FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

Understanding Persistent ZLB: Theory and Assessment

Pablo Cuba-Borda
Board of Governors of the Federal Reserve System

Sanjay R. Singh
Federal Reserve Bank of San Francisco

December 2023

Working Paper 2024-03

https://www.frbsf.org/economic-research/publications/working-papers/2024/03/

Suggested citation:

Cuba-Borda, Pablo, Sanjay R. Singh. 2023 “Understanding Persistent ZLB: Theory and Assessment,” Federal Reserve Bank of San Francisco Working Paper 2024-03. https://doi.org/10.24148/wp2024-03

Understanding Persistent ZLB: Theory and Assessment

Pablo Cuba-Borda Sanjay R. Singh*

December 2023

Abstract

We develop a theoretical framework that rationalizes two hypotheses of long-lasting low-interest rate episodes: *deflationary-expectations-traps* and *secular stagnation* in a unified setting. These hypotheses differ in the sign of the theoretical correlation between inflation and output growth that they imply. Using the data from Japan over 1998:Q1-2019:Q4, we find that the data favor the expectations-trap hypothesis. The superior model fit of the expectations trap relies on its ability to generate the observed negative correlation between inflation and output growth.

Keywords: Expectations-driven trap, secular stagnation, zero lower bound.

JEL Classification: E31, E32, E52.

*Cuba-Borda: Division of International Finance, Federal Reserve Board, Washington DC (pablo.a.cubaborda@frb.gov). Singh: Federal Reserve Bank of San Francisco (sanjay.singh@sf.frb.org) and Department of Economics, University of California Davis (sjrsingh@ucdavis.edu). We thank the editor Giorgio Primiceri, three anonymous referees, Borağan Aruoba, Florin Bilbiie, Carlos Carvalho, Gauti Eggertsson, Jean-Paul L'Huillier, Pascal Michaillat, Jean-Baptiste Michau, Taisuke Nakata, Giovanni Nicolò, Sebastian Schmidt, Kevin Sheedy, Johannes Wieland, and participants at various presentations for helpful comments and suggestions. We are particularly grateful to Yasuo Hirose for generously sharing data and codes from his paper. This paper replaces a previous version circulated with the title "Understanding Persistent Stagnation".

"I believe that for the euro area there is some risk of *Japanification*, but it is by no means a foregone conclusion." — Mario Draghi (January, 2020).[1]

Since the global financial crisis of 2008-2009, concerns about prolonged near-zero interest rates and meager inflation became predominant across many advanced economies, most notably in Europe and the United States. Such concerns, dubbed as *Japanification*,[2] relate to the decades-long stagnation of the Asian economy following the collapse of a real-estate bubble in the early 1990s. As a result, nominal interest declined to zero, and deflation emerged, leaving the central bank unable to fight recessions.

Two predominant hypotheses rationalize interest rates near zero and inflation below the central bank's target. The first hypothesis is that of a *deflationary-expectations-driven* liquidity trap whereby deflationary expectations become self-fulfilling in the presence of the zero lower bound (ZLB) constraint on short-term nominal interest rates (Benhabib, Schmitt-Grohé and Uribe, 2001, 2002). The second hypothesis is *secular stagnation* that entails a persistently negative natural interest rate constraining the central bank at the ZLB (Hansen, 1939; Summers, 2013; Eggertsson, Mehrotra and Robbins, 2019). Existing research has shown that these two hypotheses offer contrasting policy implications (Bilbiie, 2022; Nakata and Schmidt, 2022).

This paper builds a theoretical framework that rationalizes *deflationary-expectations-driven* liquidity trap and *secular stagnation* in a unified setting. We analytically show that the empirical correlation of output growth and inflation can be used to distinguish the two hypotheses. Secular stagnation, by policy rates permanently stuck at zero, generates a positive correlation between inflation and output growth. On the other hand, because of local indeterminacy, deflationary expectation-trap models can generate a negative correlation between inflation and output growth. In Japan, the correlation between inflation and output growth is negative. We find that the superior model fit of the expectations-trap model relies on its ability to generate this negative correlation.

We augment the textbook New Keynesian model with a bonds-in-utility specification (Michaillat and Saez, 2021; Michau, 2018; Ono and Yamada, 2018). This modification breaks the tight connection between the natural interest rate and the discount factor, thus allowing

[1]ASSA Annual Meeting Panel Session: *"Japanification, Secular Stagnation, and Fiscal and Monetary Policy Challenges,"* January 2020

[2]Financial Times, *"Japanification: investors fear malaise is spreading globally,"* August 26, 2019.

for a permanently negative natural interest rate. Within this framework, we define three steady-state equilibria. A *targeted-inflation* steady state at which the central bank can meet its inflation target, output is at potential, and the nominal interest rate is positive. In addition, there are two liquidity trap steady states at which inflation is below the central bank's intended target. The level of output is below potential, and the nominal interest rate is at the ZLB. The focus of our analysis is to compare these two liquidity traps.

The liquidity trap steady states arise due to the ZLB constraint on short-term nominal interest rates. In one case, combined with long-run money non-neutrality, a shift in the agent's inflation expectations makes the liquidity trap equilibrium self-fulfilling. For this reason, we label it deflationary-expectations-driven liquidity trap or *expectations trap* for short.[3] Alternatively, when prices are rigid, the economy can settle in a liquidity trap because of a permanent decline in the natural interest rate. In this case, the liquidity trap arises because of a change in the economy's fundamentals and not because of a shift in expectations. For this reason, we label this situation as a *secular stagnation* steady state or *fundamentals-driven* liquidity trap. In the absence of discounting, the natural interest rate is constant and equal to the inverse of the household's discount factor, and the model cannot accommodate the *secular stagnation* hypothesis.

We use the Japanese experience from 1998 to 2019 as a laboratory to contrast the two hypotheses and offer the first quantitative assessment of expectation-driven liquidity traps versus secular stagnation. We use a New Keynesian framework to assess if a policymaker can use the data to discern the predominant hypothesis. Using Bayesian prediction pools, we estimate the probabilistic assessment of the relevant model (Geweke and Amissano, 2011; Del Negro, Hasengawa and Schorfheide, 2016). Our quantitative analysis offers two main findings. First, we find evidence that Japan is more likely to be in an expectations-driven liquidity trap. Second, considerable real-time uncertainty exists between secular stagnation and the expectations-driven trap models, especially during Japan's first decade near the ZLB.

We find that equilibrium indeterminacy is central to tilting our quantitative assessment in favor of the expectations-trap hypothesis. This result emerges because the dynamic properties

[3] Benigno and Fornaro (2018), and Ravn and Sterk (2021) present complementary models of expectations trap where pessimism about growth rate or unemployment risk can push the economy to the liquidity trap. In this paper, our focus is solely on the deflationary expectations trap.

of the ZLB equilibrium differ across the two narratives. Under secular stagnation, the ZLB equilibrium exhibits locally determinate dynamics. In contrast, the expectation traps model features locally indeterminate dynamics around the ZLB steady state. Thus, the equilibrium dynamics are consistent with a multiplicity of stable paths. Because our quantitative analysis focuses on a long-lasting ZLB episode, equilibrium selection implies restrictions for the response of output growth and inflation to structural disturbances. Our full-information approach ensures that the data selects the best-fitting equilibrium. At the same time, our Bayesian procedure intrinsically penalizes the likelihood function for the presence of additional parameters in the expectations-trap equilibria (Schwarz, 1978; Lubik and Schorfheide, 2004).

What accounts for the better fit of the expectations-trap hypothesis in Japan? The negative correlation between output growth and inflation in Japanese data is a key empirical moment for equilibrium selection and model fit. The equilibrium dynamics around the secular stagnation steady state cannot deliver the observed negative correlation. With interest rates pegged at the ZLB, any shock that generates a persistent increase in the inflation rate lowers the real interest rate and increases consumption; therefore, output and inflation positively co-move. In contrast, local indeterminacy of the expectations-trap steady state implies that inflation can adjust in any direction. Our estimation procedure allows the data to pin down this response. Expectation traps can generate an unconditional correlation between inflation and output growth close to that observed in the data.

We further investigate our empirical results along three dimensions. First, we investigate the importance of non-fundamental i.i.d. shocks—known as sunspots—that emerge due to indeterminate model dynamics in the expectations-trap model. Our benchmark result indexes equilibrium multiplicity through the correlation between fundamental and sunspot shocks using the method of Bianchi and Nicolò (2021). We find that restricting the correlation between price-markup and sunspot shocks to non-positive values worsens the fit of the expectations-trap model and favors secular stagnation. Thus, using data to discipline equilibrium selection is central to our results. We augment our estimation of the expectations-trap model with inflation expectations data to further discipline equilibrium selection. The prediction pool analysis still favors the estimated expectations-trap model over secular stagnation. Second, we check whether departures from rational expectations modeling enhance the ability of the secular stagnation model to fit the data. We find that of the three departures we considered, cognitive discounting (Gabaix, 2020) helps improve the model fit for the secular stagnation model. However, it still falls short of the model fit implied by the expectations-trap hypothesis. Third, we verify that the expectations-trap hypothesis outperforms secular stagnation in an estimated medium-scale new Keynesian model. The correlation of markup shocks and sunspot shocks remains central to the model fit despite added complexity. This exercise implies that our analytical insights carry over to a wide class of models commonly used for policy analysis.

Relation to the literature. Our work complements the recent analyses of Michaillat and Saez (2021), Michau (2018), and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique secular stagnation scenario. Relative to this literature, our paper considers the two narratives of persistent ZLB and offers quantitative and analytical insights.

This paper is also related to the work by Mertens and Ravn (2014), Aruoba, Cuba-Borda and Schorfheide (2018), Bilbiie (2022), and Nakata and Schmidt (2022), who contrast expectations-driven and fundamental-driven liquidity traps using the standard Euler equation without discounting. Their setup can only accommodate a short-lived fundamentals-driven liquidity trap, while our modified Euler equation allows the possibility of secular stagnation as a competing hypothesis. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), which analyzes the case of permanent expectations-driven liquidity traps. Coyle and Nakata (2019) characterizes optimal inflation target in the presence of expectations-liquidity traps. We build on these papers to show that policies that impose a lower bound on inflation preclude the expectations-driven traps. Benigno and Fornaro (2018)'s stagnation trap, which focuses on the role of pessimism about the economy's growth rate, is complementary to the inflation pessimism we study in this paper.

Our framework allows agents in the model to expect ZLB episodes of permanent duration under both hypotheses. This feature stands in contrast to models that use transitory declines in the natural interest rate to generate ZLB episodes where agents' expectations have to be consistent with recovery to the full-employment steady state in the medium run (Bianchi and Melosi, 2017; Nakata, 2017; Nakata and Schmidt, 2019).

We use the principal-agent decision framework of Del Negro et al. (2016) to identify the relevant hypothesis in Japan, combining the predictive densities derived from observed time series. The Bayesian nature of our approach allows us to measure the uncertainty about the contrasting hypotheses with a structural model. Our prediction pool analysis is related to Lansing (2021) in which a model with endogenous regime switching generates data from a time-varying mixture of two models. In this paper, we construct a time-varying probability on the predictive densities of two alternative models. Our paper also relates to Mertens and Williams (2021) that uses the implications of changes in the natural interest rate on the distribution of interest rates and inflation in the options data from the U.S. financial markets, to discern between fundamentals- and expectations-driven liquidity. We find that the sign of inflation and output growth correlation can help distinguish between the hypotheses in a long-lasting liquidity trap.

