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October 2025

Working Paper 2025-24

https://doi.org/10.24148/wp2025-24

Suggested citation:

Cúrdia, Vasco. 2025. "Monetary Policy and The Medium-Run Natural Rate." Federal Reserve Bank of San Francisco Working Paper 2025-24. https://doi.org/10.24148/wp2025-24

Monetary Policy and The Medium-Run Natural Rate Interest*

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October 9, 2025

Abstract

The natural rate of interest is an elusive concept in theory and practice. However, it is essential for central banks' calibration of the policy rate. Model consistent measures are often too extreme to be used in practice. On the other hand, empirical measures lack the full backing of theory to make them proper benchmarks. This paper proposes a medium-run measure of the natural rate that averages out some excessive fluctuations, while retaining enough connection to economic theory to make it optimal under certain circumstances. The discussion also provides a framework on how to evaluate and meaningfully address concerns by policymakers regarding the natural rate. The results suggest that a medium-run measure that concentrates on natural rate fluctuations in the range of two to five years ahead can be reasonable empirically and theoretically.

JEL classification codes: C32, E43, E52, E58

Keywords: DSGE Models, r-star, neutral rate of interest, central banks

*The content of this document does not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

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1 Introduction

Central banks usually have well defined objectives, to stabilize inflation and, sometimes, a second objective related to real activity. To achieve those objectives, conventionally the central bank sets the interest rate in the economy. In theory we know model's discount rate in steady state, and we can solve the social planner or Ramsey problem and determine the optimal path of the interest rate. However, in practice it is more complex because policymakers do not know the full model of the economy. For the models used to support policy there is also parameter uncertainty, and the risk of structural change over time. Therefore, getting the path of the interest rate that achieves the policy goals is a daunting task.

A large portion of the macroeconomic and monetary economics literature has followed Taylor (1993) in commonly representing central bank's interest rate setting behavior as a simple rule responding to inflation and the output gap. Even in the simplest models, there are concerns about the intercept term. This intercept is often assumed to be constant, corresponding to the steady state nominal interest rate. Woodford (2003) revived the idea of the natural rate of interest, developing the work of Wicksell (1898) in modern macroeconomic models. A key result is that such a simple rule can be optimal under certain assumptions, but only if the intercept is the time varying natural rate of interest—the real interest rate that would prevail in the natural equilibrium when wages and prices are flexible. This implies that the intercept is not a constant but rather a time-varying term.

The theoretical concept of the natural rate of interest is simple, but it turns out that estimates of this critical input for monetary policy is elusive at best. Cúrdia et al. (2015) show that for a simple New Keynesian (NK) model similar to the one used in this paper the policy rule that fits best the data from 1980 through 2009 actually responds to the Wicksellian natural rate of interest. However, the general consensus in the literature is that estimates of the natural rate are usually model and sample dependent. Small changes in assumptions can lead to substantial changes in the estimate of this latent variable. Furthermore, it is often very volatile, to the point that it is difficult to tell a policymaker to take an estimate of the short-run time-varying natural rate at face value without triggering major reservations.

As a result of the this state of affairs, policymakers and a substantial part of the literature, shy away from following very carefully any specific estimates of the natural rate of interest, and mostly assume that the intercept in the policy rule is constant. Some do this because the estimates are so volatile that they don't know what to make of it. Others simply do not trust the specific numbers to calibrate policy in practice. They rather prefer to focus their attention to understanding changes in underlying low frequency trends that could support time-varying long-run natural rate, due to changes in trend productivity, demographics, fiscal policy, or other factors—see for example Carvalho, Ferrero and Nechio (2016).

Laubach and Williams (2003) proposed a simple macroeconomic model in which they set policy rate to respond to inflation and output gap and the intercept—"r-star"—was modeled as responding to permanent or very persistent factors. So in this framework, the policy rule does not respond to the natural rate of interest but rather only to some sort of a persistent measure of the natural rate, and this was left to an empirical semi-structural part of the model. The result is a measure of the natural rate of interest that has been relatively reliable over time and across different countries. However, this measure explicitly abstracts from short-run transitory fluctuations, and thus is not the one consistent with the Wicksellian natural rate of interest. Nevertheless it is a measure, both in its nature and in its quantitative estimates, more palatable for policymakers than the DSGE model estimates available.

Because of the demand for low-frequency estimates of the natural rate, there are numerous contributions to the literature using semi-structural time-series models, as well as empirical finance models. Lubik and Ho (2024) and Christensen and Rudebusch (2012) discuss some of these measures. Most of these focus on longer-run estimates of the natural rate, "r-star," the "neutral rate", or the "long-run interest rate." Regardless of the exact naming of these estimates they are not aimed at characterizing where the policy interest rate ought to be in the short run, but rather where we forecast the interest rate to eventually go. As such, it is hard to argue that these measures are useful to evaluate the stance of monetary policy at business cycle frequency.

This paper is focused on exploring the middle ground between the theoretical consistent natural rate of interest that policymakers can't make anything about, and the more palatable alternative measures, that lack the theoretical backing to be considered proper benchmarks for the stance of monetary policy.

Namely, I propose a forward looking average of the short-run natural rate of interest as a simple approach to obtain more palatable estimates of the natural rate but still linked to the concept that we know is an appropriate measure of stance. I refer to this as the medium-run natural rate. The aim is to provide a measure that can be useful for policymakers. With that in mind we perform several normative explorations based on a simple quadratic loss function.[1]

[1] In the simplest of the NK models, this is consistent with the quadratic approximation of the model welfare criterion. For more general versions of the model, that is not necessarily the case. The analysis can be replicated with the model consistent welfare quadratic approximation. However, that can easily become

I find that in a calibrated model we can formalize the discussion by considering loss function penalties for excess policy rate volatility, excessive natural rate uncertainty, and measurement error considerations. We find that in this simple model addressing all the concerns of policymakers implies the desire to average out natural rate fluctuations to some extent. How much will depend on the perceived persistence of shocks that impact the natural rate, how much measurement error policymakers assign to their estimates of the natural rate, and how much weight they put into having a stable measure of the natural rate.

In the empirical version of the model analysis shows that aiming for medium-run natural rate measure that averages the projected path on the short-run natural rate over horizons of two to five years out are reasonable and retain some good normative properties once the above concerns are incorporated into the loss function of policymakers.

The remainder of the paper is organized as follows. Section 2 describes the model used. Section 3 describes the proposed medium-run rate and discusses how the choice of averaging can better address possible concerns of policymakers. Section 4 reports the empirical model, and discusses estimated paths of the natural rate and proposed medium-run rate and re-applies the normative considerations to the empirical model. Section 5 concludes.

2 The Model

I consider a simple New Keynesian model with households that consume, save in government bonds, and supply labor. Firms produce intermediate goods using labor and sell to final goods firms with nominal rigidities a la Calvo (1983).

