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Asset Purchases in a Monetary Union with Default and Liquidity Risks*

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Abstract

Using a two-country monetary union framework with financial frictions, we quantify the efficacy of targeted asset purchases, as well as expectations of such programs, in the presence of sovereign default and financial liquidity risks. The risk of default increases with the level of government debt and shifts in investors' perception of fiscal solvency. Liquidity risks increase when the probability of default affects the tightness of credit markets. We calibrate the model to Italy during the 2012 European debt crisis and compare it to key features of the data. We find that changes in investors' perception played a more significant role than increases in government debt in affecting the macroeconomy. When a debt crisis occurs, asset purchases help stabilize both financial markets and the economy. This stabilization effect can occur even if asset purchases are expected but never implemented. Moreover, expectations of potential asset purchases during a crisis alter the level of economic activity in periods when there are no crises.

JEL Classification: E58, E63, F45

Keywords: Monetary and fiscal policy interactions; Unconventional monetary policy; Monetary union; Financial frictions; Regime-switching model

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1 Introduction

Over a decade has passed since the European Central Bank (ECB) initially launched its targeted asset purchases in response to the 2012 European debt crisis. At that time, high levels of government debt in countries such as Greece and Italy created the risk of sovereign default, which, in turn, generated financial turmoil and a severe macroeconomic slowdown. In response, the ECB introduced Outright Monetary Transactions (OMT) in 2012, through which it could purchase government debt of member nations.[1] Later, in June 2022, the ECB introduced a new policy program, the Transmission Protection Instrument (TPI), which allows for asset purchases of securities issued in jurisdictions with deterioration in financing conditions not driven by country-specific fundamentals, see European Central Bank (2022).[2] Together, these programs highlight the focus of the ECB on using targeted asset purchases to mitigate both default and financial liquidity risks.

While broad-based quantitative easing programs have been implemented for an extended period of time after the 2012 crisis, the more targeted asset purchase programs remain unused, generating some uncertainty about their efficacy. The OMT or TPI asset purchase programs only apply during times of acute stress limited to specific countries, and hence have unique features that distinguish them from broader quantitative easing purchases. Importantly, not only does the implementation of asset purchases matter, but so do market expectations of these interventions, which can affect the economy even if the intervention never materializes. These expectations can be meaningful with the attention given to the ECB's announcements of these programs.

In this paper, we quantify the effects of a debt crisis and examine the stabilizing role of asset purchases by the central bank. We proceed in three steps. First, we focus on building a monetary union model where 1) sovereign default may occur, 2) the probability of default can drive liquidity risks in financial markets, and 3) these effects can produce a macroeconomic slowdown. We use Italy's 2012 debt crisis to inform the features of the model. Second, we use the model to study the efficacy of targeted asset purchases during debt crises; in effect, this exercise is akin to a counterfactual as if OMT had been used in Italy during 2012. Third, given the fact that these programs have been announced with some fanfare but have not been used, we study the implications of market expectations of such programs.

In order to capture the relevant features of a debt crisis in a monetary union, we build a model with sovereign default. Following Bi (2012) and Bi and Traum (2014), default

[1] Starting in 2014, the ECB embarked on a broad-based quantitative easing program, where they purchased private and public debt spread across member countries. This type of purchase program was less targeted and not a direct response to crises in individual countries.

[2] A key goal of the TPI is to ensure financial stability, as it was created to prevent financial market fragmentation and ensure a smooth transmission of monetary policy, see Schnabel (2023).

is determined endogenously by fiscal limits—the government's willingness and capacity to service its debt—as well as the underlying fiscal fundamentals. In this framework, the probability of default increases with the level of government indebtedness. Importantly, the default probability also changes when investors' perception of fiscal risks shifts. We model these changes in perception as a downward shift in the fiscal limit: a level of government debt that was previously sustainable may now be viewed as unsustainable, leading to a rapid rise in government bond yields.

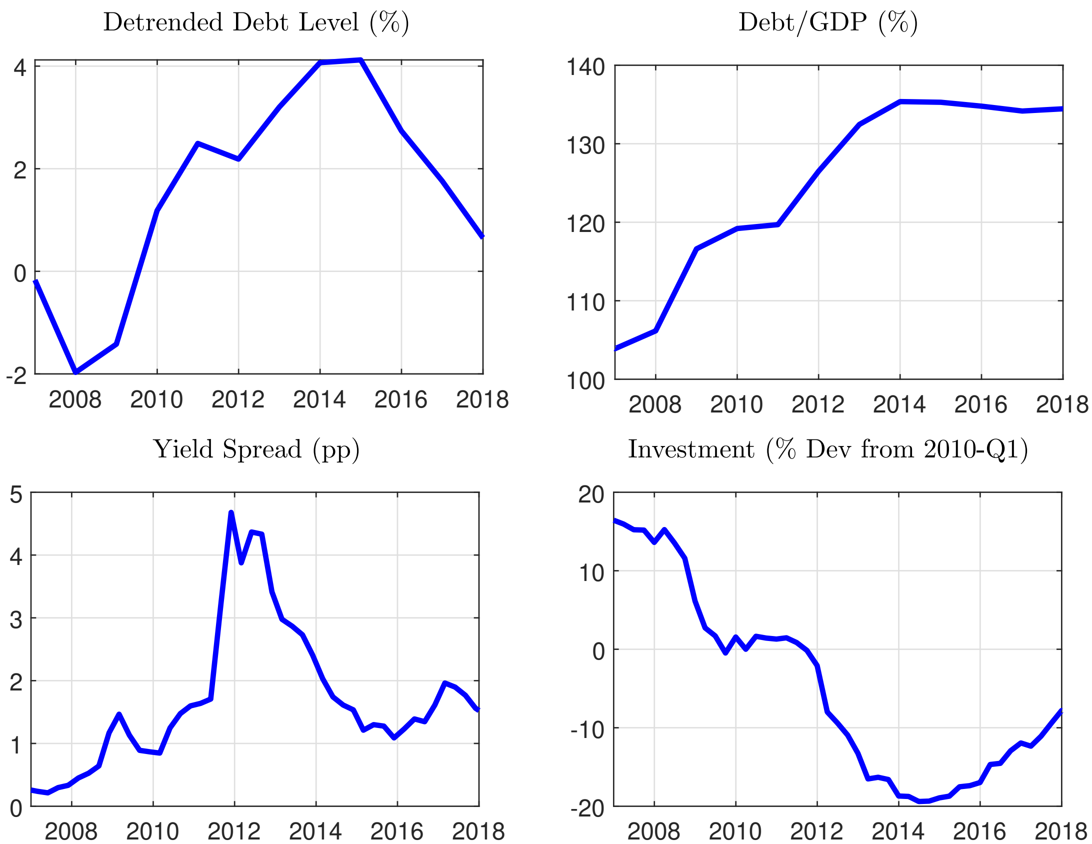
The dynamics of default are motivated by Italy's experience during the 2012 debt crisis. Figure 1 highlights the notable increase in the level of Italian debt, relative to its trend and the size of Italy's economy, between 2008 and 2014. However, the government yield dynamics, in particular the sudden and sharp increases in 2012, do not closely follow the debt dynamics. We interpret these dynamics as reflecting investors' beliefs in Italy's fiscal space that went beyond pure fiscal fundamentals; in particular, a sharp deterioration in the fiscal outlook of the Greek government may have undermined investors' beliefs in Italy's fiscal situation. In addition, the spillovers from financial stress to the macroeconomy were substantial, as investment fell sharply by about 20 percent from 2012 to 2014.

To capture the financial aspect of the European debt crisis, our model extends the two-country monetary union framework of Nakamura and Steinsson (2014) to incorporate financial intermediaries as in Gertler and Karadi (2011) and Sims and Wu (2021). The two countries differ in the safety of their government debt: the Home country (e.g., Italy) issues debt that carries some default risk, while the Foreign country (e.g., Germany) issues safe debt. In both countries, banks collect deposits from households and use these funds, along with their own net worth, to lend to domestic firms and the governments of both Home and Foreign countries. This intermediation has real economic consequences, as firms are required to have external financing to buy capital goods. In this environment, a higher risk of the Home government defaulting prompts the financial intermediary to demand a lower government bond price, depressing the financial intermediary's net worth and raising its leverage. These deteriorating conditions can prompt the sale of private bonds as the financial intermediary must adjust its assets to satisfy its balance sheet constraint. This adjustment, in turn, depresses private lending, investment, and economic activity. The chain reaction to economic activity can occur regardless of the source of default risk, i.e., either from fiscal fundamentals or investors' perception of fiscal risks.[3]

We also consider an additional channel of higher default risk stemming from liquidity issues: In anticipation of a possible default and its associated fire sale of assets, financial

[3]Bottero, Lenzu, and Mezzanotti (2020) document a tightening of credit supplied by Italian banks following the debt crisis.

Figure 1: Debt, Yield Spreads, and Investment in Italy, 2008-2018



Notes: Variables plotted are the annual debt level relative to a linear trend from 1998-2008, the quarterly debt-GDP ratio, the quarterly average of yields of Italian debt relative to German debt, and HP-filtered quarterly real investment expressed as percent deviation from the 2010-Q1 value.

intermediaries tighten liquidity conditions before default, see Bocola (2016). To capture this notion, we allow the overall tightness of the credit market to endogenously vary with the probability of a sovereign default, which we refer to as the liquidity risk channel. In this case, rising default risk raises liquidity risk and tightens financial conditions, further depressing asset prices and the financial intermediary's net worth.

To provide a quantitative evaluation of the importance of these risks, we explicitly account for the nonlinearity associated with default when solving the model. Our model maps into an endogenous regime-switching framework, which we can solve using perturbation methods proposed in Benigno, Foerster, Otrok, and Rebucci (2024). After calibrating the Home country to capture the key aspects of financial and macroeconomic data in Italy as highlighted in Figure 1, we show that the combination of higher debt plus a downward shift in the fiscal limit triggers default and liquidity risks that capture the Italian experience.

We decompose the influence of default and liquidity risks, showing that the presence of the latter is quantitatively crucial to the outcomes. Although the possibility of default lowers asset prices, net worth of financial intermediaries, investment, and output, the quantitative impact is modest when there is no liquidity risk channel. With the liquidity risk channel, tighter financial conditions directly amplify the decline in asset prices and net worth, further depressing economic activity. In other words, our model indicates that default risk has relatively modest direct effects, but reverberations through the financial system and the effect on liquidity are a key amplification mechanism.

Having established the transmission in a debt crisis, we turn to an investigation of the effectiveness of targeted asset purchases by the central bank, whereby the monetary authority conducts Home country debt in response to movements in the Home credit spread. Following an increase in Home default and liquidity risks, the spread between Home government bonds and the central bank's interest rate target rises, inducing the central bank to purchase Home sovereign debt. This policy action reduces the decline in the price of government bonds, which in turn dampens the negative effects reverberating throughout the financial market. With less severe constraints, private lending contracts less, moderating the decline in economic activity. Importantly, the model suggests that the stabilizing effects of asset purchases are more pronounced in scenarios with elevated liquidity stress and deterioration in investor sentiment. This point is of great relevance to the ECB asset purchase policy, as containing non-fundamental risks were a key consideration when the ECB rolled out the TPI program.

Since much attention surrounded the unveiling of the OMT and TPI programs, while neither have yet to be used, we study the implications of market expectations of such programs. Our Markov-switching environment is well suited for such policy analysis. We show that expectations of future asset purchases matter at the onset of a debt crisis and mute the recessionary effect. As agents form probabilities of default and credit interventions simultaneously, the anticipation of targeted asset purchases prompts financial intermediaries to be more willing to extend loans to the private sector, even when the credit intervention is never used. This, in turn, dampens the recessionary effect of a debt crisis.

In addition, expecting the central bank to purchase assets during a crisis changes the level of economic activity in periods when there are no crises. The impact depends on the likelihood of a crisis, as anticipation of a crisis has two opposing effects on bank lending: on the one hand, households are more inclined to have precautionary savings, raising bank lending; on the other hand, a crisis tightens financial conditions, lowering the incentive of the financial intermediary to lend. When crises occur infrequently, the precautionary savings motive dominates, and anticipation of a crisis increases investment, output, and consumption in non-crisis periods relative to an economy without such expectations. Expecting a crisis to be accompanied by a credit intervention dampens these incentives. However, when crises occur more frequently, the financing condition channel dominates, and the effects are reversed: anticipating tighter financial conditions in a crisis lowers lending and, in turn, economic activity. Expecting a crisis to be accompanied by a credit intervention dampens the effects, raising economic activity.

Related Literature This paper is related to several strands of the literature. First, it is related to a theoretical literature utilizing New Keynesian models of a monetary union to study fiscal policy (see Erceg and Lindé, 2013; Nakamura and Steinsson, 2014; Farhi and Werning, 2017, among many others). Maćkowiak and Schmidt (2022) study the fiscal theory of the price level in a monetary union with heterogeneous fiscal policies, while Bianchi, Melosi, and Rogantini-Picco (2023) explore the interactions of monetary and fiscal policy following the issuance of euro bonds.

Second, the paper is closely related to a large literature that studies the effects of asset purchases through the lens of DSGE models with segmented asset markets and financial frictions following Gertler and Karadi (2011), Carlstrom, Fuerst, and Paustian (2017), and Sims and Wu (2021).[4] For instance, Kirchner and Wijnbergen (2016) highlight the crowding-out mechanism through reduced private access to credit when leverage-constrained banks accumulate government debt. Krenz (2022) studies unconventional monetary policy in a monetary union without sovereign default and focuses on the welfare implications of union-wide versus country-specific asset purchases.

In addition, our paper contributes to the literature on the sovereign-bank diabolic loop. Some papers explore the interaction of sovereign default risk and liquidity risk in a closed economy.[5] For instance, Bocola (2016) estimates such a model with exogenous sovereign default, while Coimbra (2020) models financial frictions through an occasionally binding Value-at-Risk constraint. Gennaioli, Martin, and Rossi (2014) and Arellano, Bai, and Bocola (2017) show large output losses can result from financial intermediaries tightening credit after sovereign default. Other studies focus on the interactions from the perspective of banking regulations, for instance Abad (2019) and Fueki, Huertgen, and Walker (2024).

More closely related, Bianchi, Callegari, Hitaj, and Theodoridis (2024) introduce default risk and financial frictions into a two-country model of the Eurozone. They show that higher default risk raises borrowing costs, magnifying the unpleasant effects of adverse energy shocks, and argue countries would be better off deviating from fiscal rules and accommo-

[4] Kollmann, Enders, and Muller (2011), Kollmann (2013), Dedola, Karadi, and Lombardo (2013), and Wu, Xie, and Zhang (2023) explore the importance of financial connections and financial participants for the transmission of shocks across countries.

[5] Bianchi and Mondragon (2022) study the interaction of default and liquidity risks by focusing on rollover debt crises and monetary independence.

dating the adverse shock. In addition, Auray, Eyquem, and Ma (2018) introduce an asset purchase program into a perfect-foresight, two-country model without inter-region trades to study responses to a sovereign debt crisis.

The remainder of the paper is as follows. Section 2 describes our model of a monetary union with financial frictions and sovereign default. Section 3 discusses how we calibrate the model to match Italy's experience during the 2012 debt crisis, and how we solve the model. In Section 4, we study a baseline model without asset purchases, which allows us to study the effects of both default and liquidity risks. Section 5 introduces targeted asset purchases into our framework, and Section 6 explores anticipation effects, both during and outside of debt crises.