Our analysis also builds on the important work by Hirose (2020) and Iiboshi, Shintani and Ueda (2022). Hirose (2020) estimates a model of deflationary-expectations-trap for Japan comparing equilibrium dynamics between the full-employment steady state before 1999 and the dynamics around the expectations-trap steady post-1999 in Japan. Iiboshi et al. (2022) uses non-linear methods to estimate a model in which the economy moves from the full-employment steady state into a temporary liquidity trap driven only by economic fundamentals. Relative to these estimated models for Japan, we tease out a testable implication to distinguish fundamental-driven from deflationary-expectations traps using the sign of an observed correlation of output growth and inflation. We show that this correlation allows us to separate the two hypotheses in Japanese data and is crucial for model fit.

A common theme of the papers that study expectation-driven liquidity traps is that policy implications may be the opposite of the ones derived from fundamentals-driven liquidity traps. As a result, it becomes central to assess what hypothesis is dominant in the data and develop policies that can always be stabilizing.[4] Our analysis is thus related to the design of robust policies such as fiscal policy rules that prevent the decline of real marginal costs (Schmidt, 2016), or fiscal stabilization policies that eliminate expectation-traps (Nakata and Schmidt, 2022). Similarly, research and development (R&D) subsidies advocated by Benigno

[4]One can develop expectations-traps equilibria with similar comparative statics as the fundamentals-driven liquidity traps, see Eggertsson et al. (2019, Figure 6 A). Our analysis does not focus on those, as they may not generate policy dilemmas.

and Fornaro (2018) that affect aggregate supply in an endogenous growth environment can eliminate expectations-driven liquidity traps.

Layout. Section I analytically teases testable predictions from both hypotheses. Section II presents our estimation results. In Section III, we investigate the role of equilibrium selection. Section IV extends our analysis to an estimated medium-scale DSGE environment. Section V concludes. All proofs and additional results are in the online appendix.

I. Stylized new Keynesian model

We begin with a stylized new Keynesian setup that entertains both the deflationary-expectations-driven liquidity trap and a permanent fundamentals-driven liquidity trap, labeled as *secular stagnation*. The model is a standard new Keynesian model augmented with a bonds-in-utility specification. The production side features monopolistically competitive firms facing nominal price rigidity. We use this setup to tease out a contrasting empirical implication of secular stagnation and deflationary expectation traps.

The central result is that secular stagnation generates a positive correlation between output growth and inflation. In contrast, the deflationary expectations trap can generate a negative correlation. The local indeterminacy of the expectations-trap equilibrium is key to obtaining a negative correlation.

A. Household

Time is discrete, and a representative agent maximizes the following lifetime utility:

$$\max_{\{C_t,h_t,B_{t+1}\}} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\log\left(C_t\right) - \frac{\omega}{1+\frac{1}{\eta}} h_t^{1+\frac{1}{\eta}} + \frac{\delta_t}{Z_t}\frac{B_{t+1}}{P_t} \right]$$

where C_t is consumption, h_t is hours supplied to work, η is the Frisch elasticity of labor supply, ω is a constant set to normalize the steady state hours equal to one, B_{t+1} is the stock of one-period nominal risk-free government bonds, $\delta_t \geq 0$ regulates the marginal utility from holding the risk-free bonds supplied by the government, and Z_t is non-stationary level of total factor productivity (TFP) introduced in the utility function to get a stationary balanced growth path. Since our objective is to allow the possibility of a permanently negative natural rate, we adopt this modeling of bonds-in-utility following Michaillat and Saez (2021).[5]

The household earns nominal wage income $W_t h_t$, interest income on past bond holdings of risk-free government bonds B_t at gross nominal interest rate R_{t-1}, dividends D_t from firms' ownership, and receives transfers T_t from the government. The period-by-period budget constraint faced by the household is given by $P_t C_t + \frac{B_{t+1}}{R_t} = W_t h_t + B_t + D_t + T_t$. An interior solution to household optimization yields the consumption Euler equation

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{R_t}{\Pi_{t+1}} \right] + \delta_t \left(\frac{C_t}{Z_t} \right). \tag{1}$$

where Π_t denotes the gross inflation rate.

B. Production

A perfectly competitive final-good-producing firm combines a continuum of intermediate goods indexed by $j \in [0,1]$ using the CES Dixit-Stiglitz technology: $Y_t = \left(\int_0^1 Y_t(j)^{1-\nu_t} dj \right)^{\frac{1}{1-\nu_t}}$, where $1/\nu_t > 1$ is the time-varying elasticity of substitution across varieties, and $\frac{1}{1-\nu}$ is the steady-state price markup. The shock ν_t evolves according to: $\log(\nu_t) = (1-\rho_\nu)\log(\nu) + \rho_\nu \log(\nu_{t-1}) + \epsilon_{\nu,t}$, where ϵ_ν are *iid* $N(0, \sigma_\nu)$ innovations.

The price of the final good is given by $P_t = \left(\int_0^1 P_t(j)^{\frac{\nu_t - 1}{\nu_t}} dj \right)^{\frac{\nu_t}{\nu_t - 1}}$ and profit maximization gives the demand for intermediate good j as a function of good j's price relative to the final good price level: $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t} Y_t$.

Intermediate good j is produced by a monopolist with a linear production technology: $Y_t(j) = Z_t H_t(j)$, where the growth rate of the aggregate TFP, $G_{z,t}$, follows: $\log(G_{z,t}) \equiv \log Z_t - \log Z_{t-1} = (1-\rho_z)\log(G_z) + \rho_z \log(G_{z,t-1}) + \epsilon_{z,t}$ and G_z denotes the steady state growth rate and $\epsilon_{z,t}$ are *iid* $N(0, \sigma_z)$ innovations.

Intermediate goods producers buy differentiated labor services $H_t(j)$, at a nominal price of W_t, and face quadratic adjustment costs when setting prices. These adjustment costs, expressed as a fraction of total output, are defined by the function $\Phi_t \equiv \frac{\phi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi^* \right)^2 Y_t$. In a

[5] As $\delta \to 0$, this equation nests the textbook Euler equation as a special case. We use the parameter δ to target empirical estimates of the natural interest rate in Japan. The calibration will depend on the particular hypothesis, and we describe our strategy later in this section.

symmetric equilibrium, profit maximization yields the following price Phillips curve relation:[6]

$$(1-\nu_t) - \omega h_t^{1/\eta}\frac{C_t}{Z_t} + \nu_t \Phi'(\Pi_t)\Pi_t = \nu_t \beta \mathbb{E}_t \left[Q_{t+1|t} \Pi_{t+1} \Phi'(\Pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] \tag{2}$$

C. Government and resource constraint

The desired policy rate is set according to the following rule $\tilde{R}_t = \left[r\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \right]$. Here, r is the steady-state real interest rate, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate, and Π^* is the target inflation rate, which in equilibrium coincides with the steady-state inflation rate. The actual policy rate relevant to agents' decisions is subject to the zero lower bound constraint:

$$R_t = \max\left\{1, \tilde{R}_t\right\} \tag{3}$$

The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by $P_t G_t + B_t = T_t + \frac{B_{t+1}}{R_t}$, where $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ captures autonomous sources of aggregate demand, including government expenditure. The shock g_t evolves according to: $\log(g_t) = (1-\rho_g)\log(g) + \rho_g \log(g_{t-1}) + \epsilon_{g,t}$, where ϵ_g are *iid* $N(0,\sigma_g)$ innovations.

We assume that the price adjustment costs are rebated back to the household in a lump-sum fashion as part of the government transfers. The labor market clears: $h_t = \int_0^1 H_t(j)dj$, and the nominal bonds are in net zero supply $B_t = 0$. The market-clearing resource constraint is:

$$C_t + G_t = Y_t. \tag{4}$$

D. Steady-State Equilibrium and Calibration

The competitive equilibrium in the non-linear model is given by the sequence of four endogenous processes $\{Y_t, C_t, R_t, \Pi_t\}$ that satisfy the conditions 1, 2, 3, and 4 for a given exogenous sequence of processes $\{\delta_t, G_{z,t}, \nu_t, g_t\}_{t=0}^{\infty}$ and an initial level of aggregate TFP Z_0.

We define the steady-state natural interest rate as the real rate that prevails without nominal rigidities. The natural rate r is given by $\frac{1-\delta}{\beta}$. The natural rate (net) is negative if and only if

[6] We are substituting the aggregate labor market clearing condition: $h_t = \int_0^1 H_t(j)dj$ in this derivation.

$\delta < 1 - \beta$. It is non-negative otherwise. When the natural rate is positive, a unique positive interest rate steady state exists with output at potential and inflation at the central bank's target.[7] We label this steady state as the *targeted* steady state.

Depending on the parameter values regulating the natural rate and the nominal rigidities, this framework admits two kinds of ZLB steady states. One is a liquidity trap steady state (à la Schmitt-Grohé and Uribe, 2017) that co-exists with the targeted steady state when the natural rate is positive and prices are not rigid enough. This steady state was discovered in the seminal work by Benhabib et al. (2001), and we label it as the *deflationary-expectations trap*. The main identifying feature of such a steady-state equilibria is that the dynamics in its neighborhood are locally indeterminate. It features inflation below the central bank's target and output below potential. Pessimistic deflationary expectations can push the economy to this steady state without any change in fundamentals.

The second type of steady state with a binding zero lower bound emerges due to the presence of discounting in the consumption Euler equation and depends on the fundamentals driving the natural rate of interest. When the steady state natural rate is negative and nominal rigidities are severe enough, the economy can be permanently at zero nominal rates with below-target inflation and output below potential. We define the *secular stagnation steady state* as the steady state featuring a negative natural rate and zero nominal interest rate. The steady secular stagnation state exhibits locally determinate equilibrium dynamics in its neighborhood. This local determinacy property is the main difference between the secular stagnation narrative and the deflationary expectations-driven narrative that we seek to separate empirically.[8]

In our baseline model with forward looking Phillips curve, we numerically evaluate the steady state equilibria. In Online Appendix E, we analytically prove the existence of the steady states of the model using a static Phillips curve. That analytical characterization shows that two parameters—δ (regulating the natural interest rate), and ϕ_p (regulating the slope of the Phillips curve)—determine the existence of the secular stagnation steady state and the deflationary expectations trap steady state.

[7] Potential output is defined as the level of output that would prevail under flexible prices.

[8] Note that multiple steady states at zero lower bound may coexist if the Phillips curve is sufficiently non-linear. Alternately, it may be possible to model the possibility of secular stagnation steady state coexisting with the full-employment steady state as in Eggertsson et al. (2019, Figure 6A) with a sufficiently high inflation target and a sufficiently non-linear Phillips curve.

