are sensitive to bond-specific liquidity risks because their pricing is performed with the following discount function:

$$\bar{r}_t^i = r_t + \beta^i (1 - e^{-\lambda^{R,i}(t-t_0^i)}) X_t^{liq} = L_t + S_t + \beta^i (1 - e^{-\lambda^{R,i}(t-t_0^i)}) X_t^{liq}. \quad (4)$$

CR show that the net present value of one unit of consumption paid by bond i at time $t+\tau$ has the following exponential-affine form

$$\begin{aligned} P_t(t_0^i, \tau) &= E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} \bar{r}^i(s, t_0^i) ds} \right] \\ &= \exp\Big(B_1(\tau) L_t + B_2(\tau) S_t + B_3(\tau) C_t + B_4(t, t_0^i, \tau) X_t^{liq} + A(t, t_0^i, \tau) \Big). \end{aligned}$$

This result implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all i, we recover the AFNS model.

Now, consider the whole value of bond i issued at time t_0^i with maturity at $t + \tau^i$ that pays an annual coupon C^i semiannually. Its price is given by[11]

$$P_t(t_0^i,\tau^i,C^i) = C^i(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^i(s,t_0^i)ds}\Big] + \sum_{j=2}^{N}\frac{C^i}{2}E^{\mathbb{Q}}\Big[e^{-\int_t^{t_j}\overline{r}^i(s,t_0^i)ds}\Big]$$
$$+E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\overline{r}^i(s,t_0^i)ds}\Big].$$

Unlike U.S. Treasury Inflation-Protected Securities (TIPS), inflation-indexed bonds in our three markets have no embedded deflation protection option, which makes their pricing straightforward. There is only one minor omission in the bond pricing formula above. It does not account for the lag in the inflation indexation of the bond payoff. The potential error from this omission should be modest (see Grishchenko and Huang 2013), especially as we exclude bonds from our sample when they have less than one year remaining to maturity.

Finally, to complete the description of the AFNS-L model, we again specify an essentially affine risk premium structure, which implies that the risk premia Γ_t take the form

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^4$ and $\gamma^1 \in \mathbf{R}^{4\times 4}$ contain unrestricted parameters. Thus, the resulting unrestricted four-factor AFNS-L model has $\mathbb{P}$-dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^{liq} \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \\ dW_t^{liq,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

3.3 Model Estimation and Econometric Identification

Due to the nonlinear relationship between the state variables and the bond prices, the model cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012); see CR for details. Furthermore, to make the fitted errors comparable across bonds of various maturities, we scale each bond price by its duration. Thus, the measurement equation for the bond prices take the following form

$$\frac{P_t^i(t_0^i,\tau^i)}{D_t^i(t_0^i,\tau^i)} = \frac{\widehat{P}_t^i(t_0^i,\tau^i)}{D_t^i(t_0^i,\tau^i)} + \varepsilon_t^i,$$

[11] This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

where $\widehat{P}_t^i(t_0^i, \tau^i)$ is the model-implied price of bond i and $D_t^i(t_0^i, \tau^i)$ is its duration, which is calculated before estimation. See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equation.

Furthermore, since the liquidity risk factor is a latent factor that we do not observe, its level is not identified without additional restrictions. For the Brazilian market, we let the first 40-year bond issued on September 15, 2004, and maturing on May 15, 2045, with 6 percent coupon have a unit loading on this factor, that is, $\beta^i = 1$ for this bond. For the Chilean market, we follow CCR and use the first 20-year bond issued on September 11, 2002, and maturing on September 1, 2022, with 5 percent coupon and let it have a unit loading on the bond-specific risk factor. Finally, for the Mexican market, we follow Beauregard et al. (2023) and let the first 30-year, 4.5 percent coupon Mexican udibonos bond, which was issued on January 5, 2006, and matures on November 22, 2035, have a unit loading on this factor.

Finally, we note that the $\lambda^{L,i}$ parameters can be hard to identify if their values are too large or too small. As a consequence, we follow ACR and impose the restriction that they fall within the range from 0.01 to 10, which is without practical consequences. Also, for numerical stability during model optimization, we impose the restriction that the β^i parameters fall within the range from 0 to 100, which turns out not to be a binding constraint in optimum.

3.4 Model Selection

For estimation of the natural real rate and associated real term premia, the specification of the mean-reversion matrix $K^{\mathbb{P}}$ is critical, as explained in CCR. To select the best fitting specification of each model's real-world dynamics, we use a general-to-specific modeling strategy in which the least significant off-diagonal parameter of $K^{\mathbb{P}}$ is restricted to zero and the model is re-estimated. This strategy of eliminating the least significant coefficient is carried out down to the most parsimonious specification, which has a diagonal $K^{\mathbb{P}}$ matrix. The final specification choice is based on the value of the Bayesian information criterion (BIC) as in Christensen et al. (2014).[12] The summary statistics of the model selection process for each country are reported in Appendix B.

For the Brazilian bond data, the BIC is minimized by specification (10), which has a $K^{\mathbb{P}}$ matrix given by

$$K_{BRA}^{\mathbb{P}} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & 0 & 0 & 0 \\ 0 & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & 0 \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & 0 \\ 0 & 0 & 0 & \kappa_{44}^{\mathbb{P}} \end{pmatrix}.$$

[12]The Bayesian information criterion is defined as $\text{BIC} = -2\log L + k\log T$, where k is the number of model parameters and T is the number of monthly data observations. The Brazilian, Chilean, and Mexican samples have 215, 240, and 173 monthly observations, respectively.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	0.1901	0	0	0	0.0708	σ_{11}	0.0088
	(0.0980)				(0.0083)		(0.0003)
$K^{\mathbb{P}}_{2,\cdot}$	0	1.2557	0.8674	0	-0.0200	σ_{22}	0.0255
		(0.1470)	(0.1386)		(0.0124)		(0.0017)
$K^{\mathbb{P}}_{3,\cdot}$	4.3983	-1.9075	0.7850	0	-0.0238	σ_{33}	0.0355
	(0.1644)	(0.1477)	(0.1435)		(0.0164)		(0.0024)
$K^{\mathbb{P}}_{4,\cdot}$	0	0	0	0.0010	1.1578	σ_{44}	0.0532
				(0.0139)	(0.0850)		(0.0043)

Table 1: **Estimated Dynamic Parameters of the Preferred Brazilian AFNS-L Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the preferred Brazilian AFNS-L model according to the BIC. The estimated value of λ is 0.3042 (0.0093), while $\kappa^{\mathbb{Q}}_{liq} = 2.7175$ (0.0950), and $\theta^{\mathbb{Q}}_{liq} = 0.0167$ (0.0010). The maximum log likelihood value is 11,411.93. The numbers in parentheses are the estimated parameter standard deviations.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	0.2745	0.4015	0	0	0.0607	σ_{11}	0.0064
	(0.0533)	(0.0511)			(0.0145)		(0.0003)
$K^{\mathbb{P}}_{2,\cdot}$	0	0.4598	0	0	-0.0591	σ_{22}	0.0317
		(0.0687)			(0.0127)		(0.0022)
$K^{\mathbb{P}}_{3,\cdot}$	3.9147	2.0574	2.5917	0	-0.0442	σ_{33}	0.0320
	(0.0691)	(0.0704)	(0.0658)		(0.0153)		(0.0023)
$K^{\mathbb{P}}_{4,\cdot}$	0	0	0	1.6322	0.0166	σ_{44}	0.0732
				(0.0651)	(0.0145)		(0.0044)

Table 2: **Estimated Dynamic Parameters of the Preferred Chilean AFNS-L Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the preferred Chilean AFNS-L model according to the BIC. The estimated value of λ is 0.3954 (0.0096), while $\kappa^{\mathbb{Q}}_{liq} = 3.2517$ (0.0669), and $\theta^{\mathbb{Q}}_{liq} = 0.0096$ (0.0004). The maximum log likelihood value is 36,579.89. The numbers in parentheses are the estimated parameter standard deviations.