2.1 Households

Households are ex ante identical. Each aims to maximize an expected discounted utility

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \bar{\lambda}_T \left[\frac{\left(\frac{C_T(i)}{Z_T} - \eta \frac{C_{T-1}}{Z_{T-1}} \right)^{1-\sigma}}{1-\sigma} - \bar{h} \frac{(h_T(i))^{1+\nu}}{1+\nu} \right], \tag{2.1}$$

where η is the consumption habits parameter, σ is the parameter controlling the intertemporal elasticity of substitution for households, and ν is the parameter controlling the convexity

like a black box that is nebulous, hard to understand, and too model dependent to be of practical use to policymakers. As such this paper considers that to be beyond the scope of the analysis, even if it is in principle an interesting academic exercise.

of labor disutility. Following An and Schorfheide (2007), we consider that utility is bderived from consumption divided by the productivity factor Z_t.

We consider an *inter-temporal* shock to utility of households, $\bar{\lambda}_t$. This shock is determined by

$$\xi_{\lambda,t} \equiv \; log(\bar{\lambda}_t/\bar{\lambda}),$$

The productivity factor is determined by stochastic growth in which the growth rate itself follows an AR(1) process,

$$\xi_{z,t} \equiv log(Z_t/Z_{t-1}) - \gamma.$$

There is a continuum of differentiated goods; $C_t(i)$ is a Dixit-Stiglitz aggegator of the household's purchases of differentiated goods indexed by j,

$$C_t(i) \equiv \left[\int_0^1 C_t(j,i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{2.2}$$

for some constant elasticity of substitution $\theta > 1$.

Each household supplies labor and take wages as given.

The budget constraint of households is

$$P_t C_t(i) + A_t(i) = R_{t-1} A_{t-1}(i) + W_t h_t(i) + P_t \mathcal{D}_t + T_{g,t} \tag{2.3}$$

where A_t is the nominal assets held by households, W_t denotes the labor wage received by households from the employment agency; $\mathcal{D}_t$ denotes the real distributions—the same for each household i, as there is no trading in shares of the firms—from goods producing firms; $T_{g,t}$ denotes net nominal (lump-sum) government transfers (also the same for each i).

The first order conditions with respect to $C_t(i)$ is

$$\tilde{\lambda}_t = \bar{\lambda}_t Z_t^{-1} \left(\frac{C_t}{Z_t} - \eta \frac{C_{t-1}}{Z_{t-1}} \right)^{-\sigma},$$

where $\tilde{\lambda}_t$ is the Lagrange multiplier associated with the budget constraint when expressed in real terms in period t. We can simplify and rewrite as

$$\lambda_t = \bar{\lambda}_t \left(c_t - \eta c_{t-1} \right)^{-\sigma}, \tag{2.4}$$

where $\lambda_t \equiv \tilde{\lambda}_t Z_t$ is the trend-normalized multiplier; and $c_t \equiv C_t/Z_t$ is the detrended level of consumption.

Households set their labor supply according to

$$0 = -\bar{\lambda}_t \bar{h}_t (h_t)^{\nu} + \tilde{\lambda}_t \frac{W_t}{P_t}$$

which we can simplify to

$$w_t = \bar{\lambda}_t \bar{h}_t \lambda_t^{-1} h_t^{\nu} \tag{2.5}$$

where $w_t \equiv W_t/(P_t Z_t)$ is the detrended real wage received by households for each unit of labor.

2.2 Final Goods Producers

The final good Y_t is a composite made of a continuum of goods indexed by $i \in (0, 1)$

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\mu_p}} di \right]^{1+\mu_p}. \tag{2.6}$$

The final goods producers buy the intermediate goods on the market, package Y_t, and resell it to consumers and capital producers. These firms maximize profits in a perfectly competitive environment. The implied demand for each variety is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\mu_p}{\mu_p}} Y_t. \tag{2.7}$$

The implied price aggregator is

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\mu_p}} di \right]^{-\mu_p}, \tag{2.8}$$

with $\mu_p \in (0, \infty)$.

2.3 Intermediate Goods Producers

There is a continuum of intermediate goods firms that rent capital, hire labor and produce varieties of intermediate goods subject to nominal rigidities *a la* Calvo, which they sell to the final goods firms.

Technology is given by

$$Y_t(i) = Z_t h_t(i). \tag{2.9}$$

The marginal cost, MC_t, is the same for all firms and constant w.r.t. $Y_t(i)$:

$$MC_t = \left(\frac{W_t}{Z_t}\right), \tag{2.10}$$

and the real marginal cost, mc_t, is

$$mc_t = w_t \tag{2.11}$$

Profits are given by

$$\mathcal{D}_t^f(i) = \left[(1-\bar{\tau})P_t(i) - \mu_{p,t} MC_t\right] Y_t(i), \tag{2.12}$$

where $\bar{\tau}$ is a constant tax rate on sales; and $\mu_{p,t}$ is a price markup shock exogenously determined, following an ARMA(1,1) process.

We assume that a fraction α_p of firms are not able to re-optimize their price, in which case they simply index the current price as follows

$$P_t(i) = P_{t-1}(i)\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}, \tag{2.13}$$

where $\iota_p \in (0,1)$ is the indexation parameter.

Firms distribute their profits as dividends to all the households. This implies that if they are able to re-optimize the price of their variety in period t then they set that price in order to maximize the expected stream of dividends to the households,

$$E_t \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} \frac{\tilde{\lambda}_T}{P_T} \left[(1-\bar{\tau})\bar{P}_T(i) - \mu_{p,T} MC_T\right] \left(\frac{\bar{P}_T(i)}{P_T}\right)^{-\frac{1+\mu_p}{\mu_p}} Y_T,$$

where $\bar{P}_T(i)$ is the price in period $T \geq t$ for a firm that re-optimized its price in period t.

