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Inflation Expectations, Liquidity Premia and Global Spillovers in Japanese Bond Markets

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and

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Abstract

We provide market-based estimates of Japanese inflation expectations using an arbitrage-free dynamic term structure model of nominal and real yields that accounts for liquidity premia and the deflation protection afforded by Japanese inflation-indexed bonds, known as JGBi's. We find that JGBi liquidity premia exhibit significant variation, and even switch sign. Properly accounting for them significantly lowers the estimated value of the indexed bonds' deflation protection and affects inflation risk premium estimates. After liquidity adjustment, long-term Japanese inflation expectations have remained relatively stable at levels modestly exceeding one percent during the pandemic period. We then utilize our estimated liquidity measure to confirm the existence of statistically significant and economically meaningful spillovers to the JGBi market from global bond market illiquidity, as proxied by periods of low U.S. Treasury market depth.

JEL Classification: C32, E43, E52, G12, G17

Keywords: affine arbitrage-free term structure model, deflation risk, deflation protection, liquidity spillovers

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1 Introduction

Breakeven inflation (BEI)—the difference between yields on comparable-maturity nominal and real debt—is a popular market-based indicator of inflation expectations. Long-term BEI is frequently used to measure the credibility of the central bank's inflation objective.[1] However, BEI contains both inflation and other risk premia, which need to be purged from estimated BEI to obtain accurate measurements of investors' inflation expectations. In this paper, we make such adjustments to prices of inflation-indexed Japanese government bonds, commonly known as JGBi.

Japanese inflation has been quite low for decades, and has averaged far below two percent since the Bank of Japan's (BOJ) formal adoption of that inflation target in 2013. This failure to achieve the announced target appears to have weighed on Japanese inflation expectations, which have remained around one percent in surveys such as Consensus Forecasts over this period. Inflation expectations were likely further depressed following the onset of the COVID-19 pandemic.[2] Most recently, Japanese inflation has risen above the BOJ two-percent target, in tandem with elevated inflation levels in other advanced economies that pursued aggressive fiscal policy to combat the pandemic. This recent movement in realized inflation elevates the need to properly gauge Japanese inflation expectations in the current environment.

This paper extends the joint model of nominal and real yields described in Christensen and Spiegel (2022, henceforth CS (2022)) to account for liquidity risk premia in Japanese indexed bonds. Our previous model only accounted for the value of deflation protection offered by JGBi's issued since 2013.[3] As discussed in Cardozo and Christensen (2023), inflation-indexed bonds are likely to be much less traded than nominal bonds, given that they provide a natural hedge against inflation risk. Moreover, the inflation hedge would apply disproportionately to domestic investors, whose consumption expenditures track the local CPI. As a consequence, the trading of inflation-indexed bonds should be concentrated among relatively "patient" domestic buy-and-hold investors, such as domestic pension funds and insurance companies.[4]

The relatively modest trading in these instruments likely raises potential liquidity issues,

[1]Provided the inflation objective is credible, long-term inflation expectations should remain proximate to the central bank's target, as any current inflation shocks should be considered temporary and not affect long-run inflation expectations.

[2]The pandemic, and associated shutdowns adopted to combat the virus, disrupted economic activity and led to volatility in global financial markets. The BOJ responded to the turmoil with a number of monetary policy easings, including both policies designed to lower interest rates and policies aimed at easing financial conditions. Fiscal policy also responded aggressively. However, Christensen and Spiegel (2023) find that announcements of policy changes yielded only modest immediate responses in inflation expectations.

[3]These bonds implicitly offer "deflation protection," in the form of paying off the original nominal principal at maturity when deflation has occurred since issuance. These enhancements are particularly important during our sample, which contains extended periods with low and often negative Japanese inflation. CS (2022) accounts for the value of this deflation protection option using an adaptation of the approach of Christensen et al. (2012).

[4]Cardozo and Christensen (2023) confirm this conjecture for the Colombian inflation-indexed bond market, while Beauregard et al. (2022) report similar findings for the Mexican inflation-indexed bond market.

and they should offer investors some premium for bearing the associated liquidity risk. We therefore use our extended model to examine the extent and time variation of this liquidity premium, as well as the level of segmentation between the JGBi and global bond markets.

To account for any JGBi price distortions arising from such concentration among the investors in these bonds, we follow Andreasen et al. (2021, henceforth ACR (2021)) and augment the model used by CS (2022) with a bond-specific risk factor to adjust for any bond-specific risk premia in JGBi prices. The identification of the bond-specific risk factor comes from its unique loading, which is a function of both the time since issuance and the remaining time to maturity. The time since issuance serves as a proxy for the notion that, over time, an increasing fraction of the outstanding value of a given security gets locked up in buy-and-hold investors' portfolios. This increases the sensitivity of the security to variation in the market-wide bond-specific risk factor. By observing a cross section of security prices over time, this factor can be separately identified. Importantly, our analysis also accounts for the value of the deflation protection option embedded in Japanese inflation-indexed bond contracts issued since 2013. Our model also allows us to identify bond investors' underlying inflation expectations and inflation risk premia, as in Christensen et al. (2010). Finally, to obtain the appropriate persistence of the dynamic factors in the model, we follow Kim and Orphanides (2012) and incorporate long-term forecasts of inflation from surveys of professional forecasters.

Our results demonstrate that the average estimated JGBi liquidity premium series exhibits significant variation, and even switches sign. Furthermore, properly accounting for these liquidity premia significantly lowers the estimated value of the indexed bonds' deflation protection. After adjustment for both of these distortions in Japanese BEI rates, we find that long-term Japanese inflation expectations have remained relatively stable at levels modestly exceeding one percent during the pandemic period. In addition, the estimated inflation risk premia are more stable and less negative than they appear without accounting for the liquidity risk of JGBi's.

Focusing on the liquidity risk component, we find that bond illiquidity spiked towards the end of our sample, probably due to the Bank of Japan "yield curve control" policy, under which the Bank of Japan stood ready to purchase Japanese Government Bonds (JGBs) as needed to maintain its target for the 10-year JGB yield. This policy removed a significant amount of JGBs from the open market, and lowered JGB liquidity, particularly in the 10-year segment of the market. We find that the yield curve control policy was associated with sharp increases in JGBi liquidity premia. Properly accounting for these liquidity premia significantly lowers the bonds' estimated value of their deflation protection and affects the estimated inflation risk premia during this unique period.

We then utilize our estimated series for JGBi liquidity premia to investigate the degree of effective market segmentation in the Japanese inflation-indexed bond market. Japanese asset markets are commonly believed to exhibit some degree of segmentation due to "home bias" among Japanese investors in their asset holdings. Moreover, the guarantees afforded by inflation-indexed Japanese JGBi's are likely disproportionately valuable to domestic agents, whose consumption is more exposed to Japanese inflation risk. We use weekly liquidity estimates of U.S. Treasury market depth, measured as the number of open orders from surveyed dealers for the most liquid on-the-run U.S. Treasury securities from December 2009 through December 30, 2022, as a proxy for global bond market liquidity. The U.S. market is commonly considered the most liquid global bond market, and movements in that market have been characterized as reflecting overall global financial conditions [e.g. Rey (2015)].

We examine the implications of illiquidity in U.S. Treasury markets, defined as market depth two standard deviations or more below mean levels for the series, for JGBi liquidity premia. As these shocks directly measure liquidity movements, they are distinct from classic flight-to-safety events, which tend to improve both Japanese and U.S. bond market conditions.[5] We regress our generated JGBi liquidity premium series on indicators of low levels of market depth in U.S. Treasuries at various maturities, as well as other conditioning variables to account for general movements in U.S. and Japanese market conditions. Note that while our generated liquidity premium series is estimated with error, the series' role as the dependent variable in our specification implies that estimation errors are unlikely to cause bias in the coefficient estimates of interest in our specification beyond attenuation bias.

Our results indicate that exceptionally low levels of market depth for on-the-run U.S. Treasuries for a variety of maturities are associated with elevated estimated JGBi liquidity premia. Our point estimates indicate that this relationship is economically important, as low levels of market depth in U.S. Treasuries at the 2-, 5-, or 7-year maturities are associated with over 2 standard deviation predicted increases in the estimated Japanese JGBi liquidity premia, while low market depth in the 10-year segment of the U.S. Treasury market is associated with a still notable 0.90 standard deviation increase in the estimated Japanese JGBi liquidity premia. We interpret these results as evidence that illiquidity and market stress in the U.S. Treasury market does spill over into the JGBi market, implying that segmentation of the Japanese JGBi market is incomplete.

The remainder of this paper is structured as follows. Section 2 describes the data. Section 3 details our benchmark model used to decompose the nominal and real bond yields into underlying expectations and residual risk premia, while accounting for bond-specific liquidity risk premia and the values of deflation protection offered by indexed bonds. It then estimates our model and summarizes its results. Section 4 examines the estimated bond-specific liquidity risk premia and values of deflation protection, while Section 5 contains the result of the empirical BEI decomposition. Section 6 examines the evidence for global spillovers from U.S. Treasury markets to JGBi liquidity risk. Lastly, concluding comments

[5] In Japan, this is due to Japanese investors moving funds back to Japan, while in the U.S. the improvement is driven by foreigners seeking safety in times of high financial market volatility.

are provided in Section 7.

2 Japanese Government Bond Data

The Japanese government bond market is large and liquid by international standards. As of December 2022, the total outstanding notional amount of marketable bonds issued by the government in Japan was 1,197.6 trillion yen, of which close to 1 percent represented inflation-indexed bonds.[6] In total, Japanese government debt reached 263.9% of Japanese nominal GDP in 2022, far above the level of any other major industrialized country.[7]

2.1 Nominal Bonds

We follow CS (2022) and extend the Japanese nominal government bond yield series in Kim and Singleton (2012), which originally ended in March 2008, with Japanese nominal government zero-coupon yields through December 2022.[8] This data set contains six maturities: six-month yields and one-, two-, four-, seven-, and ten-year yields, with all yields being continuously compounded and available at daily frequency. We examine the data at weekly frequency using Friday observations or the most recently available whenever Friday observations are missing.

Figure 1 shows the persistent drop in yields since the mid-1990s for four of our nominal yields.[9] We also observe a persistent decline in the yield spreads. The spread between the ten- and one-year yield was larger than 200 basis points at the start of the sample and less than 50 basis points at the end of the sample. We follow Kim and Singleton (2012), who find that a two-factor model is adequate to fit their data, and use a two-factor model for the nominal yields. Furthermore, we take the compressed term structure of the past decade as evidence that liquidity premia are likely to play only a modest role for the nominal yield series.[10] As a consequence, we make no further adjustments to the nominal yields.

2.2 Real Bonds

The Japanese government has issued inflation-indexed bonds—known as JGBi—since the spring of 2004. These are all ten-year bonds, which were issued in two separate periods. From March 2004 until June 2008, a total of 16 bonds were issued with a nearly quarterly

[6]Source: https://www.mof.go.jp/english/policy/jgbs/publication/newsletter/jgb2023_04e.ps

[7]Source: tradingeconomics.com/japan/government-debt-to-gdp

[8]Extension data through the end of October 2022 is downloaded from Bloomberg, as in Christensen and Rudebusch (2015), while the data for November and December 2022 are constructed directly from nominal JGB prices downloaded from Bloomberg using the Svensson (1995) yield curve, see Andreasen et al. (2019) for details.

