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A Macroeconomic Model of Central Bank Digital Currency

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Abstract

We develop a quantitative New Keynesian DSGE model with monopolistic banks to study the macroeconomic effects of introducing a central bank digital currency (CBDC). Households benefit from an expansion of liquidity services and higher deposit rates as bank deposit market power is curtailed, while bank profitability and lending decline. We assess this trade-off for a wide range of economies that differ in their level of interest rates. We find substantial welfare gains from introducing a CBDC with an optimal rate that can be approximated by a simple rule of thumb: the maximum between 0% and the policy rate minus 1%.

JEL codes: E3, E4, E5, G21, G51.

Keywords: Central bank digital currency, Banking, DSGE, Monetary policy.

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1 Introduction

The introduction of a central bank digital currency (CBDC) is one of the most far-reaching innovations that central banks have considered in recent decades. By 2024, 134 countries and currency unions, accounting for 98% of global GDP, are exploring CBDCs. This group includes most of the G20 economies, with particularly prominent initiatives such as the digital euro in the Euro Area.[1] The launch of such a new currency could fundamentally reshape the financial landscape and raises a number of salient questions. First and foremost, is the introduction of a CBDC beneficial for an economy as a whole? Second, how should central banks set the interest rate on CBDCs, and how does this rate depend on the state of an economy, in particular the level of interest rates? And third, how does the presence of a CBDC affect the conduct of monetary policy and the behavior of an economy over the business cycle? In this paper, we seek to answer these questions by proposing a new general equilibrium model that features a realistic banking sector and is closely calibrated to empirical evidence.

To preview the key mechanisms, we start with a static partial equilibrium model of deposit intermediation. This simple framework has two important features. First, cash, deposits, and CBDC provide households with liquidity benefits, and the three instruments are imperfectly substitutable. Second, banks are monopolistic and set the deposit rate as a variable markdown on the policy rate. As a result, banks' deposit market power and the competition between the three liquidity-providing instruments jointly determine the difference between the policy rate and the deposit rate, which is the deposit spread that banks charge.

Our static framework illustrates the following features. In the absence of a CBDC, the deposit spread rises with the level of the policy rate, since banks gain market power as the rate on cash is fixed at zero (see, e.g., Drechsler et al., 2017). When a CBDC is introduced, the deposit spread decreases, as households value the liquidity benefits that a CBDC provides and lower their deposit holdings. The deposit spread falls the most if the CBDC rate is close to the policy rate, since a CBDC is a stronger competitor to deposits within that range. Thus, in an environment with an elevated policy rate and a large deposit spread, a CBDC that pays interest can be a strong competitive force, lowering banks' deposit market power.

Besides the behavior of the deposit spread, the static framework also points to the key trade-off that determines the impact of introducing a CBDC on welfare in general equilibrium. On the one hand, households benefit from CBDC since it is a liquidity-providing

[1] See, e.g., the Atlantic Council's CBDC tracker: https://www.atlanticcouncil.org/cbdctracker/.

instrument that they desire and because it provides competition to bank deposits, which lowers deposit spreads. On the other hand, banks not only have to raise their deposit rates but also face deposit outflows, both lowering their profitability and decreasing bank intermediation capacity.

To fully explore this trade-off, we enrich the static framework with a set of features that are particularly relevant in this context: a banking sector that intermediates between deposit and loan markets, financial frictions that imply bank profitability influences bank capital, which is in turn a determinant of credit supply, a corporate production sector, a bond market that can substitute for bank financing, and nominal price rigidities building on the New Keynesian tradition. We tightly calibrate the model to U.S. data and show that it successfully matches loan and bond spreads, as well as historical deposit rates for various levels of the policy rate.

We use the model as a laboratory to explore the effects of introducing a CBDC and its role in monetary policy transmission. First, we investigate how the impact of CBDC introduction varies with the level of CBDC remuneration, that is, its interest rate. Interestingly, the welfare change displays an inverted U-shape. If a CBDC pays a very low interest rate, households hold a negligible amount of CBDC in their portfolios and banks' deposit market power is largely unaffected, limiting the potential gains from CBDC introduction. By contrast, if a CBDC pays a very high interest rate, households flock to the CBDC, deposits pour out of the banking sector, and bank profitability and bank lending contract substantially. As a result, the welfare impact of a CBDC turns negative as the bank disintermediation effect that leads to lower aggregate investment and output dominates the CBDC's beneficial effects. Thus, the model delivers a unique optimal CBDC rate. For our baseline economy that is calibrated to U.S. data, this rate is different from zero and lies at around 0.8% per year.

Second, instead of studying the introduction of a CBDC for a specific economy, we analyze the effects of introducing CBDCs in many economies that differ solely in the level of their steady-state policy rate. To start, we assess the introduction of a CBDC that pays zero interest, as often envisioned by countries that plan to introduce one. For a large range of negative and positive policy rates, we find positive welfare gains; those gains are smaller in high interest rate economies where households would hold only small amounts of CBDC and consequently bank deposit market power would barely be challenged. While encouraging, the previous exercise conceals the fact that a remunerated CBDC can lead to substantially higher welfare gains. To explore this possibility, we determine the CBDC rate that maximizes welfare for each of these economies. For policy rates below 1%, the optimal CBDC rate is slightly negative and can even be higher than the policy rate. For policy rates above 1%, the optimal CBDC rate lies between 80 and 120 basis points below the policy rate. We show that this welfare-maximizing CBDC rate can be well approximated by a simple rule of thumb: it is the maximum between 0% and the steady-state policy rate minus 1%. The simplicity of this rule is appealing, since it can be applied to many economies that differ substantially in their level of interest rates. Additionally, central banks can easily communicate this remuneration schedule to households and avoid the political-economy concerns that could arise when a CBDC pays negative interest.

Introducing a CDBC with such a remuneration schedule has far-reaching effects on the banking system in our model. Particularly striking is how banks' deposit market power is curtailed in high interest rate environments. At a policy rate of 5%, banks charge a substantial deposit spread of around 2.5% in the absence of a CBDC. If a CBDC is introduced at its optimal rate, the deposit rate increases from around 2.5% to 4.3%, diminishing the deposit spread to only 70 basis points. In fact, for the range of policy rates between 2% and 7%, we find that the positive relation between the deposit spread and the level of the policy rate vanishes after the introduction of a CBDC with the welfare-maximizing rate, and that the deposit spread stabilizes around the aforementioned 70-basis-point level. These results connect with the intuition from our static framework: while cash is a weak competitor to deposits at high interest rates, a CBDC that pays interest can substantially curtail bank market power in deposit.

The scaling down of bank market power in deposit markets at high interest rates is also reflected in the welfare changes from CBDC introduction across policy rates. For policy rates below 2%, we find positive but modest welfare gains of around 0.25%—measured as the multiplicative consumption-equivalent variation required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady states. However, this number increases in high interest rate environments. For example, for a policy rate of 6%, we find a sizable welfare gain of around 1%.

Finally, we explore the role of CBDCs in potentially altering the response of an economy to typical business cycle innovations. Across a wide range of CBDC remuneration schedules, the reactions of various macroeconomic indicators to standard monetary policy and technology shocks are remarkably similar. Thus, even though the introduction of a CBDC can lead to significant welfare effects and changes in the financial landscape, responses to transitory shocks remain roughly unaltered. This is due to the fact that the two main neoclassical channels of interest rate transmission in our model—the intertemporal substitution channel and the investment-based channels—are not significantly altered by the presence of CBDC or its remuneration schedule.

Having established our main results, we discuss further aspects that help paint a richer picture of the potential impacts of CBDC. First, we decompose the welfare impacts of CBDC into the three channels discussed above: liquidity benefits, curtailing commercial bank monopoly power in deposits, and the bank disintermediation effect, finding that they are of roughly similar importance for our results. Second, we comment on the potential costs of issuing a CBDC and illustrate how they introduce a crucial non-separability between the decision whether to introduce a CBDC and the interest rate to pay on this instrument. Third, we perform an alternative calibration for the Euro Area and find that the gains from introducing a CBDC are lower than those for the U.S. due to the higher importance of banks in overall lending in the Euro Area. Fourth, we discuss how bank reserves and wholesale funding would interact after CBDC introduction in an extended framework where banks have access to such an alternative funding source. Fifth, we discuss bank subsidies as an alternative to curtailing bank monopoly power and how they differ from introducing a CBDC. Finally, we briefly touch upon CBDC holding limits.

The remainder of the paper, following a brief literature review, is organized as follows. Section 2 presents a stylized static model to build intuition. Section 3 introduces our quantitative DSGE framework and Section 4 describes its calibration. Section 5 presents our main quantitative results and policy analysis. Section 6 discusses additional issues, while Section 7 concludes.

Related Literature. Our paper contributes to the new and rapidly emerging literature on the macroeconomics of CBDCs.[2] In particular, our work is closely related to studies that examine the impact of CBDCs on bank disintermediation in a macroeconomic framework.

Most existing studies base their analysis on the New Monetarist approach. For example, Keister and Sanches (2022) show that a CBDC causes bank disintermediation as it crowds out bank deposits, leading to a decline in investment. However, they find that CBDC introduction often raises welfare by improving payment efficiency. Williamson (2022b) develops a model of banking and payments in which firms are subject to collateral constraints and a CBDC is introduced through a narrow banking facility. He finds that a CBDC can be welfare-improving as it promotes more efficient safe asset usage and helps mitigate a capital over-accumulation problem. In contrast to these models with competitive banking, Andolfatto (2021) considers a model with monopolistic banks and finds that the introduction of a CBDC can increase a bank's deposit rate and thus increase deposit financing while not necessarily impacting bank lending. Chiu et al. (2023) use a micro-founded model of payments where banks engage in oligopolistic competition in

[2] See, e.g., Chapman et al. (2023), Infante et al. (2023), and Ahnert et al. (2022) for recent surveys.

the deposit market. In their model, the introduction of a CBDC in fact crowds in bank deposits as long as the CBDC rate is not set too high. This effect is due to the assumption of perfect substitutability between deposits and CBDC. Relative to these contributions, we consider a New Keynesian dynamic stochastic general equilibrium (DSGE) model with imperfect substitutability between bank deposits and CBDC, bank market power in deposits and loans, and where bank profitability matters for bank lending.

Up to this point, relatively few papers have studied the macroeconomic effects of introducing CBDC in a DSGE model of the type that is commonly used by central banks. Barrdear and Kumhof (2022) find that CBDC issuance of 30% of GDP against government bonds could lower the real interest rate and thus increase GDP by 3%. Most closely related to our work is the paper by Burlon et al. (2023), who find that the introduction of CBDC can lead to substantial welfare gains. In comparison, our model features bank market power in deposit markets, which gives rise to the endogenous deposit spread that we highlight, as well as nonbank lending through the bond market. As a result, our model allows for two realistic additional channels through which CBDC can lead to relatively higher welfare gains.

The welfare gains of introducing CBDC may also be higher if the bank disintermediation effect is dampened, which may occur for two reasons. Using a banking industry equilibrium model, Whited et al. (2023) show that banks largely replace lost deposits with wholesale funding, such that bank lending only contracts by a fourth of the deposits lost. Relatedly, Abad et al. (2023) find that banks mainly decrease their excess reserves when deposits leave, as opposed to contracting their lending. In our framework, banks are able to replace lost deposits with borrowing from the central bank or wholesale funding. However, unlike deposits, these alternative funding sources do not carry a spread that is favorable to banks. Therefore, bank profitability declines and bank lending contracts. We further emphasize that what matters for welfare is not necessarily bank lending disintermediation per se but rather the change in overall lending. For example, if firms can easily substitute from bank to nonbank borrowing, bank disintermediation can be relatively large but the change in total lending, and hence output, can be comparatively muted.

Several other papers study optimal monetary policy and CBDC design. Brunnermeier and Niepelt (2019) formulate conditions under which a swap of private money for CBDC is irrelevant to economic allocations. Davoodalhosseini (2021) explores optimal monetary policy in a model where agents use cash and CBDC as payment instruments. Agur et al. (2022) consider the optimal design of CBDC in the presence of network effects. Closely related to our work, Niepelt (2023) studies the optimal quantity of CBDC in a standard growth and business cycle model where banks are monopsonists in deposit markets. He finds that the welfare-maximizing share of CBDC in payments generally exceeds that of deposits. In comparison, our framework features nominal rigidities, bank market power in loan markets, nonbank lending, and a role for bank profitability to determine credit supply.

The modeling differences that we highlight distinguish our paper from the literature and allow us to assess the quantitative importance of the aforementioned channels following the introduction of a CBDC. One key contribution is to show that the welfare-maximizing CBDC rates for economies that differ in their levels of interest rates can be approximated by a simple rule of thumb. We further reveal that economies with higher policy rates can obtain larger welfare gains from introducing CBDC. That is because bank deposit market power—an important feature of our model—is reduced relatively more in such environments.

2 A Static Bank Deposit Model

How does the introduction of a CBDC affect the deposit rate and its spread relative to the policy rate? And how does this relationship change with the level of the policy rate and the interest rate on CBDC? In this section, we present a static partial equilibrium model of deposit intermediation with monopolistic banks to answer these questions. This simple model facilitates analytical tractability and helps to build intuition for the results of the larger quantitative DSGE model that we discuss in Section 3.

2.1 Deposit Supply Functions

To start, we take the household's deposit supply schedule as given. Section 3 shows how such a schedule can be formally derived from the household's optimization problem. The household has access to three liquidity-providing instruments: cash (m), aggregate deposits (d), and CBDC. Their returns are zero, i^d, and i^{cbdc}, respectively. The aggregate deposit supply function is

$$d \;=\; \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta} \boldsymbol{\mathcal{L}}, \tag{2.1}$$

where θ is the elasticity of substitution between the three aggregate liquidity-providing instruments, γ_d is described below, and $\mathcal{L}$ is the real aggregate liquidity supplied by the household, which we take as given for now and endogenize in Section 3. Equation (2.1) specifies that deposit supply depends positively on the ratio of the gross deposit rate to the gross rate on liquid instruments, defined as

$$1 + i^{\mathcal{L}} = \left(\gamma_m + \gamma_d(1 + i^d)^{\theta+1} + \gamma_{cbdc}(1 + i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}. \tag{2.2}$$

The coefficients γ_m, γ_d, and γ_{cbdc} determine the importance of each of the instruments to the household due to exogenous non-interest-rate characteristics, and they satisfy $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Aggregate deposits d, in turn, are comprised of deposits in n individual banks, each of which is indexed by j. Bank j pays a deposit rate of i_j^d and faces an individual deposit supply function given by

$$d_j = \frac{1}{n}\left(\frac{1 + i_j^d}{1 + i^d}\right)^{\varepsilon^d} d, \tag{2.3}$$

where ε^d is the elasticity of substitution between different banks. Equation (2.3) indicates that the supply of deposits to bank j depends positively on the ratio of its gross deposit rate to the aggregate gross deposit rate, which is defined as

$$1 + i^d = \left(\sum_{j=1}^{n} \frac{1}{n}(1 + i_j^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}. \tag{2.4}$$

2.2 Banks

At the beginning of the period, each individual bank is endowed with equity f_j and issues deposits d_j. The bank uses these funds to finance its holding of reserves h_j, which pay the policy rate i. For simplicity, reserves are the only asset that banks invest in, an assumption that we relax below. Bank j's balance sheet condition is therefore:

$$h_j = f_j + d_j. \tag{2.5}$$

The bank maximizes its end-of-period equity

$$\max_{i_j^d, d_j, h_j} \quad (1 + i)h_j - (1 + i_j^d)d_j,$$

subject to the deposit-supply equations (2.1)-(2.4) and the balance sheet constraint (2.5). Each bank has some monopoly power, and it chooses the interest rate it pays on deposits, the amount of deposits it takes on, and how many reserves to hold. The first-order condition for this bank problem is

$$1 + i_j^d = \frac{\epsilon_j^d}{\epsilon_j^d + 1}(1 + i), \tag{2.6}$$

where ϵ_j^d is the *endogenous* elasticity of deposits with respect to the deposit rate, that is, $\epsilon_j^d \equiv \partial \ln d_j / \partial \ln(1 + i_j^d)$; see Appendix A for derivations. Equation (2.6) highlights that bank j sets its deposit rate as a markdown on the policy rate. Assuming all banks are symmetric, we can express the endogenous elasticity of the representative bank as

$$\epsilon^d = \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n}(1 - \omega_{\mathcal{L}}^d), \tag{2.7}$$

where

$$\omega_{\mathcal{L}}^d = \frac{(1+i^d)d}{(1+i^{\mathcal{L}})\mathcal{L}} = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \tag{2.8}$$

is the endogenous share of liquidity that stems from deposits at the end of the period, which we label the "endogenous deposit share" for short. When cash, deposits, and CBDC pay the same interest rate, the endogenous share coincides with the exogenous share, $\omega_{\mathcal{L}}^d = \gamma_d$.

Equation (2.7) shows that the endogenous elasticity of bank deposits with respect to the deposit rate is a combination of two elasticities. With weight $(n-1)/n$, it simply reflects the exogenous elasticity ε^d with which depositors substitute across different banks. With the complementary weight $1/n$, it depends on how aggregate deposit supply reacts to changes in the aggregate deposit rate, which individual banks partially internalize because of their market power and non-infinitesimal size.[3] Given that all banks face the same endogenous elasticity, equation (2.6) can be expressed as

$$\frac{i - i^d}{1 + i^d} = \frac{1}{\epsilon^d}, \tag{2.9}$$

where $(i - i^d)/(1 + i^d)$ represents the spread that banks make when they accept deposits at rate i^d and keep them at the central bank earning the policy rate, normalized by $1 + i^d$. This deposit spread is solely determined by the endogenous deposit elasticity. Taken together, equations (2.2) and (2.7)-(2.9) form a system that determines i^d, ϵ^d, $\omega_{\mathcal{L}}^d$, and $i^{\mathcal{L}}$

[3] Atkeson and Burstein (2008) derive a similar equation, but their focus is on the goods market, whereas we study bank deposits.

simultaneously.

2.3 How Central Bank Interest Rates Affect Deposit Rates

We first inspect how interest rates controlled by the central bank, namely the rate on reserves (i.e., the policy rate) and the CBDC interest rate, affect the deposit rate and the deposit spread.

Proposition 1.

1. *The deposit rate increases with the policy rate and the CBDC rate.*
2. *The deposit spread increases with the policy rate but decreases with the CBDC rate.*
3. *Aggregate deposits increase with the policy rate but decrease with the CBDC rate.*

Proof: see Appendix A.2.

The result on the deposit rate is intuitive, it shows the spillover from the central bank's policy instruments to the rates that are relevant for banks and households. But why do the two rates have opposite effects on the deposit spread and the amount of deposits? Equations (2.7), (2.8), and (2.9) are the key expressions that capture the transmission mechanism. When the rate on reserves increases, banks pass a fraction of this higher rate to their depositors. A higher deposit rate increases the endogenous deposit share $\omega^d_{\mathcal{L}}$, which has two effects. First, a higher deposit share translates directly into more aggregate deposits, given the exogeneity of aggregate liquidity. Second, a higher deposit share lowers the endogenous elasticity ϵ^d and increases the deposit spread. With a higher policy rate, banks gain market power relative to alternative liquid instruments and therefore charge a higher spread. This mechanism is also present in Drechsler et al. (2017). On the other hand, when the CBDC rate increases, CBDC poses more competition to banks and the endogenous deposit share decreases, which decreases both aggregate deposits and the deposit spread.

2.4 Effects of Introducing CBDC

Next, we turn to the core question of interest: what happens to the deposit rate and the deposit spread when the central bank introduces a CBDC? We capture the introduction of a CBDC by changing the CBDC interest rate from -100% to some higher percent that is roughly in the vicinity of 0%. We choose -100% as a starting point because that corresponds to the case where CBDC is not used at all in our larger DSGE model.

According to Proposition 1, the introduction of a CBDC increases the deposit rate, decreases the deposit spread, and induces an outflow of deposits. Based on a calibration that corresponds to the one used in Section 3, Panel A of Figure 2.1 plots the change in the deposit spread when CBDC is introduced with a 0% interest rate, as a function of the policy rate. Interestingly, it displays a U-shape. Per equations (2.7), (2.8), and (2.9), the underlying intuition works through the endogenous elasticity via the endogenous deposit share $\omega^d_{\mathcal{L}}$, which is plotted in Panel B of Figure 2.1. When CBDC pays a 0% interest rate and the policy rate is high, CBDC and cash barely compete with deposits, and hence $\omega^d_{\mathcal{L}}$ is close to one regardless of whether or not a CBDC exists. Therefore, the introduction of a CBDC leaves $\omega^d_{\mathcal{L}}$ mostly unaffected, and hence the deposit spread remains roughly unchanged. In the other extreme, for a fairly negative policy rate, deposits are undesirable compared with cash or CBDC. Therefore, $\omega^d_{\mathcal{L}}$ is close to zero regardless of the existence of CBDC. Deposits and CBDC are good substitutes for each other mostly when the policy rate is at intermediate levels. In this case, introducing a CBDC affects the endogenous

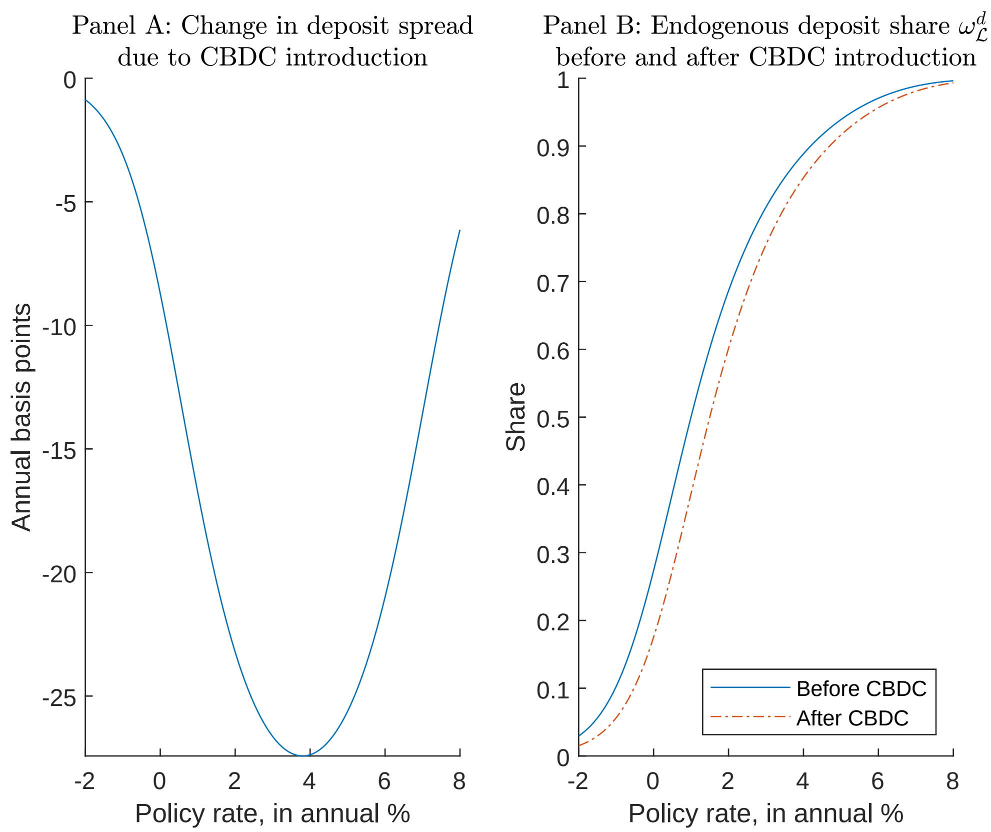

Figure 2.1: Panel A: Change in the deposit spread following the introduction of a CBDC across different values of the policy rate. Panel B: Endogenous deposit share ($\omega^d_{\mathcal{L}}$) across different values of the policy rate before and after the introduction of CBDC. The figure uses the baseline calibration described in Section 4.

deposit share and hence the endogenous elasticity of deposits substantially. Therefore, the deposit spread drops the most for moderate levels of the policy rate.

The U-shape is not unique to a CBDC that pays a zero interest rate. Panel A of Figure 2.2 shows that this shape holds as long as CBDC pays a constant interest rate. A higher interest rate on CBDC shifts the minimum of the curve towards the southeast: it increases the policy rate where CBDC introduction affects the deposit spread the most while also increasing the maximum change in the spread in absolute value.

Alternatively, when the CBDC rate is pegged to the policy rate, the U-shape disappears, as shown in Panel B of Figure 2.2. In this case, the change in the deposit spread is a decreasing function of the policy rate. This occurs because CBDC becomes more competitive with deposits the higher the policy rate is.

Thus, this simplified model already points to an important trade-off of a potential CBDC introduction. While such a policy can benefit households and shield them from the monopolistic power of banks, it can lower deposit spreads and therefore affect com-

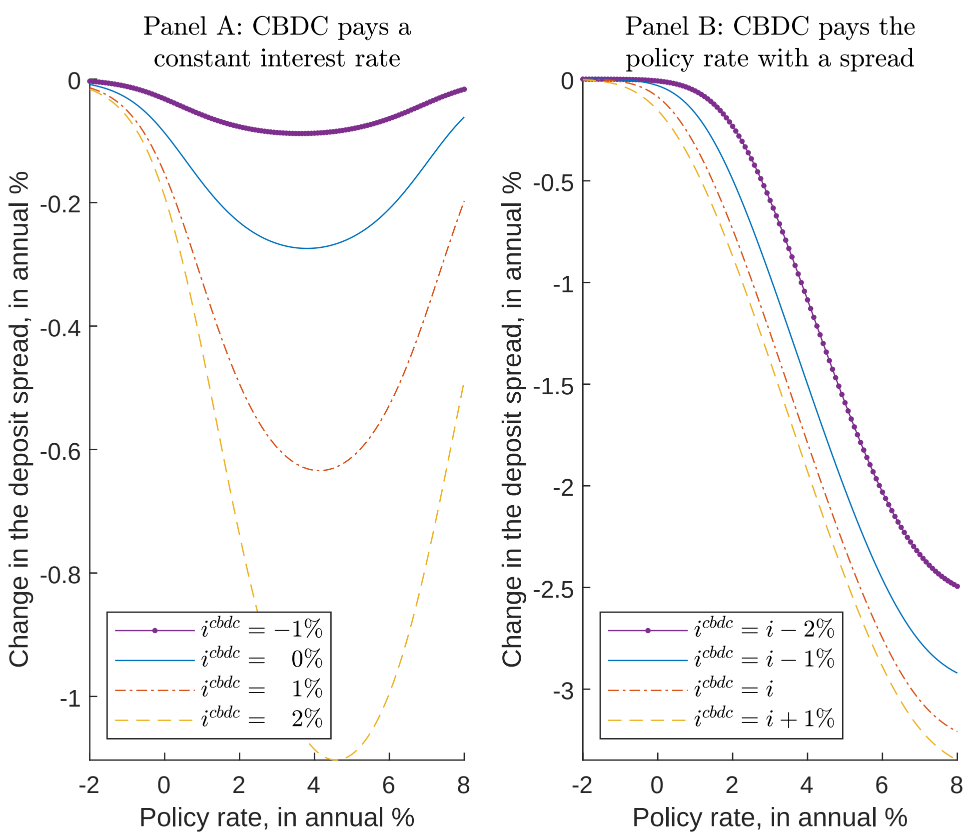

Figure 2.2: Change in the deposit spread following the introduction of a CBDC, across different values of the policy rate, for different choices on the CBDC interest rate (i^{cbdc}). Panel A depicts a CBDC that pays a constant interest rate; Panel B depicts a CBDC that pays the policy rate with a fixed spread.

mercial bank profitability negatively. In dynamic models where commercial bank equity is slow moving and relevant for lending, a fall in bank profitability can have a negative impact on the economy. In the following section, we embed the static model into a New Keynesian DSGE model to quantify this trade-off and further study aggregate welfare effects.

3 The DSGE Model

In this section, we introduce a full-fledged DSGE model for quantitative analyses. The key players in the model are a representative household, banks with monopoly power, a production sector, and a government.

The deposit side of the banking sector builds upon the ingredients laid out in Section 2. In addition, banks also issue corporate loans and face several operational costs. The household has access to four saving instruments: bonds, cash, bank deposits, and CBDC, where the last three instruments provide liquidity services with imperfect substitution.

The production sector consists of a representative intermediate good firm, a representative capital producer, monopolistically competitive retail firms, and a representative final good producer. The intermediate good firm purchases capital from the capital producer and combines it with labor from the household to produce an intermediate good. Its capital input is aggregated from two types of capital with non-unitary substitution: "non-pledgeable capital," which is financed by unsecured bond borrowing, and "pledgeable capital," which is financed through bank loans.

Retail firms face the standard Calvo price rigidity and transform the intermediate good into differentiated retail goods, which are then aggregated into a final good by the final good producer. The government includes a central bank that conducts monetary policy and a fiscal authority with a balanced budget.

3.1 Household

Setup. The household's lifetime utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t) - v(N_t) \right),$$

where β is the discount factor, C_t is consumption, and N_t is labor supply. The household's budget constraint is given by

$$P_tC_t + B_t + \Phi(\mathcal{L}_t)P_t \quad = \quad W_tN_t + AH_{t-1} + T_t,$$

where P_t is the aggregate price level, B_t are nominal bond holdings, W_t is the nominal wage, and $\Phi(\mathcal{L}_t)$ is described below. T_t captures transfers that are exogenous from the household's perspective, including net transfers from the government and profits from firms and banks. AH_{t-1} refers to "assets in hand" that the household enters period t with, given by

$$AH_{t-1} = M_{t-1} + (1+i_{t-1})B_{t-1} + \sum_{j=1}^{n}(1+i^d_{j,t-1})D_{j,t-1} + (1+i^{cbdc}_{t-1})CBDC_{t-1}.$$

The household can save in cash (M_t), bonds (B_t), deposits with any of the n different commercial banks ($D_{j,t}$), and CBDC ($CBDC_t$) if available, where capital letters denote nominal terms. The associated net nominal returns for these instruments are zero, i_t, $i^d_{j,t}$, and i^{cbdc}_t, respectively. The variable $\mathcal{L}_t$ in the budget constraint aggregates the various liquidity-providing instruments (cash, deposits, and CBDC), and is defined as

$$\mathcal{L}_t \quad = \quad \left(\gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} + \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} + \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}}, \tag{3.1}$$

where lowercase letters denote real variables (e.g., $m_t = M_t/P_t$). The parameter θ is the elasticity of substitution between liquid instruments and $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Additionally, real deposits d_t are an aggregate of deposits in n banks:

$$d_t = \left(\sum_{j=1}^{n} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{\frac{\varepsilon^d}{\varepsilon^d+1}},$$

where $\sum_{j=1}^{n}\alpha_j = 1$ and $\varepsilon^d \geq \theta$. The fact that cash, deposits, and CBDC are not perfect substitutes within $\mathcal{L}_t$ captures the possibility that the household uses them for different types of transactions because of their different properties. For example, bank deposits and CBDC are useful for online transactions, while cash is not; cash provides better anonymity than deposits and CBDC; cash and CBDC are government-backed while bank deposits are not necessarily insured; cash is more likely to be subject to theft. For these reasons, among others, the representative household might want to hold a combination of liquidity-providing instruments instead of simply holding the one with the highest return. A similar argument holds for deposits from different banks.[4]

[4]Note that, unlike the traditional CES aggregator, the exponents within the $\mathcal{L}_t$ and d_t aggregators are greater than one instead of smaller than one. This occurs because these aggregators enter the budget constraint in-

Lastly, $\Phi(\mathcal{L}_t)$ captures a potentially nonlinear cost function of acquiring liquidity. If $\Phi(\cdot)$ is linear and $\theta \to \infty$ (making the three liquid instruments perfect substitutes), then $\Phi(\mathcal{L})P = M + D + CBDC$, which is the standard way that savings instruments enter the budget constraint. By contrast, the $\Phi(\cdot)$ function that we use in our baseline specification is flexible enough to capture the feature that when the household has few liquid instruments the cost of acquiring liquidity could be less than one-for-one, $\Phi(\mathcal{L}_t) < \mathcal{L}_t$, reflecting the convenience benefit of holding liquidity. Eventually, if agents get "satiated" with liquidity services, it can instead be the case that $\Phi(\mathcal{L}_t) > \mathcal{L}_t$.[5] We choose to introduce $\Phi(\cdot)$ directly in the budget constraint for simplicity. However, as shown in Appendix B.2, one can obtain the same first-order conditions for the liquidity-providing instruments by allowing them to enter the utility function instead.[6]

Equilibrium Conditions. Our setup delivers convenient equilibrium conditions. First, the optimality conditions with respect to labor and bonds are the usual intratemporal condition for labor supply and the Euler equation:

$$v'(N_t) = u'(C_t)\left(\frac{W_t}{P_t}\right), \tag{3.2}$$

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). \tag{3.3}$$

Next, the holding schedules of the liquidity-providing instruments are

$$m_t = \gamma_m\left(\frac{1}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.4}$$

$$d_t = \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.5}$$

$$cbdc_t = \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t. \tag{3.6}$$

These holding schedules are well defined, even for negative values of the interest rates on deposits, CBDC, or overall liquidity. The interest rate for liquidity and aggregate deposits

stead of the utility function. Therefore, they must be convex (instead of concave) to prevent the household from bunching its choice into a single liquidity-providing instrument or a single bank.

[5]Balloch and Koby (2019) use a related cost function of liquidity in the context of negative nominal interest rates in Japan. The "satiation" embedded in our baseline functional form for $\Phi(\mathcal{L}_t)$ is similar to the one described in Rognlie (2016), but for liquidity as a whole rather than just for cash.

