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Ding Dong
Hong Kong Baptist University

Zheng Liu
Federal Reserve Bank of San Francisco

Pengfei Wang,
Peking University HSBC Business School
PHBS Sargent Institute of Quantitative Economics and Finance

Min Wei
Federal Reserve Board of Governors

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INFLATION DISAGREEMENT WEAKENS THE POWER OF MONETARY POLICY

DING DONG, ZHENG LIU, PENGFEI WANG, AND MIN WEI

Abstract. Households often disagree in their inflation outlooks. We present novel empirical evidence that inflation disagreement weakens the power of forward guidance and conventional monetary policy. These empirical observations can be rationalized by a model featuring heterogeneous beliefs about the central banks' inflation target. An agent who perceives higher future inflation also perceives a lower real interest rate and thus would like to borrow more to finance consumption, subject to borrowing constraints. Higher inflation disagreement would lead to a larger share of borrowing-constrained agents, resulting in more sluggish responses of aggregate consumption to changes in both current and expected future interest rates. This mechanism also provides a microeconomic foundation for Euler equation discounting that helps resolve the forward guidance puzzle.

I. Introduction

Individual forecasters often disagree in their inflation outlooks (Mankiw et al., 2003; Andrade et al., 2016; Weber et al., 2022; Fofana et al., 2024). For example, Figure 1 shows the cross-sectional distribution of households' inflation expectations (measured by expectations

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Dong: Hong Kong Baptist University. Email: dingdong@hkbu.edu.hk. Liu: Federal Reserve Bank of San Francisco. Email: Zheng.Liu@sf.frb.org. Wang: Peking University HSBC Business School and PHBS Sargent Institute of Quantitative Economics and Finance. Email: pfwang@phbs.pku.edu.cn. Wei: Federal Reserve Board of Governors. Email: min.wei@frb.gov. We are grateful to Eric Swanson for sharing his updated data on monetary policy shocks, to Damjan Pfajfar for sharing the survey data on perceived inflation target, and to Stefania D'Amico for sharing the inflation uncertainty measure. We have benefited from helpful discussions with and comments from Michael Bauer, Olivier Coibion, Marc Dordal i Carreras, Leland Farmer, Oscar Jorda, Sylvain Leduc, Geng Li, Yang Lu, Alisdair McKay, Guangyu Pei, Sanjay Singh, Eric Swanson, Michael Weber, Johannes Wieland, Dora Xia, Shengxing Zhang, and Sonya Zhu. Xi Jin and Yuhan Zhang provided excellent research assistance. Pengfei Wang acknowledges financial support from National Science Foundation of China (project No.72150003, No.72125007) and Research Grant Council of Hong Kong (Project T35-710/20-R). The views expressed herein are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of San Francisco, the Federal Reserve Board, or the Federal Reserve System.

of one-year ahead changes in the consumer price index, or CPI) from the University of Michigan Survey of Consumers in June 2023 (left panel). Consumers' inflation expectations vary from 0% to over 20%, with a median of 3.3%. Consumers disagree not just in their inflation forecasts, but also in their perceived inflation target of the Federal Reserve. The right panel of Figure 1 shows the distribution of consumers' perceived inflation target of the FOMC.[1] Despite the Fed's frequent communications of its 2% inflation goal, consumers' perceived inflation target ranges from 0% to over 9%, with a median of 3.0%. These observations illustrate pervasive inflation disagreement among consumers.

Inflation disagreement is also time-varying. Figure 2 shows the inter-quartile range (IQR, which is the difference between the 75th percentile and the 25th percentile of inflation expectations) of inflation expectations over both the one-year (in red) and the five-to-ten-year (in blue) horizons from the Michigan survey. Evidently, inflation disagreement displays frequent time variations, with occasional spikes such as those during the 2008-2009 global financial crisis and those during the post-pandemic period.

Studies have shown that inflation expectations are important for the transmission of monetary policy (Orphanides and Williams, 2004; Galí, 2015; Gargiulo et al., 2024). However, less is known about the role of inflation disagreement. This paper examines how inflation disagreement affects the transmission of monetary policy, both empirically and theoretically.

To examine the empirical importance of inflation disagreement for the transmission of monetary policy, we estimate a local projections model in the spirit of Jordà (2005). In particular, we estimate the effects of a monetary policy shock on real activity and inflation, both on average and during periods with high inflation disagreement. We measure inflation disagreement using the IQR of inflation forecasts over the one-year horizon from the Michigan survey, normalized by the median of inflation forecasts. We consider two types of monetary policy shocks, one is a shock to forward guidance and the other a shock to the federal funds rate, both constructed and updated by Swanson (2021) based on high-frequency identification.[2] Our sample covers the period from July 1991 to December 2023.

[1]The underlying data for the distribution of consumers' perceived inflation target are provided by Pfajfar and Winkler (2024), who conducted a special survey in June 2023 as a part of the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York.

[2]We are grateful to Eric Swanson for sharing his updated series of both federal funds rate shocks and forward guidance shocks. Swanson (2021) builds on the earlier work on measuring monetary policy shocks using high-frequency changes in asset prices around FOMC announcements (Kuttner, 2001; Gürkaynak et al., 2005). Many recent studies have used high-frequency changes in interest rates around the FOMC policy announcements to identify the effects of monetary policy. Examples include Cochrane and Piazzesi (2002); Faust et al. (2004); Gertler and Karadi (2015); Ramey (2016); Stock and Watson (2018); Bauer and Swanson (2023).

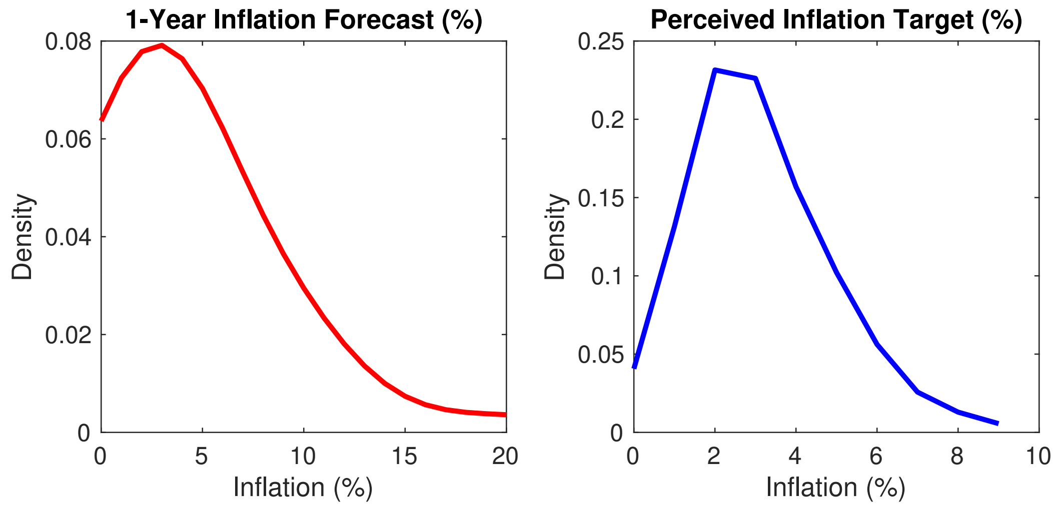

FIGURE 1. Cross-sectional distribution of inflation beliefs.

This figure shows the cross-sectional distribution of inflation beliefs. In each panel, the horizontal axis shows the inflation rate (in percent). The left panel (red line) plots the kernel density of one-year ahead CPI inflation expectations from the Michigan Survey of Consumers in June 2023. The mean, the median, and the IQR of the inflation expectations are 5.2, 3.3, and 6.1 percent, respectively. The right panel (blue line) plots the kernel density of the consumers' perceived inflation target of the FOMC (with values of perceived inflation target above 9% trimmed). The mean, the median, and the standard deviation of the perceived inflation target are 2.9, 3.0, and 1.2 respectively. The data are taken from a special survey conducted by Pfajfar and Winkler (2024) in June 2023 as a part of the Survey of Consumer Expectations of the Federal Reserve Bank of New York.

We find that, absent inflation disagreement, a forward guidance shock that signals a tightening of future monetary policy leads to persistent declines in both consumption and inflation. However, these effects are substantially attenuated in periods with high inflation disagreement. We find similar attenuating effects of inflation disagreement on the power of federal funds rate shocks.

The attenuation effects of inflation disagreement are robust and they are not driven by demographic factors or common shocks. Individual inflation expectations can be affected by both aggregate shocks and individual experiences (D'Acunto et al., 2021; Malmendier and Nagel, 2016). Thus, the measured inflation disagreement (e.g., based on the IQR of inflation expectations) does not necessarily reflect exogenous variations in inflation beliefs (Ahn and Farmer, 2024; Fofana et al., 2024). To examine whether the sources of inflation disagreement matter for our empirical findings, we use the cross-sectional archives of individual responses

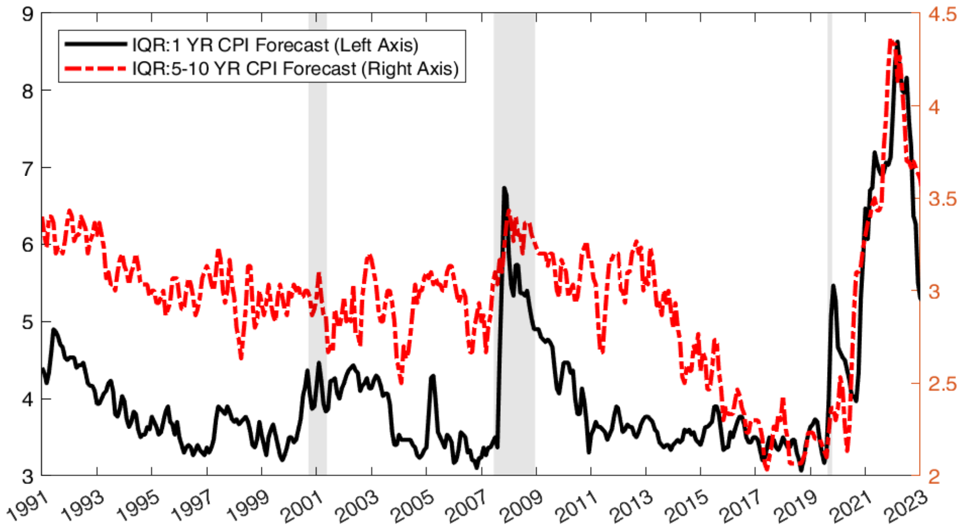

FIGURE 2. Time-varying inflation forecast dispersion from the Michigan Survey of Consumers.

This figure shows the time series of inflation disagreement, measured by the IQR (i.e., the differences between the 75th percentile and the 25th percentile) of CPI inflation forecasts over the one-year horizon from Michigan Survey of Consumers (red line, right axis). The mean, persistence and standard deviation of this time series from July 1991 to December 2023 are 4.09, 0.89 and 1.05 respectively. The series is highly correlated with IQR of inflation forecasts over five-to-ten year horizon (blue line, left axis), with a correlation coefficient of 0.60.

in the Michigan survey to construct an alternative measure of inflation disagreement that is purged of the effects of idiosyncratic factors and aggregate shocks on inflation expectations. We first regress individual one-year ahead inflation expectations on a set of demographic factors (such as income, education, marital status, and residence location), with a time fixed effect included to capture the effects of aggregate shocks. We then construct a "purified" measure of inflation disagreement based on the IQR of the regression residuals. We also construct an "orthogonized" measure by further removing the effects of three common sources of shocks — a federal funds rate shock, a forward guidance shock, and an oil supply shock — on inflation disagreement. With these alternative measures of inflation disagreement, we obtain impulse responses of consumption and inflation that are similar, both qualitatively and quantitatively, to those obtained using the baseline measure.

We also obtain similar results when we use alternative measures of real activity and inflation or when we include additional control variables in the local projections. We show that the attenuating effect of inflation disagreement is not driven by the level of inflation expectations, inflation uncertainty, consumption uncertainty, consumer sentiment, or income growth expectations.

To understand the mechanism through which inflation disagreement can weaken the power of monetary policy, we generalize the standard New Keynesian framework to incorporate belief heterogeneity and borrowing constraints. In the model economy, the central bank has a particular inflation target. However, different agents hold different beliefs about that target, reflecting, for example, imperfect credibility of the central bank or individual inattention to monetary policy. At a given nominal interest rate, an agent with a higher perceived inflation target has also a lower perceived real interest rate, resulting in higher marginal propensity to consume (MPC). For the same reason, an agent with a lower perceived inflation target would like to save more, resulting in lower MPC. High-MPC agents finance consumption using both internal funds and external debt, subject to a borrowing constraint. When inflation beliefs become more dispersed, a greater mass of agents would hold beliefs that lie at the upper tail of the belief distribution. Those high-MPC agents borrow to consume and, once they hit the borrowing limit, they cannot adjust consumption spending freely in response to exogenous shocks.[3] Thus, with greater dispersion of inflation beliefs—or equivalently, with greater inflation disagreement—there would be more borrowing-constrained agents, causing current consumption to adjust less than one-for-for to changes in future consumption or in the real interest rate.

In line with the empirical evidence, our model predicts that inflation disagreement attenuates the effects of forward guidance policy on consumption spending. With a discounted Euler equation, signaling a future reduction in the real interest rate through forward guidance would have a smaller effect on current consumption than does a reduction in the current real interest rate. The magnitude of the Euler-equation discounting increases with the magnitude of inflation disagreement. Absent inflation disagreement, the Euler equation in our model coincides with that in the standard model with no discounting. In that case, a decline in the real interest rate in arbitrarily distant future would have the same stimulative effect on current consumption as does a decline in the current real interest rate, giving rise to the forward-guidance puzzle (Del Negro et al., 2023; McKay et al., 2016). In the more general case with inflation disagreement, however, current consumption responds to expected future consumption less than one-for-one (hence, the Euler equation is "discounted"). Furthermore, the sensitivity of current consumption to future consumption declines monotonically with the magnitude of inflation disagreement as our analytical results reveal.

[3]Higher inflation disagreement also implies a larger share of low-MPC agents, whose beliefs about the inflation target lie in the lower tail of the belief distribution. However, those agents can adjust consumption optimally in response to shocks because they are unconstrained.

In our model, inflation disagreement not only reduces the sensitivity of current consumption to changes in future consumption, but also reduces the sensitivity of consumption to changes in the contemporaneous interest rate. Thus, consistent with the empirical evidence, our model predicts that higher inflation disagreement leads to more muted effects of shocks to the conventional interest rate policy.

II. Related Literature

Our paper contributes to the literature on heterogeneity in household inflation expectations (Mankiw et al., 2003; Andrade et al., 2016; Coibion et al., 2020; Ropele et al., 2024). Pervasive inflation disagreement has stimulated much interest in recent empirical studies, most of which focus on understanding potential drivers of such disagreement. For example, Ahn and Farmer (2024) decompose disagreement about inflation expectations into three sources: prior beliefs, responses to common information, and idiosyncratic information. Fofana et al. (2024) find that inflation disagreement can be driven by demographic factors such as age, sex, marital status, income, and education. They also find that inflation disagreement responds to aggregate shocks to monetary policy and to supply and demand conditions.

Our paper has a different focus: How does inflation disagreement affect the transmission of monetary policy, including forward guidance and the conventional interest rate policy? In this aspect, our study is closely related to Falck et al. (2021), who examine the implications of inflation disagreement in the Survey of Professional Forecasters (SPF) on the transmission of conventional monetary policy shocks. Our work is also related to Barbera et al. (2023), who exploit information in the term structure of inflation forecast disagreement by decomposing the disagreement into disagreement about trend inflation and about cyclical inflation. They find that cyclical inflation disagreement weakens the responses of asset prices to conventional monetary policy shocks, while disagreement about trend inflation does not. Our paper develops a theoretical framework that can rationalize some of the findings in these empirical studies. More importantly, we document novel empirical evidence that inflation disagreement attenuates the responses of real activity and inflation to both forward guidance and the conventional interest rate policy.

Our work also contributes to the literature on the forward guidance puzzle, which has been a challenge to the standard New Keynesian framework. In the standard New Keynesian models with rational expectations, forward guidance policy that promises changes in interest rates in the distant future would have implausibly large effects on output and inflation relative to the effects of shocks to the current interest rate (Del Negro et al., 2023; Hagedorn et al., 2019). Previous studies have proposed potential resolutions of the forward guidance puzzle in a representative-agent framework by introducing information frictions (Angeletos and Lian, 2018), bounded rationality (Farhi and Werning, 2019; Gabaix, 2020), imperfect central bank credibility (Andrade et al., 2019; Campbell et al., 2019), or the extensive margin of durable goods purchases (McKay and Wieland, 2022). In our model, the presence of heterogeneous beliefs about the central bank's inflation target—a form of disagreement about long-run inflation—could substantially weaken the transmission of unconventional monetary policy such as forward guidance. In this sense, the model provides an alternative microeconomic foundation for a discounted Euler equation, which helps resolve the forward guidance puzzle.

Our model highlights the importance of heterogeneity in households' beliefs for weakening the efficacy of forward guidance. Thus, our model mechanism is complementary to that in the heterogeneous-agent New Keyesian (HANK) framework. In an important contribution, McKay et al. (2016) study a HANK model with incomplete markets, where agents face uninsurable income risks and liquidity constraints. They show that a precautionary-savings effect partially offsets the intertemporal substitution effects, dampening the responses of current consumption to changes in future interest rates and therefore helps resolve the forward guidance puzzle (see also McKay et al. (2017)). Werning (2015) argues that the precautionary-savings channel may depend on the assumptions about the cyclicality of idiosyncratic income risks and liquidity. If idiosyncratic income risks are countercyclical or liquidity relative to income is procyclical, then forward guidance policies would be as powerful as in representative agent models.

Relative to the HANK model of McKay et al. (2016), our model generates heterogeneity in MPCs and Euler-equation discounting through a different mechanism. In our model with inflation disagreement, agents with higher inflation expectations have lower perceived real interest rates. Thus, they are more likely to be borrowing constrained. Greater inflation disagreement results in a larger share of borrowing-constrained agents and thus more sluggish adjustments in aggregate consumption in response to forward-guidance shocks.

Our model highlights the importance of households' debt capacity for the transmission of monetary policy. A monetary policy easing can effectively stimulate consumption spending only if households with high MPC have access to unused debt capacity. However, as pointed out by Sufi (2015), this credit extension channel was extraordinarily weak after the 2008-09 global financial crisis, rendering monetary policy ineffective during that period (see also Beraja et al. (2019)). Our model captures the essence of this "limited credit access" channel in accounting for the ineffectiveness of monetary policy. In our model, the households with higher inflation expectations have higher MPC and they are more likely to face binding borrowing constraints. They cannot further adjust their borrowing or spending upward even when monetary policy reduces the current or expected interest rates.[4] We show that limited credit access—measured by the net percentage of tightening of lending standards in consumer loans from the Federal Reserve's Senior Loan Officer Opinion Survey (SLOOS)—does reinforce the attenuation effects of inflation disagreement.

Our model implies a positive relation between inflation expectations and current consumption spending at the individual household level. This implication is consistent with empirical evidence. One strand of this literature looks at household survey responses and shows that there is a positive correlation between household inflation expectations and their willingness to spend, at least for highly educated respondents or respondents with high cognitive skills (Vellekoop and Wiederholt (2019), Bachmann et al. (2015), D'Acunto et al. (2023) and Andrade et al. (2023)). However, it is difficult to establish causal effects of changes in inflation expectations on consumption spending using survey data alone. By exploiting a quasi-natural experiment in Germany and using a difference-in-differences approach, D'Acunto et al. (2021) document evidence that the announcement of value-added tax increases in 2005, to be implemented in 2007, raised German consumers' inflation expectations, leading to an immediate increase in consumers' readiness to buy durable goods. Coibion et al. (2022) use a range of randomized information treatments in a large-scale survey of U.S. households to study how different types of communications affect consumers' inflation expectations and ultimately their spending decisions. They find that higher inflation expectations arising from information treatments lead to a rise in household spending on non-durable goods, although not on durable goods, over the next 6 months.

III. Empirical Evidence

We examine how inflation disagreement affects the transmission of monetary policy in the data. In particular, we study the effects of both forward guidance shocks and federal funds rate shocks on real activity and inflation in the presence of inflation disagreement.