The estimated parameters of the preferred Brazilian specification are reported in Table 1.

For the Chilean data, the BIC is minimized by specification (10), which has a $K^{\mathbb{P}}$ matrix given by

$$K^{\mathbb{P}}_{CHL} = \begin{pmatrix} \kappa^{\mathbb{P}}_{11} & \kappa^{\mathbb{P}}_{12} & 0 & 0 \\ 0 & \kappa^{\mathbb{P}}_{22} & 0 & 0 \\ \kappa^{\mathbb{P}}_{31} & \kappa^{\mathbb{P}}_{32} & \kappa^{\mathbb{P}}_{33} & 0 \\ 0 & 0 & 0 & \kappa^{\mathbb{P}}_{44} \end{pmatrix}.$$

The estimated parameters of the preferred Chilean specification are reported in Table 2, which are similar to those reported in CCR.

For the Mexican bond data, the BIC is minimized by specification (12), which has a $K^{\mathbb{P}}$

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	1.5608	0	0	1.3082	0.0382	σ_{11}	0.0087
	(0.6121)			(0.9169)	(0.0129)		(0.0004)
$K^{\mathbb{P}}_{2,\cdot}$	0	0.3866	0	0	-0.0097	σ_{22}	0.0286
		(0.2865)			(0.0244)		(0.0027)
$K^{\mathbb{P}}_{3,\cdot}$	0	0	0.8490	0	-0.0058	σ_{33}	0.0401
			(0.5103)		(0.0173)		(0.0056)
$K^{\mathbb{P}}_{4,\cdot}$	0	0	0	0.1803	-0.0084	σ_{44}	0.0063
				(0.2435)	(0.0166)		(0.0021)

Table 3: **Estimated Dynamic Parameters of the Preferred Mexican AFNS-L Model**
The table shows the estimated parameters of the $K^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal Σ matrix for the preferred Mexican AFNS-L model according to the BIC. The estimated value of λ is 0.8673 (0.0446), while $\kappa^{\mathbb{Q}}_{liq} = 0.0493$ (0.0558), and $\theta^{\mathbb{Q}}_{liq} = 0.0400$ (0.0362). The maximum log likelihood value is 7,625.03. The numbers in parentheses are the estimated parameter standard deviations.

matrix given by

$$K^{\mathbb{P}}_{MEX} = \begin{pmatrix} \kappa^{\mathbb{P}}_{11} & 0 & 0 & \kappa^{\mathbb{P}}_{14} \\ 0 & \kappa^{\mathbb{P}}_{22} & 0 & 0 \\ 0 & 0 & \kappa^{\mathbb{P}}_{33} & 0 \\ 0 & 0 & 0 & \kappa^{\mathbb{P}}_{44} \end{pmatrix}.$$

The estimated parameters of the preferred Mexican specification are reported in Table 3.

These results provide a couple of notable takeaways. First, the outcomes from the model selection procedures show that the real yield curves in the three considered countries each operate in unique ways, as reflected in the different preferred mean-reversion $K^{\mathbb{P}}$ matrices. Second, thanks to the latent nature of our models, it is in general challenging to compare the estimated parameters. However, the λ parameter can be meaningfully interpreted across models as it determines the pace of decay in the loading of the slope factor and the maturity at which the factor loading of the curvature factor peaks. Hence, its relatively low value for the Brazilian and Chilean data implies that the models for these two countries put more emphasis on fitting long-term bond prices, whereas the Mexican model, with a value for λ that is more than twice the size of those in the two other models, puts relatively more emphasis on fitting short-term bond prices.

3.5 Model Fit

As a final exercise, we briefly examine the model fit. To that end, Table 4 evaluates the ability of our three preferred models to match the market prices of the coupon bonds in their respective sample. Note that the pricing errors are computed based on the implied yield on

Maturity bucket	Brazil			Chile			Mexico		
	Obs.	Mean	RMSE	Obs.	Mean	RMSE	Obs.	Mean	RMSE
0-2	227	-0.08	3.83	604	0.22	7.27	126	-0.14	3.02
2-4	429	0.67	7.30	1,149	0.85	7.31	250	0.52	5.86
4-6	333	1.34	5.29	906	-1.11	6.08	186	-0.08	5.59
6-8	195	-1.83	5.17	677	0.94	5.69	157	-0.10	6.07
8-10	192	0.53	6.50	674	0.04	5.36	136	0.29	6.14
10-12	81	1.20	4.61	339	-0.67	5.61	53	-0.02	5.77
12-14	42	0.68	3.98	309	0.76	4.95	46	1.23	6.75
14-16	43	-2.09	7.89	334	1.14	4.25	48	-0.53	6.53
16-18	56	2.94	6.15	385	0.15	3.46	42	-0.86	2.99
18-20	40	2.72	8.81	405	0.28	4.32	48	1.68	4.35
20-22	18	-0.79	2.67	139	-0.47	6.03	54	1.21	3.39
22-24	32	-0.89	2.19	144	-0.04	4.17	59	1.05	3.37
24-26	28	3.12	4.51	144	-0.05	2.51	72	-1.29	4.23
26-28	46	1.63	3.24	157	0.04	3.49	65	1.06	4.00
28<	369	0.64	3.79	180	1.07	4.70	119	1.04	3.79
All bonds	2,131	0.53	5.55	6,546	0.22	5.79	1,461	0.29	5.16

Table 4: **Summary Statistics of Fitted Errors of Indexed Government Bond Yields**
This table reports the number of observations, the mean pricing errors (Mean), and the root mean-squared pricing errors (RMSE) for the preferred AFNS-L model for each country. The pricing errors are reported in basis points and computed as the difference between the observed implied yield on each coupon bond and its model-implied yield.

each coupon bond to make these errors comparable across securities.[13] Table 4 reports the summary statistics for the fit to all bonds in each sample broken down into maturity buckets. The number of observations in each maturity bucket in each market is also reported.

Overall, the fit is really tight with very low root mean-squared errors for all bonds combined in all three models. This is comforting as it suggests that there is no overlooked risk component buried in the unexplained residuals. Furthermore, the fit is very balanced across maturities in all three models. Hence, there is no part of the yield curve that the models are not able to fit, which is very comforting as well.

As for the individual samples, we note that the Brazilian data are relatively sparse in between the 10- and 30-year maturity buckets. As alluded to above in discussing the λ-values, this does not prevent the model from putting a lot of weight on fitting bond prices in that segment of the yield curve via its low value for λ, as also indicated by the relatively modest fitted errors for the associated maturity buckets.

As for the Chilean data, we stress their good coverage across the entire maturity range from the very short end all the way up to the 30-year point. Furthermore, as suggested by the low value of λ in our Chilean model, it also puts more weight on fitting the long-term

[13]The implied yields calculated from the observed bond prices are identical to those shown in Figure 1.

bond prices. This explains its small fitted errors for maturities from 10 years and above.