[6]This holds as long as one assumes a non-separable utility function between consumption and liquidity in the style of Greenwood et al. (1988), as shown in Appendix B.2.

are defined as

$$1+i_t^{\mathcal{L}} \equiv \left(\gamma_m + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \tag{3.7}$$

and

$$1+i_t^d \equiv \left(\sum_{j=1}^{n} \alpha_j (1+i_{j,t}^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}.$$

Furthermore, the amount of deposits the household supplies to an individual bank is given by

$$d_{j,t} = \alpha_j \left(\frac{1+i_{j,t}^d}{1+i_t^d}\right)^{\varepsilon^d} d_t, \tag{3.8}$$

and the equilibrium condition for the aggregator $\mathcal{L}_t$ is as follows:

$$\frac{1+i_t^{\mathcal{L}}}{1+i_t} = \Phi'(\mathcal{L}_t). \tag{3.9}$$

Appendix B.1 provides details on the derivations of the equilibrium conditions. Note that equations (2.1)-(2.4) are a special case of the equilibrium conditions above. Besides being static and considering $\mathcal{L}$ exogenous, Section 2 imposes the symmetry restriction that $\alpha_j = 1/n \ \forall j$.

3.2 Intermediate Good Firm

The intermediate good firm uses labor and capital to produce intermediate output. The production function is Cobb-Douglas:

$$Y_t^m = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{3.10}$$

where $0 < \alpha < 1$, Y_t^m is the amount of intermediate output produced, A_t is productivity, and K_t is capital input. The intermediate good firm purchases capital from a capital producer and finances its purchases via two possible channels. It borrows from the bond market to finance capital that cannot be used as collateral, denoted non-pledgeable capital K_t^{NP}, which reflects the empirical observation that bond borrowing is typically unsecured (Schwert, 2020). Alternatively, the firm can borrow from banks to purchase pledgeable capital K_t^P that can be used as collateral. Aggregate capital is a CES combination of these two types:

$$K_t = \left((1-\psi)^{\frac{1}{\theta^k}}(K_t^{NP})^{\frac{\theta^k-1}{\theta^k}} + \psi^{\frac{1}{\theta^k}}(K_t^P)^{\frac{\theta^k-1}{\theta^k}}\right)^{\frac{\theta^k}{\theta^k-1}}, \tag{3.11}$$

where θ^k captures the elasticity of substitution between the two types. K_t^P is itself an aggregate of the pledgeable capital financed by each of the n banks:

$$K_t^P = \left(\sum_{j=1}^{n}(\alpha_j^l)^{\frac{1}{\varepsilon^l}}(K_{j,t}^P)^{\frac{\varepsilon^l-1}{\varepsilon^l}}\right)^{\frac{\varepsilon^l}{\varepsilon^l-1}},$$

where ε^l is the loan elasticity of substitution among banks and α_j^l captures the exogenous importance of a particular bank in the loan portfolio, with $\sum_{j=1}^{n}\alpha_j^l = 1$ and $\varepsilon^l \geq \theta^k$.

Capital is predetermined. In period $t-1$, the intermediate good firm borrows from the bond market or banks in order to purchase capital for next period's production at price Q_{t-1}. At time t, it sells its depreciated capital stock $(1-\delta)$ back to the capital producer after production. Meanwhile, it pays back the lenders who charge different interest rates: bank j charges the loan rate $i_{j,t-1}^l$, while the bond market charges the risk-free rate i_{t-1} plus a spread ϱ. The intermediate good firm's period t profit is

$$\begin{aligned}\Pi_t^m &= P_t^m Y_t^m - W_t N_t + (1-\delta)Q_t\sum_{j=1}^{n}K_{j,t}^P + (1-\delta)Q_t K_t^{NP} \\ &- \sum_{j=1}^{n}(1+i_{j,t-1}^l)Q_{t-1}K_{j,t}^P - (1+i_{t-1}+\varrho)Q_{t-1}K_t^{NP}.\end{aligned}$$

The intermediate good firm maximizes the present value of profits (discounted using the household's stochastic discount factor) by choosing labor and capital inputs. The associated optimality conditions are given in Appendix B.3, and they depend on the real wage, as well as on the effective one-period user costs of aggregate capital, pledgeable capital, and non-pledgeable capital, which we denote with z_t, z_t^P, and z_t^{NP}, respectively.

3.3 Capital Good Producer

The capital producer faces the capital accumulation equation:

$$\left[K_{t+1}^{P}+\sum_{j=1}^{n}K_{j,t+1}^{NP}\right]=(1-\delta)\left[K_{t}^{P}+\sum_{j=1}^{n}K_{j,t}^{NP}\right]+I_{t}\left(1-\Xi\left(\frac{I_{t}}{I_{t-1}}\right)\right), \tag{3.12}$$

where I_t is overall investment and the function $\Xi(\cdot)$ captures investment adjustment costs and satisfies $\Xi(1)=\Xi'(1)=0$ and $\Xi''(1)\geq 0$. The problem of the capital producer in t is:

$$\max_{I_t}\ \mathbb{E}_t\sum_{\tau=0}^{\infty}\Lambda_{t,t+\tau}\left[Q_{t+\tau}I_{t+\tau}\left(1-\Xi\left(\frac{I_{t+\tau}}{I_{t+\tau-1}}\right)\right)-P_{t+\tau}I_{t+\tau}\right],$$

where $\Lambda_{t,t+\tau}$ denotes the household's stochastic discount factor for discounting nominal flows from $t+\tau$ back to t. The first-order condition of the capital producer is given in Appendix B.4.

3.4 Banks

The liability side of the bank balance sheet is similar to the one in Section 2.2; on the asset side, we also consider the possibility of lending to the production sector. Therefore, the nominal balance sheet constraint of bank j takes the following form

$$L_{j,t}+H_{j,t}=F_{j,t}+D_{j,t}, \tag{3.13}$$

where $L_{j,t}$ represents lending to the intermediate firm, $H_{j,t}$ are reserves issued by the central bank, $F_{j,t}$ is bank equity, and $D_{j,t}$ are household deposits, all in nominal terms.

Besides adding bank lending, we introduce three additional features.[7] First, in each period, a bank returns an exogenous fraction, $1-\omega$, of its profits to the household as dividends and spends a fraction ς of its nominal net worth to operate the managerial side of the bank. This setup implies that bank equity is partially determined by bank profitability. Second, a bank pays a quadratic cost, denoted by $\Psi(L_{j,t}/F_{j,t})$, when its loan-to-equity ratio, $L_{j,t}/F_{j,t}$, deviates from a target value. This cost captures the idea that regulators discourage banks from having high levels of leverage by imposing punishments when banks breach certain capital requirements, while market forces incentivize banks to avoid levels of leverage that are too low. Together, the previous two assumptions imply that a fall in bank profitability stemming from the introduction of a CBDC can impact bank equity, which in turn can affect bank lending. Finally, banks face exogenous costs of issuing loans, μ^l, and obtaining deposits, μ^d, expressed per dollar of loan or deposit issued. These costs are used to match the deposit and lending spreads without having to neces-

[7] These features are adopted from Ulate (2021).

sarily assume their existence is solely due to the presence of monopoly power.

With the assumptions described in the previous paragraph, the nominal resources that bank j has available when entering period $t+1$ are given by

$$S_{j,t+1} = (1 + i^l_{j,t} - \mu^l)L_{j,t} + (1 + i_t)H_{j,t} - (1 + i^d_{j,t} + \mu^d)D_{j,t} - \varsigma F_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}.$$

These total resources have to be used either to pay dividends or as next-period equity:

$$S_{j,t+1} \quad = \quad F_{j,t+1} + DIV_{j,t+1},$$

where dividends $DIV_{j,t+1}$ are a fraction $1-\omega$ of a bank's profit $X_{j,t+1}$:

$$DIV_{j,t+1} = (1-\omega)X_{j,t+1},$$

and profits $X_{j,t+1}$ are, in turn, defined as

$$X_{j,t+1} \quad \equiv i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} - \Psi\left(\tfrac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}.$$

We define $X_{j,t+1}$ as the net profit before paying managerial costs but after adjusting for inflation $\pi_{t+1} \equiv P_{t+1}/P_t - 1$. The inflation adjustment is purely for convenience, because it delivers a clean and interpretable expression for the law of motion of real bank equity, which takes the form

$$\frac{F_{j,t+1}}{P_{t+1}} \quad = \quad \frac{F_{j,t}}{P_t}(1-\varsigma) + \omega\frac{X_{j,t+1}}{P_{t+1}}. \tag{3.14}$$

If $\omega = \varsigma = 0$, then a bank's real equity is constant. The larger ω is, the more bank equity depends on profits and the more volatile it becomes.

A bank seeks to maximize the present discounted value of future dividends that it returns to the household. Hence, bank j's problem is:

$$\max \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau+1} DIV_{j,t+\tau+1}.$$

As shown in Appendix B.5.1, the solution to the bank's problem can be broken down into a deposit and a loan sub-problem that we discuss next.

Deposit Sub-problem. The deposit sub-problem amounts to

$$\max_{i^d_{j,t}} \; (i_t - i^d_{j,t} - \mu^d) D_{j,t},$$

subject to the deposit supply schedule $D_{j,t}(i^d_{j,t})$ of the household given by equation (3.8). Assuming that a bank takes the decisions of all other banks as given, it sets its deposit rate as follows:

$$1 + i^d_{j,t} \;=\; \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t} + 1}(1 + i_t - \mu^d). \tag{3.15}$$

Expression (3.15) shows that banks set their gross deposit rate as a markdown on the gross policy rate minus the cost of issuing deposits. The markdown is determined by $\epsilon^d_{j,t}$, the endogenous elasticity of bank j's deposits with respect to its deposit rate:

$$\epsilon^d_{j,t} \;\equiv\; \frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})} \frac{1 + i^d_{j,t}}{d_{j,t}}.$$

As shown in Appendix B.5.2, for the case with identical banks, this endogenous elasticity takes the form:

$$\epsilon^d_t \;=\; \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1 - \omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t} \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1 + i^{\mathcal{L}}_t)}\right], \tag{3.16}$$

where $\omega^d_{\mathcal{L},t}$ is again the endogenous deposit share

$$\omega^d_{\mathcal{L},t} \equiv \frac{(1 + i^d_t) d_t}{(1 + i^{\mathcal{L}}_t)\mathcal{L}_t} = \gamma_d \left(\frac{1 + i^d_t}{1 + i^{\mathcal{L}}_t}\right)^{\theta+1}. \tag{3.17}$$

Note that we can recover the expression in equation (2.7) from equation (3.16) when $\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t) = 0$, which is imposed in Section 2, where $\mathcal{L}$ is assumed to be constant. Thus, even in the larger model, the interpretation of ϵ^d_t remains similar: it is a weighted average of the exogenous elasticities ε^d and θ, as well as $\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t)$, where the weights for the last two terms are endogenous and can vary with the introduction of a CBDC.

Loan Sub-problem. The loan sub-problem of bank j is:

$$\max_{i^l_{j,t}} \; (i^l_{j,t} - i_t - \mu^l) L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right) F_{j,t},$$

subject to the loan demand schedule of the intermediate firm and $L_{j,t} = Q_t K^P_{j,t+1}$. As opposed to the markdown on the deposit rate, each individual bank sets its gross loan rate as a markup on the cost-adjusted policy rate:

$$1 + i^l_{j,t} \quad = \quad \frac{\epsilon^l_{j,t}}{\epsilon^l_{j,t} - 1}\left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right], \tag{3.18}$$

where $\epsilon^l_{j,t}$ denotes (the negative of) the endogenous loan elasticity of $l_{j,t}$ with respect to $1 + i^l_{j,t}$:

$$\epsilon^l_{j,t} \quad \equiv \quad -\frac{\partial l_{j,t}}{\partial(1 + i^l_{j,t})}\frac{1 + i^l_{j,t}}{l_{j,t}}.$$

As shown in Appendix B.5.3, for the case of identical banks, this endogenous elasticity takes the form:

$$\epsilon^l_t \quad = \quad \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[\theta^k(1 - \omega^{K_P}_{K,t}) + \frac{\omega^{K_P}_{K,t}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1 + i^l_t}{1 + i_t}\frac{1}{z^P_t}, \tag{3.19}$$

where $\omega^{K_P}_{K,t}$ is the expenditure on pledgeable capital as a share of total capital expenditure

$$\omega^{K_P}_{K,t} \quad \equiv \quad \frac{z^P_t K^P_t}{z_t K_t} = \psi\left(\frac{z^P_t}{z_t}\right)^{1-\theta^k}. \tag{3.20}$$

Equations (3.18)-(3.20) provide some intuition on the response of loan spreads to the introduction of a CBDC, which disintermediates banks and thereby decreases $\omega^{K_P}_{K,t}$. If θ^k is greater than $1/(1-\alpha)$, then the introduction of a CBDC increases ϵ^l_t and therefore lowers the loan spread.

Finally, we discuss the similarities and differences between equations (3.16) and (3.19). For both, the endogenous elasticity puts a weight $(n-1)/n$ on the exogenous elasticity (ε^d or ε^l). The remaining weight of $1/n$ is split between the two elasticities inside the square brackets: an elasticity of substitution (θ for deposits or θ^k for loans) and the elasticity of total liquidity ($\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t)$) or capital ($\partial \ln K / \partial \ln z = 1/(1-\alpha)$) with respect to its price.[8]

[8] A further difference between the two equations is that (3.19) features a term outside the curly bracket to reflect the fact that loan demand reacts to $z^P_{j,t}$ (which is a function of $1 + i^l_{j,t}$) instead of to $1 + i^l_{j,t}$ directly; see the intermediate firm problem in Section 3.2.

3.5 Retail Firms and Final Good Producer

The setup of retail firms and the final good producer follows the typical modeling approach in the New Keynesian literature. A continuum of retail firms indexed by $s \in [0,1]$ transform intermediate output Y_t^m into differentiated retail goods $Y_t(s)$, which are aggregated into a final good Y_t by the final good producer via a CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(s)^{\frac{\varphi-1}{\varphi}} ds \right)^{\frac{\varphi}{\varphi-1}},$$

where φ is the elasticity of substitution between the differentiated retail goods. The optimization problem of the final good producer implies the following demand function for good s and the aggregate price index:

$$Y_t(s) = \left(\frac{P_t(s)}{P_t} \right)^{-\varphi} Y_t, \qquad P_t = \left(\int_0^1 P_t(s)^{1-\varphi} ds \right)^{\frac{1}{1-\varphi}}.$$

Each period, a retail firm is able to freely adjust its price with probability $1-\gamma$ as in the Calvo setup, and it chooses the optimal reset price P_t^* to solve:

$$\max_{P_t^*} \quad \mathbb{E}_t \sum_{\tau=0}^{\infty} \gamma^\tau \beta^\tau \frac{u'(C_{t+\tau})}{u'(C_t)} \frac{P_t}{P_{t+\tau}} \left[P_t^* - P_{t+\tau}^m \right] Y_{t+\tau|t},$$

where $Y_{t+r|t}$ is the amount sold in period $t+\tau$ by a firm that last reset its price in period t. The conditions describing the optimal behavior of retail firms are given in Appendix B.6.

3.6 Government

Monetary policy is characterized by a Taylor rule with interest rate smoothing:

$$i_t = (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon_t^i, \tag{3.21}$$

where $\bar{\imath}$ is the steady-state nominal rate, $\rho_i \in [0,1]$ reflects interest rate inertia, and ϵ_t^i is an exogenous shock to monetary policy. Note that the policy rate is also the rate on reserves, which is the same as the return on bonds. For simplicity, we assume government spending is a constant fraction of output

$$G_t = gY_t. \tag{3.22}$$

We also assume that the government balances its budget period-by-period. Therefore, the lump sum transfers from the government to the household are given by the proceeds from seigniorage (covering cash, reserves, and CBDC) net of government expenditures.

3.7 Resource Constraint and Shocks

Output is divided between consumption, investment (for the two types of capital), government expenditure, and adjustment costs. The economy-wide resource constraint is thus given by

$$Y_t \;=\; C_t + I_t + G_t + \Gamma_t, \tag{3.23}$$

where Γ_t represents all additional costs:

$$\begin{aligned}\Gamma_t \;&=\; \mu^l \frac{L_{t-1}}{P_t} + \mu^d \frac{D_{t-1}}{P_t} + \varsigma \frac{F_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \varrho \frac{Q_{t-1}}{P_t} K_t^P \\ &+\; \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t} + \Omega\left(\frac{CBDC_t}{P_t Y_t}\right).\end{aligned} \tag{3.24}$$

In the previous expression, Ω represents the costs for the government of issuing CBDC, expressed as a function of the ratio of real CBDC to output. In our baseline specification we set $\Omega = 0$, but we discuss the consequences of this CBDC issuance cost in Section 6. Finally, we assume that the technology process follows an AR(1):

$$A_t \;=\; A_{t-1}^{\rho_a} \exp(\epsilon_t^a). \tag{3.25}$$

The full set of dynamic equations characterizing the equilibrium of the model is given in Appendix B.8.

4 Calibration

We calibrate the model to the U.S. economy at a quarterly frequency. The parameters associated with the financial block are particularly important for the quantitative realism of the model. We lay out our calibration in four parts. First, we discuss parameters that are set externally or are relatively standard in the literature. Next, we collect parameters related to the deposit side of the model, followed by the ones associated with the loan side, and finally we discuss all other bank parameters. Table 4.1 lists the full set of parameters, their values, and calibration targets.

4.1 Nonbank Parameters

The quarterly discount factor, β, is set to 0.995, giving an annualized policy rate of 2%, which is consistent with the low interest rates that prevailed in the United States before the COVID-19 pandemic. We use the standard functional forms for $u(c)$ and $v(n)$:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \qquad v(n) = \chi \frac{n^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \tag{4.1}$$

and set the intertemporal elasticity of substitution, $1/\sigma$, and the Frisch elasticity, η, both to one. The former is consistent with balanced growth in our model, while the latter is consistent with the upper bound for macro elasticities in Chetty et al. (2011). The disutility from labor, χ, is chosen such that steady-state labor is normalized to one-third.

The capital income share, α, is one-third and the depreciation rate, δ, is 0.02 quarterly, or 8% annually. The functional form for the investment adjustment cost function is $\Xi(x) = \kappa_I/2 \cdot (x-1)^2$, where κ_I is set to 2 as in Sims and Wu (2021). We set the elasticity of substitution between differentiated retail goods, φ, to 6, which is consistent with a steady-state markup of 20%. The Calvo parameter, γ, capturing the probability that a retail firm is not allowed to adjust its price, is set to the typical value of 0.75, implying an average duration between price updates of one year. The Taylor rule parameters are set to standard values: the persistence parameter, ρ_i, is 0.8 and the response to inflation, ψ_π, is 1.5. Steady state inflation is assumed to be zero for simplicity. Finally, the ratio of government spending to GDP, g, is set to 0.2, roughly consistent with historical U.S. data.

4.2 Deposit Parameters

On the deposit side, it is important that our model matches empirical estimates for deposit rates at different levels of the policy rate, and therefore deposit spreads, to align the model with the strength of bank deposit market power in the data. Namely, we target four moments: (i) a deposit rate of 0% at a policy rate of 0.5% (taken from Ulate, 2021), (ii) a deposit rate of 0.75% at a policy rate of 2%, (iii) a deposit rate of 1.25% at a policy rate of 3%, and (iv) a deposit rate of 2% at a policy rate of 4.5%, where the last three targets are estimated from historical Ratewatch data.[9] We match these four moments

[9] We compute a historic deposit rate series that resembles the one in our model by using data from Ratewatch on checking and saving deposit rates and weighting those by the historical shares of such deposits based on data from the H.6 releases from the Federal Reserve Board of Governors (sample: 2000:M1-2020:M4). Comparing the resulting series to the federal funds rate yields approximately the calibration targets between the policy rate and the deposit rate stated in the text.

Table 4.1: Calibration.

Param.	Value	Description	Target or source
Panel A. Nonbank			
β	0.9950	Discount factor	2% policy rate
χ	8.8487	Disutility of labor	One-third S.S. labor
η	1.0000	Frisch elasticity	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
α	0.3333	Capital share	Standard
δ	0.0200	Depreciation rate	8% annual dep.
κ_I	2.0000	Investment adjustment cost	Sims and Wu (2021)
φ	6.0000	Elasticity of subs. b/t diff. goods	20% mark-up
γ	0.7500	Prob. of keeping prices fixed	One-year duration
ψ_π	1.5000	Inflation coefficient, Taylor rule	Standard
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard
g	0.2000	Steady state G/Y	Standard
Panel B. Deposit side			
n	1.1685	Number of banks	Deposit rate target #1
θ	554.21	E.o.S. between liquid instruments	Deposit rate target #2
ε^d	661.36	E.o.S. between banks in deposits	Deposit rate target #3
μ^d	-0.20%	Cost of issuing deposits	Deposit rate target #4
γ_m	0.3005	Importance of cash in liquidity	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3990	Importance of deposits in liquidity	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3005	Importance of CBDC in liquidity	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
a	0.8764	Parameter in liquidity function Φ	$\mathcal{L}/Y = 2.4$ quarterly
b	1.0700	Parameter in liquidity function Φ	Estimation
q	-0.1615	Parameter in liquidity function Φ	S.S. relationship
Panel C. Loan side			
ψ	0.3000	Importance of pledgeable capital	Crouzet (2021)
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)
ε^l	40.013	E.o.S. between banks in loans	$i^l = i + \rho \Rightarrow \theta^k = f(\varepsilon^l)$
θ^k	5.0000	Subs. between NP and P capital	Feasible region
Panel D. Joint bank side			
ω	0.6780	Fraction staying in bank	$L/F = \nu$ in S.S.
ς	0.0474	Bank managerial cost	2.25% S.S. ROE
ν	9.0000	Loan-to-equity ratio target	Ulate (2021)
κ	0.0012	Cost of deviating from target ratio	Ulate (2021)

Notes: This table contains the parameter values used in the calibration, together with their description and their source or target. All interest rates are annualized.

by jointly calibrating n, the number of banks, θ, the elasticity of substitution between different liquidity-providing instruments, ε^d, the elasticity of substitution between banks in deposits, and μ^d, the cost of issuing deposits. This exercise yields estimates of $n = 1.16$, $\theta = 554$, $\varepsilon^d = 661$, and $\mu^d = -0.0020$ (20 basis points quarterly).

Two points are noteworthy about these estimates. First, our calibration requires a fairly low value of n. While this estimate is not an integer, and it is certainly lower than the actual number of U.S. banks, we do not intend it to be taken literally. Rather, it allows the model to match the relationship between the deposit rate and the policy rate while remaining parsimonious.[10] Second, the negative value of μ^d implies a "benefit" of issuing deposits instead of a cost, and the calibrated value is close to the one in Ulate (2021) of -0.0025. In reduced form, the negative μ^d could capture complementarities between deposit taking and lending, fees charged to depositors, or benefits of using a relatively stable source of funding (see also Abadi et al., 2022).[11]

The exogenous shares of cash, deposits, and CBDC (γ_m, γ_d, and γ_{cbdc}) are set to match two targets together with the model-implied restriction that $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. The first target is the pre-CBDC deposit-to-liquidity ratio $d/\mathcal{L}$ in steady state. We obtain an estimate for this ratio using historical data on checking deposits, savings deposits, and currency holdings, and constructing $\mathcal{L}$ based on equation (3.1) given our calibration for θ. For the sample 1975:Q1-2020:Q1, we find that $d/\mathcal{L}$ is approximately 0.8 on average. The second target is taken from surveys such as Bidder et al. (2024). When asked about their potential CBDC usage, people report that they would hold roughly the same amount of CBDC and cash if CBDC paid no interest, which implies $\gamma_{cbdc} = \gamma_m$. Appendix C.1 discusses how our results change if we vary this target to alternative ratios like $\gamma_{cbdc} = 0.5\gamma_m$ or $\gamma_{cbdc} = 2\gamma_m$. As expected, the higher γ_{cbdc}, the greater the welfare benefits of introducing CBDC. Putting all of our targeted moments together, our baseline calibration yields $\gamma_m = \gamma_{cbdc} = 0.3005$ and $\gamma_d = 0.3990$.[12]

The cost-of-liquidity function is parameterized as $\Phi(\mathcal{L}) = a\mathcal{L}^b - q$. The elasticity

[10] In particular, n is crucially related to the pass-through of the policy rate to the deposit rate, which is found to be less than unity. Drechsler et al. (2017), for example, document a pass-through of 0.39 among large banks and 0.46 on average (see pages 1821 and 1824 therein). In Appendix A.3, we obtain a closed-form expression for the pass-through of the policy rate to the deposit rate and show that the crucial parameter that governs this relation is the number of banks, n.

[11] Note that any further fixed costs of operating the deposit franchise are incorporated in the managerial costs of operating the bank, ς, which are substantial in our baseline calibration as described in the text.

[12] An alternative to assess the demand for CBDC uses demand system approaches. For Canada, Li (2023) finds that people would hold between 4% and 20% of their liquid assets in CBDC when taking into account that banks adjust deposit rates. For the Euro area, Lambert et al. (2024) find similar estimates of 3% and 28%. Our model produces a number of 13% for a zero CBDC rate and a 2% policy rate (see Section 5.1), which lies approximately in the middle of those estimates.

parameter b is calibrated starting from equilibrium condition (3.9),

$$\frac{1 + i_t^{\mathcal{L}}}{1 + i_t} = ab\mathcal{L}_t^{b-1}. \tag{4.2}$$

We proceed to take logs and subtract the resulting equation from its lagged counterpart, giving

$$s_t - s_{t-1} = (b - 1) \cdot [\ln(\mathcal{L}_t) - \ln(\mathcal{L}_{t-1})], \tag{4.3}$$

where $s_t \approx i_t^{\mathcal{L}} - i_t$. As described above, we construct a time series for $\mathcal{L}_t$ using equation (3.1). Similarly, we measure $i_t^{\mathcal{L}}$ based on equation (3.7) using a historic deposit rate series (as described in footnote 9). We estimate (4.3) for the sample 2000:M1-2020:M4, which is the maximum time span across all data series, and obtain $b = 1.07$.

Finally, the other parameters a and q inside the cost function for liquidity (Φ) are selected to match a liquidity-over-GDP ratio $\mathcal{L}/Y$ of 2.4 at the quarterly frequency, and the relationship that $\Phi(\cdot) = m + d + cbdc$ in steady state. This approach yields the estimates $a = 0.8764$ and $q = -0.1615$, respectively.

4.3 Loan Parameters

Next, we turn to parameters related to the loan side of the model. The parameter ψ governs the importance of pledgeable capital for aggregate capital in (3.11) and therefore pins down the share of bank borrowing. Crouzet (2021) shows that this share has declined to around 30% for the most recent years, and we calibrate ψ accordingly.

For the costs of bank and bond borrowing, we obtain estimates from Schwert (2020) who compares bank loan rates and secondary bond quotes for the same firms on the same date. Schwert (2020) finds that loan and bond spreads are similar for investment-grade firms. However, estimations suggest that the average bond-implied loan spread should be around 50% of the average all-in-drawn spread of 2.8% since loans are less risky due to higher recovery rates in bankruptcy. Schwert (2020) associates the remaining premium to banks' market power in the loan market. To match these numbers, we assume that bond and loan spreads are the same in steady state, that is, $\varrho = i^l - i = 2.8\%$ annually. However, banks face half of the costs of issuing credit compared with the bond market, resulting in $\varrho = 0.7\%$ for the costs of issuing bonds and $\mu^l = 0.35\%$ for the costs of issuing loans, both at the quarterly frequency.

Based on equations (3.19) and (3.20), the equivalence between bond and loan spreads in steady state implies the following relationship between ε^l and θ^k :

$$n(i+\varrho+\delta) \;=\; \left[(n-1)\varepsilon^l + \theta^k - \psi\left(\theta^k - \frac{1}{1-\alpha}\right)\right](\varrho - \mu^l), \tag{4.4}$$

where all other parameters apart from ε^l and θ^k are pinned down. Therefore, we can interpret the elasticity of substitution between bonds and loans, θ^k, as the remaining free parameter, and, conditional on that, back out ε^l from (4.4). While we lack an empirical target to pin down θ^k exactly, the model implies that it must lie in a feasible region between 1 and 11.8.[13] For our baseline specification, we choose $\theta^k = 5$ as a suggestive value roughly in the middle of the feasible set, and show the robustness of our main results to alternative values in Appendix C.2.[14]

4.4 Other Bank Parameters

Besides parameters related to the loan and deposit sides, a few other bank-related ones remain. The function for the cost of deviating from the target loan-to-equity ratio is parameterized as: $\Psi(L/F) = \kappa\nu x\,(\ln(L/F) - \ln\nu - 1) + \kappa\nu^2$, following Ulate (2021). The loan-to-equity target ratio, ν, and the cost of deviating from this ratio, κ, are also taken from that paper, matching a steady-state loan-to-equity ratio of 9 and using cross-sectional relations between loan rates, loan amounts, and bank capital to obtain a value of κ of 12 basis points. We also check the robustness of our results across different values of κ in Appendix C.2.

A banks' managerial cost, ς, helps determine their profitability. Using Call Report data for commercial banks over 1984:Q1-2022:Q3, we find an average annualized return on assets close to 1%. Given the loan-to-equity ratio of 9, this implies a quarterly return-on-equity of 2.25% and we calibrate ς to match this moment in steady state. The fraction of bank profits that stay within the bank and are not paid out as dividends, ω, is calibrated such that, in the initial pre-CBDC steady state, the loan-to-equity ratio L/F is equal to its target ν.

4.5 Loan and Deposit Spreads

To provide some intuition for the behavior of spreads in the calibrated model, Figure 4.1 displays the loan rate and the deposit rate for different levels of the policy rate (which

[13] The lower bound of one comes from the assumption that pledgeable and non-pledgeable capital are substitutes instead of complements. The upper bound for θ^k is ε^l, due to the nested-CES structure of the model. Given our remaining calibration and equation (4.4), this implies an upper bound for θ^k of 11.8.

[14] Based on a similar model structure, Buchak et al. (2024) estimate $\theta^k = 3.87$ which lies in the possible range we consider and is close to our baseline value of $\theta^k = 5$, providing an external validation of our choice.

is also shown as the 45-degree line). The loan spread ranges between 2.3% and 3.5%. It is larger for higher levels of the policy rate. That is because banks gain market power at higher policy rates, raising their profitability and market share relative to bonds and increasing the endogenous loan elasticity and therefore their loan markup over the policy rate. We can see this mechanism from equations (3.18)-(3.20).

The deposit rate is below the policy rate for all positive values, but is close to the policy rate for rates below -1%. The deposit spread rises with higher levels of the policy rate and bank market power. However, this relation is nonlinear. For policy rates between -1% and 5%, the deposit spread widens substantially as the policy rate increases, as targeted by our calibration strategy over these values (the yellow dots in Figure 4.1 denote our calibration targets for the deposit rate as a function of the policy rate). The widening of the deposit spread becomes smaller when the policy rate is above 5% and stabilizes at higher policy rates. This behavior of the deposit spread is consistent with the data,

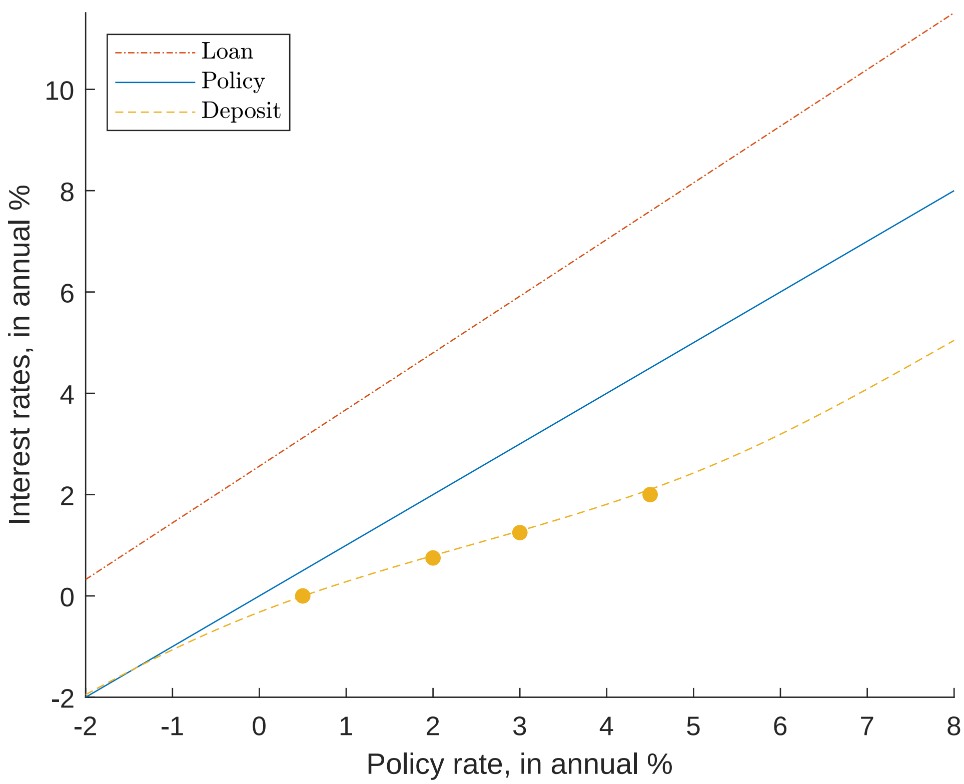

Figure 4.1: This figure shows the loan rate (dash-dot orange line) and the deposit rate (dashed yellow line) obtained in the baseline calibration of the model as a function of the policy rate. The policy rate is also plotted as the 45-degree line for comparison (solid blue line). The yellow dots denote the calibration targets for the deposit rate as a function of the policy rate.

even though we do not target deposit rates for such high levels of the policy rate in our calibration. Thus, this provides an external validation for the empirical fit of the model.[15] Appendix A.3 provides further details on the behavior of the pass-through of the policy rate to the deposit rate in our model.

5 Implications of CBDC Introduction

In this section, we discuss the implications of CBDC introduction through the lens of our full DSGE model. First, we focus on comparing how the economy differs between an initial steady state where CBDC is not used and a final steady state where CBDC is available, while considering various remuneration schedules for CBDC. Throughout this section, we frequently refer to the "welfare change" from introducing a CBDC, which is formally the multiplicative consumption-equivalent variation, in percent, required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady state (see Appendix B.10 for details). Second, we also discuss how the economy responds to shocks around the pre-CBDC and various post-CBDC steady states. Appendix C.3 discusses the transition between steady states.