2 Model

Since we are interested in the effects of asset purchases in a monetary union with government default risk, our model has a number of necessary features. It requires two countries that are linked by trade and a degree of financial flows, financial intermediaries that are subject to a financial friction, governments that issue debt, one country where the government may default as economic conditions change, and a monetary authority that sets a union-level interest rate and additionally may purchase country-level debt.

Our model builds on Bi and Traum (2023) to incorporate endogenous default risk of the government. It extends the two-country monetary union framework of Nakamura and Steinsson (2014) and incorporates financial intermediaries as in Gertler and Karadi (2011) and Sims and Wu (2021). Importantly, we model sovereign default as an endogenous regime-switching process following Bi (2012) and Bi and Traum (2014). The probability of default increases with respect to the state of government debt, such that default is driven by the underlying fiscal and macro fundamentals, as well as liquidity conditions.

We refer to each region as Home and Foreign. Both regions have the same economic structure. The financial market is segmented as households cannot hold government and private bonds directly; instead, financial intermediaries collect deposits from households and lend to the domestic private sector and governments in both regions. The two economies produce differentiated tradable goods, and trade is frictionless. Thus, Home and Foreign households pay the same nominal prices for the differentiated goods produced in each region. Both governments also set their own taxes and public expenditures and issue bonds, while a central bank conducts monetary policy at the union level. We assume that a unit of time is a quarter. Since both regions are symmetric, we focus on a description of the Home economy and list equilibrium conditions in both the Home and Foreign economies in Appendix A.

2.1 Households

The representative household maximizes the expected intertemporal utility given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \Theta_t \left[\frac{c_t^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{l_t^{1+\sigma_l}}{1+\sigma_l} \right] \right\}, \tag{2.1}$$

where c_t is composite consumption and l_t is the number of hours worked. Θ_t represents an endogenous discount factor, which in the absence of complete international markets ensures stationarity (Schmitt-Grohe and Uribe, 2003).

The composite consumption c_t aggregates Home and Foreign consumption sub-baskets, $c_{H,t}$ and $c_{F,t}$, in Armington form:

$$c_t = \left[\alpha_H^{\frac{1}{\phi^c}} \left(c_{H,t}\right)^{\frac{\phi^c-1}{\phi^c}} + (1-\alpha_H)^{\frac{1}{\phi^c}} \left(c_{F,t}\right)^{\frac{\phi^c-1}{\phi^c}} \right]^{\frac{\phi^c}{\phi^c-1}}, \tag{2.2}$$

where $\phi^c > 0$ denotes the elasticity of substitution between Home and Foreign goods, and α_H is the household's relative preference for Home goods. The subbasket $c_{H,t}$ aggregates the different domestic consumption varieties $c_{H,t}(j)$, while $c_{F,t}$ aggregates the different foreign consumption varieties $c_{F,t}(j)$, with the elasticity of substitution between the goods of $\theta^c > 1$.

The household's period budget constraint in real terms is

$$d_t + b_t^i + c_t\left(1+\tau^c\right) = \frac{R_{t-1}^d d_{t-1}}{\pi_t} + \frac{R_{t-1}^d b_{t-1}^i}{\pi_t} + w_t l_t + \Pi_t^f + div_t - x - t_t + T_t^{cb}. \tag{2.3}$$

Households make one-period savings deposits at financial intermediaries. We denote d_t as the amount of savings deposits, and R_{t-1}^d as the nominal interest rate on deposits between $t-1$ and t, which is known with certainty in $t-1$. Inflation is $\pi_t = P_t/P_{t-1}$. Households earn the real wage w_t on their labor, and pay consumption taxes τ^c to their government. In addition, households receive profits from the ownership of firms Π_t^f as well as equity from financial intermediaries div_t. In each period, households make a fixed real equity transfusion to newly born financial intermediaries, which we denote by x, and pay a lump-sum tax t_t. T_t^{cb} denotes a transfer from the monetary authority when performing asset purchases.

Financial markets are incomplete across countries, and we assume that households in each country can only exchange a nominal one-period bond b_t^i . This bond trades at the same rate as the deposit rates of the Home and Foreign countries, which also equals the interest rate set by the common central bank.[6] For reference below, we define $\Lambda_{t,t+1}$ as the household's real stochastic discount factor.

[6]Since Home deposits and the traded bond have the same interest rate, the setup is similar to allowing Foreign households to directly hold deposits at the Home financial intermediary.

2.2 Government

The Home government finances its consumption by collecting distortionary taxes on income (τ^i) and consumption, by receiving lump-sum taxes, as well as by floating government bonds b_t at price Q_t^b. Its budget constraint in real terms is given by

$$\rho_{H,t} g + (1-\Delta_t)(1+\kappa^b Q_t^b)\frac{b_{t-1}}{\pi_t} = Q_t^b b_t + t_t + \tau^i p_t^w y_t + \tau^c c_t \tag{2.4}$$

where g is a constant level of government consumption, and $\rho_{H,t}$ is the relative price of Home goods. Public bonds are defined as a perpetuity with coupons that decay exponentially, as in Woodford (2001). A bond issued on date t pays $(\kappa^b)^{k-1}$ at date $t+k$ with the coupon decay factor κ^b that captures the average maturity of the bond portfolio.

Importantly, the bond contract is not enforceable. In each period, the government may default on its government bonds by taking a haircut if the debt-GDP ratio is higher than a stochastic threshold $\mathcal{B}_t^*$,

$$\Delta_t = \begin{cases} 0, & \text{if } s_{t-1} < \mathcal{B}_t^* \\ \delta_b, & \text{otherwise} \end{cases} \tag{2.5}$$

where $s_{t-1} = \frac{Q_{t-1}^b b_{t-1}}{4y_{t-1}}$ denotes the debt to annual GDP ratio. $\mathcal{B}_t^*$ can be interpreted as the fiscal limit that captures the government's capacity to service debt. Bi (2012) shows that fiscal limits can arise endogenously from dynamic Laffer curves and depend on the underlying macroeconomic shocks. Therefore, the likelihood of the government defaulting on its debt depends on the fiscal limit and on its outstanding debt liability: it increases with a higher level of outstanding debt and a lower level of the fiscal limit.

Following Bi and Traum (2014), we specify the conditional probability of a government's default at time t as a logistic function of existing debt-GDP ratio s_{t-1} with parameters η_0^{FL} and η_s^{FL} dictating its shape:

$$P(s_{t-1} \geq \mathcal{B}_t^*) = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_{t-1}^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_{t-1}^P)]}. \tag{2.6}$$

The fiscal limit shifter ϵ_{t-1}^P, assumed to follow an AR(1) process, plays a key role in our analysis. Representing a shock to the fiscal limit distribution, it directly affects the default probability without a movement in the debt-GDP ratio. The shock reflects investors' perception of the government's willingness and capability to service its debt and may not be driven by underlying macroeconomic conditions. In the absence of the shock, the probability of default increases only when the government debt-GDP ratio increases. With a positive realization of ϵ_t^P, the fiscal limit distribution shifts down, increasing the probability of default, even if the debt-GDP ratio remains unchanged. This feature captures certain aspects of the European debt crisis — namely that the sharp deterioration in the fiscal outlook of the Greek government may have undermined investors' beliefs in Italian government fiscal space, effectively shifting down the distribution of Italian fiscal limits. In such a scenario, a debt level that would have been sustainable during normal times may become unsustainable during the crisis, leading to a rapid rise in interest rates. In Section 4, we explore the importance of a shift in the fiscal limit and highlight the transmission mechanism through default versus liquidity risks.

Finally, in addition, we assume that the lump-sum tax follows a rule to ensure debt is at least partially financed over time:

$$\frac{t_t - t}{t} = \phi^T \frac{Q^b_{t-1} b_{t-1} - Q^b b}{Q^b b}, \tag{2.7}$$

where ϕ^T denotes the response of lump-sum taxes to deviations of the value of debt from its steady state level.

2.3 Production The model includes three different types of production firms, similar to Sims and Wu (2021). A representative wholesale firm produces output using labor and its own capital. These firms also face a loan-in-advance constraint and issue long-term bonds to finance a portion of their capital purchases. Retail firms repackage wholesale output for resale and are subject to a pricing friction. In addition, a representative investment producer generates new capital using Home and Foreign retail goods.

Investment Producers Competitive investment producing firms use Home and Foreign goods, in the same Armington form as consumption, to obtain composite investment I_t. In turn, composite investment is used to produce new capital I^w_t and is sold to wholesale firms at a price P^k_t, or in real terms $p^k_t \equiv P^k_t / P_t$. Investment producers make optimal decisions on I_t to maximize the present value of expected future profits. The production function is $I^w_t = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$, where $S(.)$ denotes an investment adjustment cost.

Wholesale Firms A representative wholesale firm produces output y^w_t according to

$$y^w_t = l_t^{1-\alpha} K^\alpha_{t-1}. \tag{2.8}$$

Private capital K_{t-1} is owned by wholesale firms and evolves according to a standard law of motion with depreciation rate δ,

$$K_t \;=\; I_t^w + (1-\delta)K_{t-1}. \tag{2.9}$$

Similar to Sims and Wu (2021), we assume that the wholesale firm must issue perpetual bonds to finance a fraction η^I of new physical capital, I_t. The loan-in-advance constraint is

$$Q_t^f\left(f_t - \kappa^f\frac{f_{t-1}}{\pi_t}\right) \;\geq\; \eta^I p_t^k I_t^w, \tag{2.10}$$

where f_t denotes the amount of private bonds and Q_t^f their price. The firm chooses labor, investment, and bond issuance to maximize the present value of their profits,

$$\sum_{t=0}^{\infty} E_0\left[\Theta_t\Lambda_{t,t+1}\left(p_t^w y_t^w(1-\tau^i) - w_t l_t - p_t^k I_t^w - \frac{f_{t-1}}{\pi_t} + Q_t^f\left(f_t - \kappa^f\frac{f_{t-1}}{\pi_t}\right)\right)\right], \tag{2.11}$$

subject to equations (2.8), (2.9), and (2.10).

Retail Firms The law of one price holds and implies

$$\rho_{H,t} = rer_t\rho_{H,t}^* \quad \text{and} \quad \rho_{F,t}^* = \frac{\rho_{F,t}}{rer_t}, \tag{2.12}$$

where $rer_t = P_t^*/P_t$ denotes the real exchange rate, $\rho_{H,t}^* = P_{H,t}^*/P_t^*$ and $\rho_{F,t}^* = P_{F,t}^*/P_t^*$.[7]

The retail firm h repackages the wholesale output, $y_t(h) = y_t^w(h)$ and sells it for the price $P_t(h)$. The firm faces a Rotemberg price adjustment cost, where the real cost is denoted by: $\frac{\psi}{2}\left(\frac{P_t(h)}{P_{t-1}(h)}\frac{1}{\pi} - 1\right)^2 y_t$. The firm h sets its price while taking into account the demand for its product, which comes from multiple sources: Home and Foreign private consumers and investment firms and the Home government. The firm chooses labor and its price to optimize real profits. Total demand for good h is given by

$$y_t(h) = p_t(h)^{-\theta^c}\left(c_{H,t} + c_{H,t}^* + i_{H,t} + i_{H,t}^* + g\right) \tag{2.14}$$

[7] In general, the law of one price implies that for a nominal exchange rate ϵ_t,

$$P_{F,t} = \epsilon_t P_{F,t}^*, \quad P_{HF,t}^* = \frac{P_{H,t}}{\epsilon_t}. \tag{2.13}$$

In the above expressions, we have used the fact that the nominal exchange rate in our currency union is one.

where $p_t(h) = P_t(h)/P_t^H$. After imposing equilibrium, optimal price setting implies

$$\frac{p_t^w}{\rho_t^H} = \frac{\theta^c - 1}{\theta^c} + \frac{\psi}{\theta^c}\left(\frac{\pi_t^H}{\pi^H} - 1\right)\frac{\pi_t^H}{\pi^H} - \frac{\psi}{\theta^c}E_t\Theta_t\Lambda_{t,t+1}\left(\frac{\pi_{t+1}^H}{\pi^H} - 1\right)\frac{\pi_{t+1}^H}{\pi^H}\frac{y_{t+1}}{y_t}. \tag{2.15}$$

2.4 Financial Intermediaries Financial intermediaries, represented as a continuum in the unit interval, are structured as in Gertler and Karadi (2011) and Sims and Wu (2021). Each period, a fraction of financial intermediaries stochastically exit and are replaced by the same number of new intermediaries with startup funds from households. Financial intermediaries accumulate net worth until they exit, whereby they return their net worth to households.

The intermediary j purchases Home government bonds $b_t^{H,j}$, as well as private bonds, $f_t^{H,j}$. In addition, we allow the intermediary to purchase Foreign government bonds, $b_t^{F,j}$.[8] These purchases are financed by deposits from domestic households d_t^j and the firm's net worth n_t^j. The balance sheet condition in real terms is given by

$$Q_t^b b_t^{H,j} + Q_t^f f_t^j + Q_t^{b,*} b_t^{F,j} = d_t^j + n_t^j. \tag{2.16}$$

If intermediary j survives, then its net worth evolves as,

$$\begin{aligned} n_t^j = \left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^{H,j}}{\pi_t} + \left(R_t^f - R_{t-1}^d\right)\frac{Q_{t-1}^f f_{t-1}^j}{\pi_t} \\ + \left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^{F,j}}{\pi_t} + \frac{R_{t-1}^d n_{t-1}}{\pi_t}, \end{aligned} \tag{2.17}$$

where $R_t^b - R_{t-1}^d$, $R_t^f - R_{t-1}^d$, and $R_t^{b,*} - R_{t-1}^d$ are, respectively, the excess returns relative to the cost of funding through deposits from holding Home government, Home private, and Foreign government bonds. The realized returns on holding these bonds are

$$R_t^b = (1 - \Delta_t)\frac{1 + \kappa^b Q_t^b}{Q_{t-1}^b}, \quad R_t^f = \frac{1 + \kappa^f Q_t^f}{Q_{t-1}^f}, \quad R_t^{b,*} = \frac{1 + \kappa^{b,*} Q_t^{b,*}}{Q_{t-1}^{b,*}}. \tag{2.18}$$

The return on Home government bonds reflects the possibility that the government could default, in which case there is a haircut on payments.

Following Krenz (2022), we assume that government bonds are assembled in terms of a

[8] Cross-region exchange of multiple assets could induce multiple unit-roots into the model. In the following, we assume home bias in asset preferences, which ensures that there is no unit root dynamics from cross-region asset holdings of financial intermediaries.

CES composite portfolio of Home and Foreign bonds,

$$m_t^{b,j} \;=\; \left[\gamma_b^{\frac{1}{\sigma_b}}\left(Q_t^b b_t^{H,j}\right)^{\frac{\sigma_b-1}{\sigma_b}} + (1-\gamma_b)^{\frac{1}{\sigma_b}}\left(Q_t^{b,*} b_t^{F,j}\right)^{\frac{\sigma_b-1}{\sigma_b}}\right]^{\frac{\sigma_b}{\sigma_b-1}}. \tag{2.19}$$

The parameter $\sigma_b < 0$ denotes the interest rate elasticity of asset demand, and the parameter γ_b denotes the home bias in asset holdings. The assumption of imperfect substitutability between Home and Foreign assets can be motivated by the owners' preference for different asset types, different attitudes towards risks across regional assets, and differential convenience benefits due to institutional differences across countries, see Alpanda and Kabaca (2020) and Krenz (2022).