Using the indexation rule we can write

$$\bar{P}_T(i) = \bar{P}_t(i) \left(\frac{P_{T-1}}{P_{t-1}}\right)^{\iota_p} \bar{\Pi}^{(1-\iota_p)(T-t)}.$$

This setup implies that firms maximize the following

$$E_t \sum_{T=t}^{\infty} (\alpha_p \beta)^{T-t} \lambda_T Y_T P_T^{\frac{1}{\mu_p}} \Bigg\{ (1-\bar{\tau}) \left[\bar{P}_t(i) \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\iota_p} \bar{\Pi}^{(1-\iota_p)(T-t)} \right]^{-\frac{1}{\mu_p}} - \mu_{p,T} MC_T \left[\bar{P}_t(i) \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\iota_p} \bar{\Pi}^{(1-\iota_p)(T-t)} \right]^{-\frac{1+\mu_p}{\mu_p}} \Bigg\}.$$

The first order conditions for firms can be represented in recursive form by

$$X_t^{pn} = \frac{1+\mu_p}{1-\bar{\tau}} y_t \mu_{p,t} mc_t + \alpha_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\Pi_{t+1}}{\Pi_t^{\iota_p} \bar{\Pi}^{1-\iota_p}} \right)^{\frac{1+\mu_p}{\mu_p}} X_{t+1}^{pn} \right], \tag{2.14}$$

$$X_t^{pd} = y_t + \alpha_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\Pi_{t+1}}{\Pi_t^{\iota_p} \bar{\Pi}^{1-\iota_p}} \right)^{\frac{1}{\mu_p}} X_{t+1}^{pd} \right]. \tag{2.15}$$

and the law of motion of prices is given by

$$1 = (1-\alpha_p) \left(\frac{X_t^{pn}}{X_t^{pd}} \right)^{-1/\mu_p} + \alpha_p \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}} \right)^{1/\mu_p} \tag{2.16}$$

2.4 Government

The government budget constraint is given by

$$T_t^g = \bar{\tau} P_t Y_t - P_t G, \tag{2.17}$$

where we assume for simplicity constant real government spending G; and T_t^g are lump sum net transfers to households that balance the budget.

Following Smets and Wouters (2007), monetary policy is set according to a simple interest rate rule:

$$R_t = (R_{t-1})^{\rho_r} \left[r_t^n \Pi_t^* \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\phi_\pi} x_t^{\phi_x/4} \right]^{1-\rho_r} \left(\frac{x_t}{x_{t-1}} \right)^{\phi_{\Delta x/4}} e^{\xi_{m,t}}, \tag{2.18}$$

where r_t^n is the natural rate of interest; Π_t^* is a time-varying inflation target, exogenously given; x_t is the natural output gap; and $\xi_{m,t}$ is an exogenous shock to the policy rule, following a AR(1).

2.5 Aggregate Resources Constraint

The aggregate resources condition is

$$Y_t = C_t + G,$$

and using normalized variables,

$$y_t = c_t + g. \tag{2.19}$$

We consider that for each exogenous variable x we can write it in log-linear terms

$$\xi_{x,t} \equiv \log(x_t/x) = \rho_x \xi_{x,t-1} + \sigma_x \varepsilon_{x,t} \tag{2.20}$$

Following Smets and Wouters (2007), we assume that the price markup follows an ARMA(1,1) process, which we can write as

$$\xi_{\mu,t} = \xi_{\mu,t} + \rho_\mu \xi_{\mu,t-1} + \sigma_\mu \varepsilon_{\mu,t} - \psi_\mu \sigma_\mu \varepsilon_{\mu,t-1} \tag{2.21}$$

2.6 Log-linear Representation

The online appendix summarizes all equilibrium conditions, steady state and detailed log-linearization around the steady state. Here I show the reduced set of equations.

The output gap is defined as

$$x_t = \hat{y}_t - \hat{y}_t^n \tag{2.22}$$

but due to normalization by the level of productivity, and in the absence of labor supply shocks, we get

$$\hat{y}_t^n = 0 \tag{2.23}$$

which means that the natural level of output is always equal to the productivity level in this simple model.

The households Euler equation can be represented in terms of the output gap

$$x_t = \frac{1}{1+\eta} E_t x_{t+1} + \frac{\eta}{1+\eta} x_{t-1} - \frac{\bar{\sigma}^{-1}}{1+\eta} \left(\hat{R}_t - E_t \pi_{t+1} - \hat{r}_t^n \right) \tag{2.24}$$

The natural rate of interest is given by

$$\hat{r}_t^n = E_t \left[\bar{\sigma} \xi_{z,t+1} - (\xi_{\lambda,t+1} - \xi_{\lambda,t}) \right] \tag{2.25}$$

The Phillips curve is represented as

$$\pi_t = \frac{\iota_p}{1+\beta\iota_p}\pi_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\pi_{t+1} + \kappa\left(x_t - \frac{\eta\bar{\sigma}}{\nu+\bar{\sigma}}x_{t-1}\right) + \xi_{p,t} \tag{2.26}$$

where the slope is determined by

$$\kappa \equiv \frac{1-\alpha_p}{\alpha_p}\frac{1-\alpha_p\beta}{1+\beta\iota_p}(\nu+\bar{\sigma})$$

Finally, monetary policy is given by

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\left[\hat{r}_t^n + \phi_\pi(\pi_t - \pi_t^*) + \frac{\phi_x}{4}x_t\right] + \frac{\phi_{\Delta x}}{4}(x_t - x_{t-1}) + \xi_{m,t} \tag{2.27}$$

3 A Medium-Run Natural Rate of Interest

The goal is to get a measure of the natural rate that is linked to the theoretical concept, removes some of the unappealing excess volatility, but retains enough content to be useful for the conduct of monetary policy. Towards that goal we consider a forward looking weighted average of the natural rate as follows:

$$\hat{r}_t^{n,d} = \frac{1}{d}\hat{r}_t^n + \left(1-\frac{1}{d}\right)E_t\hat{r}_{t+1}^{n,d} \tag{3.1}$$

where d is the parameter that controls how much weight this measure puts on the contemporaneous natural rate, vis-a-vis its future path. With this form, the weights on the natural rate projections at different horizons follow a geometric progression, with more weight in near future. Note that this is the same formula as the term premium used in Chen, Cúrdia and Ferrero (2012), and d would refer to the long bond duration. Similarly, in the present paper we can think of d as the average duration of the natural rate deviations from steady state, akin to the duration of a bond cash flows. Figure 1 shows the weights at different horizons for alternative choices of d.

Computationally, this approach is very simple. This measure of the natural rate can be added to the model system with only one more state variable and one more state equation. This implies minimal impact on performance when solving and simulating the model.

To understand why this paper focuses on a forward looking average of the natural rate, we can consider what optimal policy attempts to do. Cúrdia and Woodford (2016) show that in the simple NK model, similar to the one presented in here the model consistent loss