5.1 CBDC Introduction for Different CBDC Rates

We first focus on our baseline calibration, where the steady-state policy rate is 2%, and analyze outcomes of CBDC introduction for different levels of the interest rate paid on CBDC. Figure 5.1 shows the welfare change from CBDC introduction, the deposit-to-GDP ratio, and the CBDC-to-GDP ratio across CBDC rates between -1% and 3% annually. As the rate paid on CBDC increases, the CBDC-to-GDP ratio rises and the deposit-to-GDP ratio decreases monotonically. Intuitively, as the CBDC interest rate becomes more negative, the CBDC-to-GDP ratio tends to zero, since households do not want to use a very unattractive liquid instrument. In the limit, when the CBDC rate is -100% quarterly, households do not use CBDC at all, which corresponds to our pre-CBDC scenario.

Most importantly, the welfare change from CBDC introduction displays an inverted U-shape with respect to the interest rate paid on CBDC. It tends to zero when the CBDC rate approaches -100%, becomes negative for very high CBDC rates, and achieves a positive maximum of approximately 27 basis points (of initial steady-state consumption) when the CBDC rate is approximately 0.8% per year. This welfare gain is higher than

[15]Such a behavior of deposit rates is reminiscent of deposit betas that are not constant but rise with higher market rates, as documented in Greenwald et al. (2023), for example.

the one of approximately 22 basis points when the CBDC rate is 0%, an often-discussed remuneration level by central banks that consider introducing a CBDC. Interestingly, the welfare-maximizing CBDC interest rate of approximately 0.8% per year is very close to the deposit rate in the pre-CBDC steady state.

The impact of CBDC on welfare in our model depends on three different channels. First, a CBDC can curtail commercial bank monopoly power and thereby increase the deposit rate that households get paid. Second, households like some of the characteristics that CBDC has to offer. For example, a CBDC can be used for electronic transactions while it is also a direct liability of the central bank and therefore fully insured. Such benefits are jointly captured in the model with a positive γ_{CBDC} (which is "present" even in the pre-CBDC steady state). Households therefore benefit when a CBDC is introduced because it allows them to better distribute their usage across the available liquid instruments. Third, despite a higher deposit rate, some deposits flow out of the banking system when a CBDC is introduced. The higher deposit rate and the reduced amount of deposits imply

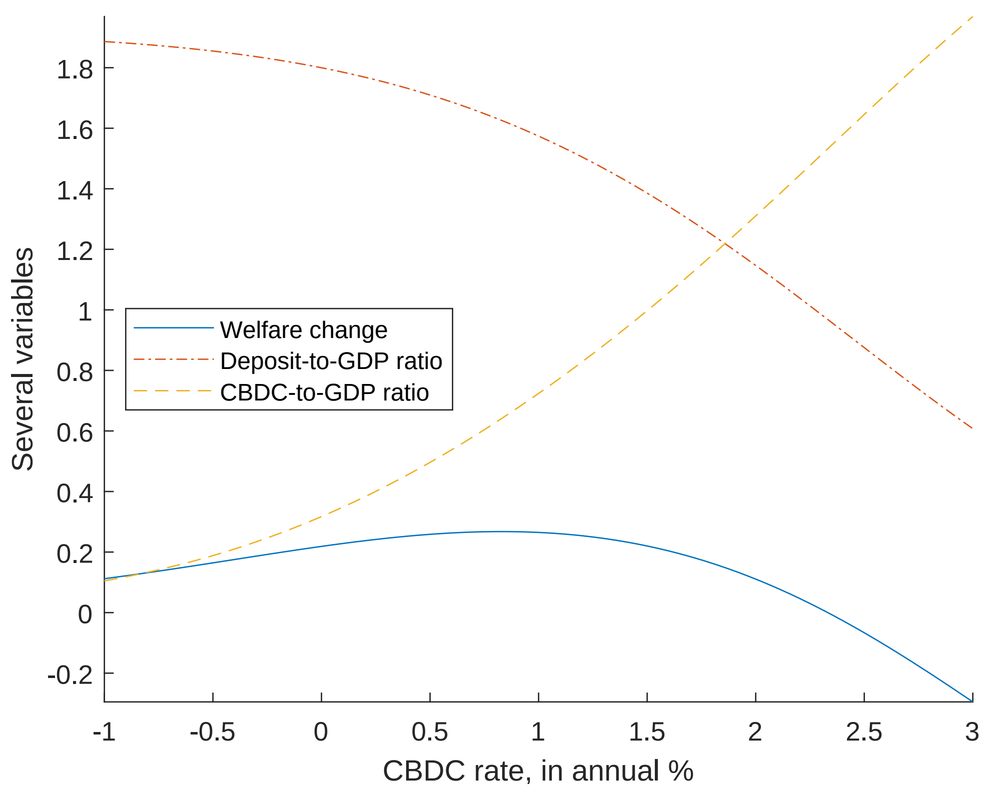

Figure 5.1: This figure displays the behavior of some important variables for different levels of the CBDC interest rate. The welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, is in blue, the deposit-to-GDP ratio is in orange, and the CBDC-to-GDP ratio is in yellow.

that bank equity declines, which in turn reduces credit supply, raises the cost of capital for firms, and lowers welfare. For an in-depth discussion on the intuition of these results, see Section 2.

For low and moderate levels of the CBDC rate, the first two channels described in the previous paragraph dominate the third one, leading to an increase in overall welfare due to CBDC introduction. However, for high levels of the CBDC rate, the bank disintermediation channel dominates, leading to a fall in overall welfare as observed in the right tail of the blue line in Figure 5.1. Section 6 provides a full decomposition of the quantitative importance of the three channels.

Figure 5.2 plots how some other variables of interest behave before and after the introduction of CBDC for different levels of the CBDC rate. The deposit spread is 120 basis points before CBDC. It falls to 96 basis points when CBDC is introduced with a rate of 0%, but to 72 basis points when CBDC is introduced with the optimal rate of 0.8%. Bank leverage is nine in the initial steady state but increases when CBDC is introduced, a pattern that intensifies as the rate on CBDC increases. When bank leverage increases, banks charge a higher loan rate, which explains the negative welfare impact of a CBDC that

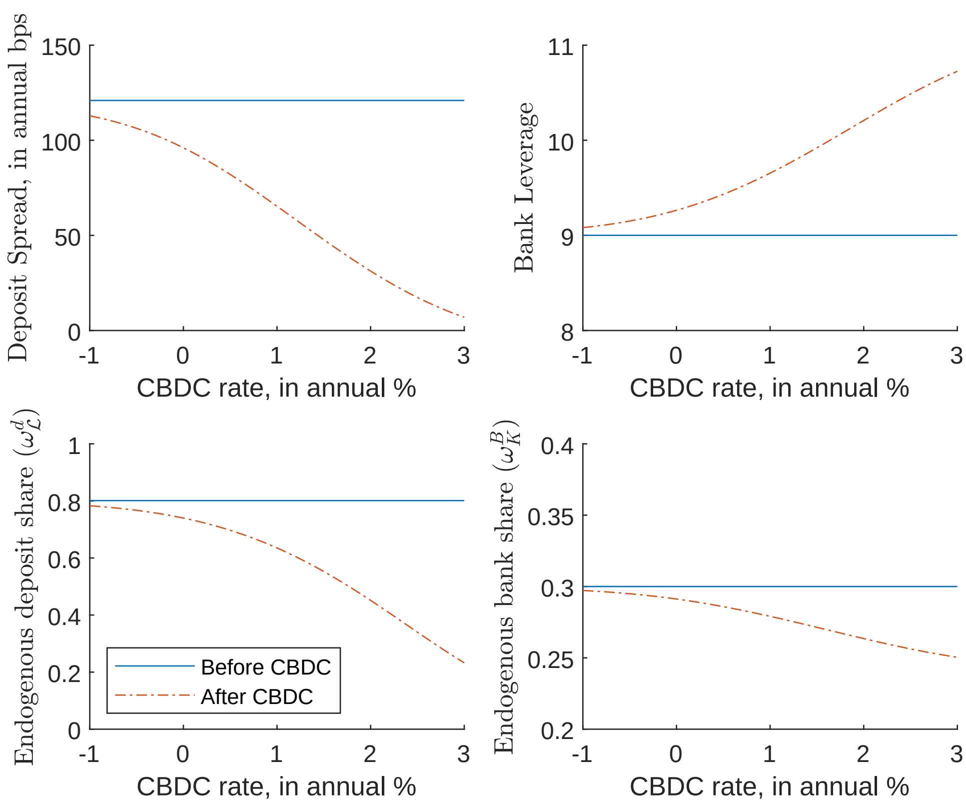

Figure 5.2: This figure shows different variables of interest before and after the introduction of a CBDC for different levels of the CBDC interest rate.

pays a very high interest rate. Both the endogenous deposit share and the share of bank lending (0.8 and 0.3, respectively, in the pre-CBDC steady state) decrease with the introduction of CBDC, and fall more as the rate on CBDC increases.

In Appendix C.2, we discuss the ratio of changes in bank credit to deposit losses in the aggregate banking sector due to the introduction of CBDC in our model. We elaborate on how this ratio depends on the elasticity of substitution between bank and nonbank borrowing, θ^k. Furthermore, we provide evidence that this ratio is not necessarily a good measure of the welfare implications of introducing a CBDC.

5.2 CBDC Introduction for Different Policy Rates

Next, we change the nature of the exercise that we perform. Instead of analyzing CBDC introduction for a given steady-state policy rate but different levels of the interest rate on CBDC, we keep the interest rate on CBDC constant at 0%, as envisioned by many central banks that consider introducing a CBDC, and change the steady-state level of the policy rate. We achieve the different steady-state levels of the policy rate by recalibrating the discount factor β, while keeping the rest of the parameters of our baseline calibration constant (however, our results are robust to recalibrating a larger set of parameters; see Appendix C.4).[16]

Figure 5.3 illustrates how several outcome variables of interest behave for steady-state policy rates between -2% and 8% annually. As in Figure 5.1, we consider the welfare change from CBDC introduction, the CBDC-to-GDP ratio, and the deposit-to-GDP ratio. As the steady-state policy rate increases, the CBDC-to-GDP ratio decreases monotonically, whereas the deposit-to-GDP ratio increases monotonically. Additionally, when the policy rate increases, the CBDC-to-GDP tends to zero, since households do not want to use a liquid instrument that pays relatively little compared to deposits. The welfare gains from CBDC introduction have an approximately monotonic shape: they roughly fall with the steady-state policy rate and tend to zero as the policy rate rises, precisely because CBDC is mostly unused in such a scenario.

Figure 5.4 replicates Figure 2.1 for the full DSGE model instead of the simple static model in Section 2. While some magnitudes change slightly due to various new ingredients and the general equilibrium nature of the model, the intuition carries over from

[16]Notice that the most important parameters in our model, namely the deposit-side banking parameters ε^d, θ, n, and μ^d, are calibrated to match deposit rates across levels of the policy rate. Therefore, these parameters do not need to be recalibrated when the discount factor is changed. Appendix C.4 shows that our results in this subsection and the next are robust to recalibrating additional parameters such that certain targets continue to be matched across different levels of the policy rate.

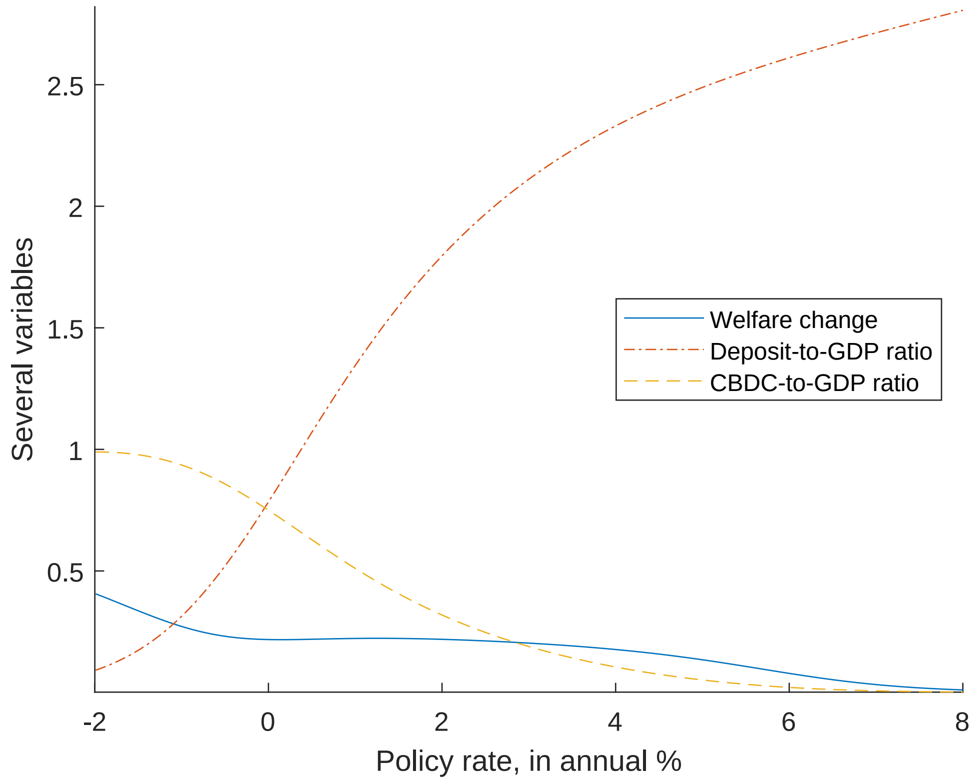

Figure 5.3: This figure displays the behavior of some important variables for different levels of the policy rate. The welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, is in blue, the deposit-to-GDP ratio is in orange, and the CBDC-to-GDP ratio is in yellow.

Section 2. The deposit spread falls the most due to the introduction of CBDC for intermediate levels of the steady-state policy rate of approximately 2.7% annually. For very high or very low levels of the policy rate, the endogenous deposit share changes little with the introduction of CBDC and the deposit spread therefore does not react much.

5.3 Welfare-Maximizing CBDC Rate across Policy Rates

In Section 5.1, we showed that, for our baseline steady-state policy rate of 2%, the welfare-maximizing level of the CBDC interest rate is around 0.8% per year. However, the effects of introducing a CBDC for a given interest rate also vary substantially depending on the steady-state level of the policy rate, as shown in Section 5.2. Therefore, a natural question that emerges is: what is the CBDC interest rate that maximizes the welfare change of introducing CBDC for each level of the steady-state policy rate? Figure 5.5 displays the answer to this question. In orange, the policy rate is shown as the 45-degree line, and in blue, the welfare-maximizing CBDC rate is plotted.

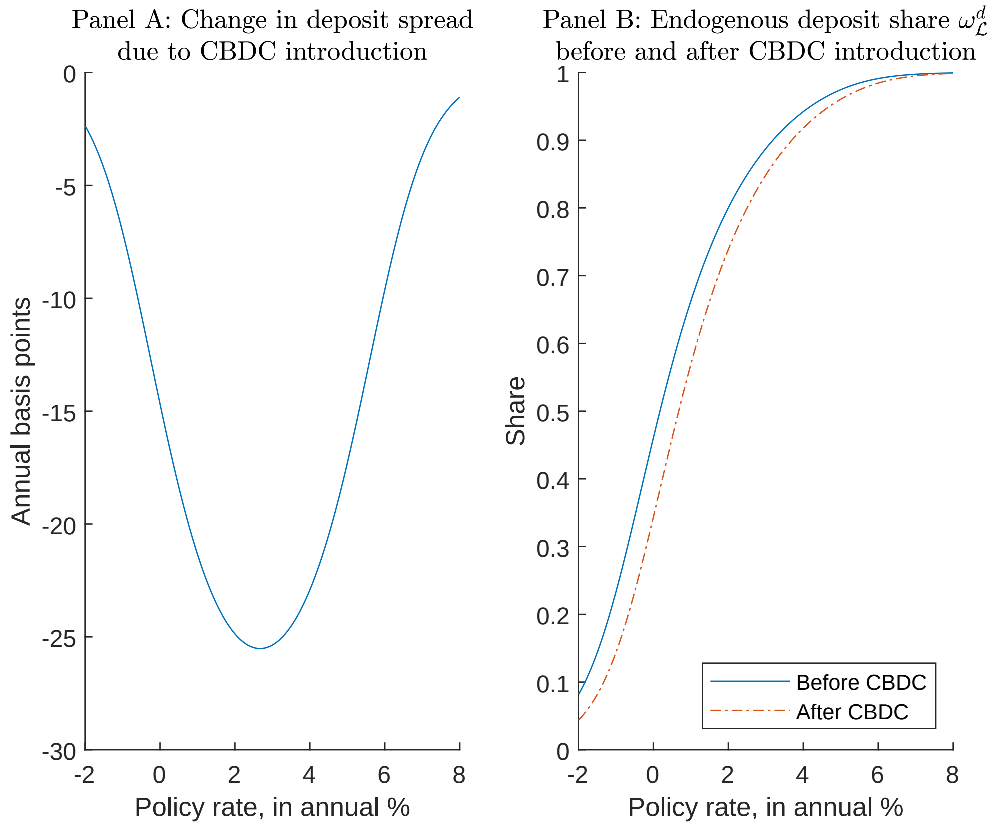

Figure 5.4: Panel A: Change in the deposit spread following the introduction of CBDC across different values of the policy rate. Panel B: Endogenous deposit share ($\omega^d_{\mathcal{L}}$) across different values of the policy rate before and after the introduction of CBDC. The figure uses the baseline calibration described in Section 4.

Starting on the left, for negative levels of the policy rate, the welfare-maximizing CBDC rate is negative and above the policy rate. The two cross at around -40 basis points annually. Subsequently, the welfare-maximizing CBDC rate is below the policy rate by roughly 1% annually. This welfare-maximizing CBDC rate as a function of the steady-state policy rate can be approximated fairly well by a rule-of-thumb CBDC rate that is the maximum between 0% and the policy rate minus 1%, as illustrated by the yellow line in Figure 5.5. While this approximate welfare-maximizing CBDC rate does not capture all the intricacies of the full welfare-maximizing CBDC rate (like being negative for negative levels of the policy rate), it is a rule of thumb that could easily be communicated by central banks and, in welfare terms, does almost as well as the welfare-maximizing rate, as shown below. This rule of thumb also has the benefit of avoiding negative rates on CBDC, which present a political economy concern for central banks due to the fear of the public that CBDC would be used to "expropriate their savings" with below-zero interest rates.

What is the intuition for the fact that the welfare-maximizing CBDC rate increases with

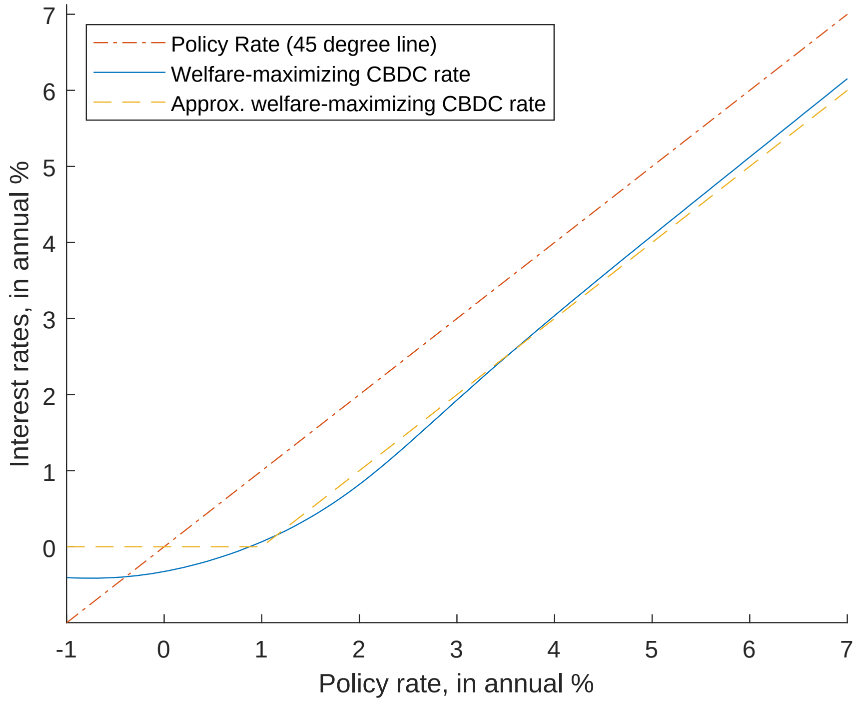

Figure 5.5: This figure displays the policy rate in orange (in both axes, so it is the 45-degree line), the welfare-maximizing level of the CBDC rate in blue, and an approximate welfare-maximizing rule-of-thumb rate, which is the maximum between 0 and the policy rate minus 1%, in yellow.

the policy rate? The higher the policy rate, the higher the CBDC rate needs to be to take a given share of the liquid-instruments market and therefore to curtail commercial-bank market power by a given amount.

To provide further intuition on this point, Figure 5.6 plots the deposit spread in the top row and the endogenous share of deposits in the bottom row—before and after the introduction of CBDC—across levels of the policy rate (on the x-axis) and for different CBDC remuneration schedules. In the left column, we present CBDCs that pay a constant interest rate, while in the right column, we present a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC rate, and a CBDC that pays the approximately welfare-maximizing rule-of-thumb rate described in Figure 5.5.

Importantly, for levels of the policy rate that are roughly above 2% per year, the welfare-maximizing policy rate achieves a stabilization of the deposit spread at around 70 basis points. Similarly, the endogenous deposit share is stabilized at around 65%. In contrast, a CBDC that pays a constant interest rate (regardless of the policy rate), can neither stabilize the deposit spread nor the deposit share, as visible from the left column of Figure 5.6. On the other hand, a CBDC that pays the policy rate reduces the deposit spread and the endogenous deposit share by too much relative to the welfare optimum.

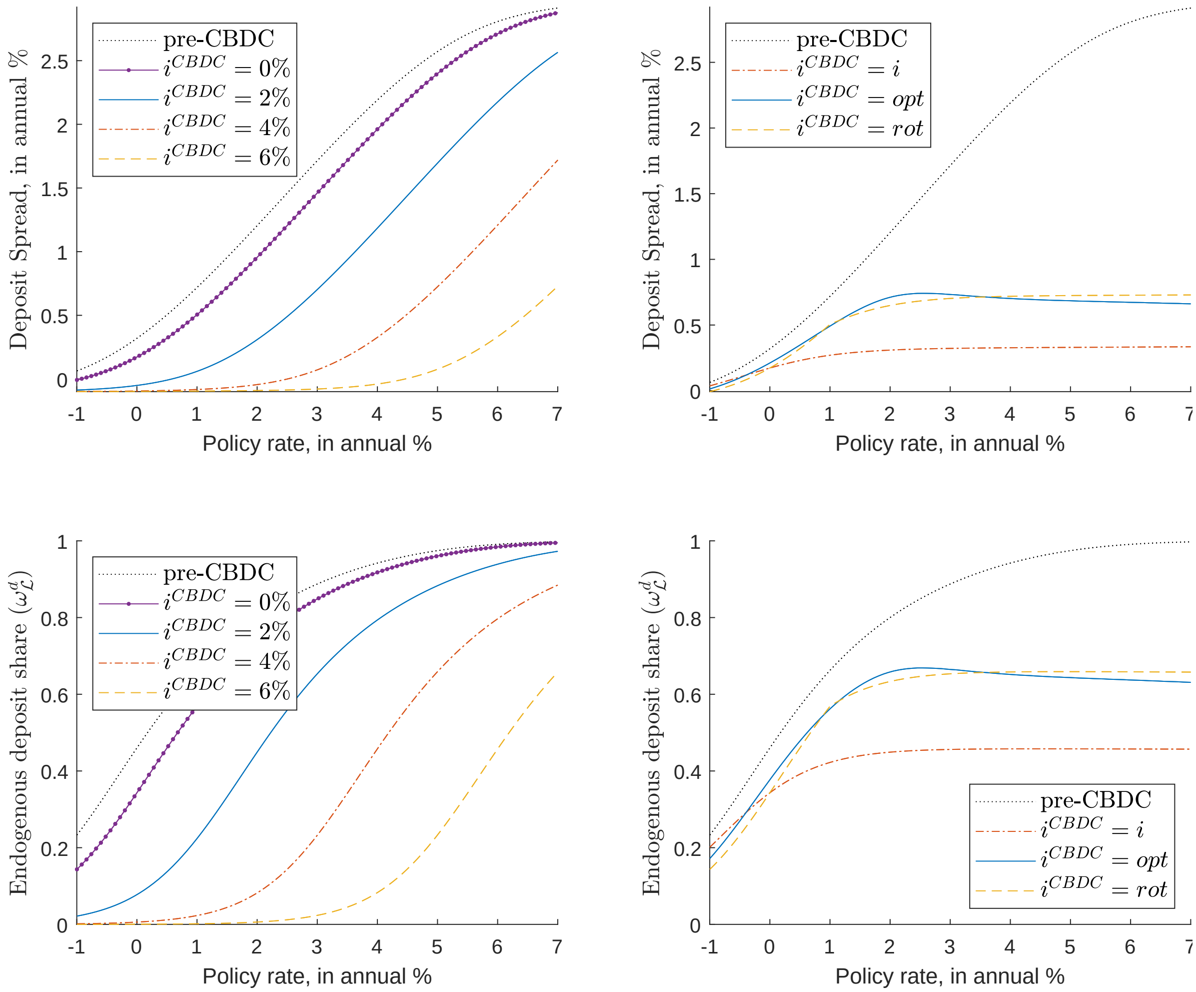

Figure 5.6: This figure shows the deposit spread (in the top row) and the endogenous share of deposits (bottom row), before and after the introduction of CBDC, across levels of the policy rate (on the x-axis), for different CBDC remuneration schedules. In the left column, we present CBDCs that pay a constant interest rate, while the right column shows a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC interest rate for each level of the policy rate, and a CBDC that pays the approximately welfare-maximizing rule-of-thumb (denoted "rot") rate described in Figure 5.5.

Figure 5.7 plots the welfare change from introducing CBDC across different levels of the policy rate (on the x-axis). The different lines represent the alternative CBDC remuneration schedules considered in Figure 5.6. As expected, the welfare change from the welfare-maximizing CBDC rate is the envelope of the other lines. Echoing the message from Figure 5.5, the line for the optimal CBDC rate touches the line for a constant CBDC rate at a level of the policy rate that is around 1% higher (e.g., the line for the optimal rate touches the line for $i^{CBDC} = 4\%$ at a policy rate of roughly 5%). Interestingly, the welfare

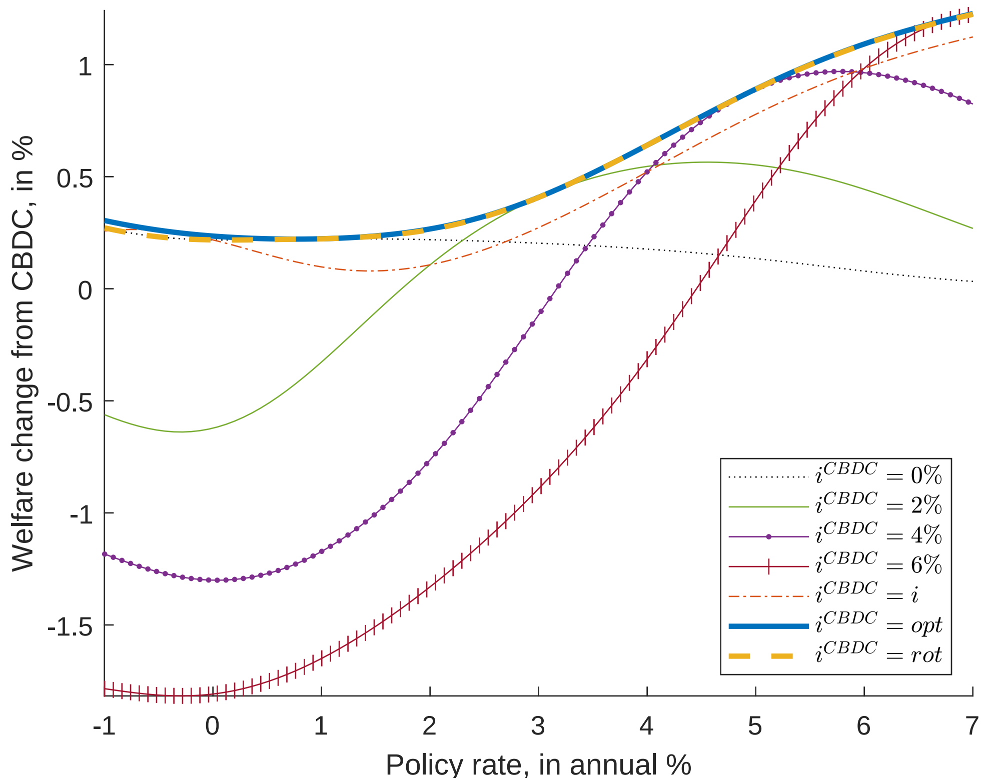

Figure 5.7: This figure shows the welfare change from the introduction of CBDC, in percent, across levels of the policy rate (in the x-axis), for different CBDC remuneration schedules described in Figure 5.6.

change of the rule-of-thumb rate is almost identical to the one of the welfare-maximizing rate. In contrast, a CBDC that pays the policy rate is only optimal when the policy rate is about -0.4% because that is the point at which the welfare-maximizing policy rate intersects the policy rate in Figure 5.5.

Finally, Figure 5.8 plots the change in the deposit spread from the introduction of CBDC across levels of the policy rate for the same CBDC remuneration schedules considered thus far. Note that the constant CBDC rates display a U-shape like the ones discussed in the left panel of Figure 2.2 (or Figure 5.4). By contrast, CBDCs that pay the policy rate, the welfare-maximizing CBDC rate, or the rule-of-thumb CBDC rate have downward-sloping lines like the ones discussed in the right panel of Figure 2.2. However, a CBDC that pays the policy rate decreases the deposit spread by substantially more than a CBDC that pays the welfare-maximizing rate. In turn, bank disintermediation is stronger and the welfare change is lower.

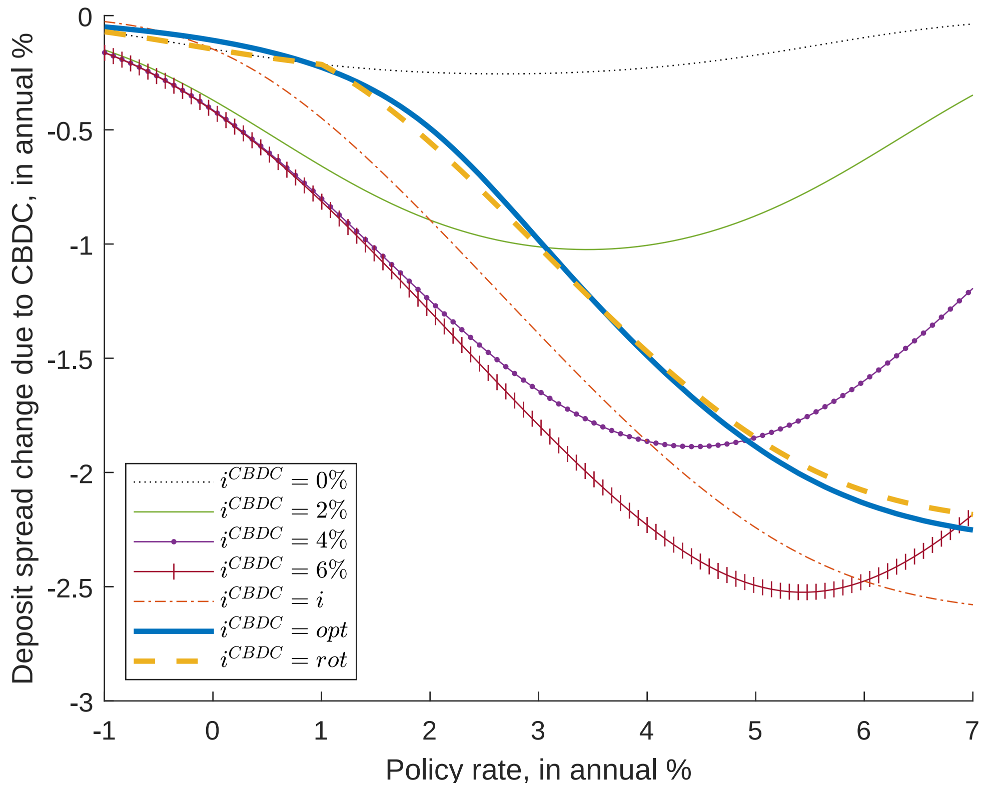

Figure 5.8: This figure plots the change in the deposit spread from the introduction of CBDC across levels of the policy rate (in the x-axis), for different CBDC remuneration schedules described in Figure 5.6.

5.4 Responses to Monetary Policy Shocks

Having already examined how the economy reacts to the introduction of CBDC by comparing the initial pre-CBDC steady state with the final post-CBDC steady state, we now turn to analyzing how the two economies differ in their response to transitory shocks around the respective steady states. We focus on the impulse responses to a monetary policy shock in this section, but our main findings also apply to technology shocks, as illustrated in Appendix C.5.

Figure 5.9 depicts the impulse responses of several important variables to a 50 basis point expansionary monetary policy shock for different CBDC remuneration schedules. The dotted black line shows the pre-CBDC case, which we compare to the following cases: (i) a CBDC that pays a constant interest rate of 0%, (ii) a CBDC that pays a constant rate at the welfare-maximizing level (of roughly 0.8% annually) for a policy rate of 2%, (iii) a CBDC that pays the policy rate minus 1%, corresponding to the approximated rule-of-thumb CBDC rate, and (iv) a CBDC that pays the policy rate.