Table 1: *Steady State Parameters*

Externally Calibrated (Common Parameters)

β Discount factor	η Inverse Frisch elasticity	$1/(1-\nu)$ Price markup	Π^* Inflation target	g Autonomous spending	G_z TFP growth rate
0.942	0.85	1.2	1.0025	1.81	0.25

Endogenously Calibrated (Model-specific Parameters)

	Secular stagnation		**Expectations-trap**	
Parameters	δ	ϕ	δ	ϕ
	Mg. utility bonds	Price adj. cost	Mg. utility bonds	Price adj. cost
	0.1132	4825	0.1088	2524
Targets	r^*	$\bar{\pi}$	r^*	$\bar{\pi}$
	Natural Rate (a.r.)	Steady state deflation (a.r.)	Natural Rate (a.r.)	Steady state deflation (a.r.)
	-1.1	-1.06	0	-1.06

Calibration. The top part of Table 1 summarizes the steady-state parameters that are common across models. We fix the discount factor β to 0.942 consistent with structural estimates of Galı and Gertler (1999). While this estimate is lower than the standard calibrated value of 0.99 in the literature, a low β is needed for the model to generate a positive natural interest rate in the presence of a positive bond premium. In studies that have estimated the discount rate using field and laboratory experiments, the estimates for β are dispersed but point to high discount rates. Michaillat and Saez (2021) choose an annual discount rate of 43% from the median value of these estimates in the experimental literature (Frederick, Loewenstein and O'Donoghue, 2002; Andersen, Harrison, Lau and Rutström, 2014).

We fix the Frisch labor supply elasticity at 0.85 following (Kuroda and Yamamoto, 2008). The elasticity of demand for intermediate goods $1/\nu$, is set to 6 to generate a steady-state markup of 20%. Japan did not officially adopt an inflation target until 2013:Q2, but the inflation rate averaged 1.1% in the two decades before entering the ZLB. Thus we assume the central bank was pursuing an inflation target of 1% and use that target rate as the reference value for price adjustment($\Pi^* = 1.0025$).[9] We determine the values of steady-state TFP growth rate G_z

[9]Our results are robust to choosing a zero inflation target as well.

such that the model matches the average output growth during the period 1998:Q1-2012:Q4. In the model, autonomous expenditure subsumes investment, net exports, and government spending. Consistent with this definition, we set g to match a consumption-output ratio of 55%.[10]

Two steady-state parameters, δ, and ϕ, are specific to each model. The bottom panel of Table 1 summarizes their calibration. These parameters match targets for Japan's natural interest rate and average inflation. We adopt two targets for the natural rate depending on the regime. Under secular stagnation, we choose an annual rate of -1.1%. This choice is based on two studies by Fujiwara, Iwasaki, Muto, Nishizaki and Sudo (2016) and Iiboshi et al. (2022) that separately estimate a series for the natural interest rate in Japan based on Laubach and Williams (2003). They find that the quarterly estimate was often -0.5% since the late 1990s and -2% at the lowest level. In contrast, we calibrate the expectations-trap steady state to imply an annualized long-run real interest rate of 0%. Our calibrated values of the natural interest rate imply a unique value for δ noted in Table 1. Using the Phillips curve equation 2, we calibrate ϕ to match the average inflation rate of -1.06% for both steady states, which is the average inflation rate in Japan over our estimation sample period.[11] Our calibration results in a somewhat larger value of the price adjustment parameter, ϕ, compared with econometric estimates of DSGE models for Japan (Iiboshi et al., 2022). Nonetheless, this value lies within the range of plausible estimates found in the literature—see Aruoba, Bocola and Schorfheide (2017). Finally, the implied output gap under both calibrations (not shown) is close to the estimates of 5% in Hausman and Wieland (2014).

E. Approximate Equilibrium

We approximate the equilibrium conditions (1–4) around secular stagnation and deflationary expectations trap steady states. We denote all liquidity-trap steady-state parameters by $\bar{x}$ and denote $\hat{x}_t$, the log deviations of stationary variables relative to the steady state. Online Appendix A provides the derivation of the following log-linearized equations that summarize

[10] As an alternative, it is straightforward to make g_t in the model track actual government spending in the data by defining consumption appropriately. Results are available upon request.

[11] Combined with zero nominal rates, this calibration targets the average real rate over the estimation sample.

the dynamics of consumption, inflation, output, and the nominal interest rate:

$$\hat{c}_t = \bar{\mathcal{D}}\mathbb{E}_t(\hat{c}_{t+1} - \hat{R}_t + \hat{\pi}_{t+1} + \hat{G}_{z,t+1}) \tag{5}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \bar{\varphi}\mathbb{E}_t\left(\hat{g}_{t+1} - \hat{g}_t\right) + \bar{\kappa}\left[\left(\frac{1}{\eta} + 1\right)\hat{y}_t - \hat{g}_t\right] + \bar{\lambda}\hat{\nu}_t \tag{6}$$

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \tag{7}$$

$$\hat{R}_t = 0 \tag{8}$$

The coefficients entering equations 5 and 6 are functions of the following structural parameters: $\bar{\mathcal{D}} = \frac{\beta}{\beta + \bar{\pi}G_z\delta\bar{c}}$; $\bar{\kappa} = \frac{(1-\nu)g_*\bar{c}\bar{y}^{1/\eta}}{\nu\phi(2\bar{\pi}-\pi_*)\bar{\pi}}$; $\bar{\varphi} = \frac{\bar{\pi}-\pi_*}{2\bar{\pi}-\pi_*}$; and $\bar{\lambda} = \frac{1-\phi(\bar{\pi}-\pi_*)\bar{\pi}(1-\beta)}{\phi(2\bar{\pi}-\pi_*)\bar{\pi}}$, where ϕ and δ correspond to the cost of price adjustment and the marginal utility of bonds, these are the only structural parameters specific to each model. The shocks in the approximate equilibrium are (i) government expenditure, $\hat{g}_t$, (ii) the growth rate of productivity, $\hat{G}_{z,t}$, and (iii) price markups, $\hat{\nu}_t$, each following an AR(1) process.[12]

F. Properties of Approximate Equilibrium

We briefly discuss some analytical properties of the approximate equilibrium. The main takeaway is that secular stagnation generates a positive correlation between inflation and output growth while expectations trap can generate a negative correlation.

We begin with establishing the necessary and sufficient condition for determinacy in the approximate equilibrium.

Proposition 1. (Local Determinacy). Assume $\beta < 1$. The system 5 - 8 is locally determinate if and only if $\frac{\bar{\pi}G_z\delta\bar{c}}{\beta} > \frac{1+\eta}{\eta(1-\beta)}\bar{\kappa}$.

The secular stagnation steady state is defined to be a steady state at zero nominal interest rates which exhibits local determinacy. In Proposition 1, we provide conditions under which the approximate equilibrium exhibits local determinacy. Obtaining local determinacy requires

[12] A caveat of the deflationary expectations trap hypothesis is that it may not generate significant decline in real rate endogenously as the economy transitions from full employment regime to the liquidity trap regime. In our model, TFP growth rate shocks help match the observed real interest rate dynamics in Japan. Growth traps studied in Benigno and Fornaro (2018) present a complementary mechanism that can generate endogenous decline in real rate. They also note that the possibility of a self-fulfilling expectations trap is more likely when multiple sources of pessimism (growth, deflation) are allowed in the same model. We leave the quantitative analysis of this generalized hypothesis to future research.

a sufficiently flat Phillips curve (low $\bar{\kappa}$), or high enough discounting (high δ).[13] In contrast, an expectations-driven liquidity trap has locally indeterminate dynamics, with low enough discounting or a sufficiently steep Phillips curve. Our steady-state calibrations for secular stagnation and expectations trap steady states to satisfy these restrictions.

Given the local determinacy, a unique solution under secular stagnation equilibrium can be derived. The solution is summarized in Proposition 2.

Proposition 2. (Unique Solution under Secular Stagnation). Let $\mathbb{X} = \{g, v, G_z\}'$ collect all fundamental state variables of the model, and let a and b be vectors of unknown coefficients. Assume the local determinacy condition in Proposition 1 is satisfied. The unique solution of the approximate equilibrium under secular stagnation is given by:

$$\hat{y}_t(\mathbb{X}) = a_1\hat{G}_{z,t} + a_2\hat{g}_t + a_3\hat{\nu}_t; \quad \hat{\pi}_t(\mathbb{X}) = b_1\hat{G}_{z,t} + b_2\hat{g}_t + b_3\hat{\nu}_t.$$

The coefficients (a_i, b_i) are reported in Online Appendix A.4.

Using this analytical solution for secular stagnation equilibrium, we establish in Proposition 3 that the correlation between inflation and output growth is positive.

Proposition 3. (Positive Correlation under Secular Stagnation). Consider the locally unique solution for the secular stagnation model shown in Proposition 2. The unconditional correlation between output growth and inflation is always positive.

The result follows from the fact that the steady-state aggregate demand relationship between inflation and output (derived from combining the Euler equation and the resource constraint) is upward-sloping under secular stagnation. Along with an upward-sloping Phillips curve, this steady-state equilibrium implies that price-markup shocks and TFP growth rate shocks only shift one schedule – either the Phillips curve or the Euler equation. These shifters unequivocally induce a positive correlation between inflation and output, given local determinacy. As long as the Phillips curve is sufficiently flat (low enough $\bar{\kappa}$ relative to other structural parameters), the government spending shock also induces a positive correlation between inflation and output, as

[13] Definition 1 in Michaillat and Saez (2021) impose a similar restriction for obtaining a permanent fundamentals-driven ZLB episode.

well as inflation and output growth. The required restriction on the slope of the Phillips curve follows from the local determinacy restriction on parameters discussed in Proposition 1.

On the other hand, the deflationary expectations trap steady state features local indeterminacy. This implies that extraneous innovations, known as sunspot shocks (ζ_t), that are not part of the original description of agents' optimization problems, can determine the equilibrium outcomes (Lubik and Schorfheide, 2004; Canova and Gambetti, 2010). To characterize the multiplicity of equilibrium, we apply the methods in Bianchi and Nicolò (2021). This method allows one to index equilibria in locally indeterminate linear models by explicitly specifying a correlation structure between the i.i.d. sunspot shocks and the fundamental shocks.

Using their insight, we demonstrate that the expectations-trap model is capable of generating a negative correlation between inflation and output growth. Following Bilbiie (2019), we consider the following static Phillips curve to illustrate this result:

$$\hat{\pi}_t = \tilde{\kappa}\hat{y}_t + \bar{\lambda}\hat{\nu}_t \tag{9}$$

where we have shut down government spending shocks. For exposition reasons, we directly postulate this static Phillips curve and relegate its derivation from first principles to Online Appendix E. With this static Phillips curve, we can analytically show how a negative correlation between inflation and output growth emerges in several of the infinite solutions to the expectations-trap model.