III.1. **Inflation disagreement and forward guidance shocks.** We first examine how inflation disagreement influences the macroeconomic effects of forward guidance shocks. We measure consumption and price level using real personal consumption expenditure (PCE) and PCE price index from Bureau of Economic Analysis. We use the measure of forward

[4]Some empirical studies find that consumption spending of more indebted households is more responsive to interest rate changes (Cloyne et al., 2020; Cumming and Hubert, 2023). This evidence does not necessarily contradict our model's implication (and the empirical evidence of Sufi (2015) and Beraja et al. (2019)) that limited debt capacity can dampen the effects of monetary policy shocks. If indebted households have unused debt capacity, lowering interest rates would boost their consumption spending. However, such effects would be muted if these households have limited debt capacity.

guidance shocks from Swanson (2021) and a measure of inflation disagreement based on the one-year ahead inflation forecasts from the University of Michigan Survey of Consumers. We follow the approach of Jordà (2005) and compute the impulse responses of the real variables and inflation to a forward guidance shock by estimating the following local projections specification

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^\pi_{t-1} + \alpha^h_3 IQR^\pi_{t-1} * FG_t + \alpha^h_4 \Gamma_{t-1} + \varepsilon_{t+h} \quad (1)$$

with $h = 0, 1, 2, ..., 48$. In this specification, $\log(y^j_{t+h}) - \log(y^j_{t-1})$ denotes cumulative log changes in real consumption ($j = 1$) and the price level ($j = 2$) from period $t-1$ to period $t+h$. FG_t denotes the forward guidance shock constructed by Swanson (2021) based on high-frequency changes of short- and long-term interest rates around FOMC announcements. Inflation disagreement, denoted as IQR^π_{t-1}, is the interquartile range of inflation forecasts over a one-year horizon from the Michigan Survey of Consumers at month $t-1$, normalized by the median of inflation forecast. We use lagged inflation disagreement to avoid complications from potential endogeneity of the inflation forecast dispersion.[5] Γ_{t-1} is a set of macroeconomic control variables, all lagged by one period. Those control variables include the log growth rates of consumption and industrial production, the PCE inflation rate, the unemployment rate, and the shadow federal funds rate constructed by Wu and Xia (2016). The term ε_{t+h} denotes the regression residuals. The monthly sample covers the period from July 1991 to December 2023.

Table 1 presents the summary statistics of the variables used in our regressions. On average, there is substantial inflation disagreement in our sample, with the IQR of the one-year ahead inflation forecasts modestly above the the median of those forecasts (with a ration of 1.38, which is our measure of inflation disagreement). Inflation disagreement is also persistent and volatile, with a first-order autocorrelation of 0.55 and a standard deviation of 0.50. The measures of the FG shock and the conventional monetary policy shock from Swanson (2021) are both highly volatile, although they are both close to an i.i.d. process, with little persistence.

The coefficient α^h_1 captures the average effects of forward guidance shocks on macroeconomic variables of interest. The coefficient α^h_3 captures the marginal effect of high inflation expectation dispersion on monetary policy transmission. When α^h_1 and α^h_3 have opposite signs, it suggests that high inflation disagreement may weaken or even overturn the effect of forward guidance.

[5]We focus on inflation disagreement based on one-year ahead inflation forecasts. In the data, the dispersion of long-term inflation forecasts is strongly correlated with the short-term forecasts (Andrade et al., 2016; Weber et al., 2022). Thus, our results are robust to using long-term inflation forecasts to measure inflation disagreement.

Table 1. Summary statistics

Variable	Mean	SD	AutoCorr
Inflation Disagreement	1.39	0.50	0.55
Real PCE Growth (month-over-month, log changes, %)	0.23	0.94	0.08
PCE Inflation (month-over-month, log changes, %)	0.17	0.20	0.47
FG Shocks	0.00	0.94	-0.05
MP Shocks	-0.01	0.79	-0.07
Shadow Federal Funds Rate	2.18	2.62	0.99

Note: The monthly sample covers the period from July 1991 to December 2023.

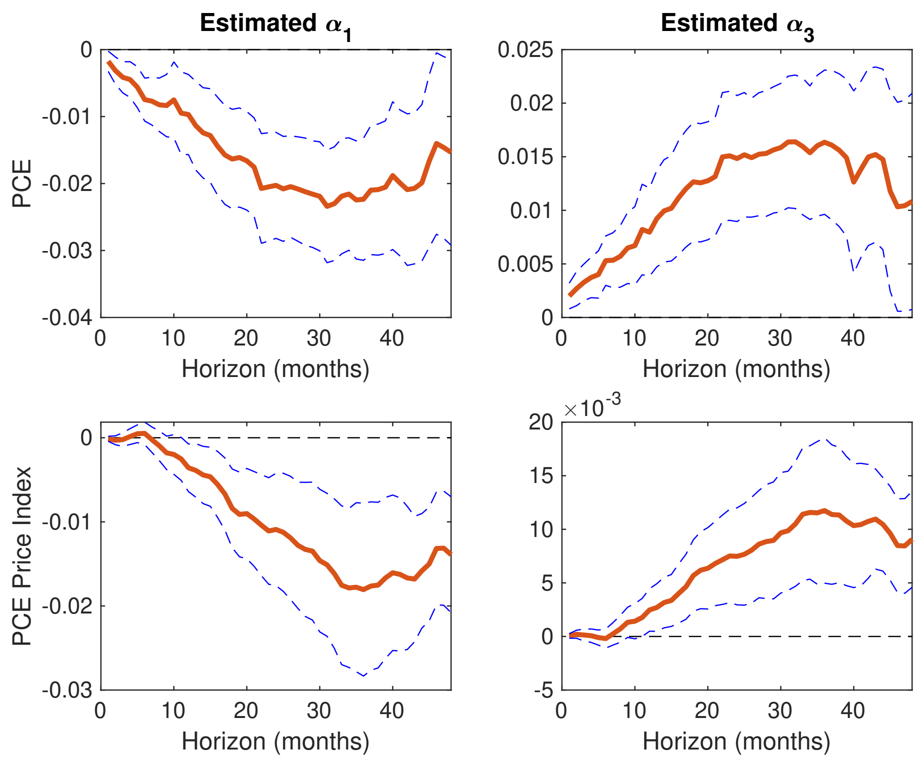

Figure 3. Estimated impulse response to a forward guidance shock

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock estimated from the local projections model (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

The upper panels of Figure 3 shows that an identified forward guidance shock is followed by a decline in real consumption ($\alpha_1^h < 0$), but the effect is mitigated if the current state is characterized by high inflation disagreement ($\alpha_3^h > 0$). The point estimates imply that a one-standard-deviation FG shock leads to a cumulative reduction in real consumption of 1.9 percent over a period of two years.[6] However, in periods with inflation disagreement that is one standard deviation above its mean, the cumulative effect of FG on consumption at the two-year horizon would be attenuated by about 37.5% (relative to the 1.9 percent direct effect of FG on consumption).[7]

The bottom panels of Figure 3 shows that an FG shock that signals future monetary policy tightening reduces inflation (i.e., $\alpha_1^h < 0$), but the effects are partially blunted in periods with inflation disagreement (i.e. $\alpha_3^h > 0$). The point estimates imply that, absent inflation disagreement, a one-standard-deviation increase in the FG shock would reduce the PCE price index by 1.02 percent over a two-year period ($h = 24$). However, when inflation disagreement rises by one standard deviation above its mean, the cumulative effect of the FG shock on the price level would be weakened by about 34.4% at the two-year horizon.[8]

III.2. **Inflation disagreement and conventional monetary policy shocks.** We estimate a simiar empirical specification for conventional monetary policy shocks:

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \beta_0^h + \alpha_1^h MP_t + \alpha_2^h IQR^{\pi}_{t-1} + \alpha_3^h IQR^{\pi}_{t-1} * MP_t + \alpha_4^h \Gamma_{t-1} + \varepsilon_{t+h}. \quad (2)$$

Here, as in the case with the forward guidance shocks, the dependent variable (y_t^j) is either real PCE ($j = 1$) or the PCE price index ($j = 2$), both in terms of cumulative log-changes from the pre-shock period $t-1$ to period $t+h$, where $h = 1, 2, \dots, 48$ denotes the time horizon (in months). The term MP_t denotes the the federal funds rate shock constructed by Swanson (2021). In estimating the specification (2), we control for the same set of lagged macroeconomic variables summarized in Γ_{t-1}, as for the specification (1).

[6]Swanson (2021) shows that a one-standard-deviation FG shock raises the two-year Treasury yield by about 4.6 basis points on average.

[7]From the summary statistics presented in Table 1, the FG shock has a standard deviation of 0.94 and the inflation disagreement measure has a standard deviation of 0.50. The point estimate of $\alpha_1 = -0.020$ at the two-year horizon (i.e., $h = 24$) implies that a one-standard-deviation shock to FG reduces consumption by $0.020 \times 0.94 \times 100 = 1.9$ percent. The point estimate of $\alpha_3 = 0.015$ at the two-year horizon implies that a one-standard-deviation increase in inflation disagreement attenuates the negative effect of FG on consumption by $0.50 \times (0.015/0.020) \times 100 = 37.5$ percent.

[8]The point estimate of $\alpha_1^{h=24} = -0.0109$ implies that a one-standard-deviation shock to FG reduces the PCEPI by $-0.0109 \times 0.94 \times 100 = 1.02\%$ in periods with no inflation disagreement. The estimated $\alpha_3^{h=24} = 0.0075$ implies that, in periods with inflation disagreement one standard deviation above its mean, the effects of the FG shock on the PCEPI would be weakened by $0.50 * 0.0075/0.0109 * 100 = 34.4\%$.

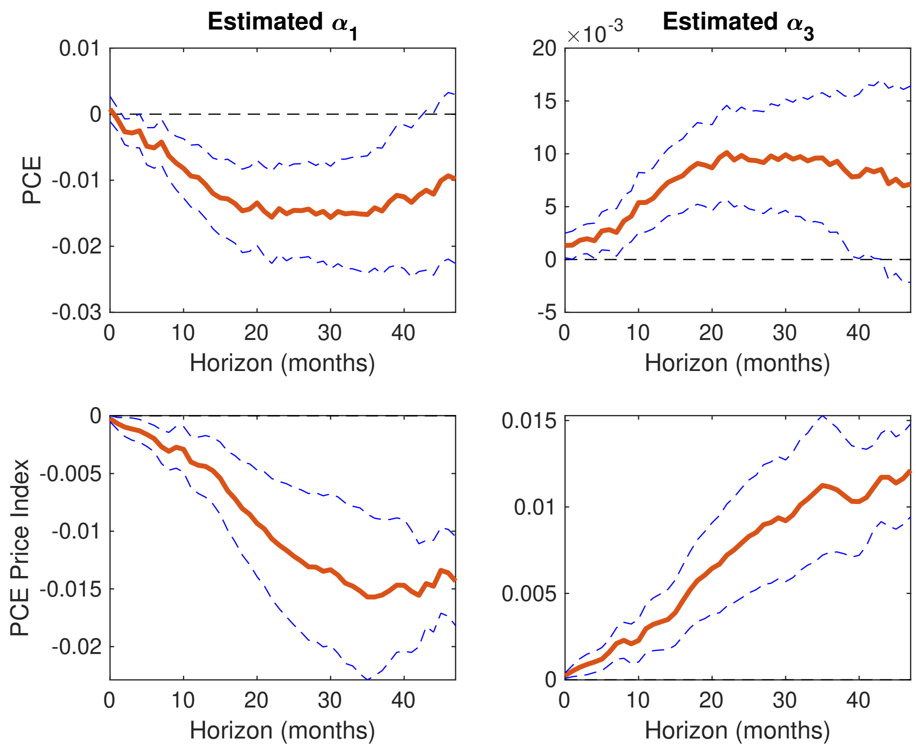

FIGURE 4. Estimated impulse response to a federal fund rate shock

Note: This figure shows the cumulative impulse responses of real personal consumption expenditure (PCE), and the PCE price index following a federal fund rate shock estimated from the local projections model (2). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

Figure 4 shows the impulse responses of real PCE (upper panel) and of the PCE price index (lower panel) to a one-standard-deviation fed funds rate shock.[9] Absent inflation disagreement, a tightening of monetary policy lowers both consumption and inflation ($\alpha_1^h < 0$). The presence of inflation disagreement mitigates the contractionary effects of the monetary policy shock ($\alpha_3^h > 0$). These impulse responses to a fed funds rate shock are qualitatively similar to those following an FG shock. Quantitatively, a one-standard-deviation increase in inflation disagreement would weaken the cumulative effects of a fed funds rate shock on consumption expenditures and the price level by 32.7% and 34.1% respectively.[10]

[9] Swanson (2021) shows that a one-standard-deviation fed funds rate shock raises the current-month fed funds target rate by about 8.4 basis points on average.

[10] The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE change are -.015 and 0.0098 respectively. A one-standard-deviation higher inflation disagreement would thus reduce the effects of a fed funds rate shock by

IV. Robustness of empirical results

Our baseline empirical results suggest that inflation disagreement attenuates the power of forward guidance and fed funds rate shocks. We now show that these empirical results are robust to alternative measures of inflation disagreement, alternative measures of real activity and inflation, and the inclusion of additional control variables.

IV.1. **Measures of inflation disagreement.** Individual inflation expectations can be affected by both aggregate shocks and idiosyncratic factors (such as age, income, the region of residence, education, marital status, and home ownership status). To mitigate the influence of those idiosyncratic and aggregate factors on measured inflation disagreement, we use the cross-sectional archives of individual responses in the Michigan survey from July 1991 to December 2023 to estimate the panel-data specification[11]

$$\pi^e_{jt} = \gamma_t + \beta Z_{jt} + \varepsilon_{jt}, \tag{3}$$

where the term π^e_{jt} denotes the one-year ahead CPI inflation expectations of individual j in period t; γ_t denotes a time fixed effect, capturing the responses of individual inflation expectations to changes in aggregate economic conditions; Z_{jt} is a vector of individual demographic characteristics, including income (in log units), home ownership status, region of residence, education, sex, and marital status; and ε_{jt} denotes the regression residual, which measures the individual inflation expectations that are not explained by the demographic factors and aggregate shocks. We construct a "purified" measure of inflation disagreement using the IQR of the regression residuals in Eq. (3).

Inflation disagreement may also respond to monetary policy shocks and oil supply shocks (Fofana et al., 2024). To mitigate this concern, we construct an "orthogonalized" measure of inflation disagreement, which is the part of our purified measure not explained by current and lagged values (for up to 12 months) of the fed funds rate and forward guidance surprises constructed by Swanson (2021) and those of the oil supply shocks constructed by Känzig (2021).

Figure 5 plots the purified measure (blue dashed line) and the orthogonalized measure (black dashed line). Evidently, those measures are both highly correlated with the raw measure of inflation disagreement used in our baseline regressions (red solid line).

$0.50 * 0.0098/0.015 * 100 = 32.7\%$. The estimated $\alpha_1^{h=24}$ and $\alpha_3^{h=24}$ on cumulative PCE price level change are -0.0116 and 0.0079 respectively, implying an attenuation effect of $0.50 * 0.0079/0.0116 * 100 = 34.1\%$.

[11]The data is accessible via https://data.sca.isr.umich.edu/findings/findings.php. Information on home-ownership is not available for surveys in 1991 and 1992, so we drop the regressants for these two years. We exclude individual responses with recorded inflation expectation above 20% or below -20%.

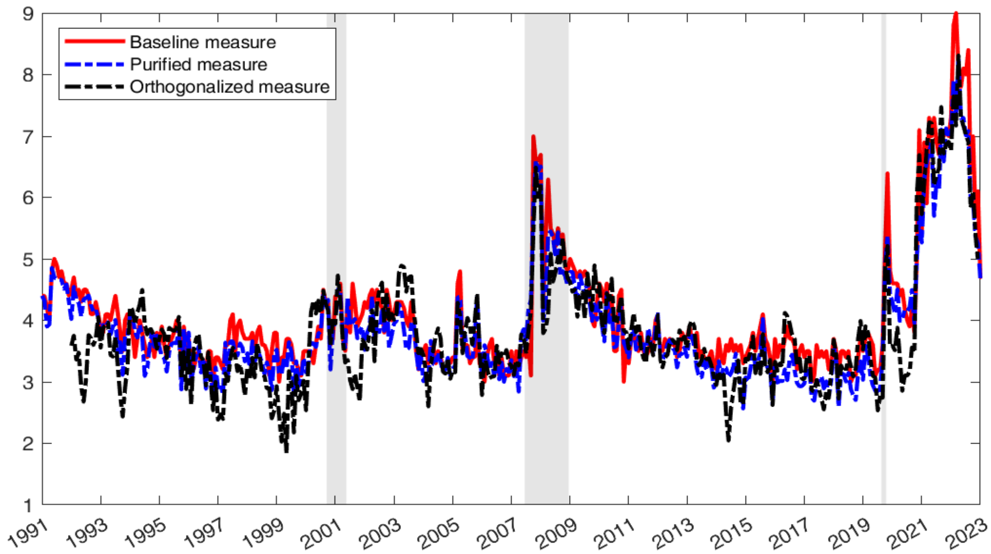

FIGURE 5. Alternative measures of inflation disagreement.

This figure shows three measures of inflation disagreement. The red solid line shows the baseline measure of inflation disagreement, defined as the IQR of the one-year ahead CPI inflation forecasts from the Michigan Survey of Consumers. The blue dashed line shows the purified measure of inflation disagreement, defined as the IQR of the one-year ahead inflation forecasts that are unexplained by demographics and aggregate shocks (i.e., the IQR of ε_{jt} in Eq. (3)). The black dashed line shows the orthogonalized inflation disagreement, defined as the components of the purified inflation disagreement that are unexplained by current and lagged values of monetary policy surprises and oil supply shocks.

When we use either the purified measure or the orthogonalized measure of inflation disagreement (each scaled by the median of inflation expectations) to re-estimate the baseline local projections model in Eq. (1), we obtain impulse responses of real PCE and inflation that are similar—both qualitatively and quantitatively—to those obtained in our baseline regressions (see Figures A.1 and A.2 in Appendix A.1).

IV.2. **Disagreement about long-term inflation.** In our baseline regression, we measure inflation disagreement based on the dispersion of one-year ahead inflation forecast from the Michigan Survey of Consumers. Nevertheless, our results hold for disagreement about long-term inflation. When we re-estimate the local projections specifications (1) and (2) using inflation disagreement measured by the IQR of 5-10 year ahead inflation expectations in the Michigan survey (normalized by the median of the long-term inflation expectations, we find that long-term inflation disagreement also attenuates the power of forward guidance and conventional policy, similar to what we find in our baseline regressions using the short-term (1-year ahead) inflation disagreement (see Figures A.3 and A.4 in Appendix A.2).

IV.3. **Measures of economic activity and inflation.** In the baseline model, we measure real activity by personal consumption expenditures and inflation by changes in the PCE price index. The results are robust to alternative measures of real activity, including industrial production and unemployment. They are also robust to measuring inflation using the consumer price index (see the results reported in Appendix A.3).

IV.4. **Additional control variables.** The attenuating effect of inflation disagreement can be potentially confounded by the effects from other factors, such as the average level of inflation expectations, consumer sentiment, income growth expectations, consumers' perceived uncertainty for durable goods purchases, or inflation uncertainty. To alleviate this concern, we include these additional control variables–one at a time—and the interaction of each additional control variable with the monetary policy shock (FG or fed funds rate shocks) in estimating our local projections.

Figure 6 reports the estimated cumulative effects of the FG shock and its interactions with inflation disagreement on consumption (upper panel) and the price level (lower panel) at the two-year horizon estimated from the baseline local projections (model 0) and those from 5 alternative models. The alternative models each has a different additional control variable and its interactions with the FG shock. The first 4 models (Models 1-4) include, respectively, the median one-year ahead inflation expectations, the consumer sentiment index, the median income growth expectations of consumers, and consumers' perceived uncertainty concerning vehicle purchasing conditions, all taken from the Michigan survey. Studies have shown that inflation disagreement is related to but different from inflation uncertainty (Gambetti et al., 2023). Thus, we also estimate a model (Model 5) that includes an updated measure of inflation uncertainty constructed by Breach et al. (2020) using the density forecasts from the SPF of the Federal Reserve Bank of Philadelphia.[12]

In the figure, the horizontal axis shows the baseline model (Model 0) and the 5 alternative model specifications. The vertical axis hows the point estimates of the coefficients of interest (α_1 and α_3) at the two-year horizon ($h = 24$).[13] The figure shows that the attenuation

[12]The Michigan survey does not provide a good measure of inflation uncertainty. So we use the SPF instead, with two caveats. First, the SPF is a quarterly survey of macroeconomic forecasts. We interpolate the measure of inflation uncertainty into monthly, assuming that it remains the same within each quarter. Second, studies have shown that forecasts by professional forecasters can exhibit very different properties from those by households (Candia et al., 2020).