Even though these regimes have significantly different welfare implications, the im-

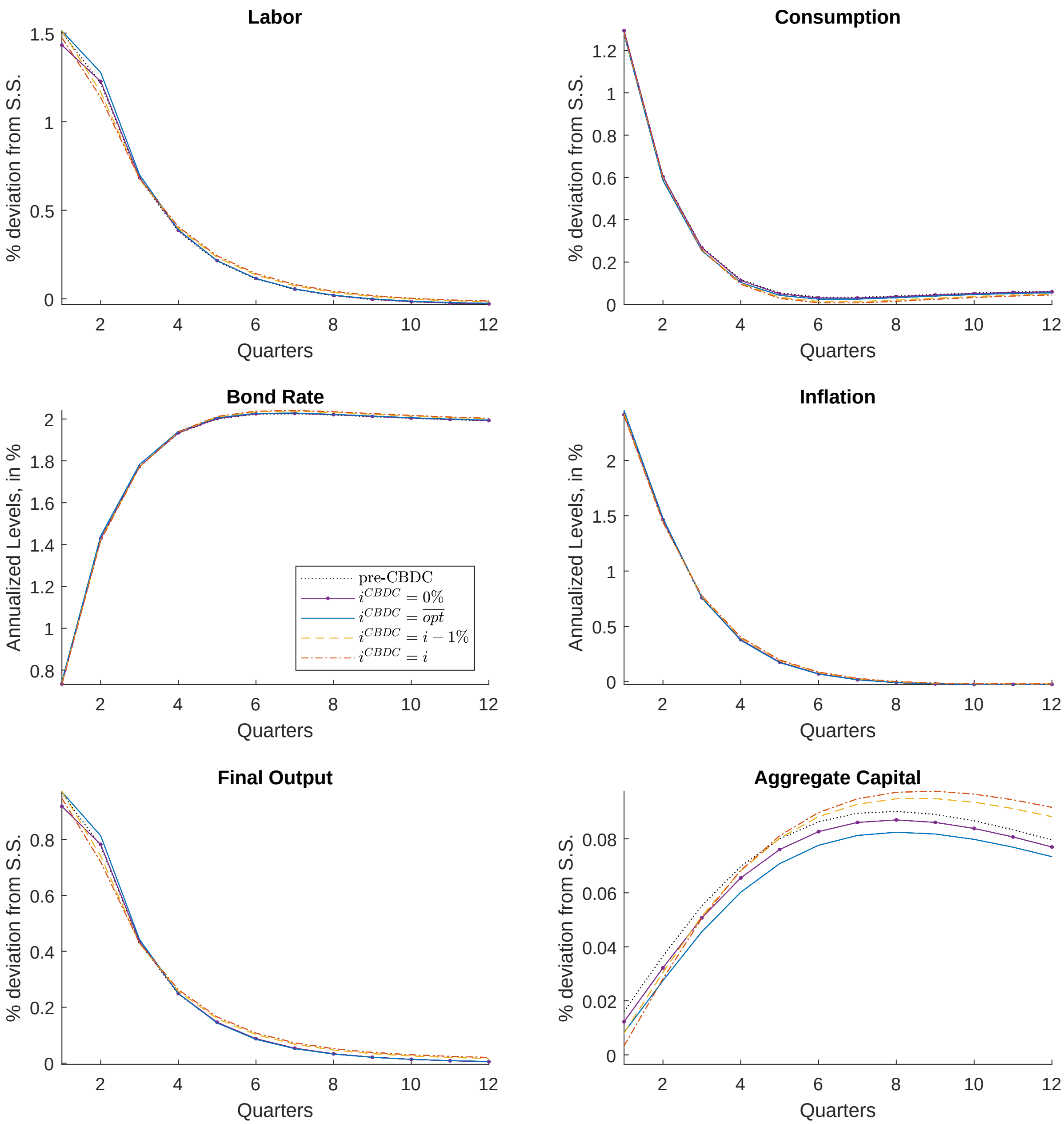

Figure 5.9: This figure depicts the impulse response functions to a 50 basis point expansionary monetary policy shock for different CBDC remuneration schedules. The line denoted $i^{CBDC} = \overline{opt}$ stands for a CBDC that pays a rate that is constant at the welfare-maximizing level for a steady-state policy rate of 2%, which is 0.8% per year as described in Section 5.1.

pulse responses are remarkably similar. To understand why the IRFs to transitory shocks do not change much with CBDC introduction or its remuneration schedule, it is useful to discuss the two main neoclassical channels of interest rate transmission in our model: the intertemporal substitution channel and the investment-based channels.[17]

Since the pass-through of the policy rate to the deposit rate improves after the introduction of CBDC, one could think that the intertemporal substitution channel could be stronger after CBDC introduction, particularly if the CBDC rate varies with the policy rate. However, the important thing to note here is that households in our model are allowed to save in bonds (although bonds do not provide liquidity benefits). Therefore, the relevant interest rate governing the consumption-saving trade-off in our model's Euler equation, expression (3.2), is the bond rate, which coincides with the policy rate. Since the pass-through of the policy rate to the interest rate governing the consumption-saving decision is already perfect, it cannot be improved by the introduction of CBDC.[18]

The investment-based channels also do not get substantially altered with CBDC introduction or its remuneration schedule. This is due to the fact that transitory monetary or technology shocks have a limited impact on bank equity and lending capacity.[19] Therefore, these shocks do not lead to major differences in investment and capital accumulation from interactions between the banking system and the existence of CBDC or its remuneration schedule.[20] To establish this in a scenario where bank lending mechanisms are given the best possible chance of making a difference, we perform an alternative calibration where the importance of banks in overall lending is much higher than in our baseline calibration (as in the European calibration we discuss in Section 6.3, where $\psi = 82\%$), and where the substitution between bank lending and bond borrowing is lower ($\theta^k = 1.1$). Even in such an economy where bank lending is substantially less dispensable than in

[17] We refer to Table 1 of Boivin et al. (2010) for a general classification of the transmission channels of monetary policy. What we call the "investment-based channels" are those referred to as "interest rate/cost-of-capital/Tobin's q" in Boivin et al. (2010), which may in turn be affected by loan rates and bank lending capacity. Asset-price channels based on wealth effects and exchange rate effects are not present in our model due to the absence of long-lived assets and the closed-economy nature of the model.

[18] In an alternative model where all households, or a substantial fraction of them, were only allowed to save in the liquid instruments, then the pass-through of the policy rate to these instruments would become more relevant for the intertemporal substitution channel and the IRFs to monetary policy shocks are likely to be more impacted by the introduction of CBDC. Importantly, even in an extended model with heterogeneous agents, relatively high-wealth agents would be the ones driving the bulk of the consumption-saving margin, and in the "real world" these high-wealth agents certainly have access to bonds and other financial instruments that move very closely with the policy rate, which justifies our modeling choice.

[19] Even though bank profitability and equity evolve differently across different CBDC remuneration scenarios, a 50 basis points monetary policy shock leads at most to a 2% deviation in bank equity from its steady-state value across a range of CBDC remuneration scenarios and values for the importance of banks in overall lending (ψ) and the elasticity of substitution between pledgeable and non-pledgeable capital (θ^k).

[20] We note that, in our baseline calibration for the U.S., the importance of banks in overall lending is relatively low ($\psi = 30\%$) and bank and bond borrowing are relatively good substitutes ($\theta^k = 5$). As a consequence, even if banks were differentially impacted by transitory shocks once CBDC is present and their lending capacity was affected to a different extent, this would not have major consequences for overall investment in the economy.

our baseline, the responses to transitory monetary policy shocks are still very similar across the different CBDC scenarios.

6 Discussion

In this section, we discuss additional issues that are relevant to paint a richer picture of the potential impacts of CBDC. First, we decompose the welfare impacts of CBDC into three channels: liquidity benefits, curtailing commercial bank monopoly power in deposits, and the bank disintermediation effect. Second, we consider potential costs of issuing a CBDC and how they can affect the welfare analysis. Third, we perform an alternative calibration for the Euro Area and analyze how it impacts our results. Fourth, we explore a link between bank reserves and wholesale funding in an extended framework where banks have access to such alternative funding sources. Fifth, we discuss bank taxes or subsidies as an alternative to curtailing bank monopoly power and the political viability (or lack thereof) of this strategy. Finally, we briefly touch upon CBDC holding limits.

6.1 Decomposing the Welfare Impacts of CBDC

There are three key channels driving the welfare effects of CBDC. First, introducing CBDC affects the amount of liquidity services that households obtain through the $\Phi(\cdot)$ function. Second, introducing CBDC can curtail the market power that commercial banks have in the deposit market. Third, CBDC introduction can reduce commercial bank profitability and decrease lending activity through a bank disintermediation effect. So far, our welfare analysis combined all of these channels. In this section, we decompose the welfare impact to better understand the quantitative importance of each channel.

Starting from the full model, we can shut down the bank disintermediation effect simply by setting the parameter κ, which determines the importance of bank equity for lending, to zero. The orange dash dotted line in Figure 6.1 depicts the welfare effect of introducing CBDC in an alternative economy where $\kappa = 0$. This economy only incorporates the effects of the liquidity services channel and the curtailing of commercial bank market power in deposits (we therefore denote it "Liq+MPD", for "Liquidity" and "Market Power in Deposits"). When $\kappa = 0$, bank equity is irrelevant for lending, and the bank disintermediation effect, which reduces the welfare gains of CBDC, is therefore absent. As a result, the orange line is substantially higher than the blue solid line, which contains all three channels and replicates the blue solid line in Figure 5.1. The difference between the orange dash dotted line and the blue solid line can be interpreted as the welfare impact

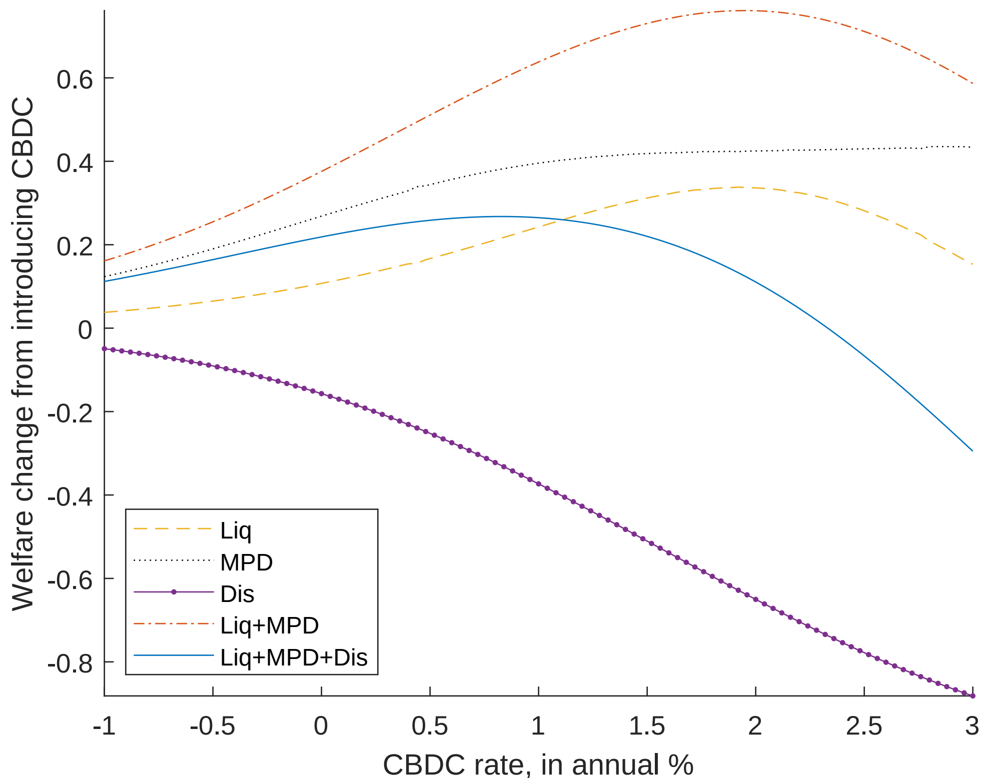

Figure 6.1: This figure decomposes the welfare gain from issuing CBDC for the baseline calibrated economy across three channels: increased access to liquid savings instruments (denoted "Liq" and depicted by the yellow dashed line), the effects of curtailing commercial bank market power in deposits (denoted "MPD" and depicted by the black dotted line), and the effects of disintermediating commercial bank lending activity (denoted "Dis" and depicted by the purple line with circular markers). Combinations of these channels are depicted by the orange dashed-dotted line ("Liq + MPD") and the blue solid line ("Liq + MPD + Dis") which contains all three channels present in the baseline model and therefore replicates the solid blue line in Figure 5.1. The main text contains details on how the decomposition across channels is performed.

of introducing CBDC solely through the bank disintermediation channel, and is given by the purple line with circular markers in Figure 6.1.[21]

Starting from the economy with $\kappa = 0$, we can further make the deposit market fully competitive by tending the number of banks (n) and the elasticity of substitution between banks in deposits (ε^d) to infinity.[22] The yellow dashed line in Figure 6.1 depicts the wel-

[21]While the pre-CBDC steady states are different between the economy with $\kappa = 0$ and our baseline with $\kappa > 0$, the differences for key variables (like consumption, labor, output, etc.) are generally smaller than 1%. Thus, differences in the welfare effects across these various steady states are still informative.

[22]We also set the banking costs ς, μ^d, and μ^l to zero and the elasticity of substitution between banks in loans (ε^l) to infinity to achieve a fully frictionless banking sector. The extra cost of borrowing in corporate bonds (ρ) is also set to zero, and the fraction of end-of-period-resources staying in the bank (ω) is chosen

fare effect of introducing CBDC in this alternative economy. With a perfectly competitive banking sector, the economy only incorporates the effects of the liquidity services channel (we therefore simply denote it "Liq") captured by the $\Phi(\cdot)$ function. The benefits of curtailing commercial bank market power are no longer present, and therefore the yellow dashed line is consistently below the orange dashed-dotted line for all levels of the CBDC rate. The difference between the orange dashed-dotted line and the yellow dashed line, which can be interpreted as the welfare impact of introducing CBDC just through the curtailment of commercial bank market power, is given by the black dotted line in Figure 6.1.

Besides the level differences, Figure 6.1 also shows how the welfare impacts of the various channels differ across CBDC rates. As expected, the disintermediation channel (purple line with circular markers) leads to negative welfare effects of introducing CBDC. These negative impacts increase with the CBDC rate, as a higher fraction of liquidity moves from deposits to CBDC, reducing commercial bank profitability and lending capacity. The effects of introducing CBDC through the liquidity channel (yellow dashed line) display an inverted U-shape, with a maximum around a CBDC rate of 2%. The decreasing portion to the right of 2% occurs because CBDC holdings are being inefficiently subsidized as the CBDC rate goes far above the policy rate.[23] Finally, the effects of introducing CBDC just from the curtailment of commercial bank monopoly power (black dotted line) increase with the CBDC rate. The slope of the black dotted line is steep when the CBDC rate is below 1.5% per year and relatively flat thereafter, as the majority of the gains from closing the deposit spread have been achieved once the CBDC rate hits 1.5%. For the welfare-maximizing CBDC rate of 0.8% per year, the welfare gains from the liquidity channel are 22 basis points, those from curtailing bank deposit market power are 38 basis points, and those from the disintermediation channel are -33 basis points, indicating that (in absolute value), all three channels are of roughly similar importance for our results.

6.2 Costs of Issuing CBDC

Our model has the flexibility to incorporate a cost of issuing CBDC through the function $\Omega(\cdot)$ introduced in equation (3.24) although our baseline specification set this cost to zero

to ensure a well-defined steady state where $L/F = \nu$. In this economy, the deposit rate, the loan rate, and the cost of borrowing in corporate bonds all equal the policy rate (i.e., $i^d = i^l = i + \rho = i$).

[23] Recall that the functional form for the Φ function displays satiation, so setting a rate on CBDC that is too high can lead households to use CBDC to very high levels that are individually optimal but socially inefficient. See Appendix A.4 for a closed form expression of the welfare gains of introducing CBDC stemming just from the liquidity channel.

for simplicity. In this section, we choose the cost of issuing CBDC at an illustrative fixed level of 20 basis points of initial steady state consumption since there is substantial uncertainty around the overall costs of CBDC issuance at this point. Under this calibration, the welfare gains from introducing CBDC are depicted in Figure 6.2.

While the level of CBDC issuance costs that we have chosen is illustrative, it serves to make the following point. If a CBDC is implemented with a rate that is too low or negative, it can lead to negative welfare effects, as the costs of operating the CBDC system have to be paid but there is little to no tangible gain due to low usage of CBDC. Importantly, this exercise also indicates that it is not appropriate to separate the questions of whether to issue a CBDC or not, and what interest rate to pay on CBDC. The two questions are fundamentally connected. Depending on parameter values and the costs of issuing CBDC, it is possible for a zero-interest CBDC to be welfare reducing in a situation where an interest-bearing CBDC is welfare improving. This is relevant, as some

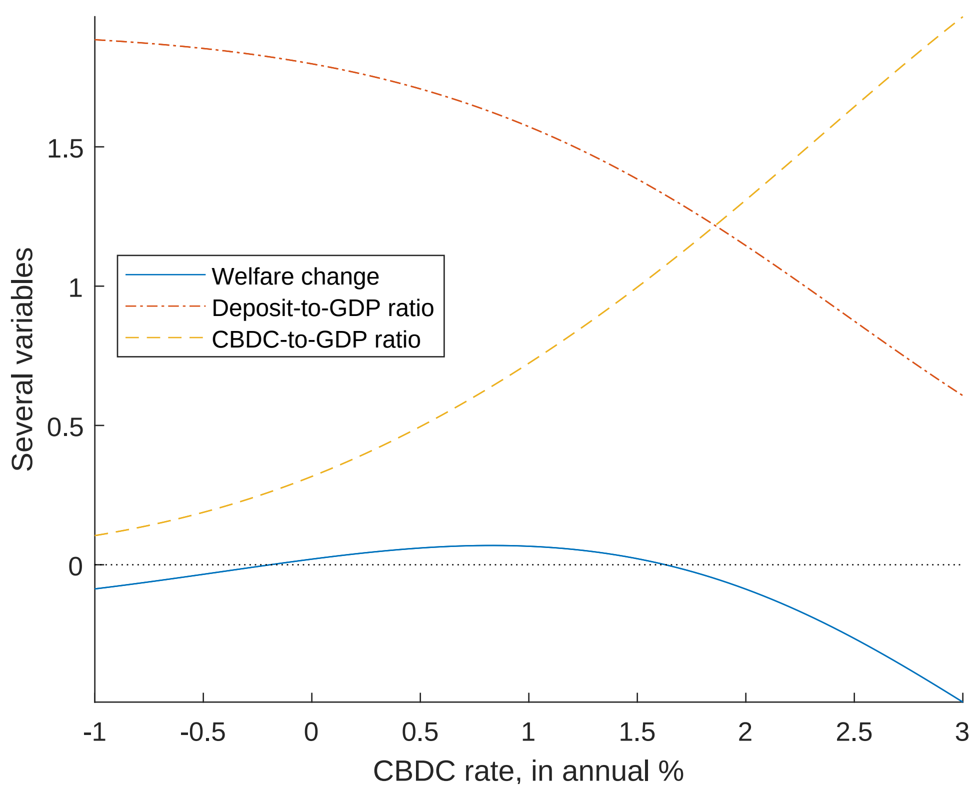

Figure 6.2: This figure displays the behavior of some important variables for different levels of the CBDC interest rate in a calibration with positive costs of issuing CBDC that equal 20 basis points of initial steady-state consumption. The welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, is in blue, the deposit-to-GDP ratio is in orange, and the CBDC-to-GDP ratio is in yellow.

central banks or policy makers have announced that they are tying their hands to issuing a zero-interest CBDC. Without enough research into the costs and benefits of CBDC, such a restriction might be counterproductive and can lead to a welfare reduction from the issuance of a poorly-designed CBDC.

6.3 Euro-Area Calibration

Our main calibration so far has focused on the U.S. However, the Euro Area is more advanced in its research of CBDC and is currently in the preparation phase of the Digital Euro project, which began in November 2023 and is laying the foundations for the potential issuance of a Euro-Area CBDC.[24] Additionally, the economy of the Euro Area has some characteristics that differ from those in the U.S., particularly regarding the importance of commercial banks for total lending. For these reasons, we perform an alternative calibration for the Euro Area.

While several calibrated parameters change slightly, as detailed in the table given in Appendix C.6, the key difference is an increase in the importance of pledgeable capital, ψ, from 30% in the U.S. calibration to 82% in the Euro Area calibration. This change amplifies the importance of the bank disintermediation channel since commercial banks account for a larger fraction of overall lending and reducing their lending capacity has a more substantial negative impact on welfare. As a consequence, the welfare gains from introducing CBDC are lower in the Euro Area, as depicted in Figure 6.3. The results under the baseline calibration are given by the solid blue line (which replicates the solid blue line in Figure 5.1) and the results for the Euro-Area calibration are given by the dashed-dotted orange line. The solid blue line for the United States peaks at a 26.8 basis points welfare gain for a CBDC rate of 84 basis points. The orange line, which peaks at a 17.8 basis points welfare gain for a CBDC rate of 64 basis points, indicates that the Euro-Area can achieve a maximum of 9 basis points lower welfare gain than the United States, and this happens at a CBDC rate that is 20 basis points lower. While the Euro Area has expressed more interest in pursuing a CBDC than the United States, our model indicates the welfare gains from issuing a CBDC may actually be higher in the United States.

6.4 Disintermediation, Reserves, and Wholesale Funding

So far, we have referred to the key negative consequence of introducing CBDC as a "Bank Disintermediation" channel. However, it is important to notice that banks in our model

[24] See, e.g., https://www.ecb.europa.eu/euro/digital_euro/html/index.en.html.

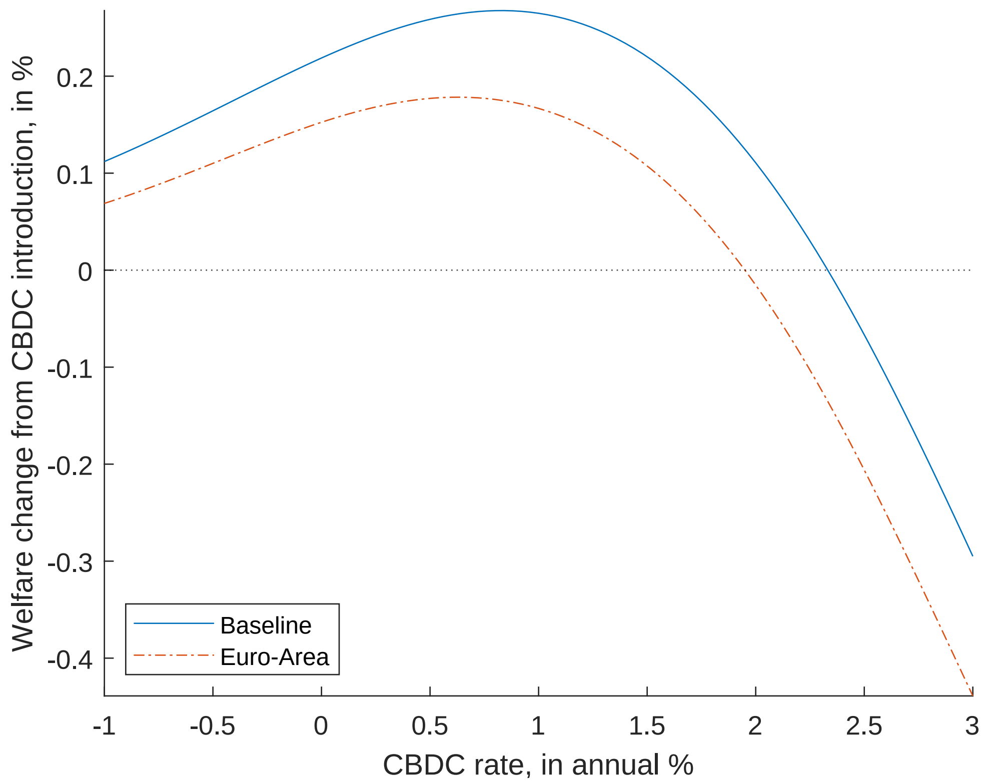

Figure 6.3: This figure displays the welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, across levels of the interest rate on CBDC for two different calibrations. The results under the baseline calibration are in blue (replicating the solid blue line in Figure 5.1) and the results under the Euro-Area calibration are given by the dashed-dotted orange line.

are free to substitute any lost deposit funding with borrowing from the central bank at the policy rate. Therefore, the "disintermediation" does not take place because banks do not have enough funds to lend, it happens instead because deposit funding earns a spread that contributes positively to bank profitability, while funds borrowed from the central bank do not earn this spread. If the central bank were to compensate commercial banks for this lost spread after the introduction of CBDC, this channel could be mitigated or eliminated. However, such a subsidization of banks is likely politically untenable.

We also note that the representative bank in the post-CBDC steady state is a net borrower from the central bank (i.e., $\overline{H}_j < 0$). At first glance, this may be considered unrealistic, since the banking sector as a whole does not continuously borrow at the discount window in practice. However, this outcome stems from the fact that we did not explicitly allow banks to access wholesale funding. If banks had such funding available at the policy rate, then our post-CBDC steady state can be expressed instead as banks having a positive amount of wholesale funding and positive excess reserves simultaneously, with no changes to our results.[25]

6.5 Bank Taxes and Subsidies

One of the positive effects of introducing CBDC in our model is to curtail commercial bank market power in deposits. We acknowledge that there are alternative tools to address this issue, namely taxes or subsidies on deposits. Importantly, just using taxes or subsidies to curtail commercial bank market power does not achieve the liquidity benefits of introducing CBDC, lowering the overall attainable welfare gains as illustrated in Figure 6.1.

Additionally, the tax/subsidy remedy for commercial bank market power in deposits entails *subsidizing* commercial banks (not *taxing* them) which could be politically unpopular, as banks are not necessarily well regarded among the American public.[26] While in our model with homogeneous households subsidizing banks would have no effect on the wealth distribution, in an extended model with heterogeneous households where banks are mostly owned by agents in the upper end of the income distribution, subsidizing banks could exacerbate inequality.

Another issue arises in the case of heterogeneous banks. The subsidies required to address market power in deposits would vary by bank and could potentially require the central bank to have a lot of information about individual banks in order to set the correct subsidy. By contrast, CBDC carries a single interest rate, requiring less information, while already addressing market power in deposits in a way that varies across banks depending on their market power.

[25] To illustrate this point, consider the expression for the bank balance sheet $L + H = F + D$ and divide through by bank equity to obtain $L/F + H/F = 1 + D/F$. In the post-CBDC steady state, these quantities take the values $L/F \approx 9.25$, $H/F \approx -1.6$, and $D/F \approx 6.65$, which indicates that banks borrow a substantial amount from the discount window. However, if we extend the balance sheet to $L/F + \tilde{H}/F = 1 + D/F + W/F$, where W represents wholesale funding, then the same values for the loan and deposit ratios in the post-CBDC steady state can be supported by $\tilde{H}/F \approx 0.4$ and $W/F \approx 2$, for example, so that banks hold excess reserves but borrow from the wholesale market. Since the household's supply of funds at the policy rate is perfectly elastic in steady state based on the Euler equation, changing the amount that banks demand in that market makes no difference to our results.

[26] See, e.g., https://bankingjournal.aba.com/2024/07/survey-u-s-public-confidence-remains-low-for-banks-other-institutions/.

6.6 Holding Limits

Even though research about the impacts of CBDC is still in its early stages, many central banks are coalescing around imposing holding limits on CBDCs that they might issue.[27] In our model, the introduction of CBDC leads to positive welfare effects for a wide range of CBDC interest rates, even without holding limits. Adopting binding holding limits has the potential to reduce the ability of a CBDC to curtail commercial bank market power in deposits and may reduce its overall welfare benefits.

To assess whether some commonly proposed holding limits might be binding in our model, we perform the following back-of-the-envelope calculation. Quarterly U.S. GDP per capita in 2024 was around 21 thousand dollars, so a limit of three thousand dollars, if fully utilized by the population, would imply a CBDC-to-GDP ratio of around 15%, while a limit of five thousand dollars would imply a ratio of 24%.[28] As illustrated in Figure 5.1, the CBDC-to-GDP ratio in our model is around 32% when the CBDC interest rate is 0%, and reaches 64% for the welfare maximizing CBDC interest rate of 0.8%. Therefore, holding limits of the considered amounts would be insufficient to achieve the maximum benefits in our model.

That being said, holdings limits may be beneficial in times of financial distress when uninsured depositors withdraw substantial funds from banks and convert them to CBDC, which may in turn exacerbate financial turmoil. While our model abstracts from the impact of CBDC on financial stability, a line of research that studies this relation often builds on the traditional work of Diamond and Dybvig (1983), and includes Fernandez-Villaverde et al. (2021), Schilling et al. (2020), Williamson (2022a), Keister and Monnet (2022), and Bidder et al. (2024), though these models exclude bank market power. A salient path for future research would be to integrate financial crises into our DSGE framework with bank market power to study how the introduction and remuneration of CBDC might affect the frequency and severity of crises, as well as the role of holding limits in this regard.

7 Conclusion

Many countries are currently considering the introduction of a central bank digital currency and debating what the effects on their economies might be. Since practical expe-

[27] See, e.g., page 10 of the latest BIS survey on CBDC: https://www.bis.org/publ/bppdf/bispap147.pdf.

[28] Of course, in an extended model with heterogeneous households that have different levels of CBDC usage, for example stemming from a discrete choice microfoundation, not everyone would max-out their CBDC account and the holding limits would have to be greater to achieve the same CBDC-to-GDP ratio.

rience with CBDCs remains scarce, policymakers turn to analysis based on theoretical economic models for insights. Our paper provides such guidance and delivers a practical message that can be applied to various economies around the world.

We develop a New Keynesian DSGE model to assess the introduction of a CBDC. Three competing channels determine the welfare effects in our model. On the positive side, households benefit from the introduction of a CBDC in two ways. First, they value the expansion of liquidity services that the new saving instrument provides. Second, households receive higher deposit rates since CBDC competes with bank deposits, thus reducing banks' deposit market power. On the negative side, banks face deposit outflows and cut their lending, which in turn reduces aggregate investment and output.

We assess this welfare trade-off for a wide range of economies that differ in their level of interest rates. We find substantial welfare improvements of introducing CBDC if countries follow a simple rule that determines the rate of interest on CBDC: it pays the maximum between 0% and the policy rate minus 1%. The simplicity of this rule is appealing in that it can easily be communicated to the public and avoids political-economy concerns related to paying negative rates on CBDC. Interestingly, we also find that the introduction of a CBDC is most beneficial for economies with high interest rates. In such environments, banks have substantial market power in deposit markets which is sharply curtailed once a CBDC is introduced.

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Appendix A Solving the Static Bank Model

First, substitute the balance sheet condition (2.5) into the objective function and write d_j as an implicit function of $1 + i_j^d$, then the bank's problem becomes

$$\max_{i_j^d} \quad (1+i)(f_j + d_j) - (1 + i_j^d) d_j,$$

Take the first-order condition with respect to $1 + i_j^d$

$$-d_j + \left((1+i) - (1+i_j^d)\right) \epsilon_j^d \frac{d_j}{1+i_j^d} = 0,$$

where $\epsilon_j^d \equiv \partial \ln d_j / \partial \ln(1+i_j^d)$. Rearranging this equation we can obtain (2.6). Next, use (2.3) to express ϵ_j^d as:

$$\begin{aligned}
\epsilon_j^d &= \frac{\partial d_j}{\partial(1+i_j^d)} \frac{1+i_j^d}{d_j} \\
&= d_j \frac{\varepsilon^d}{1+i_j^d} \frac{1+i_j^d}{d_j} - d_j \frac{\varepsilon^d}{1+i^d} \frac{\partial(1+i^d)}{\partial(1+i_j^d)} \frac{1+i_j^d}{d_j} + d_j \frac{1}{d} \frac{\partial d}{\partial(1+i^d)} \frac{\partial(1+i^d)}{\partial(1+i_j^d)} \frac{1+i_j^d}{d_j} \\
&= \varepsilon^d - \varepsilon^d \frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)} + \frac{\partial \ln d}{\partial \ln(1+i^d)} \frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)}.
\end{aligned} \tag{A.1}$$

Define the elasticity of the aggregate gross deposit rate w.r.t. an individual gross deposit rate and use (2.4) and (2.3) to write it as:

$$\omega_d^{d_j} \equiv \frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)} = \frac{1}{n} \left(\frac{1+i_j^d}{1+i^d}\right)^{\varepsilon^d+1} = \frac{(1+i_j^d) d_j}{(1+i^d) d}. \tag{A.2}$$

We can interpret this as the share of overall deposits that are maintained at bank j. Using (2.1), the elasticity of aggregate deposits w.r.t the gross deposit rate can be expressed as:

$$\frac{\partial \ln d}{\partial \ln(1+i^d)} = \theta \left(1 - \frac{\partial \ln(1+i^{\mathcal{L}})}{\partial \ln(1+i^d)}\right) \equiv \theta \left(1 - \omega_{\mathcal{L}}^d\right), \tag{A.3}$$

where the last equality is by definition. Using (2.2) and (2.1) we can further express $\omega_{\mathcal{L}}^d$ as:

$$\omega_{\mathcal{L}}^d \equiv \frac{\partial \ln(1+i^{\mathcal{L}})}{\partial \ln(1+i^d)} = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} = \frac{(1+i^d) d}{(1+i^{\mathcal{L}}) \mathcal{L}}.$$

Finally, substitute (A.2) and (A.3) into (A.1) to obtain

$$\epsilon_j^d = (1 - \omega_d^{d_j}) \varepsilon^d + \omega_d^{d_j} (1 - \omega_{\mathcal{L}}^d) \theta. \tag{A.4}$$

When all banks are identical, in a symmetric equilibrium, they all pay the same deposit rate $i_j^d = i^d$, face the same elasticity $\epsilon_j^d = \epsilon^d$, and obtain one n-th of total deposit. Consequently, $\omega_d^{d_j} = 1/n$, and we obtain equation (2.7).