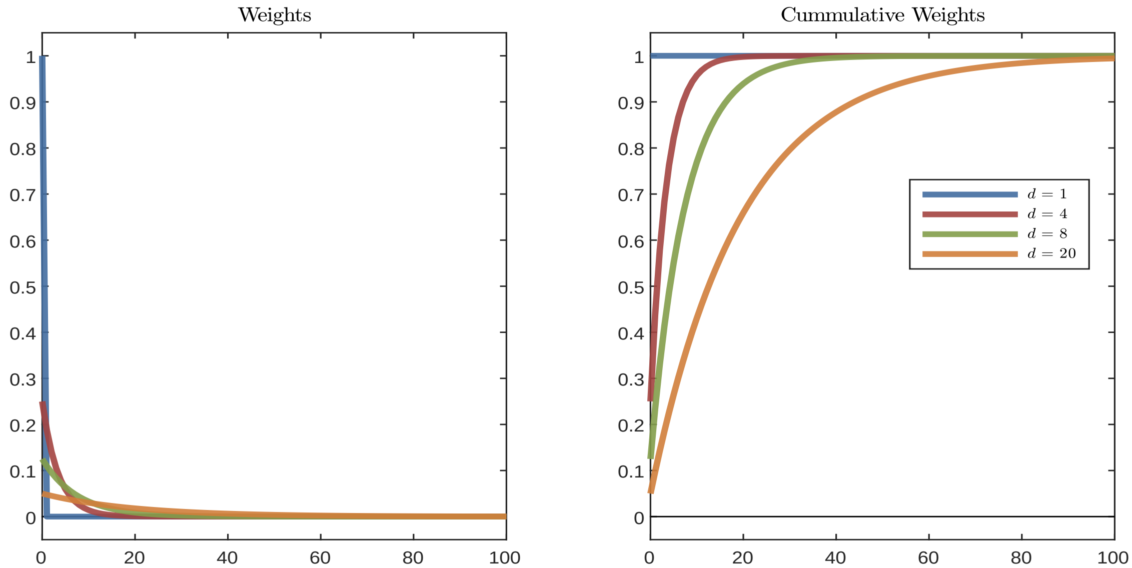

Figure 1: Natural Rate Weights

function takes the following form

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2], \tag{3.2}$$

which leads to optimal policy characterized by a flexible inflation targeting criterion of the type

$$\pi_t + \frac{\lambda_x}{\kappa}(x_t - x_{t-1}) = 0.$$

It is also shown in that paper that the interest rate rule that implements the optimal inflation targeting criterion takes the following form,

$$\hat{R}_t = \hat{r}_t^n + \phi_{\xi_p}^* \xi_{p,t} + \phi_\pi^* E_t \pi_{t+1} + \phi_x^* E_t x_{t+1} - \phi_{x,L}^* x_{t-1}.$$

There are two key takes to take out of this. First, optimal policy rule responds to the natural rate of interest, as already discussed. Second, this is a forward looking rule, with expected inflation and output gaps. So it makes sense to have a forward-looking measure of the natural rate. Furthermore, using a measure of the natural rate that is backward looking does not appear very appealing because it is a benchmark of where interest rates should have been in the past (aside inflation and output gap deviations), rather than where it is supposed to be going forward. Policymakers have to be forward looking, and thus averaging over future projections appears more reasonable.

We turn next to how the proposed medium-run rate responds to shocks, compared to the natural rate. Furthermore, we can also evaluate how responding systematically to the proposed measure differs from responding to the natural rate.

3.1 Numerical Evaluation of the Medium-Run Natural Rate

In order to proceed we set the model parameters as follows: and note that in this version we

β	0.999	σ	2	ρ_r	0
400γ	1.5	η	0	ϕ_π	1.5
$400r$	1.7	ν	1	ϕ_x	0.5
τ	0.2	α_p	0.7	$\phi_{\Delta x}$	0
g	0.2	ι_p	0	λ_x	1/16
μ_p	0.2	ρ_ξ	0.8		

Table 1: Simple model calibration

consider the very basic NK version of the model, without habits, no price indexation, and no interest rate smoothing in the policy rule.

In additional to the simple rules, I also consider optimal policy subject to a loss function as in (3.2), with a weight on output gap, $\lambda_x = 1/16$, which is equivalent to giving equal weight to annualized inflation and the output gap. The responses of nominal interest rate, inflation, output gap and the natural rate used in the policy rule are shown in Figure 2. In the figure, the different measures of the medium-run natural rate are shown as the natural rate that the central bank follows, $r^{n,cb}$, where optimal policy does not follow any measure of the natural rate and is thus shown as zero.

The figure shows that using a 4-quarter duration reduces substantially the response of the medium-run natural rate to the shock, and as we increase the duration even more. At a duration of 20 (equivalent to 5 years) the response is about one fourth, but it does not fully dissipate.

In terms of outcomes from alternative policies, not surprisingly, optimal policy fully stabilizes both output and inflation, and responding to the natural rate is the policy that gets us closest to the optimal outcomes. In this basic version of the model the policy rule that responds to the natural rate implements the optimal policy. As we consider policies responding to the medium-run natural rate at higher durations we get less and less stabilization of inflation and output gaps. Ignoring completely fluctuations in the natural rate (No r^n) yields the worst outcomes.

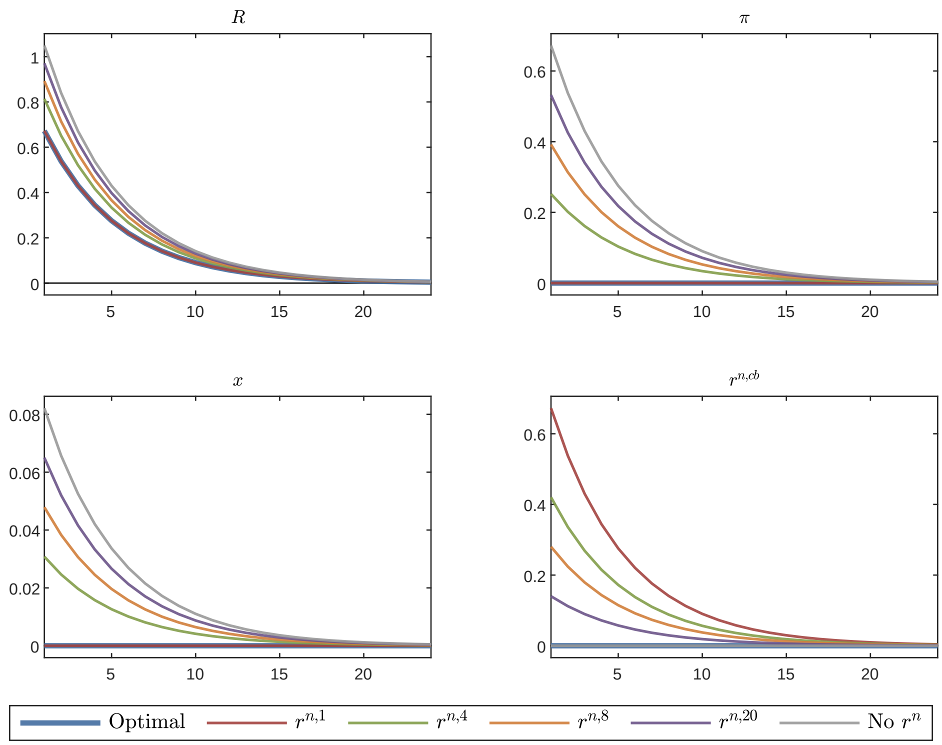

Figure 2: Responses to a productivity shock for alternative policy rules.