[13]Further details about the additional control variables and the full set of impulse responses are reported in Appendix A.4.

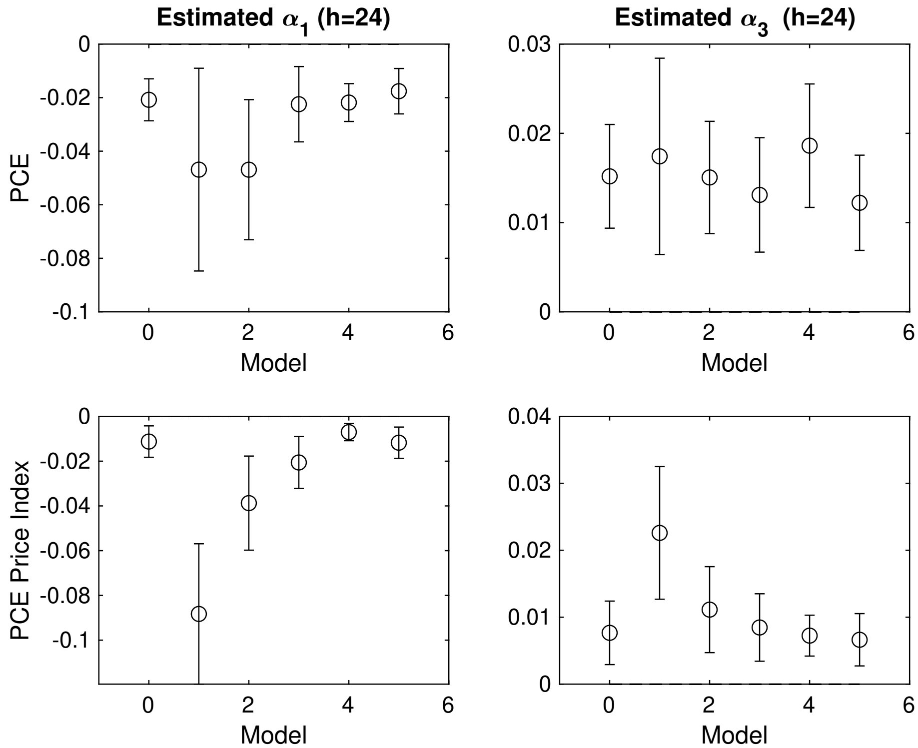

FIGURE 6. Impulse responses to a forward guidance shock at the two-year horizon: Robustness

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index at the two-year horizon ($h = 24$) following a forward guidance shock under alternative specifications of the local projections model. Model 0 represents the baseline specification. The other model specifications differ in the set of control variables. These include (1) the median of one-year ahead inflation expectations from the Michigan survey (model 1); (2) the consumer sentiment index from the Michigan survey (model 2); (3) the median income growth expectation of consumers from the Michigan survey (model 3); (4) consumers' perceived uncertainty concerning vehicle purchases from the Michigan survey (model 4); (5) inflation uncertainty from the SPF (model 5). The circles represent the point estimates at two-year horizon ($h = 24$) and the whiskers represent the 68% confidence bands (with Newey-West standard errors).

effects of inflation disagreement for FG shocks are robust across these alternative model specifications. Similarly, the attenuating effects of inflation disagreement are also robust for the fed funds rate shocks, as shown in Figure 7.[14]

[14]The full response paths are reported in Appendix A.4.

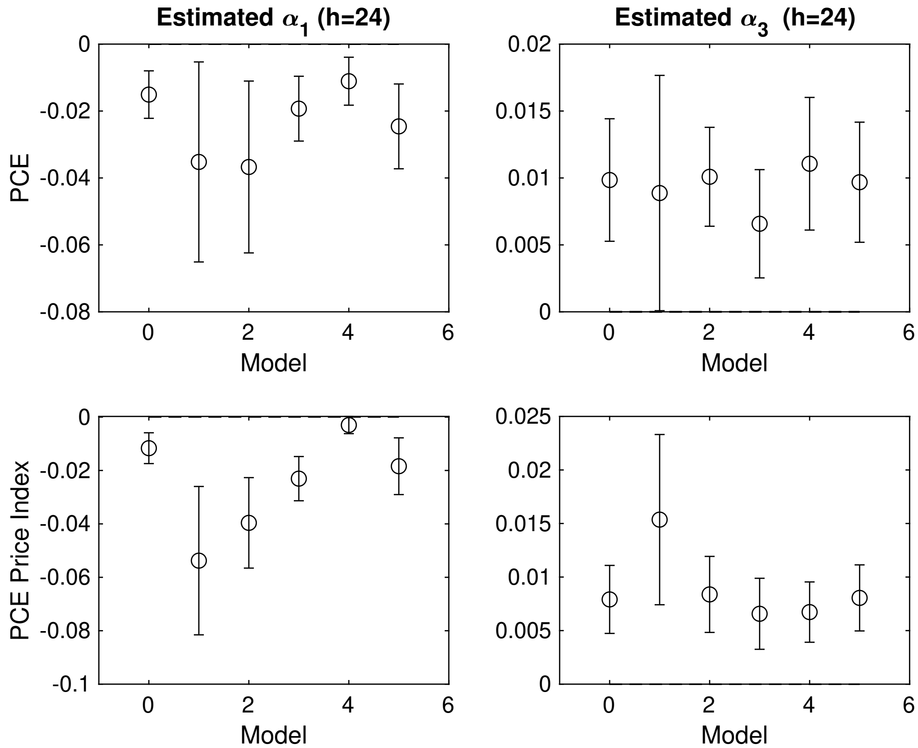

FIGURE 7. Impulse responses to a federal funds rate shock at the two-year horizon: Robustness

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index at the two-year horizon ($h = 24$) following a federal funds rate shock under alternative specifications of the local projections model. Model 0 represents the baseline specification. The other model specifications differ in the set of control variables. These include (1) the median of one-year ahead inflation expectations from the Michigan survey (model 1); (2) the consumer sentiment index from the Michigan survey (model 2); (3) the median income growth expectation of consumers from the Michigan survey (model 3); (4) consumers' perceived uncertainty concerning vehicle purchases from the Michigan survey (model 4); (5) inflation uncertainty from the SPF (model 5). The circles represent the point estimates at two-year horizon ($h = 24$) and the whiskers represent the 68% confidence bands (with Newey-West standard errors).

V. A New Keynesian model with heterogeneous beliefs

To understand the mechanism through which inflation disagreement can weaken the power of monetary policy shocks, we generalize the standard New Keynesian model to incorporate heterogeneous beliefs and borrowing constraints.

V.1. **The forward guidance puzzle in a representative-agent model.** We first illustrate the forward guidance (FG) puzzle, a challenge facing the standard representative-agent New Keynesian models. In those models, news about future real interest rates at any horizon——however distant in the future——has an equally powerful effect on current consumption as a change in the current interest rate. This implication seems implausible, and hence the FG puzzle (Del Negro et al., 2023).

To put the FG puzzle into context, consider the intertemporal Euler equation derived from the standard model with a logarithmic utility function:

$$\frac{1}{C_t} = \beta R_{ft} E_t \frac{1}{C_{t+1}} \frac{1}{\Pi_{t+1}}, \tag{4}$$

where C_t denotes real consumption in period t, R_{ft} denotes the risk-free nominal interest rate, Π_{t+1} denotes the inflation rate from t to $t+1$, $\beta \in (0,1)$ is a subjective discount factor, and E_t is a conditional expectation operator. Log-linearizing the Euler equation around the steady state and iterating forward leads to

$$\hat{C}_t = -\sum_{j=0}^{\infty} E_t(\hat{R}_{ft+j} - E_t \hat{\Pi}_{t+j+1}), \tag{5}$$

where the hatted variables denote the log-deviations the corresponding variables from their steady-state levels.

Since there is no discounting on the right-hand of Eq (5), expected policy rate changes in the future—no matter how distant it is from the present—have equally powerful effects on current consumption as does a change in the current interest rate, an implication that seems implausible.

One way to attenuate the power of forward guidance within the representative-agent framework is to introduce a time-varying discount factor (denoted by β_t) in the Euler equation. For example, consider a log-linearized Euler equation given by

$$\hat{C}_t = -\hat{\beta}_t + E_t \hat{C}_{t+1} - (\hat{R}_{ft} - E_t \hat{\Pi}_{t+1}),$$

where

$$\hat{\beta}_t \equiv \frac{1-\rho}{\rho} \hat{C}_t, \quad \rho \in (0,1). \tag{6}$$

This modification results in a "discounted Euler equation" given by

$$\hat{C}_t = \rho E_t \hat{C}_{t+1} - \rho(\hat{R}_{ft} - E_t \hat{\Pi}_{t+1}). \tag{7}$$

If $\rho \in (0,1)$, a future interest rate change has a smaller effect on current consumption than does a current interest rate change of the same magnitude.[15] This can be seen more directly

[15]The discounting of the Euler equation can arise from bounded rationality, such as myopia of agents toward future surprises in the economy (Gabaix, 2020).

by iterating Eq. (7) forward to obtain

$$\hat{C}_t = -\rho \sum_{j=0}^{\infty} E_t \rho^j (\hat{R}_{ft+j} - E_t \hat{\Pi}_{t+j+1}), \quad \rho \in (0,1).$$

So, in principle, a discounted Euler equation can resolve the forward guidance puzzle. But the question remains: What is behind the discounting of the Euler equation?

V.2. **A model with heterogeneous beliefs.** We now provide a micro-foundation for the discounted Euler equation by introducing heterogeneous beliefs about the central bank's inflation target.

Assume that the central bank follows a Taylor rule

$$R_{ft} = r^* \Pi_t^* \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\varphi} \exp(\xi_t), \qquad \varphi > 1, \tag{8}$$

where r^* denotes the natural real interest rate, Π_t^* denotes the inflation target, and ξ_t denotes a monetary policy shock. The parameter $\varphi > 1$ measures the responsiveness of the policy rate to deviations of inflation from the target.[16]

Assume that the true process of the inflation target is a random walk such that

$$\Pi_{t+1}^* = \Pi_t^* \exp(\varepsilon_{t+1}), \tag{9}$$

where ε_{t+1} is an i.i.d. random variable with a mean of $-\frac{1}{2}\sigma_r^2$ and a variance of σ_r^2. In the special case with no fluctuations in ε_{t+1} (i.e., with $\sigma_r = 0$), the inflation target would be a constant (e.g. a 2 percent annual rate).

The household family consists of a large number of members indexed by $j \in [0,1]$. Each member has her own belief about the stochastic process of the inflation target, which might be different from the true process specified in Eq. (9). We assume that member j's conditional expectation of the growth rate of the inflation target is given by

$$E_t^j \frac{\Pi_{t+1}^*}{\Pi_t^*} = e_{jt}, \quad j \in [0,1], \tag{10}$$

where E_t^j is a conditional expectations operator for member j and e_{jt} is a random variable drawn from a time-varying distribution with the cumulative density function $G_t(e)$. Absent belief heterogeneity, rational expectations would imply that $e_{jt} = 1$ for all j and t. In general, however, e_{jt} is a random variable that might differ across members and across time.[17]

[16]For analytical tractability, we do not include output gap in the Taylor rule. Putting output gap in the Taylor rule would not affect the main results, which depend mainly on households' heterogeneous beliefs about the inflation target.

[17]Since the inflation target follows a random-walk process, our assumption about the individual belief process in Eq. (10) implies that inflation disagreement is highly persistent. This model feature is consistent

In the beginning of each period t, the household receives labor income, dividends, and returns from savings. The household makes equal lump-sum transfers of the net worth to all members of the family. The family members then make individual consumption-saving decisions in decentralized markets. As we shall see, household members with higher inflation expectations will choose to consume more today, financed by both the internal net worth and external debt, subject to a borrowing constraint. By contrast, household members with lower inflation expectations will prefer to save more today and consume more in the future, and they make interior optimal choices because they do not face a binding borrowing constraint.

The expected utility function of the household family is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \log C_{jt} dj - \psi \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

where C_{jt} is consumption by agent j, and N_t is a homogeneous labor supply. The family has a beginning-of-period net worth (denoted by A_t) given by

$$A_t = \frac{\int_0^1 B_{jt} dj}{P_t} + \frac{W_t}{P_t} N_t + D_t, \tag{11}$$

where B_{jt} is value of member j's net savings (i.e., bond holdings) from period $t-1$ to t, P_t is the aggregate price level, W_t is the nominal wage rate, and D_t is the real dividend income from the firms that the household owns.

Each individual member receives a lump-sum transfer A_t from the household family. She then chooses her consumption and savings based on her own inflation expectations, subject to the flow-of-funds constraint

$$C_{jt} + \frac{B_{jt+1}/R_{ft}}{P_t} \leq A_t, \tag{12}$$

and the borrowing constraint

$$\frac{B_{jt+1}/R_{ft}}{P_t} \geq -\bar{B} \tag{13}$$

where $\bar{B}$ is an exogenous borrowing limit that cannot exceed A_t.

Denote by $w_t \equiv W_t/P_t$ the real wage rate. The optimizing labor supply decision is given by

$$\Lambda_t w_t = \psi N_t^{\gamma}, \quad where \quad \Lambda_t = \int_0^1 \Lambda_{jt} dj = \int_0^1 \frac{1}{C_{jt}} dj, \tag{14}$$

with the empirical evidence in Andrade et al. (2016), who find that the term structure of inflation disagreement is almost flat, meaning that inflation disagreement for long horizons (such as 5 to 10 years ahead) is almost as large as that for shorter horizons (such as 1 year ahead). The model feature is also consistent with the evidence in Weber et al. (2022), who find a strong positive correlation between short-term and long-term inflation expectations for households, firms, and professional forecasters. In the Michigan Survey data that we use, the correlation between the 1-year ahead inflation expectations and the 5-to-10 year ahead expectations is also positive, at 0.60.

and Λ_{jt} is the multiplier associated with the budget constraint (12) and equals the marginal utility from consumption.

The first-order condition with respect to nominal savings is given by

$$\Lambda_{jt} = \beta R_{ft} \mathbb{E}_t^j \frac{\Lambda_{t+1}}{\Pi_{t+1}} + \Omega_{jt} \quad \forall j, \tag{15}$$

where Ω_{jt} is the Lagrangian multiplier associated with the borrowing constraint (13). Define $r_{ft} = R_{ft}/\Pi_t^*$ and $\pi_t = \Pi_t/\Pi_t^*$. The Euler equation of individual j, who believes that $\frac{\Pi_{t+1}^*}{\Pi_t^*} = e_{jt}$ in expectations, can be written as

$$\Lambda_{jt} = \beta r_{ft} \mathbb{E}_t^j \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \frac{\Pi_t^*}{\Pi_{t+1}^*} \right] + \Omega_{jt} \tag{16}$$

Since aggregate normalized inflation π_{t+1} (derived from the firms' decisions) and the aggregate Lagrangian multiplier Λ_{t+1} are both uncorrelated with individual beliefs e_{jt}, we can integrate out the individual beliefs such that $\mathbb{E}_t^j \frac{\Lambda_{t+1}}{\pi_{t+1}} = \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}}$ for all j. Thus, the Euler equation (16) can be rewritten as

$$\Lambda_{jt} = \beta \frac{1}{e_{jt}} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] + \Omega_{jt}. \tag{17}$$

Denote by e_t^* the belief of the marginal agent, who is indifferent between borrowing or saving. Since the marginal agent's borrowing constraint is not binding (i.e., $\Omega_t^* = 0$), her Euler equation is given by

$$\frac{1}{\bar{C}_t} = \frac{\beta}{e_t^*} r_{ft} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \tag{18}$$

where $\bar{C}_t = A_t + \bar{B}$ is the maximum consumption attainable with internal funds A_t and external debt up to the borrowing limit $\bar{B}$.

For an agent j with a perceived inflation target higher than that of the marginal agent (i.e., with $e_{jt} > e_t^*$), her perceived real interest rate would be lower. Accordingly, she would choose to consume more, partly financed by external debt. Since the agent faces a binding borrowing constraint, her maximum amount of consumption is given by $\bar{C}_t$. For an agent who has a belief lower than that of the marginal agent (i.e., $e_{jt} \leq e_t^*$), his optimal consumption choice is not constrained by the borrowing limit, and the level of consumption would depend on his perceived inflation target relative that of the marginal agent. For those unconstrained agents, a higher perceived inflation target (relative to that of the marginal agent) implies a lower perceived real interest rate and thus a higher level of consumption.

The consumption decision rule is given by

$$C_{jt} = \begin{cases} C_t + \bar{B}, & \text{for } e_{jt} > e_t^* \\ \frac{e_{jt}}{e_t^*}(C_t + \bar{B}), & \text{for } e_{jt} \leq e_t^* \end{cases}, \tag{19}$$

where $C_t \equiv \int_0^1 C_{jt} dj$ denotes aggregate consumption.[18]

Given the consumption decision rule (19), we can express the average marginal utility Λ_t as a function of aggregate consumption and the distribution of individual beliefs. We can then rewrite the Euler equation (18) for the marginal agent as

$$1 = \beta r_{ft} \mathbb{E}_t \frac{C_t + \bar{B}}{C_{t+1} + \bar{B}} \frac{1}{\pi_{t+1}} \frac{e^*_{t+1}}{e^*_t} F(e^*_{t+1}), \tag{20}$$

where $F(e^*)$ is a function of the belief distribution and is given by

$$F(e^*) = \left[\frac{1 - G(e^*)}{e^*} + \int_{e_{\min}}^{e^*} \frac{1}{e} dG(e) \right]. \tag{21}$$

The optimal labor supply decisions imply that

$$\Lambda_t W_t = \psi N_t^{\gamma}, \qquad \text{where} \quad \Lambda_t = \frac{1}{C_t + \bar{B}} e^*_t F(e^*_t),$$

Aggregate production function is given by

$$Y_t = Z_t N_t, \tag{22}$$

where Y_t denotes aggregate output and Z_t denotes labor productivity.

As in the standard New Keynesian model, we assume that firms producing differentiated intermediate goods face monopolistic competition in the product markets and each firm sets a price for its own product, taking as given the demand schedule derived under a CES aggregation technology, and price adjustment incurs a quadratic cost in the spirit of Rotemberg (1982).[19] Firms' optimizing decisions lead to the Phillips curve relation (in log-linearized form)

$$\hat{\pi}_t = \varphi_y [\hat{w}_t - \hat{Z}_t] + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \tag{23}$$

[18] In deriving the consumption decision rule, we have used the relation $A_t = C_t$, which is obtained by aggregating the flow-of-funds constraint (12) over all consumers and imposing the bond market clearing condition that $\int_0^1 B_{jt} dj = 0$.

[19] We drive the Phillips curve relation in Appendix B.2. For analytical tractability, we assume that firms are fully rational and do not disagree about the future inflation target. In reality, of course, firms may also disagree about their inflation expectations. For example, the Survey of Firms' Inflation Expectations conducted by the Cleveland Fed shows that firms' one-year ahead inflation expectations have a mean of 5% and a cross-sectional standard deviation of 1.6% in the second quarter of 2023. In comparison, in the same quarter, consumers' one-year ahead inflation expectations from the Michigan survey have a mean of about 6% and a standard deviation of about 9.6%. This observation suggests that inflation disagreement among firms, while substantive, may not be as pervasive as that among consumers. In our view, allowing for firms' inflation disagreement would make the model more realistic and would likely introduce additional frictions in the Phillips curve. However, it would unlikely alter our model's mechanism through which belief heterogeneity on the consumer side leads to a discounted Euler equation and thus attenuating the power of forward guidance.

where a hatted variable denotes the log-deviations of the variable from its steady-state value and the slope parameter φ_y is a function of the fundamental parameters including the relative risk aversion, the Frish elasticity of labor supply, the elasticity of substitution between differentiated products, and the size of price adjustment costs.