Once symmetry across banks has been imposed in the model of Section 2, the equilibrium system for the determination of the endogenous deposit rate is composed of equation (2.6) for the representative bank, the definition of $\omega_{\mathcal{L}}^d$ in equation (2.8), the definition of the interest rate on liquidity in equation (2.2), as well as the equation for the behavior of the endogenous deposit markdown (2.7). Reproducing those here, we have the following equilibrium system of equations:

$$\begin{aligned}
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i)\\
\omega_{\mathcal{L}}^d &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}\\
1+i^{\mathcal{L}} &= \left(\gamma_m+\gamma_d(1+i^d)^{\theta+1}+\gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}\\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}(1-\omega_{\mathcal{L}}^d).
\end{aligned}$$

Introduce the third into the second and simplify to obtain:

$$\begin{aligned}
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i)\\
\omega_{\mathcal{L}}^d &= \frac{\gamma_d}{\gamma_m\left(\frac{1}{1+i^d}\right)^{\theta+1}+\gamma_d+(1-\gamma_m-\gamma_d)\left(\frac{1+i^{cbdc}}{1+i^d}\right)^{\theta+1}}\\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}(1-\omega_{\mathcal{L}}^d).
\end{aligned}$$

This is a system of three equations in three endogenous variables $(i^d, \omega_{\mathcal{L}}^d, \epsilon^d)$ and several exogenous variables $(i, i^{cbdc}, \gamma_m, \gamma_d, n, \varepsilon^d)$. The system is implicit and cannot be solved in closed form. Therefore, we apply the implicit function theorem to determine how changes in exogenous variables affect the endogenous variables. First, we show that in a special case, there is a known solution to the system that we can apply the implicit function theorem around.

Appendix A.1 A Special Case

In the special case where cash and CBDC pay zero interest rate, we can solve in closed form for the level of the policy rate where the deposit rate reaches zero percent. In this case, the equilibrium equations are:

$$\begin{aligned}
i^d &= i^{\mathcal{L}} = 0\\
\omega_{\mathcal{L}}^d &= \gamma_d\\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}(1-\gamma_d)\\
1 &= \frac{\epsilon^d}{\epsilon^d+1}(1+i),
\end{aligned}$$

which, from the last equation, allows us to obtain the required level of the policy rate for this to be an equilibrium:

$$
\begin{aligned}
\frac{\epsilon^d + 1}{\epsilon^d} &= 1 + i \\
\frac{1}{\epsilon^d} &= i \\
i_{i^d=0} &= \frac{n}{\varepsilon^d(n-1) + \theta(1-\gamma_d)}.
\end{aligned}
$$

This is useful because we know that there is an actual solution to the system of equations that we can then approximate the system around (this is technically a requirement for the implicit function theorem). It is also useful to know what the level of the policy rate is and where the deposit rate becomes zero, both before and after the introduction of CBDC.

Appendix A.2 Implicit Function Theorem Application

Denote with x all the exogenous variables and with y the three endogenous ones, simplify the notation of $\omega^d_{\mathcal{L}}$ to just ω, and define:

$$
\begin{aligned}
F_1(x,y) &= 1 + i^d - \frac{\epsilon^d}{\epsilon^d + 1}(1+i) \\
F_2(x,y) &= \omega - \frac{\gamma_d}{\frac{\gamma_m + (1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d} \\
F_3(x,y) &= \epsilon^d - \frac{n-1}{n}\varepsilon^d - \frac{\theta}{n}(1-\omega).
\end{aligned}
$$

Then we can apply the implicit function theorem to our system of equations that can be represented by $F(x,y) = 0$. We can write the matrix of derivatives of the F's w.r.t. the endogenous variables as:

$$
D_y F = \begin{bmatrix} \frac{\partial F_1}{\partial i^d} & \frac{\partial F_1}{\partial \omega} & \frac{\partial F_1}{\partial \epsilon^d} \\ \frac{\partial F_2}{\partial i^d} & \frac{\partial F_2}{\partial \omega} & \frac{\partial F_2}{\partial \epsilon^d} \\ \frac{\partial F_3}{\partial i^d} & \frac{\partial F_3}{\partial \omega} & \frac{\partial F_3}{\partial \epsilon^d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ b & 1 & 0 \\ 0 & c & 1 \end{bmatrix},
$$

where:

$$
\begin{aligned}
a &= -\frac{1+i}{(\epsilon^d+1)^2} < 0 \\
b &= -\frac{\gamma_d(1+\theta)(1+i^d)^{-\theta-2}\left[\gamma_m + (1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}\right]}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} < 0 \\
c &= \frac{\theta}{n} > 0.
\end{aligned}
$$

The determinant of D_yF is $1 + abc$, which is positive because of the signs of a, b, and c. Moreover, we can also calculate the inverse of D_yF (using the transpose of the matrix of cofactors divided by the determinant):

$$
(D_yF)^{-1} = \frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix},
$$

We can also write:

$$D_xF = \begin{bmatrix} \frac{\partial F_1}{\partial i} & \frac{\partial F_1}{\partial i^{cbdc}} & \frac{\partial F_1}{\partial \gamma_m} & \frac{\partial F_1}{\partial \gamma_d} & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial \varepsilon^d} \\ \frac{\partial F_2}{\partial i} & \frac{\partial F_2}{\partial i^{cbdc}} & \frac{\partial F_2}{\partial \gamma_m} & \frac{\partial F_2}{\partial \gamma_d} & \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial \varepsilon^d} \\ \frac{\partial F_3}{\partial i} & \frac{\partial F_3}{\partial i^{cbdc}} & \frac{\partial F_3}{\partial \gamma_m} & \frac{\partial F_3}{\partial \gamma_d} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial \varepsilon^d} \end{bmatrix} = \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix},$$

where:

$$\begin{aligned} e &= \frac{\gamma_d(1-\gamma_m-\gamma_d)(1+\theta)(1+i^{cbdc})^{\theta}(1+i^d)^{-\theta-1}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} > 0 \\ f &= \frac{\gamma_d\frac{1-(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} \lesseqgtr 0 \\ g &= -\frac{\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d-\gamma_d\left(1-\frac{(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}\right)}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} \\ &= -\frac{\frac{\gamma_m+(1-\gamma_m)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} < 0. \end{aligned}$$

Notice that f has the opposite sign of i^{cbdc}. That is, if i^{cbdc} is positive, then f is negative, if $i^{cbdc}=0$, then $f=0$, and if i^{cbdc} is negative then f is positive. We can use the implicit function theorem to write:

$$\begin{aligned} D_xy &= \begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varepsilon^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varepsilon^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varepsilon^d} \end{bmatrix} \\ &= -(D_yF)^{-1}D_xF \\ &= -\frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix} \\ &= -\frac{1}{1+abc}\begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & ace & acf & acg & a\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & a\left(1-\frac{1}{n}\right) \\ b\frac{\epsilon^d}{\epsilon^d+1} & e & f & g & -ab\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & ab\left(\frac{1}{n}-1\right) \\ -bc\frac{\epsilon^d}{\epsilon^d+1} & -ce & -cf & -cg & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix}. \end{aligned}$$

Knowing the sign of all the letters ($a<0, b<0, c>0, e>0, f \lesseqgtr 0, g<0$) we can sign these derivatives:

$$\begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varepsilon^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varepsilon^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varepsilon^d} \end{bmatrix} = \begin{bmatrix} + & + & ? & - & + & + \\ + & - & ? & + & + & + \\ - & + & ? & - & + & + \end{bmatrix},$$

where we required $\varepsilon^d > \theta(1-\omega)$ to sign the fifth column, but this requirement is less stringent than $\varepsilon^d > \theta$ which we should assume anyway (more substitutability in the inner nest than the outer nest, saying that banks are more substitutable with each other than deposits are substitutable with cash and CBDC). The signs of the third column are -, + , - if $i^{cbdc} > 0$, all zero if $i^{cbdc} = 0$, and +, - , + if $i^{cbdc} < 0$.

For the proof of the reaction of aggregate deposits, d, to any exogenous variable, notice that equations (2.1) and (2.8) can be combined to obtain

$$d \quad = \quad \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}} \right)^{\theta} \boldsymbol{\mathcal{L}} = \gamma_d \left(\frac{\omega^d_{\mathcal{L}}}{\gamma_d} \right)^{\frac{\theta}{\theta+1}} \boldsymbol{\mathcal{L}}.$$

Since $\mathcal{L}$ is constant in this setup, and $\frac{\theta}{\theta+1} > 0$, then the sign of $\frac{\partial d}{\partial z}$ is the same as the sign of $\frac{\partial \omega^d_{\mathcal{L}}}{\partial z}$, for any exogenous variable z (other than γ_d).

Appendix A.3 Pass-Through of the Policy Rate to the Deposit Rate

In this appendix, we analyze the pass-through of the policy rate to the deposit rate and how that depends on parameters, and whether this pass-through has a minimum, what that minimum is, and how it depends on parameters. Relative to the static model in Section 2, we introduce the μ^d cost of issuing deposits that we adopt in the full model, and we also allow total liquidity to be endogenous as in the full model, denoting $\varepsilon^{\mathcal{L}} \equiv (\partial \ln \mathcal{L})/(\partial \ln(1+i^{\mathcal{L}}))$. To start, notice that, in the pre-CBDC scenario where $i^{cbdc} = 0$ and where cash pays zero percent, the deposit rate depends on just four equations (we ignore the time subscripts for notational convenience and because all variables are dated the same):

$$(1+i^{\mathcal{L}})^{\theta+1} \quad = \quad \gamma_m + \gamma_d (1+i^d)^{\theta+1} \tag{A.5}$$

$$\omega^d_{\mathcal{L}} \quad = \quad \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}} \right)^{\theta+1} \tag{A.6}$$

$$\epsilon^d \quad = \quad \frac{n-1}{n} \varepsilon^d + \frac{\theta}{n} - \frac{\omega^d_{\mathcal{L}}}{n} \left(\theta - \varepsilon^{\mathcal{L}} \right) \tag{A.7}$$

$$1+i^d \quad = \quad \frac{\epsilon^d}{\epsilon^d + 1} (1 + i - \mu^d). \tag{A.8}$$

We can combine all of these into a single equation in i and i^d and then use the implicit function theorem to compute the derivative of i^d w.r.t. i, and then we can see how this object (which is the pass-through) behaves. Start with equation (A.8) and simplify:

$$1+i^d \quad = \quad \epsilon^d (i - \mu^d - i^d). \tag{A.9}$$

Then, introduce equation (A.7) and simplify:

$$n(1+i^d) \quad = \quad \left[(n-1)\varepsilon^d + \theta - \omega^d_{\mathcal{L}} \left(\theta - \varepsilon^{\mathcal{L}} \right) \right] (i - \mu^d - i^d). \tag{A.10}$$

Furthermore, introduce equation (A.6) and simplify:

$$n(1+i^d) \quad = \quad \left[(n-1)\varepsilon^d + \theta - \frac{\gamma_d (1+i^d)^{\theta+1}}{(1+i^{\mathcal{L}})^{\theta+1}} \left(\theta - \varepsilon^{\mathcal{L}} \right) \right] (i - \mu^d - i^d). \tag{A.11}$$

Finally, introduce equation (A.5) and simplify:

$$n(1+i^d) = \left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i - \mu^d - i^d). \tag{A.12}$$

Notice that this is an equation in the variables i and i^d and the parameters n, ε^d, θ, γ_m, γ_d, $\varepsilon^{\mathcal{L}}$, and μ^d. Now we write:

$$F(i^d, i) = n(1+i^d) - \left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i - \mu^d - i^d). \tag{A.13}$$

So, the equilibrium equation for i^d as a function of i can be written as:

$$F(i^d, i) = 0. \tag{A.14}$$

Then, if the assumptions of the implicit function theorem are satisfied, we know that:

$$\frac{di^d}{di} = -\frac{F_i}{F_{i^d}}. \tag{A.15}$$

Since this is the derivative of i^d w.r.t. to i, it has the interpretation of the pass-through of the policy rate to the deposit rate, which is an important object in papers like Drechsler et al. (2017, 2021). Notice, also, that if we want the second derivative of i^d w.r.t. i we can also use the implicit function theorem for this:

$$\frac{d^2 i^d}{di^2} = \frac{2F_i F_{i^d} F_{ii^d} - F_{ii} F_{i^d}^2 - F_{i^d i^d} F_i^2}{F_{i^d}^3}. \tag{A.16}$$

We want to study if there is a value of the policy rate for which $\frac{d^2 i^d}{di^2} = 0$, and then we can obtain the value of the pass-through, $\frac{di^d}{di}$, at that value of the policy rate, to obtain the minimum pass-through and evaluate how it depends on parameters.

Start with F_i:

$$F_i = -\left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right] \equiv -\aleph(i^d) < 0, \tag{A.17}$$

where the Aleph function denotes the expression inside the brackets which is a function of i^d only. Notice that $\aleph(i^d) > 0$ everywhere. Then compute F_{i^d}:

$$F_{i^d} = n + \aleph(i^d) - \aleph'(i^d)(i - \mu^d - i^d), \tag{A.18}$$

and notice that:

$$\aleph'(i^d) = -\left(\theta - \varepsilon^{\mathcal{L}}\right)\frac{(\theta+1)\frac{\gamma_m}{\gamma_d}(1+i^d)^{-\theta-2}}{\left(1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}\right)^2} < 0. \tag{A.19}$$

Notice then that $F_{i^d} > 0$, so we can apply the implicit function theorem safely. We also know that $\frac{di^d}{di}$ is positive, so that pass-through is always positive (this is proved in proposition 1 of the paper in Section

2, but that was with exogenous total liquidity, while here liquidity is endogenous, and there we had the possibility of CBDC, whereas here we are necessarily in the pre-CBDC scenario). Now lets investigate the second derivative of i^d w.r.t. i and when it is zero. Notice that $F_{ii} = 0$. Given this, we know that $\frac{d^2 i^d}{di^2} = 0$ iff:

$$F_i F_{i^d i^d} \quad = \quad 2 F_{i^d} F_{i i^d} \tag{A.20}$$

Using the expressions above, we can re-write this as:

$$-\aleph(i^d)\left[\aleph'(i^d) - \aleph''(i^d)(i - \mu^d - i^d) + \aleph'(i^d)\right] = 2\left[n + \aleph(i^d) - \aleph'(i^d)(i - \mu^d - i^d)\right](-\aleph'(i^d)) \tag{A.21}$$

Simplifying, we get:

$$2n \quad = \quad \aleph(i^d)(i - \mu^d - i^d)\left(2\frac{\aleph'(i^d)}{\aleph(i^d)} - \frac{\aleph''(i^d)}{\aleph'(i^d)}\right) \tag{A.22}$$

Using the equilibrium condition $F(i^d, i) = 0$, which can be re-written as $n(1 + i^d) = \aleph(i^d)(i - \mu^d - i^d)$, we can write the previous equation as:

$$\frac{2}{1 + i^d}\frac{\aleph(i^d)}{\aleph'(i^d)} \quad = \quad 2 - \frac{\aleph''(i^d)\aleph(i^d)}{(\aleph'(i^d))^2} \tag{A.23}$$

Next, we have to calculate $\aleph''(i^d)$. We first write $\aleph(i^d)$ as a function that depends on constants a and b (these are unrelated to the a and b parameters inside the Φ function that we use in the full model) and another function of i^d denoted $y(i^d)$:

$$\aleph(i^d) \quad = \quad a - by(i^d)^{-1} \tag{A.24}$$

where $a = (n - 1)\varepsilon^d + \theta$, $b = \theta - \varepsilon^{\mathcal{L}}$, and $y(i^d)$ is defined as follows:

$$y(i^d) \quad = \quad 1 + g(1 + i^d)^h \tag{A.25}$$

where $g = \gamma_m / \gamma_d$ and $h = -(\theta + 1)$. This seems convoluted, but it will make computing derivatives much easier. First, notice that:

$$\begin{aligned} \aleph'(i^d) &= by(i^d)^{-2} y'(i^d) \\ \aleph''(i^d) &= -2by(i^d)^{-3}(y'(i^d))^2 + by(i^d)^{-2} y''(i^d) \end{aligned} \tag{A.26}$$

Hence:

$$\frac{\aleph(i^d)\aleph''(i^d)}{(\aleph'(i^d))^2} \quad = \quad \left(\frac{ay(i^d)}{b} - 1\right)\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2} - 2\right) \tag{A.27}$$

And:

$$\frac{\aleph(i^d)}{\aleph'(i^d)} \quad = \quad \frac{ay(i^d)^2}{by'(i^d)} - \frac{y(i^d)}{y'(i^d)} = \frac{y(i^d)}{y'(i^d)}\left(\frac{ay(i^d)}{b} - 1\right) \tag{A.28}$$

With this, equation (A.23) can be written as:

$$\begin{aligned}
\frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)}\left(\frac{ay(i^d)}{b}-1\right) &= 2-\left(\frac{ay(i^d)}{b}-1\right)\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2}-2\right) \\
\frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)} &= \frac{2b}{ay(i^d)-b}-\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2}-2\right) \qquad \text{(A.29)}
\end{aligned}$$

Next, notice that:

$$\begin{aligned}
y(i^d) &= 1+g(1+i^d)^h \\
y'(i^d) &= gh(1+i^d)^{h-1} \\
y''(i^d) &= gh(h-1)(1+i^d)^{h-2} \qquad \text{(A.30)}
\end{aligned}$$

So:

$$\frac{y''(i^d)y(i^d)}{(y'(i^d))^2} = (1+g(1+i^d)^h)\frac{gh(h-1)(1+i^d)^{h-2}}{g^2h^2(1+i^d)^{2h-2}} = \frac{(h-1)}{gh(1+i^d)^h}+1-\frac{1}{h} \qquad \text{(A.31)}$$

Hence:

$$\frac{y''(i^d)y(i^d)}{(y'(i^d))^2}-2 = \frac{(h-1)}{gh(1+i^d)^h}-1-\frac{1}{h} \qquad \text{(A.32)}$$

With this, equation (A.29) can be written as:

$$\begin{aligned}
\frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)} &= \frac{2b}{ay(i^d)-b}-\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2}-2\right) \\
1+h+(1-h)g(1+i^d)^h &= \frac{2bhg(1+i^d)^h}{a+ag(1+i^d)^h-b} \qquad \text{(A.33)}
\end{aligned}$$

For convenience, notice that $a-b=(n-1)\varepsilon^d+\theta-\theta+\varepsilon^{\mathcal{L}}=(n-1)\varepsilon^d+\varepsilon^{\mathcal{L}}\equiv k$, so we get:

$$1+h+(1-h)g(1+i^d)^h = \frac{2bhg(1+i^d)^h}{k+ag(1+i^d)^h} \qquad \text{(A.34)}$$

Use the definition of $h=-\theta-1$ to simplify:

$$k(2+\theta)g(1+i^d)^h+a(2+\theta)g^2(1+i^d)^{2h}-\theta k-\theta ag(1+i^d)^h = 2bhg(1+i^d)^h \qquad \text{(A.35)}$$

So, we can finally simplify this into a quadratic equation in $z=g(1+i^d)^h$:

$$a(2+\theta)z^2+\left[k(2+\theta)-\theta a-2bh\right]z-\theta k = 0 \qquad \text{(A.36)}$$

Simplify the middle coefficient:

$$k(2+\theta)-\theta a-2bh = 2a+b\theta \qquad \text{(A.37)}$$

With this, we can express the quadratic equation just in terms of a, b, and θ:

$$a(2+\theta)z^2 + (2a+b\theta)z - \theta(a-b) \quad = \quad 0 \tag{A.38}$$

The discriminant for this quadratic equation is:

$$\Delta \quad = \quad (2a\theta + 2a - b\theta)^2 \tag{A.39}$$

Therefore, the two solutions are:

$$z_{1,2} \quad = \quad \frac{-(2a+b\theta) \pm (2a\theta + 2a - b\theta)}{2a(2+\theta)} \tag{A.40}$$

The correct solution is the one with the plus (the one with the minus would lead to $1+i^d$ being negative, which would lead to an extremely negative i^d that is implausible), so we get:

$$z^* \quad = \quad \frac{2a\theta + 2a - b\theta - 2a - b\theta}{2a(2+\theta)} = \frac{a\theta - b\theta}{2a + a\theta} \tag{A.41}$$

Since we know the minimizer z^* in closed form, we can use it to obtain the minimizer i^{d*} in closed form as well:

$$\begin{aligned} z^* &= g(1+i^d)^h \\ i^{d*} &= \left(\frac{z^*}{g}\right)^{\frac{1}{h}} - 1 \end{aligned} \tag{A.42}$$

We want to obtain the value of the pass-through at this pass-through minimizer i^{d*}. Notice that since $y^* = 1 + z^*$, then we get:

$$y^* \quad = \quad 1 + \frac{a\theta - b\theta}{2a + a\theta} = \frac{2a + 2a\theta - b\theta}{2a + a\theta} \tag{A.43}$$

And since $\aleph^* = a - b(y^*)^{-1}$, then we get:

$$\aleph^* \quad = \quad a - b\frac{2a + a\theta}{2a + 2a\theta - b\theta} = \frac{2a(1+\theta)(a-b)}{2a(1+\theta) - b\theta} \tag{A.44}$$

And then the pass-through at the minimizer is:

$$\left(\frac{di^d}{di}\right)^* \quad = \quad -\frac{F_i^*}{F_{i^d}^*} = \frac{\aleph^*}{n + \aleph^* - (\aleph')^*(i - \mu^d - i^d)} \tag{A.45}$$

Recall that $n(1+i^d) = \aleph(i^d)(i - \mu^d - i^d)$, so we can rewrite the previous expression as:

$$\left(\frac{di^d}{di}\right)^* \quad = \quad \frac{\aleph^*}{n + \aleph^* - (\aleph')^* \frac{n(1+i^d)}{\aleph^*}} = \frac{(\aleph^*)^2}{n\aleph^* + (\aleph^*)^2 - (\aleph')^* n(1+i^d)} \tag{A.46}$$

Recall that:

$$(\aleph')^* \quad = \quad b(y^*)^{-2}(y')^* \tag{A.47}$$

Then relate y' to y using the equations in (A.30):

$$\begin{aligned} y'(i^d) &= gh(1+i^d)^{h-1} \\ (y')^* &= \frac{h(y^*-1)}{1+i^d} \end{aligned} \tag{A.48}$$

Introducing this into our equation for the minimum pass-through, we get:

$$\left(\frac{di^d}{di}\right)^* = \frac{(\aleph^*)^2}{n\aleph^* + (\aleph^*)^2 + b(y^*)^{-2}(\theta+1)(y^*-1)n} \tag{A.49}$$

Compute the inverse of the pass-through for convenience:

$$\begin{aligned} \left(\left(\frac{di^d}{di}\right)^*\right)^{-1} &= \frac{n}{\aleph^*} + 1 + \frac{b}{(y^*)^2}\frac{\theta+1}{(\aleph^*)^2}nz^* \\ &= 1 + \frac{n}{a-b} + \frac{nb\theta^2}{4a(1+\theta)(a-b)} \end{aligned} \tag{A.50}$$

Finally, substituting what a and b are, we obtain an expression for the inverse minimum pass-through as a function of just four parameter values n, ε^d, $\varepsilon^{\mathcal{L}}$, and θ:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varepsilon^d + \varepsilon^{\mathcal{L}}} + \frac{n(\theta-\varepsilon^{\mathcal{L}})\theta^2}{4[(n-1)\varepsilon^d + \theta](1+\theta)[(n-1)\varepsilon^d + \varepsilon^{\mathcal{L}}]} \tag{A.51}$$

This is an exact expression for the (inverse) minimum pass-through as a function of four relevant parameters n, ε^d, $\varepsilon^{\mathcal{L}}$, and θ. This tells us that the inverse minimum pass-through is always between 1 and infinity, so the minimum pass-through is always between 0 and 1. It is easy to see that when $n \to \infty$, the inverse minimum pass-through tends to $1 + \frac{1}{\varepsilon^d}$.

The expression for the pass-through tells us that a higher $\varepsilon^{\mathcal{L}}$ always increases the minimum pass-through while a higher θ decreases the minimum pass-through. Therefore, if one intended to find the parameter values that lower the minimum pass-through, one would pick the lowest possible $\varepsilon^{\mathcal{L}}$ and the highest possible θ. However, we also require $0 \leq \varepsilon^{\mathcal{L}} \leq \theta \leq \varepsilon^d$, so in order to obtain the lowest possible minimum pass-through w.r.t. $\varepsilon^{\mathcal{L}}$ and θ one can pick $\varepsilon^{\mathcal{L}} = 0$ and $\theta = \varepsilon^d$. In this case, the expression for the inverse minimum pass-through is:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varepsilon^d} + \frac{\varepsilon^d}{4(1+\varepsilon^d)(n-1)} \tag{A.52}$$

While this expression depends both on n and ε^d, if n is too big, then there are no reasonable values of ε^d that can achieve a minimum pass-through of 50% or lower. Therefore, the model requires a low n to be able to match a low minimum pass-through.

Appendix A.4 Optimal CBDC Rate for Liquidity Provision

In this subsection, we explore what is the optimal interest rate on CBDC rate just from the perspective of liquidity management. As discussed in Section 6.1, CBDC impacts welfare through three main channels in our model, it affects the liquidity services that household receive, it affects commercial bank monopoly power in deposits, and it affects bank lending. Imagine that we abstract from the last two effects (by setting κ, the importance of equity for bank lending, to zero and furthermore making the banking sector frictionless and perfectly competitive so that the deposit rate equals the policy rate), and focus just on liquidity provision. Furthermore, we can focus on a steady state and abstract from time subscripts. What is the optimal interest rate on CBDC in that case?

One can show that under these conditions, the optimal CBDC rate solves the following problem:

$$\max_{1+i^{cbdc}} \qquad m + d + cbdc - \Phi(\mathcal{L}(m, d, cbdc)).$$

The F.O.C. for this problem is:

$$\begin{aligned} 0 &= \frac{\partial m}{\partial 1 + i^{cbdc}} + \frac{\partial d}{\partial 1 + i^{cbdc}} + \frac{\partial cbdc}{\partial 1 + i^{cbdc}} - \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial m} \frac{\partial m}{\partial 1 + i^{cbdc}} \\ &- \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial d} \frac{\partial d}{\partial 1 + i^{cbdc}} - \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial cbdc} \frac{\partial cbdc}{\partial 1 + i^{cbdc}}. \end{aligned}$$

We simplify this to:

$$\begin{aligned} 0 &= \frac{\partial m}{\partial 1 + i^{cbdc}} \left(1 - \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial m}\right) + \frac{\partial d}{\partial 1 + i^{cbdc}} \left(1 - \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial d}\right) \\ &+ \frac{\partial cbdc}{\partial 1 + i^{cbdc}} \left(1 - \Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial cbdc}\right). \end{aligned}$$

Recalling that the F.O.C.'s of the household problem (equations B.3-B.5 below) take the form:

$$\Phi'(\mathcal{L}) \frac{\partial \mathcal{L}}{\partial x} = \frac{1 + i^x}{1 + i},$$

and using this in the previous expression, we obtain:

$$0 = \frac{\partial m}{\partial 1 + i^{cbdc}} (i - i^m) + \frac{\partial d}{\partial 1 + i^{cbdc}} (i - i^d) + \frac{\partial cbdc}{\partial 1 + i^{cbdc}} (i - i^{cbdc}). \tag{A.53}$$

Now we need to obtain the expressions for the derivatives of all three liquid instruments w.r.t. to $1 + i^{cbdc}$. Recall that:

$$\frac{1 + i^{\mathcal{L}}}{1 + i} = ab\mathcal{L}^{b-1},$$

and that (for $x = m, d, cbdc$):

$$x = \gamma_x \left(\frac{1 + i^x}{1 + i^{\mathcal{L}}}\right)^\theta \mathcal{L}.$$

We can combine these two to obtain:

$$\begin{aligned} x &= \gamma_x \left(\frac{1 + i^x}{1 + i^{\mathcal{L}}}\right)^\theta \left(\frac{1}{ab} \frac{1 + i^{\mathcal{L}}}{1 + i}\right)^{\frac{1}{b-1}} \\ &= \gamma_x (1 + i^x)^\theta (ab(1 + i))^{\frac{1}{1-b}} (1 + i^{\mathcal{L}})^{\frac{1}{b-1} - \theta}. \end{aligned}$$

We can also obtain the derivative of $1+i^{\mathcal{L}}$ w.r.t. $1+i^{cbdc}$, assuming that i^m and i^d are taken as given:

$$\begin{aligned}
1+i^{\mathcal{L}} &= \left(\gamma_m(1+i^m)^{\theta+1} + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{\partial 1+i^{\mathcal{L}}}{\partial 1+i^{cbdc}} &= \frac{1}{\theta+1}\left((1+i^{\mathcal{L}})^{\theta+1}\right)^{\frac{1}{\theta+1}-1}(\theta+1)\gamma_{cbdc}(1+i^{cbdc})^{\theta} \\
&= (1+i^{\mathcal{L}})^{-\theta}\gamma_{cbdc}(1+i^{cbdc})^{\theta} \\
&= \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta} = \frac{cbdc}{\mathcal{L}}.
\end{aligned}$$

With this, we can finally obtain the three derivatives we care about:

$$\begin{aligned}
\frac{\partial m}{\partial 1+i^{cbdc}} &= \gamma_m(1+i^m)^{\theta}(ab(1+i))^{\frac{1}{1-b}}\left(\frac{1}{b-1}-\theta\right)(1+i^{\mathcal{L}})^{\frac{1}{b-1}-\theta-1}\gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta} \\
\frac{\partial d}{\partial 1+i^{cbdc}} &= \gamma_d(1+i^d)^{\theta}(ab(1+i))^{\frac{1}{1-b}}\left(\frac{1}{b-1}-\theta\right)(1+i^{\mathcal{L}})^{\frac{1}{b-1}-\theta-1}\gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta} \\
\frac{\partial cbdc}{\partial 1+i^{cbdc}} &= \gamma_{cbdc}\theta(1+i^{cbdc})^{\theta-1}(ab(1+i))^{\frac{1}{1-b}}(1+i^{\mathcal{L}})^{\frac{1}{b-1}-\theta} \\
&+ \gamma_{cbdc}(1+i^{cbdc})^{\theta}(ab(1+i))^{\frac{1}{1-b}}\left(\frac{1}{b-1}-\theta\right)(1+i^{\mathcal{L}})^{\frac{1}{b-1}-\theta-1}\gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}.
\end{aligned}$$

Using these in equation (A.53), we obtain:

$$\begin{aligned}
0 &= (i-i^m) + \frac{\gamma_d}{\gamma_m}\left(\frac{1+i^d}{1+i^m}\right)^{\theta}(i-i^d) \\
&+ \frac{\theta(1+i^{\mathcal{L}})^{1+\theta}}{\gamma_m(1+i^m)^{\theta}\left(\frac{1}{b-1}-\theta\right)(1+i^{cbdc})}(i-i^{cbdc}) + \frac{\gamma_{cbdc}}{\gamma_m}\left(\frac{1+i^{cbdc}}{1+i^m}\right)^{\theta}(i-i^{cbdc}),
\end{aligned}$$

which we can write as:

$$0 = \frac{m}{\mathcal{L}}(i-i^m) + \frac{d}{\mathcal{L}}(i-i^d) + \frac{cbdc}{\mathcal{L}}(i-i^{cbdc}) + \frac{\theta}{\frac{1}{b-1}-\theta}\frac{1+i^{\mathcal{L}}}{1+i^{cbdc}}(i-i^{cbdc}). \quad \text{(A.54)}$$

This expression (which is an extension of the Friedman rule to multiple instruments with imperfect substitutability, under satiation, and where the steady state policy rate is considered exogenous and determined, for example, by real factors and ZLB considerations), is very intuitive, and it indicates that the social planner "seeks" to achieve four things simultaneously. It seeks to promote the optimal level of cash usage, deposit usage, CBDC usage, and overall liquidity usage.

While the expression above is useful, if we do some approximations we can obtain even more intuition. We will approximate $x/\mathcal{L}$ with $\omega^x_{\mathcal{L}}$, this is not exact but only differs by terms like $(1+i^x)/(1+i^{\mathcal{L}})$, which will be very close to one. We will also approximate $\theta/(1/(b-1)-\theta)$ by minus one, which is close to being true because $\frac{1}{b-1}$ is around 14 while θ is around 550. Finally, we will also approximate $(1+i^{\mathcal{L}})/(1+i^{cbdc})$ by one. In this case, we get (recall that all three omegas sum to one):

$$
\begin{aligned}
0 &= \omega_{\mathcal{L}}^{m}(i-i^{m})+\omega_{\mathcal{L}}^{d}(i-i^{d})+\omega_{\mathcal{L}}^{cbdc}(i-i^{cbdc})-(i-i^{cbdc}) \\
i^{cbdc} &= \frac{\omega_{\mathcal{L}}^{m}}{\omega_{\mathcal{L}}^{m}+\omega_{\mathcal{L}}^{d}}i^{m}+\frac{\omega_{\mathcal{L}}^{d}}{\omega_{\mathcal{L}}^{m}+\omega_{\mathcal{L}}^{d}}i^{d}
\end{aligned}
$$

So, under this approximation, the result is that, just from the perspective of liquidity provision, the optimal rate on CBDC is set as a weighted average of i^m (which is zero in our model) and i^d, where the weights sum to one and are given by $\omega_{\mathcal{L}}^{x}/(\omega_{\mathcal{L}}^{m}+\omega_{\mathcal{L}}^{d})$. When the policy rate is very low or negative, the weight of cash is higher, so the optimal rate of CBDC tracks the rate on cash (zero) more closely. Conversely, when the policy rate is high, deposits have most of the weight, so the optimal CBDC rate tracks the deposit rate closely.