[9]To maintain the readability of Figure 1, we display four of the six yield series we use in the model estimation. The nominal yields not shown exhibit a similar pattern.

[10]While the BOJ's purchases of close to 45 percent of all outstanding JGBs by the end of our sample raises the possibility of illiquidity in this market, both Kurosaki et al. (2015) and Sakiyama and Kobayashi (2018) find no evidence of market impairment during our sample period.

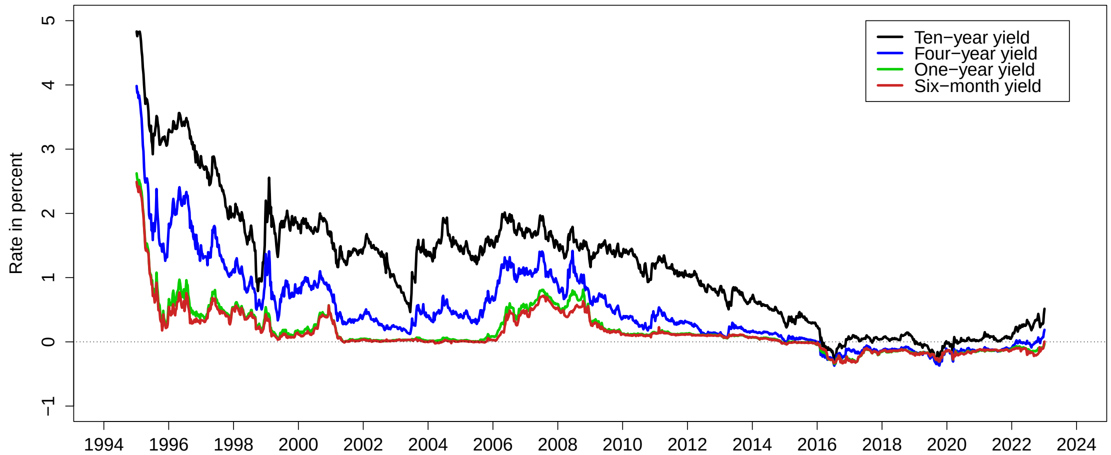


Figure 1: **Japanese Nominal Government Bond Yields**
Illustration of the Japanese nominal government zero-coupon bond yields with maturities of six months, one year, four years, and ten years. The data series are weekly covering the period from January 6, 1995, to December 30, 2022.

frequency. The program was then temporarily halted in the aftermath of the global financial crisis. However, new inflation-indexed bonds have been issued roughly once a year since the fall of 2013. These are government bonds whose principal amount fluctuates in proportion with the consumer price index (CPI) excluding fresh food.

This latter period of issuance included the deflation protection enhancement noted in the introduction. These bonds are guaranteed to pay off at least at par at maturity, even if there was net deflation between the issuance and maturity dates, effectively embedding a deflation protection option into the bond contract.[11] Table 1 contains the contractual details of all 27 JGBi's in our sample as well as their individual number of weekly observations.

The distribution of individual JGBi's for each date in our sample is illustrated in Figure 2(a). Each bond's trajectory over time in terms of remaining years to maturity is represented by a diagonal solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. The two waves of issuances of JGBi's are clearly visible.

The solid grey rectangle in Figure 2(a) indicates the sub-sample of bonds used in our empirical analysis. The sample is restricted to start on January 7, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity.

Figure 2(b) shows the distribution across time of the number of JGBi's included in the sample. Our sample starts with three bonds and increases to sixteen bonds by 2008. The number of bonds available then gradually declined beginning in 2011, as bonds from the first

[11] See https://www.mof.go.jp/english/jgbs/topics/bond/10year_inflation/index.htm

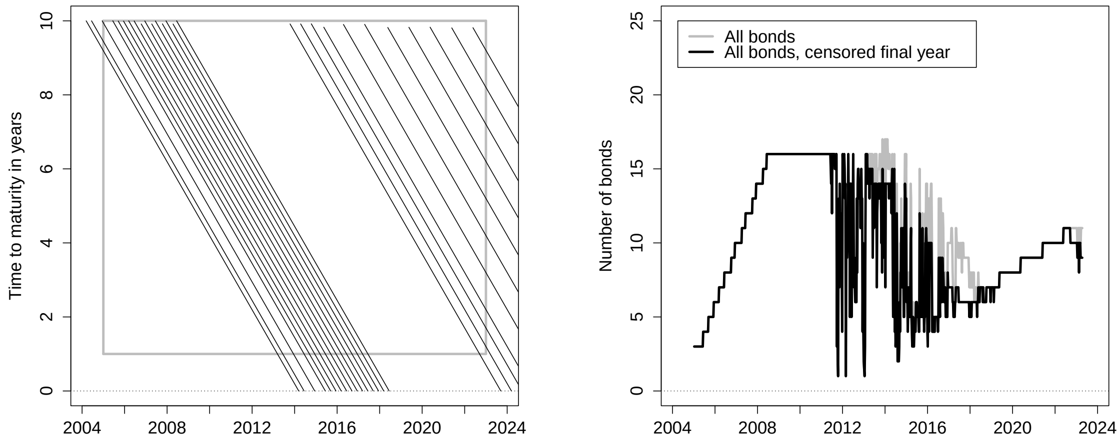
JGBi (coupon, maturity)	No. obs.	Issuance Date	Issuance amount	Number of auctions	Total notional amount
(1) 1.2% 3/10/2014	376	3/10/2004	100	1	100
(2) 1.1% 6/10/2014	389	6/10/2004	300	1	300
(3) 0.5% 12/10/2014	428	12/10/2004	500	1	500
(4) 0.5% 6/10/2015	432	6/10/2005	500	1	500
(5) 0.8% 9/10/2015	419	9/12/2005	500	1	500
(6) 0.8% 12/10/2015	402	12/12/2005	500	1	500
(7) 0.8% 3/10/2016	414	3/10/2006	500	1	500
(8) 1% 6/10/2016	394	6/12/2006	500	2	1000
(9) 1.1% 9/10/2016	395	10/11/2006	500	1	500
(10) 1.1% 12/10/2016	388	12/12/2006	500	2	1000
(11) 1.2% 3/10/2017	369	4/10/2007	500	1	500
(12) 1.2% 6/10/2017	395	6/12/2007	500	2	1000
(13) 1.3% 9/10/2017	382	10/10/2007	500	1	500
(14) 1.2% 12/10/2017	375	12/11/2007	500	2	1000
(15) 1.4% 3/10/2018	356	4/10/2008	500	1	500
(16) 1.4% 6/10/2018	358	6/10/2008	500	2	1000
(17) 0.1% 9/10/2023	463	10/10/2013	300	2	600
(18) 0.1% 3/10/2024	451	4/10/2014	400	2	800
(19) 0.1% 9/10/2024	420	10/10/2014	500	2	1000
(20) 0.1% 3/10/2025	398	5/12/2015	500	4	2000
(21) 0.1% 3/10/2026	348	4/14/2016	400	4	1600
(22) 0.1% 3/10/2027	296	4/13/2017	400	4	1600
(23) 0.1% 3/10/2028	242	5/11/2018	400	4	1600
(24) 0.1% 3/10/2029	190	5/13/2019	400	4	1600
(25) 0.2% 3/10/2030	138	5/11/2020	200	4	800
(26) 0.005% 3/10/2031	84	5/18/2021	200	4	800
(27) 0.005% 3/10/2032	33	5/17/2022	200	3	700

Table 1: **Sample of Japanese Real Government Bonds**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of Japanese yen for the sample of Japanese inflation-indexed government bonds (JGBi). Also reported are the number of weekly observation dates for each bond during the sample period from January 7, 2005, to December 30, 2022.

wave of issuances started to mature. However, since 2018, the number of outstanding bonds has been gradually increasing with the second issuance wave. At the end of our sample there are nine bonds. The number of inflation-indexed bonds $n_R(t)$ combined with the time variation in the cross-sectional dispersion in the maturity dimension observed in Figure 2(a) provides the identification of the real factors in our model.[12]

Figure 3 shows the yields to maturity for all 27 Japanese inflation-indexed bonds. We see notable changes in the level and slope of the Japanese real yield curve, which motivates our choice to model the inflation-indexed data with three real yield factors. Note also that

[12] Finlay and Wende (2012) represent an early example of analysis like ours based on prices from a limited number of Australian inflation-indexed bonds.



(a) Distribution of inflation-indexed bonds

(b) Number of inflation-indexed bonds

Figure 2: **Real Japanese Government Bond Sample**
Panel (a) shows the maturity distribution of available Japanese inflation-indexed government bonds (JGBi) on any given date. The solid grey rectangle indicates the sample used in our empirical analysis. The sample is restricted to start on January 7, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity. Panel (b) reports the number of outstanding inflation-indexed bonds available at a given point in time for various samples.

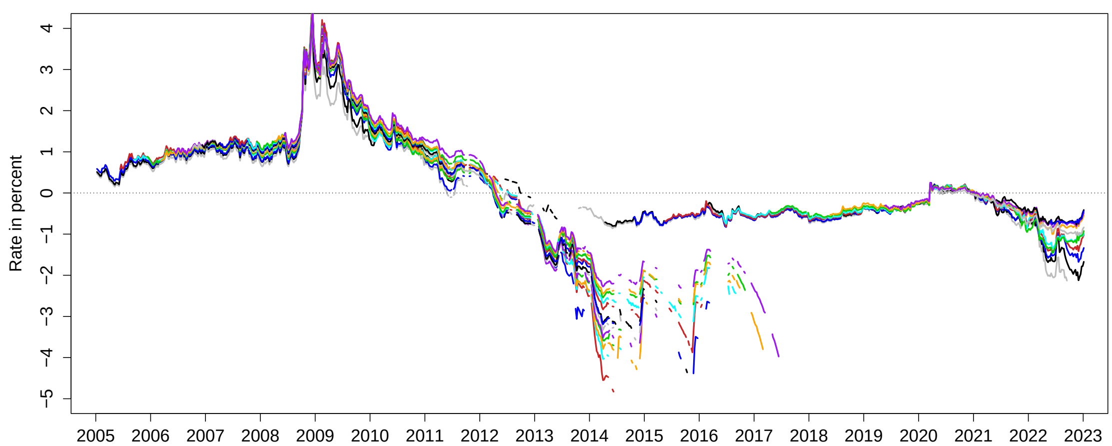


Figure 3: **Yield to Maturity of Japanese Real Government Bonds**
Illustration of the yield to maturity of the Japanese inflation-indexed bonds considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 7, 2005, to December 30, 2022; (2) censoring of a bond's price when it has less than one year to maturity.

the series for individual bonds show gaps as the bonds approach maturity. Our use of all available bond price information in combination with the Kalman filter is designed to handle such data gaps. More importantly, the more erratic pricing patterns observed as these bonds approach maturity suggests the presence of bond-specific liquidity premia, motivating the model structure presented in the following section.

3 Model Estimation and Results

In this section, we first detail our benchmark model, which we use to decompose the nominal and real bond yields into underlying expectations and residual risk premia, while evaluating the value of the inflation-indexed bond deflation enhancement. We then describe our identification restrictions, estimate the model, and summarize our results.

3.1 An Arbitrage-Free Model of Nominal and Real Yields with Bond-Specific Liquidity Risk

In order to precisely measure both the value of deflation protection offered by the inflation-indexed bonds and their individual bond-specific liquidity risk premia, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R. We focus on a tractable affine dynamic term structure model of nominal and real yields briefly summarized below. This model can be viewed as a restricted Gaussian version of the affine term structure models introduced by Dai and Singleton (2000).