It is instructive to work through the steps in this analytical case. Since the Phillips curve is assumed to be static, and we model the price-markup shocks as the only fundamental shocks, the system of equations (5), (7), (8), and (9) simplifies to a univariate system:

$$\hat{\pi}_t = \Lambda E_t \hat{\pi}_{t+1} + \bar{\lambda}\hat{\nu}_t$$

where $\Lambda \equiv \bar{\mathcal{D}}\,(1+\tilde{\kappa})$. Because we are analyzing the system around a locally indeterminate steady state, note that $\Lambda > 1$. We define the one-step ahead forecast error associated with the

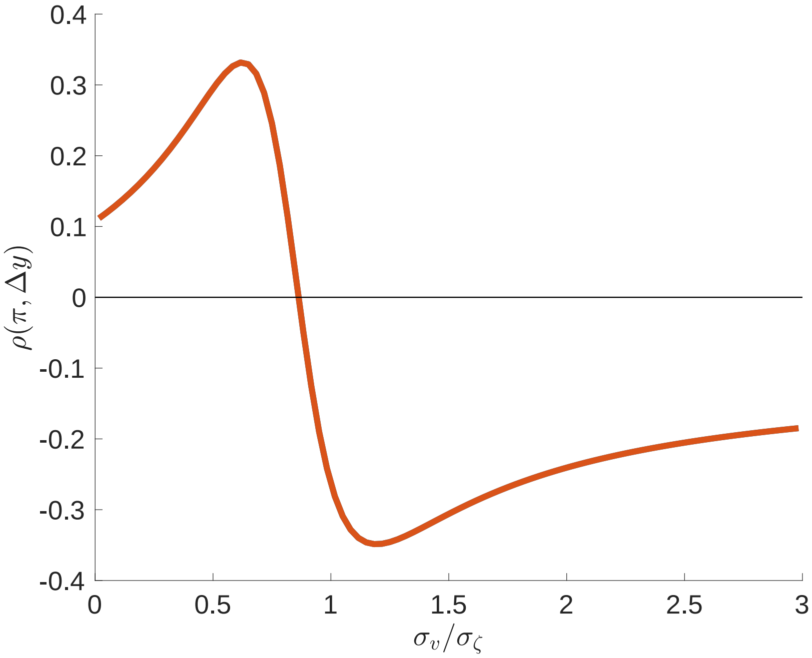

Figure 1: *Inflation-Output Growth Correlation under Expectations-trap Hypothesis*

Notes: Theoretical correlations for stylized model under the calibration provided in Table 1 with the following additional parameters: $\sigma_\zeta = 0.0030$, and $\rho(\epsilon_\nu, \zeta) = 0.96$. Other shock parameters are set to zero.

expectational variable $\hat{\pi}_t$, as:[14]

$$\zeta_t \equiv \hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t \tag{10}$$

Since $\Lambda > 1$, we combine this equation with equation (10), to get the solutions of the following form:

$$\hat{\pi}_t = \Lambda^{-1}\ \hat{\pi}_{t-1} - \Lambda^{-1}\ \bar{\lambda}\hat{\nu}_{t-1} + \zeta_t$$

$$\hat{y}_t = \tilde{\kappa}^{-1}\hat{\pi}_t - \bar{\lambda}\tilde{\kappa}^{-1}\hat{\nu}_t$$

There are multiple solutions for the evolution of output, each indexed by the sunspot shock, ζ_t. In Proposition 4, we show that this system can produce a negative correlation between inflation and output growth if the exogenous correlation between the sunspot and the fundamental shock is large enough.

Proposition 4. (Negative Correlation under Deflationary-Expectations Trap). Consider the

[14]To be consistent with the idea of deflationary trap, we impose the sunspot on inflation forecast error. Online Appendix A.6 extends the proof to the case of sunspots on the output forecast error.

equilibrium of the expectations-trap model described by Equations (5), (7), (8), and (9) with only the i.i.d price-markup $\hat{\nu}_t$ shocks as the only fundamental. Let $\zeta_t \equiv \hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t$, denote a mean zero sunspot shock with variance σ_ζ^2. Let $\rho_{\nu,\zeta}$ denote the correlation between the sunspot and the markup shock. The correlation of output growth and inflation is negative if and only if $1 > \rho_{\nu,\zeta} > \frac{\Lambda\sigma_\zeta^2 - \bar{\lambda}\sigma_\nu^2}{(\Lambda-1)\bar{\lambda}\sigma_\zeta\sigma_\nu}$.

This result of a negative correlation between inflation and output growth with a static Phillips curve also extends to our baseline calibrated model with a forward-looking Phillips curve (Equation 6). To illustrate, we compute theoretical moments from the expectations-trap model assuming a correlation between sunspot and markup shocks equal to $\rho(\epsilon_\nu, \epsilon_\zeta) = 0.96$. In the subsequent section, we estimate this correlation with Japanese data. Figure 1 plots the correlation between output growth and inflation against the relative standard deviation of markup and sunspot shocks. In this example, for values of $\frac{\sigma_\nu}{\sigma_\zeta} > 0.9$, the expectations trap model generates a negative correlation between inflation and output growth.

In the rest of the paper, we investigate whether this particular correlation matters for model fit in the case of twenty years of near-zero interest rates in Japan.

II. Model Evaluation During Japan's Liquidity Trap

We now present a quantitative analysis based on the small-scale New Keynesian model of Section I. We briefly discuss the estimation of remaining parameters and evaluate model fit using Bayesian methods.

A. Data and estimation

Conditional on our calibration of steady state parameters, we estimate the vector of the following parameters $\Theta = [\rho_g, \rho_z, \rho_\nu, \sigma_g, \sigma_z, \sigma_\nu]'$. For the expectations-trap model, in addition to the parameters listed in the vector Θ, we also estimate the standard deviation of the sunspot shock, σ_ζ, and the correlation between the structural and the sunspot shocks, denoted with $\rho(x, \zeta)$, for $x = \{\epsilon_z, \epsilon_g, \epsilon_\nu\}$. Because our model is linear we can construct the exact likelihood and use a standard Bayesian approach to estimate the parameters of the model. We obtain draws from the posterior distribution by a single-block random walk Metropolis–Hastings (RWMH) algorithm

(An and Schorfheide, 2007). Online Appendix C reports additional estimation details.

For parameter estimation, we use quarterly data on output growth, consumption growth, and GDP deflator-based inflation rate in Japan for the period 1998:Q1 – 2012:Q4.[15] We focus on this sample period for two reasons. First, from 1995 to 1998 the Bank of Japan (BOJ) held the monetary policy rate at 0.5%. We start our analysis in 1998 to parallel the assumption in our model that the economy starts at the ZLB and agents expect near-zero interest rates for a prolonged period. The BOJ lowered its policy rate to zero in the first quarter of 1999, and it remained between 0% and 0.5%. We consider the economy to be at the ZLB for the entire period. Second, in 2013, the BOJ introduced a new monetary policy program that included an explicit inflation target, asset and bond purchase programs as well as started considering negative nominal interest rates. None of these policies are explicitly modeled in our framework.

B. Estimation Results

Table 2 summarizes the estimated posterior distribution of parameters that fit the respective model to Japan's output growth, consumption growth, and inflation data. The marginal prior distributions for the estimated parameters are tabulated in Online Appendix C.3. The posterior estimates for the common parameters are remarkably similar across model specifications. For the expectations-trap model, the standard deviation of the sunspot shock is statistically different from zero and with a magnitude similar to that of the shock to the technology growth rate. The estimated correlation between the fundamental and the sunspot shocks varies substantially. The data favors a robust positive correlation between the price markup and the sunspot shocks and a small correlation between the sunspot shock with the other two fundamental shocks.

The last row in Table 2 shows that the log-data density favors the expectations-trap hypothesis in terms of overall fit. To gauge the difference in fit, we construct Bayes factors of expectations-traps relative to secular stagnation, $\mathcal{F} = p(Y^T|\mathcal{M}_b)/p(Y^T|\mathcal{M}_s)$. As a test statistic, we compute $2 \times \log(\mathcal{F})$ because it resembles the familiar likelihood-ratio test. In our estimation, we find that this test statistic is equal to 77, implying "very strong" evidence in favor of the expectations-trap hypothesis over secular stagnation according to standard criteria

[15] Our findings are robust to using data from 1998:Q1-2019:Q4 in estimation. We use the longer sample for our assessment of the mechanism in section III.

Table 2: *Stylized DSGE Model: Posterior Estimates*

Parameters	Description	$\mathcal{M}_b$: Exp. trap		$\mathcal{M}_s$: Sec. Stag.	
		Mean	[05 95]	Mean	[05 95]
ρ_g	Persistence gov. spending shock	0.91	[0.86 0.96]	0.88	[0.83 0.92]
ρ_ν	Persistence markup shock	0.18	[0.08 0.28]	0.17	[0.08 0.25]
ρ_z	Persistence tech. growth shock	0.50	[0.27 0.74]	0.49	[0.25 0.72]
$100 \times \sigma_g$	Std dev. gov. spending shock	0.92	[0.81 1.04]	0.94	[0.82 1.06]
$100 \times \sigma_\nu$	Std dev. markup shock	0.35	[0.30 0.40]	0.38	[0.32 0.44]
$100 \times \sigma_z$	Std dev. tech. growth shock	0.36	[0.17 0.54]	0.64	[0.36 0.92]
$100 \times \sigma_\zeta$	Std dev. sunspot shock	0.36	[0.31 0.41]	-	-
$\rho(\epsilon_z, \zeta)$	Corr. sunspot and tech. growth shocks	-0.11	[-0.26 0.03]	-	-
$\rho(\epsilon_\nu, \zeta)$	Corr. sunspot and markup shocks	0.98	[0.96 1.00]	-	-
$\rho(\epsilon_g, \zeta)$	Corr. sunspot and gov. spending shocks	0.04	[-0.00 0.08]	-	-
$\log\left[p\left(Y^T\right)\right]$	Log-data density	-415.42		-453.90	

Notes: The estimation sample is 1998:Q1 - 2012:Q4. We use $Y^T = [y_1, \ldots, y_T]$ to denote all the available data in our sample. For each model, we report posterior means and 90% highest posterior density intervals in square brackets. All posterior statistics are based on the last 50,000 draws from a RWMH algorithm, after discarding the first 50,000 draws.

(Kass and Raftery, 1995).[16]

In our application, the sunspot shock and the correlation parameters necessary to select equilibria in the expectations-trap model come at a cost from the perspective of the log-data density as a penalty for additional parameters.[17] Nonetheless, one may still be concerned that the expectations-trap model always "edges over" secular stagnation because of the multiplicity of equilibria or the presence of "free" parameters. To allay this concern, we conduct an exercise where we simulate data from the secular stagnation model using the parameters in Table 2. Then, we re-estimate both models on simulated data and conduct a model comparison. We find that $2 \times \log(\mathcal{F})$ is equal to -32, which indicates that when data comes from secular stagnation, our estimation procedure finds "very strong" evidence in its favor.

C. Expectation traps or secular stagnation?

We now compare the relative importance of the two competing hypotheses in explaining the persistent liquidity trap episode in Japan over time. We use Bayesian prediction pools, as in

[16]According to Kass and Raftery (1995), values of $2 \times \log(\mathcal{F})$ above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 positive evidence, while values below 2 are "not worth more than a bare mention."

[17]The log-data densities intrinsically penalize the likelihood function for the presence of additional parameters as in the Bayesian Information Criterion (Lubik and Schorfheide, 2004, Footnote 11).

Geweke and Amissano (2011) and Del Negro et al. (2016), that rely on predictive densities to construct recursive estimates of model weights. These time-varying model weights can be interpreted as a policymaker's views on the most relevant model using the information available in real-time.