In each period, a fraction $1-\sigma$ of financial intermediaries exit and return their net worth to domestic households. The objective of the intermediary j is to maximize its expected terminal net worth according to,

$$V_t^j = (1-\sigma)E_t\Theta_t\Lambda_{t,t+1}n_{t+1}^j + \sigma\beta E_t\Theta_t\Lambda_{t,t+1}V_{t+1}^j, \tag{2.20}$$

where it discounts future net worth by the stochastic discount factor of households.

If it can generate positive excess returns from investing in bonds, an intermediary would want to expand its assets indefinitely by collecting deposits from households. To limit their ability to do so, we assume that financial intermediaries face a costly enforcement problem as in Gertler and Karadi (2011): At the end of a period, an intermediary may divert a portion of its assets to households, leaving depositors to claim what is left, and forcing the intermediary into bankruptcy. To ensure depositors are willing to lend, an incentive constraint must therefore be satisfied. The constraint is given by,

$$V_t^j \geq \eta_t^v(Q_t^f f_t^j + \theta^b m_t^{b,j}). \tag{2.21}$$

The above condition implies that the value of continuing as an intermediary V_t^j should be larger or equal to the funds that the intermediary j can divert. Should it choose to go bankrupt, the incentive constraint implies that the intermediary can keep a fraction η_t^v of its bonds. This variable captures the tightness of the overall credit market: the higher η_t^v is, the more funds financial intermediaries can divert, making depositors less willing to lend funds. $0 \leq \theta^b \leq 1$ implies that it is easier for the intermediary to divert private bonds than public bonds.

The financial intermediary solves its optimization problem and has the following first-order conditions:

$$E_t \Theta_t \Lambda_{t,t+1} \Omega_{t+1} \frac{R^f_{t+1} - R^d_t}{\pi_{t+1}} = \frac{\lambda^v_t}{1+\lambda^v_t} \eta^v_t, \tag{2.22}$$

$$E_t \Theta_t \Lambda_{t,t+1} \Omega_{t+1} \left(\frac{R^b_{t+1} - R^d_t}{\pi_{t+1}} \frac{Q^b_t b^H_t}{m^b_t} + \frac{R^{b,*}_{t+1} - R^d_t}{\pi_{t+1}} \frac{Q^{b,*}_t b^F_t}{m^b_t} \right) = \frac{\lambda^v_t}{1+\lambda^v_t} \eta^v_t \theta^b, \tag{2.23}$$

$$E_t \Theta_t \Lambda_{t,t+1} \frac{\Omega_{t+1}}{\pi_{t+1}} R^d_t = \frac{\phi_t}{1+\lambda^v_t} \eta^v_t, \tag{2.24}$$

where λ^v_t is the multiplier on the incentive constraint. Without the incentive constraint, the expected returns on the two assets equal the cost of funds. In our analysis, the incentive constraint binds, meaning that there are excess returns of long-term public and private bonds over the deposit rate. If $\theta^b < 1$, then the excess returns of the government bonds will be lower than those of private bonds. In addition, $\Omega_t = 1 - \sigma + \sigma \eta^v_t \phi_t$, where $\phi_t = \frac{Q^f_t f_t + \theta^b m^b_t}{n_t}$ is an endogenous leverage ratio and all financial intermediaries make the same optimal decisions.

We model liquidity risk in the financial sector as fluctuations in η^v_t. Importantly, the liquidity risk can depend on the probability of default, a mechanism we call the *liquidity risk channel*:

$$\frac{\eta^v_t}{\bar{\eta}^v} = 1 + \phi^\eta \left(\exp\left(\phi^\eta_s \left(\frac{s_{t-1} + \epsilon^P_t}{\bar{s}} - 1 \right) \right) - 1 \right). \tag{2.25}$$

If $\phi^\eta > 0$, the liquidity risk channel is present and η^v_t increases with the government debt-GDP ratio, as well as with the change in the fiscal limit. That is, the liquidity risk increases with the probability of sovereign default. This channel captures the idea that a deterioration in fiscal fundamentals or investors' perception of the fiscal limit can directly tighten the financial intermediary's incentive constraint. If $\phi^\eta = 0$, the liquidity risk channel is not present and the default probability does not have a direct impact on liquidity conditions.[9]

Finally, we assume that newly entrant intermediaries receive start-up funds from households, denoted by x in real terms. The aggregate net worth in the financial intermediary sector evolves as,

$$\begin{aligned} n_t \;\; = \;\; & \sigma \left[\left(R^b_t - R^d_{t-1}\right) \frac{Q^b_{t-1} b^H_{t-1}}{\pi_t} + \left(R^f_t - R^d_{t-1}\right) \frac{Q^f_{t-1} f_{t-1}}{\pi_t} + \left(R^{b,*}_t - R^d_{t-1}\right) \frac{Q^{b,*}_{t-1} b^F_{t-1}}{\pi_t} \right] \\ & + \sigma R^d_{t-1} \frac{n_{t-1}}{\pi_t} + (1-\sigma) x. \end{aligned} \tag{2.26}$$

[9] Bocola (2016) shows that, in a model where financial intermediaries' incentive constraints do not always bind, sovereign default can force a fire sale of their assets. Thus, anticipation of a possible default and the associated fire sale can tighten the liquidity conditions of financial intermediaries prior to default. Our liquidity risk channel captures this spillover from fiscal risks to liquidity conditions.

2.5 Policy Interventions The central bank sets a common monetary policy for the two regions by following a Taylor-rule for the economy-wide nominal interest rate:

$$\ln \frac{R_t^d}{R^d} = \phi_\pi \ln \frac{\pi_t^{ag}}{\pi^{ag}} + \phi_y \ln \frac{y_t^{ag}}{y^{ag}} \tag{2.27}$$

The monetary authority responds to variation in the weighted average of consumer price inflation, $\ln \frac{\pi_t^{ag}}{\pi^{ag}} = 0.5 \ln \frac{\pi_t}{\pi} + 0.5 \ln \frac{\pi_t^*}{\pi^*}$, and the weighted average of output in each region, $\ln \frac{Y_t^{ag}}{Y^{ag}} = 0.5 \ln \frac{Y_t}{Y} + 0.5 \ln \frac{Y_t^*}{Y^*}$.

We model the unconventional policy of liquidity injections directly on the issuance of new public bonds. In equilibrium, the total public bonds satisfy

$$b_t = b_t^{cb} + b_t^H + b_t^{H,*} rer_t. \tag{2.28}$$

The central bank's holdings of public bonds are denoted as $Q_t^b b_t^{cb} = re_t$. Following Sims and Wu (2021), we assume that the operating surplus is returned to households via a lump-sum transfer:

$$T_t^{cb} = R_t^b Q_{t-1}^b \frac{b_{t-1}^{cb}}{\pi_t} - Q_t^b b_t^{cb}. \tag{2.29}$$

In steady state, the central bank does not hold any public bonds with $b_t^{cb} = 0$. During a crisis, the central bank can raise reserves to inject liquidity into the financial market. We model this as

$$re_t = re + \phi_{cb}(R_t^{spread} - R^{spread}), \tag{2.30}$$

where $R_t^{spread} \equiv E_t R_{t+1}^b - R_t^d$ measures the spread between the expected returns on government bonds and deposits. When $\phi_{cb} > 0$, the central bank injects credit when there is an increase in the spread on government bonds. Gertler and Karadi (2011) consider a similar rule for a closed-economy model without default risk.

2.6 Goods Market Clearing and Additional Equilibrium Conditions The Home goods market clearing condition implies that $y_t = c_{H,t} + c_{H,t}^* + i_{H,t} + i_{H,t}^* + g$. Adjustment of relative prices between countries is summarized by the condition linking the real exchange rate to relative inflation (since the nominal exchange rate is fixed): $rer_t / rer_{t-1} = \pi_t^* / \pi_t$.

The equation for net foreign asset accumulation can be written as follows:

$$
\begin{aligned}
b_t^i + Q_t^{b,*} b_t^F - Q_t^b b_t^{H,*} rer_t &= \frac{R_{t-1}^d b_{t-1}^i}{\pi_t} + \frac{R_t^{b,*} Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t} \\
-R_t^b Q_{t-1}^b \frac{b_{t-1}^{H,*} rer_{t-1}}{\pi_t} &+ \rho_t^H (c_{H,t}^* + i_{H,t}^*) - \rho_t^{F,*} rer_t (c_{F,t} + i_{F,t})
\end{aligned} \tag{2.31}
$$

Appendix A lists all the equilibrium conditions for the model.

3 Calibration and Solution Method

Having laid out our theoretical framework, in this section, we discuss how we calibrate and solve the model.

3.1 Calibration We apply a symmetric calibration to the deep parameters that are common across countries. To have a steady-state price markup of 10 percent, we let $\theta^c = 11$. The price adjustment cost parameter, ψ, is calibrated to replicate firms adjusting prices 25% of the time in a Calvo-type Phillips curve in the absence of strategic price complementary.[10] The Frisch elasticity is set to 1, and we also assume logarithmic preferences ($\sigma_c = 1$). The investment adjustment cost $S\left(\frac{I_t}{I_{t-1}}\right)$ takes the functional form of $\frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, and the investment adjustment cost parameter ω_I is set to 2.

For the endogenous discount factor, we assume $\Theta_{t+1}/\Theta_t = \beta_c(1 + c_t)^{-\omega_\beta}$ with $\Theta_0 = 1$; we calibrate ω_β to 0.01 as in Devereux and Sutherland (2009). The small value ensures that the endogenous discount factor has only a limited impact on the dynamics. We assume that the steady-state real interest rate is 2 percent annualized, which implies $\beta(c) = 0.995$. Given our calibration of ω_β, we set β_c to ensure the calibration of $\beta(c)$. The elasticity of substitution between goods is set to $\phi^c = 1.3$ and the degree of home bias α_H to 0.7, in the range of estimates of Albonico, Calès, Cardani, Croitorov, Ferroni, Giovannini, Hohberger, Pataracchia, Pericoli, Raciborski, and Rat (2017). For monetary policy, the Taylor rule parameters are set to standard values with $\phi_\pi = 1.5$ and $\phi_y = 0.15$.

The financial and government sectors play a key role in our results. For these two sectors, we calibrate the Home country using Italian data and the Foreign counterpart using German data, as summarized in Table 1. The steady-state value of the public debt-GDP ratio is set to 105 percent for Home and 60 percent for Foreign, roughly in line with the average level of government debt in Italy and Germany between 2000 and 2008. The rest of the fiscal variables are calibrated following Bianchi, Melosi, and Rogantini-Picco (2023).[11] The

[10] The mapping implies $\psi = \theta^c/[(1-\xi_p)/\xi_p(1-\beta\xi_p)]$, where $\xi_p = 0.75$ is the Calvo adjustment parameter.
[11] See the European Commission, DG Taxation and Customs Union, Taxes in Europe database and IBFD

government expenditures-to-GDP ratio is calibrated to 19 percent, the consumption tax is set to 0.22, and the income tax is calibrated to 0.2 for the Home country to match Italian fiscal data. The average duration of government debt, $1 - 1/\kappa^b$, is set to 7 years. In the Foreign country, government expenditures are set to 20 percent of GDP, the consumption tax to 0.19, and the income tax to 0.25 to be consistent with German fiscal data. The average duration of government debt, $1 - 1/\kappa^b$, is set to 6 years. We calibrate the response of lump sum taxes to debt, ϕ^T, as 3. We assume that the haircut parameter, δ_b, is 0.1, implying that upon default, the Home government takes a haircut of 40 percent at an annual rate.

Turning to the financial sector, we target an excess return over the deposit rate, $R^f - R^d$, of 3 percent annualized in both Home and Foreign to match the spread between the non-financial lending and deposit rates. Following Sims and Wu (2021), we use the total credit to private non-financial sector to calibrate the outstanding private debt to annualized GDP, $\frac{Q^f f}{4y}$, which is set to 1.1 for Home to match Italian data and 1.2 for Foreign to match German data. With investment accounting for 17 percent of GDP in the Home country, the fraction of investment the wholesale firms must finance by issuing debt, η^I, is 0.65. For the Foreign country, investment accounts for 16 percent of GDP and 75 percent of investment is financed through corporate debt. We calibrate the private debt maturity, $1 - 1/\kappa^f$, to 10 years in both countries. The interest rate elasticity of asset demand, σ_b, is calibrated to -2 for both countries in the baseline case, in line with Poutineau and Vermandel (2015). We also calibrate the parameter governing home bias in asset holdings, γ_b, to 0.7 for Home and 0.8 for Foreign, which respectively match the shares of government debt held by the domestic financial sectors in Italy and Germany according to Eurostat. The rest of the financial variables are calibrated using similar metrics from the existing literature (Sims and Wu, 2021; Gertler and Karadi, 2011). Financial intermediaries have a survival probability, σ, of 0.95. The value of the initial funding for new financial intermediaries, x, is chosen to be consistent with a leverage ratio of 4.

Finally, we calibrate the fiscal limit distribution to match the Italian data during the European debt crisis, as shown in Figure 1. Before the crisis, the Italian government debt was 105 percent of GDP, while its long-term yield spread against the German counterpart was around 0.2 percentage points. During the crisis, Italian debt rose to 120 percent of GDP in 2012 when its yield spread peaked close to 5 percentage points. After the crisis, the yield spread declined and reached 1 percentage point in 2015, while Italian debt remained elevated at 135 percent of GDP. We calibrate the parameters that govern the logistic function, equation (2.6), and the fiscal limit shock to match these periods in the Italian data before, during, and after the debt crisis.

data.