Let $X_t = (L_t^N, S_t^N, L_t^R, S_t^R, X_t^L)$ denote the state vector of our five-factor model, which we refer to as the $G^L(5)$ model using the terminology of ACR (2021). (L_t^N, S_t^N) represent level and slope factors in the nominal yield curve, while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve.[13] Finally, X_t^L represents the added liquidity risk factor in the pricing of JGBi's. Our model represents an augmented version of the four-factor $G(4)$ model used by CS (2022).

The instantaneous nominal and real risk-free rates are defined as

$$r_t^j = L_t^j + S_t^j, \quad j = N, R. \tag{1}$$

The risk-neutral $\mathbb{Q}$-dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dL_t^R \\ dS_t^R \\ dX_t^L \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda^N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda^R & 0 \\ 0 & 0 & 0 & 0 & \kappa_L^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_L^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t^N \\ L_t^R \\ S_t^R \\ X_t^L \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \\ dW_t^{X^L,\mathbb{Q}} \end{pmatrix},$$

where Σ continues to be a lower triangular matrix.

[13] Chernov and Mueller (2012) provide evidence of a hidden factor in the U.S. nominal yield curve that is observable from real yields and inflation expectations. Our joint model accommodates this stylized fact via the factors (L_t^R, S_t^R).

Based on this specification of the $\mathbb{Q}$-dynamics, *frictionless* nominal and real zero-coupon bond yields preserve a simplified Nelson and Siegel (1987) factor loading structure:

$$y_t^j(\tau) = L_t^j + \left(\frac{1 - e^{-\lambda^j \tau}}{\lambda^j \tau}\right) S_t^j - \frac{A^j(\tau)}{\tau}, \quad j = N, R, \tag{2}$$

where $\frac{A^j(\tau)}{\tau}$ represents deterministic yield-adjustment terms.

As our model allows JGBi's to be sensitive to bond-specific liquidity risks, their pricing is not performed with the standard frictionless real discount function above, but with a discount function that accounts for the bond-specific liquidity risk:

$$\overline{r}_t^{R,i} = r_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^L = L_t^R + S_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^L, \tag{3}$$

where t_0^i denotes the date of issuance of the specific JGBi and β^i is its sensitivity to the variation in the bond-specific risk factor. Furthermore, the decay parameter $\lambda^{L,i}$ is assumed to vary across securities.

By Christensen and Rudebusch (2019), the net present value of one unit of consumption paid by JGBi bond i at time $t + \tau$ has the following exponential-affine form

$$\begin{aligned} P_t(t_0^i, \tau) &= E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} \overline{r}^{R,i}(s, t_0^i) ds}\right] \\ &= \exp\Big(B_1(\tau) L_t^R + B_2(\tau) S_t^R + B_3(t, t_0^i, \tau) X_t^L + A(t, t_0^i, \tau)\Big). \end{aligned}$$

This result implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all i, we recover the $G(4)$ model.

We next evaluate the value of the deflation protection enhancement that has been embedded in Japanese inflation-indexed bonds issued since 2013. Consider an inflation-indexed bond issued at time t_0 with a reference price index value equal to Π_{t_0}. By time t, its accrued inflation compensation is $\frac{\Pi_t}{\Pi_{t_0}}$, which we define as the "inflation index ratio." There are then two mutually exclusive scenarios: First, the net price index change from time t to maturity T could be sufficiently positive that the inflation index ratio is greater than one. Given this outcome, the bond will pay off its principal adjusted for inflation between inception and maturity ($\frac{\Pi_T}{\Pi_{t_0}}$).

Alternatively, the net price index change between t and T may be insufficient to bring the inflation index ratio for the entire period from inception to maturity to be greater than one. Given that outcome, the deflation protection option will be in the money, as the inflation-indexed bond will return its original principal. The value of the deflation protection option, DOV_t, is then given by[14]

[14]For derivation, see CS (2022).

$$DOV_t\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = \Bigg[E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^N ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big] - E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big]\Bigg].$$

The option value will be lower when accrued inflation compensation is larger, as it is less likely that the net price index change over the bond's remaining life will be sufficiently low (or negative) to bring the option back into the money. Moreover, when accrued inflation is larger, the option value is lower the shorter is the remaining time to maturity, as the probability of bringing the option back into the money at maturity is reduced.

The "clean price" of a bond i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C^R semi-annually, i.e. that which does not account for any accrued interest and maps to our observed real bond prices, then satisfies

$$\begin{aligned}\overline{P}_t^{R,i}\Big(t_0^i,\tau^i,C^R,\frac{\Pi_t}{\Pi_0}\Big) \quad &= \quad C^R(t_1-t)E^{\mathbb{Q}}\Big[e^{-\int_t^{t_1}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + \sum_{j=2}^N \frac{C^R}{2}E^{\mathbb{Q}}\Big[e^{-\int_t^{t_j}\overline{r}^{R,i}(s,t_0^i)ds}\Big] \\ &+E^{\mathbb{Q}}\Big[e^{-\int_t^{t+\tau^i}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + DOV_t\Big(\frac{\Pi_t}{\Pi_0}\Big).\end{aligned} \qquad (4)$$

Note that we only include the option value for the inflation-indexed bonds that have this contractual feature and compute it using the four frictionless factors, $(L_t^N, S_t^N, L_t^R, S_t^R)$, within our model following an approach similar to that outlined in Christensen et al. (2012).[15]

To implement our model empirically, we need to specify the risk premia that connect these factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the real-world $\mathbb{P}$-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical $\mathbb{P}$-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In a Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^5$ and $\gamma^1 \in \mathbf{R}^{5\times 5}$ contain unrestricted parameters. Thus, the resulting unrestricted five-factor joint model of nominal and real yields has $\mathbb{P}$-dynamics given by

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an unrestricted 5×5 mean-reversion matrix, $\theta^{\mathbb{P}}$ is a 5×1 vector of mean levels, and Σ is a 5×5 lower triangular volatility matrix. This is the transition equation in the

[15] See CS (2022) for details. We also do not account for the approximately 2.5 month lag in Japanese inflation indexation. Grishchenko and Huang (2013) and D'Amico et al. (2018) find that this adjustment normally is within a few basis points for the implied yield on U.S. TIPS. It is likely to be very small for our Japanese data as well.

extended Kalman filter estimation of our model.

As shown in online Appendix A, nominal and real zero-coupon yields can be expressed as

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real zero-coupon yields as described in the previous section, while the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right] \tag{5}$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2012).

Now, the BEI rate is defined as the difference between nominal and real yields of the same maturity

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau).$$

Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

3.2 Model Estimation and Econometric Identification

We estimate the model using a conventional likelihood-based approach, where we extract latent pricing factors from our observed data, nominal zero-coupon yields and inflation-indexed mid-market yields to maturity. The functional form for nominal yields is specified as affine and provided in equation (2), whereas the expression for the yield to maturity $\hat{y}_t^R$ of an inflation-indexed bond with maturity at T that pays an annual coupon C^R semi-annually is given by the solution to the following fixed-point problem

$$\overline{P}_t^{R,i} \quad = \quad C^R(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^R\right\} + \sum_{k=2}^{n}\frac{C^R}{2}\exp\left\{-(t_k-t)\hat{y}_t^R\right\} + \exp\left\{-(T-t)\hat{y}_t^R\right\}, \quad (6)$$

where $\overline{P}_t^{R,i}$ is the model-implied inflation-indexed bond price in equation (4).

Following Joslin et al. (2011), all nominal yields have independent Gaussian measurement errors $\varepsilon_t^{N,i}$ with zero mean and a common standard deviation σ_ε^N, denoted $\varepsilon_{y,t}^i \sim \mathcal{NID}\left(0,(\sigma_\varepsilon^N)^2\right)$ for $i=1,2,\ldots,n_N$. We also account for measurement errors in the yields to maturity of the inflation-indexed bonds through $\varepsilon_t^{R,i}$, where $\varepsilon_t^{R,i} \sim \mathcal{NID}\left(0,(\sigma_\varepsilon^R)^2\right)$ for $i=1,2,\ldots,n_R(t)$. We follow CS (2022) and restrict the volatility matrix Σ to a diagonal matrix. ACR (2021) show that this has at most a very small impact on the estimated liquidity premia.

Finally, as the bond-specific risk factor is a latent factor, it is not identified without additional restrictions. We let the third JGBi, which was issued right before the start of our sample period, have a unit loading on this factor, that is, the 10-year JGBi issued on December 12, 2004, and maturing on December 10, 2014, with 0.5% coupon has $\beta^i = 1$. This implies that the β^i sensitivity parameters measure bond-specific liquidity risk sensitivity relative to that of this 10-year 2014 JGBi.

3.2.1 Survey Forecasts

The inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics, which can otherwise suffer from significant finite-sample bias.[16] We therefore also incorporate long-term forecasts of inflation from surveys of professional forecasters, using the projected ten-year CPI inflation ex fresh-food series that can be constructed semi-annually from Consensus Forecasts.[17] We only include forecasts for every other year, i.e., the April forecasts from 2005, 2007, 2009, 2011, 2013, 2015, 2017, 2019, and 2021, to minimize the impact of the survey information on our model results.

The measurement equation for the survey expectations incorporating these long-term forecasts takes the form

$$\pi_t^{CF}(10) = \pi_t^e(10) + \varepsilon_t^{CF},$$

where $\pi_t^e(10)$ is the model-implied ten-year expected inflation calculated using equation (5), which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\left(0,(\sigma_\varepsilon^{CF})^2\right)$ with σ_ε^{CF} fixed at 0.0075, as recommended by Kim and Orphanides (2012).

[16] For discussions, see Kim and Orphanides (2012) and Bauer et al. (2012).

[17] We do not include inflation data in the model estimation. This omission is expected to, at most, have a small impact on our results due to the relatively long maturities of most of our real yield observations. See D'Amico et al. (2018).

Maturity in months	$G(4)$ model				$G^L(5)$ model			
	w.o. opt. adj.		w. opt. adj.		w.o. opt. adj.		w. opt. adj.	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
6	5.41	8.95	5.41	9.02	5.40	8.92	5.41	8.94
12	1.07	5.19	1.04	5.28	1.07	5.16	1.07	5.20
24	-3.99	6.70	-4.10	6.85	-3.99	6.70	-4.01	6.76
48	-5.39	10.02	-5.56	10.16	-5.39	10.01	-5.41	10.06
84	-0.56	10.81	-0.70	11.01	-0.56	10.78	-0.56	10.83
120	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
All maturities	-0.58	7.84	-0.65	7.96	-0.58	7.83	-0.58	7.87

Table 2: **Pricing Errors of Nominal Yields**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese nominal yields in the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment. All errors are reported in basis points.

Finally, we note that the model is estimated with the standard extended Kalman filter due to the nonlinear measurement equations for the inflation-indexed bond yields.[18]

3.3 Estimation Results

In this section, we examine the results under our $G^L(5)$ model, contrasting them with the earlier CS (2022) $G(4)$ model that did not adjust for the bond-specific liquidity risk premia in JGBi prices. For both models we perform the estimation without and with adjustment for the value of the deflation protection offered by JGBi's issued since 2013.