The log-linearized monetary policy rule in equation (8) implies that

$$\hat{r}_{ft} = \varphi\hat{\pi}_t + \xi_t \tag{24}$$

In an equilibrium, the markets for final goods, bonds, and labor all clear. Final goods market clearing implies that

$$C_t + \frac{\chi_p}{2}\left[\pi_t - 1\right]^2 Y_t = Y_t, \tag{25}$$

where χ_p measures the size of the price adjustment costs. Bond market clearing implies that

$$\int_0^1 B_{jt} dj = 0. \tag{26}$$

An equilibrium consists of allocations $\{C_t, N_t, Y_t\}$ and prices $\{w_t, r_{ft}, \pi_t\}$ such that (i) taking all prices as given, the allocations solve the households' utility maximization problem; (ii) taking all prices but its own as given, the allocations and each firm's price solve its profit maximizing problems; (iii) final goods market, bond market, and labor market all clear.

V.3. **Analytical results from the model with heterogeneous beliefs.** Log-linearizing the Euler equation (20) around the deterministic steady state, we obtain

$$\hat{C}_t = \underbrace{\frac{\mu + (1-\theta)\kappa}{\mu+\kappa}}_{\equiv\beta_1} \mathbb{E}_t\hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv\beta_2} \left(\hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1}\right), \tag{27}$$

where a hatted variable denotes the log-deviations of that variable from its steady-state value. The parameter $\kappa \equiv \frac{\bar{B}}{A} = \frac{\bar{B}}{C} \in (0,1)$ denotes the steady-state loan-to-value ratio; the parameter $\theta \equiv -\frac{F'(e^*)e^*}{F(e^*)} \in [0,1)$ denotes the inverse elasticity of $F(\cdot)$ with respect to the cutoff belief e^*, evaluated at the steady state equilibrium; and $\mu \in (0,1]$ denotes the inverse elasticity of the leverage ratio $\frac{C_t}{C_t+\bar{B}}$ with respect to the cutoff belief e^*, also evaluated at the steady state.[20]

In the special case with $\theta = 0$ and $\mu = 1$, the model nests the standard Euler equation in the representative-agent models, such that $\beta_1 = \beta_2 = 1$, and there is no "discounting" of the Euler equation. In the more general case with heterogeneous beliefs about the central bank's inflation target, we have $\theta \in (0,1)$ and $\mu \in (0,1)$. In this case, the Euler equation would be discounted in the sense that $\beta_1 < 1$, such that aggregate consumption in the current period changes less than one-for-one with expected future consumption. This result is formally stated in Proposition V.3 below.

[20]We derive the log-linearized Euler equation (20) in Appendix B.1.

Proposition V.1. *(Euler-equation discounting) With belief heterogeneity, aggregate current consumption responds less than one-for-one to changes in future consumption.*

Proof. In Appendix B.1, we show that $\theta \in (0,1)$, $\mu \in (0,1)$, and $\kappa \in (0,1)$ with heterogeneous beliefs. It follows immediately that

$$\beta_1 \equiv \frac{\mu + (1-\theta)\kappa}{\mu + \kappa} = 1 - \frac{\theta\kappa}{\mu + k} \in (0,1). \tag{28}$$

□

Recall that in the standard New Keynesian framework *a la* section V.1, the intertemporal discount factor in linearized equation (5) equals 1. In our framework with heterogeneous expectation about future inflation, by contrast, the coefficient β_1 is less than 1.

To obtain sharper characterizations of how the magnitude of Euler equation discounting (i.e., the size of β_1) depends on the dispersions in inflation beliefs, we assume that the households' idiosyncratic beliefs follow a Pareto distribution with the cumulative density function (cdf)

$$G(e) = \begin{cases} 1 - (\frac{e_{min}}{e})^\alpha & \text{if } e \geq e_{min} \\ 0 & \text{if } e < e_{min} \end{cases} \tag{29}$$

We fix the scale parameter at $e_{min} \equiv \frac{\alpha-1}{\alpha}$ such that the mean stays constant at $E(e) = 1$, implying that the agents' expectations on average are rational. The shape parameter α measures the thickness of the tail, with a smaller α corresponding to more dispersed beliefs.

Proposition V.2. *Assuming that individual beliefs of the inflation target follow the Pareto distribution* (29)*. Then, β_1 increases with α, implying that more dispersed beliefs lead to greater Euler equation discounting.*

Proof. In Appendix B.3, we prove in Lemma B.2 and B.3 that θ decreases with α and that μ increases with α. It follows that $\beta_1 \equiv 1 - \frac{\theta\kappa}{\mu+k}$ increases with α. □

Heterogeneous beliefs about the inflation target and the resulting inflation disagreement can also weaken the elasticity of aggregate consumption to the contemporaneous real interest rate (i.e., $\beta_2 < 1$). Furthermore, more dispersed beliefs lead to smaller responses of aggregate consumption to changes in the real interest rate (i.e., β_2 increases with α). These results are formally established in Proposition V.3 below.

Proposition V.3. *Assuming that individual beliefs of the inflation target follow the Pareto distribution* (29)*. Then, β_2 increases with α, implying that an increase in the mean-preserving dispersion in beliefs lead to more muted responses of aggregate consumption to changes in the contemporaneous real interest rate.*

Proof. The parameter β_2 is given by

$$\beta_2 = \frac{(1+\kappa)(\mu+\kappa-\kappa)}{\mu+\kappa} = 1+\kappa - \frac{(1+\kappa)\kappa}{\mu+\kappa}, \qquad \kappa \in (0,1).$$

In Appendix B.3 we prove that μ increases with α. It immediately follows that β_2 also increases with α. □

Proposition V.3 also implies that inflation disagreement weakens the response of real activity to other demand shocks, such as a shock to the natural real interest rate.[21]

Finally, we evaluate the effectiveness of monetary policy in stabilizing inflation. We show that an increase in inflation disagreement reduces the sensitivity of inflation to changes in the output gap. Or equivalently, inflation disagreement flattens the Phillips curve. This result is formally stated in Proposition V.4.

Proposition V.4. *The effectiveness of contemporaneous monetary policy shocks on inflation decreases with inflation disagreement.*

Proof. Using Equations (23), (24), and (27), we can derive the following inflation response to a monetary policy shock (assuming that $\hat{Z}_t = 0$)

$$\begin{aligned}
\hat{\pi}_t &= \varphi_y \hat{w}_t + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu} \hat{C}_t + \gamma \hat{N}_t \right] + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[\frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu} \hat{C}_t + \gamma \hat{C}_t \right] + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[(\gamma + \frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu})[\beta_1 E_t \hat{C}_{t+1} - \beta_2(\hat{r}_{ft} - \beta E_t \hat{\pi}_{t+1})] \right] + \beta E_t \hat{\pi}_{t+1} \\
&= \varphi_y \left[(\gamma + \frac{\mu+\kappa(1-\theta)}{(1+\kappa)\,\mu})[\beta_1 E_t \hat{C}_{t+1} - \beta_2(\varphi \hat{\pi}_t + \xi_t - \beta E_t \hat{\pi}_{t+1})] \right] + \beta E_t \hat{\pi}_{t+1} \\
&\equiv \varphi_y \left[\beta_3 [\beta_1 E_t \hat{C}_{t+1} - \beta_2(\varphi \hat{\pi}_t + \xi_t - \beta E_t \hat{\pi}_{t+1})] \right] + \beta E_t \hat{\pi}_{t+1} \qquad (31)
\end{aligned}$$

where $\beta_3 \equiv \gamma + \frac{\mu+\kappa(1-\theta)}{(1+\kappa)\mu}$. Thus, we can write

$$\hat{\pi}_t = -\frac{\varphi_y \beta_3 \beta_2}{(1+\varphi_y \beta_3 \beta_2 \varphi)} \xi_t + \frac{\varphi_y \beta_3 \beta_1}{(1+\varphi_y \beta_3 \beta_2 \varphi)} E_t \hat{C}_{t+1} + \frac{\beta(1+\varphi_y \beta_3 \beta_2)}{(1+\varphi_y \beta_3 \beta_2 \varphi)} E_t \hat{\pi}_{t+1}. \qquad (32)$$

[21] Introducing stochastic natural real interest rate (denoted as r_t^n) into the model obtains

$$\hat{C}_t = \underbrace{\frac{\mu+(1-\theta)\kappa}{\mu+\kappa}}_{\equiv \beta_1} \mathbb{E}_t \hat{C}_{t+1} - \underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv \beta_2} (\hat{r}_{ft} - \mathbf{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n) \qquad (30)$$

where $\hat{r}_t^n$ denotes deviation of natural real interest rate from steady state. According to Prop. V.3 that β_2 is an increasing function of α, it is implied that higher inflation disagreement weakens the effects of shocks to r_t^n.

It's sufficient to prove that $\frac{\varphi_y \beta_3 \beta_2}{1+\varphi_y \beta_3 \beta_2 \varphi}$ increases with α, or that $\beta_3\beta_2$ increases with α.

$$\beta_3\beta_2 = \gamma\beta_2 + \frac{\mu + \kappa(1-\theta)}{\mu+\kappa} = \gamma\beta_2 + 1 - \frac{\kappa\theta}{\mu+\kappa}$$

We have proved that β_2 increases with α (Prop. V.3), that θ is a decreasing function of α (Lemma B.2), and that μ is an increasing function of α (Lemma B.3). It's immediate then that $\beta_3\beta_2$ increases with α. □

Overall, our model predicts that inflation disagreement attenuates the power of both forward guidance and conventional monetary policy. These predictions are consistent with our empirical evidence.

VI. Supporting evidence for the model mechanism

The model mechanism relies on the interactions between inflation disagreement and borrowing constraints. Borrowing constraints introduce an asymmetry in the responses of agents with different inflation beliefs. An agent who has higher perceived future inflation also has a lower perceived real interest rate; as such, the agent has a high MPC and is thus more likely to face binding borrowing constraints. In contrast, an agent who has a lower perceived future inflation is unconstrained. Such asymmetric behaviors stemming from the interactions between belief heterogeneity and borrowing constraints implies that the skewness of inflation expectations is also an important driver of aggregate consumption responses to monetary policy shocks. We now present some evidence that supports the model mechanism.

VI.1. **The role of positive skewness of inflation expectations.** According to our theory, consumption of the households with higher inflation expectations is less sensitive to changes in the real interest rate, because they face binding borrowing constraints. Thus, the model implies that the attenuation effects of inflation disagreement on the power of monetary policy should be stronger with a more positively-skewed distribution of inflation expectations.

To test this model implication, we modify the baseline empirical specification (1) by including a measure of positive skewness of inflation expectations (denoted by $Skew^{\pi}_{t-1}$) and its interactions with the FG shocks as two additional explanatory variables. The modified empirical specification is given by

$$\begin{aligned}\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^{\pi}_{t-1} + \alpha^h_3 IQR^{\pi}_{t-1} * FG_t \\ + \alpha^h_4 Skew^{\pi}_{t-1} + \alpha^h_5 Skew^{\pi}_{t-1} * FG_t + \alpha^h_6 \Gamma_{t-1} + \varepsilon_{t+h} \qquad (33)\end{aligned}$$

where we measure the positive skewness of inflation expectations by the difference between the upper quartile (the 75^{th} percentile minus the median) and the lower quartile (the median minus the 25^{th} percentile) of the one-year ahead inflation forecast distribution in the Michigan Survey, and we scale the measure of skewness by the IQR of the inflation forecasts. We also estimate a similar specification with the FG shock replaced by a federal funds rate shock (MP).

The estimated results shown in Figures 8 and 9 are consistent with our model's implications. The figures show that inflation disagreement weakens the power of both forward guidance and conventional policy ($\alpha_3 > 0$), and the attenuation effects are stronger with a more positively skewed distribution of inflation forecasts ($\alpha_5 > 0$). These effects are statistically significant and economically important.

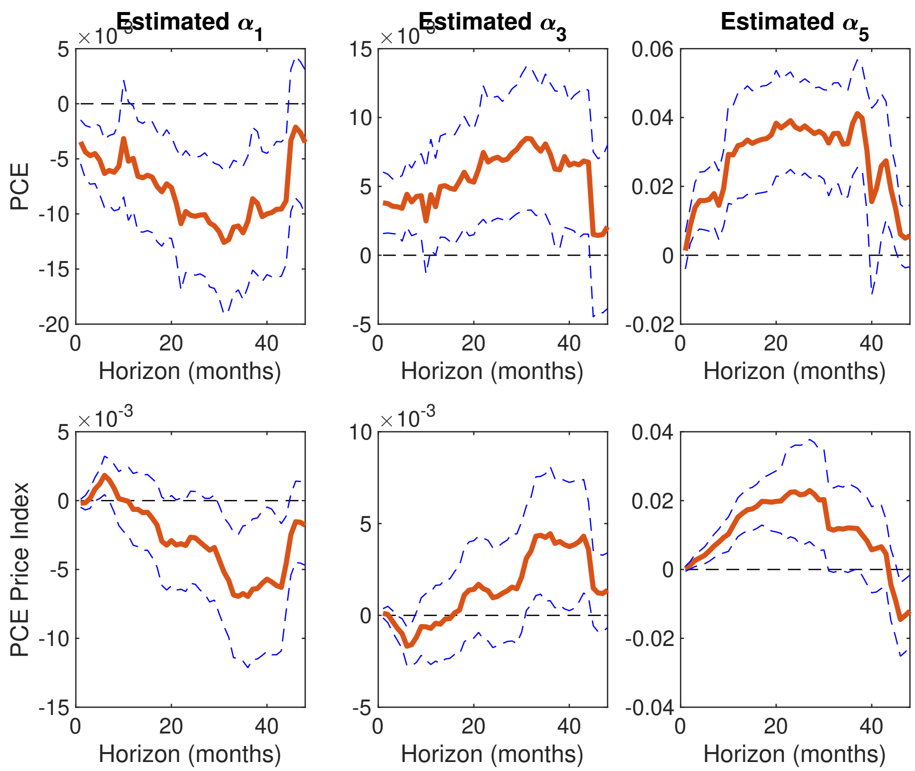

FIGURE 8. Estimated response to a forward guidance shock: effects of positive skewness of the inflation forecast distribution.

Note: This figure shows estimated cumulative responses of monthly real personal consumption expenditure (PCE) and the PCE price index to identified forward guidance shock from the local projections model (33). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

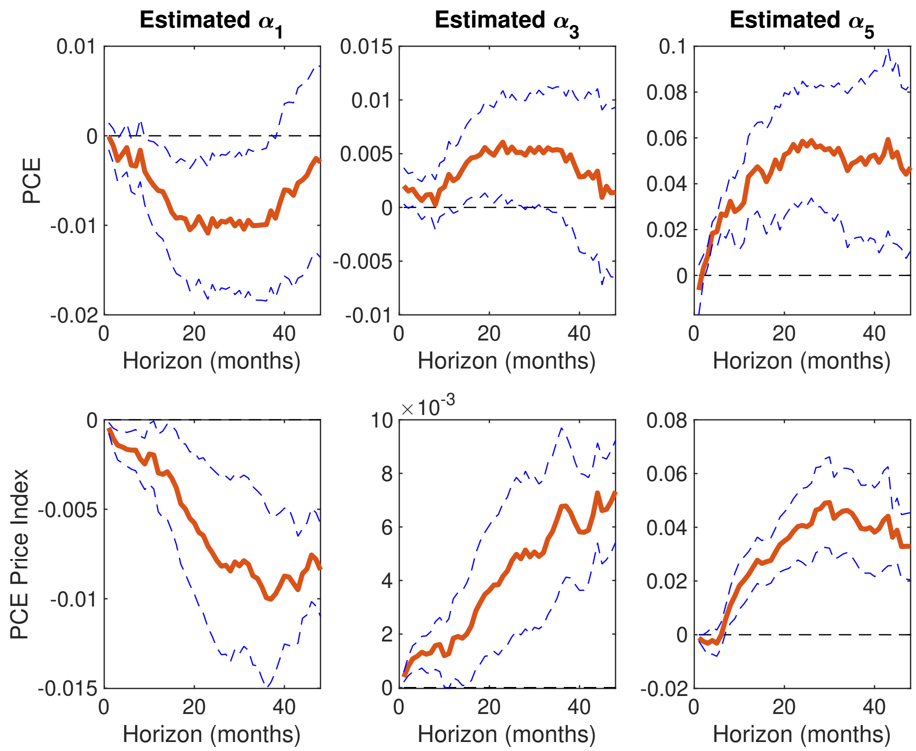

FIGURE 9. Estimated response to a federal funds rate shock: effects of positive skewness of the inflation forecast distribution.

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to federal fund rate shocks from the local projections model (33) (where FG is replaced by MP). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

VI.2. **The importance of credit access.** Our model shows that the attenuation effects of inflation disagreement depend crucially on households' credit access. When agents have easier access to credit, inflation disagreement can still attenuate the power of monetary policy shocks, but the attenuation effects would be weaker. We now provide some empirical evidence supporting this model mechanism.

To implement this idea, we estimate a variation of our baseline local projection specification by including a tipple interaction term:

$$\begin{aligned} \log(y^j_{t+h}) - \log(y^j_{t-1}) &= \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^{\pi}_{t-1} + \alpha^h_3 IQR^{\pi}_{t-1} * FG_t + \alpha^h_4 LoanStd_{t-1} * FG_t \\ &+ \alpha^h_5 IQR^{\pi}_{t-1} * LoanStd_{t-1} * FG_t + \alpha^h_6 \Gamma_{t-1} + \varepsilon_{t+h}. \end{aligned} \quad (34)$$

Here, the variable *LoanStd* denotes the lending standards measured by the net percentage of domestic banks reporting increased willingness to make consumer installment loans, with data obtained from the Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices (SLOOS).[22] A higher value of *LoanStd* indicates more favorable lending standards; and according to our theory, it should mitigate the attenuation effects of inflation disagreement (i.e. $\alpha_5 < 0$).[23] We also estimate a similar specification with the FG shock replaced by a federal funds rate shock (MP).

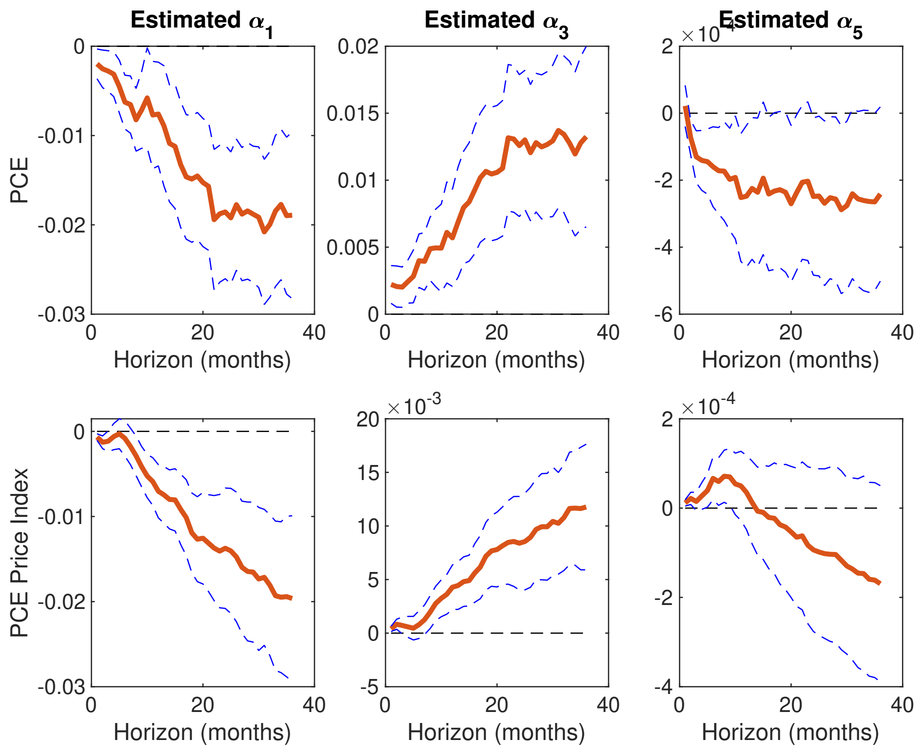

FIGURE 10. Estimated response to a forward guidance shock: effects of lending standard

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to forward guidance shocks from the local projections model (34). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

[22]The time series of lending standards is available from FRED (series ID: DRIWCIL) at the quarterly frequency from the second quarter of 1982. Since the survey is conducted at quarterly frequency, we interpolate the data into monthly, assuming that the lending standards remain the same within each quarter.

[23]We include $LoanStd_{t-1}$ in the set of macroeconomic controls (Γ_{t-1}).