4 The Inflation-Indexed Bond Liquidity Premium

In this section, we analyze the inflation-indexed bond-specific liquidity premia implied by the estimated preferred AFNS-L models described in the previous section. First, we formally define the bond-specific liquidity risk premia before we study their historical evolution.

4.1 The Estimated Inflation-Indexed Bond Liquidity Premium

We now use the estimated AFNS-L models to extract the liquidity premium in each inflation-indexed bond market. To compute this premium, we first use the estimated parameters and the filtered states $\{X_{t|t}\}_{t=1}^{T}$ to calculate the fitted inflation-indexed bond prices $\left\{\hat{P}_t^i\right\}_{t=1}^{T}$ for all outstanding securities in our sample for the market at hand. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ by solving the fixed-point problem

$$\begin{aligned}\hat{P}_t^i &= C(t_1 - t)\exp\left\{-(t_1 - t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n} \frac{C}{2}\exp\left\{-(t_k - t)\hat{y}_t^{c,i}\right\} \\ &\quad + \exp\left\{-(T - t)\hat{y}_t^{c,i}\right\},\end{aligned} \tag{5}$$

for $i = 1, 2, ..., n_t$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^{T}$ is approximately the real rate of return on the ith bond if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields with correction for the liquidity risk, we compute a new set of model-implied bond prices from the estimated AFNS-L model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^{liq} = 0$ for all t as well as $\sigma_{44} = 0$ and $\theta_{liq}^{Q} = 0$. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^{T}$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using equation (5). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the ith bond is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{6}$$

Figure 2 shows the average inflation-indexed bond liquidity premium $\bar{\Psi}_t$ across the outstanding bonds at each point in time for the three countries in our sample. As one could expect of a risk premium, the three series are all positive with notable variation over time. Furthermore, we note that they are non-negligible in size, with a mean of 144 basis points, 115 basis points, and 60 basis points for Brazil, Chile, and Mexico, respectively. Thus, liquidity premia represent a notable component in all these real yields as anticipated by the structural arguments laid out in Cardozo and Christensen (2023). For comparison, that study reports

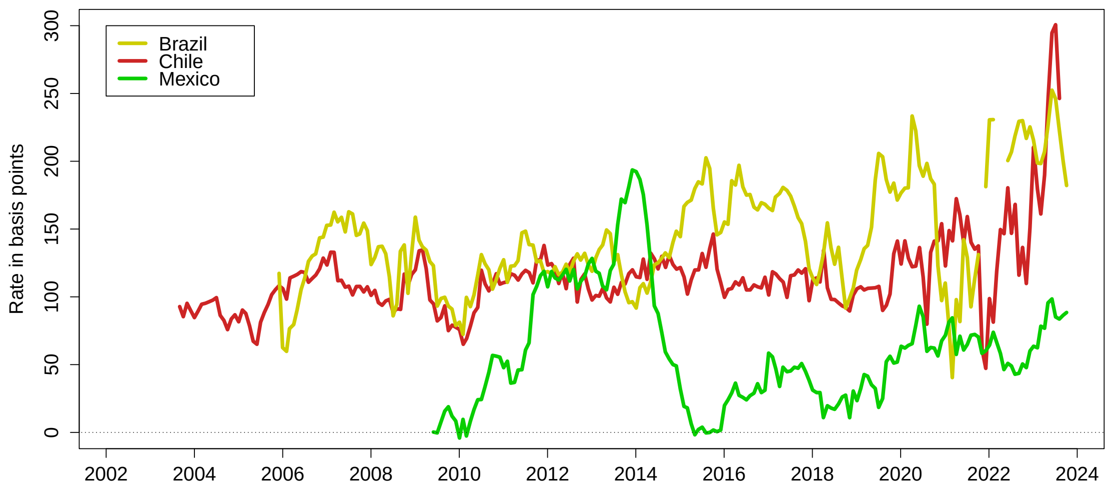


Figure 2: **Average Estimated Inflation-Indexed Bond Liquidity Premia**
Illustration of the average estimated inflation-indexed bond liquidity premium for each observation date implied by the preferred AFNS-L model for Brazil, Chile, and Mexico. The inflation-indexed bond liquidity premia are measured as the estimated yield difference between the fitted yield to maturity of individual inflation-indexed bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off.

estimated liquidity premia for Colombian inflation-indexed bonds, so-called bonos UVR, that average 225 basis points with a standard deviation of 32 basis points during the 2005-2020 period. Overall, we take these results to imply that our estimated liquidity premia for the inflation-indexed bonds are of reasonable size and fall within the range of estimates reported for other comparable markets of inflation-indexed bonds.

5 Market-Based Estimates of the Natural Real Rate

In this section, we first introduce our market-based definition of the natural real rate before we use our preferred AFNS-L models to account for liquidity and term premia in the inflation-indexed bond prices and obtain expected real short rates and the associated measure of the equilibrium real rate for the three countries in our sample.

5.1 Definition of the Natural Real Rate

Our working definition of the equilibrium real interest rate r_t^* is

$$r_t^* = \frac{1}{5}\int_{t+5}^{t+10} E_t^{\mathbb{P}}[r_s^R]ds, \tag{7}$$

that is, the average expected real short rate over a five-year period starting five years ahead where the expectation is with respect to the objective $\mathbb{P}$-probability measure. As explained in CR, this 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks. Alternatively, r_t^* could be defined as the expected real short rate at an infinite horizon. However, this quantity will depend crucially on whether the factor dynamics exhibit a unit root. As is well known, the typical spans of time series data that are available do not distinguish strongly between highly persistent stationary processes and nonstationary ones. Our model follows the finance literature and adopts the former structure, so strictly speaking, our infinite-horizon steady-state expected real rate is constant. However, we do not view our data samples as having sufficient information in the 10-year to infinite horizon range to definitively pin down that steady state, so we prefer our definition with a medium- to long-run horizon.

5.2 Estimates of the Natural Real Rate

Our market-based measure of the natural real rate is the average expected real short rate over a five-year period starting five years ahead. This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks and well positioned to capture the persistent trends in the natural real rate.

To illustrate the decomposition underlying our definition of r_t^*, recall that the real term premium is defined as

$$TP_t(\tau) = y_t(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s]ds. \tag{8}$$

That is, the real term premium is the difference in expected real returns between a buy-and-hold strategy for a τ-year real bond and an instantaneous rollover strategy at the risk-free real rate r_t. We stress that we use the frictionless real yield in calculating the term premia. Figure 3 shows the AFNS-L model decomposition of the 5yr5yr forward real yield into its underlying components for all three countries. In each case, the 5yr5yr fitted real yield without any adjustments is shown with a solid black line.[14] The 5yr5yr frictionless real yield, which is the fitted real yield from the AFNS-L model, is shown with a solid gray line. The difference between the 5yr5yr forward real yield and the 5yr5yr frictionless real yield represents a measure of the liquidity premium frictions at the 5yr5yr maturity point and is highlighted with yellow shading. Note that this synthetic constant-maturity measure of liquidity premia is different from the bond-specific measures shown in Figure 2 as the latter is affected by heterogeneity in coupon sizes and remaining time to maturity across the bonds in our samples. Finally, thanks to its theoretical consistency, the AFNS-L model provides a decomposition

[14] These fitted real yields are obtained by estimating the standard AFNS model described in Section 3.1 on each sample. They are our equivalent to raw observed real yields without any adjustments.

of the 5yr5yr frictionless real yield into its real short rate expectations component and the residual real term premium based on equation (8). In each chart, the solid green line is the 5yr5yr forward real term premium, while the solid blue line is the expectations component, which is identical to our definition of r_t^*.