Notice that the responses to the demand shock are very similar because that shock impacts the natural rate and the economy in a similar way, up to a different scale—see equation (2.25). Similarly, the price markup shock is not shown because by definition, it does not impact the natural rate, hence irrelevant for this discussion, but needed later when the model is estimated.

In this simple version of the model we thus get that responding to the medium-run natural rate is better than not responding at all to the natural rate. However, the higher the duration chosen, the furthest we move from optimal, thus getting worse welfare levels.

This approach is naive in the sense that it ignores the reasons why policymakers are reluctant to track the natural rate to start with. Namely, uncertainty and excess volatility, which can lead to excess volatility in the policy rate. For this reason, it is often prescribed that the policy rate adjusts gradually to its intended target, leading to interest rate smoothing.

We thus consider the case in which the policy rule includes interest rate smoothing, and we set $\rho_r = 0.8$ which is commonly found in empirical estimates of DSGE models. Appendix figure A1 shows the responses in this case.[2] It is still the case that ignoring fluctuations of

[2] To avoid crowding the main part of the paper, I relegate this and other figures to the appendix.

the natural rate yields the worst outcomes. However, it is no longer clear from the figure that the policy rules that responds to the natural rate yields better outcomes than the rule that responds to the medium-run rate with duration of 4 quarters.

To properly compare the desirability of alternative durations of the medium-run natural rate used by the central bank, I evaluate the loss function (3.2), normalize it in steady state consumption equivalent, and take the difference relative to optimal policy. Figure 3 shows three panels, each with a different assumption for the persistency of productivity and demand shocks, with the AR coefficient, ρ_ξ, set at 0.2, 0.5, and 0.8. In each panel I show the welfare level at different natural rate durations for alternative assumptions about the policy interest rate smoothing, ρ_r.

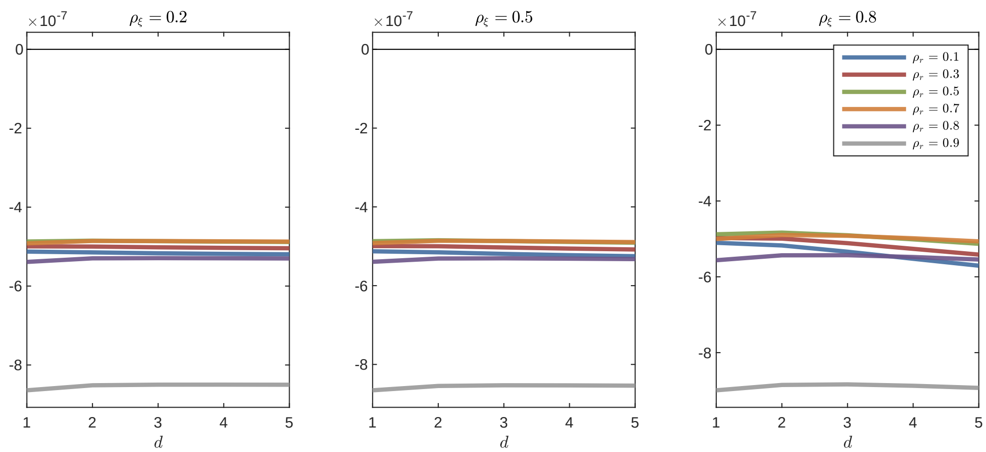

Figure 3: Welfare levels for rules that respond to the medium-run natural rate at different durations, for alternative levels of interest rate smoothing and different persistency of shocks.

The main takeaway from this figure is that in the basic NK model, it is desirable to stay close to the natural rate, even if there is a small amount of the interest rate smoothing, regardless of the persistency of the shocks. However, if there is a high degree of interest rate smoothing, then it is desirable to use a measure of the natural rate that is smoother. The reason is that with interest rate smoothing we are moving away from optimal policy as stated in the loss function. This means that the more interest rate smoothing there is then the deviations from optimal rates persist more. Interest rate smoothing can be a double edged blade. It responds less to the natural rate (in addition to the gaps), but once some response is baked into the current level of the interest rate, then future interest rates will stay close by unless the target rate changes substantially. Therefore, with high degree of interest rate smoothing, it is desirable to use a less volatile measure of the natural rate of interest. For an interest rate smoothing coefficient of 0.8, commonly found in empirical estimates, the optimal d^* is 3 regardless of the persistence of shocks.[3]

3.2 Concerns about Excess Policy Rate Volatility

The previous results suggest that setting d above one may be desirable when policymaker want to act somewhat cautiously, but not by much more, sticking to duration choices of less than a year. A more direct approach to address some of central banks concerns about the natural rate is that they want to avoid too much volatility in the policy rate. One approach commonly used,[4] is to add a quadratic term in the loss function to penalize large changes in the policy rule:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta R} \left(\hat{R}_t - \hat{R}_{t-1} \right)^2 \right]. \tag{3.3}$$

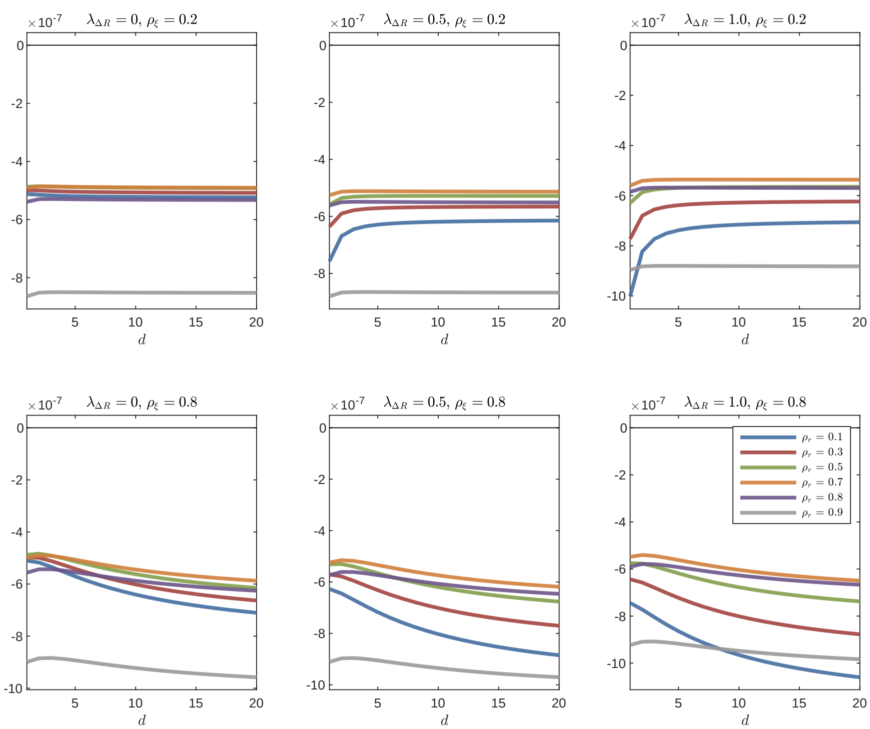

Figure 4: Welfare levels for rules that respond to the medium-run natural rate at different durations, for alternative levels of interest rate smoothing and different persistency of shocks.