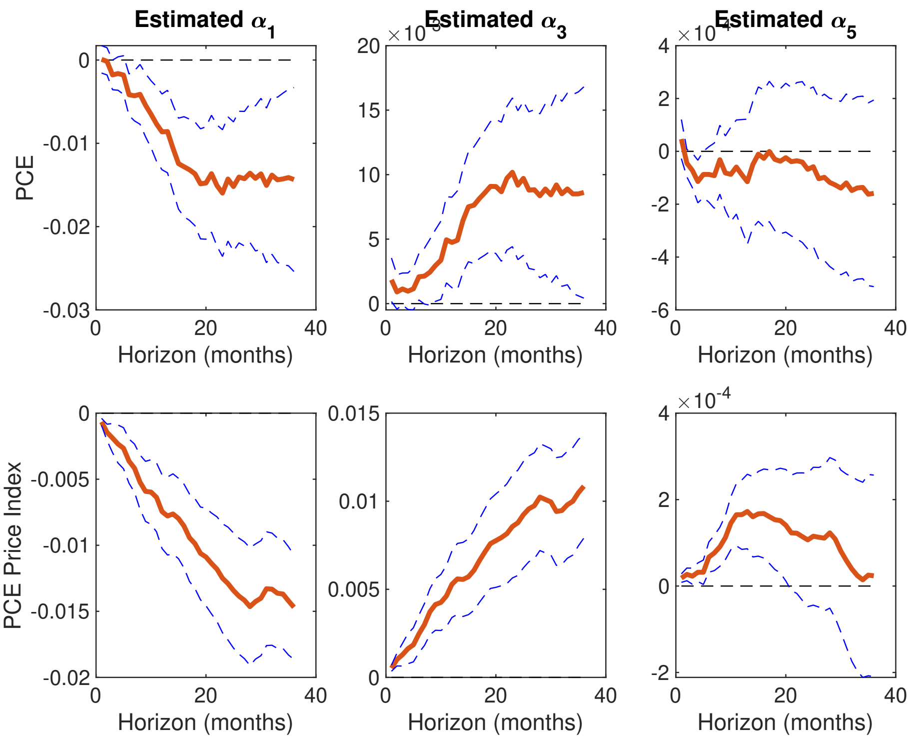

FIGURE 11. Estimated response to a federal fund rate shock: effects of lending standard

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to federal fund rate shocks from the local projections model (34) (where FG is replaced by MP). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

The estimated results shown in Figures 10 and 11 are consistent with our model's mechanism. The figures show that inflation disagreement attenuates the power of both forward guidance and conventional policy ($\alpha_3 > 0$), but the attenuation effects are mitigated by more favorable lending conditions to consumers ($\alpha_5 < 0$). These effects are statistically significant and economically important.[24]

[24]For robustness, we present results in the appendix using an alternative measure of lending standards from the SLOOS, with the lending standards measured by the net percentage of domestic banks tightening standards on household loans, weighted by banks' outstanding loan balances by category. Consistent with our theory, we find that the attenuation effects of inflation disagreement are amplified by the tightening of lending standard on household loans.

VII. Conclusion

Survey data shows pervasive and time-varying disagreement among consumers in their inflation expectations. We provide empirical evidence that inflation disagreement weakens the power of both forward guidance and conventional monetary policy. Absent inflation disagreement, a forward guidance shock that signals future policy tightening would lead to persistent and significant declines in consumer spending and inflation. However, in periods with high inflation disagreement, the recessionary effects of such a forward guidance shock would be significantly attenuated. We find similar attenuating effects of inflation disagreement for conventional monetary policy shocks (such as a surprise increase in the fed funds rate). These empirical findings are robust to using alternative measures of real activity and inflation, purging inflation disagreement of demographic factors and common sources of shocks, and considering other potential drivers of consumer spending.

We also provide a theoretical framework for understanding the mechanism through which inflation disagreement can influence the transmission of monetary policy. The model shows that households' heterogeneous beliefs about the central bank's inflation target, along with occasionally binding borrowing constraints, dampen the spending and inflation response to macroeconomic shocks and attenuate the power of both forward guidance and conventional monetary policy. This model also provides a microeconomic foundation for a discounted Euler equation that solves the forward guidance puzzle.

Our model has important policy implications. For example, in response to the post-pandemic surge in inflation, the Federal Reserve aggressively tightened monetary policy by rapidly raising the federal funds rate target from near zero to a range between 5.25 percent and 5.5 percent. In addition, the Fed signaled that policy would remain tight until substantial progress has been made toward the 2 percent inflation goal. Despite these policy actions, consumer spending remained resilient and inflation remained persistently above 2 percent. Our findings suggest a potential resolution to this apparent puzzle: inflation disagreement increased and remained elevated during much of the post-pandemic period, reducing the impact of monetary policy.

To maintain analytical tractability, we have intentionally kept the model simple by abstracting from many realistic features of the economy. For example, the agents in our model are extremely naive, with their beliefs following an i.i.d. distribution. In a more realistic environment with persistent beliefs, agents could learn from their past mistakes, and such learning may have important consequences for the transmission of monetary policy. Our model also abstracts from other plausible sources of heterogeneity, such as heterogeneity in income or wealth that may give rise to precautionary savings, a crucial feature of the HANK models that helps alleviate the forward guidance puzzle (McKay et al., 2016). We conjecture that incorporating those realistic features into our model would provide a richer framework for studying the quantitative importance of belief heterogeneity in explaining the empirical observations. It would also make the framework more useful for evaluating alternative policies, such as the role of fiscal and monetary policy coordination in stabilizing inflation and macroeconomic fluctuations. Our work represents a first step toward that promising avenue for future research.

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Appendix A. Robustness of empirical results

A.1. **Alternative measures of inflation disagreement.** Our baseline empirical results are robust to alternative measures of inflation disagreement. We re-estimate the baseline local projections specification (1) using either a purified measure or an othogonalized measure of inflation disagreement (see the main text for a description of how those measures are constructed).

Figure A.1 shows that, following a forward guidance shock that signals future tightening of policy, real PCE and inflation both falls on average. However, in periods with high inflation disagreement, the declines in real consumption and inflation are both significantly mitigated. These patterns are similar—both qualitatively and quantitatively—to those obtained using the baseline measure of inflation disagreement shown in Figure 3.

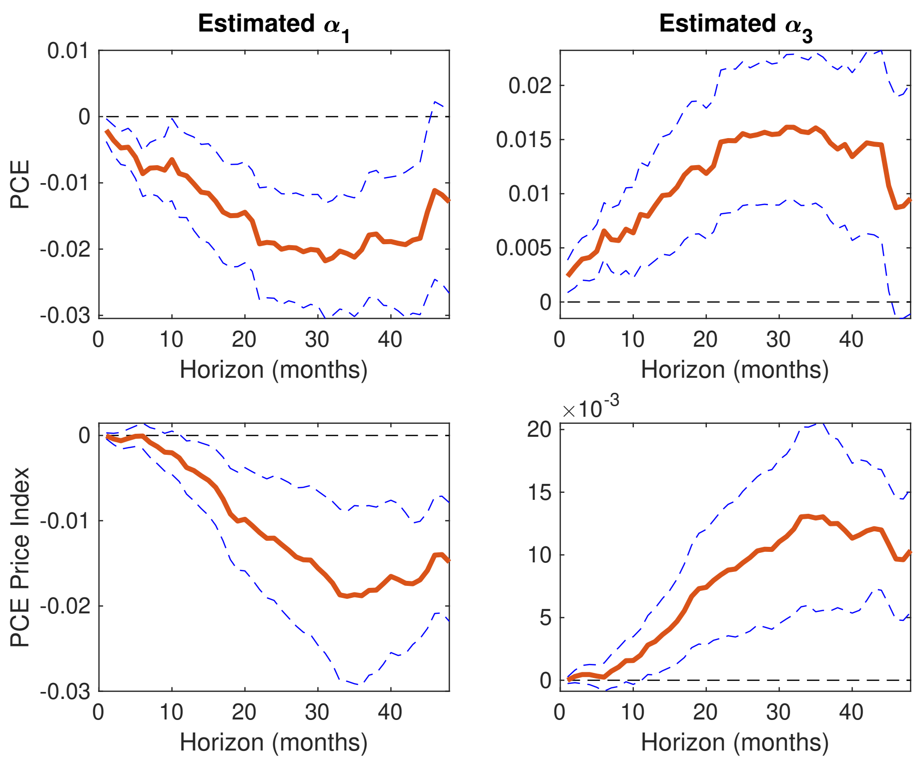

Figure A.1. Estimated impulse response to a forward guidance shock (purified measure of inflation disagreement)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using the purified inflation disagreement measure (i.e., the IQR of ε_{jt} in Eq. (3) across individuals for each month). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

Figure A.2 shows that the impulse responses of real PCE and inflation are also similar to those obtained using the raw measure of inflation disagreement.

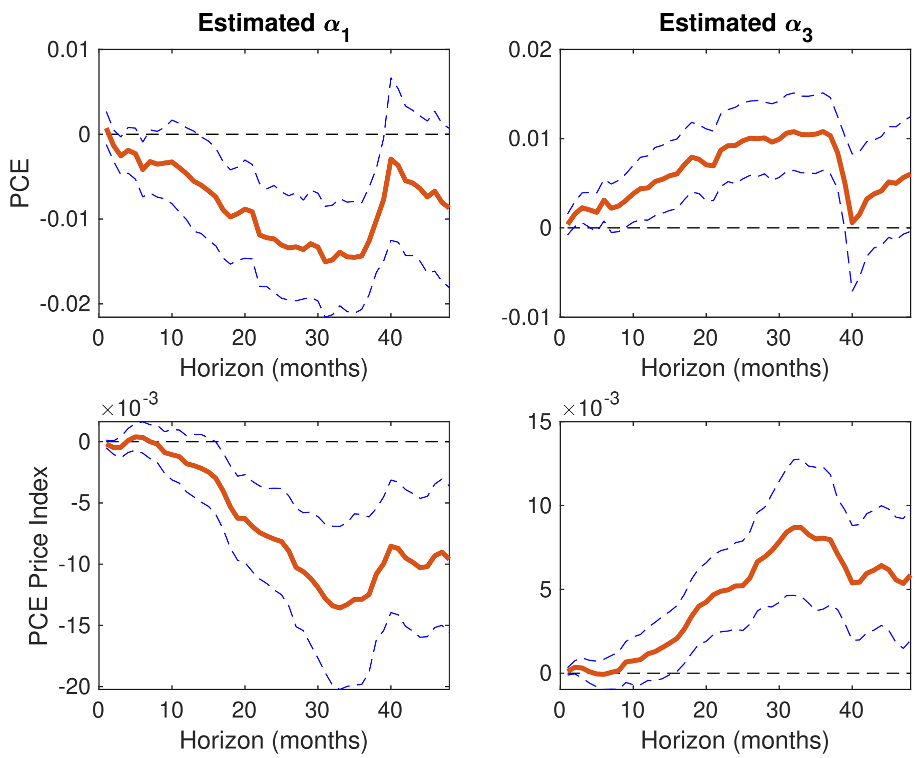

FIGURE A.2. Estimated impulse response to forward guidance shocks (orthogonalized measure of inflation disagreement)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using the orthogonalized inflation disagreement measure based on the residuals from the regression of the purified measure (i.e., the IQR of ε_{jt} in Eq. (3)) on current and lagged values of monetary policy surprises and oil supply shocks. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

A.2. **Disagreement about long-term inflation.** Figures A.3 and A.4 show the impulse responses to a forward guidance shock and a federal funds rate shock, respectively, from the estimated local projections with inflation disagreement measured by the IQR of 5-10 year ahead inflation forecasts in the Michigan survey (normalized by the median of the 5-10 year ahead inflation expectations). Evidently, the attenuation effects of long-term inflation disagreement are similar to those of the short-term disagreement in our baseline empirical model.

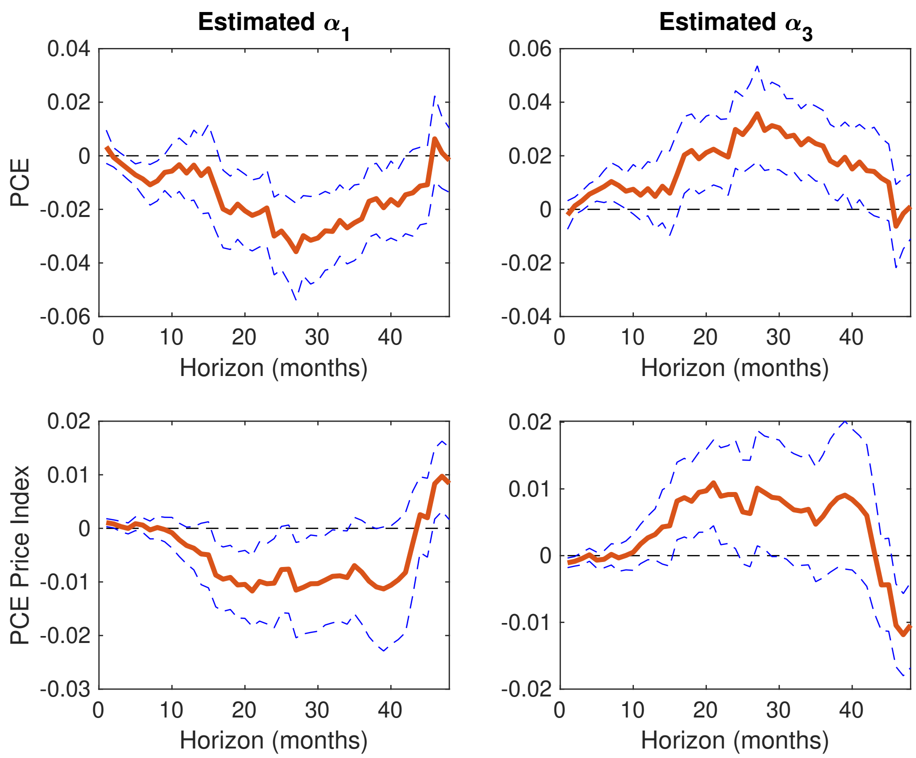

FIGURE A.3. Estimated impulse response to a forward guidance shock (disagreement about long-term inflation)

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using disagreement measure based on 5-10 year ahead inflation forecast. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

A.3. **Alternative measures of real activity and inflation.** We replace the LHS of Eq. (1) by cumulative growth rates of unemployment, industrial production and CPI. Figure A.5 shows the impulse responses of monthly unemployment rate (upper panel), industrial production (mid panel) and consumer price index (lower panel) to a forward guidance shock.

The upper panel of Figure A.5 shows that an identified forward guidance shock is followed by a rise in the unemployment rate ($\alpha_1^h > 0$), but the effect is mitigated if the current state is characterized by high inflation disagreement ($\alpha_3^h < 0$). Similar results are obtained from regressions for industrial production (mid panel), indicating that a positive forward guidance shock predicts a decline in output ($\alpha_1^h < 0$), but the effect is again mitigated in states with high inflation disagreement ($\alpha_3^h > 0$). For example, a one-standard-deviation higher inflation forecast disagreement will reduce the effects of forward guidance shocks on

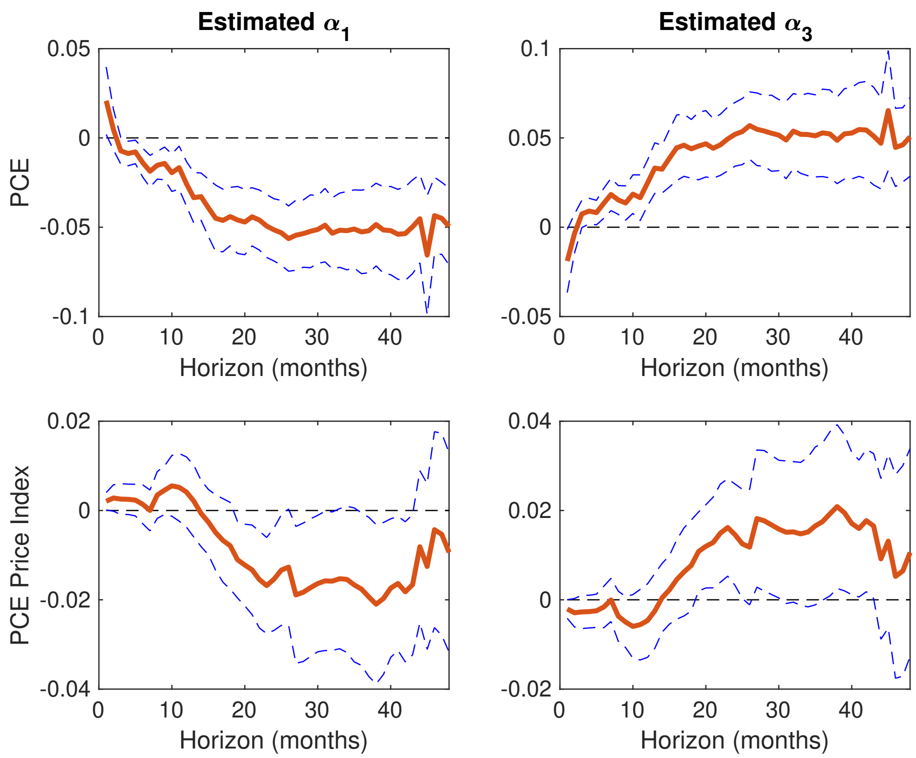

FIGURE A.4. Estimated impulse response to federal fund rate shocks (disagreement about long-term inflation))

Note: This figure shows the cumulative responses of real personal consumption expenditure (PCE) and the PCE price index following a forward guidance shock using disagreement measure based on 5-10 year ahead inflation forecast. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

2-year ahead unemployment and industrial production by 34.7% and 34.2% respectively. Forward guidance policy is also less effective in stabilizing inflation when it is carried out during times of high disagreement in inflation expectations *a la* Prop. V.4. The bottom panel of Figure A.5 supports this prediction: news about future monetary tightening helps stabilize the price level ($\alpha_1^h < 0$), but the effect is mitigated if the current economy features high inflation disagreement (i.e. $\alpha_3^h > 0$).

Similarly, we replace the LHS of Eq. (2) by cumulative growth rate of unemployment, industrial production and CPI, and estimate their responses to federal fund rate shocks.

The upper panel of Figure A.6 shows that a positive policy rate shock predicts a rise in the unemployment rate ($\alpha_1^h > 0$), but the effect is mitigated in a state with high inflation disagreement ($\alpha_3^h < 0$). Similar results are obtained from regression on output (middle

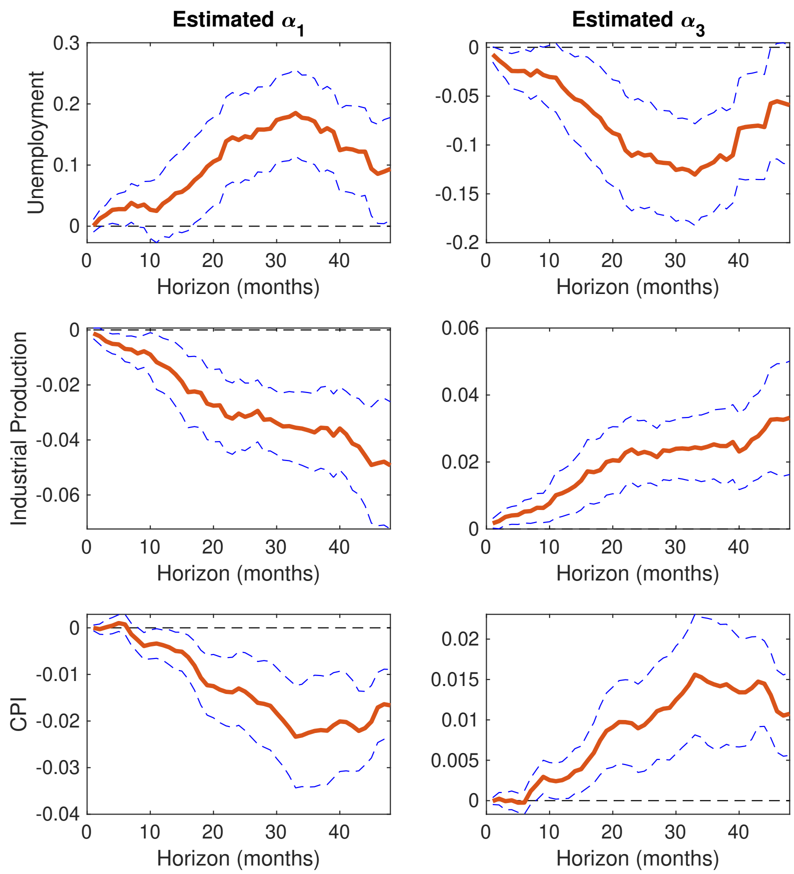

FIGURE A.5. Estimated response to forward guidance shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production and consumer price index (CPI) to forward guidance shock from the local projections model (1). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

panel): a positive policy rate shock predicts a decline in industrial production ($\alpha_1^h < 0$), but the effect is mitigated in state with high inflation disagreement ($\alpha_3^h > 0$). The attenuating effects on stabilizing inflation (lower panel) are also consistent with previous results on PCE price level.