First and importantly, we note the sizable inflation-indexed bond liquidity premia that drive a large wedge between the observed 5yr5yr real yield shown with a solid black line and the lower 5yr5yr frictionless real yield shown with a solid gray line in all three countries. Thus, without the liquidity premium adjustment, one might be led to believe that real yields are much higher than what is actually the case, a point also made by Andreasen and Christensen (2016) in the context of U.S. TIPS.

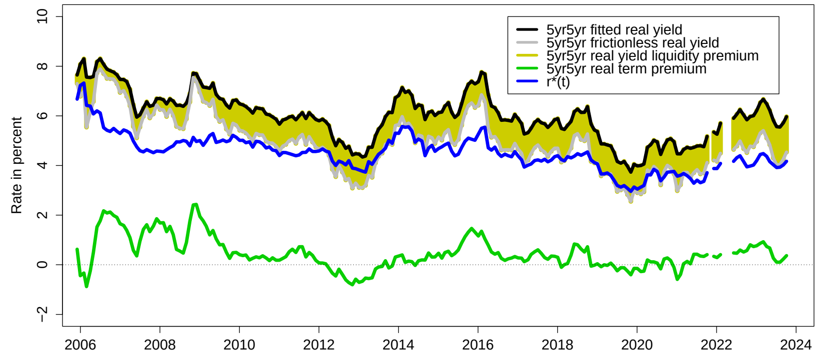
For Brazil, the model decomposition produces an estimate of r_t^* that fluctuates significantly with the business cycle around an overall lower trend. It starts out close to 7 percent in early 2006 but then persistently falls to reach a level below 4 percent in the months before the COVID-19 pandemic in spring 2020.[15] Following the economic reopening after the pandemic, our estimate of r_t^* suggests that the natural real rate in Brazil has trended back up to reach a level close to 4 and a half percent by the end of our sample. In contrast, the Brazilian 5yr5yr real term premium is relatively stable except for some sharp gyrations in the first few years of our sample. Lastly, the upward trend in the average estimated bond-specific liquidity premium in Figure 2 is also evident in the implied Brazilian 5yr5yr real liquidity premium.

For Chile, we note that, although volatile, the 5yr5yr forward real term premium has fluctuated around a fairly stable level since the early 2000s. Although theory suggests that this premium is countercyclical and elevated during economic recessions, our Chilean estimate only partially aligns with these characteristics. In contrast, the Chilean estimate of the natural rate of interest implied by the AFNS-L model—the blue line—shows a gradual decline from around 0.5 percent in the early 2000s to below -2 percent by mid-2022. Importantly, it has remained low since then despite the recent large increases in bond yields. By the end of our sample, the Chilean estimate of r_t^* stands at -1.30 percent.

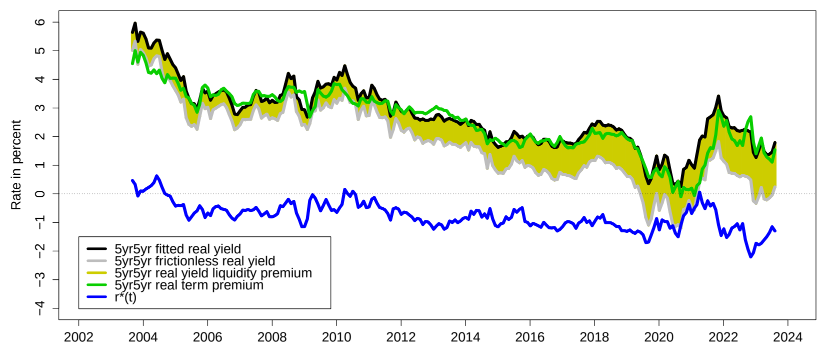
In the case of Mexico, our r_t^* estimate shows a fairly constant and acyclical pattern for the natural real rate. As a consequence, the sizable fluctuations in Mexican 5yr5yr frictionless real yields, that is, after accounting for changes in the liquidity premia, are due to changes in the Mexican 5yr5yr real term premium, which frequently switches sign, unlike its Chilean counterpart, which remains positive throughout our sample.

Next, we provide a direct comparison of our three estimates of the natural real rate, which are shown in Figure 4. The comparison reveals a stark and fundamental difference between these three important Latin American economies. The large and varying r_t^* in Brazil points to

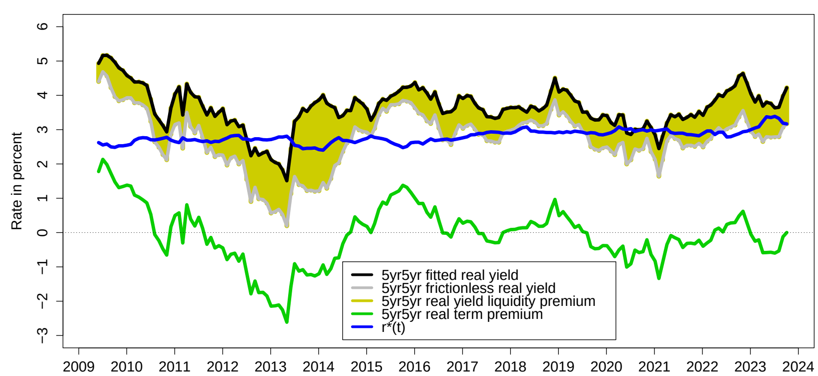
[15] Perrelli and Roache (2014) report estimates of the natural real rate for Brazil based on a structural macro model. Although uncertain, they report an average r_t^* estimate of 7.2 percent for the 2005-2008 period, while their average estimate for the 2010-2013 period is 5.7 percent.



(a) Brazil



(b) Chile



(c) Mexico

Figure 3: **5yr5yr Real Yield Decompositions**

somewhat unstable fundamental economic conditions in that country. This contrasts sharply with the results for Mexico, which suggest that fundamental economic conditions are more stable there. However, the elevated level of r_t^* in Mexico with a value close to 3 percent