In Figure 4 I show the welfare levels for alternative weights, $\lambda_{\Delta R}$, for different shock persistency and different interest rate smoothing. When shocks are persistent, welfare falls

[3]In the analysis I consider only integer values of d. We could consider a more refined optimization of the duration, but the qualitative results would be the same, and that level of detail is likely to be model dependent, hence besides the point of this analysis.

[4]See for example Ajello et al. (2021).

as we increase the choice of d. With a bigger loss function weight on policy changes, the decay is more pronounced. Table 2 shows the optimal choice of d dropped from 3 to 2 when interest rate smoothing is set at 0.8, as we set $\lambda_{\Delta R} > 0$. This means that if policy rate volatility is a concern, and shocks are persistent, responding to higher frequency fluctuations in the natural rate is helpful, because otherwise policy is not responding fast enough.

	$\rho_r = 0.1$	$\rho_r = 0.3$	$\rho_r = 0.5$	$\rho_r = 0.7$	$\rho_r = 0.8$	$\rho_r = 0.9$
$\lambda_{\Delta R} = 0,\ \rho_\xi = 0.2$	1	1	2	2	3	4
$\lambda_{\Delta R} = 0.5,\ \rho_\xi = 0.2$	100*	100*	11	4	3	4
$\lambda_{\Delta R} = 1.0,\ \rho_\xi = 0.2$	100*	100*	100*	6	4	4
$\lambda_{\Delta R} = 0,\ \rho_\xi = 0.8$	1	1	2	2	3	3
$\lambda_{\Delta R} = 0.5,\ \rho_\xi = 0.8$	1	1	2	2	2	3
$\lambda_{\Delta R} = 1.0,\ \rho_\xi = 0.8$	1	1	1	2	2	3

Table 2: Optimal choice of medium-run d, for alternative levels of interest rate smoothing, different weights of loss function on interest rate changes, and different persistency of shocks. Note: 100* means that the choice of d was trunctated at 100.

Instead, if shock persistency is low, then concerns about policy rate volatility suggest responding less to high frequency changes in the natural rate and increase the choice of d. With interest rate smoothing set to 0.7, choice of medium-run duration increases from 2 to 6 and with interest rate smoothing of 0.8, duration increases from 3 to 4. For low interest rate smoothing, then, policy rate volatility concerns suggest not responding much at all to the transitory fluctuations in the natural rate, which choice of d rising above 100. However, notice that the level of welfare does not change very substantially, as shown by the relatively flat lines in the top panels of Figure 4.

3.3 Concerns about Natural Rate Volatility

Another way to look at why policymakers may be reluctant to use the natural rate to calibrate policy too closely is because of communication considerations, in addition to concerns about policy rate volatility. A big part of monetary policy setting is to communicate not just what the current interest rate is going to be, but also how it will behave over time. In order to do so effectively central banks need to ground chances to the policy rates and their paths on analysis and forecasts. With that in mind, announcing that they will respond to a benchmark that is volatile poses obvious concerns. This is one of the reasons central banks respond more to core inflation, rather than headline, not because they do not care for energy and food prices, but rather because those are very volatile categories, and changes in those are not necessarily informative about future underlying inflation.

For these reasons then we can formalize the choice of d with these concerns by considering an additional penalty for volatility in the natural rate measure used by the central bank:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta R} \left(\hat{R}_t - \hat{R}_{t-1} \right)^2 + \lambda_{r^{n,cb}} \left(r_t^{n,cb} \right)^2 \right], \tag{3.4}$$

and notice that choosing this loss function penalizes the variance of the natural rate measure used by the central bank, but it does not impact optimal policy per se, because it does not impact the natural rate. It also gives a very clear advantage to not responding to the natural rate, unless the benefits in terms of inflation and output outcomes overcome the losses from having a measure of the natural rate that is volatile.

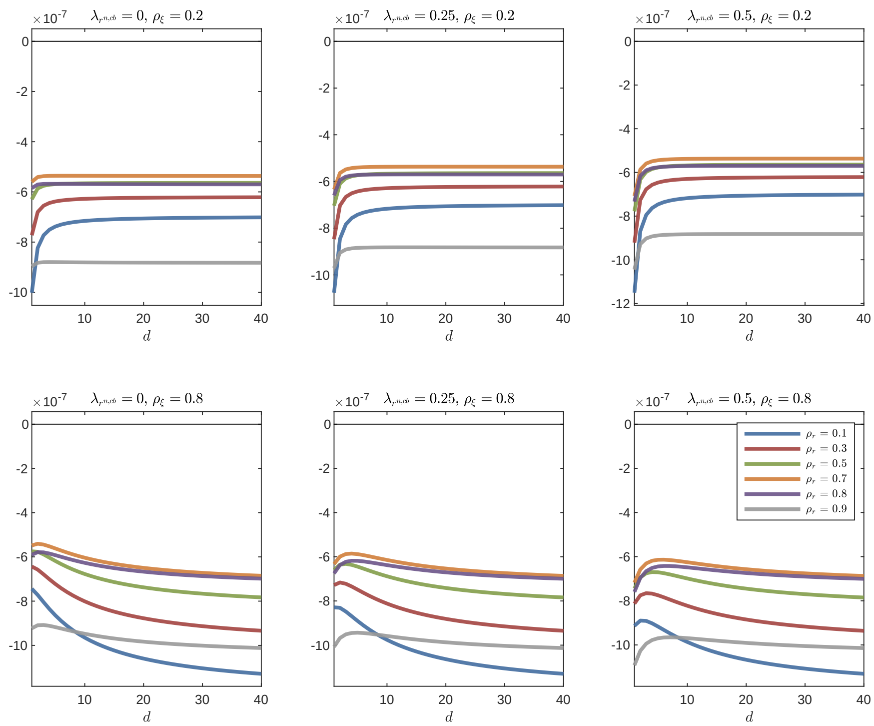

Figure 5: Welfare levels for rules that respond to the medium-run natural rate at different durations, for alternative levels of interest rate smoothing, different weights of loss function on the natural rate of interest used, and different persistency of shocks.