To begin our comparison, we first examine the models' ability to fit nominal yields. To that end, Table 2 reports the summary statistics of the fitted errors of the 6 nominal yields in our sample. The table documents that our benchmark $G^L(5)$ model with option adjustment fits all of the nominal yields well, as the overall root mean-squared error (RMSE) is only 7.87 basis points. Moreover, all four specifications fit the nominal yields about equally well.

The summary statistics of the fitted errors for each JGBi calculated as described in equation (6) are reported in Table 3. The RMSE for all yield errors combined is 5.59 basis points in our $G^L(5)$ model that adjusts for the bond-specific liquidity premia in the JGBi prices. In contrast, the $G(4)$ model without adjustment for these premia produces a RMSE for all yield errors of 10.07 basis points. Thus, the $G^L(5)$ model achieves a markedly better fit to the JGBi data. We also find that the estimated measurement error standard deviations within our benchmark model, $\sigma_\varepsilon^N = 0.0010$ and $\sigma_\varepsilon^R = 0.0006$, are consistent with these results.

Second, we report the estimated dynamic parameters of our benchmark model in Table 4. The volatility parameters in the Σ matrix are estimated with precision. For the mean-reversion parameters in the $K^{\mathbb{P}}$ matrix and the mean parameters in the $\theta^{\mathbb{P}}$ vector, the results

[18]See Andreasen et al. (2019) for evidence of the robustness of this approach.

JGBi (coupon, maturity)	$G(4)$ model				$G^L(5)$ model			
	w.o. opt. adj.		w. opt. adj.		w.o. opt. adj.		w. opt. adj.	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
(1) 1.2% 3/10/2014	-6.33	15.24	-6.61	15.76	-2.08	6.31	-2.13	6.36
(2) 1.1% 6/10/2014	7.39	16.11	7.27	16.39	0.44	4.44	0.56	4.54
(3) 0.5% 12/10/2014	-1.64	10.04	-1.93	10.25	3.49	6.03	2.84	5.90
(4) 0.5% 6/10/2015	6.81	11.15	6.59	11.31	1.94	5.51	1.86	5.40
(5) 0.8% 9/10/2015	2.69	8.29	2.67	8.19	-1.41	5.78	-1.20	5.70
(6) 0.8% 12/10/2015	0.02	10.97	0.12	10.78	-1.19	9.12	-0.51	9.00
(7) 0.8% 3/10/2016	-1.99	8.15	-1.73	8.32	-1.65	6.24	-1.88	6.16
(8) 1% 6/10/2016	0.84	10.51	1.11	10.70	2.38	7.83	2.36	7.83
(9) 1.1% 9/10/2016	-5.08	8.51	-4.80	8.51	-0.42	4.11	-0.22	4.07
(10) 1.1% 12/10/2016	-5.10	7.60	-4.78	7.54	-0.76	3.33	-0.31	3.45
(11) 1.2% 3/10/2017	-6.48	12.96	-6.03	12.04	-4.11	7.46	-3.81	7.20
(12) 1.2% 6/10/2017	1.03	5.33	1.37	5.67	2.32	4.01	2.77	4.46
(13) 1.3% 9/10/2017	-1.92	5.90	-1.61	5.69	-1.60	3.50	-1.33	3.45
(14) 1.2% 12/10/2017	0.84	7.79	0.97	7.69	0.11	3.56	0.15	3.69
(15) 1.4% 3/10/2018	-2.84	11.43	-3.08	11.67	-3.01	6.15	-3.78	6.75
(16) 1.4% 6/10/2018	7.99	14.01	7.46	13.56	4.72	7.19	3.49	6.75
(17) 0.1% 9/10/2023	4.00	10.37	2.16	10.41	-0.04	4.20	0.21	4.95
(18) 0.1% 3/10/2024	2.22	6.27	-0.49	6.79	0.79	3.79	0.30	4.55
(19) 0.1% 9/10/2024	-0.71	5.25	3.60	9.07	-1.23	3.96	0.89	4.48
(20) 0.1% 3/10/2025	-0.05	6.10	2.31	7.77	1.08	2.96	0.96	3.21
(21) 0.1% 3/10/2026	-2.20	5.45	-0.83	6.45	0.29	3.57	-0.72	3.87
(22) 0.1% 3/10/2027	-5.78	10.77	-5.61	12.25	-1.16	7.21	-1.55	8.58
(23) 0.1% 3/10/2028	2.88	5.02	2.79	5.77	-0.12	2.76	-0.29	4.17
(24) 0.1% 3/10/2029	4.77	6.54	4.12	7.38	0.27	3.32	0.26	4.95
(25) 0.2% 3/10/2030	-9.13	11.18	-11.22	12.53	-0.90	3.31	-0.13	4.95
(26) 0.005% 3/10/2031	6.15	7.27	5.83	7.54	1.24	4.57	1.30	4.94
(27) 0.005% 3/10/2032	-1.43	2.85	1.68	3.14	-0.27	3.40	0.02	3.18
All yields	-0.03	9.74	0.09	10.07	-0.01	5.38	0.02	5.59
Max $\mathcal{L}^{EKF}$	97,297.21		97,132.59		101,674.4		101,504.6	

Table 3: **Pricing Errors of Japanese Real Government Bond Yields to Maturity**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese inflation-indexed bond (JGBi) yields to maturity in the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment. The errors are computed as the difference between the observed yield to maturity from Bloomberg and the corresponding model-implied yield. All errors are reported in basis points.

are more mixed, in that some of them are highly statistically significant, while others are clearly insignificant.

In Figure 4, we plot the estimated real yield factors from all four models studied so far. While the two nominal factors, L_t^N and S_t^N, are essentially identical across all four models and therefore only reported in online Appendix B, the most noticeable differences are observed for the estimated path of the real yield level and slope factors, L_t^R and S_t^R.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$K^{\mathbb{P}}_{\cdot,5}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	2.8269	2.9431	-0.3028	-0.1396	-0.0072	0.0056	σ_{11}	0.0033
	(0.0895)	(0.0978)	(0.1049)	(0.0590)	(0.0394)	(0.0084)		(0.0000)
$K^{\mathbb{P}}_{2,\cdot}$	0.0751	0.1820	0.1103	0.1036	-0.0028	-0.0085	σ_{22}	0.0034
	(0.0974)	(0.1062)	(0.0989)	(0.0704)	(0.0461)	(0.0084)		(0.0001)
$K^{\mathbb{P}}_{3,\cdot}$	-2.2619	-2.5178	0.4049	0.2966	0.0067	-0.0100	σ_{33}	0.0057
	(0.1059)	(0.0000)	(0.0965)	(0.0554)	(0.0343)	(0.0199)		(0.0002)
$K^{\mathbb{P}}_{4,\cdot}$	3.1529	3.5006	0.2192	0.4906	-0.0831	-0.0032	σ_{44}	0.0167
	(0.1308)	(0.0000)	(0.1197)	(0.0638)	(0.0352)	(0.0241)		(0.0004)
$K^{\mathbb{P}}_{5,\cdot}$	0.4554	0.3522	0.7118	-0.1521	0.5676	0.0388	σ_{55}	0.0300
	(0.1394)	(0.1462)	(0.1506)	(0.1308)	(0.1213)	(0.0396)		(0.0018)

Table 4: **Estimated Dynamic Parameters in Benchmark Model**
The estimated parameters for the mean-reversion matrix $K^{\mathbb{P}}$, the mean vector $\theta^{\mathbb{P}}$, and the volatility matrix Σ in the option-adjusted $G^L(5)$ model. The $\mathbb{Q}$-related parameters are estimated at $\lambda^N = 0.1010$ (0.0000), $\lambda^R = 0.3240$ (0.0000), $\kappa_L^{\mathbb{Q}} = 2.4086$ (0.1262), and $\theta_L^{\mathbb{Q}} = 0.0015$ (0.0000). The numbers in parentheses are the estimated standard deviations.

This seems reasonable given that the liquidity premium adjustment affects the prices of JGBi most directly. As a consequence, we also see some differences in the estimated path for the frictionless instantaneous inflation rate $\pi_t = L_t^N + S_t^N - L_t^R - S_t^R$ across the four shown models (see panel (d) of Figure 4).

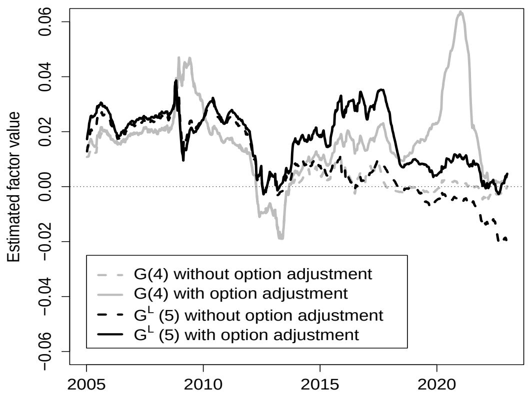
4 The JGBi Bond-Specific Premia

In this section, we first analyze the JGBi bond-specific liquidity risk premium implied by the estimated $G^L(5)$ model described in the previous section before we proceed to a detailed analysis of the estimated values of the deflation protection options, which represent another component specific to the price of each JGBi.

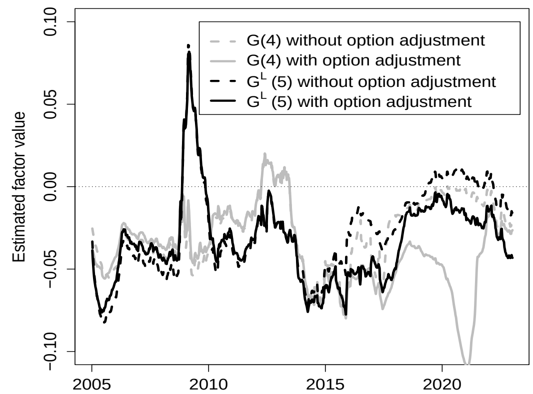
4.1 The Estimated JGBi Bond-Specific Liquidity Risk Premia

To compute the novel bond-specific liquidity risk premia in the JGBi market, we first use the estimated parameters and the filtered states $\left\{X_{t|t}\right\}_{t=1}^T$ to calculate the fitted JGBi prices $\left\{\hat{P}_t^i\right\}_{t=1}^T$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^T$ by solving the fixed-point problem

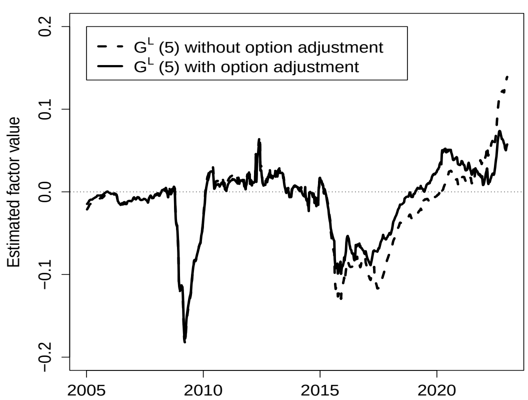
$$\begin{aligned}
\hat{P}_t^{R,i} &= C^R(t_1 - t)\exp\left\{-(t_1 - t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^{n} \frac{C^R}{2} \exp\left\{-(t_k - t)\hat{y}_t^{c,i}\right\} \\
&\quad + \exp\left\{-(T - t)\hat{y}_t^{c,i}\right\},
\end{aligned} \qquad (7)$$