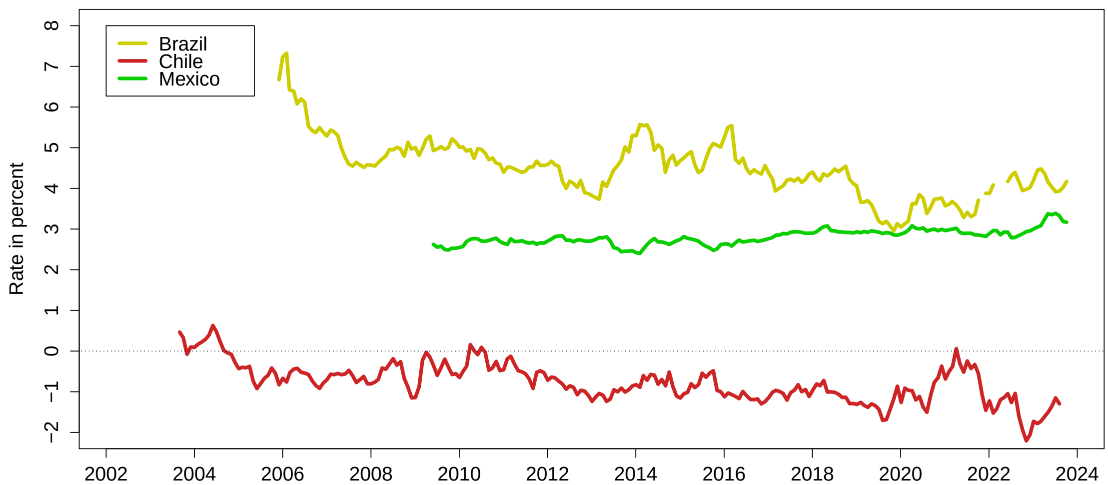


Figure 4: **Comparison of Market-Based Estimates of r***

still hints at some economic fragility relative to advanced economies, where r_t^* is generally lower; see Holston et al. (2017). Against that background and given that Chile is the most advanced major economy in Latin America, it is not all that surprising to see that the Chilean natural real rate is much lower and, with few exceptions, has remained negative for almost 20 years. Furthermore, we note that CCR find that their Chilean r_t^* estimate, which is similar to ours, follows broadly similar trends to a macro-based estimate of r_t^* for Chile using the approach of Holston et al. (2017). They also find their r_t^* estimate to be similar to a Colombian market-based estimate calculated from an update of the analysis in Cardozo and Christensen (2023). We add that the r_t^* estimate from our Mexican preferred AFNS-L model is very close to the market-based estimate reported by Beauregard et al. (2023), although they use a combination of Mexican nominal bond prices and prices of Mexican inflation-indexed bonds for their analysis. Thus, we consider our r_t^* estimates to be of reasonable size and representative of the estimates reported in the literature for our sample of countries.

Finally and interestingly, we note that the increases in interest rate levels and the associated tightening of financial conditions in response to the spell of high inflation following the economic reopening after the COVID-19 pandemic appear to have left only relatively minor prints on our r_t^* estimates. This suggests that interest rates in these three major Latin American countries are likely to settle back down at lower levels once inflation has been brought back to the target set by their respective central bank. Moreover, we add that the estimated factor dynamics of our preferred AFNS-L models could be leveraged to make projections of r_t^* into the future, as done in CCR. However, although such projections could be of tremendous importance to both fiscal and monetary policy, as well as long-term asset management, we leave that task for future work.

On a practical note, we stress that these estimates come with significant uncertainty. Unfortunately, there is no standard way of calculating their confidence bands. However, CR use simulations to examine the role of parameter uncertainty for their r_t^* estimates and find its impact to be quite sizable. To that, one must then add the unquantified effects of model and measurement uncertainty. Thus, the general recommendation would be to avoid overemphasizing any individual point estimate and instead focus on the broader time series trends similar to our description above.

5.3 Stance of Monetary Policy

In this section, we use our r_t^* estimates as an input to produce a measure of the stance of monetary policy in the three economies under analysis. This also offers an alternative way to validate our approach to estimating r_t^*.

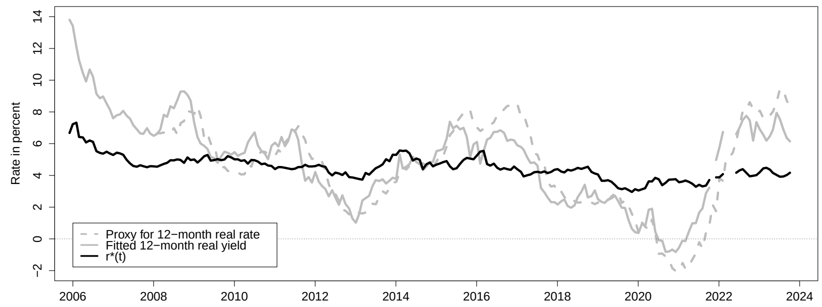
As noted in the introduction, estimates of the natural real rate serve a key role in monetary policy rules as the indicator above which the short-term real rate r_t would be considered restrictive, while the ones below it would be associated with an accommodative stance of monetary policy. Thus, the stance of monetary policy can be measured by the difference between the two as follows

$$\zeta_t = r_t - r_t^*.$$

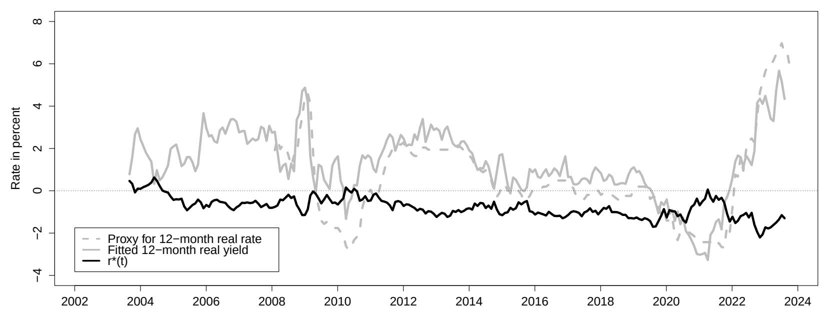
Importantly, though, thanks to the forward looking behavior of firms, their workers, and investors in general, it is not the current real short rate that matters for a wide variety of economic decisions, but rather the one likely to prevail over coming quarters. To quantify the stance of monetary policy, we therefore consider two proxies for r_t. One is a measure taken from Werner (2023) that exploits the Fischer equation by discounting the annualized nominal policy rate with the 12-month expected inflation, while the other is the fitted 12-month real yield from our bond data without any adjustments, as in Christensen and Mouabbi (2024).[16] These measures of r_t and the associated estimate of r_t^* are shown in Figure 5 for each of the three countries in our sample.

Some common traits are notable. First, monetary policy reached an accommodative stance relatively early during the pandemic period in all three countries. Furthermore, by the end of our sample, monetary policy appears to be highly restrictive by historical standards. This underscores the point made by Werner (2023) that, as inflation gradually declines towards the official inflation target in all three economies, the overnight policy rates need to be lowered

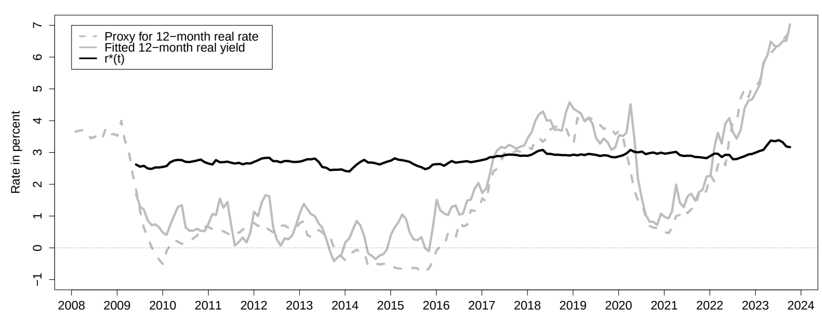
[16]The fitted real yields are obtained by estimating the standard AFNS model using the sample of inflation-indexed bond prices for each country.



(a) Brazil



(b) Chile



(c) Mexico

Figure 5: **Measures of Short-Term Real Yields and r***

in tandem to prevent the stance of monetary policy from becoming overly restrictive.

Finally, despite the large and outsized gyrations in short-term real yields in all three economies since 2020, our estimates of the natural real rate r_t^* have varied remarkably little, and by the end of our samples they are all close to where they were in early 2020. This suggests that neither the pandemic period nor the unexpected spike in inflation following the economic reopening and the associated tightening of monetary policy has left any meaningful marks on the steady-state real interest rate in these three countries. Given that they are representative of both Latin America specifically and emerging market economies more broadly, our findings may extend well beyond the sample of countries examined here. However, we leave it for future research to explore that conjecture.