We consider a policymaker that has access to the sequence of one-period-ahead predictive densities $p\left(y_t|y_{1:t-1},\mathcal{M}_s\right)$ under secular stagnation and $p\left(y_t|y_{1:t-1},\mathcal{M}_b\right)$ under the expectations-trap hypothesis.[18] We are interested in constructing an estimate of the model weight, λ, that pools the information of each individual model:

$$p\left(y_t|\lambda,\mathcal{P}\right)=\lambda p\left(y_t|y_{1:t-1},\mathcal{M}_b\right)+(1-\lambda)p\left(y_t|y_{1:t-1},\mathcal{M}_s\right),\quad 0\leq\lambda\leq 1 \tag{11}$$

where $p\left(y_t|\lambda,\mathcal{P}\right)$ is the predictive density obtained by pooling the two competing models for a given weight λ and pool $\mathcal{P}=\{\mathcal{M}_b,\mathcal{M}_s\}$. The policymaker is Bayesian and has a prior density $p(\lambda|\mathcal{P})$ of the weight assigned to each model in the pool. The posterior distribution of the model weights, $p(\lambda|\mathcal{I}_t^{\mathcal{P}},\mathcal{P})$, can be updated recursively conditional on the information available to the pool in the previous period $\mathcal{I}_{t-1}^{\mathcal{P}}$:

$$p(\lambda|\mathcal{I}_t^{\mathcal{P}},\mathcal{P})\propto p\left(y_t|\lambda,\mathcal{P}\right)p(\lambda|\mathcal{I}_{t-1}^{\mathcal{P}},\mathcal{P}) \tag{12}$$

We estimate the posterior distribution in Equation (12) recursively, starting in 1998:Q1. The estimated model weights are shown in Figure 2 together with 90% posterior credible sets to capture model and parameter uncertainty. The Japanese data imply roughly similar weights on both models in the early part of the sample and through the early 2000s. Afterward, the data lean in favor of the specification $\mathcal{M}_b$, indicating a better fit of the expectations-trap hypothesis. Uncertainty about the model weight's posterior distribution is substantial but decreases later in the sample as more information favoring the expectations-trap model accumulates. Starting in 2015, the data put at least 90% weight on the expectations-trap hypothesis as the best-fitting explanation.

[18] The predictive density is constructed sampling from the posterior distribution of the DSGE parameters of the baseline model of Section I and averaging the predictive densities across draws.

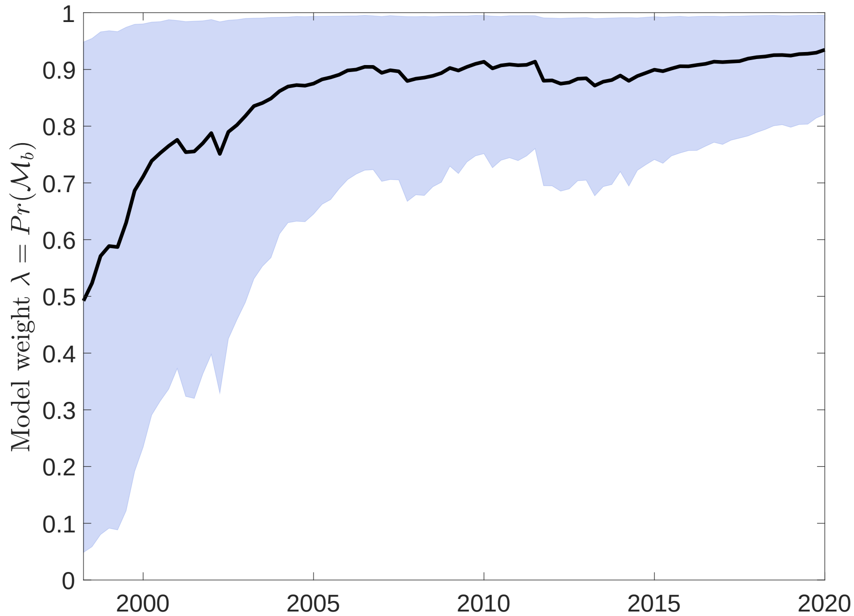

Figure 2: *Model Weights: Expectations Traps vs Secular Stagnation*

Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2019:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

III. Inspecting the Mechanism

A. Correlation between Inflation and Output Growth

We first discuss the data moments that favor the expectations-trap hypothesis in our estimation. We find that the superior model fit of the expectations trap is related to its ability to generate a negative unconditional correlation between inflation and output growth, as observed in the data.

The two columns in Figure 3 show the range of theoretical correlations between inflation and output growth implied by the posterior parameter distribution of the expectation traps and secular stagnation model respectively. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively. The red dots in the figure represent the same moments in the Japanese data used in estimation.

The expectations-trap model generates a negative unconditional correlation between inflation and output growth consistent with the data. In contrast, as shown analytically in Proposition 3,

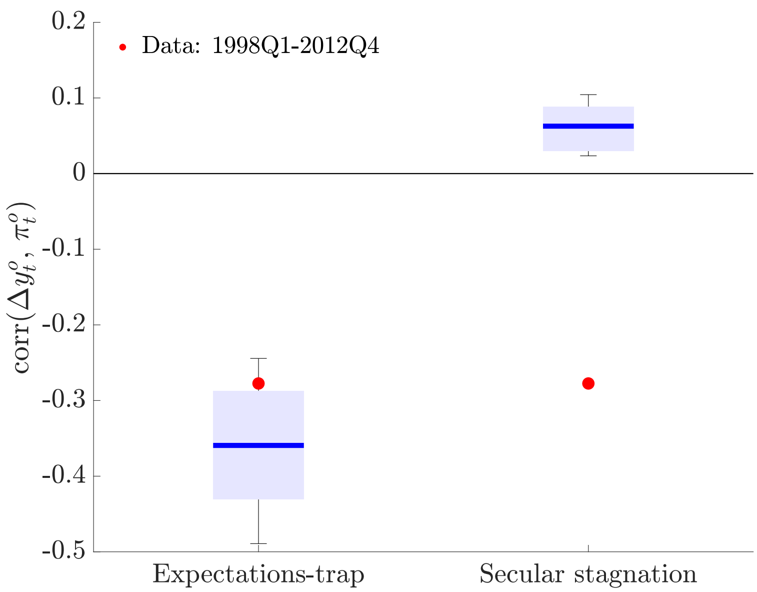

Figure 3: *Moments: models vs data*

Notes: Dots correspond to sample moments in Japan's data. Solid horizontal lines indicate medians of theoretical moments of the posterior distributions for parameter estimates. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively.

the secular stagnation model can only generate a positive inflation output growth correlation.[19] We next explore the source of this negative correlation in the expectations trap model.

B. Indeterminacy and sunspot shocks

The presence of equilibrium indeterminacy relaxes the tight co-movement between inflation and output that afflicts the secular stagnation model, and inflation in the expectations-trap model can arbitrarily jump in response to fundamental shocks. In section I, we introduced i.i.d. sunspot shocks as exogenous expectational errors on inflation to select among the multiple equilibria. Using the technique of Bianchi and Nicolò (2021), we allowed sunspot shocks to be correlated with fundamental shocks in the model to index various equilibria. We now investigate which correlations with structural shocks are essential for our results. And how do our results change if we discipline the equilibrium selection with data on inflation expectations?

[19] Datta, Johannsen, Kwon and Vigfusson (2021) document a positive correlation between oil and equity prices in the U.S. post-2008. This measure is a proxy of the correlation between inflation and output growth in our model. We leave a formal quantitative assessment for the U.S. in our framework for future work.

B.1 Which Equilibrium?

The correlation between sunspots and structural shocks is crucial because it characterizes all admissible solutions under indeterminacy (Bianchi and Nicolò, 2021) while disciplining equilibrium selection using data.[20]

To understand which of the multiple equilibrium paths plays a role in discriminating between expectation traps and secular stagnation, we re-estimate the prediction pool under four restrictions on the correlation between the sunspot and fundamental shocks. Figure 4 displays the estimated time-varying model weights. Panel (a) sets the correlation between the sunspot and productivity shocks to zero. Panel (b) sets the correlation between the sunspot shock and the government expenditure shock to zero. Panel (c) sets the correlation of the sunspot shock and markup shock to zero. Lastly, panel (d) sets all the correlations to zero.

When price-markup and sunspot shocks are uncorrelated, as in panels (c) and (d), secular stagnation explains the data better. Conversely, when the correlation between sunspots with productivity or government spending shocks is zero while letting the data choose the correlation between sunspot and markup shocks, we obtain results similar to our baseline specification, i.e., expectation traps as the more likely explanation for the observed data. Combined with our analytical results presented in Section I, these equilibrium selection results imply that the correlation between price markup and sunspot shocks is crucial for the expectations trap hypothesis because it allows the model to generate a negative correlation between inflation and output growth. This correlation implies that markup shocks are contractionary at the zero lower bound—see impulse responses in Online Appendix C.5. This finding echoes the evidence presented in Wieland (2019), which shows that the oil supply shocks in Japan, which are equivalent to price markup shocks in our model, generate a negative correlation between inflation and output at the ZLB.[21]

[20]While we discipline equilibrium selection using the empirical model fit, a caveat of expectation traps is that we do not explicitly model how this expectations formation occurs, and how agents coordinate on one of the many multiple equilibria.

[21]Relatedly, Cohen-Setton, Hausman and Wieland (2017) provide partial equilibrium evidence from a supply-side policy restricting hours worked in France. They find that a policy-mandated reduction in weekly hours worked adversely affected industrial production in sectors exposed to the law.