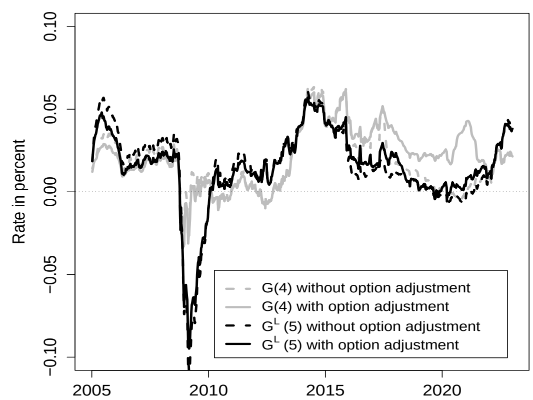
(a) L_t^R



(b) S_t^R



(c) X_t^L



(d) π_t

Figure 4: **Estimated Real Yield State Variables and Instantaneous Inflation**
Illustration of the estimated real yield state variables, (L_t^R, S_t^R, X_t^L), and the instantaneous inflation rate π_t from the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment.

for $i = 1, 2, ..., n_{R,t}$, meaning that $\left\{\hat{y}_t^{c,i}\right\}_{t=1}^T$ is approximately the real rate of return on the ith JGBi if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields with correction for the bond-specific liquidity risk premia, a new set of model-implied bond prices are computed from the estimated $G^L(5)$ model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^L = 0$ for all t as well as $\sigma_{55} = 0$ and $\theta_L^Q = 0$. These prices are denoted $\left\{\tilde{P}_t^i\right\}_{t=1}^T$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using equation (7). They represent estimates of the prices that would prevail in a world without any financial frictions or special demands for certain bonds.[19] The bond-specific liquidity risk premium

[19]We stress that, for the $G^L(5)$ model estimated with option adjustment, DOV_t^i is included in the calculation of $\hat{P}_t^i$ and $\tilde{P}_t^i$ for JGBi's issued since 2013.

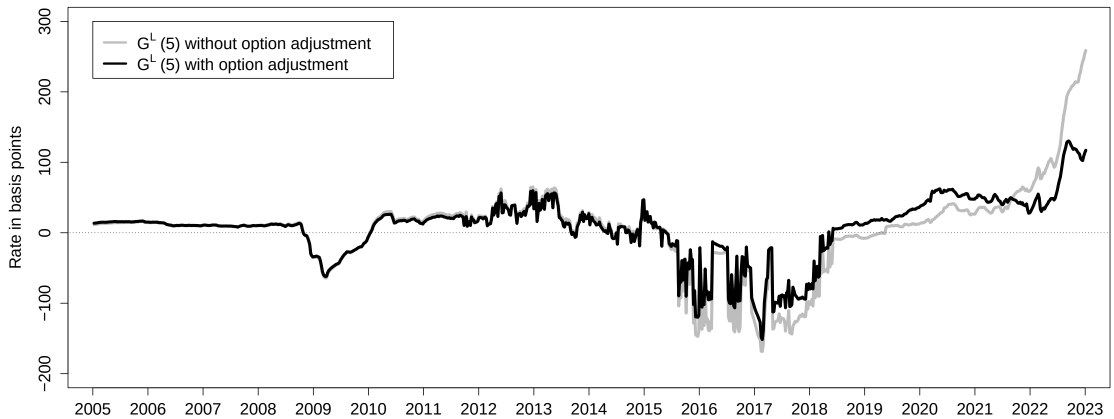


Figure 5: **Average Estimated JGBi Bond-Specific Liquidity Risk Premium**
Illustration of the average estimated bond-specific liquidity risk premium of JGBi's for each observation date implied by the $G^L(5)$ model estimated without and with option adjustment. The bond-specific liquidity risk premia are measured as the estimated yield difference between the fitted yield to maturity and the corresponding frictionless yield to maturity of individual JGBi's with the liquidity risk factor turned off as described in the text. The data cover the period from January 7, 2005, to December 30, 2022.

for the ith JGBi is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \tag{8}$$

Figure 5 shows the average JGBi bond-specific risk premium $\bar{\Psi}_t$ across the outstanding JGBi's at a given point in time without and with adjustment for the value of the deflation protection offered by JGBi's. Beyond some level difference in the 2018-2022 period, the estimated average bond-specific liquidity premia have mostly modest sensitivity to the inclusion of the deflation protection option adjustment. Note that the average estimated JGBi bond-specific risk premium varies over time and takes on both positive and negative values. They tend to turn negative and become convenience premia when uncertainty abroad is elevated, as in 2009 in the aftermath of the global financial crisis and in the 2015-2018 period when there were concerns about the growth outlook for China and trade disputes between it and the United States.

4.2 The Estimated JGBi Deflation Option Values

Figure 6 shows the inflation index ratios for all 27 JGBi's in our sample. Several bonds issued since 2013 when the deflation protection clause was introduced have been exposed to periods of deflation. However, despite extended spells of deflation in Japan since 2005, so far no JGBi has reached maturity with an index ratio below one, i.e. with the deflation option

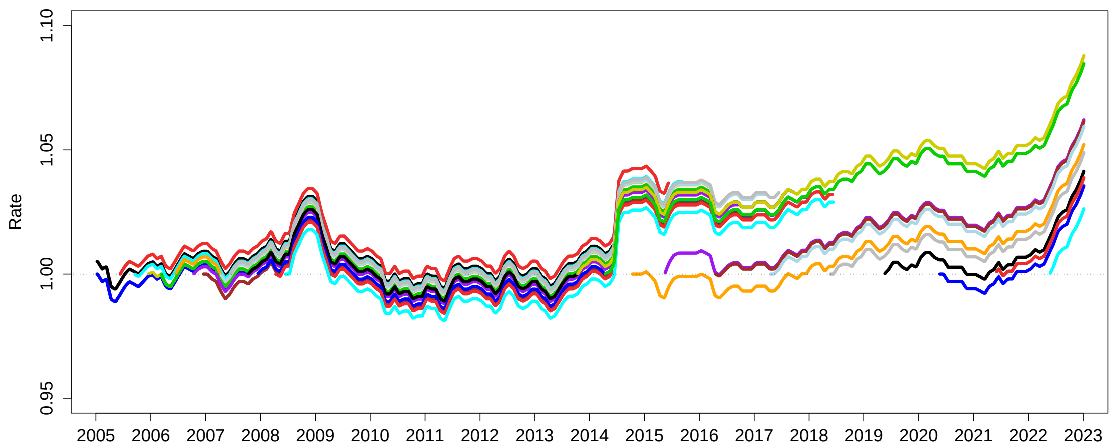


Figure 6: **Inflation Index Ratios of JGBi's**

in the money.

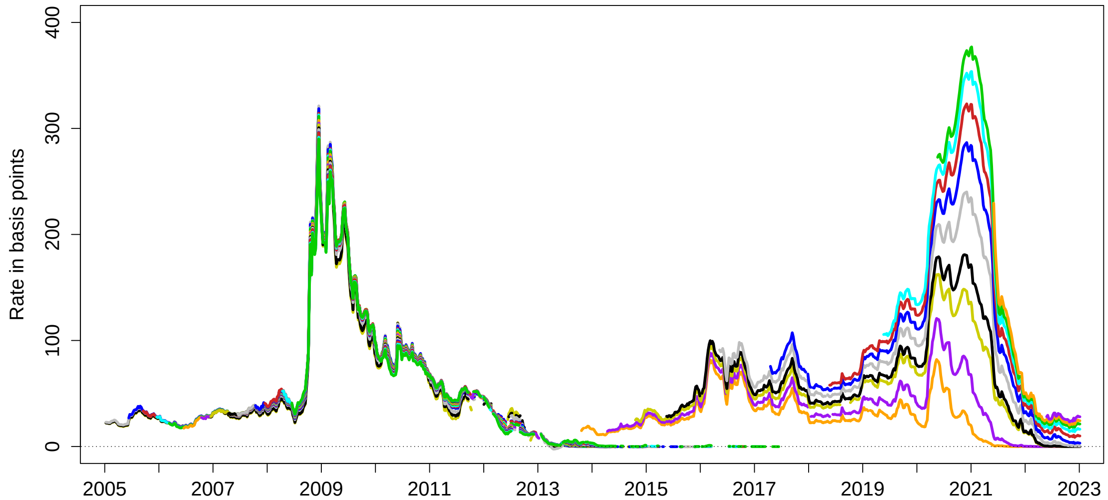
Figure 7 shows the option value for each JGBi implied by the $G(4)$ and $G^L(5)$ models, each estimated with option adjustment. We note that we measure the option value as a yield spread between the model-implied yield to maturity based on the fitted bond price *without* the option value and the model-implied yield to maturity based on the fitted bond price *with* the option value included.

The results show that the liquidity premium adjustment significantly alters the estimated value of the deflation protection option starting in mid-2015. As we document below, this has dramatic implications for BEI decompositions over our sample period.

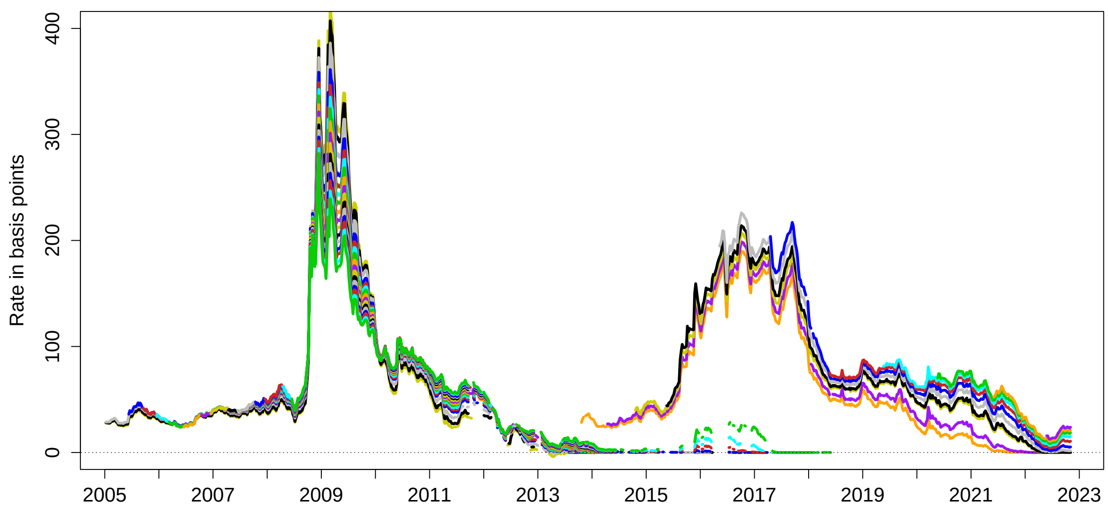
4.2.1 Deflation Option Values Measured as Par Yield Spreads

To have a consistent measure of deflation values across time, we follow Christensen et al. (2012) and construct synthetic ten-year real par-coupon yield spreads as described in the following. We compare prices of newly issued JGBi's without any accrued inflation compensation and similar seasoned JGBi's with sufficient accrued inflation compensation to make its deflation option value entirely negligible. First, consider a hypothetical seasoned JGBi with T years remaining to maturity that pays an annual coupon C semi-annually. Assume this bond has accrued sufficient inflation compensation so it is practically impossible to reach the deflation floor before maturity. The par-coupon bond satisfying these criteria has a coupon rate determined by the equation[20]

[20]In the $G^L(5)$ model, the relevant expectations under the risk-neutral measure are calculated using its frictionless yield curves.