6 Conclusion

In the existing literature, most estimates of the steady-state level of the short-term real interest rate are based on *macroeconomic* models and data. However, uncertainty about the correct macroeconomic specification, in particular during the COVID-19 pandemic shock, raises questions about the resulting macro-based estimates of the natural real rate. In this paper, we avoid this issue by focusing on a finance-based measure of the equilibrium real rate that is based on empirical dynamic term structure models estimated solely on the prices of inflation-indexed bonds. By adjusting for both inflation-indexed bond liquidity premia and real term premia, we uncover investors' expectations for the underlying frictionless real short rate for the five-year period starting five years ahead. Moreover, our study is unique in that we apply this approach to three major Latin American economies, namely Brazil, Chile, and Mexico.

First, we document large and time-varying liquidity risk premia in all three bond markets with few commonalities. Hence, our study provides empirical support for the arguments laid out in Cardozo and Christensen (2023), which entail that inflation-indexed bond holdings should be concentrated among domestic patient buy-and-hold investors such as pension funds. In turn, inflation-indexed bonds should be less traded than standard nominal bonds and face significant liquidity risks consistent with our findings. Importantly, this logic also implies that all three indexed bond markets are likely to be dominated by domestic investors. As a consequence, the bond price information should mainly reflect domestic investors' real rate expectations and risk appetites—a key assumption underlying our choice to analyze each country separately.

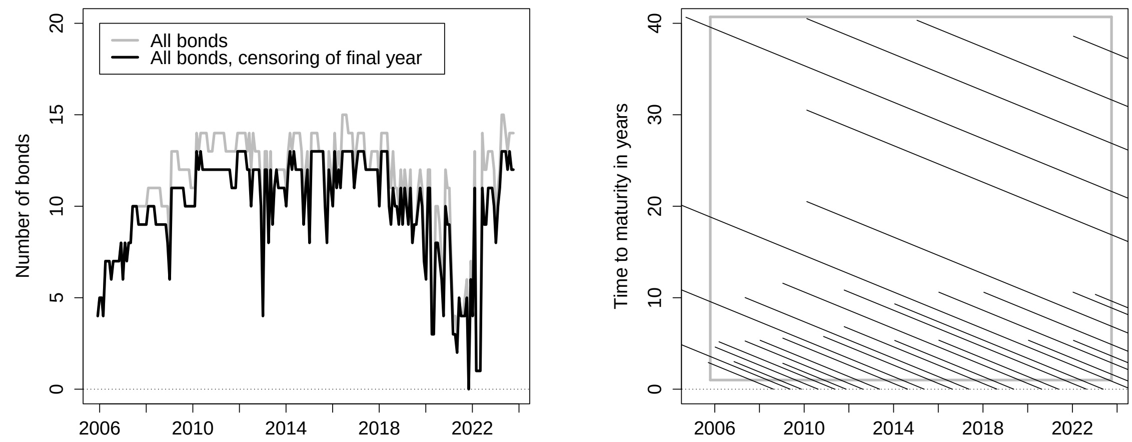
In a second step, we then proceed to adjust the liquidity-adjusted *frictionless* real yields for the embedded real term premia to produce our estimates of the natural real rate of interest for each country. Consistent with the intuition above, our r_t^* estimates indeed appear to be unique to each country and quite disperse. Our estimate for Brazil is high on average with large cyclical fluctuations. In contrast, our Mexican estimate is very stable, although elevated by international standards. Finally, our r_t^* estimate for Chile resembles those reported for advanced economies, say, as in CR by being low and characterized by a persistent gradual decline the past 20 years. The large dispersion in the estimates of the natural real rate across these three economies would seem to be consistent with the notable differences in their economic fundamentals and general level of economic development. However, we leave it for future research to parse out how exactly the documented differences in our r_t^* estimates correlate with economic fundamentals.

The documented dispersion in r_t^* is also likely to have quite different implications for the conduct of monetary policy in these three countries. However, to that end, the notable uncertainty surrounding these estimates should be kept in mind, as also emphasized in Holston et al. (2017).

Furthermore, we stress that our market-based approach to estimating r_t^* can be applied to both advanced and emerging economies. However, we again leave it for future research to verify whether our findings extend beyond the three major Latin American economies examined here.

Finally, given that our measures of the natural real rate of interest are based on the forward-looking information priced into the active inflation-indexed bond markets in these three countries and can be updated at daily frequency, they could serve as an important input for real-time monetary policy analysis. However, we also leave it for future research to explore such applications.

A Appendix: Brazilian Inflation-Indexed Bond Data



(a) Number of inflation-indexed bonds

(b) Distribution of inflation-indexed bonds

Figure A.1: **Overview of the Brazilian Inflation-Indexed Bond Data**
Panel (a) reports the number of outstanding Brazilian inflation-indexed bonds at a given point in time. Panel (b) shows the maturity distribution of all Brazilian inflation-indexed bonds issued since 2006. The solid gray rectangle indicates the sample used in our analysis, where the sample is restricted to start on November 30, 2005, and limited to inflation-indexed bond prices with more than one year to maturity after issuance.

Due to its experience with long spells of extremely elevated inflation, Brazil has a long history of issuing inflation-indexed bonds as a key instrument to fund government debt. These bonds are known as Notas do Tesouro Nacional Serie B, or NTN-B, and their coupon and principal payments are indexed to IPCA with a lag of 15 days. IPCA is the main Brazilian consumer price index and the one targeted by the Central Bank of Brazil in setting monetary policy. Furthermore, as noted by Kubudi and Vicente (2016), the market for these bonds is the fifth largest inflation-indexed bond market in the world and hence quite sizable.

The total number of outstanding Brazilian inflation-indexed bonds over time in our sample is shown as a solid gray line in Figure 1(a). At the end of our sample period—which runs from November 2003 to September 2023—14 bonds were outstanding. However, as noted by Gürkaynak et al. (2010) and ACR, prices of inflation-indexed bonds near their maturity tend to be somewhat erratic because of the indexation lag in their payoffs. Therefore, to facilitate model estimation, we drop inflation-indexed bonds from our sample when they have less than one year to maturity. Using this cutoff, the number of bonds in the sample is modestly reduced as shown with a solid black line in Figure 1(a). Finally, Table A.1 contains the contractual details of all 31 NTN-Bs in our data, as well as the number of monthly observations for each. One unique feature about this market is that all bonds have the same annual coupon rate of 6 percent, which is paid on a semiannual basis just like in the Chilean and Mexican bond