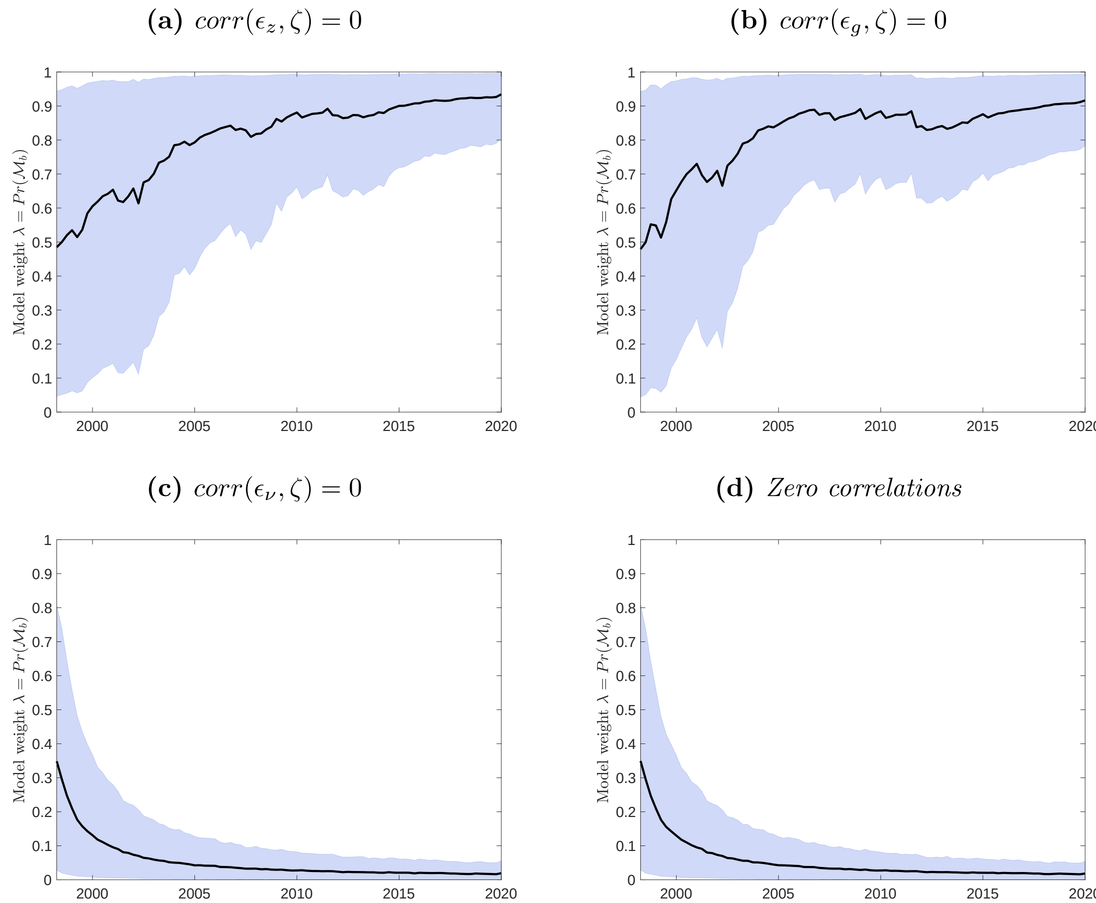

Figure 4: *Model Weights: Role of Sunspots*

Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

B.2 Inflation Expectations Data and Indeterminacy

We now investigate if data on inflation expectations as a source of additional information helps discipline equilibrium selection in the expectations-trap model. We collect Japan's 6- to 10-year inflation expectations data from Consensus Economics. Inflation expectations data is semi-annual and we use linear interpolation to obtain quarterly figures. We relate quarterly deviations from the sample mean in the data to the implied steady-state deviations of 6- to 10-year inflation expectations in the model. Formally we construct: $\hat{\pi}^e_t = \frac{1}{20}\mathbb{E}\left[\sum_{h=21}^{40} \hat{\pi}_{t+h}\right]$ and augment the expectations-trap model with the following measurement equation $\pi^{e,o}_t = 400 \times \hat{\pi}^e_t$ and re-estimate the posterior distribution of parameters associated with the expectations-trap model. Detailed estimation results are presented in Online Appendix C.6.

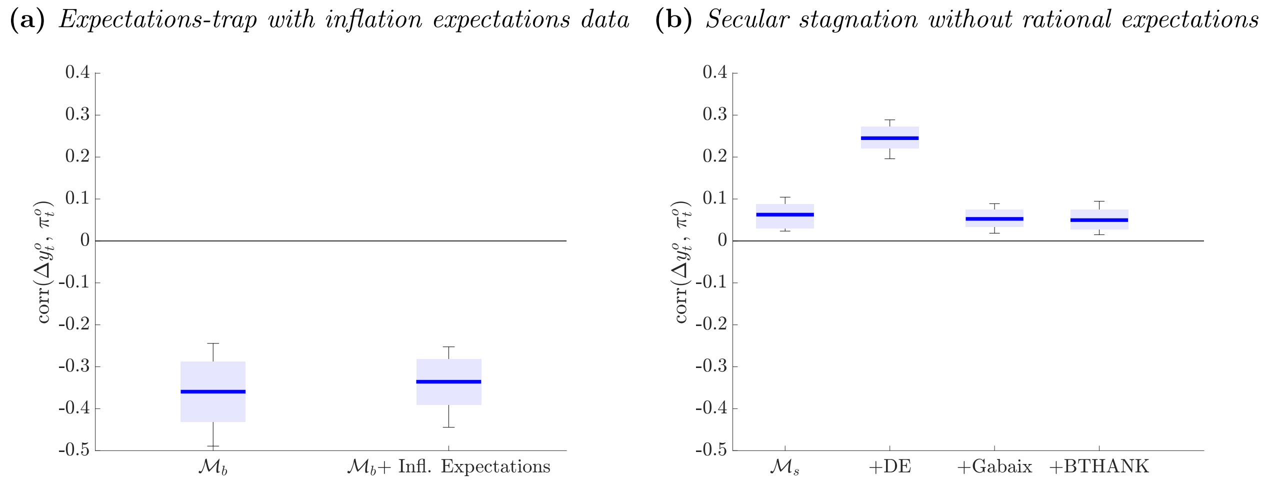

Figure 5: *Robustness: inflation-output correlation*

Notes: Solid horizontal lines indicate medians of theoretical moments of the posterior distributions for parameter estimates. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively.

The left panel in Figure 5 compares the posterior distribution of the theoretical correlation between inflation and output growth in our benchmark estimation and in the expectations-trap model re-estimated using data on inflation expectations. When using the data on inflation expectations, the model matches this key empirical correlation between inflation and output growth. Consistent with tighter posterior intervals on the estimated correlation parameters, reported in Table C.2 in Online Appendix C, we recover a somewhat tighter posterior range for correlation between inflation and output growth relative to the first column where we do not use data on inflation expectations in the estimation.

Since the expectations trap model is estimated using additional data, we cannot directly compare the log data density with the baseline secular stagnation model. Nonetheless we construct the Bayesian prediction pools to study which model explains the observed data series better. In Online Appendix C.7, we report these results. We find that the expectations trap model, even when estimated with additional data restrictions, continues to be superior to the secular stagnation model in explaining the three data series used in the baseline estimation.

C. Secular stagnation without rational expectations

We investigate the ability of departures from the rational expectations assumption in their ability to enhance secular stagnation's empirical fit relative to the expectations trap hypothesis. We consider three dominant alternatives: (i) diagnostic agents à la Bordalo, Gennaioli and Shleifer (2018), (ii) "behavioral" agents à la Gabaix (2020), and (iii) behavioral heterogeneous agents à la Bilbiie (2021) and Pfäuti and Seyrich (2022). The stagnation steady state is unchanged in all three extensions. We provide a detailed derivation of the log-linearized equilibrium conditions of these model extensions in Online Appendix B, while additional estimation results are available in Online Appendix C.6.

We find neither of these departures from rational expectations allows secular stagnation equilibrium to match the negative correlation between inflation and output growth. Our first extension considers diagnostic expectations, in which agents extrapolate current innovations thus distorting beliefs about future states.[22] Relative to the baseline, there is a new parameter $\theta \geq 0$ that regulates the departure from rational expectations. A value of $\theta = 0$ simplifies the model to the rational expectations benchmark. We set $\theta = 1$, following Bordalo et al. (2018). The second column in Figure 5b shows that the estimated secular stagnation model with diagnostic expectations also exhibits a positive correlation between output growth and inflation. The marginal data density of this model is 20 log points lower than that of the rational expectations counterpart.

In contrast to extrapolation, Gabaix (2020) proposes a model of "cognitive discounting", whereby the representative agent is partly myopic about innovations in the distant future.[23] In this model we introduce a new parameter, $\bar{m} \in [0, 1]$, that regulates the behavioral discounting. Following Gabaix (2020), we set $\bar{m} = 0.85$, and re-estimate the baseline secular stagnation model with this configuration. The case of $\bar{m} = 1$ reduces the model to the rational expectations benchmark. The third column in Figure 5b shows that the estimated secular stagnation model with cognitive discounting exhibits a lower correlation between output growth and inflation than the baseline case in the first column. The marginal data density of this model is 2 log points higher than the benchmark secular stagnation model with rational expectations. Still, the model fit remains substantially lower relative to the expectations-trap model.

Finally, we consider Bilbiie (2021)'s tractable heterogeneous agents New Keynesian (THANK) model combined with Gabaix (2020) behavioral friction as formally studied by Pfäuti and

[22] We follow Bianchi, Ilut and Saijo (2023) and L'Huillier, Singh and Yoo (2023) in their approach to solving diagnostic expectations in linear general equilibrium settings.

[23] Angeletos, Huo and Sastry (2020, Sec. 6.4) show that leading departures from rational expectations such as level-k thinking, dogmatic higher-order beliefs, and cognitive discounting imply under-extrapolation.

Seyrich (2022). The resulting behavioral-THANK (B-THANK) model analytically captures key ingredients of rich heterogeneity often considered in quantitative models while allowing for local determinacy through cognitive discounting. To keep the steady state unchanged, we consider the baseline version in Bilbiie (2021) featuring a zero-inequality steady state which isolates the role of cyclical inequality. The steady state of the model is thus the same as before. The new parameters $\chi = 1.48$ (a composite measure of cyclical income inequality),[24] $\lambda = 0.37$ (an unconditional measure of "hand-to-mouth" agents), and $1 - s = 0.04$ (transition probability to move from being a saver to a "hand-to-mouth" agent) are set following Bilbiie (2021)'s baseline calibration. The final column in Figure 5b shows that the estimated secular stagnation model with cognitive discounting and heterogeneous agents also exhibits a somewhat lower correlation between output growth and inflation than the baseline case depicted in the first column. However, the marginal data density of this model is 7 log points lower than the representative rational agent counterpart.

In sum, while the cognitive discounting friction offers a better model fit at the margin for the secular stagnation model, neither of the three departures allows the secular stagnation model to generate the negative correlation between inflation and output growth observed in the data. Our results suggest that the fit of the secular stagnation hypothesis is impaired because the model cannot generate the empirical correlation of inflation and output growth even with some of the departures from rational expectations.[25]

IV. A Medium-Scale DSGE Model

We assess the robustness of our findings using a medium-scale model similar to Smets and Wouters (2007), which extends our benchmark New Keynesian model along several dimensions. We use this model to explore two dimensions of our results. First, the medium-scale model features additional cross-equation restrictions and data relative to the model in Section I. Do additional complexity and more data diminish the role of the negative correlation between output growth and inflation as the key moment driving model fit? Second, we use the quantitative

[24] $\chi > 1$ is the empirically relevant case of counter-cyclical income inequality in this framework.

[25] It is possible to relax model misspecification by allowing the correlation of fundamental shocks in the secular stagnation model, thus generating a negative inflation-output correlation. We do not see a clear economic interpretation to pursue such an approach.

model to ask if the correlation of markup shocks with sunspot shocks is still one of the main drivers of the model fit for the expectations trap. Relaxing misspecification through a more elaborate model structure or additional data does not overturn the main insights we derived with the baseline model.

The medium-scale extension of the baseline model is relatively standard. We briefly describe the main ingredients and defer a detailed derivation to Online Appendix F. There are three important differences relative to the model in Section I. First, we allow for internal habits in consumption. Second, we introduce nominal wage stickiness as in Erceg, Henderson and Levin (2000). Third, we introduce capital into production, with costly capital utilization and investment adjustment costs as in Christiano, Eichenbaum and Evans (2005). Along each of these modifications, we introduce three new structural shocks: time-varying wage markups ($\frac{1}{1-\nu_{w,t}}$), a risk-premium shock (η_t), and a shock to the marginal efficiency of investment (μ_t) as in Justiniano, Primiceri and Tambalotti (2010).