(a) $G(4)$ model



(b) $G^L(5)$ model

Figure 7: **Value of Deflation Protection Options in JGBi's**

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q[e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q[e^{-\int_t^{T} r_s^R ds}] = 1. \tag{9}$$

The first term is the sum of the present value of the $2T$ coupon payments using the model's fitted real yield curve at day t. The second term is the discounted value of the principal payment. We denote the coupon payment for the seasonal bond that solves this equation as C_S.

Next, consider a new JGBi with no accrued inflation compensation and T years to maturity. Since the coupon payments are not protected against deflation, the difference is in accounting for the deflation protection on the principal payment:

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q [e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q \Big[\frac{\Pi_T}{\Pi_t} \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} > 1\}} \Big] + E_t^Q \Big[1 \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}} \Big] = 1.$$

The first term is the same as before. The second term represents the present value of the principal payment conditional on a positive net change in the price index over the bond's maturity; i.e., $\frac{\Pi_T}{\Pi_t} > 1$. Under this condition, full inflation indexation applies, and the price change $\frac{\Pi_T}{\Pi_t}$ is placed within the expectations operator and weighted by the probability of accumulated inflation at time T. The third term represents the present value of the *floored* JGBi principal conditional on accumulated net deflation; i.e., when the price level change is below one, $\frac{\Pi_T}{\Pi_t}$ is replaced by a value of one to provide the promised deflation protection. Since

$$\frac{\Pi_T}{\Pi_t} = e^{\int_t^T (r_s^N - r_s^R) ds},$$

the equation can be rewritten as

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q [e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q \Big[e^{-\int_t^T r_s^R ds} \Big] + \Bigg[E_t^Q \Big[e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}} \Big] - E_t^Q \Big[e^{-\int_t^T r_s^R ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq 1\}} \Big] \Bigg] = 1,$$

where the last term on the left-hand side represents the net present value of the deflation protection of the principal in the JGBi contract. The par-coupon yield of a new hypothetical JGBi that solves this equation is denoted as C_0.

The difference between C_S and C_0 is a direct measure of the advantage of being at the inflation adjustment floor for a newly issued JGBi. Figure 8 shows this par yield difference at the ten-year maturity from the same two model estimations studied above. From 2005 until 2013 there is relatively little difference between the two estimated deflation risk premia. However, in the 2015-2018 period, the average bond-specific liquidity premia turned negative and JGBi's traded with a convenience premium *even after* accounting for the value of the deflation protection. In turn, this implies that the frictionless option-adjusted BEI is *below* the fitted option-adjusted BEI. As a consequence, the estimated priced risk of deflation is notably higher according to the $G^L(5)$ model during this period than it was under our earlier $G(4)$ model without the liquidity adjustment.

In contrast, when the average bond-specific liquidity premia switched back into positive territory during 2018, the frictionless option-adjusted BEI moved back up above the fitted option-adjusted BEI, which explains the relatively low the deflation risk premium implied by the $G^L(5)$ model during the remaining years of our sample. In particular, we only obtain a modest uptick at the onset of the COVID-19 pandemic, which stands in sharp contrast to

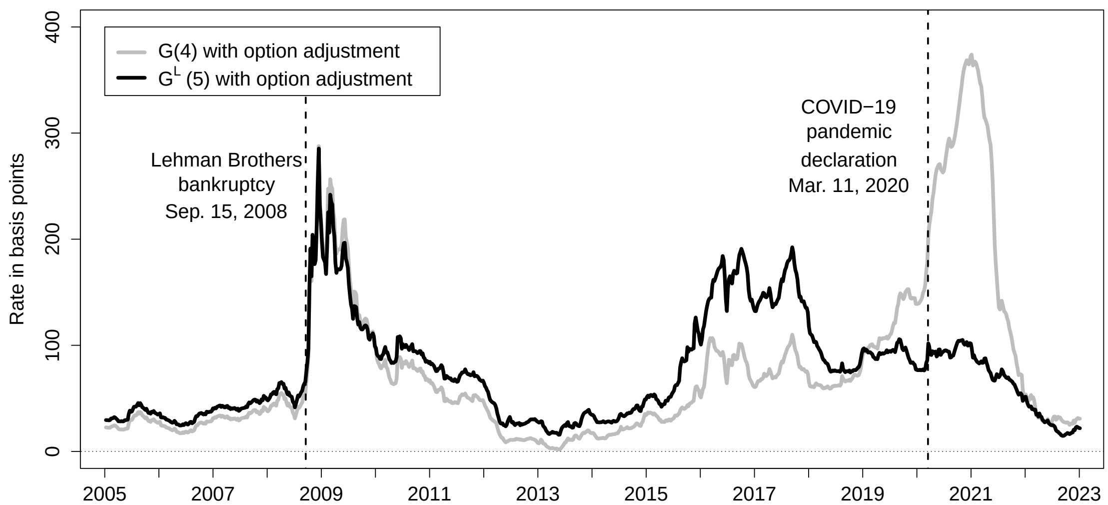


Figure 8: **Ten-Year Deflation Risk Premium**

the much higher estimated values implied by our earlier $G(4)$ model.

5 Empirical BEI Decomposition

In this section, we examine the implications for the decomposition of BEI that follow from our $G^L(5)$ model. We focus on the ten-year maturity, which is the benchmark in the literature, but our results are robust to using other maturities.

Figure 9 shows the ten-year BEI decomposition implied by the $G^L(5)$ model and the $G(4)$ model without liquidity adjustment from CS (2022), both estimated with deflation protection option adjustment. Figure 9 also shows the semiannual ten-year inflation forecasts from the Consensus Forecasts survey that we use in the model estimation. As discussed above we only use the April-forecast every other year, highlighted in the figure with thick black dots.

The wedge between the ten-year fitted BEI and the ten-year liquidity- and option-adjusted BEI implied by our $G^L(5)$ model represents the net effect of the deflation protection and liquidity adjustments. This adjusted measure of BEI fell below fitted BEI in the 2015-2019 period, while it was above it during the last year of our sample. In contrast, by only accounting for the value of deflation protection, the adjustment produced by the $G(4)$ model leads to a distorted measure of adjusted BEI, and yields an expected inflation component that is clearly high relative to available survey evidence and a residual inflation risk premium that is low and overly volatile. In comparison, the $G^L(5)$ model yields an expected inflation estimate that is closer to the forecasts reported in the surveys of professional forecasters and a more stable inflation risk premium.

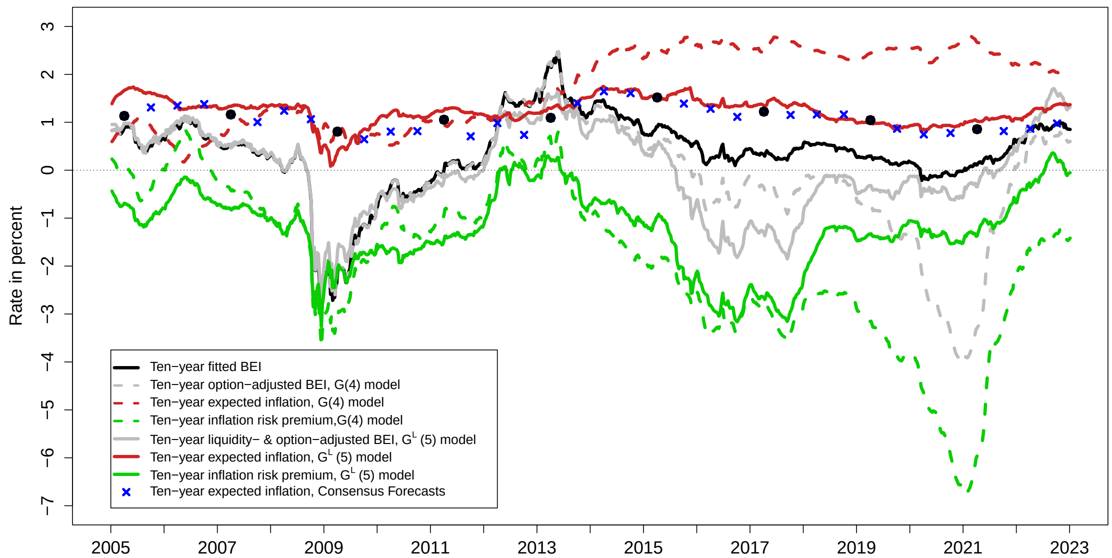


Figure 9: **Decomposition of Ten-Year BEI**
Illustration of the fitted ten-year BEI obtained by fitting the $G(4)$ model to nominal and real yields without any option adjustment or survey forecasts. Also shown are (i) the ten-year fitted option-adjusted break-even inflation (BEI) calculated as the difference between the fitted ten-year nominal yield and the fitted ten-year option-adjusted real yield from the $G(4)$ model and as the difference between the fitted ten-year nominal yield and the fitted ten-year liquidity- and option-adjusted real yield from the $G^L(5)$ model, both estimated with deflation protection option adjustment, (ii) the estimated ten-year expected inflation, and (iii) the residual ten-year inflation risk premium. Finally, the semi-annual ten-year expected inflation series from the Consensus Forecasts survey is shown with blue crosses with the biannual ones used in the model estimations highlighted in black.

Specifically, between 2005 and 2020, the $G^L(5)$ model's ten-year expected inflation closely tracks the survey forecasts even though they are only included every other year in the model estimation and with a high measurement error of 0.75 percent. This suggests that investors' inflation expectations were by and large in agreement with those of the professional forecasters. In contrast, a quite sizable wedge opened up in the 2021-2022 period between the survey and the model-implied ten-year expected inflation. As of October 2022, the survey forecast was 0.97 percent, while the model expects inflation to average 1.39 percent over the next ten years. Hence, long-term expected inflation in Japan may be closer to the Bank of Japan's two percent inflation target than either the surveys or unadjusted BEI may suggest.

The uptick in the liquidity- and option-adjusted BEI also implies an increase in the ten-year inflation risk premium since 2021 according to our $G^L(5)$ model. This seems reasonable in light of the significant spike in Japanese and global inflation during this period. However, by the end of our sample, the Japanese inflation risk premium is close to zero. This may be an early sign that fears about a material undershooting of the inflation target have abated.

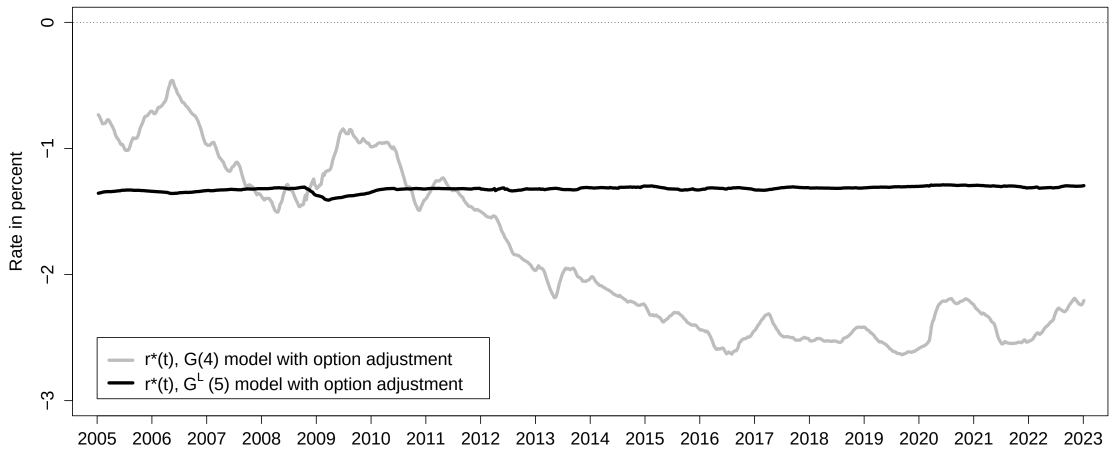


Figure 10: **Market-Based Estimates of r^***

To explore that question further, we follow Christensen and Rudebusch (2019) and define the equilibrium real rate of interest r_t^* as

$$r_t^* = \frac{1}{5}\int_{t+5}^{t+10} E_t^{\mathbb{P}}[r_{t+s}^R]ds, \tag{10}$$

that is, the average expected real short rate over a five-year period starting five years ahead where the expectation is with respect to the objective $\mathbb{P}$-probability measure. This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks.