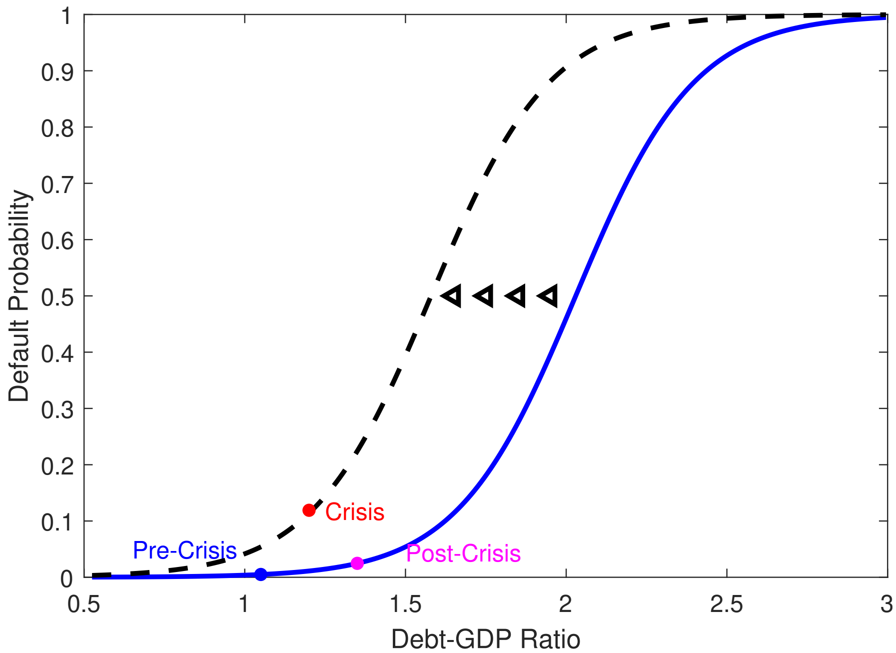
Table 1: Calibration

Parameter	Value	Description
		Home Country
κ^f	$1-40^{-1}$	Coupon decay parameter for private bonds
κ^b	$1-28^{-1}$	Coupon decay parameter for government bond
η^I	0.65	Fraction of investment from debt
ϕ	4	Leverage ratio
η^v	0.59	Recoverability parameter
$\frac{Q^f f}{4y}$	1.1	Private bonds as share of GDP
$\frac{Q^b b}{4y}$	1.05	Government bonds as share of GDP
τ^c	0.22	Consumption tax rate
τ^i	0.2	Income tax rate
$\frac{g^c}{y}$	0.19	Government consumption as share of GDP
		Foreign Country
$\kappa^{f,*}$	$1-40^{-1}$	Coupon decay parameter for private bonds
$\kappa^{b,*}$	$1-24^{-1}$	Coupon decay parameter for government bond
$\eta^{I,*}$	0.75	Fraction of investment from debt
ϕ^*	4	Leverage ratio
$\eta^{v,*}$	0.59	Recoverability parameter
$\frac{Q^{f,*} f^*}{4y^*}$	1.2	Private bonds as share of GDP
$\frac{Q^{b,*} b^*}{4y^*}$	1.05	Government bonds as share of GDP
$\tau^{c,*}$	0.19	Consumption tax rate
$\tau^{i,*}$	0.25	Income tax rate
$\frac{g^{c,*}}{y^*}$	0.2	Government consumption as share of GDP

Specifically, we calibrate η_0^{FL} and η_s^{FL} by targeting the debt-GDP ratio and the yield spread prior to and after the crisis: when debt-GDP was 1.05, the probability of default was 0.5 percent, consistent with a yield spread of 0.2 percentage points assuming a haircut of 40 percent annually; when debt-GDP was 1.35, the probability of default was 2.5 percent, consistent with a yield spread of 1 percentage point.[12] As shown in Figure 2, the solid blue line shows the fiscal limit distribution in the case without a shock to the distribution, $\epsilon_t^P = 0$, with the blue dot representing the data point prior to the crisis and the purple dot after the crisis. Finally, to calibrate the shock to the fiscal limit distribution, we match the data during the crisis with $(\hat{s}_c, \hat{p}_c) = (1.2, 0.12)$. The black dashed line shows the shifted fiscal limit distribution during the crisis with the red dot representing the data point in 2012.

[12] A property of the logistic function is that for any given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters η_0^{FL} and η_s^{FL} can be uniquely determined by $\eta_s^{FL} = \frac{1}{\tilde{s}-\hat{s}} \log\left(\frac{\tilde{p}}{\hat{p}}\frac{1-\hat{p}}{1-\tilde{p}}\right)$, and $\eta_1^{FL} = \log\frac{\tilde{p}}{1-\tilde{p}} - \eta_s^{FL}\tilde{s}$.

Figure 2: Fiscal Limit Distribution



3.2 Solving the Model Given the evolution of the haircut (2.5) and the default probability (2.6), the model is an endogenous regime-switching model. We introduce a regime variable def_t, and note that there are two regimes: one in which there is no default and one with default, denoted $def_t = 0$ and $def_t = 1$, respectively. The haircut can then be written as

$$\Delta_t = \begin{cases} 0, & \text{if } def_t = 0 \\ \delta_b, & \text{if } def_t = 1 \end{cases} \tag{3.1}$$

The transition matrix is time varying depending on the state $s_t = \frac{Q_t^b b_t}{4y_t}$ and the underlying macroeconomic shocks, and has elements $\mathbb{P}_{ij,t} = \Pr\left(def_{t+1} = j | def_t; s_t, \epsilon_t^P\right)$. Using our assumption that the transition follows a logistic function, we have

$$\mathbb{P}_t = \begin{bmatrix} \mathbb{P}_{00,t} & \mathbb{P}_{01,t} \\ \mathbb{P}_{10,t} & \mathbb{P}_{11,t} \end{bmatrix} = \begin{bmatrix} 1 - pdef_t & pdef_t \\ 1 - pdef_t & pdef_t \end{bmatrix},$$

where the probability of default in $t + 1$, which is a function of time t variables, is

$$pdef_t = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_t + \epsilon_t^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_t + \epsilon_t^P)]}.$$

In order to solve the model, we use the perturbation approach to solve endogenous regime-switching models in Benigno, Foerster, Otrok, and Rebucci (2024), which generates a set of approximated decision rules conditional on each regime. Since Benigno, Foerster, Otrok, and Rebucci (2024) show that first-order approximations to the decision rules are insufficient for capturing behavior induced by endogenous probabilities, we approximate our model to the second order. This second-order approximation captures important features of our model, such as households and firms internalizing the fact that default becomes more likely as the debt-GDP ratio increases. Appendix B provides additional details on the solution procedure.

4 Anatomy of a Debt Crisis

In this section, we investigate the importance of default risk and its transmission through various channels. In all of the following scenarios, we consider an unexpected exogenous increase in the debt level combined with a downward shift in the fiscal limit (i.e., $\epsilon_t^p > 0$).[13] Both of these factors trigger a "debt crisis" by raising the default risk in the Home country, and, by extension, increasing the liquidity risk. This exercise is motivated by the dynamics in Italy during the 2012 European debt crisis, when there was a rapid accumulation of debt coupled with a deterioration in market sentiment, as shown in Figure 1.

We start by illustrating the effects of the debt crisis in our model under the baseline assumption that the central bank does not perform targeted asset purchases. We then decompose our baseline results by highlighting the quantitative importance of two key features of the model. First, we show that liquidity risks crucially amplify the effects of default risk on both the financial market and macroeconomy. Second, we show that shifts in the fiscal limit, rather than outright increases in debt, have more quantitative significance, suggesting that market sentiment was key to understanding the 2012 crisis.

4.1 Baseline Consider increases in Home default and liquidity risks brought forth by an increase in Home government debt coupled with a leftward shift in the Home fiscal limit distribution. Figure 3 shows the responses following this change for the Home financial intermediary. In this baseline case, we assume that the central bank does not conduct asset purchases (i.e., $\phi_{cb} = 0$ in equation 2.30) to highlight the transmission mechanism.

A higher supply of government bonds tightens financial conditions. At $t = 0$, government debt increases from its steady state by 6 percent, in line with the Italian data in Figure 1. The higher bond supply depresses the price of government bonds and induces a fire sale of private bonds so that the financial intermediary can meet its balance sheet constraint. Asset prices decline for both private and government bonds, lowering the financial intermediary's

[13] Abstracting from the drivers of this accumulation, such as changes in government spending or taxes, enables us to solely focus on the economic effects of higher debt rather than the effects of its sources.

net worth. With a larger amount of government debt to absorb and a lower net worth, the financial intermediary faces a higher leverage ratio, and thus demands a higher excess return on government bonds, which is the spread between the expected return on government bonds and the risk-free deposit rate paid to households.

This transmission mechanism is strengthened by two factors: higher default risk and higher liquidity risk. As shown in the bottom right panel, the default probability increases to close to 7 percent with the higher level of government debt and accompanying shift in the fiscal limit. A higher likelihood of receiving a haircut prompts the financial intermediary to demand a lower price for the government bond, further tightening financial conditions. Importantly, with $\phi_\eta > 0$, the liquidity risk channel further amplifies this effect. The higher probability of default transmits to a higher fraction of divertable assets, and, subsequently, a more binding incentive constraint further tightens the credit market.

Turning to the impact beyond the financial sector, Figure 4 compares the macroeconomic responses between the Home and Foreign economies. The solid blue lines show that in the Home economy, the higher government bond supply crowds out private investment through tighter financial conditions. The labor supply also declines, and the real wage falls. Lower investment, coupled with a lower labor supply, leads to a sustained decline in output. Consumption increases initially, but quickly declines as income falls. Tighter financial conditions also increase the relative price of Home goods ($\rho_{H,t}$), as firms face higher financing costs. With home bias in goods, this implies higher overall inflation.

In contrast, the Foreign economy, shown in dashed blue lines in Figure 4, sees an increase in investment. Tightened home financial conditions prompt an increase in Foreign asset holdings. The capital outflow raises foreign investment. Supported by higher investment, Foreign output increases over the medium term. Its consumption also rises, and stronger demand raises Foreign inflation.

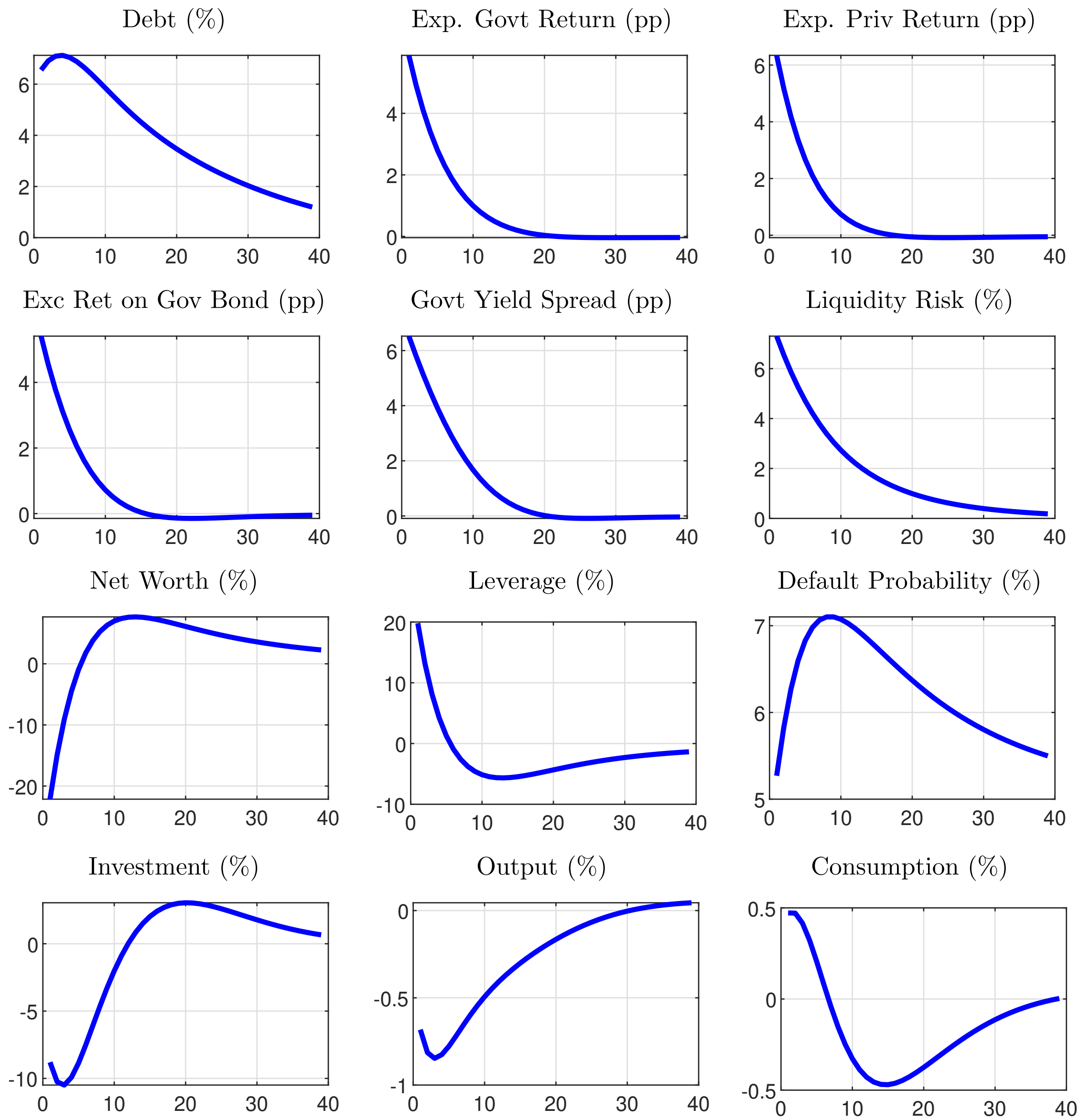
4.2 Default risks vs. liquidity risks

In the baseline case, both default and liquidity risks contribute to the deterioration in Home financial conditions and macroeconomic performance. Figure 5 distinguishes the impact of these various risks.

The direct impact from the risk of Home sovereign default is modest. To see this, the dashed black lines show the case with only default risk (that is, the liquidity risk channel is turned off, $\phi_\eta = 0$). The possibility of receiving a haircut in the future prompts the financial intermediary to demand a higher expected return on government bonds, tightening financial conditions. Net worth decreases, while leverage rises. The tighter financial conditions weigh on the private bond market and depress investment.

When interacted with the liquidity risk channel, higher default risk implies a more pro-

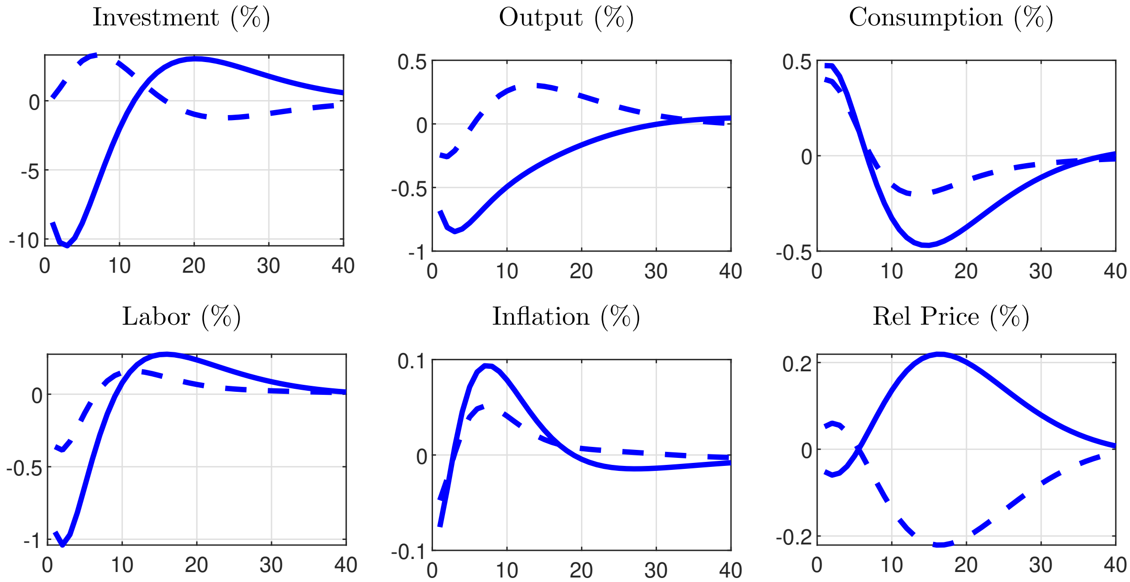
Figure 3: Response to a Default Risk Crisis, Home Country



Notes: All variables are plotted as deviations from their stochastic steady states. Expected spread and deposit rate are shown in annualized rates, while default probability is in level.

nounced deterioration in the financing conditions and economic outlook. To see this, the gap between the solid and dashed lines reflects the amplification from the liquidity risk channel. In this case, higher government debt directly tightens the incentive constraint since $\phi_\eta > 0$. A more binding constraint amplifies the decline in asset prices and net worth, as well as the rise in leverage. The quantitative impact is much more significant in this case. For instance, the default risk reduces investment by 3 percent without the liquidity risk channel, but it lowers investment by 10 percent when the default risk also triggers liquidity risks.