A.4. **Additional control variables.** According to standard Euler equation, the household's consumption-saving decisions are affected by many factors, including nominal saving rate, expected inflation, discount factor, and expected income change etc. While this paper focuses on the effects from inflation disagreement on aggregate consumption and its response

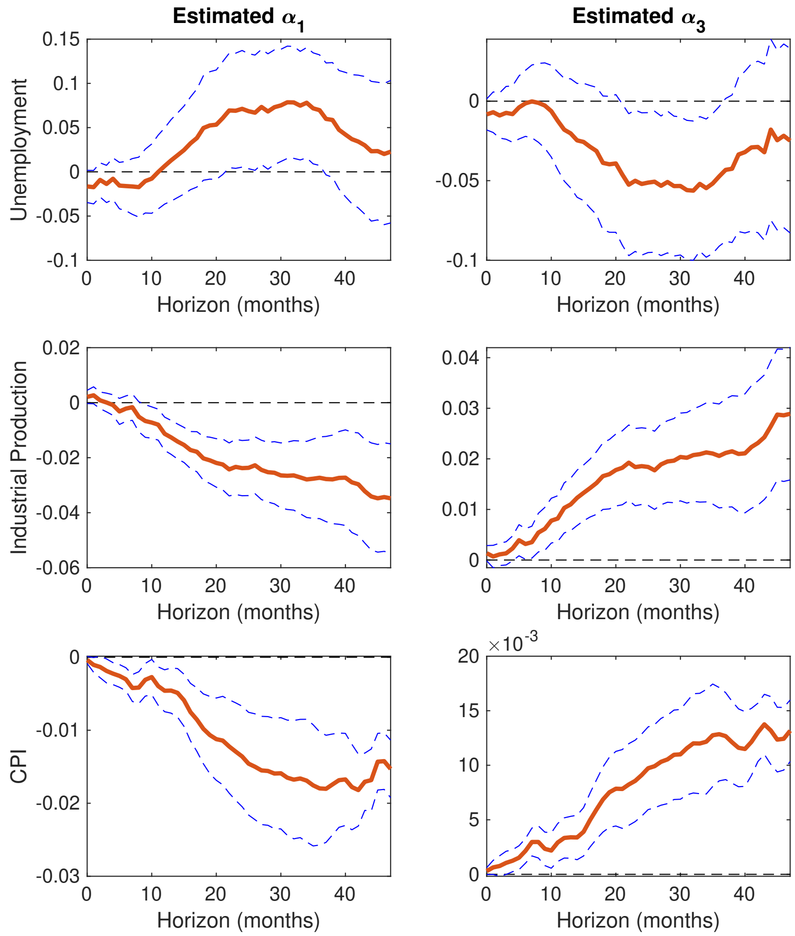

Figure A.6. Estimated response to federal fund rate shocks

Note: This figure shows estimated impulse responses of monthly unemployment rate, industrial production, and CPI inflation to federal fund rate shock from the local projections model (2). The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

to monetary policy shocks, the expectation and disagreement about other factors could confound the attenuating effect of inflation disagreement. For example, the households with high income expectation may increase current spending to smooth consumption. Leduc and Liu (2016) shows a negative effect of consumer uncertainty on aggregate consumption and price level due to a real option-value channel.

To test the robustness of our empirical results, we include additional control variables in the baseline specification (1). In particular, we consider the empirical specification

$$\log(y^j_{t+h}) - \log(y^j_{t-1}) = \alpha^h_0 + \alpha^h_1 FG_t + \alpha^h_2 IQR^\pi_{t-1} + \alpha^h_3 IQR^\pi_{t-1} * FG_t + \alpha^h_4 X_{t-1} + \alpha^h_5 X_{t-1} * FG_t + \alpha^h_6 \Gamma_{t-1} + \varepsilon_{t+h}, \tag{A.1}$$

where X_t denotes the additional control variable. The set of those control variables that we consider include (1) the median of one-year ahead inflation expectations in the Michigan survey; (2) the consumer sentiment index from the Michigan survey; (3) the median income growth expectation in the Michigan survey; (4) consumers' perceived uncertainty concerning vehicle purchases from the Michigan survey;[1]; and (5) an updated measure of inflation forecast uncertainty from Breach et al. (2020), constructed using the density forecasts from the Survey of Professional Forecasters (SPF).[2]. Those 5 alternative model specifications are reported in the main text. In addition, we consider 3 measures of disagreement from the Blue Chip as additional controls, including the forecast dispersion in real GDP growth, in the federal funds rate path, and in the two-year Treasury yields. We include an additional control variable and its interaction with the policy shock one at a time.

The impulse responses to a forward guidance shock in these 8 alternative specifications are shown in Figures A.7 through A.14 The impulse responses to a federal funds rate shock in these alternative specifications are shown in Figures A.15 through A.22.

In each case, we find that inflation disagreement attenuates the power of forward guidance shocks and the fed funds rate shocks.

A.5. **Alternative measures of liquidity constraint.** To test the robustness of empirical support for our model mechanism, we use an alternative measure of lending standard from the same survey based on the net percentage of domestic banks tightening standards on household loans, weighted by banks' outstanding loan balances by category. [3] We estimate

[1]We measure consumer uncertainty based on Michigan Survey of Consumers following Leduc and Liu (2016). One question in the survey asks for reason why consumer thinks it is a good or bad time to buy a vehicle. The survey tallies the fraction of respondents who report that "uncertain future" is a reason, which we use to measure consumer uncertainty.

[2]The SPF by the Federal Reserve Bank of Philadelphia is a quarterly survey of macroeconomic forecasts. We interpolate the measure of inflation uncertainty into monthly, assuming that inflation uncertainty remains the same within each quarter. We also estimate a similar specification of Eq. (A.1) at quarterly frequency, replacing inflation disagreement measure from Michigan Survey by that from SPF, and obtain consistent results (not reported).

[3]The series is available at quarterly frequency from the third quarter of 1991 from FRED (series ID: SUBLPDMHSXWBNQ). Again we interpolate the data into monthly, assuming the lending standards remain the same within each quarter.

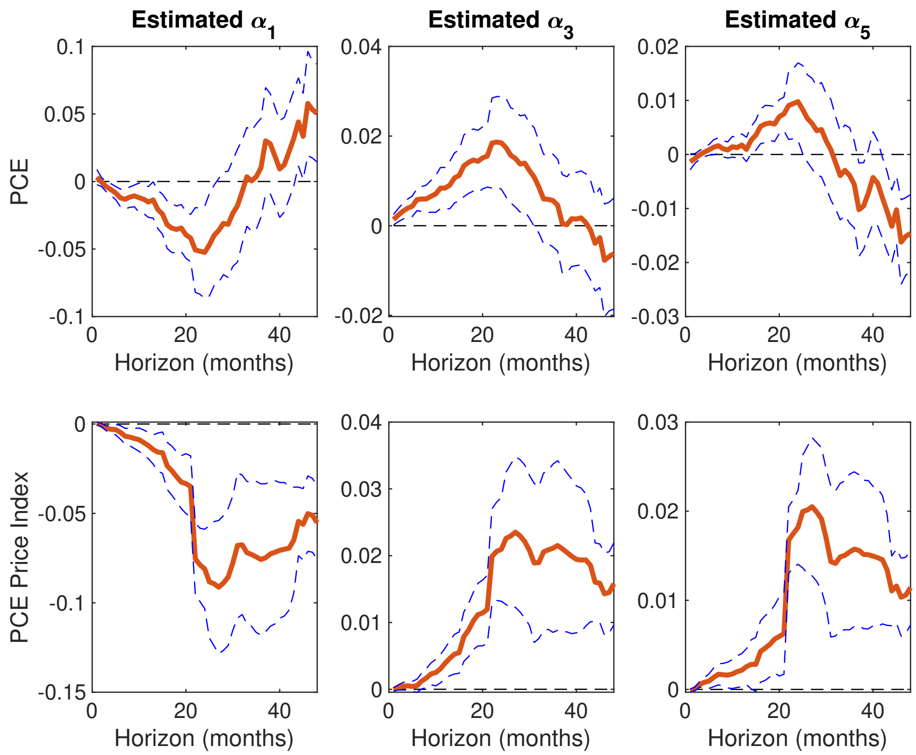

FIGURE A.7. Estimated response to forward guidance shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where we include the median inflation expectation and its interactions with the forward guidance shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

a similar specification of Eq. (34) with *LoanStd* replaced by this measure. Consistent with our theory, Figure A.23 and A.23 show that the attenuation effects of inflation disagreement are amplified by the tightening of lending standard on household loans ($\alpha_5 > 0$).

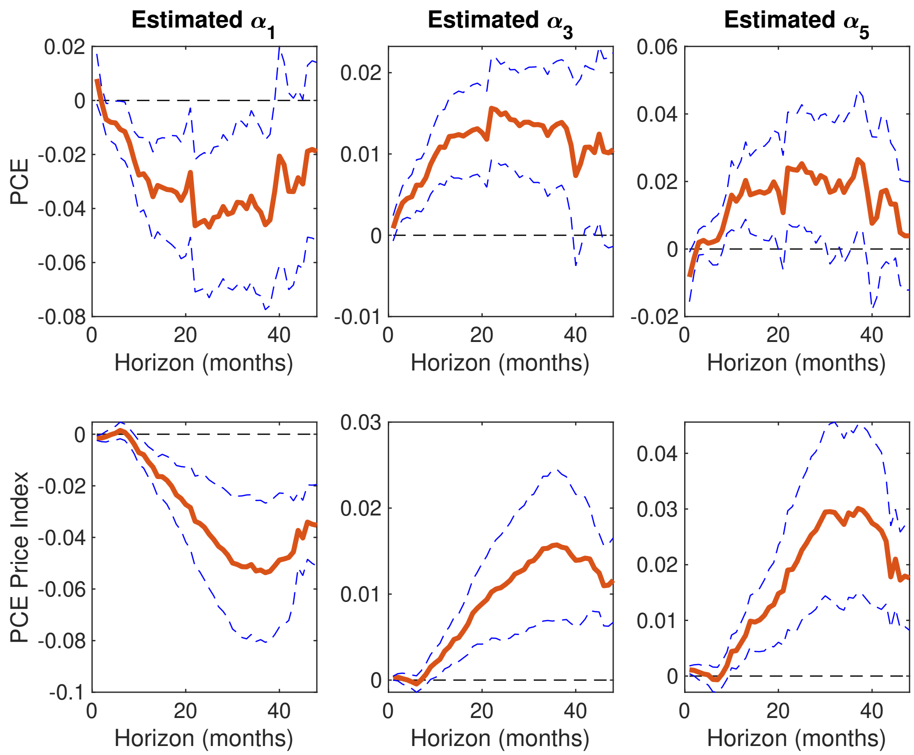

FIGURE A.8. Estimated response to forward guidance shocks (controlling for consumer sentiment)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the consumer sentiment index from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

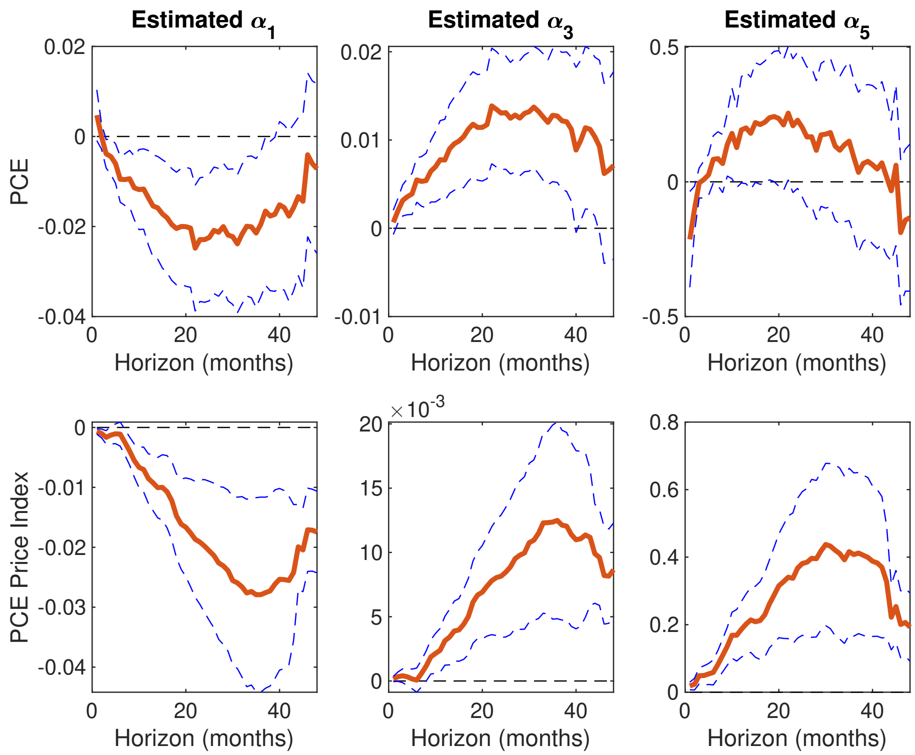

FIGURE A.9. Estimated response to forward guidance shocks (controlling for expected income growth)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the median income expectation from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

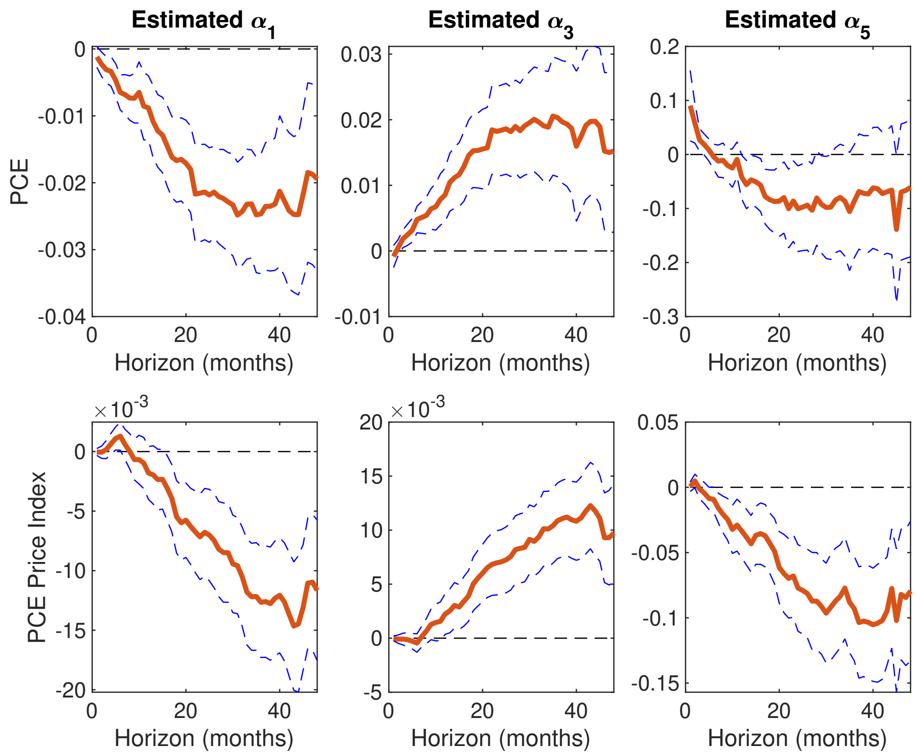

FIGURE A.10. Estimated response to forward guidance shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is consumer uncertainty from Michigan Survey of Consumers. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

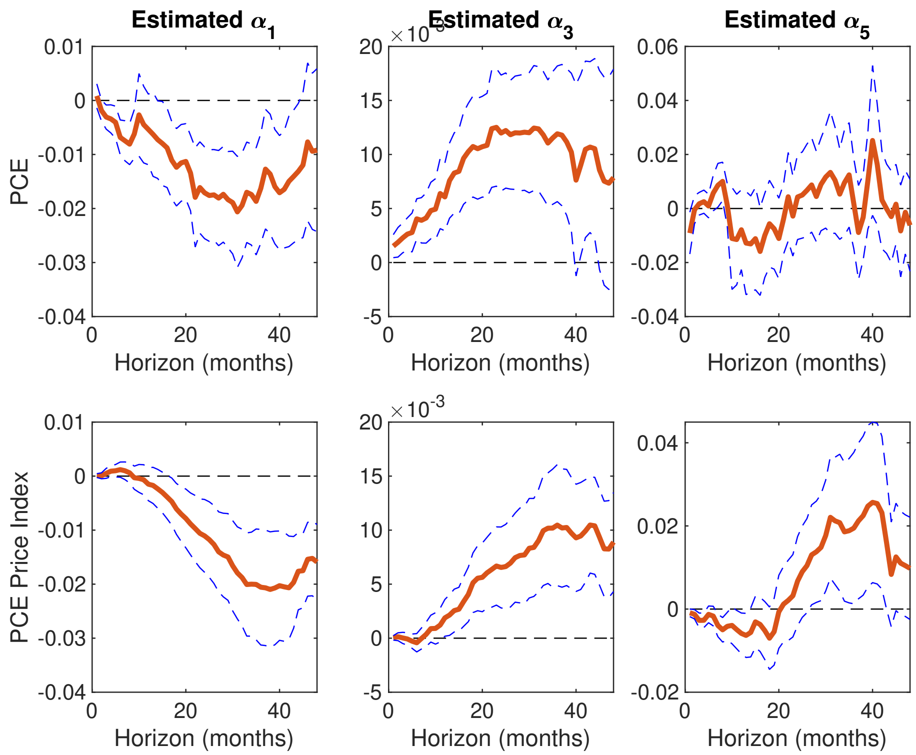

FIGURE A.11. Estimated response to forward guidance shocks (controlling for inflation uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the inflation uncertainty measure from the Survey of Professional Forecasters. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

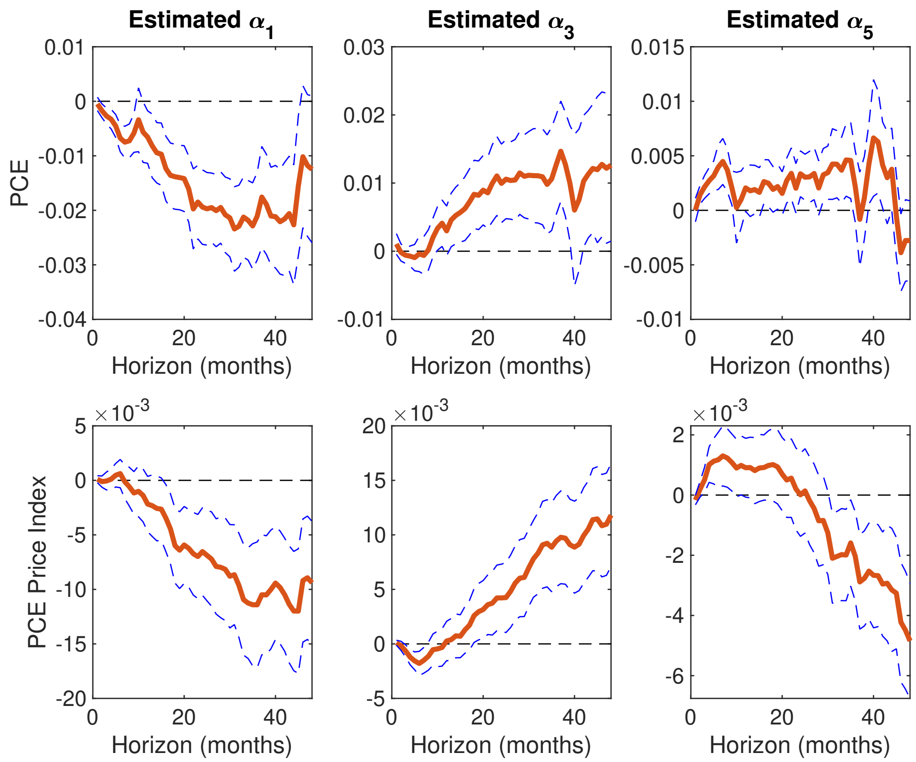

Figure A.12. Estimated response to forward guidance shocks (controlling for forecast dispersion of GDP)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the one-year ahead forecast dispersion of real GDP growth from the Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