Inflation-indexed bond	Issuance Date	No. obs.
(1) 6% 5/15/2009	9/15/2003	29
(2) 6% 8/15/2024	10/15/2003	193
(3) 6% 5/15/2015	10/15/2003	101
(4) 6% 5/15/2045	9/15/2004	171
(5) 6% 8/15/2008	9/15/2005	19
(6) 6% 8/15/2010	1/5/2006	41
(7) 6% 5/15/2011	3/9/2006	50
(8) 6% 11/15/2009	11/9/2006	24
(9) 6% 5/15/2017	5/9/2007	107
(10) 6% 8/15/2012	5/9/2007	51
(11) 6% 5/15/2013	1/9/2008	52
(12) 6% 8/15/2020	1/14/2009	126
(13) 6% 11/15/2011	1/14/2009	22
(14) 6% 8/15/2014	1/14/2009	55
(15) 6% 8/15/2050	2/10/2010	117
(16) 6% 8/15/2030	2/10/2010	129
(17) 6% 8/15/2040	2/10/2010	116
(18) 6% 8/15/2016	11/10/2010	57
(19) 6% 8/15/2022	10/13/2011	114
(20) 6% 8/15/2018	10/13/2011	68
(21) 6% 5/15/2023	1/15/2014	88
(22) 6% 5/15/2019	1/15/2014	52
(23) 6% 5/15/2055	1/14/2015	85
(24) 6% 8/15/2026	1/6/2016	76
(25) 6% 5/15/2021	1/6/2016	52
(26) 6% 8/15/2028	1/10/2018	50
(27) 6% 5/15/2025	1/8/2020	29
(28) 6% 8/15/2032	1/5/2022	17
(29) 6% 8/15/2060	1/12/2022	21
(30) 6% 5/15/2027	1/12/2022	16
(31) 6% 5/15/2033	1/4/2023	6

Table A.1: **Sample of Brazilian Inflation-Indexed Bonds**
The table reports the characteristics, first issuance date, and the number of monthly observation dates for each bond during the sample period from November 30, 2005, to September 30, 2023.

markets.

Generally the Brazilian government has issued a variety of inflation-indexed bonds with original maturities ranging from 3 years to 41 years. The maturity distribution of the 31 bonds in our sample is shown in Figure 1(b). Each bond is represented by a single downward-sloping line that plots its remaining years to maturity for each date. For the five- to ten-year maturities of particular interest for our analysis, the available universe of bonds provides good coverage throughout our sample.

B Appendix: Model Selection Results

Alternative specifications	Goodness of fit statistics			
	$\log L$	k	p-value	BIC
(1) Unrestricted $K^{\mathbb{P}}$	11,419.88	89	n.a.	-22,361.77
(2) $\kappa_{24}^{\mathbb{P}} = 0$	11,419.79	88	0.67	-22,366.96
(3) $\kappa_{24}^{\mathbb{P}} = \kappa_{13}^{\mathbb{P}} = 0$	11,418.77	87	0.15	-22,370.29
(4) $\kappa_{24}^{\mathbb{P}} = \kappa_{13}^{\mathbb{P}} = \kappa_{14}^{\mathbb{P}} = 0$	11,418.76	86	0.89	-22,375.65
(5) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{34}^{\mathbb{P}} = 0$	11,418.71	85	0.75	-22,380.92
(6) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{43}^{\mathbb{P}} = 0$	11,418.57	84	0.60	-22,386.01
(7) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{12}^{\mathbb{P}} = 0$	11,418.27	83	0.44	-22,390.78
(8) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{21}^{\mathbb{P}} = 0$	11,417.15	82	0.13	-22,393.91
(9) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{41}^{\mathbb{P}} = 0$	11,413.55	81	< 0.01	-22,392.08
(10) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{42}^{\mathbb{P}} = 0$	11,411.93	80	0.07	**-22,394.21**
(11) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{31}^{\mathbb{P}} = 0$	11,405.87	79	< 0.01	-22,387.46
(12) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{32}^{\mathbb{P}} = 0$	11,403.93	78	0.05	-22,388.95
(13) $\kappa_{24}^{\mathbb{P}} = \ldots = \kappa_{23}^{\mathbb{P}} = 0$	11,391.67	77	< 0.01	-22,369.80

Table B.1: **Evaluation of Alternative Specifications of the Brazilian AFNS-L Model**
There are 13 alternative estimated specifications of the Brazilian AFNS-L model. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Bayesian information criterion (BIC). The period analyzed covers monthly data from November 30, 2005, to September 30, 2023.

Alternative specifications	Goodness of fit statistics			
	$\log L$	k	p-value	BIC
(1) Unrestricted $K^{\mathbb{P}}$	36,592.28	171	n.a.	-72,247.37
(2) $\kappa_{23}^{\mathbb{P}} = 0$	36,592.10	170	0.55	-72,252.49
(3) $\kappa_{23}^{\mathbb{P}} = \kappa_{14}^{\mathbb{P}} = 0$	36,592.09	169	0.89	-72,257.95
(4) $\kappa_{23}^{\mathbb{P}} = \kappa_{14}^{\mathbb{P}} = \kappa_{13}^{\mathbb{P}} = 0$	36,590.90	168	0.12	-72,261.05
(5) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{34}^{\mathbb{P}} = 0$	36,589.66	167	0.12	-72,264.05
(6) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{42}^{\mathbb{P}} = 0$	36,589.54	166	0.62	-72,269.29
(7) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{24}^{\mathbb{P}} = 0$	36,584.91	165	< 0.01	-72,265.51
(8) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{21}^{\mathbb{P}} = 0$	36,582.49	164	0.03	-72,266.16
(9) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{43}^{\mathbb{P}} = 0$	36,581.54	163	0.17	-72,269.74
(10) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{41}^{\mathbb{P}} = 0$	36,579.89	162	0.07	**-72,271.92**
(11) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{12}^{\mathbb{P}} = 0$	36,570.83	161	< 0.01	-72,259.28
(12) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{32}^{\mathbb{P}} = 0$	36,564.36	160	< 0.01	-72,251.82
(13) $\kappa_{23}^{\mathbb{P}} = \ldots = \kappa_{31}^{\mathbb{P}} = 0$	36,559.13	159	< 0.01	-72,246.84

Table B.2: **Evaluation of Alternative Specifications of the Chilean AFNS-L Model**
There are 13 alternative estimated specifications of the Chilean AFNS-L model. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Bayesian information criterion (BIC). The period analyzed covers monthly data from August 31, 2003, to July 31, 2023.

Alternative specifications	Goodness of fit statistics			
	$\log L$	k	p-value	BIC
(1) Unrestricted $K^{\mathbb{P}}$	7,633.29	67	n.a.	-14,921.31
(2) $\kappa_{13}^{\mathbb{P}} = 0$	7,633.28	66	0.87	-14,926.43
(3) $\kappa_{13}^{\mathbb{P}} = \kappa_{21}^{\mathbb{P}} = 0$	7,633.24	65	0.79	-14,931.52
(4) $\kappa_{13}^{\mathbb{P}} = \kappa_{21}^{\mathbb{P}} = \kappa_{24}^{\mathbb{P}} = 0$	7,633.22	64	0.84	-14,936.63
(5) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{12}^{\mathbb{P}} = 0$	7,633.02	63	0.53	-14,941.39
(6) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{43}^{\mathbb{P}} = 0$	7,632.00	62	0.15	-14,944.50
(7) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{23}^{\mathbb{P}} = 0$	7,631.36	61	0.26	-14,948.37
(8) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{42}^{\mathbb{P}} = 0$	7,629.45	60	0.05	-14,949.71
(9) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{32}^{\mathbb{P}} = 0$	7,628.63	59	0.20	-14,953.21
(10) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{31}^{\mathbb{P}} = 0$	7,628.01	58	0.27	-14,957.13
(11) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{34}^{\mathbb{P}} = 0$	7,627.30	57	0.23	-14,960.86
(12) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{41}^{\mathbb{P}} = 0$	7,625.03	56	0.03	**-14,961.47**
(13) $\kappa_{13}^{\mathbb{P}} = \ldots = \kappa_{14}^{\mathbb{P}} = 0$	7,621.58	55	< 0.01	-14,959.73

Table B.3: **Evaluation of Alternative Specifications of the Mexican AFNS-L Model** There are 13 alternative estimated specifications of the Mexican AFNS-L model. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Bayesian information criterion (BIC). The period analyzed covers monthly data from May 31, 2009, to September 29, 2023.