A. Data and Calibration

We follow Hirose (2020) to map the medium-scale model to Japanese data. Data for estimation includes time series on output growth, personal consumption expenditure growth, investment growth, GDP deflator-based inflation, wage growth, and hours worked. As in the analysis of Section II, we estimate the relevant parameters in the medium-scale model using data from 1998:Q1 - 2012:Q4, while we use the sample 1998:Q1 - 2019:Q4 to conduct the prediction pool analysis and assess the likelihood of expectation-traps vis-a-vis secular stagnation. Detailed data sources are available in Online Appendix C.

The parameters that govern the capital share in production, depreciation rate of capital, habit persistence in consumption, price and wage indexation, capital utilization elasticity, and investment adjustment costs are sourced from Hirose (2020). To be consistent with the stylized model of Section II, we have set the discount factor $\beta = 0.942$, and calibrated the steady-state level of price and wage markups to 20 percent. Likewise, we have set the growth rate of productivity and the share of government expenditure to GDP to their observed values during the estimation sample from 1998:Q1 to 2012:Q4. For a detailed description of model calibration, please refer to Online Appendix F.

We separately estimate the parameters governing each model's structural shocks and equilibrium selection. The expectations-trap and secular stagnation models have six structural shocks in common, labeled as follows: technology growth shocks ($G_{z,t}$); domestic absorption shocks (g_t), which include government spending and foreign demand; shocks to the marginal efficiency of investment (μ_t); shocks to price markup ($\nu_{p,t}$) and wage markup ($\nu_{w,t}$); and shocks to households risk premia (η_t) that shift the marginal utility of bonds. All the shocks follow a first-order auto-regressive process. The expectations-trap model has seven additional parameters: a non-fundamental or sunspot shock (ζ_t) and six parameters for the correlation of the sunspot shock with the structural shocks that select the equilibrium path under indeterminacy.

B. Estimation Results

Table 3 reports the estimated parameters for both models. We report posterior mean estimates and 90% credible sets. In the bottom row, we report the marginal likelihoods of both models. Additional estimation details, such as the prior distribution and the configuration of the posterior sampler, are available in Online Appendix C.

The estimated posterior mean for the common parameters across the two models is similar. In terms of overall fit, although the medium-scale model has additional propagation and shocks, we still find "strong" evidence favoring the expectations-trap model over secular stagnation. The Bayes factor is 17. The experiments in the remainder of this section suggest that this difference in fit is also related to the central moment of our analysis, the negative correlation between inflation and output growth when interest rates are at the ZLB.

Turning to the estimated parameters specific to the expectations-trap model, we make two observations. First, the sunspot shock's standard deviation is similar to our estimates in Table 2. This result suggests that sunspot shocks remain a statistically important source of fluctuations despite the additional structural shocks in the model. Second, we find that the correlation of the sunspot shock with price and wage markup shocks is positive and tightly estimated. In particular, the mean estimate of the correlation between sunspot and price markup shock is remarkably close to our estimates in the stylized model of Section II. Both results suggest that parameters governing the dynamics of the expectations trap model are robustly estimated using additional data and a more complex model.

Table 3: *Medium Scale Model: Posterior Estimates*

Parameters	Description	$\mathcal{M}_b$: Exp. trap Mean	[05 95]	$\mathcal{M}_s$: Sec. Stag. Mean	[05 95]
ρ_w	Persistence wage markup shock	0.20	[0.19 0.21]	0.17	[0.06 0.27]
ρ_p	Persistence price markup shock	0.20	[0.19 0.21]	0.09	[0.03 0.15]
ρ_g	Persistence gov. spending shock	0.99	[0.98 1.00]	0.89	[0.85 0.93]
ρ_μ	Persistence MEI shock	0.42	[0.41 0.43]	0.90	[0.87 0.94]
ρ_η	Persistence risk premium shock	0.35	[0.34 0.37]	0.87	[0.82 0.93]
ρ_z	Persistence tech. growth shock	0.06	[0.05 0.06]	0.18	[0.10 0.26]
$100 \times \sigma_w$	Std dev. wage markup shock	0.55	[0.49 0.60]	0.49	[0.41 0.56]
$100 \times \sigma_p$	Std dev. price markup shock	0.49	[0.47 0.52]	0.48	[0.42 0.54]
$100 \times \sigma_g$	Std dev. gov. spending shock	0.72	[0.65 0.77]	0.77	[0.67 0.86]
$100 \times \sigma_\mu$	Std dev. MEI shock	14.17	[12.01 16.16]	7.94	[6.87 8.91]
$100 \times \sigma_z$	Std dev. tech. growth shock	1.49	[1.32 1.66]	1.43	[1.24 1.62]
$100 \times \sigma_\eta$	Std dev. risk premium shock	0.57	[0.14 1.16]	4.01	[2.08 5.87]
$100 \times \sigma_\zeta$	Std dev. sun. shock	0.49	[0.47 0.51]	-	-
$\rho(\epsilon_z, \zeta)$	Corr. sun. and tech. growth shocks	-0.04	[-0.06 -0.02]	-	-
$\rho(\epsilon_g, \zeta)$	Corr. sun. and gov. spending shocks	-0.08	[-0.09 -0.06]	-	-
$\rho(\epsilon_\mu, \zeta)$	Corr. sun. and MEI shocks	-0.09	[-0.11 -0.07]	-	-
$\rho(\epsilon_p, \zeta)$	Corr. sun. and price markup shocks	0.95	[0.93 0.96]	-	-
$\rho(\epsilon_w, \zeta)$	Corr. sun. and wage markup shocks	0.29	[0.25 0.32]	-	-
$\rho(\epsilon_\eta, \zeta)$	Corr. sun. and risk premium shocks	0.07	[0.05 0.09]	-	-
$\log \left[p\left(Y^T\right)\right]$		-944.67		-953.07	

Notes: The estimation sample is 1998:Q1 - 2012:Q4. We use $Y^T = [y_1, \ldots, y_T]$ to denote all the available data in our sample. We report posterior means and 90% the highest posterior density intervals for each model in square brackets. All posterior statistics are based on the last 50,000 draws from an RWMH algorithm after discarding the first 50,000 draws.

C. Expectations-trap vs secular stagnation

Next, we construct recursive weights using the predictive densities from the medium-scale specification of both models. Figure 2 shows the posterior mean estimates of the model weights, λ_t, and the associated 90 percent credible sets. The estimated value of λ_t reflects the probability that the six data series observed in period-t come from the expectations-trap hypothesis. Our results confirm that both models provided an equally plausible explanation of the data in the initial part of the sample. With additional information accumulating over time, the expectations-trap model emerges as a more likely explanation, with our estimates of λ_t rising to about 90 percent by 2010 and remaining at that level. Although the estimated mean probability of the expectation-trap model kept rising over time, uncertainty about the source of stagnation remained elevated for several years.

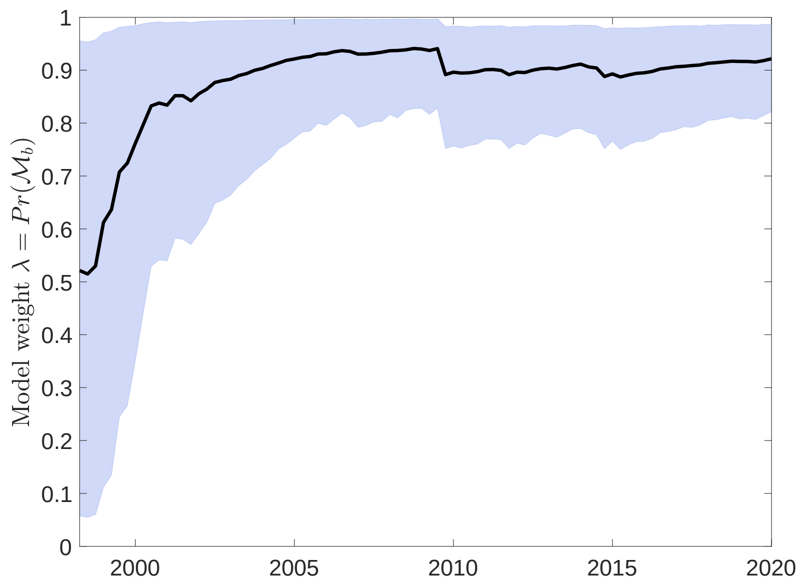

Figure 6: *Expectations Traps vs Secular Stagnation in Medium Scale Model*

Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2019:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

D. Discussion

Using a small-scale model, we argued that the correlation between inflation and output growth is central to tilting the prediction pool weights in favor of the expectations trap. Using the estimated medium-scale model, we now revisit the importance of the correlation between inflation and output growth.

In Figure 7, we re-estimate the model weights, λ_t while restricting the correlation structure between the sunspot and the structural shocks. In all experiments, we draw the estimated parameters from the posterior distributions presented in Table 3 but set correlations between the sunspot and particular structural shocks to zero when computing predictive densities.

The left panel in Figure 7 shows the estimated model weights when the sunspot shock is allowed to have a non-zero correlation with the price and the wage markup shocks.[26] The mean estimates of λ_t indicate that the expectations-trap model remains the preferred model.

[26] In Online Appendix C.7 we present the reverse exercise in which we restrict the correlation of the sunspot shock with the markup shocks to be zero and allow the correlation of sunspot shocks with other structural shocks to be unrestricted. We find that the recursive model weights favor the secular stagnation model, consistent with our results from the estimated small-scale model. The correlation between sunspot and markup shocks is the key driver for model fit in the expectations-trap model.

(a) *Model weights: markup shocks only* **(b)** *Theoretical moments*

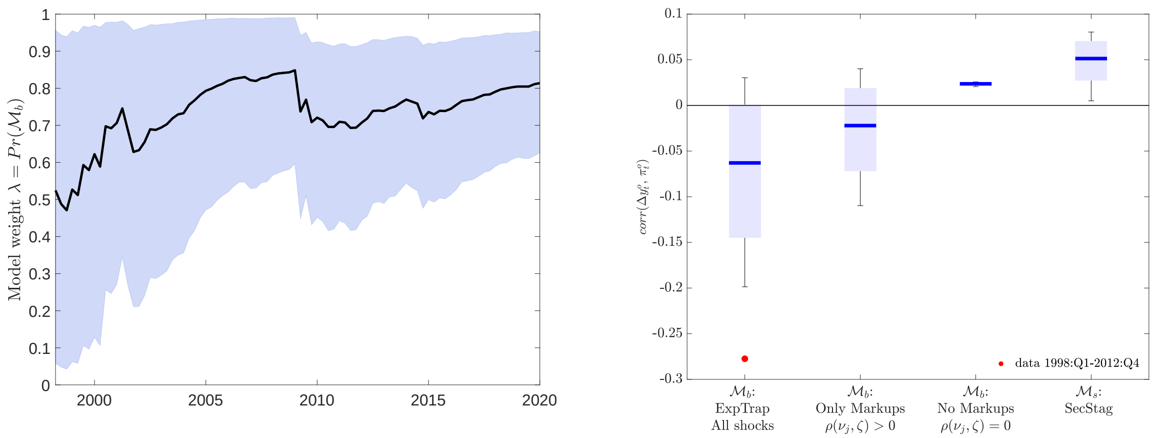

Figure 7: *Inspecting the Mechanism in the Medium Scale Model*

Notes: The solid black line is the posterior mean of λ estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

Although the weight of the expectations-trap model increases steadily over our sample, the wider credible sets show more uncertainty about the dominant model.