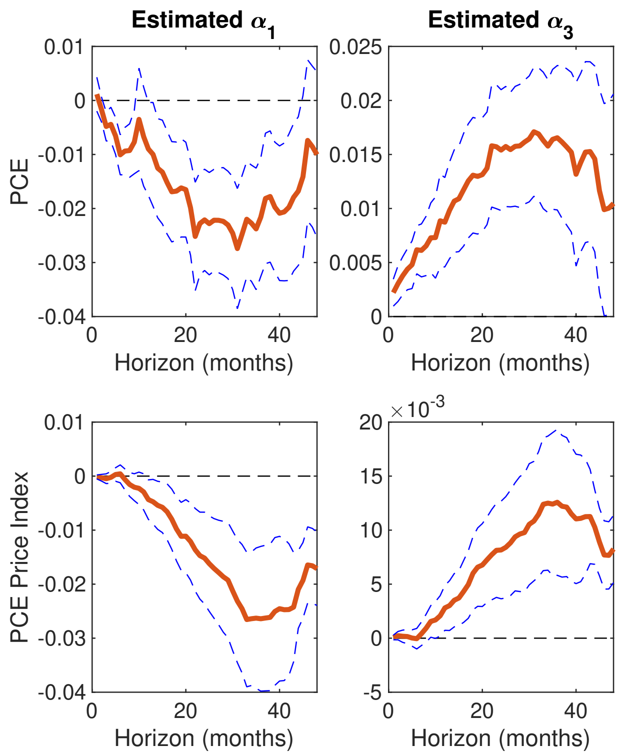

FIGURE A.13. Estimated response to forward guidance shocks (controlling for forecast dispersion of federal funds rate)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the one-year ahead forecast dispersion of the federal funds rate from the Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

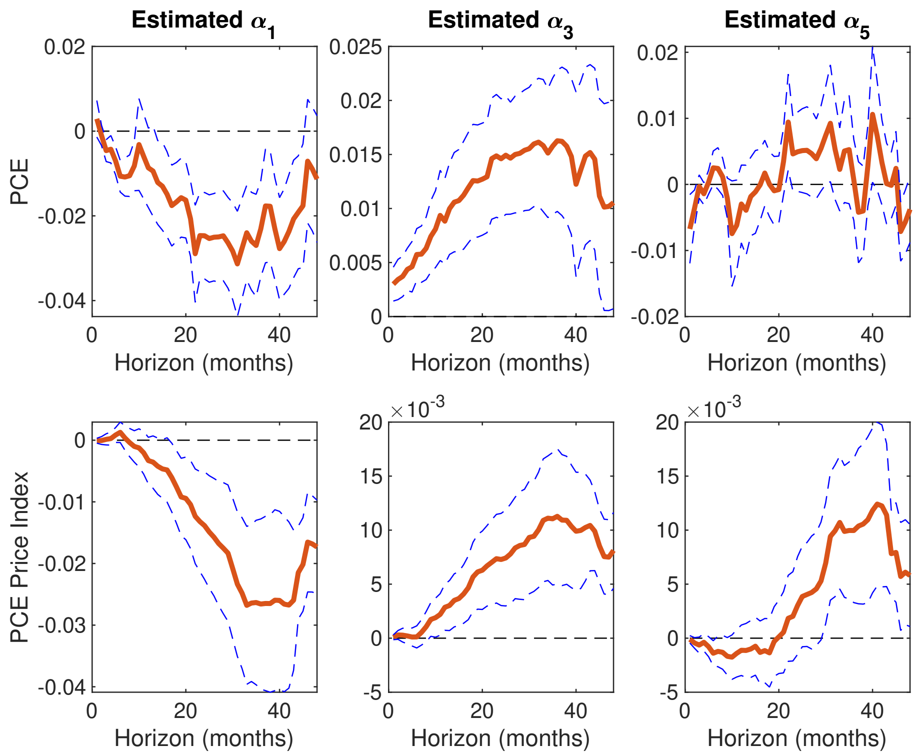

FIGURE A.14. Estimated response to forward guidance shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to identified forward guidance shock from the local projections model (A.1), where the additional control variable X_t is the one-year ahead forecast dispersion of the 2-year Treasury yields from the Blue Chip Financial Forecasts Database. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

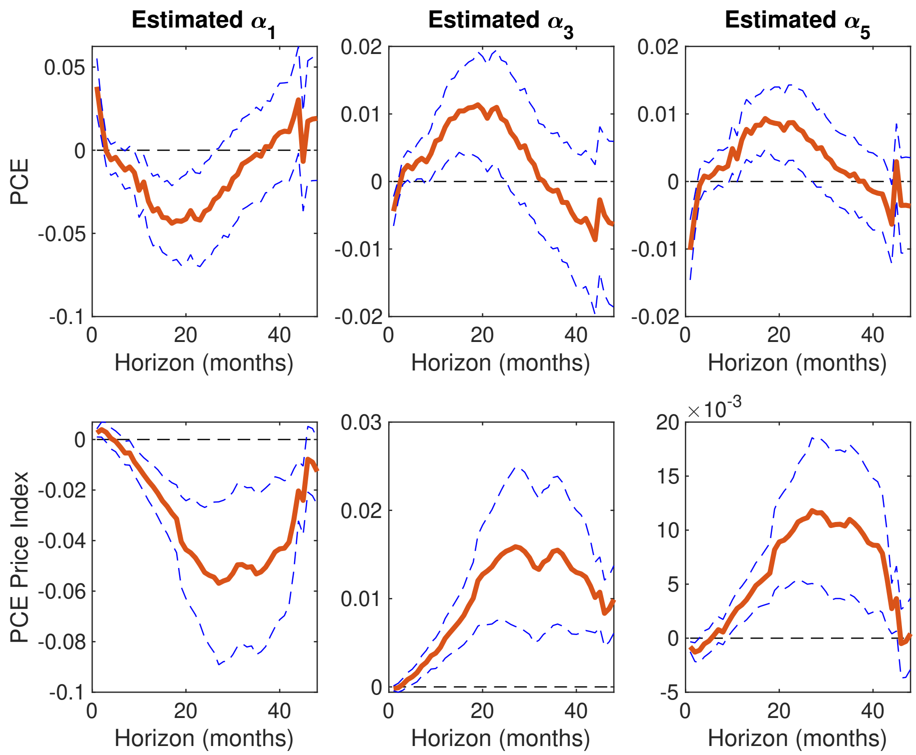

Figure A.15. Estimated response to federal fund rate shocks (controlling for inflation expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the median inflation expectation and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

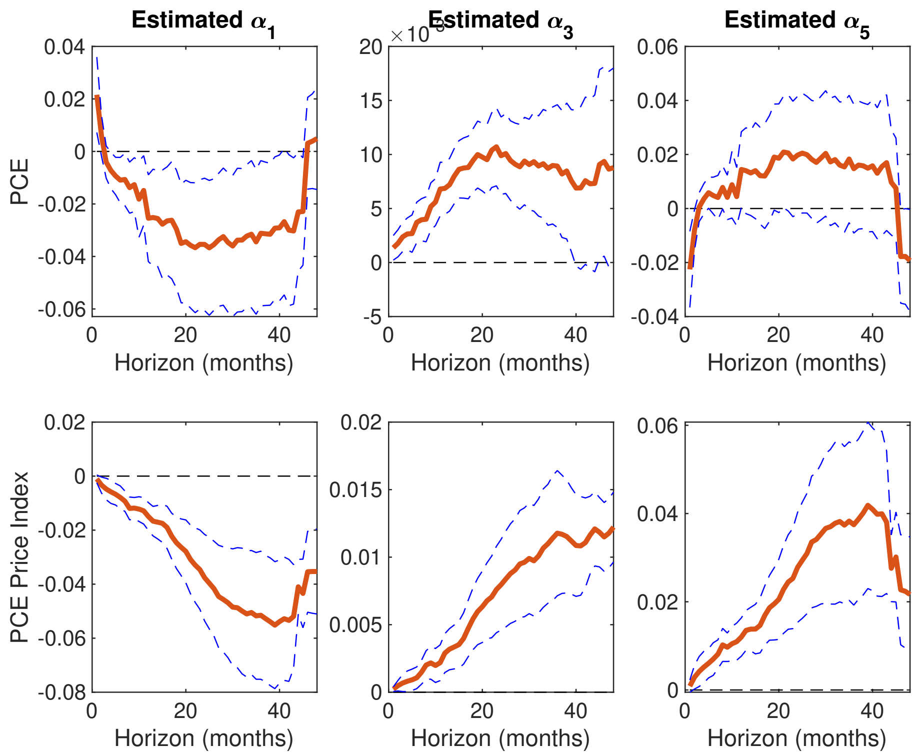

FIGURE A.16. Estimated response to federal fund rate shocks (controlling for consumer sentiment)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the consumer sentiment index from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

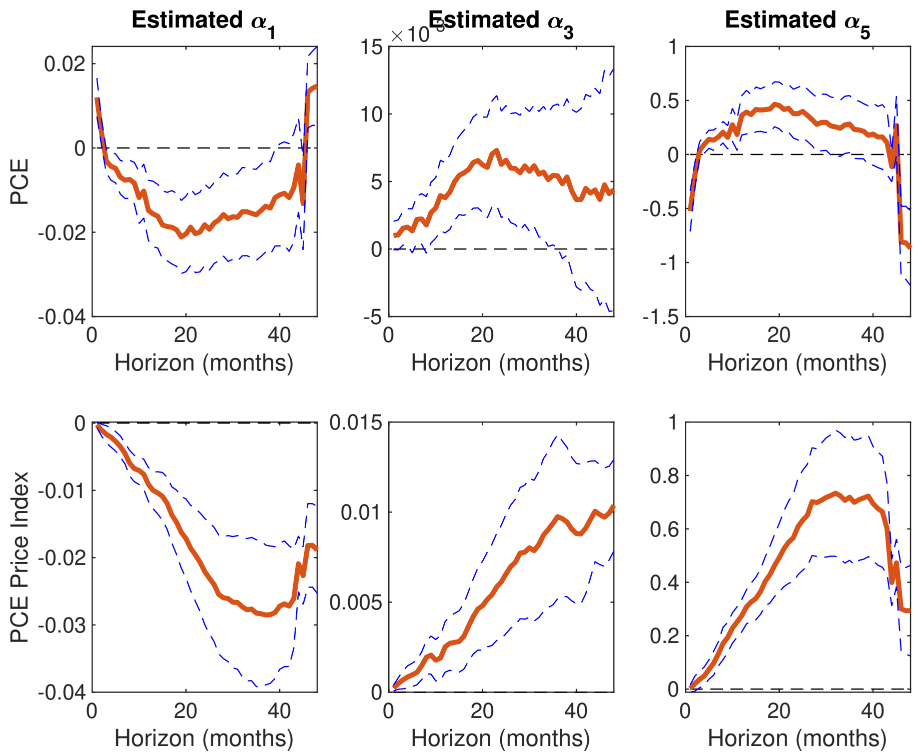

FIGURE A.17. Estimated response to federal fund rate shocks (controlling for income growth expectation)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the median income expectation from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

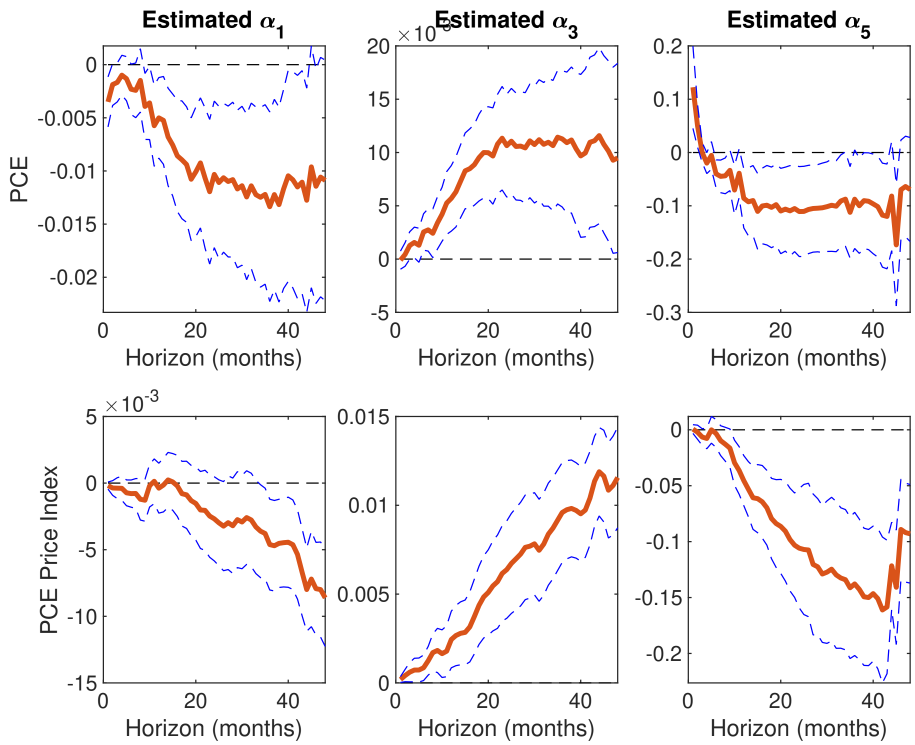

FIGURE A.18. Estimated response to federal fund rate shocks (controlling for consumer uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include consumer uncertainty from the Michigan Survey of Consumers and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

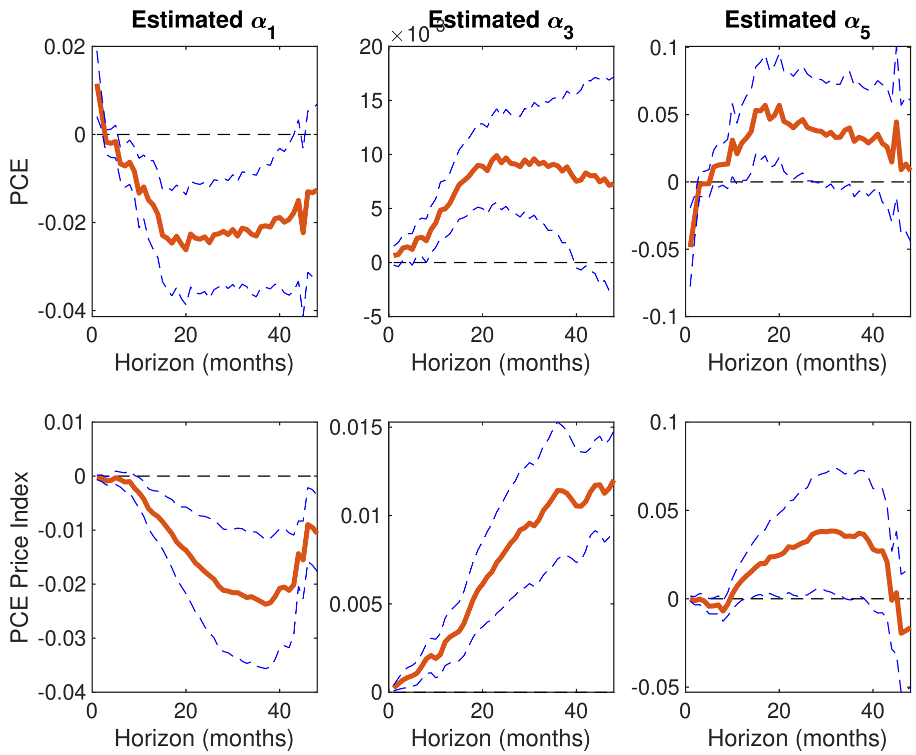

FIGURE A.19. Estimated response to federal fund rate shocks (controlling for inflation uncertainty)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the inflation uncertainty measure from the Survey of Professional Forecasters and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

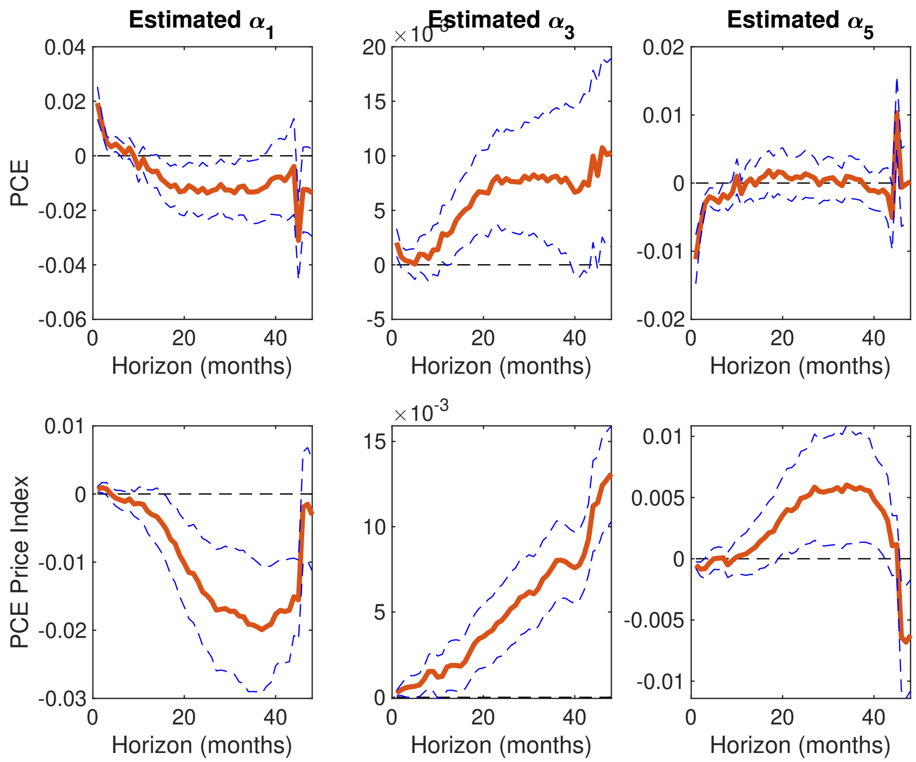

FIGURE A.20. Estimated response to federal fund rate shocks (controlling for forecast dispersion of GDP)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the one-year ahead forecast dispersion of real GDP growth from the Blue Chip Financial Forecasts Database and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

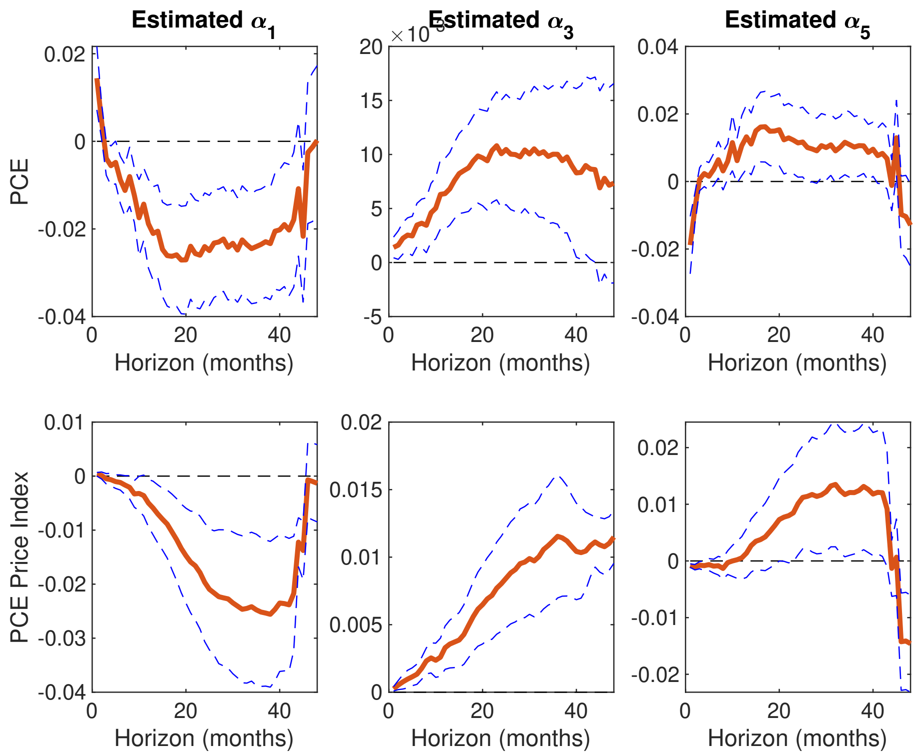

FIGURE A.21. Estimated response to federal fund rate shocks (controlling for forecast dispersion of federal funds rate)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the one-year ahead forecast dispersion of the federal funds rate from the Blue Chip Financial Forecasts Database and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