References

Andreasen, Martin M. and Jens H. E. Christensen, 2016, "TIPS Liquidity and the Outlook for Inflation," *FRBSF Economic Letter* 2016-35, 1-5.

Andreasen, Martin M., Jens H. E. Christensen, and Simon Riddell, 2021, "The TIPS Liquidity Premium," *Review of Finance*, Vol. 25, No. 6, 1639-1675.

Andreasen, Martin M., Jens H. E. Christensen, and Glenn D. Rudebusch, 2019, "Term Structure Analysis with Big Data: One-Step Estimation Using Bond Prices," *Journal of Econometrics*, Vol. 212, 26-46.

Arida, Persio, Edmar L. Bacha, and André Lara-Resende, 2005, "Credit, Interest, and Jurisdictional Uncertainty: Conjectures on the Case of Brazil," in Inflation Targeting, Debt, and the Brazilian Experience, 1999 to 2003, edited by F. Giavazzi, I. Goldfajn, and S. Herrera. Cambridge, MA: MIT Press, 265-293 (Chapter 8).

Bacha, Edmar L., Márcio Holland, and Fernando M. Gonçalves, 2009, "A Panel-Data Analysis of Interest Rates and Dollarization in Brazil," *Revista Brasileira de Economia*, Vol. 63, No. 4, 341–360.

Barbosa, Fernando de Holanda, 2006, "The Contagion Effect of Public Debt on Monetary Policy: The Brazilian Experience," *Brazilian Journal of Political Economy*, Vol. 26, No. 2, 231-238.

Beauregard, Remy, Jens H. E. Christensen, Eric Fischer, and Simon Zhu, 2023, "Inflation Expectations and Risk Premia in Emerging Bond Markets: Evidence from Mexico," forthcoming *Journal of International Economics.*

Blake, Andrew P., Gareth R. Rule, and Ole J. Rummel, 2015, "Inflation Targeting and Term Premia Estimates for Latin America," *Latin American Economic Review*, Vol. 24, No. 3.

Cardozo, Cristhian H. R. and Jens H. E. Christensen, 2023, "The Benefit of Inflation-Indexed Debt: Evidence from an Emerging Bond Market," Working Paper 2023-04, Federal Reserve Bank of San Francisco.

Ceballos, Luis, Jens H. E. Christensen, and Damian Romero, 2024, "A Post-Pandemic New Normal for Interest Rates in Emerging Bond Markets? Evidence from Chile," *mimeo.*

Ceballos, Luis, Alberto Naudon, and Damian Romero, 2016, "Nominal Term Structure and Term Premia: Evidence from Chile," *Applied Economics*, Vol. 48, No. 29, 2721-2735.

Christensen, Jens H. E., Francis X. Diebold, and Glenn D. Rudebusch, 2011, "The Affine Arbitrage-Free Class of Nelson-Siegel Term Structure Models," *Journal of Econometrics*, Vol. 164, No. 1, 4-20.

Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, 2014, "Do Central Bank Liquidity Facilities Affect Interbank Lending Rates?," *Journal of Business and Economic Statistics*, Vol. 32, No. 1, 136-151.

Christensen, Jens H. E. and Sarah Mouabbi, 2024, "The Natural Rate of Interest in the Euro Area: Evidence from Inflation-Indexed Bonds," *mimeo*.

Christensen, Jens H. E. and Glenn D. Rudebusch, 2019, "A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt," *Review of Economics and Statistics*, Vol. 101, No. 5, 933-949.

Clarida, Richard, 2014, "Navigating the New Neutral," *Economic Outlook,* PIMCO, November.

Dai, Qiang and Kenneth J. Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, Vol. 55, No. 5, 1943-1978.

De Pooter, Michiel, Patrice Robitaille, Ian Walker, and Michael Zdinak, 2014, "Are Long-Term Inflation Expectations Well Anchored in Brazil, Chile, and Mexico?," *International Journal of Central Banking*, Vol. 10, No. 2, 337-400.

Duffee, Gregory R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, Vol. 57, No. 1, 405-443.

Eggertsson, Gauti B., Neil R. Mehrotra, Sanjay R. Singh, and Lawrence H. Summers, 2016, "A Contagious Malady? Open Economy Discussions of Secular Stagnation," *IMF Economic Review*, Vol. 64, No. 4, 581-634.

Finlay, Richard and Sebastian Wende, 2012, "Estimating Inflation Expectations with a Limited Number of Inflation-Indexed Bonds," *International Journal of Central Banking*, Vol. 8, No. 2, 111-142.

Gonçalves, Fernando M., Márcio Holland, and A. D. Spacov, 2007, "Can Jurisdictional Uncertainty and Capital Controls Explain the High Level of Real Interest Rates in Brazil? Evidence from Panel Data," *Revista Brasileira de Economia*, Vol. 61, No. 1, 49-75.

Grishchenko, Olesya V. and Jing-Zhi Huang, 2013, "Inflation Risk Premium: Evidence from the TIPS Market," *Journal of Fixed Income*, Vol. 22, No. 4, 5-30.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2010, "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, Vol. 2, No. 1, 70-92.

Hamilton, James D., Ethan S. Harris, Jan Hatzius, and Kenneth D. West, 2016, "The Equilibrium Real Funds Rate: Past, Present, and Future," *IMF Economic Review*, Vol. 64, No. 4, 660-707.

Holston, Kathryn, Thomas Laubach, and John C. Williams, 2017, "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, Vol. 108, 559-575.

Kim, Don H. and Kenneth J. Singleton, 2012, "Term Structure Models and the Zero Bound: An Empirical Investigation of Japanese Yields," *Journal of Econometrics*, Vol. 170, No. 1, 32-49.

Kubudi, Daniela and Jose Vicente, 2016, "A Joint Model of Nominal and Real Yield Curves," Banco Central do Brasil Working Paper Series No. 452.

Nelson, Charles R. and Andrew F. Siegel, 1987, "Parsimonious Modeling of Yield Curves," *Journal of Business*, Vol. 60, No. 4, 473-489.

Orphanides, Athanasios, and John C. Williams, 2002, "Robust Monetary Policy Rules with Unknown Natural Rates," *Brookings Papers on Economic Activity* 2002(2), 63-145.

Perrelli, Roberto and Shaun K. Roache, 2014, "Time-Varying Neutral Interest Rate—The Case of Brazil," IMF Working Paper 14/84.

Rudebusch, Glenn D., 2001, "Is the Fed Too Timid? Monetary Policy in an Uncertain World," *Review of Economics and Statistics*, Vol. 83, No. 2, 203-217.

Sack, Brian and Robert Elsasser, 2004, "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Federal Reserve Bank of New York Economic Policy Review*, Vol. 10, No. 1, 47-63.

Segura-Ubiergo, Alex, 2012, "The Puzzle of Brazil's High Interest Rates," IMF Working Paper 12/62.

Werner, Alejandro, 2023, "Monetary Policy in Latin America: The Easing Cycle Has Begun," Policy Brief 23-16, Peterson Institute for International Economics.