Figure 10 shows these finance-based estimates of r_t^* from both the $G(4)$ and $G^L(5)$ models. Based on the $G(4)$ model the equilibrium real rate has experienced a persistent decline of about 1 percentage point on net since 2005. This would imply that monetary policy is likely to have been less stimulative than widely perceived. Furthermore, it would entail that monetary policy has been restrictive for a while already in Japan.

In contrast, our novel $G^L(5)$ model produces an estimate of r_t^* estimates, which is very stable and has remained close to -1.35 percent for our entire sample period.[21] Hence, the missing liquidity premium adjustment in the earlier $G(4)$ model distorts its assessment of the outlook for short-term real rates.

[21]Given that the model-implied long-term expected inflation shown in Figure 9 does vary over time, the stable r_t^* estimates are *not* a result of upward bias in the estimated mean-reversion rates as could be a concern, see Bauer et al. (2012).

6 Global JGBi Liquidity Spillovers

We next use our estimated time series of liquidity premia in the JGBi market to evaluate whether or not liquidity conditions in Japanese bond markets are linked to disruptions in other important global bond markets. We concentrate on the U.S. Treasury market, the most liquid bond market in existence and one that is commonly considered a determinant of global financial conditions, e.g. Rey (2015). In principle, one might think that the Japanese market would be less exposed than other national bond markets to disruptions in the U.S. Treasury market due to its own large size and the general perception that investors in Japan exhibit substantive home bias in their asset holdings; see Tesar and Werner (1995). If this were the case, liquidity shocks in the U.S. Treasury market should not give rise to dislocations or illiquidity conditions in Japanese bond markets.

To measure the timing of such liquidity disruptions in the U.S. Treasury market, we obtain market depth data for the two-, five-, seven-, and ten-year on-the-run U.S. Treasury securities from December 4, 2009, through December 30, 2022. Market depth is measured as the number of open orders from surveyed dealers at the stated maturity. We take values as of the close on Fridays, if available. In a few cases this data is unavailable due to market closure, in which case we use the latest trading day of the week.[22] We then compare movements in these U.S. Treasury market depth measures to our estimated JGBi liquidity premium series.

6.1 Specification

To determine if liquidity spillovers from U.S. Treasury markets are disrupting the Japanese JGBi market, we start from our continuous measures of U.S. Treasury market depth, denoted $US(X)_{depth}$, where $X \in \{2, 5, 7, 10\}$ refers to the maturity in years of the considered on-the-run U.S. Treasury security. We examine time-series specifications for all four depth measures separately as well as combinations of measured disruptions in all four series.

We characterize Treasury market depth as shallow if the measured depth is two standard deviations below the mean for that series or less. We then evaluate the implications of an indicator variable, denoted $I^{US}(X)_{shallow}$, $X \in \{2, 5, 7, 10\}$, which takes a value of one during weeks exhibiting shallow market depth in the X-year on-the-run U.S. Treasury security, and 0 otherwise. We also consider two summary indicators, $I^{US}(any)_{shallow}$, which takes a value of one in weeks in which any of the individual market indicators of shallowness signals low market depth, and an in between "counting" indicator $C^{US}_{shallow}$, which is calculated as the sum of the four individual maturity indicators, and so takes values 1 through 4 based on the number of maturities indicating shallow market depth conditions, and 0 otherwise.

To isolate the impact of low market depth and control for potentially confounding factors, we include variables indicating general market conditions in both the U.S. and Japan. We in-

[22]Unfortunately, this data is proprietary and cannot be released to the public.

clude both U.S. and Japanese yields of comparable maturities, $y^{US}(X)$ with $X \in \{2,5,7,10\}$, and $y^{JPN}(X)$ with $X \in \{2,4,7,10\}$ measured in years for the U.S. and Japanese bond markets, respectively. Note that while we match the 2-, 7, and 10-year shallow market depth indicators with identical maturity yields, we pair the 5-year U.S. shallow market depth indicators with the 4-year Japanese maturity yield. Furthermore, we include VIX^{JGB} and $MOVE^{US}$ to proxy for volatility in Japanese and U.S. financial markets, respectively.

For example, our baseline specification for the impact of shallow market depth in the 2-year U.S. Treasury market satisfies:

$$\bar{\Psi}_t = c + \beta_1 I^{US}(2yr)_{shallow,t} + \beta_2 y^{US}(2yr)_t + \beta_3 y^{JPN}(2yr)_t + \beta_4 VIX_t^{JPN} + \beta_5 MOVE_t^{US} + \epsilon_t, \quad (11)$$

where $\bar{\Psi}_t$ is our estimate of the average JGBi liquidity premium at time t and ϵ_t is a disturbance term, with Newey-West standard errors to adjust for possible heteroskedasticity and autocorrelation. Note that while our estimates of liquidity premia likely contain errors, as they are included as the dependent variable in our estimation they should not introduce any systematic bias in our estimates of our coefficients of interest. Indeed, if anything, they lead to attenuation bias for our estimated coefficients on those variables. Specifications for shallow market conditions in other maturities as well as the composite measures are similar.

6.2 Data

Data are weekly, and obtained from Bloomberg unless otherwise indicated, from July 1, 2009 through July 1, 2023. U.S. Treasury market depth data was obtained from Brokertec. To remove outliers, we winsorize the data at the $(2.5\%, 97.5\%)$ level.

Summary statistics for our sample are shown in Table 5. As discussed above, volatility in U.S. Treasury market depth is particularly large for the 2-year U.S. Treasury maturity.

6.3 Results

Our regression results are shown in Table 6. It follows that weeks exhibiting shallow depth in U.S. Treasury markets are associated with higher estimates of liquidity premia in the Japanese JGBi market for all of the individual maturities studied, as well as for specifications 5 and 6, which uses a measure of the existence of shallowness at any maturity in the U.S. market and the total number of maturities indicating shallow treasury market depth, respectively. All of our estimates are statistically significant, with all entering positively at a 1% confidence level except for the 10 year maturity, which enters positively at a 10% confidence level.

In terms of our estimated magnitudes and the summary statistics above, our point estimates indicate that having shallow two-year U.S. Treasury markets according to our metric

Table 5: Summary Statistics

	N	Mean	Std. Dev.	Min	Max
$\bar{\Psi}_t$	939	6.524	41.153	-101.987	89.511
VIX^{JPN}	742	2.811	1.254	1.290	6.410
$US(2yr)_{depth}$	651	1054.359	747.427	22.135	3632.355
$US(5yr)_{depth}$	651	212.767	93.727	16.162	581.166
$US(7yr)_{depth}$	651	155.749	58.473	12.478	333.293
$US(10yr)_{depth}$	651	163.160	61.653	10.868	333.778
$MOVE^{US}$	893	81.204	29.025	46.230	162.500
$y^{JPN}(2yr)$	939	0.119	0.323	-0.359	1.073
$y^{JPN}(4yr)$	939	0.244	0.428	-0.371	1.416
$y^{JPN}(7yr)$	939	0.448	0.550	-0.382	1.855
$y^{JPN}(10yr)$	939	0.720	0.658	-0.281	2.014
$y^{US}(2yr)$	893	1.607	1.492	0.102	5.230
$y^{US}(5yr)$	893	2.162	1.253	0.210	5.210
$y^{US}(7yr)$	893	2.480	1.149	0.390	5.210
$y^{US}(10yr)$	893	2.893	1.135	0.552	5.257

Note: Summary statistics of the data sample for the baseline regressions.

are associated with 2.10 standard deviation increases in our estimate of JGBi liquidity premia. Similarly, our estimated coefficients for shallow market conditions for 5-, 7-, and 10-year U.S. Treasuries are associated with increases of 2.29, 2.14, and 0.90 standard deviations in the estimated JGBi liquidity premia, respectively. Our point estimate for regression model (5) indicates that having U.S. treasuries of *any* maturity exhibit shallowness is associated with a predicted increase of 1.74 standard deviations in the estimated liquidity premia, while our point estimate for regression model (6) indicates that having an additional U.S. Treasury market exhibiting shallowness is associated with a predicted increase of a 0.70 standard deviation increase in the estimated JGBi liquidity premia.

In terms of the conditioning variables, both the VIX^{JPN} and the $MOVE^{US}$ variables are insignificant. The Japanese yield variables, $y^{JPN}(2yr)$, $y^{JPN}(4yr)$, $y^{JPN}(7yr)$, and $y^{JPN}(10yr)$ all enter positively in their respective specifications, at least at a 10% significance level. This is the expected sign, as we would expect elevated interest rate periods to be associated with higher liquidity premia as well. In contrast, U.S. Treasury yields enter negatively, albeit only insignificantly for the 2-year yield, while the 5-, 7- and 10-year Treasury yields enter negatively at least at a 10% confidence level. Again, all of these variables enter with their expected signs.

Overall then, our results indicate significant and economically important spillovers between illiquidity disruptions in U.S. Treasury markets and estimated liquidity premia in the Japanese JGBi market. As a consequence, the Japanese JGBi market may be less segmented from global bond markets than could be expected based on the structural arguments outlined in Cardozo and Christensen (2023). All else being equal, this market may then be more

Table 6: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)
$I^{US}(2yr)_{shallow}$	86.423***					
	(25.107)					
$I^{US}(5yr)_{shallow}$		94.272***				
		(23.127)				
$I^{US}(7yr)_{shallow}$			87.997***			
			(21.577)			
$I^{US}(10yr)_{shallow}$				37.185*		
				(14.991)		
$I^{US}(any)_{shallow}$					71.471***	
					(20.295)	
$C^{US}_{shallow}$						28.781***
						(6.038)
VIX^{JPN}	-4.990	-7.898	-9.063	-6.149	-6.377	-7.520
	(6.194)	(6.592)	(6.580)	(7.008)	(6.419)	(6.466)
$MOVE^{US}$	0.070	0.303	0.451	0.689*	0.240	0.227
	(0.319)	(0.339)	(0.335)	(0.339)	(0.304)	(0.323)
$y^{JPN}(2yr)$	96.833*					
	(45.687)					
$y^{JPN}(4yr)$		62.541*				
		(29.633)				
$y^{JPN}(7yr)$			48.636*			
			(18.851)			
$y^{JPN}(10yr)$				33.884*	45.227**	49.034***
				(14.379)	(13.848)	(13.723)
$y^{US}(2yr)$	-4.100					
	(7.075)					
$y^{US}(5yr)$		-16.884*				
		(6.650)				
$y^{US}(7yr)$			-21.199**			
			(6.929)			
$y^{US}(10yr)$				-21.001**	-22.416**	-24.074***
				(7.510)	(6.966)	(6.692)
Constant	18.171	26.190	26.286	6.088	34.823	41.038
	(24.587)	(24.848)	(23.577)	(24.521)	(21.103)	(21.215)
N	606	606	606	606	606	606

Note: Newey-West standard errors are shown in parentheses. Statistical significance levels are indicated by the asterisks: *** p<0.01, ** p<0.05, and * p<0.10.

liquid than other inflation-indexed bond markets, as also suggested by the mostly moderate size of our estimated JGBi liquidity premia. On the downside, the integration with global bond markets implies that the JGBi market is sensitive to liquidity shortages in other major bond markets as captured in our market depth measures for on-the-run U.S. Treasury securities.