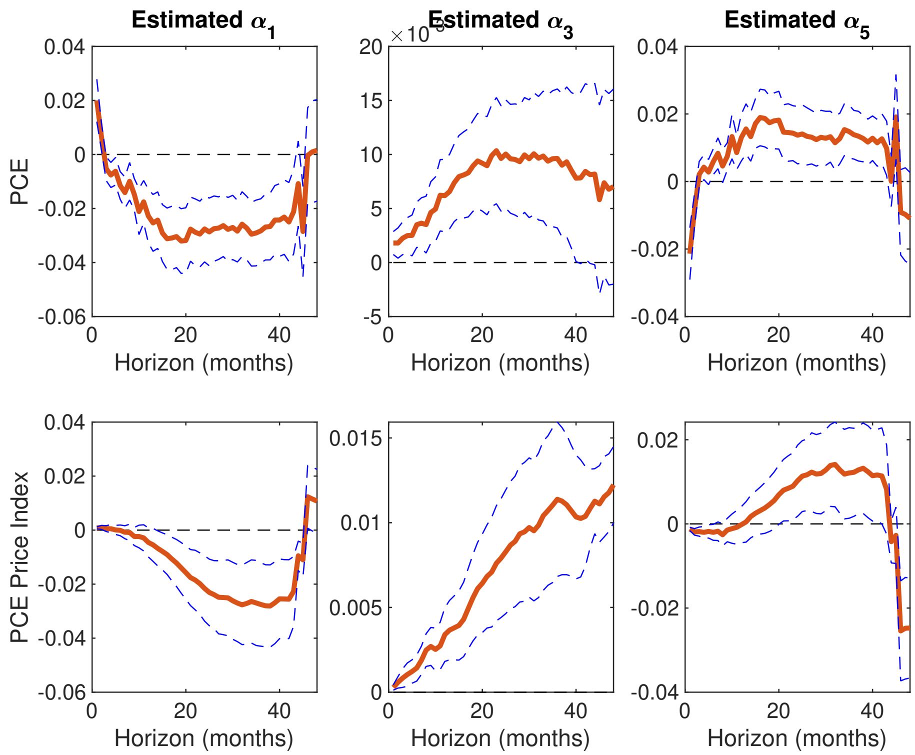

Figure A.22. Estimated response to federal fund rate shocks (controlling for forecast disagreement of 2-year Treasury yield)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE) and PCE price index to a federal funds rate shock from the local projections model (A.1) (with FG replaced by MP), where we include the one-year ahead forecast dispersion of 2-year Treasury yields from the Blue Chip Financial Forecasts Database and its interactions with the federal funds rate shock. The solid lines show the point estimates of the impulse responses. The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

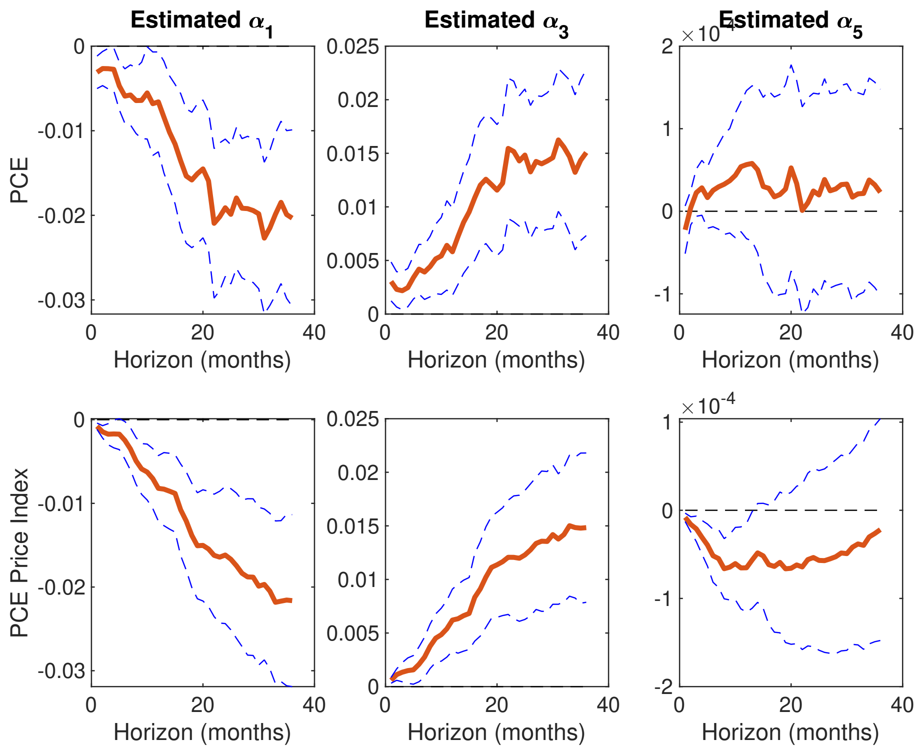

FIGURE A.23. Estimated response to a forward guidance shock: effects of lending standard (robustness)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to forward guidance shocks from the local projections model (34) (where *LoanStd* is replaced by the net percentage of domestic banks tightening standards on household loans). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

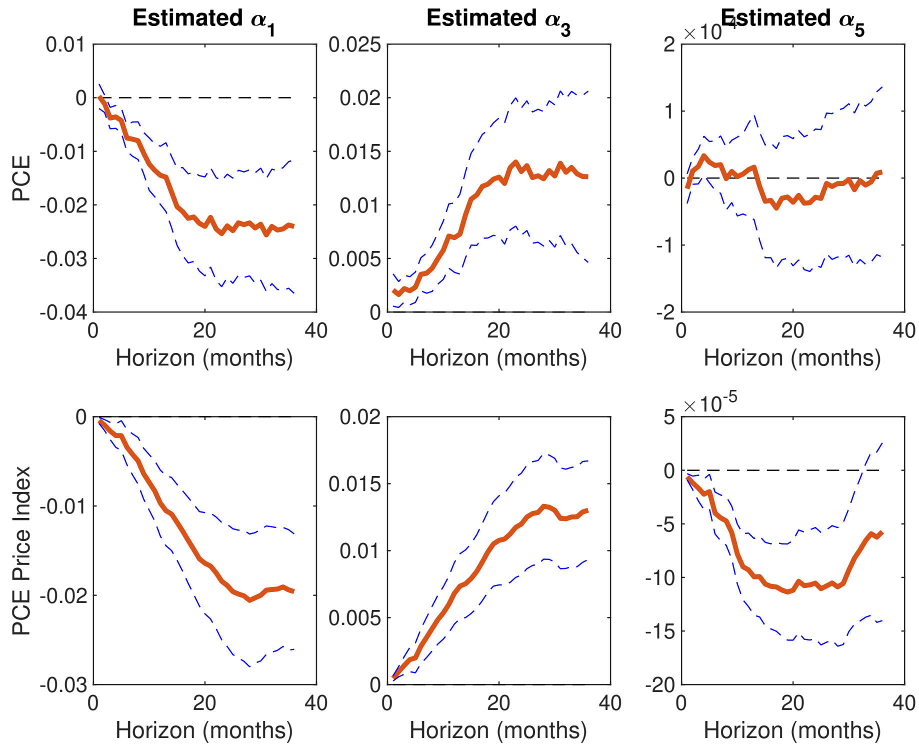

FIGURE A.24. Estimated response to a federal fund rate shock: effects of lending standard (robustness)

Note: This figure shows estimated impulse responses of monthly real personal consumption expenditure (PCE), and PCE price index to forward guidance shocks from the local projections model (34) (where FG is replaced by MP and $LoanStd$ is replaced by the net percentage of domestic banks tightening standards on household loans). The dashed lines show the 68% confidence intervals based on a Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator.

Appendix B. Derivations and proofs

We derive the log-linearized Euler equation and the Phillips curve in the baseline model and we also provide proofs of some auxiliary results.

B.1. **Derivations of the log-linearized Euler equation.** Log-linearizing equation (20), we obtains

$$-\frac{C}{C+\bar{B}}\hat{C}_t = -\hat{e}_t^* + \hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1} - \frac{C}{C+\bar{B}}E_t\hat{C}_{t+1} + E_t[1-\theta]\hat{e}_{t+1}^*,$$

where θ measures the (inverse) elasticity of $F(\cdot)$ with respect to e^*, such that

$$\theta \equiv -\frac{F'(e^*)e^*}{F(e^*)} = \frac{1-G(e^*)}{1-G(e^*)+e^*\int_{e_{\min}}^{e^*}\frac{1}{e}dG(e)} \in [0,1). \tag{B.1}$$

In the special case with no belief heterogeneity, we would have a degenerate distribution of beliefs, such that $\theta = 0$.

After rearrangement, we have

$$\hat{C}_t - \frac{C+\bar{B}}{C}\hat{e}_t^* = E_t\hat{C}_{t+1} - \frac{C+\bar{B}}{C}E_t[1-\theta]\hat{e}_{t+1}^* - \frac{C+\bar{B}}{C}\left(\hat{r}_{ft} - \mathbf{E}_t\hat{\pi}_{t+1}\right). \tag{B.2}$$

Finally, we replace $\hat{e}_t^*$ with $\hat{C}_t$ using aggregate consumption condition:

$$C_t = (C_t + \bar{B})\left[1 - G(e_t^*) + \int_{e_{\min}}^{e_t^*}\frac{e}{e_t^*}dG(e)\right],$$

or equivalently,

$$\frac{C_t}{C_t+\bar{B}} \equiv \Phi(e_t^*), \tag{B.3}$$

where

$$\Phi(e_t^*) \equiv \left[1 - G(e_t^*) + \frac{\int_{e_{\min}}^{e_t^*} edG(e)}{e_t^*}\right].$$

By definition, $\Phi(e_t^*)$ is the ratio of average consumption to consumption of the marginal consumer. Since $C_t = A_t$ in equilibrium, $\Phi(e) \equiv \frac{C_t}{C_t+\bar{B}} = \frac{A_t}{A_t+\bar{B}}$ can be interpreted as (the inverse of) the average leverage ratio.

Denote the (inverse) elasticity of $\Phi(\cdot)$ to e^* by μ, such that

$$\mu \equiv -\frac{\Phi'(e^*)e^*}{\Phi(e^*)} = \frac{\int_{e_{\min}}^{e^*} edG(e)}{[1-G(e^*)]e^* + \int_{e_{\min}}^{e^*} edG(e)} \in (0,1]. \tag{B.4}$$

Note have that $\mu = 1$ if and only if inflation expectation is homogeneous. We can derive $\hat{e}_t^*$ as a function of $\hat{C}_t$

$$\frac{\bar{B}}{C+\bar{B}}\hat{C}_t = -\mu\hat{e}_t^* \tag{B.5}$$

Plugging equation (B.5) into the Euler equation (B.2), we have

$$\hat{C}_t\left(1+\frac{\bar{B}}{\mu C}\right)=E_t\hat{C}_{t+1}[1+(1-\theta)\frac{\bar{B}}{\mu C}]-\frac{C+\bar{B}}{C}\left(\hat{r}_{ft}-\mathbf{E}_t\hat{\pi}_{t+1}\right)$$

Denoting the steady state loan-to-value ratio as $\kappa\equiv\frac{\bar{B}}{A}=\frac{\bar{B}}{C}\in(0,1)$, we derive a *discounted Euler equation* as

$$\hat{C}_t=\underbrace{\frac{\mu+(1-\theta)\kappa}{\mu+\kappa}}_{\equiv\beta_1}\mathbb{E}_t\hat{C}_{t+1}-\underbrace{\frac{(1+\kappa)\mu}{\mu+\kappa}}_{\equiv\beta_2}\left(\hat{r}_{ft}-\mathbf{E}_t\hat{\pi}_{t+1}\right). \tag{B.6}$$

Ceteris paribus, a higher θ or lower μ will reduce the responsiveness of current aggregate consumption to future interest rates and future wealth changes. Lower μ will also weaken the effect of contemporaneous interest rate changes on consumption. Intuitively, aggregate consumption is less responsive to shocks when there is a larger mass of constrained household members, who do not adjust sufficiently to changes in wealth (i.e., changes in expected future consumption) or changes in the real interest rate.

B.2. **Derivations of the Phillips curve.** We now derive the Phillips curve.

There is a continuum of intermediate goods producers index by $j\in[0,1]$, each producing a differentiated product Y_{jt}. The final consumption good is a composite of the differentiated intermediate goods, with the aggregation technology

$$Y_t=\left[\int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_t denotes the final goods output and $\sigma>1$ denotes the elasticity of substitution between differentiated intermediate goods.

Final goods producers are price takers. Their optimal production decisions lead to the downward-sloping demand schedule for each intermediate product

$$Y_{jt}=\left(\frac{P_{jt}}{P_t}\right)^{-\sigma}Y_t. \tag{B.7}$$

Each variety of intermediate goods is produced using labor as the only input. Intermediate goods producers are price takers in the input market and monopolistic competitors in the product markets. Unlike the households, we assume that firms are perfectly rational, with no heterogeneity in beliefs. Each intermediate goods producer takes as given the price level P_t, the real wage rate w_t, and the demand schedule (B.7), and chooses its own price P_{jt} to maximize the present value of its profit flows, subject to the quadratic price adjustment cost in the spirit of Rotemberg (1982).

The price adjustment cost function is given by

$$\frac{\chi_P}{2}\left[\frac{P_t(i)}{\Pi_t^* P_{t-1}(i)}-1\right]^2 Y_t$$

Define $\Phi_{t,t+\tau}=\Pi_{t+1}^*\times\cdots\times\Pi_{t+\tau}^*$, for $\tau\geq 1$. We can normalize the price as $\tilde{P}_t(i)=\frac{P_t(i)}{\Phi_{0,t}}$, and the cost becomes

$$\frac{\chi_P}{2}\left[\frac{\tilde{P}_t(i)}{\tilde{P}_{t-1}(i)}-1\right]^2 Y_t$$

In a symmetric equilibrium,

$$\frac{P_t(i)}{\Pi_t^* P_{t-1}(i)}=\frac{P_t}{\Pi_t^* P_{t-1}}=\frac{\Pi_t}{\Pi_t^*}\equiv\pi_t.$$

Firm j chooses P_{jt} to maximize the present value of profits

$$\begin{aligned} & E_t\sum\beta^\tau\frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{1-\sigma}Y_t-\frac{w_t}{Z_t}\left(\frac{P_{jt+\tau}}{P_{t+\tau}}\right)^{-\sigma}Y_t-\frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}}-1\right]^2 Y_t\right\} \\ = \; & E_t\sum\beta^\tau\frac{\Lambda_{t+\tau}}{\Lambda_t}\left\{\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{1-\sigma}Y_t-\frac{w_t}{Z_t}\left(\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{t+\tau}}\right)^{-\sigma}Y_t-\frac{\chi_P}{2}\left[\frac{\tilde{P}_{jt+\tau}}{\tilde{P}_{jt+\tau-1}}-1\right]^2 Y_t\right\}. \end{aligned}$$

The firm's optimal pricing decisions is given by

$$\begin{aligned} (1-\sigma)\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma}\frac{Y_t}{\tilde{P}_t} \; & + \; \sigma\frac{w_t}{Z_t}\left(\frac{\tilde{P}_{jt}}{\tilde{P}_t}\right)^{-\sigma-1}\frac{Y_t}{\tilde{P}_t}-\chi_P\left[\frac{\tilde{P}_{jt}}{\tilde{P}_{jt-1}}-1\right]\frac{Y_t}{\tilde{P}_{jt-1}} \\ & + \; \chi_P\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{\tilde{P}_{jt+1}}{\tilde{P}_{jt}}-1\right]\frac{\tilde{P}_{jt+1}}{\left(\tilde{P}_{jt}\right)^2}Y_{t+1}=0 \end{aligned}$$

In a symmetric equilibrium with $P_{jt}=P_t$, we have

$$\chi_P\left[\frac{\tilde{P}_t}{\tilde{P}_{t-1}}-1\right]\frac{\tilde{P}_t}{\tilde{P}_{t-1}}=\sigma\frac{w_t}{Z_t}+(1-\sigma)+\chi_P\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{\tilde{P}_{t+1}}{\tilde{P}_t}-1\right]\frac{\tilde{P}_{t+1}}{\tilde{P}_t}\frac{Y_{t+1}}{Y_t}$$

Log-linearizing this optimal pricing decision leads to the Phillips curve

$$\hat{\pi}_t=\varphi_y[\hat{w}_t-\hat{Z}_t]+\beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{B.8}$$

B.3. **Proofs of auxiliary results.** Suppose that the idiosyncratic beliefs of households follow a Pareto distribution, such that

$$G(e)=\begin{cases}1-(\frac{e}{e_{min}})^{-\alpha} & \text{if } e\geq e_{min}\\ 0 & \text{if } e<e_{min}\end{cases} \tag{B.9}$$

We fix $E(e) = 1$ by setting $e_{min} = \frac{\alpha-1}{\alpha}$. The variance of inflation expectation is a decreasing function of α:

$$Var(e) = \frac{\alpha}{\alpha-2} \cdot \left(\frac{e_{min}}{\alpha-1}\right)^2 = \frac{1}{\alpha(\alpha-2)}, \quad \alpha > 2. \tag{B.10}$$

We can prove the following Lemmas:

Lemma B.1. *e_t^* is an increasing function of α.*

Proof. Incorporating the distribution function of inflation expectations (e) and the assumption that $e_{min} \equiv \frac{\alpha-1}{\alpha}$ into Eq. (B.5), we obtain

$$\begin{aligned}\frac{1}{1+\kappa} &= (1 - G(e_t^*)) + \frac{\int_{e_{\min}}^{e_t^*} eg(e)de}{e_t^*} = (1 - (1 - (\frac{e_{min}}{e_t^*})^\alpha)) + \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de}{e_t^*} \\ &= -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{\alpha}{\alpha-1} \cdot \frac{e_{min}}{e_t^*} = -\frac{1}{\alpha-1} \cdot \left(\frac{e_{min}}{e_t^*}\right)^{-\alpha} + \frac{1}{e_t^*},\end{aligned} \tag{B.11}$$

which implies that e_t^* is an increasing function of α, or a *decreasing* function of inflation disagreement. □

Lemma B.2. *θ is a decreasing function of α.*

Proof. Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, and e_t^* to solve for θ from Eq. (B.1):

$$\begin{aligned}\theta =& \frac{1 - G(e^*)}{1 - G(e^*) + e^* \int_{e_{\min}}^{e^*} \frac{1}{e} g(e)de} = \frac{(\frac{e_{min}}{e_t^*})^\alpha}{(\frac{e_{min}}{e_t^*})^\alpha + e_t^* \int_{e_{min}}^{e_t^*} \frac{1}{e} \frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}} de} \\ =& \frac{1}{\frac{1}{\alpha+1} + \frac{\alpha}{\alpha+1} \cdot \left(\frac{e_t^*}{e_{min}}\right)^{\alpha+1}},\end{aligned} \tag{B.12}$$

which implies that θ is a decreasing function of α, or an *increasing* function of inflation disagreement. □

Lemma B.3. *μ is an increasing function of α.*

Proof. Use α, $e_{min} \equiv \frac{\alpha-1}{\alpha}$, e_t^* and θ to solve for μ from Eq. (B.4).

$$\begin{aligned}\mu =& \frac{\int_{e_{\min}}^{e^*} eg(e)de}{(1 - G(e^*))e^* + \int_{e_{\min}}^{e^*} eg(e)de} = \frac{\int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de}{(1 - (1 - (\frac{e_{min}}{e_t^*})^\alpha))e_t^* + \int_{e_{min}}^{e_t^*} e(\frac{\alpha \cdot e_{min}^\alpha}{e^{\alpha+1}})de} \\ =& \frac{\frac{\alpha e_{min}^\alpha}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})}{(\frac{e_{min}}{e_t^*})^\alpha e_t^* + \frac{\alpha e_{min}^\alpha}{-\alpha+1}(e_t^{*-\alpha+1} - e_{min}^{-\alpha+1})} = \frac{\frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})}{1 + \frac{\alpha}{-\alpha+1}(1 - (\frac{e_{min}}{e_t^*})^{-\alpha+1})}\end{aligned} \tag{B.13}$$

which implies that μ is an increasing function of α, or a *decreasing* function of inflation disagreement. □