Figure 4: Response to a Default Risk Crisis, Home vs. Foreign Country



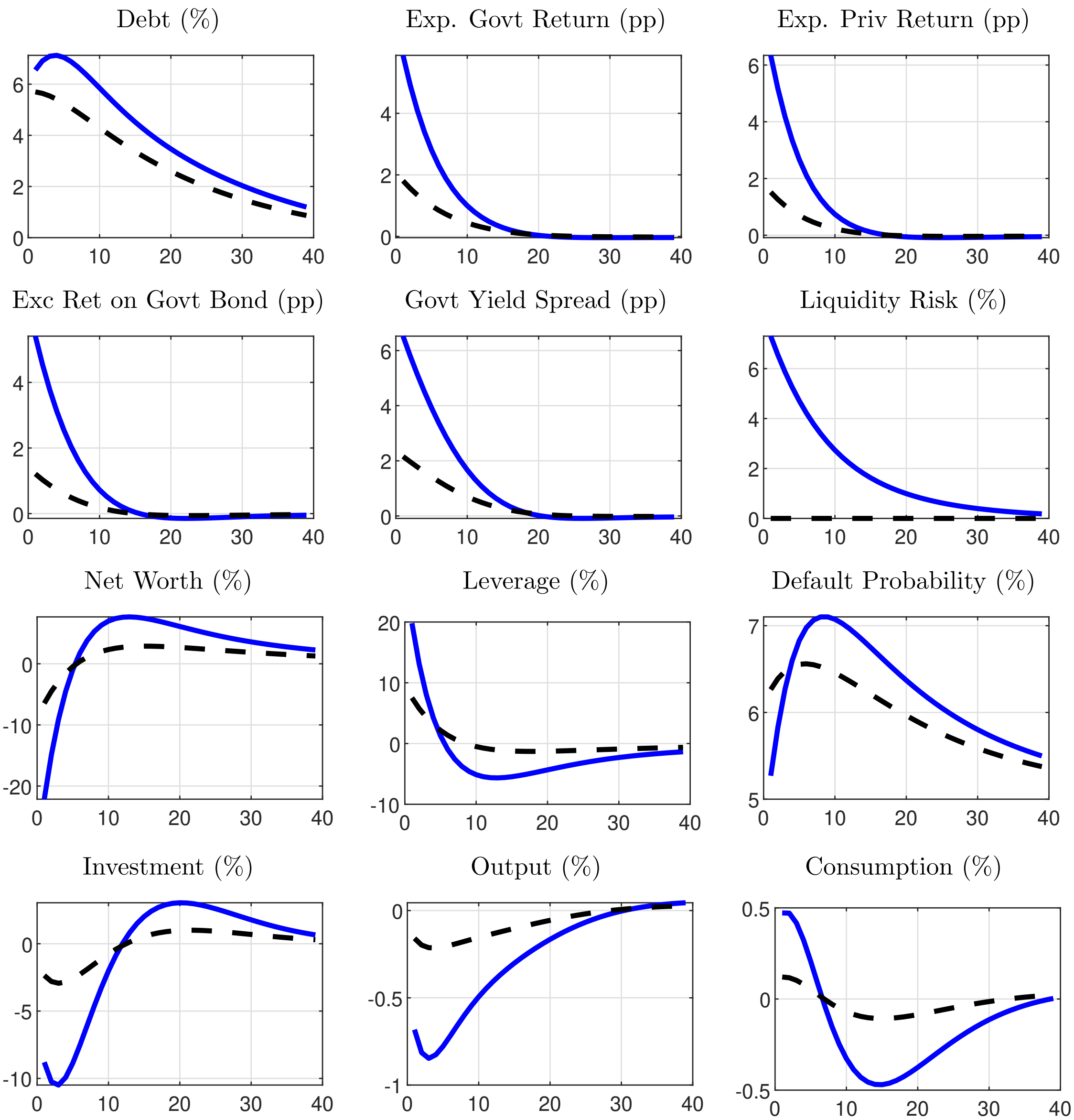
Notes: Blue solid lines are responses for the Home economy and blue dashed lines are for the Foreign economy. All variables are plotted as deviations from their stochastic steady states.

4.3 Shifts in fiscal limit Figure 6 plots an alternative scenario in which there is no change in investor perceptions during the debt crisis. To do so, the dashed black lines show an alternative case where there is no shift in the fiscal limit ($\epsilon_t^p = 0$). For comparison, the solid blue lines show the baseline case following a rise in government debt coupled with a shift in the fiscal limit. The comparison shows that the shift in the fiscal limit plays a more important role in explaining the quantitative impact, while both higher government debt and a deterioration in the fiscal limit tighten financial conditions and depress private investment. The initial increase in government debt contributes to a decline of 3 percent in investment out of the overall contraction of more than 10 percent.

The results highlight the importance of investor sentiment on the financial market as well as the macroeconomy. During the European debt crisis, the sharp deterioration in the fiscal outlook of the Greek government may have undermined investors' beliefs in Italian government fiscal space, effectively shifting down the distribution of Italian fiscal limits. In such a case, a debt level that would have been sustainable during normal times can become unsustainable during the crisis.

4.4 Discussion The effects of a debt crisis depend on the nature of the crisis. To highlight this, Table 2 summarizes the results of the various scenarios considered in this section. The table displays the maximum change for select variables in each case, as well as the change in the same measures in Italy over the period 2008-2014.

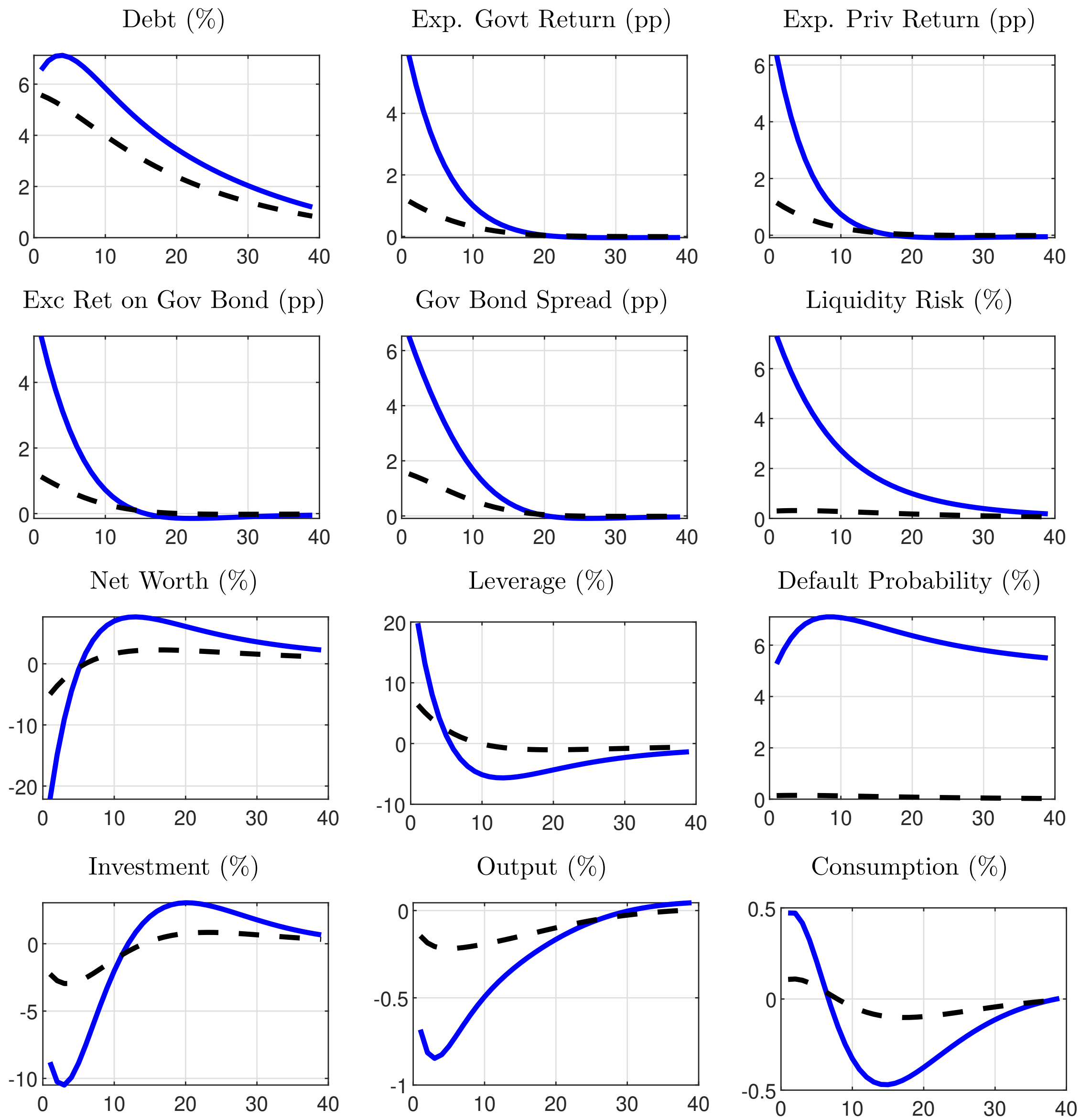
Figure 5: Response to a Default Risk Crisis, with and without Liquidity Risk



Notes: Solid blue lines show the response in the baseline case with both default and liquidity risk channels. Dashed black lines show the response in the alternative case with default risks but abstracting from the liquidity risk channel.

In the baseline model scenario, a debt crisis occurs after an increase in government debt and a leftward shift in the fiscal limit. In turn, a higher probability of default induces greater liquidity risks. By design, this case matches the change in government debt in the data well, as we assume an initial increase in government debt of 6 percent. It also matches the yield spread and excess return on government debt, with the model changes only slightly higher. In addition, it accounts for one third of the total change in investment in the data over this

Figure 6: Response to a Fiscal Limit Shift



Notes: Solid blue lines show the response in the baseline case with a shift in fiscal limits. Dashed black lines show the response in the alternative case with an initial increase in Home government debt but without a shift in fiscal limits.

period, and half of the post-2010 fall. This amount is nontrivial, as Italian investment was already depressed over this period due to the Global Financial Crisis.

The remaining model scenarios demonstrate how important select model features are to ensure that the dynamics are coherent with the data. Without the liquidity risk channel or the shift in the fiscal limit, the model fails to capture the change in the yield spread and the excess return on government debt. Moreover, the change in investment is roughly only a

Table 2: Changes Following a Debt Crisis

	Data	Baseline	No Liquidity Risk	No FL Shift	No Debt Change
Debt	6.1	7.1	5.7	5.6	2.1
Investment	-34	-10.3	-2.4	-2.2	-7.3
Yield Spread	5.0	6.5	2.2	1.5	4.9
Excess Return	5.1	5.4	1.2	1.1	4.2

quarter of the change in the baseline model. Alternatively, if there is a change in investors' sentiment without an actual change in debt (i.e., only a shift in the fiscal limit), the model cannot account for actual fiscal fundamentals.

5 Targeted Credit Interventions During a Crisis

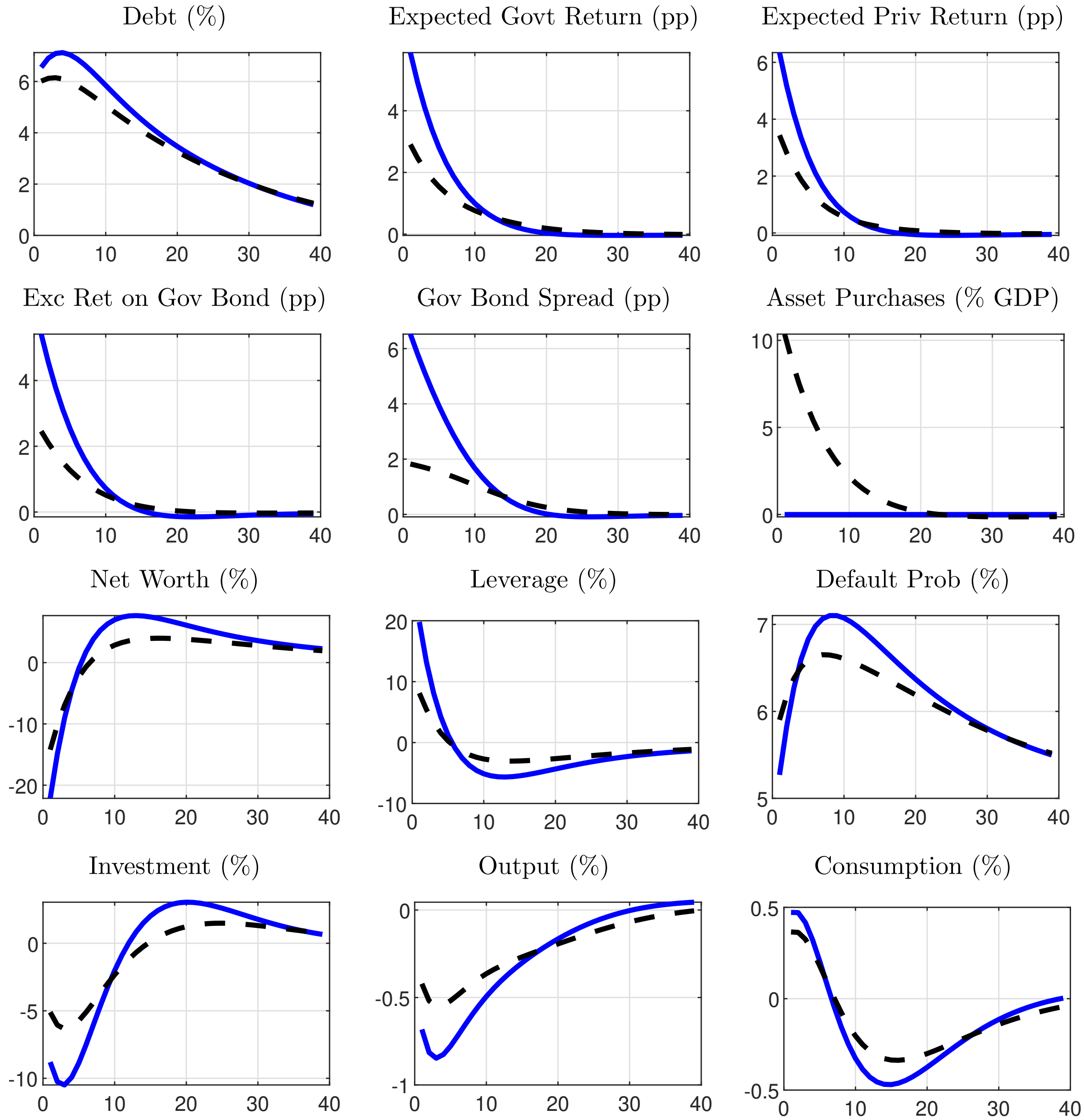
In this section, we consider the effectiveness of asset purchases by the central bank following a debt crisis. We show that a credit intervention can effectively dampen the negative effects of a debt crisis.