In this part of the analysis I consider $\lambda_{\Delta R} = 1$ and then consider different weights on the volatility of the central bank's measure of the natural rate, $\lambda_{r^{n,cb}}$. Figure 5 and Table 3 show the respective results. As before the results depend on the persistency of the shocks and the degree of interest rate smoothing in the policy rule. In all cases, higher $\lambda_{r^{n,cb}}$ leads to higher d being optimal, and this should be fairly straightforward because higher duration

	$\rho_r = 0.1$	$\rho_r = 0.3$	$\rho_r = 0.5$	$\rho_r = 0.7$	$\rho_r = 0.8$	$\rho_r = 0.9$
$\lambda_{r^{n,cb}} = 0$, $\rho_\xi = 0.2$	100*	100*	100*	6	4	4
$\lambda_{r^{n,cb}} = 0.25$, $\rho_\xi = 0.2$	100*	100*	100*	20	14	13
$\lambda_{r^{n,cb}} = 0.5$, $\rho_\xi = 0.2$	100*	100*	100*	34	24	23
$\lambda_{r^{n,cb}} = 0$, $\rho_\xi = 0.8$	1	1	1	2	2	3
$\lambda_{r^{n,cb}} = 0.25$, $\rho_\xi = 0.8$	1	2	3	4	4	5
$\lambda_{r^{n,cb}} = 0.5$, $\rho_\xi = 0.8$	2	3	4	6	6	7

Table 3: Optimal choice of medium-run d, for alternative levels of interest rate smoothing, different weights of loss function on the natural rate of interest used, and different persistency of shocks. Note: 100* means that the choice of d was truntacted at 100.

implies less volatile medium-run natural rate. The difference is by how much.

For persistent shocks, and interest rate smoothing in the 0.7 to 0.8 range then d rises from 2 to 6 quarters. This is a non-negligible increase. It is also clear that very high choices of d penalize welfare quite a bit (observed in the slope of the lines in the lower panel), despite clearly improving the additional loss-function component. So some more duration, but not too much.

Instead, if we consider relatively transitory shocks, with AR coefficient of 0.2 we get very meaningful increases in optimal choice of d, shown in the table. If we consider interest rate smoothing of 0.8, the optimal d jumps to 14 and 24 quarters as we increase the weights on the natural rate volatility. The reason is that if shocks are relatively transitory then there is not too much benefit to responding to every single fluctuation, compared to how much more volatile that makes the natural rate of interest. As such, averaging over the fluctuations in the natural rate offers considerable benefits in terms of reducing volatility in the medium-run rate, while remaining useful for policy, compared to not responding at all to natural rate fluctuations. For low levels of interest rate smoothing, then it is desirable to not respond at all to natural rate fluctuations. But notice that the welfare for those degrees of smoothing is lower. In this model it is desirable to have interest rate smoothing and some averaging of natural rate fluctuations.

3.4 Concerns with Natural Rate Measurement Error

One additional concern for policymakers is the degree of estimation uncertainty regarding the natural rate. Previously I considered implications from volatility, but the central bank knew always what exactly the true natural rate is. In this section I now consider that the measure of the natural rate of interest used by the central bank is subject to persistent measurement error:

$$r_t^{n,cb} = \hat{r}_t^{n,d} + \xi_{r^{n,cb},t}, \tag{3.5}$$

and

$$\xi_{r^{n,cb},t} = \rho_{r^{n,cb}}\xi_{r^{n,cb},t-1} + \sigma_{r^{n,cb}}\varepsilon_{r^{n,cb},t}. \tag{3.6}$$

A critical part of this analysis is to acknowledge that this measurement error ought to be smaller for more measures of the natural rate of higher duration, because they are less volatile and thus there is estimation uncertainty. In this part of the analysis simulate volatility of the medium-run rate at horizons two to five years out, and concluded that the baseline natural rate has a standard deviation of about 0.8 with shocks sizes considered, and as we increase d the standard deviation decays as a geometric progression with rate of about 0.925. This is not meant to be exact, but rather give an idea of how accounting for estimation uncertainty can impact the optimal choice of d. So avoid biasing results too much I assume a baseline standard deviation of 0.5 for $d = 1$. Formally I set

$$\sigma_{r^{n,d}} = \bar{\sigma}_{r^{n,cb}}(0.925)^{d-1} \tag{3.7}$$

Notice that in this setup the response function to shocks is not changed. However, when we consider the evaluation of the loss function across all the shocks in the economy, including measurement error shocks, it will penalize measures with more measurement error. In the simulations I assume $\lambda_{\Delta R} = 1$ and $\lambda_{r^{n,cb}} = 0.25$. Figure 6 and Table 4 show the results. Top panel as before with transitory shocks, lower panel with persistent shocks, left panels with no measurement error, and right panels with measurement error.

	$\rho_r = 0.1$	$\rho_r = 0.3$	$\rho_r = 0.5$	$\rho_r = 0.7$	$\rho_r = 0.8$	$\rho_r = 0.9$
$\bar{\sigma}_{r^{n,cb}} = 0,\ \rho_\xi = 0.2$	100*	100*	100*	20	14	13
$\bar{\sigma}_{r^{n,cb}} = 0.5,\ \rho_\xi = 0.2$	100*	100*	100*	59	55	54
$\bar{\sigma}_{r^{n,cb}} = 0,\ \rho_\xi = 0.8$	1	2	3	4	4	5
$\bar{\sigma}_{r^{n,cb}} = 0.5,\ \rho_\xi = 0.8$	4	6	9	12	14	15

Table 4: Optimal choice of medium-run d, for alternative levels of interest rate smoothing, with natural rate uncertainty, and different persistency of shocks. Note: 100* means that the choice of d was tructated at 100.

The results in the figure and table show that with the measurement error there is a clear preference for more forward averaging of natural rate fluctuations. Interestingly for the transitory shocks, with AR coefficient of 0.2, even with high interest rate smoothing, the model prefers choices of d around 55, compared to 14 in the absence of measurement error.

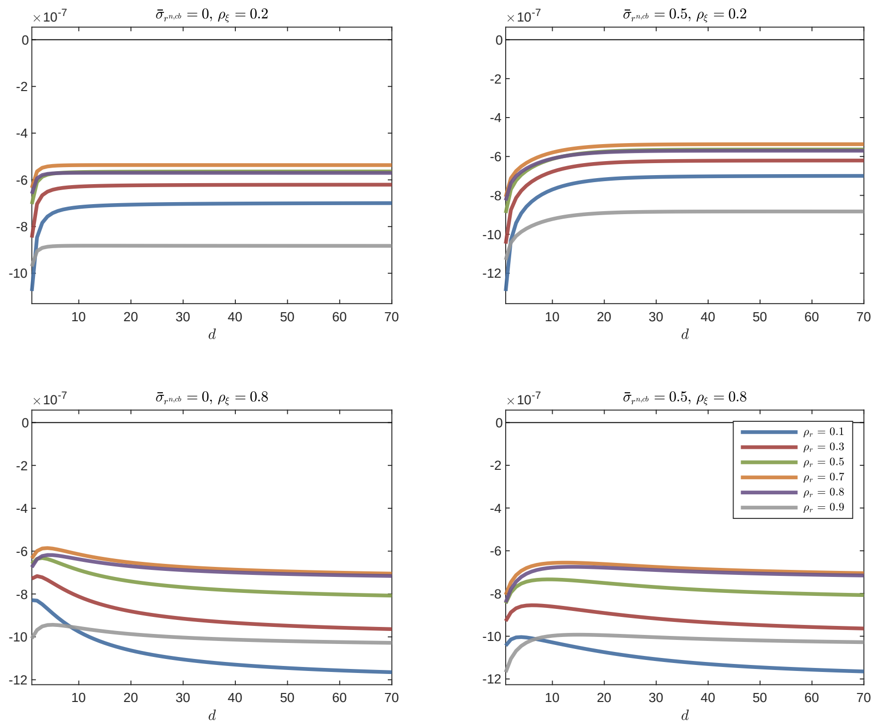

Figure 6: Welfare levels for rules that respond to the medium-run natural rate at different durations, with natural rate uncertainty, different weights of loss function on the natural rate of interest used, and different persistency of shocks.