Appendix B Details on the Full Model

Appendix B.1 The Household's Problem

The Bellman equation for the household's problem is given by:

$$
V_t(AH_{t-1})=\max_{C_t,N_t,M_t,\{D_{j,t}\}_{j=1}^{n},CBDC_t,B_t}\left\{u(C_t)-v(N_t)+\beta\mathbb{E}_t(V_{t+1}(AH_t))\right\}.
$$

We can express C_t as:

$$
C_t = \frac{W_tN_t+AH_{t-1}+T_t-B_t-\Phi(\mathcal{L}_t)P_t}{P_t},
$$

with this definition we can write the Bellman equation as a function of 4 individual choice variables and n deposit choices (the $D_{j,t}$). The first-order conditions are:

$$
\begin{aligned}
0 &= u'(C_t)\left(\frac{W_t}{P_t}\right)-v'(N_t) \\
0 &= u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial\mathcal{L}_t}{\partial m_t}\frac{1}{P_t}\right)+\beta(1+i_t^{m})\mathbb{E}_t\left(V_{t+1}'(AH_t)\right) \\
0 &= u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial\mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}}\frac{1}{P_t}\right)+\beta(1+i_{j,t}^{d})\mathbb{E}_t\left(V_{t+1}'(AH_t)\right) \\
0 &= u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial\mathcal{L}_t}{\partial cbdc_t}\frac{1}{P_t}\right)+\beta(1+i_t^{cbdc})\mathbb{E}_t\left(V_{t+1}'(AH_t)\right) \\
0 &= u'(C_t)\left(-\frac{1}{P_t}\right)+\beta(1+i_t)\mathbb{E}_t\left(V_{t+1}'(AH_t)\right).
\end{aligned}
$$

The Benveniste-Scheinkman condition is:

$$
V_t'(AH_{t-1}) = \frac{u'(C_t)}{P_t},
$$

moving this condition one period forward and introducing it into the F.O.C.'s we can rewrite them as:

$$
v'(N_t) = u'(C_t)\frac{W_t}{P_t} \tag{B.1}
$$

$$
\begin{aligned}
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t}\frac{1}{P_t} &= \beta(1+i_t^m)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}}\frac{1}{P_t} &= \beta(1+i_{j,t}^d)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}\frac{1}{P_t} &= \beta(1+i_t^{cbdc})\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
\frac{u'(C_t)}{P_t} &= \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). \qquad \text{(B.2)}
\end{aligned}
$$

The first condition is the intratemporal condition for labor supply and the fifth one is the Euler equation. The second, third, and fourth deal with the demand for cash, deposits, and CBDC respectively.

We first aggregate the individual demands for the deposits of each of the n banks into an aggregate deposit demand. If we introduce the fifth F.O.C. into the third, we obtain:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}} = \frac{1+i_{j,t}^d}{1+i_t}.
$$

The derivative of aggregate deposits w.r.t. an individual deposit is:

$$
\begin{aligned}
\frac{\partial d_t}{\partial d_{j,t}} &= \frac{\varepsilon^d}{\varepsilon^d+1}\left(\sum_{j=1}^{n}\alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{-\frac{1}{\varepsilon^d+1}} \alpha_j^{-\frac{1}{\varepsilon^d}}\frac{\varepsilon^d+1}{\varepsilon^d} d_{j,t}^{\frac{1}{\varepsilon^d}} \\
&= \left(d_t^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{-\frac{1}{\varepsilon^d+1}} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{1}{\varepsilon^d}} = \alpha_j^{-\frac{1}{\varepsilon^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varepsilon^d}}.
\end{aligned}
$$

Introducing this into the F.O.C. for deposits we get:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\alpha_j^{-\frac{1}{\varepsilon^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varepsilon^d}} = \frac{1+i_{j,t}^d}{1+i_t},
$$

raise this to the power of ε^d+1, multiply by α_j, and then add over banks:

$$
\begin{aligned}
\left(\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\right)^{\varepsilon^d+1}\alpha_j^{-\frac{\varepsilon^d+1}{\varepsilon^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{\varepsilon^d+1}{\varepsilon^d}} &= \frac{(1+i_{j,t}^d)^{\varepsilon^d+1}}{(1+i_t)^{\varepsilon^d+1}} \\
\left(\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\right)^{\varepsilon^d+1}\left(\frac{1}{d_t}\right)^{\frac{\varepsilon^d+1}{\varepsilon^d}}\sum_{j=1}^{n}\alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}} &= \frac{\sum_{j=1}^{n}\alpha_j(1+i_{j,t}^d)^{\varepsilon^d+1}}{(1+i_t)^{\varepsilon^d+1}} \\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1+i_t^d}{1+i_t},
\end{aligned}
$$

where we have defined:

$$
1+i_t^d = \left(\sum_{j=1}^{n}\alpha_j(1+i_{j,t}^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}.
$$

Using the equation $\Phi'(\mathcal{L}_t)(\partial\mathcal{L}_t/\partial d_t) = (1+i_t^d)/(1+i_t)$, we can turn the F.O.C. for individual deposits into:

$$d_{j,t} = \alpha_j \left(\frac{1+i^d_{j,t}}{1+i^d_t} \right)^{\varepsilon^d} d_t.$$

Once we have "aggregated up" deposits, we can turn to the decision between the three liquid savings instruments, where we have the following three F.O.C.s:

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{1+i^m_t}{1+i_t} \tag{B.3}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} = \frac{1+i^d_t}{1+i_t} \tag{B.4}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} = \frac{1+i^{cbdc}_t}{1+i_t}. \tag{B.5}$$

The derivative of liquidity w.r.t. real money balances is:

$$\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{\theta}{\theta+1}\left(\mathcal{L}_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}-1} \gamma_m^{-\frac{1}{\theta}} \frac{\theta+1}{\theta} m_t^{\frac{1}{\theta}} = \mathcal{L}_t^{-\frac{1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{1}{\theta}}.$$

Similar expressions are available for $\partial \mathcal{L}_t / \partial d_t$ and $\partial \mathcal{L}_t / \partial cbdc_t$. We can write demands as:

$$\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{1}{\theta}} = \frac{1+i^m_t}{1+i_t}$$

$$\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{1}{\theta}} = \frac{1+i^d_t}{1+i_t}$$

$$\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{1}{\theta}} = \frac{1+i^{cbdc}_t}{1+i_t}.$$

Raise all of these to the power of $\theta+1$ and multiply by an appropriate constant:

$$\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} = \gamma_m \frac{(1+i^m_t)^{\theta+1}}{(1+i_t)^{\theta+1}}$$

$$\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} = \gamma_d \frac{(1+i^d_t)^{\theta+1}}{(1+i_t)^{\theta+1}}$$

$$\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}} = \gamma_{cbdc} \frac{(1+i^{cbdc}_t)^{\theta+1}}{(1+i_t)^{\theta+1}},$$

by adding these three we get:

$$\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}} \mathcal{L}_t^{\frac{\theta+1}{\theta}} = \frac{(1+i^{\mathcal{L}}_t)^{\theta+1}}{(1+i_t)^{\theta+1}},$$

where the aggregate interest rate for liquidity takes the form:

$$1+i^{\mathcal{L}}_t \equiv \left(\gamma_m(1+i^m_t)^{\theta+1} + \gamma_d(1+i^d_t)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc}_t)^{\theta+1}\right)^{\frac{1}{\theta+1}}. \tag{B.6}$$

This finally allows us to write a simple demand equation for overall liquidity:

$$\frac{1+i^{\mathcal{L}}_t}{1+i_t} = \Phi'(\mathcal{L}_t). \tag{B.7}$$

And we can write the demand for each instrument as:

$$m_t = \gamma_m \left(\frac{1+i_t^m}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t \tag{B.8}$$

$$d_t = \gamma_d \left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t \tag{B.9}$$

$$cbdc_t = \gamma_{cbdc} \left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t. \tag{B.10}$$

Appendix B.2 Alternative Setup: Liquidity in Utility

In the baseline model, liquid instruments are demanded by the household because of the non-linear cost function $\Phi(\mathcal{L}_t)$ in the budget constraint. This leads to the set of tractable holding schedules in (3.4)-(3.6). This appendix provides an alternative setup where we introduce liquidity into the utility function to achieve the same holding schedules. Assuming all banks are symmetric, we focus only on the aggregate deposits.

Assume the household has the following utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U(C_t, \mathcal{L}_t) - v(N_t)),$$

while keeping the budget constraint standard:

$$P_t C_t + B_t + M_t + D_t + CBDC_t = W_t N_t + AH_{t-1} + T_t,$$

where

$$AH_{t-1} = (1+i_{t-1})B_{t-1} + (1+i_{t-1}^m)M_{t-1} + (1+i_{t-1}^d)D_{t-1} + (1+i_{t-1}^{cbdc})CBDC_{t-1}.$$

is the same as in the main text. The liquidity instruments inside $\mathcal{L}_t$ now have a one-for-one cost in the budget constraint, but they enter the utility function.

The first-order conditions are

$$\begin{aligned}
v'(N_t) &= U_C(C_t, \mathcal{L}_t) \frac{W_t}{P_t} \\
\frac{U_C(C_t, \mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t, \mathcal{L}_t)}{P_t} \frac{\partial \mathcal{L}_t}{\partial m_t} + \beta(1+i_t^m)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, \mathcal{L}_{t+1})}{P_{t+1}} \right) \\
\frac{U_C(C_t, \mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t, \mathcal{L}_t)}{P_t} \frac{\partial \mathcal{L}_t}{\partial d_t} + \beta(1+i_t^d)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, \mathcal{L}_{t+1})}{P_{t+1}} \right) \\
\frac{U_C(C_t, \mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t, \mathcal{L}_t)}{P_t} \frac{\partial \mathcal{L}_t}{\partial cbdc_t} + \beta(1+i_t^{cbdc})\mathbb{E}_t \left(\frac{U_C(C_{t+1}, \mathcal{L}_{t+1})}{P_{t+1}} \right) \\
\frac{U_C(C_t, \mathcal{L}_t)}{P_t} &= \beta(1+i_t)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, \mathcal{L}_{t+1})}{P_{t+1}} \right).
\end{aligned}$$

Introducing the last equation (Euler equation) into the second to fourth ones and simplifying, we obtain:

$$\frac{i_t - i_t^m}{1+i_t} = \frac{U_{\mathcal{L}}(C_t, \mathcal{L}_t)}{U_C(C_t, \mathcal{L}_t)} \frac{\partial \mathcal{L}_t}{\partial m_t} \tag{B.11}$$

$$
\begin{aligned}
\frac{i_t - i_t^d}{1+i_t} &= \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{U_C(C_t,\mathcal{L}_t)}\frac{\partial \mathcal{L}_t}{\partial d_t} && \text{(B.12)}\\
\frac{i_t - i_t^{cbdc}}{1+i_t} &= \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{U_C(C_t,\mathcal{L}_t)}\frac{\partial \mathcal{L}_t}{\partial cbdc_t}. && \text{(B.13)}
\end{aligned}
$$

Let's assume a non-separable utility function similar to Greenwood et al. (1988), with the utility of C and $\mathcal{L}$ taking the following form:

$$
U(C_t,\mathcal{L}_t) = \frac{(C_t + \xi(\mathcal{L}_t))^{1-\sigma} - 1}{1-\sigma},
$$

then the marginal utilities with respect to consumption and $\mathcal{L}$ take the following forms:

$$
\begin{aligned}
U_{\mathcal{L}}(C_t,\mathcal{L}_t) &= (C_t + \xi(\mathcal{L}_t))^{-\sigma}\xi'(\mathcal{L}_t)\\
U_C(C_t,\mathcal{L}_t) &= (C_t + \xi(\mathcal{L}_t))^{-\sigma}.
\end{aligned}
$$

Hence, (B.11)-(B.13) become

$$
\begin{aligned}
\frac{i_t - i_t^m}{1+i_t} &= \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} && \text{(B.14)}\\
\frac{i_t - i_t^d}{1+i_t} &= \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} && \text{(B.15)}\\
\frac{i_t - i_t^{cbdc}}{1+i_t} &= \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}. && \text{(B.16)}
\end{aligned}
$$

These equations are convenient, because they do not contain wealth effects in the demand for cash, deposits, and CBDC. This result comes from the GHH-style non-separable utility function.

Next, we assume the ξ function takes the following form:

$$
\xi(\mathcal{L}_t(m_t,d_t,cbdc_t)) = m_t + d_t + cbdc_t - \Phi(\mathcal{L}_t(m_t,d_t,cbdc_t)),
$$

Taking its derivatives with respect to money, aggregate deposits, and CBDC, we get

$$
\begin{aligned}
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t}\\
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\\
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t},
\end{aligned}
$$

turning (B.14)-(B.16) into

$$
\begin{aligned}
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} &= \frac{1+i_t^m}{1+i_t}\\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1+i_t^d}{1+i_t}\\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} &= \frac{1+i_t^{cbdc}}{1+i_t}.
\end{aligned}
$$

These equations are identical to equations (B.3)-(B.5), which then lead to the holding schedules (3.4)-(3.6).

Appendix B.3 The Intermediate Good Firm's Problem

The Bellman equation of the intermediate good firm is:

$$V_t(\{K^P_{j,t}\}^n_{j=1}, K^{NP}_t) = \max_{N_t, \{K^P_{j,t+1}\}^n_{j=1}, K^{NP}_{t+1}} \left\{ \Pi^m_t + \mathbb{E}_t(\Lambda_{t,t+1} V_{t+1}(\{K^P_{j,t+1}\}^n_{j=1}, K^{NP}_{t+1}) \right\},$$

where

$$\begin{aligned}
\Pi^m_t &= P^m_t Y^m_t - W_t N_t + (1-\delta) Q_t \sum_{j=1}^n K^P_{j,t} + (1-\delta) Q_t K^{NP}_t \\
&\quad - \sum_{j=1}^n (1 + i^l_{j,t-1}) Q_{t-1} K^P_{j,t} - (1 + i_{t-1} + \varrho) Q_{t-1} K^{NP}_t \\
Y^m_t &= A_t K^\alpha_t N^{1-\alpha}_t \\
K_t &= \left((1-\psi)^{\frac{1}{\theta^k}} (K^{NP}_t)^{\frac{\theta^k - 1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} (K^P_t)^{\frac{\theta^k-1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k - 1}} \\
K^P_t &= \left(\sum_{j=1}^n (\alpha^l_j)^{\frac{1}{\varepsilon^l}} (K^P_{j,t})^{\frac{\varepsilon^l - 1}{\varepsilon^l}} \right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}},
\end{aligned}$$

and $\Lambda_{t,t+1}$ is the stochastic discount factor that the household uses to discount nominal cash flows between $t+1$ and t. The derivatives of K_t w.r.t. to non-pledgeable and the different components of pledgeable capital are:

$$\begin{aligned}
\frac{\partial K_t}{\partial K^{NP}_t} &= (1-\psi)^{\frac{1}{\theta^k}} \left(\frac{K_t}{K^{NP}_t} \right)^{\frac{1}{\theta^k}} \\
\frac{\partial K_t}{\partial K^P_{j,t}} &= \frac{\partial K_t}{\partial K^P_t} \frac{\partial K^P_t}{\partial K^P_{j,t}} = \psi^{\frac{1}{\theta^k}} \left(\frac{K_t}{K^P_t} \right)^{\frac{1}{\theta^k}} (\alpha^l_j)^{\frac{1}{\varepsilon^l}} \left(\frac{K^P_t}{K^P_{j,t}} \right)^{\frac{1}{\varepsilon^l}}.
\end{aligned}$$

The F.O.C.'s w.r.t. labor, non-pledgeable, and all the individual types of pledgeable capital are then:

$$\begin{aligned}
0 &= (1-\alpha) P^m_t \frac{Y^m_t}{N_t} - W_t \\
0 &= \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{\partial V_{t+1}(\{K^P_{j,t+1}\}^n_{j=1}, K^{NP}_{t+1})}{\partial K^{NP}_{t+1}} \right) \\
0 &= \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{\partial V_{t+1}(\{K^P_{j,t+1}\}^n_{j=1}, K^{NP}_{t+1})}{\partial K^P_{j,t+1}} \right).
\end{aligned}$$

The Benveniste-Scheinkman conditions are:

$$\frac{\partial V_t(\{K^P_{j,t}\}^n_{j=1}, K^{NP}_t)}{\partial K^{NP}_t} = \alpha (1-\psi)^{\frac{1}{\theta^k}} P^m_t \frac{Y^m_t}{K_t} \left(\frac{K_t}{K^{NP}_t} \right)^{\frac{1}{\theta^k}} + (1-\delta) Q_t - Q_{t-1}(1 + i_{t-1} + \varrho)$$

$$\frac{\partial V_t(\{K^P_{j,t}\}^n_{j=1}, K^{NP}_t)}{\partial K^P_{j,t}} = \alpha\psi^{\frac{1}{\theta^k}} P^m_t \frac{Y^m_t}{K_t}\left(\frac{K_t}{K^P_t}\right)^{\frac{1}{\theta^k}} (\alpha^l_j)^{\frac{1}{\varepsilon^l}}\left(\frac{K^P_t}{K^P_{j,t}}\right)^{\frac{1}{\varepsilon^l}} + (1-\delta)Q_t - Q_{t-1}(1+i^l_{j,t-1}).$$

Moving these forward one period and introducing them in the capital F.O.C.s we get:

$$\begin{aligned} 0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\left(\alpha(1-\psi)^{\frac{1}{\theta^k}} P^m_{t+1}\frac{Y^m_{t+1}}{K_{t+1}}\left(\frac{K_{t+1}}{K^{NP}_{t+1}}\right)^{\frac{1}{\theta^k}} + (1-\delta)Q_{t+1} - Q_t(1+i_t+\varrho)\right)\right) \\ 0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\left(\alpha\psi^{\frac{1}{\theta^k}} P^m_{t+1}\frac{Y^m_{t+1}}{K_{t+1}}\left(\frac{K_{t+1}}{K^P_{t+1}}\right)^{\frac{1}{\theta^k}} (\alpha^l_j)^{\frac{1}{\varepsilon^l}}\left(\frac{K^P_{t+1}}{K^P_{j,t+1}}\right)^{\frac{1}{\varepsilon^l}} + (1-\delta)Q_{t+1} - Q_t(1+i^l_{j,t})\right)\right). \end{aligned}$$

Using the fact that $\Lambda_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}$, the Euler equation, and denoting the intermediate variable $\Theta_t \equiv \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P^m_{t+1}}{P_{t+1}}\frac{Y^m_{t+1}}{K_{t+1}}\right)$ we obtain:

$$\begin{aligned} \frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) &= \Theta_t(1-\psi)^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K^{NP}_{t+1}}\right)^{\frac{1}{\theta^k}} \\ \frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) &= \Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K^P_{t+1}}\right)^{\frac{1}{\theta^k}} (\alpha^l_j)^{\frac{1}{\varepsilon^l}}\left(\frac{K^P_{t+1}}{K^P_{j,t+1}}\right)^{\frac{1}{\varepsilon^l}}. \end{aligned}$$

We manipulate the second equation (of which there are a total of n versions, one for each bank), raising it to the power of $1-\varepsilon^l$, multiplying by α^l_j, and then adding over all the n equations, to obtain:

$$\sum_{j=1}^n \alpha^l_j (z^P_{j,t})^{1-\varepsilon^l} = \left(\Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K^P_{t+1}}\right)^{\frac{1}{\theta^k}}\right)^{1-\varepsilon^l} \sum_{j=1}^n (\alpha^l_j)^{\frac{1}{\varepsilon^l}}\left(\frac{K^P_{j,t+1}}{K^P_{t+1}}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}},$$

where,

$$z^P_{j,t} \equiv \frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right).$$

Defining:

$$z^P_t \equiv \left(\sum_{j=1}^n \alpha^l_j (z^P_{j,t})^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}},$$

we can rewrite the previous expression as:

$$z^P_t = \Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K^P_{t+1}}\right)^{\frac{1}{\theta^k}}.$$

We can also write demand for the individual pledgeable capital of bank j as:

$$K_{j,t+1}^P = \alpha_j^l \left(\frac{z_{j,t}^P}{z_t^P}\right)^{-\varepsilon^l} K_{t+1}^P.$$

Defining:

$$z_t^{NP} \equiv \frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right),$$

we then have two aggregate conditions for K_t^{NP} and K_t^P that we can rewrite as:

$$\begin{aligned} \Theta_t(1-\psi)^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}}(K_{t+1}^{NP})^{-\frac{1}{\theta^k}} &= z_t^{NP} \\ \Theta_t\psi^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}}(K_{t+1}^{P})^{-\frac{1}{\theta^k}} &= z_t^{P}. \end{aligned}$$

Raise these to the power of $1-\theta^k$ and multiply by ψ in the top one and $(1-\psi)$ in the bottom one to obtain:

$$\begin{aligned} \left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}}\right)^{1-\theta^k}(1-\psi)^{\frac{1}{\theta^k}}(K_{t+1}^{NP})^{\frac{\theta^k-1}{\theta^k}} &= (1-\psi)\left(z_t^{NP}\right)^{1-\theta^k} \\ \left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}}\right)^{1-\theta^k}\psi^{\frac{1}{\theta^k}}(K_{t+1}^{P})^{\frac{\theta^k-1}{\theta^k}} &= \psi(z_t^{P})^{1-\theta^k}. \end{aligned}$$

Adding both of the previous equations we get:

$$\left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}}\right)^{1-\theta^k} K_{t+1}^{\frac{\theta^k-1}{\theta^k}} = z_t^{1-\theta^k},$$

where:

$$z_t \equiv \left(\psi(z_t^P)^{1-\theta^k} + (1-\psi)(z_t^{NP})^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}}.$$

The previous equation for determining aggregate K_t as a function of z_t can then be simplified to:

$$\Theta_t = z_t.$$

With this, the F.O.C.'s for pledgeable and non-pledgeable capital can also be expressed as:

$$\begin{aligned} (1-\psi)^{\frac{1}{\theta^k}} z_t K_{t+1}^{\frac{1}{\theta^k}}(K_{t+1}^{NP})^{-\frac{1}{\theta^k}} &= z_t^{NP} \\ \psi^{\frac{1}{\theta^k}} z_t K_{t+1}^{\frac{1}{\theta^k}}(K_{t+1}^{P})^{-\frac{1}{\theta^k}} &= z_t^{P}, \end{aligned}$$

which can be rearranged to:

$$\begin{aligned} K_{t+1}^{NP} &= (1-\psi)\left(\frac{z_t^{NP}}{z_t}\right)^{-\theta^k} K_{t+1} \\ K_{t+1}^{P} &= \psi\left(\frac{z_t^{P}}{z_t}\right)^{-\theta^k} K_{t+1}, \end{aligned}$$

the usual CES expressions.

Appendix B.4 The Capital Producer

We assume that even though non-pledgeable and pledgeable capital are financed differently by intermediate good firms (one by borrowing from banks and the other by borrowing in bonds), they are produced by the same representative capital producer that treats them indistinguishably, so they have the same price of capital Q_t and there is a single investment adjustment cost. It would be straightforward to augment the model to have two different prices of capital. Denote:

$$K_t^S = K_t^{NP} + \sum_{j=1}^{n} K_{j,t}^P.$$

The representative capital producer sells $Q_t K_{t+1}^S$ dollars worth of new capital, buys $(1-\delta)Q_t K_t^S$ dollars worth of used capital, and additionally pays I_t dollars in order to increase capital from K_t^S to K_{t+1}^S. New capital K_{t+1}^S is obtained from K_t^S and I_t as follows:

$$K_{t+1}^S = (1-\delta)K_t^S + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right).$$

With these elements, the nominal period-t profits of the capital good producer are:

$$\Pi_t^K = Q_t K_{t+1}^S - (1-\delta)Q_t K_t^S - P_t I_t,$$

which, using the previous equation for K_{t+1}^S, can be expressed as:

$$\Pi_t^K = Q_t I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) - P_t I_t,$$

where the function $\Xi(\cdot)$ captures investment adjustment costs. The problem of the capital producer in period t is:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau}\left[Q_{t+\tau} I_{t+\tau}\left(1 - \Xi\left(\frac{I_{t+\tau}}{I_{t+\tau-1}}\right)\right) - P_{t+\tau} I_{t+\tau}\right],$$

where $\Lambda_{t,t+\tau}$ is the household's nominal stochastic discount factor for discounting nominal flows from $t+\tau$ back to t. The F.O.C. is:

$$0 = Q_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) - Q_t\Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \mathbb{E}_t\Lambda_{t,t+1}Q_{t+1}I_{t+1}\Xi'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}}{I_t^2} - P_t.$$

Which we rewrite as:

$$1 = \frac{Q_t}{P_t}\left[1 - \Xi\left(\frac{I_t}{I_{t-1}}\right) - \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2.$$

The $\Xi(\cdot)$ function satisfies $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) \geq 0$.

Appendix B.5 The Bank's Problem

Appendix B.5.1 Separation

Recall that the bank's problem is given by:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s+1} DIV_{j,t+s+1}.$$

As discussed in the main text, banks do not independently optimize their dividend distribution but instead take as given that a fraction $(1-\omega)$ of "profits" $X_{j,t+1}$ are distributed as dividends. The Bellman equation for the bank's problem is:

$$V(F_{j,t}, \Omega_t) = \max_{i^d_{j,t}, D_{j,t}, i^l_{j,t}, L_{j,t}} \mathbb{E}\left\{\beta\Lambda DIV_{j,t+1} + \beta\Lambda V(F_{j,t+1}, \Omega_{t+1})\right\},$$

where Ω_t denotes the aggregate state variables that influence the value of being a bank in period t and we have used Λ as short hand notation for $\Lambda_{t,t+1}$. The maximization problem is subject to the deposit supply schedule, loan demand schedule, as well as:

$$\begin{aligned}
DIV_{j,t+1} &= (1-\omega)X_{j,t+1} \\
F_{j,t+1} &= F_{j,t}(1-\varsigma)(1+\pi_{t+1}) + \omega X_{j,t+1} \\
X_{j,t+1} &= i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} \\
&- \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}.
\end{aligned}$$

The F.O.C. w.r.t. i^d_j yields the following:

$$0 = \mathbb{E}\left\{\beta\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} + \beta\Lambda\frac{\partial V(F_{j,t+1}, \Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}\right\}.$$

Since $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}$ is deterministic (known in period t), it can exit the expectation operator and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} = 0$, which is equivalent to maximizing $(i_t - \mu^d - i^d_{j,t})D_{j,t}$ w.r.t. $i^d_{j,t}$ subject to the deposit supply schedule $D_{j,t}(i^d_{j,t})$.

Similarly, the F.O.C. w.r.t. i^l_j yields the following:

$$0 = \mathbb{E}\left\{\beta\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} + \beta\Lambda\frac{\partial V(F_{j,t+1}, \Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}\right\}.$$

Since $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}$ is also deterministic, it can exit the expectation operator as well, and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} = 0$, which is equivalent to maximizing

$$(i^l_{j,t} - \mu^l - i_t)L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}$$

w.r.t. $i^l_{j,t}$ subject to the loan demand schedule $L_{j,t}(i^l_{j,t})$.

The reason the deposit and loan problems can be neatly separated, is because banks can always use their reserves $H_{j,t}$ to borrow or lend any excess funds to the central bank, so they always optimize their loan and deposit franchises separately. If there was a constraint like $H_{j,t} \geq 0$, then there are circumstances under which the deposit and loan franchises interact and the maximization problem cannot be neatly separated into the two subproblems.

Appendix B.5.2 Deposits

A bank that maximizes $(i_t - i^d_{j,t} - \mu^d)D_{j,t}$ has the following F.O.C.:

$$0 = -D_{j,t} + ((1 + i_t - \mu^d) - (1 + i^d_{j,t}))\frac{\partial D_{j,t}}{\partial d_{j,t}}\frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})}.$$

Denote with $\epsilon^d_{j,t}$ the endogenous elasticity of $d_{j,t}$ w.r.t. $(1 + i^d_{j,t})$:

$$\epsilon^d_{j,t} \equiv \frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})}\frac{1 + i^d_{j,t}}{d_{j,t}}.$$

Then we can write the previous F.O.C. as:

$$\begin{aligned} 1 &= \epsilon^d_{j,t}((1 + i_t - \mu^d) - (1 + i^d_{j,t}))\frac{1}{1 + i^d_{j,t}} \\ 1 + i^d_{j,t} &= \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t} + 1}(1 + i_t - \mu^d). \end{aligned} \tag{B.17}$$

Now, lets obtain $\epsilon^d_{j,t}$. This is not trivial because both $1 + i^d_t$ and d_t depend on $1 + i^d_{j,t}$. Lets compute the elasticity of the aggregate deposit rate w.r.t. one individual deposit rate:

$$\begin{aligned} 1 + i^d_t &= \left(\sum_{j=1}^n \alpha_j(1 + i^d_{j,t})^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}} \\ \frac{\partial(1 + i^d_t)}{\partial(1 + i^d_{j,t})} &= \frac{1}{\varepsilon^d + 1}\left(\sum_{j=1}^n \alpha_j(1 + i^d_{j,t})^{\varepsilon^d+1}\right)^{-\frac{\varepsilon^d}{\varepsilon^d+1}} \alpha_j(\varepsilon^d + 1)(1 + i^d_{j,t})^{\varepsilon^d} \\ &= (1 + i^d_t)^{-\varepsilon^d}\alpha_j(1 + i^d_{j,t})^{\varepsilon^d} \\ &= \alpha_j\left(\frac{1 + i^d_{j,t}}{1 + i^d_t}\right)^{\varepsilon^d} = \frac{d_{j,t}}{d_t} \\ \frac{\partial(1 + i^d_t)}{\partial(1 + i^d_{j,t})}\frac{1 + i^d_{j,t}}{1 + i^d_t} &= \alpha_j\left(\frac{1 + i^d_{j,t}}{1 + i^d_t}\right)^{\varepsilon^d+1} = \frac{(1 + i^d_{j,t})d_{j,t}}{(1 + i^d_t)d_t} \equiv \omega^{d_j}_{d,t}, \end{aligned} \tag{B.18}$$

where $\omega^{d_j}_{d,t}$ is the share of gross interest spending on deposits of bank j at time t. Now lets compute the elasticity of d_t w.r.t. $(1 + i^d_t)$:

$$
\begin{aligned}
d_t &= \gamma_d \left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta} \mathcal{L}_t \\
\ln d_t &= \ln \gamma_d + \theta \ln(1+i_t^d) - \theta \ln(1+i_t^{\mathcal{L}}) + \ln \mathcal{L}_t \\
\frac{\partial \ln d_t}{\partial \ln(1+i_t^d)} &= \theta - \theta \frac{\partial \ln(1+i_t^{\mathcal{L}})}{\partial \ln(1+i_t^d)} + \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i_t^{\mathcal{L}})} \frac{\partial \ln(1+i_t^{\mathcal{L}})}{\partial \ln(1+i_t^d)} \\
&= \theta \left(1 - \frac{\partial \ln(1+i_t^{\mathcal{L}})}{\partial \ln(1+i_t^d)}\right) + \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i_t^{\mathcal{L}})} \frac{\partial \ln(1+i_t^{\mathcal{L}})}{\partial \ln(1+i_t^d)}.
\end{aligned}
$$

The elasticity of $1+i_t^{\mathcal{L}}$ w.r.t. $1+i_t^d$ is:

$$
\begin{aligned}
1+i_t^{\mathcal{L}} &= \left(\gamma(1+i_t^m)^{\theta+1} + \delta(1+i_t^d)^{\theta+1} + \eta(1+i^{cbdc_t})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{\partial(1+i_t^{\mathcal{L}})}{\partial(1+i_t^d)} &= \left(1+i_t^{\mathcal{L}}\right)^{-\theta} \gamma_d (1+i_t^d)^{\theta} = \frac{d_t}{\mathcal{L}_t} \\
\frac{\partial(1+i_t^{\mathcal{L}})}{\partial(1+i_t^d)} \frac{1+i_t^d}{1+i_t^{\mathcal{L}}} &= \gamma_d \left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta+1} = \frac{(1+i_t^d) d_t}{(1+i_t^{\mathcal{L}})\mathcal{L}_t} \equiv \omega_{\mathcal{L},t}^d.
\end{aligned}
$$

With all of these things we can write:

$$
\begin{aligned}
\ln d_{j,t} &= \ln \alpha_j + \varepsilon^d \ln(1+i_{j,t}^d) - \varepsilon^d \ln(1+i_t^d) + \ln d_t \\
\frac{\partial \ln d_{j,t}}{\partial \ln(1+i_{j,t}^d)} &= \varepsilon^d - \varepsilon^d \frac{\partial \ln(1+i_t^d)}{\partial \ln(1+i_{j,t}^d)} + \frac{\partial \ln d_t}{\partial \ln(1+i_t^d)} \frac{\partial \ln(1+i_t^d)}{\partial \ln(1+i_{j,t}^d)} \\
\epsilon_{j,t}^d &= (1-\omega_{d,t}^{d_j})\varepsilon^d + \omega_{d,t}^{d_j} \left[(1-\omega_{\mathcal{L},t}^d)\theta + \omega_{\mathcal{L},t}^d \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i_t^{\mathcal{L}})}\right].
\end{aligned}
$$

If all banks are identical they all pay the same deposit rate ($i_{j,t}^d = i_t^d$), all face the same elasticity ϵ_t^d, and they all obtain one n-th of total deposits (i.e. $\omega_{d,t}^{d_j} = 1/n$), and the expression becomes:

$$
\begin{aligned}
\epsilon_t^d &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega_{\mathcal{L},t}^d)\theta + \omega_{\mathcal{L},t}^d \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i_t^{\mathcal{L}})}\right] \\
&= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n} + \frac{1}{n}\omega_{\mathcal{L},t}^d \left(\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i_t^{\mathcal{L}})} - \theta\right). \qquad \text{(B.19)}
\end{aligned}
$$

Appendix B.5.3 Loans

The F.O.C. of the loan sub-problem (w.r.t. $i_{j,t}^l$) is:

$$
\begin{aligned}
0 &= L_{j,t} + \left\{[1+i_{j,t}^l] - \left[1+i_t+\mu^l+\Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right]\right\} \frac{\partial L_{j,t}}{\partial l_{j,t}} \frac{\partial l_{j,t}}{\partial(1+i_{j,t}^l)} \\
1+i_{j,t}^l &= \frac{\epsilon_{j,t}^l}{\epsilon_{j,t}^l - 1}\left[1+i_t+\mu^l+\Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right]. \qquad \text{(B.20)}
\end{aligned}
$$

where $\epsilon_{j,t}^l$ denotes the (negative of the) elasticity of $l_{j,t}$ w.r.t. $(1+i_{j,t}^l)$:

$$\epsilon^l_{j,t} \equiv -\frac{\partial l_{j,t}}{\partial(1+i^l_{j,t})}\frac{1+i^l_{j,t}}{l_{j,t}}.$$