We now consider how central bank credit interventions can help mitigate the effects of the crisis. Following the initial increase in government debt and shift in the fiscal limit, we assume that the central bank can conduct asset purchases in response to movements in the Home credit spread, as dictated by equation (2.30). To understand the effect of asset purchases, we compare the responses of the baseline case with those from a case in which the central bank conducts asset purchases. Figure 7 displays the responses in both cases for the Home economy.

The central bank's credit policy, as modeled in this framework, can help stabilize the Home economy. To see this, Figure 7 shows the responses with the endogenous asset purchases (dashed black lines), as well as the responses from the baseline case (solid blue lines). The central bank's asset purchases significantly moderate the rise in government bond yields and improve the financial conditions relative to the baseline case. With asset purchases, there is a more subdued decline in net worth and a less sharp increase in leverage. Macroeconomic conditions are also improved, as the declines in investment and output are more muted. Endogenous asset purchases imply lower funding costs for firms, which dampens the increase in Home relative prices and Home inflation.

The impact of the credit policy depends on the financial conditions. In Figure 8, we consider two cases: 1) the baseline case with an initial increase in Home government debt as well as a shift in the fiscal limit; and 2) an alternative case with only the initial increase in debt but without a shift in the fiscal limit. In each case, we plot the impulse response

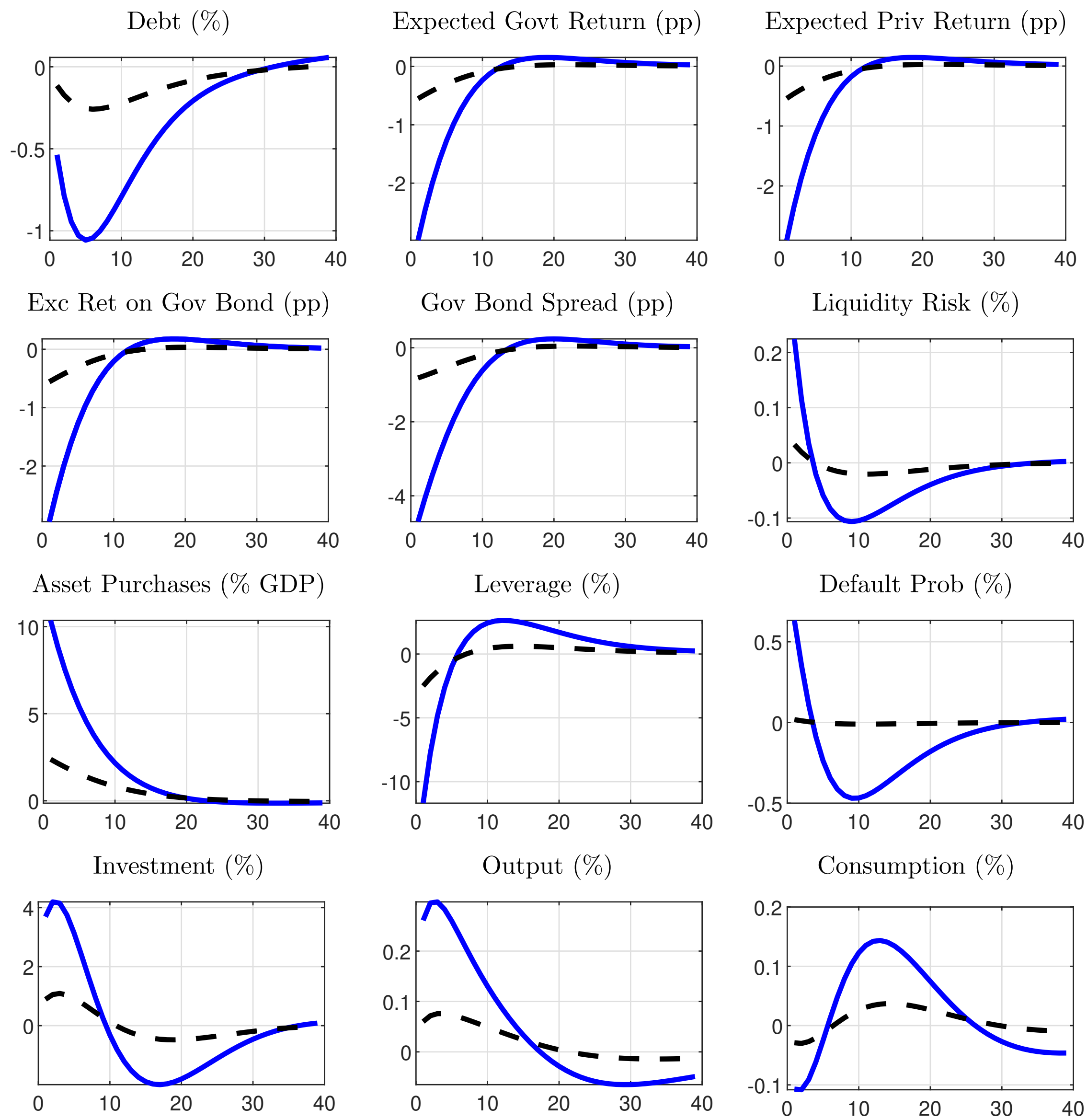
Figure 7: Response to a Default Risk Crisis, Effects of Asset Purchases



Notes: Home responses following an increase in Home government debt. Solid blue lines show the responses in the baseline case, and dashed black lines show the case with endogenous asset purchases.

differences as a result of asset purchases. The solid blue lines show the stabilization impact associated with asset purchases in the baseline case, that is, they capture the differences between the two simulations in Figure 7. The dashed black lines highlight the impact of credit policy in the case without a shift in the fiscal limit. While asset purchases still lower the government bond yield and stabilize investment in the latter case, the magnitude is much smaller than in the baseline case. This is consistent with Figure 6, as the shift in the fiscal limit perceived by investors plays a more important role in the deterioration in financial

Figure 8: Relative Effectiveness of Asset Purchases, with and without a Fiscal Limit Shift



Notes: This chart shows the response differences between the case with and without asset purchases. Solid blue lines show the baseline case with an increase in Home government debt as well as a shift in fiscal limit. Dashed black lines show the alternative case with an increase in Home government debt while fiscal limit remains unchanged.

conditions.

To summarize, Figure 7 shows that targeted asset purchases are effective in stabilizing financial markets and the macroeconomy in response to debt crises. The magnitudes of the responses indicate that if the ECB had the OMT in place prior to the 2012 debt crisis, the same shift in Italian fiscal conditions could have led to a reduction in the government bond spread that was about a third of what occurred, and the transmission to investment from this channel would have been nearly halved. In addition, Figure 8 highlights that the targeted asset purchases are especially effective when the fiscal limit shifts inward, as default and liquidity risks are partially driven by non-fundamental factors. This point is of great relevance to the ECB asset purchase policy. Investor sentiment arguably played an important role in Italy's experience during the 2012 crisis. More recently, containing non-fundamental risks were a key consideration when the ECB rolled out the TPI program.

6 Anticipation Effects During and Before Crises

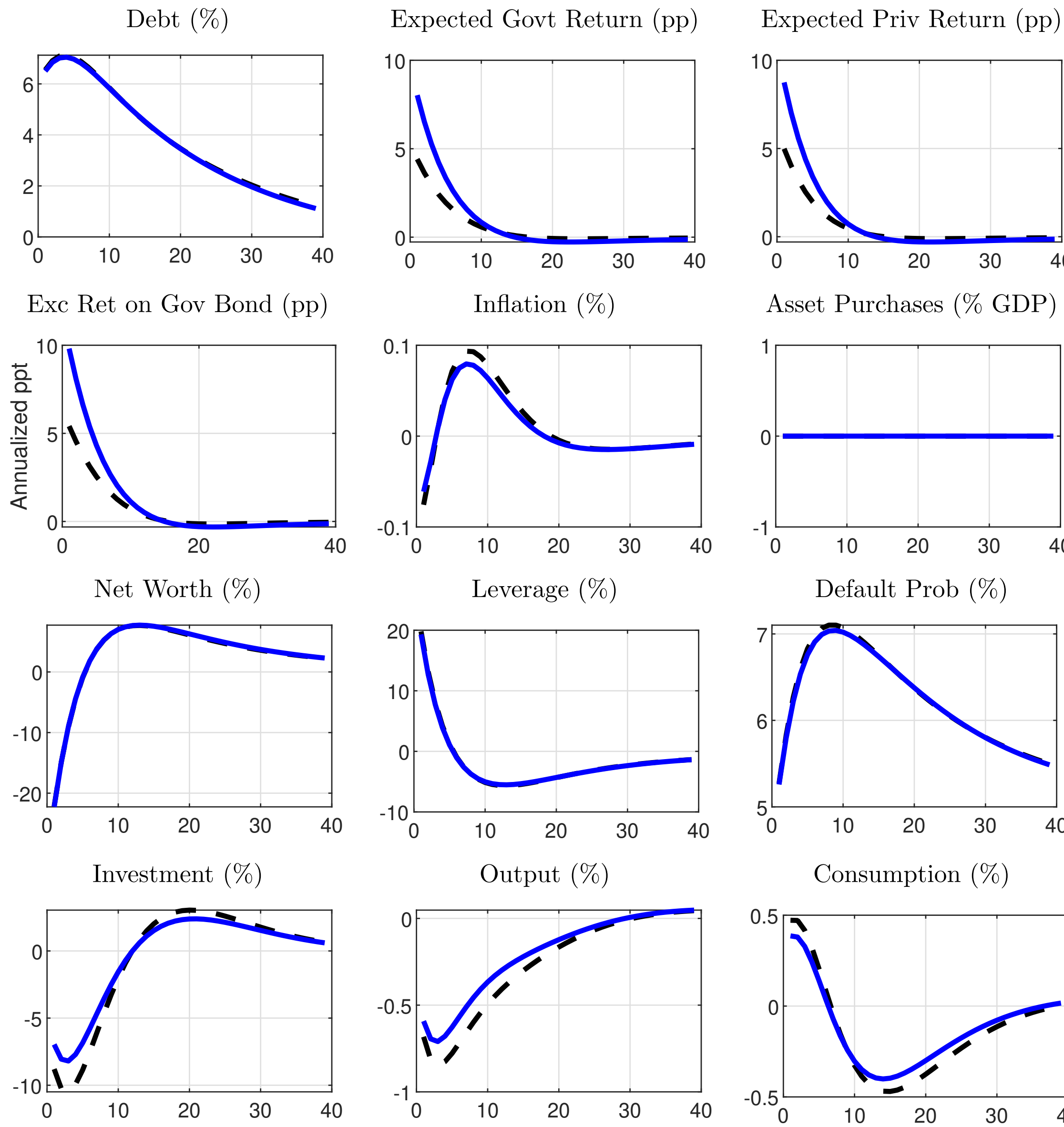
In this section, we consider how the anticipation of a credit intervention alters the macroeconomic environment. Since 2012, the ECB has adopted various asset-purchasing tools targeting default and liquidity risks. Given this, market participants may well anticipate such interventions with the possibility of the next crisis.

We consider two variants of our baseline model. First, we consider anticipation effects during a crisis: the expectation of an intervention helps mitigate the recessionary effects of a debt crisis, even if asset purchases are never implemented. Second, we consider the effect of intervention policy outside of crises, and show that the expectation of a credit intervention alters the economic environment in normal times. Appendix B provides details of these setups relative to our baseline analysis.

6.1 Anticipation Effects During a Crisis Our first model variant studies anticipation effects during a crisis. We show that the expectation of a targeted credit intervention following a debt crisis can have substantial effects. In our baseline model, there is no uncertainty about the future asset purchase policy conditional on macroeconomic conditions – following an increase in debt and shift in the fiscal limit, the central bank either uses asset purchases or does not, depending on the specified policy rule. To study anticipation effects during crises, we amend our baseline model to include a probability p_a that the central bank may conduct asset purchases after the onset of a crisis.

To understand how expectations matter after the start of a debt crisis, Figure 9 plots the responses following a scenario similar to the baseline analysis in Section 4. That is, the figure displays the responses following an unexpected exogenous increase in the level of government debt combined with a downward shift in the fiscal limit (i.e., $\epsilon_t^p > 0$). The dashed black lines show the baseline case without asset purchases ($p_a = 0$), which reproduces our initial results in Section 4. In contrast, the solid blue lines plot the responses with an expectation of a future credit intervention ($p_a = 0.5$). These responses are conditional on staying in the regime where asset purchases are not implemented, but differ from the dashed-line case in that the anticipation of the future likelihood of a credit intervention is present. Put

Figure 9: Anticipated Asset Purchases



Notes: Solid blue lines show the case with expected asset purchases in response to an increase in Home government debt as well as a shift in fiscal limit. Dashed black lines show the baseline case without asset purchase expectation.

differently, both responses have no asset purchases along the impulse response path, but the blue line shows an environment where asset purchases can come into effect in the subsequent periods along the path.

Comparing the responses in Figure 9 shows that the anticipation of asset purchases mutes the recessionary effect of the debt crisis. This result is achieved despite little change in key financial variables, such as the financial intermediary's net worth and leverage. On impact, the expectation of a credit intervention implies that the central bank may purchase government bonds in the future. This raises the expectation of the future price of government bonds, as demand from the central bank would bid up the price. A higher expected price of government bonds increases the expected rate of return today (see equation 2.18), which also causes the spread on government bonds to increase. At the same time, the financial intermediary would like to hold more private bonds once the asset purchase program begins and preemptively starts to increase its holdings today (relative to the baseline). Since the financial intermediary is relatively more willing to extend private loans, investment increases (relative to the baseline), creating an expansionary effect from the anticipation of asset purchases. These two forces – changes in expected returns and changes in private loans – mostly offset each other, accounting for the fact that net worth and leverage are similar across the two cases. Interestingly, the larger increase in the excess return on government bonds when asset purchases are anticipated implies that the central bank would have to react more strongly and purchase a larger amount of government debt in the event it started a credit intervention (see equation 2.30).

6.2 Anticipation Effects Before A Crisis Our next model variant studies anticipation effects before a crisis occurs. In the baseline analysis of section 4, the increase in debt and shift in the fiscal limit is an unexpected shock. We amend the baseline model to include the probability of a debt crisis occurring, which allows us to study how the possibility of crises, and any credit intervention policy during those crises, affects the economy before a crisis actually occurs. With probability p_c, agents expect a one-time large increase in debt coupled with a leftward shift in the fiscal limit, similar to the baseline exercise of Section 4. Following this event, the government can default on its debt, with the probability of default determined by equation (2.6).

This framework gives agents expectations that crises and default can occur, which, in turn, affect prices and quantities before a crisis is realized. Thus, we examine how the average level of the economy during normal times — when there is no crisis — changes with and without asset purchases by the central bank during crises. To do so, we compute the stochastic steady state associated with the "normal" regime, which represents the average from a long simulation where a crisis is never realized.[14] We calculate the stochastic steady state under two alternative scenarios: 1) there is never an intervention after a crisis ($\phi_{cb} = 0$) and 2) there is a credit intervention after a crisis ($\phi_{cb} > 0$). Finally, as a reference point, we calculate the stochastic steady state assuming there is never a crisis, $p_c = 0$, and, thus, no

[14]This concept is similar to the "risky steady state" where agents expect future risk but the realization of shocks (regime changes in our context) is zero (Coeurdacier, Rey, and Winant, 2011; Hills, Nakata, and Schmidt, 2019).

need for an intervention.