7 Conclusion

In this paper, we account for both bond-specific liquidity premia and the value of deflation protection in the prices of individual Japanese inflation-indexed government bonds, known as JGBi's. To do so, we extend the four-factor model of nominal and real yields introduced by CS (2022), which only accounts for the deflation protection values, by adding a bond-specific liquidity risk factor structured as in ACR (2021).

We find that both adjustments are sizable and time varying. Importantly, the liquidity adjustment has persistent trends and switches sign, meaning that it periodically transitions from being a liquidity price discount to being a convenience price premium. This adjustment is impactful, as our measure of adjusted BEI during these latter periods fall *below* observed BEI, which significantly increases the priced risk of deflation and raises the model-implied value of the deflation protection offered by JGBi's. Similarly, when the liquidity adjustment is positive, our measure of adjusted BEI is *above* observed BEI, which leads to lower values of deflation protection compared to the standard model.

Our extended model produces more stable measures of expected inflation, which are closer to the inflation forecasts reported in surveys of professional forecasters than those coming from the model without the liquidity adjustment. For that same reason the extended model also produces more stable estimates of the residual inflation risk premium. Overall, we take these results to document that the extended model represents a significant improvement relative to the existing models in the literature, including our earlier model in CS (2022).

We then utilize our estimated liquidity premia estimates to examine the degree of spillovers between U.S. and Japanese bond markets. While Japanese asset markets are commonly believed to exhibit some degree of segmentation due to "home bias" among Japanese investors, we find that exceptionally low levels of market depth for on-the-run U.S. Treasuries for a variety of maturities are associated with elevated estimated JGBi liquidity premia. Our point estimates also indicate that this relationship is economically important. We interpret these results as evidence that illiquidity and market stress in the U.S. Treasury market does spill over into the JGBi market, implying that segmentation of the Japanese JGBi market is incomplete.

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Online Appendix

Inflation Expectations, Liquidity Premia and Global Spillovers in Japanese Bond Markets

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

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Contents

A Bond Yield Decomposition

In this appendix, we describe the decomposition of nominal and real bond yields into underlying expectations and residual risk premium components using arbitrage-free term structure models.

We follow Merton (1974) and assume the existence of a continuously-traded continuum of nominal and real zero-coupon bonds. This implies that inflation risk is spanned by the nominal and real yields. This allows us to decompose the nominal and real yields into the sum of the corresponding short-rate expectations and associated term premia using our arbitrage-free term structure model.

To begin, define the nominal and real stochastic discount factors as M_t^N and M_t^R, respectively. Their dynamics are standard and given by

$$\begin{aligned} dM_t^N/M_t^N &= -r_t^N dt - \Gamma_t' dW_t^{\mathbb{P}}, \\ dM_t^R/M_t^R &= -r_t^R dt - \Gamma_t' dW_t^{\mathbb{P}}, \end{aligned}$$

where Γ_t contains the risk premia.

Under our no-arbitrage condition, the price of a nominal bond that pays one unit of currency in τ years and the price of a real bond that pays one consumption unit in τ years must satisfy

$$P_t^N(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] \quad \text{and} \quad P_t^R(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right],$$

where $P_t^N(\tau)$ and $P_t^R(\tau)$ are the prices of the zero-coupon, nominal and real bonds for maturity τ at time t and $E_t^{\mathbb{P}}[.]$ is the conditional expectations operator under the real-world (or $\mathbb{P}$-) probability measure.

The no-arbitrage condition also requires that the price of a consumption unit, denoted as the overall price level Π_t, is the ratio of the real and nominal stochastic discount factors:

$$\Pi_t = \frac{M_t^R}{M_t^N}.$$

By Ito's lemma, the dynamic evolution of Π_t is given by

$$d\Pi_t = (r_t^N - r_t^R)\Pi_t dt.$$

Thus, in the absence of arbitrage, the instantaneous growth rate of the price level is equal to the difference between the instantaneous nominal and real risk-free rates.[1] Correspondingly, we can express the stochastic price level at time t+τ as

$$\Pi_{t+\tau} = \Pi_t e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

The relationship between the yields and inflation expectations can be obtained by decomposing the price of the nominal bond as follows

$$\begin{aligned}
P_t^N(\tau) &= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R/\Pi_{t+\tau}}{M_t^R/\Pi_t}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\frac{\Pi_t}{\Pi_{t+\tau}}\right] \\
&= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] + cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] \\
&= P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).
\end{aligned}$$

Converting this price into yield to maturity using

$$y_t^N(\tau) = -\frac{1}{\tau}\ln P_t^N(\tau) \quad \text{and} \quad y_t^R(\tau) = -\frac{1}{\tau}\ln P_t^R(\tau),$$

we obtain

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right]$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or nega-

[1]Note that the price level Π_t is a stochastic process as long as r_t^N and r_t^R are stochastic processes.

tive. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences.

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

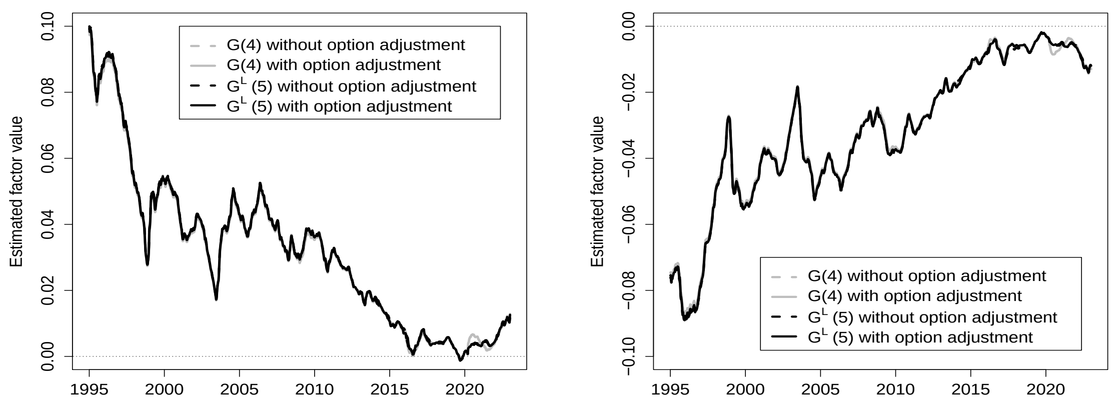
that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

Finally, we define the nominal and real term premia as

$$\begin{aligned} TP_t^N(\tau) &= y_t^N(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^N]ds, \\ TP_t^R(\tau) &= y_t^R(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^R]ds. \end{aligned}$$

That is, the nominal term premium is the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal rate r_t^N. The interpretation for the real term premium is similar. The model thus allows us to decompose nominal and real yields into their respective term premia and short-rate expectations components.

B Estimated Nominal State Variables



(a) L_t^N

(b) S_t^N

Figure 1: **Estimated Nominal State Variables**
Illustration of the estimated state nominal variables, L_t^N and S_t^N, from the $G(4)$ and $G^L(5)$ models without and with deflation protection option adjustment.

In this appendix, we plot the estimated nominal yield factors from all four models studied in the main text. These are shown in Figure 1. Note that the two nominal factors, L_t^N and S_t^N, are essentially identical across all four models. This also explains the very similar fit to the nominal yield data across all four models reported in Table 2 in the main text. Thus, the nominal side of our models has very little sensitivity to adjustments for either the liquidity premia of JGBi's or the values of the deflation protection they offer.

C Sensitivity of JGBi Liquidity Premia to Data Frequency

In this appendix, we assess whether the data frequency plays any role for our estimated JGBi liquidity premia. To do so, we estimate the $G^L(5)$ model with deflation protection option adjustment using daily, weekly, and monthly data.

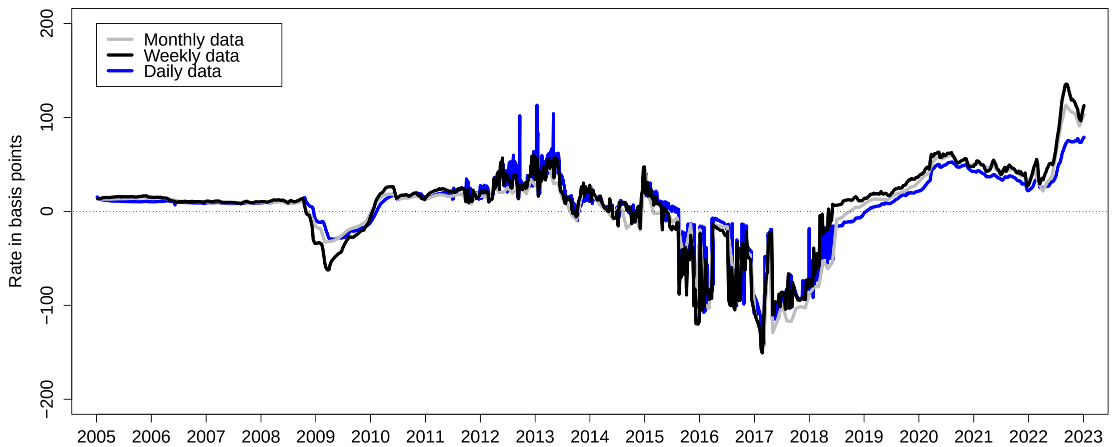


Figure 2: **Average Estimated JGBi Liquidity Premium: Data Frequency**
Illustration of the average estimated bond-specific liquidity risk premium of JGBi's for each observation date implied by the $G^L(5)$ model with deflation option adjustment when estimated using daily, weekly, and monthly data. In all three cases, the bond-specific liquidity risk premia are measured as the estimated yield difference between the fitted yield to maturity and the corresponding frictionless yield to maturity of individual JGBi's with the liquidity risk factor turned off as described in the text.

Figure 2 shows the average JGBi bond-specific liquidity risk premium from the three estimations calculated as described in Section 4.1 in the main text. Note that they are all very close to each other. Thus, we conclude that data frequency matters little for our estimated JGBi liquidity premia.

D Sensitivity of Deflation Risk Premia to Data Frequency

In this appendix, we assess whether the data frequency plays any role for our estimated ten-year deflation risk premia. To do so, we estimate the $G^L(5)$ model with deflation protection option adjustment using daily, weekly, and monthly data.

Figure 3: **Ten-Year Deflation Risk Premium**

Figure 3 shows the estimated ten-year deflation risk premia from the three estimations, each calculated as par yield spreads as described in Section 4.2.1 in the main text. Note that they are all very close to each other. Thus, we conclude that data frequency matters little for our estimated measures of deflation risk.

References

Merton, Robert C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, Vol. 29, No. 2, 449-470.