Now, lets obtain an expression for $\epsilon^l_{j,t}$ as a function of the other variables in the model. We know the following things:

$$\begin{aligned}
z_t &= \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P^m_{t+1}}{P_{t+1}}A_{t+1}N^{1-\alpha}_{t+1}\right)K^{\alpha-1}_{t+1} \\
z_t &= \left(\psi(z^P_t)^{1-\theta^k} + (1-\psi)(z^{NP}_t)^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}} \\
z^P_t &= \left(\sum_{j=1}^n \alpha^l_j (z^P_{j,t})^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}} \\
z^P_{j,t} &\equiv \frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) \\
l_{j,t} &= \alpha^l_j\left(\frac{z^P_{j,t}}{z^P_t}\right)^{-\varepsilon^l} l_t \\
l_t &= \frac{Q_t}{P_t}\psi\left(\frac{z^P_t}{z_t}\right)^{-\theta^k} K_{t+1}.
\end{aligned}$$

Lets first compute the elasticity of z^P_t w.r.t. one individual $z^P_{j,t}$:

$$\begin{aligned}
\frac{\partial z^P_t}{\partial z^P_{j,t}} &= \frac{1}{1-\varepsilon^l}\left(\sum_{j=1}^n \alpha^l_j (z^P_{j,t})^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}-1} \alpha^l_j (1-\varepsilon^l)(z^P_{j,t})^{-\varepsilon^l} \\
&= \alpha^l_j\left(\frac{z^P_{j,t}}{z^P_t}\right)^{-\varepsilon^l} = \frac{l_{j,t}}{l_t} \\
\frac{\partial z^P_t}{\partial z^P_{j,t}}\frac{z^P_{j,t}}{z^P_t} &= \alpha^l_j\left(\frac{z^P_{j,t}}{z^P_t}\right)^{1-\varepsilon^l} = \frac{z^P_{j,t} l_{j,t}}{z^P_t l_t} \equiv \omega^{l_j}_{l,t}.
\end{aligned}$$

Now, we compute the elasticity of l_t w.r.t. $(1+i^l_t)$. For simplicity, we assume that all banks take the real price of capital Q_t/P_t as given, as well as all aggregate variables that are not explicitly related to capital. Then, we have:

$$\begin{aligned}
\frac{\partial \ln K_t}{\partial \ln z_t} &= \frac{1}{\alpha-1} \\
\frac{\partial \ln l_t}{\partial \ln z^P_t} &= -\theta^k + \theta^k\frac{\partial \ln z_t}{\partial \ln z^P_t} + \frac{\partial \ln K_t}{\partial \ln z_t}\frac{\partial \ln z_t}{\partial \ln z^P_t}.
\end{aligned}$$

The elasticity of z_t w.r.t. z^P_t is:

$$\begin{aligned}
\frac{\partial z_t}{\partial z^P_t} &= z^{\theta^k}_t \psi (z^P_t)^{-\theta^k} = \frac{l_t}{K_t} \\
\frac{\partial \ln z_t}{\partial \ln z^P_t} &= \frac{\partial z_t}{\partial z^P_t}\frac{z^P_t}{z_t} = \psi\left(\frac{z^P_t}{z_t}\right)^{1-\theta^k} = \frac{l_t z^P_t}{K_t z_t} \equiv \omega^{K_{NP}}_{K,t}. \qquad \text{(B.21)}
\end{aligned}$$

We also need the elasticity of $z^P_{j,t}$ w.r.t. $(1+i^l_{j,t})$:

$$\begin{aligned}
\frac{\partial z^P_{j,t}}{\partial(1+i^l_{j,t})} &= \frac{Q_t}{P_t}\frac{1}{1+i_t} \\
\frac{\partial \ln z^P_{j,t}}{\partial \ln(1+i^l_{j,t})} &= \frac{\partial z^P_{j,t}}{\partial(1+i^l_{j,t})}\frac{(1+i^l_{j,t})}{z^P_{j,t}} = \frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t}\frac{1}{z^P_{j,t}}.
\end{aligned}$$

With all of these things we can write:

$$\begin{aligned}
\ln l_{j,t} &= \ln \alpha^l_j - \varepsilon^l \ln z^P_{j,t} + \varepsilon^l \ln z^P_t + \ln l_t \\
\frac{\partial \ln l_{j,t}}{\partial \ln(1+i^l_{j,t})} &= \left[-\varepsilon^l + \varepsilon^l \frac{\partial z^P_t}{\partial z^P_{j,t}} + \frac{\partial \ln l_t}{\partial \ln z^P_{j,t}}\right]\frac{\partial \ln z^P_{j,t}}{\partial \ln(1+i^l_{j,t})} \\
-\epsilon^l_{j,t} &= \left[-\varepsilon^l(1-\omega^{l_j}_{l,t}) + \frac{\partial \ln l_t}{\partial \ln z^P_t}\frac{\partial \ln z^P_t}{\partial \ln z^P_{j,t}}\right]\frac{\partial \ln z^P_{j,t}}{\partial \ln(1+i^l_{j,t})} \\
\epsilon^l_{j,t} &= \left[\varepsilon^l(1-\omega^{l_j}_{l,t}) + \omega^{l_j}_{l,t}\left(\theta^k(1-\omega^{K_{NP}}_{K,t}) + \frac{\omega^{K_{NP}}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t}\frac{1}{z^P_{j,t}}.
\end{aligned}$$

If all banks are identical, they all charge the same loan rate ($i^l_{j,t} = i^l_t$), face the same elasticity ϵ^l_t, and obtain one n-th of total loans (i.e. $\omega^{l_j}_{l,t} = 1/n$), and the expression becomes:

$$\epsilon^l_t = \left[\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left(\theta^k(1-\omega^{K_{NP}}_{K,t}) + \frac{\omega^{K_{NP}}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}\frac{1}{z^P_t}. \tag{B.22}$$

Appendix B.6 The Retailer's Problem

Recall that the retailer's problem is:

$$\max_{P^*_t} \ \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)}\frac{P_t}{P_{t+r}}\left[P^*_t - P^m_{t+r}\right] Y_{t+r|t}.$$

Notice that $Y_{t+r|t}$, the amount sold in period $t+r$ by a firm that last reset its price in period t, is defined as:

$$Y_{t+r|t} \equiv \left(\frac{P^*_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r}.$$

Hence, its derivative with respect to the optimal reset price is given by:

$$\frac{\partial Y_{t+r|t}}{\partial P^*_t} = -\varphi \frac{Y_{t+r|t}}{P^*_t}.$$

The F.O.C. w.r.t. to the optimal reset price is then given by:

$$0 = \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)}\frac{P_t}{P_{t+r}}\left[Y_{t+r|t} - \varphi(P^*_t - P^m_{t+r})\frac{Y_{t+r|t}}{P^*_t}\right]$$

$$= \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} \left[P_t^*(1-\varphi) + \varphi P_{t+r}^m\right].$$

Define

$$\begin{aligned}
\Gamma_t^1 &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P_{t+r}^m \\
\Gamma_t^2 &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P_t^*.
\end{aligned}$$

With this notation we can write the F.O.C. as:

$$\varphi \Gamma_t^1 = (\varphi - 1)\Gamma_t^2. \tag{B.23}$$

We can also characterize Γ_t^1 recursively as:

$$\begin{aligned}
\Gamma_t^1 &= \frac{u'(C_t)}{P_t} Y_t P_t^m + \mathbb{E}_t \sum_{r=1}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P_{t+r}^m \\
&= u'(C_t) \frac{P_t^m}{P_t} Y_t + \gamma\beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}}\right)^{-\varphi} \Gamma_{t+1}^1. \qquad \text{(B.24)}
\end{aligned}$$

Similarly, for Γ_t^2 we have:

$$\begin{aligned}
\Gamma_t^2 &= \frac{u'(C_t)}{P_t} Y_t P_t^* + \mathbb{E}_t \sum_{r=1}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P_t^* \\
&= u'(C_t) \frac{P_t^*}{P_t} Y_t + \gamma\beta \mathbb{E}_t \frac{P_t^*}{P_{t+1}^*} \left(\frac{P_t}{P_{t+1}}\right)^{-\varphi} \Gamma_{t+1}^2. \qquad \text{(B.25)}
\end{aligned}$$

From the definition of the price index we can easily derive an equation for its evolution in terms of the real optimal reset price:

$$1 = (1-\gamma) \left(\frac{P_t^*}{P_t}\right)^{1-\varphi} + \gamma \left(\frac{P_{t-1}}{P_t}\right)^{1-\varphi}. \tag{B.26}$$

Additionally, the aggregate demand for intermediate inputs is the integral over all retail firms:

$$Y_t^m = \int_0^1 Y_t(s) ds = \int_0^1 \left(\frac{P_t(s)}{P_t}\right)^{-\varphi} Y_t ds = Y_t v_t^p, \tag{B.27}$$

where v_t^p is an index of price dispersion that evolves as follows:

$$\begin{aligned}
v_t^p &= \int_0^1 \left(\frac{P_t(s)}{P_t}\right)^{-\varphi} ds \\
&= \gamma \left(\frac{P_{t-1}}{P_t}\right)^{-\varphi} v_{t-1}^p + (1-\gamma) \left(\frac{P_t^*}{P_t}\right)^{-\varphi}. \qquad \text{(B.28)}
\end{aligned}$$

Equations (B.23)-(B.28) are the ones given in the text as describing the optimal behavior of retail firms.

Appendix B.7 Resource Constraint

In this appendix, we derive the aggregate resource constraint of the model economy. Notice that the aggregate nominal profits of retail firms are:

$$\Pi_t^R = P_t Y_t - P_t^m Y_t^m.$$

This is necessarily the case because on aggregate they sell all output in the economy at price P_t, so they make revenue of $P_t Y_t$, and they buy all intermediate inputs in the economy at price P_t^m, so they have costs of $P_t^m Y_t^m$. The dividends distributed by the bank in period t are:

$$\Pi_t^B = (1-\omega) X_t.$$

Intermediate good firms (in the case with symmetric commercial banks) have nominal profits of:

$$\Pi_t^m = P_t^m Y_t^m - W_t N_t + Q_t(1-\delta)K_t^P + Q_t(1-\delta)K_t^{NP} - Q_{t-1}(1+i_{t-1}^l)K_t^P - Q_{t-1}(1+i_{t-1}+\varrho)K_t^{NP}.$$

Capital producers (in the case with symmetric commercial banks) have nominal profits of:

$$\Pi_t^K = Q_t\left[K_{t+1}^{NP} + K_{t+1}^P - (1-\delta)\left(K_t^{NP} + K_t^P\right)\right] - P_t I_t.$$

We also need the following equations:

$$\begin{aligned}
K_{t+1}^{NP} + K_{t+1}^P &= (1-\delta)\left[K_t^{NP} + K_t^P\right] + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) \\
Tr_t &= M_t - M_{t-1} + H_t - (1+i_{t-1})H_{t-1} + CBDC_t - (1+i_{t-1}^{cbdc})CBDC_{t-1} \\
&- P_t\Omega\left(\frac{CBDC_t}{P_t Y_t}\right) - P_t G_t \\
B_t &= Q_t K_{t+1}^{NP} \\
L_t &= Q_t K_{t+1}^P \\
D_t + F_t &= H_t + L_t \\
F_t &= (1+i_{t-1})F_{t-1} - (1-\omega)X_t - \varsigma F_{t-1} + (i_{t-1}^l - \mu^l - i_{t-1})L_{t-1} \\
&+ (i_{t-1} - \mu^d - i_{t-1}^d)D_{t-1} - \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)F_{t-1}.
\end{aligned}$$

Start with the budget constraint of the households and simplify:

$$\begin{aligned}
P_t C_t &= W_t N_t - P_t\Phi(\mathcal{L}_t) - B_t + (1+i_{t-1})B_{t-1} + (1+i_{t-1}^m)M_{t-1} \\
&+ (1+i_{t-1}^d)D_{t-1} + (1+i_{t-1}^{cbdc})CBDC_{t-1} + Tr_t + \Pi_t^R + \Pi_t^B + \Pi_t^m + \Pi_t^K \\
&= P_t Y_t - P_t G_t - P_t\Phi(\mathcal{L}_t) - Q_t K_{t+1}^{NP} + (1+i_{t-1})Q_{t-1}K_t^{NP} + (1+i_{t-1}^d)D_{t-1} + M_t \\
&+ H_t - (1+i_{t-1})H_{t-1} + CBDC_t + (1-\omega)X_t + Q_t(1-\delta)K_t^P + Q_t(1-\delta)K_t^{NP} \\
&- Q_{t-1}(1+i_{t-1}^l)K_t^P - Q_{t-1}(1+i_{t-1}+\varrho)K_t^{NP} + \Pi_t^K - P_t\Omega\left(\frac{CBDC_t}{P_t Y_t}\right) \\
&= P_t Y_t - P_t G_t - Q_t(K_{t+1}^{NP} - (1-\delta)K_t^{NP}) - P_t\Phi(\mathcal{L}_t) + M_t + CBDC_t - \varrho Q_{t-1}K_t^{NP} \\
&+ (1+i_{t-1}^d)D_{t-1} + D_t + F_t - L_t - (1+i_{t-1})(D_{t-1} + F_{t-1} - L_{t-1}) + (1-\omega)X_t
\end{aligned}$$

$$
\begin{aligned}
&+ \quad Q_t(1-\delta)K_t^P - Q_{t-1}(1+i_{t-1}^l)K_t^P + \Pi_t^K - P_t\Omega\left(\frac{CBDC_t}{P_tY_t}\right) \\
&= \quad P_tY_t - P_tG_t - P_tI_t - P_t\Phi(\mathcal{L}_t) + M_t + CBDC_t + D_t - \varrho Q_{t-1}K_t^{NP} \\
&- \quad \varsigma F_{t-1} - \mu^l L_{t-1} - \mu^d D_{t-1} - \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)F_{t-1} - P_t\Omega\left(\frac{CBDC_t}{P_tY_t}\right).
\end{aligned}
$$

Finally, we obtain:

$$
\begin{aligned}
Y_t &= C_t + G_t + I_t + \Phi(\mathcal{L}_t) - \frac{M_t + CBDC_t + D_t}{P_t} + \varrho\frac{Q_{t-1}}{P_t}K_t^{NP} \\
&+ \varsigma\frac{F_{t-1}}{P_t} + \mu^l\frac{L_{t-1}}{P_t} + \mu^d\frac{D_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \Omega\left(\frac{CBDC_t}{P_tY_t}\right).
\end{aligned}
$$

Which is equivalent to equations (3.23)-(3.24) in the main text.

Appendix B.8 Equilibrium Equations

We assume the following functional forms for $v(\cdot)$, $u(\cdot)$, $\Psi(\cdot)$, $\Phi(\cdot)$ and $\Xi(\cdot)$:

$$
\begin{aligned}
v(x) &= \chi\frac{x^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \\
u(x) &= \frac{x^{1-\sigma}-1}{1-\sigma} \\
\Psi(x) &= \kappa\nu x\left(\ln x - \ln\nu - 1\right) + \kappa\nu^2 \\
\Phi(x) &= ax^b - q \\
\Xi(x) &= \frac{\kappa_I}{2}\left(x-1\right)^2 \\
\Omega(x) &= N(e\mathbb{1}(\text{CBDC exists}) + fx).
\end{aligned}
$$

Note that we allow the cost of CBDC function Ω to have both a fixed component that is turned on as soon as CBDC is implemented but does not vary with the amount of CBDC, and a component that varies with the amount of real CBDC as a function of GDP. We also multiply Ω by steady state labor for numerical convenience. The function Ψ is not exactly quadratic, but it has several useful properties described in Ulate (2021). Furthermore, its second order approximation around the steady state is:

$$
\Psi(x) \approx^2 \frac{\kappa}{2}\left(x-\nu\right)^2,
$$

which is the quadratic form that has been traditionally used in the literature.

We reiterate the equilibrium equations here according to their sector. Households (7 equations):

$$
\begin{aligned}
\chi N_t^{\frac{1}{\eta}} &= C_t^{-\sigma}\frac{W_t}{P_t} \\
1 &= \beta(1+i_t)\mathbb{E}_t\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\right) \\
\frac{1+i_t^{\mathcal{L}}}{1+i_t} &= ab\mathcal{L}_t^{b-1} \\
(1+i_t^{\mathcal{L}})^{\theta+1} &= \gamma_m + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1}
\end{aligned}
$$

$$
\begin{aligned}
m_t &= \gamma_m \left(\frac{1}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t \\
d_t &= \gamma_d \left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t \\
cbdc_t &= \gamma_{cbdc} \left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t.
\end{aligned}
$$

Intermediate good firms (8 equations):

$$
\begin{aligned}
Y_t^m &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= (1-\alpha) \frac{P_t^m}{P_t} \frac{Y_t^m}{N_t} \\
z_t &= \mathbb{E}_t \left(\alpha\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_{t+1}^m}{P_{t+1}} \frac{Y_{t+1}^m}{K_{t+1}} \right) \\
z_t &= \left(\psi (z_t^P)^{1-\theta^k} + (1-\psi)(z_t^{NP})^{1-\theta^k} \right)^{\frac{1}{1-\theta^k}} \\
z_t^P &= \frac{Q_t}{P_t} \frac{1+i_t^l}{1+i_t} - (1-\delta)\mathbb{E}_t \left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \right) \\
z_t^{NP} &= \frac{Q_t}{P_t} \frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t \left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \right) \\
K_{t+1}^P &= \psi \left(\frac{z_t^P}{z_t} \right)^{-\theta^k} K_{t+1} \\
K_{t+1}^{NP} &= (1-\psi) \left(\frac{z_t^{NP}}{z_t} \right)^{-\theta^k} K_{t+1}.
\end{aligned}
$$

Capital producers (2 equations):

$$
\begin{aligned}
K_{t+1}^{NP} + K_{t+1}^P &= (1-\delta) \left[K_t^{NP} + K_t^P \right] + I_t \left(1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) \right) \\
1 &= \frac{Q_t}{P_t} \left[1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) - \Xi' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \\
&+ \mathbb{E}_t \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \Xi' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2.
\end{aligned}
$$

Banks (10 equations):

$$
\begin{aligned}
\omega_{\mathcal{L},t}^d &= \gamma_d \left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}} \right)^{\theta+1} \\
\epsilon_t^d &= \frac{n-1}{n} \varepsilon^d + \frac{1}{n} \left[(1-\omega_{\mathcal{L},t}^d)\theta + \frac{\omega_{\mathcal{L},t}^d}{b-1} \right] \\
1 + i_t^d &= \frac{\epsilon_t^d}{\epsilon_t^d + 1} (1 + i_t - \mu^d) \\
\omega_{K,t}^{K_{NP}} &= \psi \left(\frac{z_t^P}{z_t} \right)^{1-\theta^k}
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_t^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[(1-\omega_{K,t}^{K_{NP}})\theta^k + \frac{\omega_{K,t}^{K_{NP}}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1+i_t^l}{1+i_t}\frac{1}{z_t^P} \\
1+i_t^l &= \frac{\epsilon_t^l}{\epsilon_t^l-1}\left[1+i_t+\mu^l+\kappa\nu\left(\ln\left(\frac{L_t}{F_t}\right)-\ln(\nu)\right)\right] \\
\frac{L_t}{P_t} &= \frac{Q_t}{P_t}K_{t+1}^P \\
\frac{X_t}{P_t}\frac{P_t}{P_{t-1}} &= i_{t-1}\frac{F_{t-1}}{P_{t-1}} + (i_{t-1}^l-\mu^l-i_{t-1})\frac{L_{t-1}}{P_{t-1}} + (i_{t-1}-\mu^d-i_{t-1}^d)\frac{D_{t-1}}{P_{t-1}} \\
&- \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_{t-1}} - \frac{F_{t-1}}{P_{t-1}}(1-\varsigma)\pi_t \\
\frac{F_t}{P_t} &= \frac{F_{t-1}}{P_{t-1}}(1-\varsigma) + \omega\frac{X_t}{P_t} \\
\frac{H_t}{P_t} &= \frac{F_t}{P_t} + \frac{D_t}{P_t} - \frac{L_t}{P_t}.
\end{aligned}
$$

Retail firms (6 equations):

$$
\begin{aligned}
1 &= (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\varphi} + \gamma\left(\frac{P_{t-1}}{P_t}\right)^{1-\varphi} \\
\varphi\Gamma_t^1 &= (\varphi-1)\Gamma_t^2 \\
\Gamma_t^1 &= C_t^{-\sigma}\frac{P_t^m}{P_t}Y_t + \gamma\beta\mathbb{E}_t\left(\frac{P_t}{P_{t+1}}\right)^{-\varphi}\Gamma_{t+1}^1 \\
\Gamma_t^2 &= C_t^{-\sigma}\frac{P_t^*}{P_t}Y_t + \gamma\beta\mathbb{E}_t\frac{P_t^*/P_t}{P_{t+1}^*/P_{t+1}}\left(\frac{P_t}{P_{t+1}}\right)^{1-\varphi}\Gamma_{t+1}^2 \\
Y_t^m &= Y_t v_t^p \\
v_t^p &= \gamma\left(\frac{P_{t-1}}{P_t}\right)^{-\varphi} v_{t-1}^p + (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\varphi}.
\end{aligned}
$$

Others (5 equations):

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t + \Gamma_t \\
\Gamma_t &= \mu^l\frac{L_{t-1}}{P_t} + \mu^d\frac{D_{t-1}}{P_t} + \varsigma\frac{F_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \varrho\frac{Q_{t-1}}{P_t}K_t^{NP} \\
&+ \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t} + \Omega\left(\frac{CBDC_t}{P_t Y_t}\right) \\
i_t &= (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon_t^i \\
A_t &= A_{t-1}^{\rho_a}\exp(\epsilon_t^a) \\
G_t &= gY_t.
\end{aligned}
$$

Plus a value for the interest rate on CBDC (this would be -100% in the pre-CBDC scenario, but something like $i_t^{cbdc} = 0$ or $i_t^{cbdc} = i - 1$ in the post-CBDC scenario).

Appendix B.9 Steady State

In steady state, we have $Q/P = 1$, $P^* = P$, $v^p = 1$, $Y^M = Y$, $P^m/P = \frac{\varphi-1}{\varphi}$, and $i = \bar{\imath}$, we can also get rid of the investment equations. This way we can drop all the 6 equations for the retailers, 2 for the intermediate good firms, 2 for the capital producers, and 4 of the "others", to simplify the steady state system to 24 equations:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}\frac{W}{P} \\
\frac{1}{\beta}-1 &= i \\
\frac{1+i^{\mathcal{L}}}{1+i} &= ab\mathcal{L}^{b-1} \\
1+i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
m &= \gamma_m\left(\frac{1}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
d &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
cbdc &= \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
Y &= AK^{\alpha}N^{1-\alpha} \\
\frac{W}{P} &= (1-\alpha)\frac{\varphi-1}{\varphi}\frac{Y}{N} \\
z &= \alpha\beta\frac{\varphi-1}{\varphi}\frac{Y}{K} \\
z(1+i) &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k} + (1-\psi)\left(i+\varrho+\delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}} \\
K^P &= \psi\left(\frac{i^l+\delta}{z(1+i)}\right)^{-\theta^k}K \\
K^{NP} &= (1-\psi)\left(\frac{i+\varrho+\delta}{z(1+i)}\right)^{-\theta^k}K \\
\omega^d_{\mathcal{L}} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega^d_{\mathcal{L}})\theta + \omega^d_{\mathcal{L}}\frac{\partial\ln\mathcal{L}}{\partial\ln(1+i^{\mathcal{L}})}\right] \\
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i-\mu^d) \\
\omega^{K_{NP}}_K &= \psi\left(\frac{i^l+\delta}{z(1+i)}\right)^{1-\theta^k} \\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[\theta^k(1-\omega^{K_{NP}}_K) + \frac{1}{1-\alpha}\omega^{K_{NP}}_K\right]\right\}\frac{1+i^l}{i^l+\delta}
\end{aligned}
$$

$$
\begin{aligned}
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left[1+i+\mu^l+\kappa\nu\left(\ln\left(\frac{L}{F}\right)-\ln(\nu)\right)\right]\\
\frac{X}{P} &= i\frac{F}{P}+(i^l-\mu^l-i)\frac{L}{P}+(i-\mu^d-i^d)\frac{D}{P}-\Psi\left(\frac{L}{F}\right)\frac{F}{P}\\
\varsigma\frac{F}{P} &= \omega\frac{X}{P}\\
\frac{L}{P}+\frac{H}{P} &= \frac{F}{P}+\frac{D}{P}\\
Y &= C+\delta(K^P+K^{NP})+gY+\mu^l\frac{L}{P}+\mu^d\frac{D}{P}+\varsigma\frac{F}{P}+\Psi\left(\frac{L}{F}\right)\frac{F}{P}+\varrho K^{NP}\\
&+ \Phi(\mathcal{L})-\frac{M+D+CBDC}{P}+\Omega\\
\frac{L}{P} &= K^P.
\end{aligned}
$$

We can further simplify these to:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha}\\
1+i^{\mathcal{L}} &= \left(\gamma_m+\gamma_d(1+i^d)^{\theta+1}+\gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}\\
\frac{D}{P} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}}\\
\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha} &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k}+(1-\psi)\left(1/\beta-1+\varrho+\delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}}\\
K^P &= \psi\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k}K\\
K^{NP} &= (1-\psi)\left(\frac{1/\beta-1+\varrho+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k}K\\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}-\frac{\gamma_d}{n}\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}\left[\theta-\frac{1}{b-1}\right]\\
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1/\beta-\mu^d)\\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l+\frac{\theta^k}{n}-\frac{\psi}{n}\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{1-\theta^k}\left[\theta^k-\frac{1}{1-\alpha}\right]\right\}\frac{1+i^l}{i^l+\delta}\\
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left[1/\beta+\mu^l+\kappa\nu\left(\ln\left(\frac{K^P}{F/P}\right)-\ln(\nu)\right)\right]\\
\frac{\varsigma}{\omega}+\Psi\left(\frac{K^P}{F/P}\right) &= \left(\frac{1}{\beta}-1\right)\left(1+\frac{D}{F}-\frac{K^P}{F/P}\right)+(i^l-\mu^l)\frac{K^P}{F/P}-(\mu^d+i^d)\frac{D}{F}\\
(1-g)K^{\alpha}N^{1-\alpha} &= C+\delta K^P+\delta K^{NP}+\mu^l K^P+\mu^d\frac{D}{P}+\varsigma\frac{F}{P}+\Psi\left(\frac{K^P}{F/P}\right)\frac{F}{P}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \varrho K^{NP} + a\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{b}{b-1}} - \frac{D}{P} - \gamma_m\left(\frac{1+i^m}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} \\
&- \quad \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} - q + \Omega.
\end{aligned}
$$

This is a system of 12 equations in 12 unknowns: $N, C, i^{\mathcal{L}}, i^d, D/P, K, K^P, K^{NP}, i^l, \epsilon^d, \epsilon^l, F/P$. Recall that i^{cbdc} would be given by an assumption (like $i^{cbdc} = 0$ in the case of our baseline calibration).

Imagine that we are interested in a steady state where $\frac{K^P}{F/P} = \nu$. In that case, we eliminate the eleventh equation and replace it with $\frac{K^P}{F/P} = \nu$. The eleventh equation in that case would simply define the ω needed for $\frac{K^P}{F/P} = \nu$ to be consistent with equilibrium. That steady state would satisfy the simpler set of equations:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha} \\
1+i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{D}{P} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} \\
\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha} &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k} + (1-\psi)\left(1/\beta - 1 + \varrho + \delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}} \\
K^P &= \psi\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k} K \\
K^{NP} &= (1-\psi)\left(\frac{1/\beta - 1 + \varrho + \delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k} K \\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n} - \frac{\gamma_d}{n}\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}\left[\theta - \frac{1}{b-1}\right] \\
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1/\beta - \mu^d) \\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{\theta^k}{n} - \frac{\psi}{n}\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{1-\theta^k}\left[\theta^k - \frac{1}{1-\alpha}\right]\right\}\frac{1+i^l}{i^l+\delta} \\
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left(1/\beta + \mu^l\right) \\
(1-g)K^{\alpha}N^{1-\alpha} &= C + \delta K^P + \delta K^{NP} + \mu^l K^P + \mu^d\frac{D}{P} + \frac{\varsigma}{\nu}K^P + \varrho K^p \\
&+ \quad a\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{b}{b-1}} - \frac{D}{P} - \gamma_m\left(\frac{1+i^m}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} \\
&- \quad \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} - q + \Omega
\end{aligned}
$$

As an aside, notice that the equation for i^d can be simplified to:

$$\begin{aligned}
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1/\beta-\mu_d)\\
\epsilon^d(1/\beta-\mu_d) &= (\epsilon^d+1)(1+i^d)\\
1+i^d &= \epsilon^d(1/\beta-1-\mu_d-i^d)\\
&= \left[\frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}-\frac{\gamma_d}{n}\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}\left(\theta-\frac{1}{b-1}\right)\right](i-\mu_d-i^d)\\
\frac{1+i^d}{i-\mu_d-i^d} &= \frac{n-1}{n}\varepsilon^d+\frac{\theta}{n}-\frac{(\gamma_d/n)(1+i^d)^{\theta+1}}{\gamma_m+\gamma_d(1+i^d)^{\theta+1}+\gamma_{cbdc}(1+i^{cbdc})^{\theta+1}}\left(\theta-\frac{1}{b-1}\right)
\end{aligned}$$

Which is just one equation in one unknown which can be solved for i^d.