First, we only consider the anticipation effects of a crisis and assume that the central bank never performs asset purchases ($\phi_{cb} = 0$). We find that the expectation of a crisis has two opposing effects on bank lending. On the one hand, households are more inclined to have precautionary savings in order to smooth consumption in a possible crisis. This incentive increases household savings through higher deposits, which raises lending and ultimately allows more firms to borrow. On the other hand, a crisis tightens financial conditions, raises leverage, and lowers net worth. Anticipating this possibility lowers the incentive of the financial intermediary to lend.

Importantly, whether the precautionary motive or the financing condition channel dominates depends on the likelihood of a crisis. Table 3 displays the difference between the level of variables in the stochastic steady state under various scenarios relative to a baseline with no crises ($p_c = 0$). Column (A) highlights that a low probability of a crisis, namely a 10% annual probability, leads to higher macroeconomic activity, relative to the baseline without crises. In this case, the precautionary savings motive dominates in equilibrium, and higher bank lending ultimately leads to higher investment, output, and consumption in the stochastic steady state. Foerster (2015) finds similar incentives in a model with financial crises that abstracts from sovereign default. In contrast, column (C) shows that with a high probability of a crisis (20% annually), macroeconomic activity is lower in the stochastic steady state relative to the baseline with no crises, as the effect from financial conditions is stronger with a higher likelihood of a crisis and default. Inflation is also higher in the stochastic steady state in this case, as the cost of capital increases when the financial intermediary is less willing to lend, which ultimately results in inflation.

The distortions to macroeconomic spillovers can also be nontrivial. Expectations of a crisis raise the deposit rate in the stochastic steady state, relative to the baseline with no crises. Since the rate is the same across the union, the higher deposit rate makes the risk-adjusted returns lower for the Foreign financial intermediary. These lower returns lead to a lower net worth and contracts Foreign lending, investment, and output. A higher probability of a crisis strengthens this channel, enhancing the decline in Foreign economic activity, as shown by comparing column (C) relative to column (A).

Next, we evaluate how the expectation of a credit intervention affects non-crisis periods. We assume that before a crisis, the central bank does not perform credit interventions. Once a crisis occurs, the central bank may conduct asset purchases according to equation (2.30) with $\phi_{cb} > 0$. Columns (B) and (D) assume that the central bank conducts asset purchases during a crisis and repeat the analyses of columns (A) and (C). With a credit intervention, both the precautionary savings and the financial condition channels weaken; in turn, net

Table 3: Effects of Expectations of Crises and Asset Purchases, Relative to No Crises

Variable	Low Prob (10% annual) No Purch (A)	Low Prob (10% annual) Purch (B)	High Prob (20% annual) No Purch (C)	High Prob (20% annual) Purch (D)
Output	0.10	0.07	-0.06	0.01
Investment	0.18	0.09	-0.39	-0.24
Consumption	0.04	-0.04	-0.06	-0.20
Net Worth	-0.07	0.14	-0.98	-0.66
Deposit	0.52	0.40	0.46	0.33
Inflation	0.02	0.02	0.05	0.05
Foreign Output	-0.10	-0.10	-0.17	-0.24
Foreign Investment	-0.32	-0.28	-0.57	-0.63

worth improves and deposits decline. Nevertheless, the precautionary savings motive is more prevalent in the case with a low-probability crisis, while the financial condition channel plays a more important role when a crisis is a high probability event. Compared to Column (A), Column (B) shows that macroeconomic activity is lower when a low-probability crisis is accompanied by a credit intervention. In this case, a weakened precautionary savings motive dominates the financial condition channel and leads to lower output, investment, and consumption. In contrast, when a high-probability crisis is accompanied by a credit intervention, Column (D) shows that macroeconomic activity is higher relative to column (C). In this case, a credit intervention helps to improve financial conditions, leading to relatively more lending by the financial intermediary and boosting investment and output.

6.3 Discussion As noted, the ECB has announced asset purchase programs meant to target specific countries in crisis, but these programs have yet to be used. Our first set of results in this section indicate that, when debt crises occur, the mere presence of these programs can lead to less severe crises, as expectations of possible asset purchases help mute the financial fallout and subsequent macroeconomic slowdown. These expectation effects were seen in the fact that the ECB President Mario Draghi's "whatever it takes" speech, along with the announcement of the OMT program in 2012, helped stabilize markets (European Parliament, 2022). Moreover, the second set of results indicate that the presence of these programs has small but meaningful effects in how they shape economic decisions outside of crisis times, with the magnitude of the distortion depending on how frequent crises may occur.

7 Conclusion

In recent years, the European Union has grappled with multiple crises that have affected sovereign default risks and liquidity risks of Union members. In response, the European Central Bank has introduced various credit policies to stabilize the economy when subject to rising default and liquidity risks. In this paper, we quantify the efficacy of asset purchases in a two-country monetary-union framework subject to both default and liquidity risks.

Following a notable increase in the probability of sovereign default from a rise in government debt, we find that both default and liquidity risks dampen economic and financial conditions. However, the quantitative effects depend crucially on the presence of the liquidity risk channel. While the possibility of default lowers the net worth of financial intermediaries, asset prices, and economic activity, the quantitative impact is moderate when there is no liquidity risk channel.

These quantitative effects have important implications for the effectiveness of asset purchases. The model suggests that credit policies can help stabilize the economy in the presence of default or liquidity risks, as in either case the policy aims to offset the increase in excess returns on government bonds. Lowering this return helps alleviate pressures in the financial market, which eases credit access for the private sector and lessens the declines in overall economic activity. At the same time, expectations of a crisis and credit intervention can distort the economy in normal times, raising or lowering economic activity depending on how frequently crises occur.

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A Equilibrium Conditions

The Foreign economy is identical to the Home economy except that Home government debt may default while Foreign government debt doesn't. In this section, we present the full set of equilibrium conditions for the baseline model.

Household The endogenous discount factor evolves according to $\Theta_{t+1} = \Theta_t \beta(\tilde{c}_t)$ with $\Theta_0 = 1$, therefore

$$\beta(c_t) = \beta_c (1 + c_t)^{-\omega_\beta} \tag{A.1}$$

$$\beta(c_t^*) = \beta_c^* (1 + c_t^*)^{-\omega_\beta^*} \tag{A.2}$$

Definition of marginal utility of consumption:

$$U_{c,t}(1 + \tau^c) = c_t^{-\sigma_c} \tag{A.3}$$

$$U_{c,t}^*(1 + \tau^{c,*}) = (c_t^*)^{-\sigma_c^*} \tag{A.4}$$

Household's real stochastic discount factor (net of the endogenous discount factor):

$$\Lambda_{t,t+1} = U_{c,t+1}/U_{c,t} \tag{A.5}$$

$$\Lambda_{t,t+1}^* = U_{c,t+1}^*/U_{c,t}^* \tag{A.6}$$

Household Euler equation:

$$\frac{1}{R_t^d} = E_t \beta(c_t) \Lambda_{t,t+1} \frac{1}{\pi_{t+1}} \tag{A.7}$$

$$\frac{1}{R_t^d} = E_t \beta(c_t^*) \Lambda_{t,t+1}^* \frac{1}{\pi_{t+1}^*} \tag{A.8}$$

First order condition for labor:

$$\chi L_t^{\sigma_L} = U_{c,t} w_t \tag{A.9}$$

$$\chi^* L_t^{*,\sigma_L} = U_{c,t}^* w_t^* \tag{A.10}$$

Consumption allocations:

$$c_{H,t} = \alpha_H \rho_{H,t}^{-\phi} c_t \tag{A.11}$$

$$c_{F,t} = (1 - \alpha_H)(\rho_{F,t}^* rer_t)^{-\phi} c_t \tag{A.12}$$

$$c^*_{F,t} = \alpha_F(\rho^*_{F,t})^{-\phi^*}c^*_t \tag{A.13}$$

$$c^*_{H,t} = (1-\alpha_F)(\rho_{H,t}/rer_t)^{-\phi^*}c^*_t \tag{A.14}$$

$$1 = \alpha_H\left(\rho_{H,t}\right)^{1-\phi} + (1-\alpha_H)\left(\rho^*_{F,t}rer_t\right)^{1-\phi} \tag{A.15}$$

$$1 = \alpha_F\left(\rho^*_{F,t}\right)^{1-\phi^*} + (1-\alpha_F)\left(\rho_{H,t}/rer_t\right)^{1-\phi^*} \tag{A.16}$$

Government Government budget constraints:

$$\rho_{H,t}g_t + (1-\Delta_t)(1+\kappa^b Q^b_t)\frac{b_{t-1}}{\pi_t} = Q^b_t b_t + t_t + tax_t \tag{A.17}$$

$$\rho^*_{F,t}g^* + (1+\kappa^{b,*}Q^{b,*}_t)\frac{b^*_{t-1}}{\pi^*_t} = Q^{b,*}_t b^*_t + t_t + tax^*_t \tag{A.18}$$

Governments use transfers in response to changes to government debt level:

$$\frac{t_t - t}{t} = \phi_t\frac{Q^b_{t-1}b_{t-1} - Q^b b}{Q^b b} \tag{A.19}$$

$$\frac{t^*_t - t^*}{t^*} = \phi^*\frac{Q^{b,*}_{t-1}b^*_{t-1} - Q^{b,*}b^*}{Q^{b,*}b^*} \tag{A.20}$$

Default rule:

$$\Delta_t = \begin{cases} 0, & \text{if } s_{t-1} < \mathcal{B}^*_t \\ \delta_b, & \text{otherwise} \end{cases}$$

The conditional probability of a government default tomorrow,

$$P(s_{t-1} \geq \mathcal{B}^*_t) = \frac{\exp[\eta^{FL}_0 + \eta^{FL}_s(s_{t-1} + \epsilon^P_t)]}{1 + \exp[\eta^{FL}_0 + \eta^{FL}_s(s_{t-1} + \epsilon^P_t)]}. \tag{A.21}$$

with $s_{t-1} = \frac{Q^b_{t-1}b_{t-1}}{4y_{t-1}}$. Definition of tax revenue:

$$tax_t = \tau^i p^w_t y_t + \tau^c c_t \tag{A.22}$$

$$tax^*_t = \tau^{i,*}p^w_t y^*_t + \tau^{c,*}c^*_t \tag{A.23}$$

Firms Private Investment allocations:

$$i_{H,t} = \alpha_H\rho^{-\phi}_{H,t}I_t \tag{A.24}$$

$$i_{F,t} = (1-\alpha_H)(\rho^*_{F,t}rer_t)^{-\phi}I_t \tag{A.25}$$

$$i^*_{F,t} = \alpha_F(\rho^*_{F,t})^{-\phi^*}I^*_t \tag{A.26}$$

$$i^*_{H,t} = (1-\alpha_F)(\rho_{H,t}/rer_t)^{-\phi^*}I^*_t \tag{A.27}$$

Investment producer's production function:

$$I_t^w = u_t^I \left(1 - \frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{A.28}$$

$$I_t^{w*} = \left(1 - \frac{\omega_I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right) I_t^* \tag{A.29}$$

The optimization problem for investment producer at Home is given by

$$\max \quad \sum_{t=0}^{\infty} E_0 \left[\Theta_t \Lambda_{t,t+1}\left(p_t^k u_t^I \left(1 - \frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t - I_t\right)\right].$$

Thus, investment producer's first-order condition:

$$\begin{aligned} 1 = p_t^k u_t^I \left(1 - \frac{\omega^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \omega^I \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \\ E_t u_{t+1}^I \beta(c_t) \Lambda_{t,t+1} p_{t+1}^k \omega^I \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned} \tag{A.30}$$

$$\begin{aligned} 1 = p_t^{k*} \left(1 - \frac{\omega^I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2 - \omega^I \left(\frac{I_t^*}{I_{t-1}^*} - 1\right)\frac{I_t^*}{I_{t-1}^*}\right) + \\ E_t \beta(c_t^*) \Lambda_{t,t+1}^* p_{t+1}^{k*} \omega^I \left(\frac{I_{t+1}^*}{I_t^*} - 1\right)\left(\frac{I_{t+1}^*}{I_t^*}\right)^2 \end{aligned} \tag{A.31}$$

Wholesale production function (after substituting for retail production function):

$$y_t = A_t l_t^{1-\alpha} K_{t-1}^{\alpha} \tag{A.32}$$

$$y_t^* = A^* l_t^{*,1-\alpha^*} K_{t-1}^{*,\alpha^*} \tag{A.33}$$

Law of motion for private capital:

$$K_t = I_t^w + (1-\delta) K_{t-1} \tag{A.34}$$

$$K_t^* = I_t^* + (1-\delta^*) K_{t-1}^* \tag{A.35}$$

Wholesale firm's first-order condition for labor:

$$w_t = (1-\alpha)\frac{p_t^w y_t^w}{L_t}(1-\tau^i) \tag{A.36}$$

$$w_t^* = (1-\alpha^*)\frac{p_t^{w,*} y_t^{w,*}}{L_t^*}(1-\tau^{i,*}) \tag{A.37}$$

Wholesale firm's first-order condition for capital:

$$\zeta_t^1 = E_t\beta(c_t)\Lambda_{t,t+1}\left(\frac{p_{t+1}^w \alpha y_{t+1}}{K_t}(1-\tau^i) + (1-\delta)\zeta_{t+1}^1\right) \tag{A.38}$$

$$\zeta_t^{1,*} = E_t\beta(c_t^*)\Lambda_{t,t+1}^*\left(\frac{p_{t+1}^{w,*} \alpha^* y_{t+1}^*}{K_t^*}(1-\tau^{i,*}) + (1-\delta^*)\zeta_{t+1}^{1,*}\right) \tag{A.39}$$

The loan-in-advance constraints for private capital:

$$Q_t^f\left(f_t - \kappa^f\frac{f_{t-1}}{\pi_t}\right) = \eta^I p_t^k I_t^w \tag{A.40}$$

$$Q_t^{f,*}\left(f_t^* - \kappa^{f,*}\frac{f_{t-1}^*}{\pi_t^*}\right) = \eta^{I,*} p_t^{k,*} I_t^{w,*} \tag{A.41}$$

Wholesale firm's first-order conditions for I_t^w:

$$\zeta_t^1 = (1+\eta^I\zeta_t^2) \tag{A.42}$$

$$\zeta_t^{1,*} = (1+\eta^{I,*}\zeta_t^{2,*}) \tag{A.43}$$

where ζ_t^1, ζ_t^2, $\zeta_t^{1,*}$, and $\zeta_t^{2,*}$ are the Lagrangian multipliers. Wholesale firm's first-order conditions for corporate bond:

$$Q_t^f(1+\zeta_t^2) = E_t\beta(c_t)\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}\left(1+\kappa^f Q_{t+1}^f(1+\zeta_{t+1}^2)\right) \tag{A.44}$$

$$Q_t^{f,*}(1+\zeta_t^{2,*}) = E_t\beta(c_t^*)\Lambda_{t,t+1}^*\frac{1}{\pi_{t+1}^*}\left(1+\kappa^{f,*} Q_{t+1}^{f,*}(1+\zeta_{t+1}^{2,*})\right) \tag{A.45}$$