We find that the correlation between inflation and output growth remains the key moment in the tilt of model weights toward the expectations-trap model. The right panel in Figure 7 shows the theoretical correlation between inflation and output growth for three versions of the expectations-trap model. Specifically, we plot the baseline estimates (first column), a restricted model in which sunspot shocks are correlated only with markup shocks (second column), and a restricted model in which the correlation of sunspot shocks and markup shocks is zero (third column). For comparison, we also report the theoretical moments for the secular stagnation model (fourth column).

In the unrestricted expectations-trap model, shown in the first column, the median correlation between inflation and output growth is negative and the posterior intervals cover a large range of values. In contrast, the secular stagnation model has a very tight range of outcomes in the positive territory for this central correlation. In the restricted expectations trap model with only markup shocks driving the equilibrium selection, shown in the second column, the range of correlations between inflation and output growth narrows but the posterior mean estimate remains negative. In the third column, the restricted model with zero correlation between markup and sunspot shocks, the mean correlation between inflation and output growth is tightly distributed around a positive value. These results suggest that equilibrium selection through markup shocks is the key source of the correlation between inflation and output growth. Overall, our quantitative findings from the medium-scale model align with the analytical and quantitative results from the small-scale model despite the added model complexity.

V. Conclusion

In this paper, we formally test two hypotheses of stagnation: deflationary expectations traps and secular stagnation. We entertain both models within a unified framework using a modified Euler equation with discounting. Because both theories differ in their local determinacy properties, we show analytically that the correlation of output growth and inflation can distinguish the two hypotheses in the data.

We leverage the Japanese experience at the ZLB to empirically assess our theory using a quantitative New Keynesian model that embeds both hypotheses. We offer a summary measure of the likelihood of each hypothesis using Bayesian prediction pools. We find evidence that Japan's experience is consistent with the expectations trap model, but making such an assessment is subject to substantial uncertainty in the early part of the estimation sample. Consistent with our theory, the negative correlation between output growth and inflation is the key empirical moment that ultimately shifts the balance in favor of the expectations-trap hypothesis in Japan. Our quantitative findings extend to models that deviate from rational expectations, models that use inflation-expectations data, and more elaborate medium-scale models of the Japanese economy.

References

Andersen, Steffen, Glenn W Harrison, Morten I Lau, and E Elisabet Rutström. 2014. "Discounting behavior: A reconsideration." *European Economic Review*, 71: 15–33.

Angeletos, George-Marios, Zhen Huo, and Karthik A. Sastry. 2020. "Imperfect macroeconomic expectations: Evidence and theory." In *NBER Macroeconomics Annual 2020, Vol. 35.*

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian analysis of DSGE models." *Econometric reviews*, 26(2-4): 113–172.

Aruoba, Borağan, Luigi Bocola, and Frank Schorfheide. 2017. "Assessing DSGE model nonlinearities." *Journal of Economic Dynamics and Control*, 83: 34–54.

Aruoba, S Borağan, Pablo Cuba-Borda, and Frank Schorfheide. 2018. "Macroeconomic dynamics near the ZLB: A tale of two countries." *The Review of Economic Studies*, 85(1): 87–118.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe. 2001. "The Perils of Taylor Rules." *Journal of Economic Theory*, 96(1): 40–69.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe. 2002. "Avoiding liquidity traps." *Journal of Political Economy*, 110(3): 535–563.

Benigno, Gianluca, and Luca Fornaro. 2018. "Stagnation Traps." *The Review of Economic Studies*, 85(3): 1425–1470.

Bianchi, Francesco, and Giovanni Nicolò. 2021. "A generalized approach to indeterminacy in linear rational expectations models." *Quantitative Economics*, 12(3): 843–868.

Bianchi, Francesco, and Leonardo Melosi. 2017. "Escaping the great recession." *American Economic Review*, 107(4): 1030–58.

Bianchi, Francesco, Cosmin Ilut, and Hikaru Saijo. 2023. "Diagnostic Business Cycles." *The Review of Economic Studies*, rdad024.

Bilbiie, Florin O. 2019. "Optimal Forward Guidance." *American Economic Journal: Macroeconomics*, 11(4): 310–45.

Bilbiie, Florin Ovidiu. 2021. "Monetary Policy and Heterogeneity: An Analytical Framework." CEPR Discussion Papers 12601.

Bilbiie, Florin Ovidiu. 2022. "Neo-Fisherian Policies and Liquidity Traps." *American Economic Journal: Macroeconomics*, 14(4): 378–403.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer. 2018. "Diagnostic expectations and credit cycles." *The Journal of Finance*, 73(1): 199–227.

Canova, Fabio, and Luca Gambetti. 2010. "Do expectations matter? The great moderation revisited." *American Economic Journal: Macroeconomics*, 2(3): 183–205.

Christiano, L.J., M. Eichenbaum, and C.L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1).

Cohen-Setton, Jeremie, Joshua K. Hausman, and Johannes F. Wieland. 2017. "Supply-Side Policies in the Depression: Evidence from France." *Journal of Money, Credit and Banking*, 49(2-3): 273–317.

Coyle, Philip, and Taisuke Nakata. 2019. "Optimal inflation target with expectations-driven liquidity traps." Finance and Economics Discussion Series, Federal Reserve Board of Governors.

Datta, Deepa D., Benjamin K. Johannsen, Hannah Kwon, and Robert J. Vigfusson. 2021. "Oil, Equities, and the Zero Lower Bound." *American Economic Journal: Macroeconomics*, 13(2): 214–53.

Del Negro, Marco, Raiden Hasengawa, and Frank Schorfheide. 2016. "Dynamic Prediction Pools: An Investigation of Financial Frictions and Forecasting Performance." *Journal of Econometrics*, 192(1): 391–405.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins. 2019. "A Model of Secular Stagnation: Theory and Quantitative Evaluation." *American Economic Journal: Macroeconomics*, 11(1): 1–48.

Erceg, Christopher J, Dale W Henderson, and Andrew T Levin. 2000. "Optimal monetary policy with staggered wage and price contracts." *Journal of Monetary Economics*, 46(2): 281–313.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue. 2002. "Time discounting and time preference: A critical review." *Journal of Economic Literature*, 40(2): 351–401.

Fujiwara, Shigeaki, Yuto Iwasaki, Ichiro Muto, Kenji Nishizaki, and Nao Sudo. 2016. "Supplementary Paper Series for the "Comprehensive Assessment" (2): Developments in the Natural Rate of Interest in Japan." Bank of Japan Review Series 16-E-12.

Gabaix, Xavier. 2020. "A behavioral New Keynesian model." *American Economic Review*, 110(8): 2271–2327.

Galı, Jordi, and Mark Gertler. 1999. "Inflation dynamics: A structural econometric analysis." *Journal of Monetary Economics*, 44(2): 195–222.

Geweke, John, and Gianni Amissano. 2011. "Dynamic Prediction Pools: An Investigation of Financial Frictions and Forecasting Performance." *Journal of Econometrics*, 164(1): 130–141.

Hansen, Alvin H. 1939. "Economic progress and declining population growth." *American Economic Review*, 29(1): 1–15.

Hausman, Joshua K, and Johannes F Wieland. 2014. "Abenomics: preliminary analysis and outlook." *Brookings Papers on Economic Activity*, 2014(1): 1–63.

Hirose, Yasuo. 2020. "An Estimated DSGE Model with a Deflation Steady State." *Macroeconomic Dynamics*, 24(4): 1151–1185.

Iiboshi, Hirokuni, Mototsugu Shintani, and Kozo Ueda. 2022. "Estimating a Nonlinear New Keynesian Model with a Zero Lower Bound for Japan." *Journal of Money, Credit and Banking*, 54(6): 1637–1671.

Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti. 2010. "Investment shocks and business cycles." *Journal of Monetary Economics*, 57(2): 132–145.

Kass, Robert E., and Adrian E. Raftery. 1995. "Bayes Factors." *Journal of the American Statistical Association*, 90(430): 773–795.

Kuroda, Sachiko, and Isamu Yamamoto. 2008. "Estimating Frisch labor supply elasticity in Japan." *Journal of the Japanese and International Economies*, 22(4): 566–585.

Lansing, Kevin J. 2021. "Endogenous forecast switching near the zero lower bound." *Journal of Monetary Economics*, 117: 153–169.

Laubach, Thomas, and John C Williams. 2003. "Measuring the natural rate of interest." *Review of Economics and Statistics*, 85(4): 1063–1070.

L'Huillier, Jean-Paul, Sanjay R Singh, and Donghoon Yoo. 2023. "Incorporating diagnostic expectations into the New Keynesian framework." *The Review of Economic Studies*.

Lubik, Thomas A, and Frank Schorfheide. 2004. "Testing for indeterminacy: an application to US monetary policy." *American Economic Review*, 94(1): 190–217.

Mertens, Karel, and Morten O Ravn. 2014. "Fiscal policy in an expectations-driven liquidity trap." *The Review of Economic Studies*, 81(4): 1637–1667.

Mertens, Thomas M, and John C Williams. 2021. "What to Expect from the Lower Bound on Interest Rates: Evidence from Derivatives Prices." *American Economic Review*, 111(8): 2473–2505.

Michaillat, Pascal, and Emmanuel Saez. 2021. "Resolving New Keynesian Anomalies with Wealth in the Utility Function." *The Review of Economics and Statistics*, 103(2): 197–215.

Michau, Jean-Baptiste. 2018. "Secular stagnation: Theory and remedies." *Journal of Economic Theory*, 176: 552–618.

Nakata, Taisuke. 2017. "Uncertainty at the zero lower bound." *American Economic Journal: Macroeconomics*, 9(3): 186–221.

Nakata, Taisuke, and Sebastian Schmidt. 2019. "Conservatism and liquidity traps." *Journal of Monetary Economics*, 104: 37–47.

Nakata, Taisuke, and Sebastian Schmidt. 2022. "Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy." *American Economic Journal: Macroeconomics*, 14(4): 68–103.

Ono, Yoshiyasu, and Katsunori Yamada. 2018. "Difference or Ratio: Implications of Status Preference on Stagnation." *Australian Economic Papers*.

Pfäuti, Oliver, and Fabian Seyrich. 2022. "A behavioral heterogeneous agent New Keynesian model." UT Austin Working Paper.

Ravn, Morten O, and Vincent Sterk. 2021. "Macroeconomic fluctuations with HANK & SAM: An analytical approach." *Journal of the European Economic Association*, 19(2): 1162–1202.

Schmidt, Sebastian. 2016. "Lack of confidence, the zero lower bound, and the virtue of fiscal rules." *Journal of Economic Dynamics and Control*, 70: 36–53.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2017. "Liquidity traps and jobless recoveries." *American Economic Journal: Macroeconomics*, 9(1): 165–204.

Schwarz, Gideon. 1978. "Estimating the dimension of a model." *The annals of statistics*, 461–464.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review*, 97(3): 586–606.

Summers, Lawrence H. 2013. "Have we Entered an Age of Secular Stagnation." *IMF Fourteenth Annual Research Conference in Honor of Stanley Fischer, Washington, DC*, 63(1): 277–280.

Wieland, Johannes F. 2019. "Are negative supply shocks expansionary at the zero lower bound?" *Journal of Political Economy*, 127(3): 973–1007.