Going back to our full system of equations. If $\mu^l=\mu^d=0$, $n\to\infty$, and $q=a\mathcal{L}^b-\frac{M+D+E}{P}$, then the equations for $i^{\mathcal{L}}$, D/P, ϵ^d, i^d are detached from the system (they are only needed for the equation that determines ω), the equation for ϵ^l simplifies to $\epsilon^l=\varepsilon^l\frac{1+i^l}{i^l+\delta}$, and i^l is easy to find:

$$\begin{aligned}
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left(1/\beta+\mu^l\right)\\
(1+i^l)(\epsilon^l-1) &= \epsilon^l\left(1/\beta+\mu^l\right)\\
-(1+i^l) &= \epsilon^l\left(1/\beta-1+\mu^l-i^l\right)\\
-(1+i^l) &= \varepsilon^l\frac{1+i^l}{i^l+\delta}\left(1/\beta-1+\mu^l-i^l\right)\\
-(i^l+\delta) &= \varepsilon^l\left(1/\beta-1+\mu^l-i^l\right)\\
i^l(\varepsilon^l-1) &= \varepsilon^l\left(1/\beta-1+\mu^l\right)+\delta\\
i^l &= \frac{\varepsilon^l}{\varepsilon^l-1}(1/\beta-1)+\frac{\delta}{\varepsilon^l-1}
\end{aligned}$$

Going back to the remaining equations and manipulating them, we get:

$$\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha}\\
\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha} &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k}+(1-\psi)\left(1/\beta-1+\varrho+\delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}}\\
\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{\theta^k} &= \frac{\psi K}{K^P}\\
\left(\frac{1/\beta-1+\varrho+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{\theta^k} &= \frac{(1-\psi)K}{K^{NP}}\\
(1-g)K^{\alpha}N^{1-\alpha} &= C+\delta K^P+\delta K^{NP}+\frac{\varsigma}{\nu}K^P+\varrho K^p+\Omega
\end{aligned}$$

We can then substitute the second equation (which determines z as a function of z^P and z^{NP}) for a quantity equation that determines K as a function of K^P and K^{NP}, because that is a little more convenient to work with (and they are interchangeable if you also have the demands for K^P and K^{NP} in the system). We can also continue to simplify the third and fourth equations. We get:

$$\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha} \\
K &= \left(\psi^{\frac{1}{\theta^k}}\left(K^P\right)^{\frac{\theta^k-1}{\theta^k}} + (1-\psi)^{\frac{1}{\theta^k}}\left(K^{NP}\right)^{\frac{\theta^k-1}{\theta^k}}\right)^{\frac{\theta^k}{\theta^k-1}} \\
1 &= \frac{\varepsilon^l}{\varepsilon^l-1}\frac{1}{\beta} - \frac{1}{\varepsilon^l-1} + \frac{\varepsilon^l\delta}{\varepsilon^l-1} - \alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}\left(\frac{\psi K}{K^P}\right)^{\frac{1}{\theta^k}} \\
1 &= \frac{1}{\beta} + \varrho + \delta - \alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}\left(\frac{(1-\psi)K}{K^{NP}}\right)^{\frac{1}{\theta^k}} \\
C &= (1-g)K^{\alpha}N^{1-\alpha} - \delta K^P - \delta K^{NP} - \frac{\varsigma}{\nu}K^P - \varrho K^p - \Omega
\end{aligned}$$

If we assume that $\varrho = \frac{\frac{1}{\beta}+\delta-1}{\varepsilon^l-1}$, then this guarantees that in the steady state where $\frac{K^P}{F/P} = \nu$ the cost of pledgeable and non-pledgeable capital is the same, which guarantees that:

$$\begin{aligned}
\left(\frac{(1-\psi)K}{K^{NP}}\right)^{\frac{1}{\theta^k}} &= \left(\frac{\psi K}{K^P}\right)^{\frac{1}{\theta^k}} \\
\frac{(1-\psi)K}{K^{NP}} &= \frac{\psi K}{K^P} \\
\frac{K^P}{K^{NP}} &= \frac{\psi}{1-\psi}
\end{aligned}$$

This allows us to obtain:

$$\begin{aligned}
K &= \left(\psi^{\frac{1}{\theta^k}}\left(K^P\right)^{\frac{\theta^k-1}{\theta^k}} + (1-\psi)^{\frac{1}{\theta^k}}\left(\frac{1-\psi}{\psi}\right)^{\frac{\theta^k-1}{\theta^k}}\left(K^P\right)^{\frac{\theta^k-1}{\theta^k}}\right)^{\frac{\theta^k}{\theta^k-1}} \\
&= K^P\left(\psi^{\frac{1}{\theta^k}} + (1-\psi)\psi^{\frac{1-\theta^k}{\theta^k}}\right)^{\frac{\theta^k}{\theta^k-1}} \\
&= \frac{K^P}{\psi}\left(\psi^{\frac{\theta^k-1}{\theta^k}}\left[\psi^{\frac{1}{\theta^k}} + (1-\psi)\psi^{\frac{1-\theta^k}{\theta^k}}\right]\right)^{\frac{\theta^k}{\theta^k-1}} \\
&= \frac{K^P}{\psi}
\end{aligned}$$

So, we can simplify the previous equations to:

$$\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha} \\
\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{1}{\beta}-1+\delta\right) &= \alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha} \\
C &= (1-g)K^{\alpha}N^{1-\alpha} - \delta K - \frac{\varsigma}{\nu}\psi K - \varrho(1-\psi)K - \Omega
\end{aligned}$$

The second equation lets us obtain the capital-labor (K/N) ratio:

$$\frac{K}{N} = \left(\frac{\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{1}{\beta}-1+\delta\right)}{\alpha\frac{\varphi-1}{\varphi}}\right)^{\frac{1}{\alpha-1}} \equiv \aleph$$

With this we can express the other equations as:

$$\begin{aligned} \chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\aleph^{\alpha} \\ (1-g)N\aleph^{\alpha} &= C+\Omega+\left(\delta+\frac{\varsigma}{\nu}\psi+\varrho(1-\psi)\right)\aleph N \end{aligned}$$

From the first we obtain:

$$\begin{aligned} C^{\sigma} &= \frac{1}{\chi}N^{-\frac{1}{\eta}}(1-\alpha)\frac{\varphi-1}{\varphi}\aleph^{\alpha} \\ C &= N^{-\frac{1}{\eta\sigma}}\left(\frac{1-\alpha}{\chi}\frac{\varphi-1}{\varphi}\aleph^{\alpha}\right)^{\frac{1}{\sigma}} \end{aligned}$$

Introducing this into the resource constraint we get:

$$(1-g)\aleph^{\alpha} = N^{-\frac{1}{\eta\sigma}-1}\left(\frac{1-\alpha}{\chi}\frac{\varphi-1}{\varphi}\aleph^{\alpha}\right)^{\frac{1}{\sigma}} + \left(\delta+\frac{\varsigma}{\nu}\psi+\varrho(1-\psi)\right)\aleph + \Omega/N$$

Which we can simplify to:

$$N^{\sigma+\frac{1}{\eta}} = \frac{\frac{1-\alpha}{\chi}\frac{\varphi-1}{\varphi}\aleph^{\alpha}}{\left((1-g)\aleph^{\alpha}-\left(\delta+\frac{\varsigma}{\nu}\psi+\varrho(1-\psi)\right)\aleph+e\right)^{\sigma}}$$

Appendix B.10 Welfare Change Measure

We define the (multiplicative) consumption equivalent variation required to keep the representative household indifferent between an initial scenario (for example the pre-CBDC deterministic steady state) and a new scenario (for example the post-CBDC deterministic steady state) to be the scalar ζ that satisfies the following equation:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[u(C_t^{POST})-v(N_t^{POST})\right] = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[u(\zeta C_t^{PRE})-v(N_t^{PRE})\right].$$

For example, if the scalar, ζ, that satisfies the previous equation comes out to be 1.0030, this indicates that the representative household needs to be given 0.3% of its initial-scenario consumption path to be indifferent between the initial and final scenarios. In the case where $u(\cdot)=\ln(\cdot)$, and when we are comparing two steady states, the previous equation becomes:

$$\begin{aligned} \ln(\overline{C}^{POST})-v(\overline{N}^{POST}) &= \ln(\zeta\overline{C}^{PRE})-v(\overline{N}^{PRE}) \\ \zeta &= \exp\{[\ln(\overline{C}^{POST})-v(\overline{N}^{POST})]-[\ln(\overline{C}^{PRE})-v(\overline{N}^{PRE})]\}. \end{aligned}$$

In our exposition, when comparing the pre-CBDC and the post-CBDC steady states, we refer to $(\zeta - 1) \cdot 100$ as the "welfare change from CBDC introduction".

Appendix C Additional Results and Robustness

Appendix C.1 Varying the Importance of CBDC

As discussed in Section 4.2, our calibration of the γ parameters governing the importance of the three liquid instruments is based on three moments. First, the three γ's sum to one: $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Second, the deposits-to-liquidity ratio is equal to 0.8 in the pre-CBDC steady state. Third, from surveys of the potential use of CBDC we obtain that cash and CBDC would be used roughly to similar extents if CBDC paid an interest rate of zero, which implies $\gamma_{cbdc} = \gamma_m$. Out of these three moments, the least settled one is the latter one, as it is not based on "actual data" but is based instead on answers of households that might have limited information on how CBDC will operate. Therefore, in this subsection, we analyze how our results change if we modify the third targeted moment from $\gamma_{cbdc}/\gamma_m = 1$ to $\gamma_{cbdc}/\gamma_m = k$, where k is a parameter that can take values between 0.25 (so that CBDC is a fourth as important as cash) to 2 (so that CBDC is twice as important as cash). Notice that, when changing this moment, we have to recalibrate all of our internally-calibrated parameters to still match all of our targets.

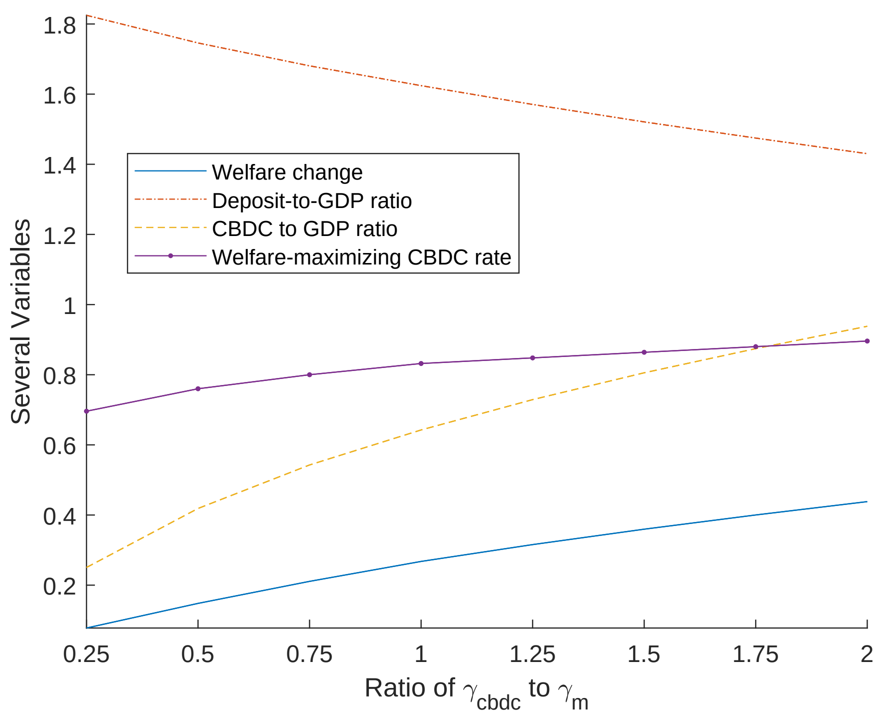

Figure C.1: This figure shows the welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital.

Figure C.1 present the results for our baseline economy with a long run steady-state policy rate of 2%. The k representing the target ratio γ_{cbdc}/γ_m is given on the x-axis, and several variables of interest are presented on the y-axis. The welfare change from introducing CBDC with its welfare-maximizing interest rate is given by the solid blue line, the deposits-to-GDP ratio is the orange dash-dotted line, the CBDC-to-GDP ratio is the dashed yellow line, and the welfare-maximizing CBDC rate is the purple line with circular markers. Certainly, the ratio of γ_{CBDC}/γ_m matters for the welfare gains of introducing CBDC, as is to be expected, but this dependence seems to be roughly linear, i.e., the gains for $k = 0.25$ are roughly a fourth of those for $k = 1$. For example, for $k = 1$, the welfare gains of introducing CBDC are 27 basis points, consistent with our baseline when choosing the welfare-maximizing CBDC rate. For $k = 0.25$ the welfare gains are 7.5 basis points, very close to one fourth of the gains in the baseline. For $k = 2$ the gains are 44 basis points, not exactly double those in the baseline, but not too far off. The optimal CBDC rate is more stable with k, 0.7% per year for $k = 0.25$, passing through 0.8% per year for $k = 1$ (replicating the baseline specification), and 0.9% for $k = 2$.

Appendix C.2 CBDC Introduction Effects Across Kappa and Theta

Figure C.2 plots the welfare change between the pre-CBDC scenario and the post-CBDC scenario for the baseline specification (where CBDC pays an interest rate of zero percent once it is introduced), for different levels of κ (the importance of bank equity for lending) and different levels of θ^k (the elasticity of substitution

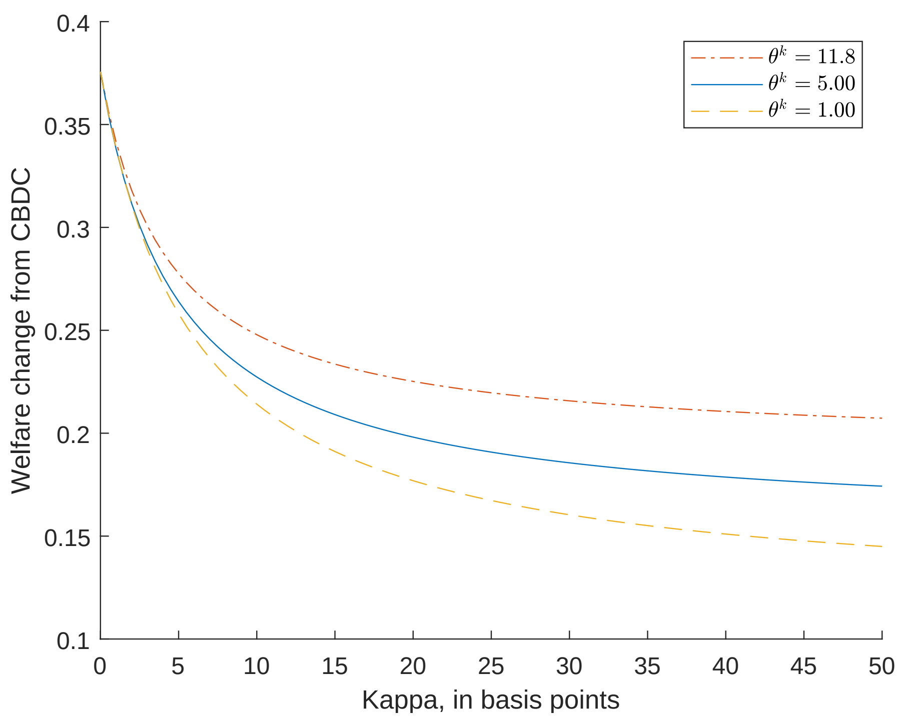

Figure C.2: This figure shows the welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital.

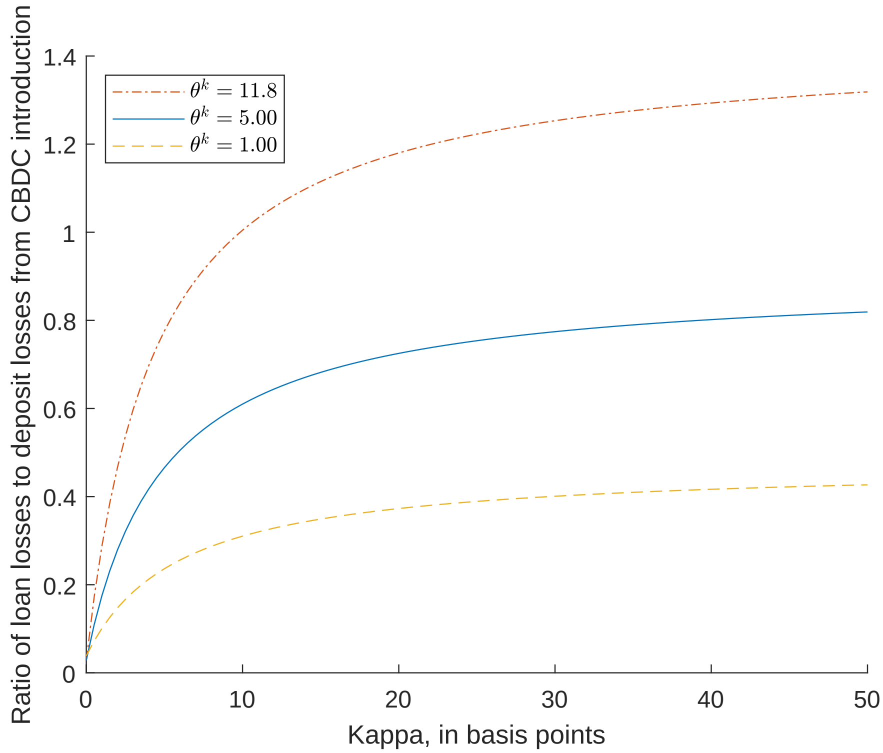

Figure C.3: This figure displays the ratio of loan losses to deposits losses in the aggregate banking sector due to CBDC introduction for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital (θ^k).

between pledgeable and non-pledgeable capital). As the importance of bank equity for lending increases, the welfare gain from introducing CBDC goes down. This makes sense because ”disintermediating” banks, by lowering their profitability through the introduction of CBDC, decreases lending more when κ is high. Recall that our baseline value is $\kappa = 12$ basis points.

Across the different lines, we see that when θ^k is higher (the orange line), the welfare gains from introducing CBDC are higher (except for $\kappa = 0$). This is also to be expected, because when the substitutability between bank and nonbank intermediation is higher, firms can more easily switch between bank and nonbank borrowing when banks are disintermediated, and the detrimental aspects of CBDC introduction are muted (leading to higher overall welfare gains).

Figure C.3 displays the ratio of loan losses to deposits losses in the aggregate banking sector due to the introduction of CBDC. This ratio increases with κ, since a higher κ indicates that bank equity is more important for lending, and the CBDC-induced bank-profitability decline has more important implications for the loan rate and bank lending. Additionally, the higher the θ^k, the higher is the ratio of loan losses to deposit losses. This is due to the fact that, when θ^k is high, even a small increase in the loan rate leads firms to heavily substitute bank borrowing with nonbank borrowing.

Interestingly, even though the three lines in Figure C.3 for the ratio of loan losses to deposits losses are relatively far apart, indicating substantial differences in the intensity of the bank disintermediation effect across levels of θ^k, the three lines in Figure C.2 for the welfare change due to CBDC introduction are much closer together. Furthermore, the case of a high θ^k leads to higher welfare gains from CBDC introduction, despite the fact that it is associated with a higher bank-disintermediation effect. This is due to the counteracting effects of changing θ^k. On the one hand, a high θ^k means that any small deviation in the interest rate on bank loans from that on corporate bonds leads to a large fall in bank lending (a large bank disintermediation effect). On the other hand, a high θ^k also implies that firms are very adept at substituting bank and nonbank borrowing, so a given bank disintermediation effect has a smaller impact on aggregate capital, output, and welfare. This logic highlights the fact that the ratio of loan losses to deposit losses due to the introduction of CBDC is not the most important object to focus on when assessing the welfare implications of CBDC introduction

Appendix C.3 Transition Between Steady States

Figures C.4 and C.5 depict the transition between the pre-CBDC and the post-CBDC steady state for several variables of interest. The transitions use our baseline calibration and a CBDC that pays an interest rate of zero percent. In orange, we have the initial steady state, in the dashed yellow line we have the final (post-

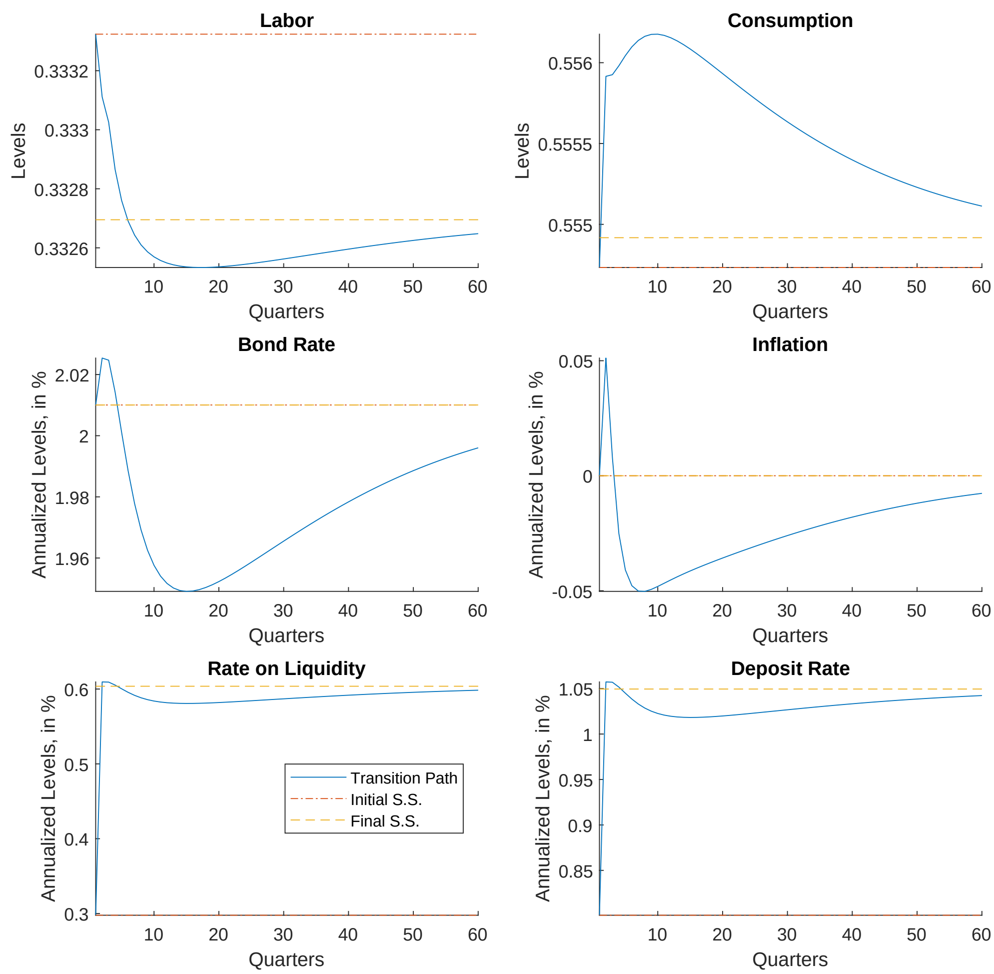

Figure C.4: This figure depicts the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for several variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

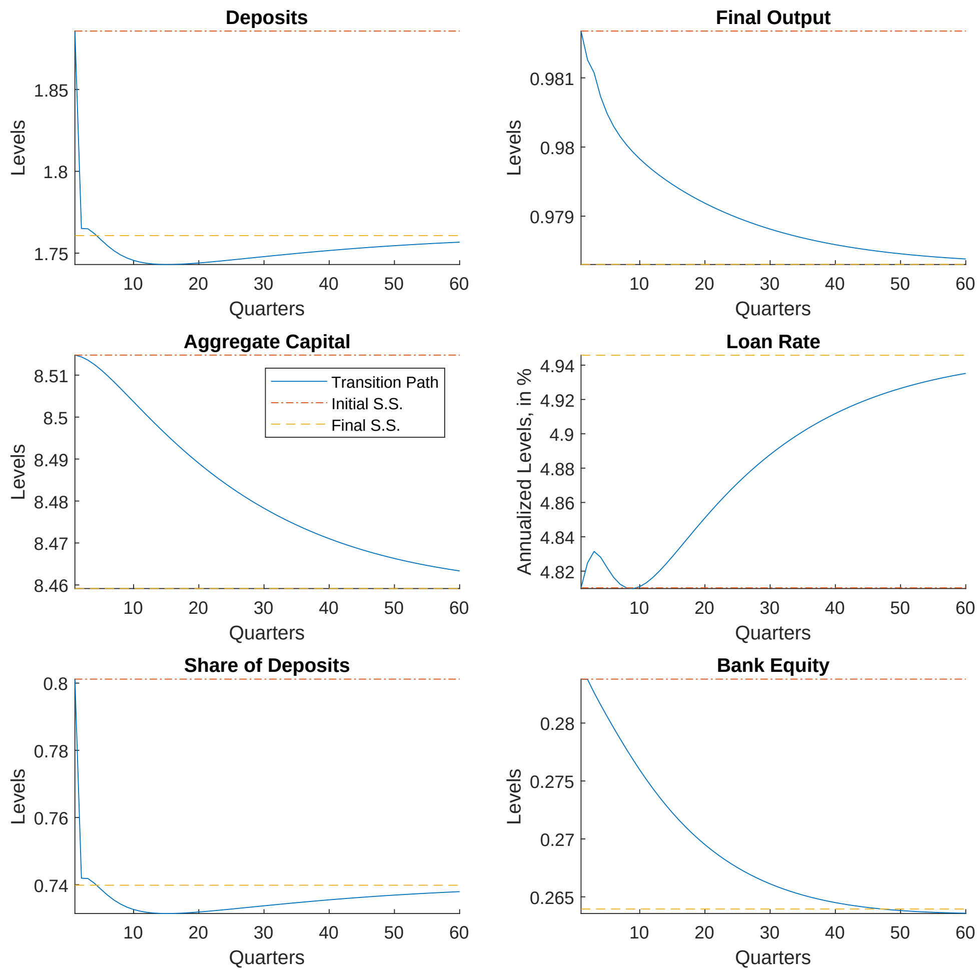

Figure C.5: This figure continues Figure C.4, depicting the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for additional variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

CBDC) steady state, and in blue with have the transition between the two.

We can see that labor falls between the initial and final steady state, and it actually falls by more than that in the transition. Similarly, consumption increases between the initial and final steady state, and increases even more in the transition. This is possible because aggregate capital actually contracts in the new steady state (so the transition has disinvestment). Final output is lower in the new steady state, both due to the lower labor and lower capital. Nevertheless, consumption can end up higher because government spending, investment, and waste all fall in the new steady state, and allow consumption to be higher despite the lower final output.

Deposits and the share of deposits in liquidity all fall in the new steady state due to the introduction of CBDC, but not by much. The fraction of deposits in liquidity ($\omega^d_{\mathcal{L}}$) falls from 80% to 74%. The loan rate increases by roughly 0.1% in the new steady state, due to commercial banks having less equity, as can be seen in the bottom right panel of Figure C.5. Both the deposit rate and the rate on liquidity increase substantially in the new steady state as can be seen from the bottom row of Figure C.4.

Importantly, even though banks pay a higher deposit rate (due to the greater competition with CBDC) and they have less equity in the new steady state, they also charge a higher loan rate and pay less operating costs in the new steady state (due to having less equity, recall that their operating costs are given as a fraction of their equity). Overall, their return on equity is essentially unchanged between the initial and final steady state. This alleviates concerns that our model is missing an entry margin in response to changes in bank profitability that could potentially change the results. Overall, labor falls by 0.18%, and consumption increases by around 0.04%. Overall, welfare is approximately 22 basis points higher in the post-CBDC steady state than in the initial (pre-CBDC steady state).

Appendix C.4 Robustness to Recalibrating Additional Parameters

Sections 5.2 and 5.3 analyzed several CBDC-related outcomes for different levels of the policy rate. In those sections, only the discount factor, β, was changing to generate the different levels of the policy rate, and no other underlying parameters where changing along with it. In this section, along with the discount factor, we vary additional parameters to continue to match some targets which we matched in our baseline calibration. Namely, we recalibrate the values of the disutility of labor parameter χ, the q parameter in the liquidity-cost function Φ, the exogenous elasticity of substitution between different banks in loans ε^l, the managerial cost of operating the bank ς, and the fraction of bank profits that stay in the bank ω. We do so to continue to match the following targets in different steady states associated with different policy-rate levels: 1) labor is equal to one third, 2) $\Phi(\cdot) = m + d + cbdc$, 3) the endogenous share of loans in firm borrowing

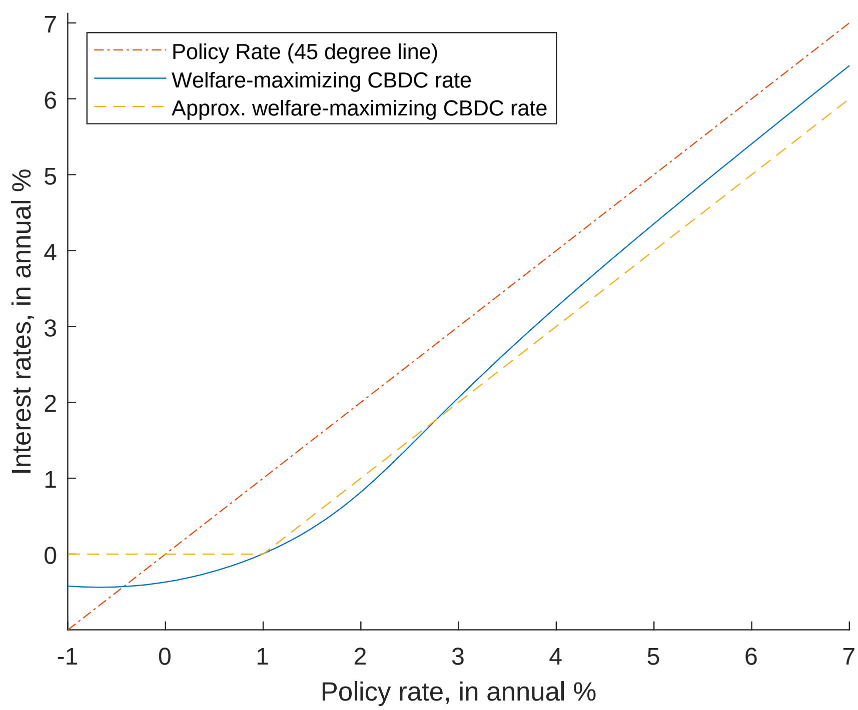

Figure C.6: This figure displays the policy rate, in orange (in both axes, so it is the 45 degree line), the welfare-maximizing level of the CBDC rate, in blue, and an approximate welfare-maximizing rule-of-thumb rate which is the maximum between 0 and the policy rate minus 1%, in yellow.

is equal to the exogenous share, $\omega_K^B = \psi$, 4) Banks are at their loan-to-equity ratio target, $\mathcal{L}/F = \nu$, and 5) bank return on equity is 2.25% quarterly.

The results that we obtained in Sections 5.2 and 5.3 are qualitatively robust to this recalibration. Quantitatively, the results do change but to a fairly small degree. As an illustration, we reproduce Figure 5.5, but now with the recalibration of the aforementioned parameters. Figure C.6 provides the results. The orange dash-dot line for the policy rate and the yellow dashed line for the rule-of-thumb CBDC rate are still the same as those in Figure 5.5. The blue line is now different, and increases a bit more steeply with the policy rate, but the differences are fairly small. Further results with this recalibration procedure are available upon request.

Appendix C.5 IRFs to a Technology Shock

Figure C.7 presents the IRFs of the economy to a 25-basis-points positive productivity shock, ϵ_t^a, with a persistence of 0.95 (see equation 3.25 for the law of motion of the technology shock). While the response of the economy to a technology shock is obviously different than the one to a monetary policy shock depicted in Figure 5.9, our main conclusion that responses to the shock are very similar across different remuneration schedules for CBDC is preserved.

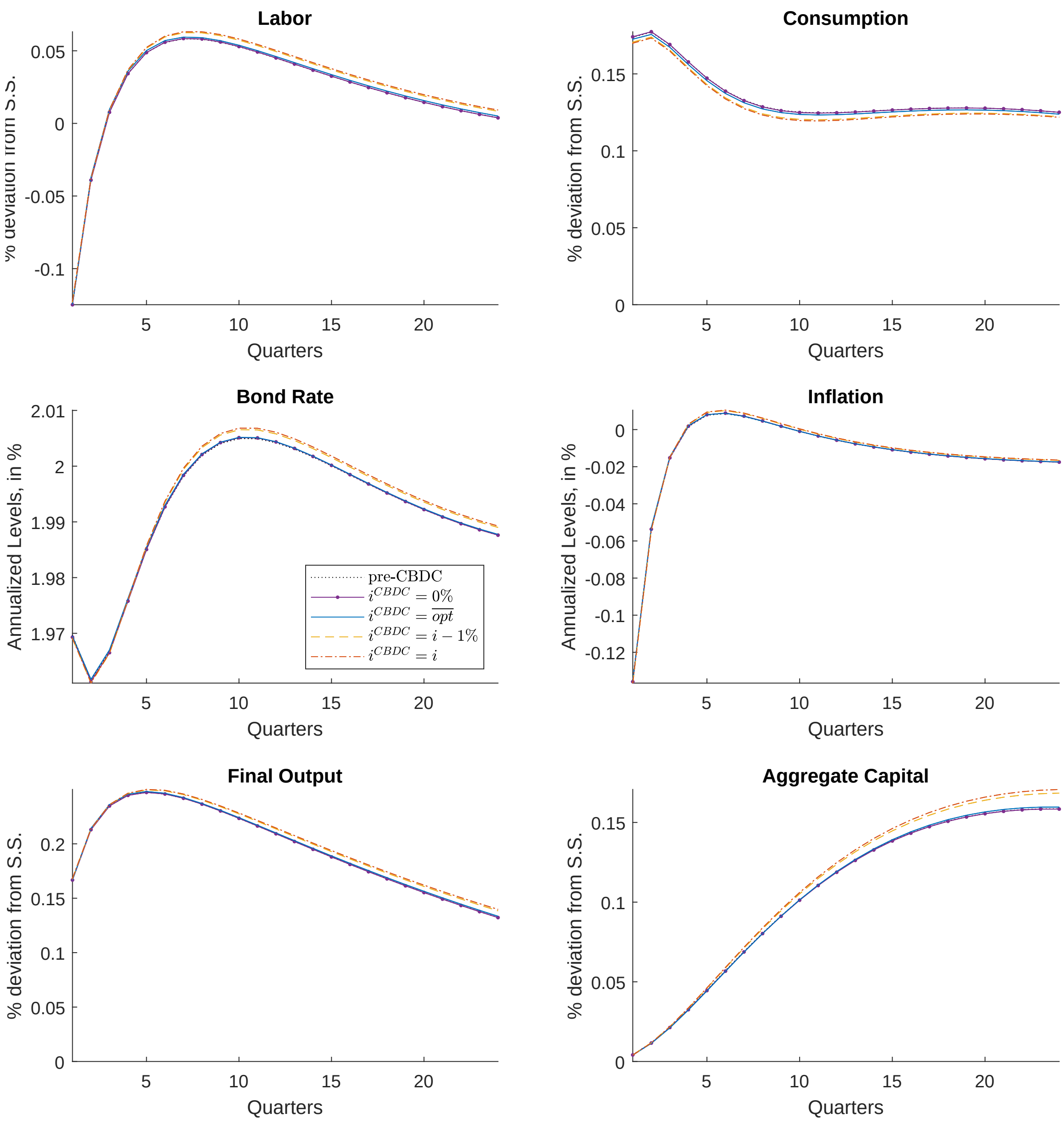

Figure C.7: This figure depicts the IRFs to a 25 basis points positive productivity shock, with a persistence of 0.95, for different CBDC remuneration schedules.

Appendix C.6 Euro-Area Calibration

Table C.1: Euro Area Calibration

Param.	Value	Description	Target or source
Panel A. Nonbank			
β	0.9950	Discount factor	2% policy rate
χ	8.7428	Disutility of labor*	One-third S.S. labor
η	1.0000	Frisch elasticity	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
α	0.3333	Capital share	Standard
δ	0.0200	Depreciation rate	8% annual dep.
κ_I	2.0000	Investment adjustment cost	Sims and Wu (2021)
φ	6.0000	Elasticity of subs. b/t diff. goods	20% mark-up
γ	0.7500	Prob. of keeping prices fixed	One-year duration
ψ_π	1.5000	Inflation coefficient, Taylor rule	Standard
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard
g	0.2000	Steady state G/Y	Standard
Panel B. Deposit side			
n	1.0863	Number of banks*	Deposit rate target #1
θ	675.10	E.o.S. between liquid instruments*	Deposit rate target #2
ε^d	852.59	E.o.S. between banks in deposits*	Deposit rate target #3
μ^d	-0.18%	Cost of issuing deposits*	Deposit rate target #4
γ_m	0.3379	Importance of cash in liquidity*	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3242	Importance of deposits in liquidity*	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3379	Importance of CBDC in liquidity*	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
a	0.9294	Parameter in liquidity function Φ*	$\mathcal{L}/Y = 2.4$ quarterly
b	1.0375	Parameter in liquidity function Φ*	Estimation
q	-0.0926	Parameter in liquidity function Φ*	S.S. relationship
Panel C. Loan side			
ψ	0.8200	Importance of pledgeable capital*	Own calculations
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)
ε^l	90.503	E.o.S. between banks in loans*	$i^l = i + \rho \Rightarrow \theta^k = f(\varepsilon^l)$
θ^k	5.0000	Subs. between NP and P capital	Feasible region
Panel D. Joint bank side			
ω	0.5886	Fraction staying in bank*	$L/F = \nu$ in S.S.
ς	0.0283	Bank managerial cost*	2.25% S.S. ROE
ν	9.0000	Loan-to-equity ratio target	Ulate (2021)
κ	0.0012	Cost of deviating from target ratio	Ulate (2021)

Notes: This table contains the parameter values used in our Euro-Area calibration, together with their description and their source or target. An asterisk in the "description" column indicates that the parameter has changed between the baseline calibration and the Euro-Area calibration. All interest rates are annualized.