Retail producers choose labor and price to optimize real profits, given by:

$$\sum_{k=0}^{\infty} E_0\left[\Theta_{t+k}\Lambda_{t,t+k}\left(p_{t+k}(h)y_{t+k}(h) - \frac{p_{t+k}^w}{\rho_{t+k}^H}y_{t+k}(h) - \frac{\psi}{2}\left(\frac{P_{t+k}(h)}{P_{t+k-1}(h)}\frac{1}{\pi^H}-1\right)^2 y_{t+k}\right)\right]$$

where $p_t(h) = P_t(h)/P_t^H$. Phillips equations from the optimization problem of retail producers:

$$\frac{p_t^w}{\rho_t^H} = \frac{\theta^c - 1}{\theta^c} + \frac{\psi}{\theta^c}\left(\frac{\pi_t^H}{\pi^H} - 1\right)\frac{\pi_t^H}{\pi^H} - \frac{\psi}{\theta^c}E_t\beta(c_t)\Lambda_{t,t+1}\left(\frac{\pi_{t+1}^H}{\pi^H} - 1\right)\frac{\pi_{t+1}^H}{\pi^H}\frac{y_{t+1}}{y_t} \tag{A.46}$$

$$\frac{p_t^{w,*}}{\rho_t^{F,*}} = \frac{\theta^{c*} - 1}{\theta^{c*}} + \frac{\psi^*}{\theta^{c*}}\left(\frac{\pi_t^{F,*}}{\pi^{F,*}} - 1\right)\frac{\pi_t^{F,*}}{\pi^{F,*}} - \frac{\psi^*}{\theta^c}E_t\beta(c_t^*)\Lambda_{t,t+1}^*\left(\frac{\pi_{t+1}^{F,*}}{\pi^{F,*}} - 1\right)\frac{\pi_{t+1}^{F,*}}{\pi^{F,*}}\frac{y_{t+1}^*}{y_t^*} \tag{A.47}$$

Financial Intermediary The financial intermediary's balance sheet conditions:

$$Q_t^b b_t^H + Q_t^f f_t + Q_t^{b,*} b_t^F = d_t + n_t \tag{A.48}$$

$$Q_t^{b,*} b_t^{F,*} + Q_t^{f,*} f_t^* + Q_t^b b_t^{H,*} = d_t^* + n_t^* \tag{A.49}$$

Evolutions of net worth:

$$\begin{aligned} n_t &= \sigma\left[\left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^H}{\pi_t} + \left(R_t^f - R_{t-1}^d\right)\frac{Q_{t-1}^f f_{t-1}}{\pi_t} + \left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t}\right] \\ &\quad + \sigma R_{t-1}^d \frac{n_{t-1}}{\pi_t} + (1-\sigma)x \end{aligned} \tag{A.50}$$

$$\begin{aligned} n_t^* &= \sigma^*\left[\left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^{F,*}}{\pi_t^*} + \left(R_t^{f,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{f,*} f_{t-1}^*}{\pi_t^*} + \left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^{H,*}}{\pi_t^*}\right] \\ &\quad + \sigma^* R_{t-1}^d \frac{n_{t-1}^*}{\pi_t^*} + (1-\sigma^*)x^* \end{aligned} \tag{A.51}$$

Definitions of rates of return:

$$R_t^b = (1-\Delta_t)\frac{1 + \kappa^b Q_t^b}{Q_{t-1}^b}, \quad R_t^f = \frac{1 + \kappa^f Q_t^f}{Q_{t-1}^f} \tag{A.52}$$

$$R_t^{b,*} = \frac{1 + \kappa^{b,*} Q_t^{b,*}}{Q_{t-1}^{b,*}}, \quad R_t^{f,*} = \frac{1 + \kappa^{f,*} Q_t^{f,*}}{Q_{t-1}^{f,*}} \tag{A.53}$$

Adjusted leverage:

$$\phi_t = \frac{Q_t^f f_t + \theta^b m_t^b}{n_t} \tag{A.54}$$

$$\phi_t^* = \frac{Q_t^{f,*} f_t^* + \theta^{b,*} m_t^{b,*}}{n_t^*} \tag{A.55}$$

Definition of Ω:

$$\Omega_t = 1 - \sigma + \sigma \eta^v \phi_t \tag{A.56}$$

$$\Omega_t^* = 1 - \sigma^* + \sigma^* \eta^{v,*} \phi_t^* \tag{A.57}$$

Portfolio manager allocations in the government bond market:

$$\frac{Q_t^b b_t^H}{m_t^b} = \gamma_b \left(\frac{E_t R_{t+1}^b}{E_t R_{t+1}^m} \right)^{-\sigma_b} \tag{A.58}$$

$$\frac{Q_t^{b,*} b_t^F}{m_t^b} = (1 - \gamma_b) \left(\frac{E_t R_{t+1}^{b,*}}{E_t R_{t+1}^m} \right)^{-\sigma_b} \tag{A.59}$$

$$\frac{Q_t^{b,*} b_t^{F,*}}{m_t^{b,*}} = \gamma_{b,*} \left(\frac{E_t R_{t+1}^{b,*}}{E_t R_{t+1}^{m,*}} \right)^{-\sigma_{b,*}} \tag{A.60}$$

$$\frac{Q_t^b b_t^{H,*}}{m_t^{b,*}} = (1 - \gamma_{b,*}) \left(\frac{E_t R_{t+1}^b}{E_t R_{t+1}^{m,*}} \right)^{-\sigma_{b,*}} \tag{A.61}$$

$$R_t^m = \left[\gamma_b \left(R_t^b \right)^{1-\sigma_b} + (1 - \gamma_b) \left(R_t^{b,*} \right)^{1-\sigma_b} \right]^{\frac{1}{1-\sigma_b}} \tag{A.62}$$

$$R_t^{m,*} = \left[\gamma_{b,*} \left(R_t^{b,*} \right)^{1-\sigma_{b,*}} + (1 - \gamma_{b,*}) \left(R_t^b \right)^{1-\sigma_{b,*}} \right]^{\frac{1}{1-\sigma_{b,*}}} \tag{A.63}$$

First-order conditions for portfolios:

$$E_t \beta(c_t) \Lambda_{t,t+1} \Omega_{t+1} \frac{R_{t+1}^f - R_t^d}{\pi_{t+1}} = \frac{\lambda_t^v}{1 + \lambda_t^v} \eta_t^v \tag{A.64}$$

$$E_t \beta(c_t) \Lambda_{t,t+1} \Omega_{t+1} \left(\frac{R_{t+1}^b - R_t^d}{\pi_{t+1}} \frac{Q_t^b b_t^H}{m_t^b} + \frac{R_{t+1}^{b,*} - R_t^d}{\pi_{t+1}} \frac{Q_t^{b,*} b_t^F}{m_t^b} \right) = \frac{\lambda_t^v}{1 + \lambda_t^v} \eta_t^v \theta^b \tag{A.65}$$

$$E_t \beta(c_t^*) \Lambda_{t,t+1}^* \Omega_{t+1}^* \frac{R_{t+1}^{f,*} - R_t^d}{\pi_{t+1}} = \frac{\lambda_t^{v,*}}{1 + \lambda_t^{v,*}} \eta_t^{v,*} \tag{A.66}$$

$$E_t \beta(c_t^*) \Lambda_{t,t+1}^* \Omega_{t+1}^* \left(\frac{R_{t+1}^{b,*} - R_t^d}{\pi_{t+1}^*} \frac{Q_t^{b,*} b_t^{F,*}}{m_t^{b,*}} + \frac{R_{t+1}^b - R_t^d}{\pi_{t+1}^*} \frac{Q_t^b b_t^{H,*}}{m_t^{b,*}} \right) = \frac{\lambda_t^{v,*}}{1 + \lambda_t^{v,*}} \eta_t^{v,*} \theta^{b,*} \tag{A.67}$$

Evolution of adjusted leverage:

$$\frac{\phi_t}{1 + \lambda_t^v} \eta^v = E_t \beta(c_t) \Lambda_{t,t+1} \frac{\Omega_{t+1}}{\pi_{t+1}} R_t^d \tag{A.68}$$

$$\frac{\phi_t^*}{1 + \lambda_t^{v,*}} \eta^{v,*} = E_t \beta(c_t^*) \Lambda_{t,t+1}^* \frac{\Omega_{t+1}^*}{\pi_{t+1}^*} R_t^d \tag{A.69}$$

The Rest Goods' market clearing:

$$y_t = c_t^H + c_t^{H,*} + g + i_t^H + i_t^{H,*} \tag{A.70}$$

$$y_t^* = c_t^{F,*} + c_t^F + g^* + i_t^{F,*} + i_t^F \tag{A.71}$$

Monetary policy:

$$\ln \frac{R_t^d}{R^d} = \phi_\pi \ln \frac{\pi_t^{ag}}{\pi^{ag}} + \phi_y \ln \frac{y_t^{ag}}{y^{ag}} \tag{A.72}$$

Asset purchase policy:

$$re_t = re + \phi_{cb}(R_t^{spread} - R^{spread}) \tag{A.73}$$

with

$$re_t = Q_t^b b_t^{cb} \tag{A.74}$$

$$R_t^{spread} = E_t R_{t+1}^b - R_t^d \tag{A.75}$$

Market clearing in asset markets:

$$b_t = b_t^{cb} + b_t^H + b_t^{H,*} rer_t \tag{A.76}$$

$$b_t^* = b_t^{F,*} + \frac{b_t^F}{rer_t} \tag{A.77}$$

$$0 = b_t^i + rer_t b_t^{i,*} \tag{A.78}$$

Net foreign assets evolution:

$$\begin{aligned} b_t^i + Q_t^{b,*} b_t^F - Q_t^b b_t^{H,*} rer_t &= \frac{R_{t-1}^d b_{t-1}^i}{\pi_t} + \frac{R_t^{b,*} Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t} \\ -R_t^b Q_{t-1}^b \frac{b_{t-1}^{H,*} rer_{t-1}}{\pi_t} &+ \rho_t^H (c_{H,t}^* + i_{H,t}^*) - \rho_t^{F,*} rer_t (c_{F,t} + i_{F,t}) \end{aligned} \tag{A.79}$$

Relative consumer price adjustment:

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^*}{\pi_t} \tag{A.80}$$

B Computational Details

This Appendix provides details of the solution method in both the baseline model and the extensions with anticipation effects.

B.1 Baseline Model As noted in section 3.2, the evolution of the haircut (2.5) and the default probability (2.6) implies that the model is an endogenous regime-switching model. There is a regime variable, and $def_t = 0$ indicates no default while $def_t = 1$ indicates default. The haircut can then be written as

$$\Delta_t = \begin{cases} 0, & \text{if } def_t = 0 \\ \delta_b, & \text{if } def_t = 1 \end{cases} \tag{B.1}$$

The transition matrix is time varying depending on the state $s_t = \frac{Q_t^b b_t}{4y_t}$ and the underlying macroeconomic shocks, and has elements $\mathbb{P}_{ij,t} = \Pr\left(def_{t+1} = j | def_t; s_t, \epsilon_t^P\right)$. Using our assumption that the transition follows a logistic function, we have

$$\mathbb{P}_t = \begin{bmatrix} \mathbb{P}_{00,t} & \mathbb{P}_{01,t} \\ \mathbb{P}_{10,t} & \mathbb{P}_{11,t} \end{bmatrix} = \begin{bmatrix} 1 - pdef_t & pdef_t \\ 1 - pdef_t & pdef_t \end{bmatrix}$$

where the probability of default in $t+1$, which is a function of time t variables is

$$pdef_t = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_t + \epsilon_t^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_t + \epsilon_t^P)]}.$$

We use the perturbation approach for solving endogenous regime-switching models in Benigno, Foerster, Otrok, and Rebucci (2024), which generates a set of approximated decision rules conditional on each regime. More specifically, the equilibrium conditions in Appendix A can be written as

$$E_t f\left(y_{t+1}, y_t, x_t, x_{t-1}, \chi\varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t\right) = 0 \tag{B.2}$$

where y_t denotes the non-predetermined variables, x_t the predetermined variables, ε_t the shocks, and θ_t the regime switching parameters. In our case, we follow Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) and use the partition principle to write

$$\theta_t \equiv \Delta\left(def_t\right) = \bar{\Delta} - \chi\hat{\Delta}\left(def_t\right) \tag{B.3}$$

where $\bar{\Delta}$ denotes the ergodic mean of $\Delta\left(def_t\right)$ using the steady state matrix $\mathbb{P}_{ss}$, where

$$pdef_{ss} = \frac{\exp[\eta_0^{FL} + \eta_s^{FL} s_{ss}]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL} s_{ss}]}. \tag{B.4}$$

Benigno, Foerster, Otrok, and Rebucci (2024) show that an iterative procedure can be used to solve for the steady state of the equilibrium conditions – this procedure is needed because the ergodic mean $\bar{\Delta}$ depends on the steady state debt-to-GDP ratio, which is itself a function of $\bar{\Delta}$.

After finding the steady state, we use perturbation to find second-order approximations to the regime-dependent decision rules

$$y_t = g\left(x_{t-1}, \varepsilon_t, \chi; def_t\right) \tag{B.5}$$

$$x_t = h\left(x_{t-1}, \varepsilon_t, \chi; def_t\right) \tag{B.6}$$

We then use these approximated decision rules for simulations. One relevant feature of the approximated decision rules is that second-order approximations are necessary to capture behavior induced by endogenous probabilities. Benigno, Foerster, Otrok, and Rebucci (2024) show that first-order approximations are identical to an exogenous regime-switching model with transition probabilities $\mathbb{P}_{ss}$. Intuitively, the second-order approximation is necessary to capture important features of our model, such as households and firms internalizing the fact that default becomes more likely as the debt-to-output ratio increases.

B.2 Models with Anticipation Effects In the first variant with anticipation effects, there is constant probability of asset purchases starting. In this case, there are four regimes reflecting the possibility of both default and credit interventions: 1) no default without asset purchases, 2) default without asset purchases, 3) no default but with asset purchases, 4) default with asset purchases. The regime transition matrix is given by

$$\mathbb{P}_t = \begin{bmatrix} (1-pdef_t)(1-p_a) & (1-p_a)pdef_t & p_a(1-pdef_t) & p_a pdef_t \\ (1-pdef_t)(1-p_a) & (1-p_a)pdef_t & p_a(1-pdef_t) & p_a pdef_t \\ p_s(1-pdef_t) & p_s pdef_t & (1-pdef_t)(1-p_s) & pdef_t(1-p_s) \\ p_s(1-pdef_t) & p_s pdef_t & (1-pdef_t)(1-p_s) & pdef_t(1-p_s) \end{bmatrix},$$

where p_s denotes the probability that the central bank stops its asset purchase program. We assume that agents anticipate such programs to be permanent and set $p_s = 0.0001$ to reflect this while preserving ergodicity of the transition matrix.

In the second variant, asset purchase policy is known, but there are times when a crisis does not occur and times when it does. To reflect this environment, we amend our model to have four possible regimes: 1) "normal times", 2) the debt crisis period, 3) no default following a crisis, and 4) default following a crisis. The transition matrix continues to vary in time depending on the state s_t and has elements $\mathbb{P}_{ij,t} = \Pr(s_{t+1} = j | s_t = i)$. Given our assumptions, the transition matrix is given by

$$\mathbb{P}_t = \begin{bmatrix} 1-p_c & p_c & 0 & 0 \\ 0 & 0 & 1-pdef_t & pdef_t \\ p_r & 0 & (1-pdef_t)(1-p_r) & pdef_t \\ p_r & 0 & (1-pdef_t)(1-p_r) & pdef_t \end{bmatrix}$$

where p_r denotes the probability that the central bank stops its asset purchase program, which we assume is 0.001, reflecting expectations that a credit program is likely permanent once initiated while preserving ergodicity.