Notice however that it's not shooting further which means that it is still better than not responding at all to the natural rate.

In the case of the persistent shocks, with AR coefficient of 0.8, the optimal choice of d increases for all levels of interest rate smoothing. For the more empirically relevant $\rho_r = 0.8$ the optimal d goes from 4 to 14, so from 1 year duration to nearly 5 years duration. This a big jump. Notice as well that now using 0.7 or 0.8 interest rate smoothing are very similar from a welfare perspective.

We have considered several different concerns regarding the response of policy to the natural rate in the simpler model. Adding habits, price indexation, or changing the degree of nominal rigidities or policy rule coefficients impacts quantitative results somewhat but do not change the qualitative results so far. The online appendix includes some fo those robustness considerations.

What we get is that in this simple model addressing all the concerns of policymakers implies the desire to average out natural rate fluctuations to some extent. How much will depend on the persistence of shocks that impact the natural rate, how much measurement error policymaker perceive in their estimates of the natural rate, and how much weight they put into having a stable measure of the natural rate. Some of those are choices of policymakers, but the exact model parameters and persistence of shocks can be estimated. Therefore, the next section estimates the model and uses the estimated model to review the analysis above.

4 Empirical Analysis

In this section we consider the full model, and estimate it with Bayesian estimation methods to reevaluate how much averaging of the natural rate fluctuations may be warranted.

4.1 Data

To estimate the model I use data from 1987 through the fourth quarter of 2019 for core personal consumption expenditures (PCE) price inflation; real GDP growth in per-capita terms; the effective federal funds rate; the 10-year ahead inflation expectations from the Survey of Professional Forecaster; and federal funds rate expectations derived from financial data based on Christensen and Rudebusch (2012). The obersvation equations mapping data to the model are as folows:

$$DGDPPC_t = \gamma + 400dy_t + \xi^{me}_{dY,t} \tag{4.1}$$

$$COREPCEPI_t = \pi + 400\pi_t + \xi^{me}_{\pi,t} \tag{4.2}$$

$$FFR_t = r^n + \pi + 400\hat{R}_t \tag{4.3}$$

$$PTR_t = \pi + 400\pi^*_t + \xi^{me}_{\pi^*,t} \tag{4.4}$$

The FFR expectations are introduced as

$$FFRE_t(j) = r^n + \pi + 400\hat{R}^{e,j}_t \tag{4.5}$$

where

$$\hat{R}^{e,j}_t = E_t\hat{R}^{e,j-1}_{t+1} \tag{4.6}$$

and

$$\hat{R}^{e,1}_t = E_t\hat{R}_{t+1} \tag{4.7}$$

hence a recursive formulation for FFR expectations. Finally the policy rule also has up to 6 quarter ahead anticipated shocks, meant to be allow for forward guidance in the period post-financial crisis.

4.2 Priors and Posterior Distribution of Model Parameters

Priors are standard in the NK DSGE literature and are presented in Table 5 together with the posterior distribution of each parameter. Inflation with a 2% average normal prior. Loose Gamma priors on growth and real interest rate.

	Prior				Posterior					
	Dist	5%	Median	95%	Mode	Mean	SD	5%	Median	95%
$400(\Pi-1)$	N	1.836	2.000	2.164	1.996	1.992	0.093	1.838	1.993	2.145
400γ	G	0.875	1.548	2.501	2.081	2.092	0.157	1.845	2.085	2.359
$400(\beta^{-1}-1)$	G	0.068	0.184	0.388	0.174	0.210	0.101	0.074	0.195	0.397
σ	G	1.254	1.958	2.887	1.351	1.440	0.248	1.067	1.421	1.876
η	B	0.524	0.707	0.853	0.572	0.557	0.060	0.452	0.560	0.650
ν	G	0.342	0.918	1.938	0.592	0.794	0.412	0.261	0.723	1.572
α_p	B	0.524	0.707	0.853	0.972	0.973	0.005	0.964	0.973	0.980
ι_p	B	0.240	0.397	0.570	0.511	0.457	0.106	0.264	0.468	0.613
ρ_r	B	0.570	0.760	0.897	0.863	0.868	0.015	0.843	0.869	0.892
ϕ_π	G	1.114	1.486	1.934	1.800	1.823	0.257	1.423	1.809	2.267
ϕ_x	G	0.222	0.474	0.868	0.463	0.552	0.194	0.267	0.533	0.902
$\phi_{\Delta x}$	G	0.222	0.474	0.868	0.111	0.123	0.037	0.067	0.120	0.188
ρ_m	B	0.172	0.500	0.828	0.948	0.948	0.010	0.930	0.949	0.964
ρ_{π^*}	B	0.903	0.954	0.983	0.969	0.969	0.011	0.951	0.969	0.988
ρ_z	B	0.172	0.500	0.828	0.941	0.942	0.009	0.926	0.942	0.956
ρ_λ	B	0.172	0.500	0.828	0.025	0.040	0.025	0.009	0.035	0.088
ρ_p	B	0.172	0.500	0.828	0.123	0.194	0.112	0.050	0.172	0.413
$400\sigma_m$	IG1	0.082	0.170	0.622	0.160	0.167	0.031	0.121	0.164	0.222
$400\sigma_{\pi^*}$	IG1	0.082	0.170	0.622	0.093	0.095	0.006	0.085	0.095	0.105
$400\sigma_z$	IG1	0.166	0.343	1.237	0.369	0.371	0.040	0.309	0.369	0.442
$100\sigma_\lambda$	IG1	0.166	0.343	1.237	0.402	0.438	0.065	0.342	0.432	0.553
$100\sigma_p$	IG1	0.166	0.343	1.237	0.492	0.505	0.056	0.424	0.500	0.604
ψ_p	B	0.172	0.500	0.828	0.741	0.765	0.047	0.691	0.764	0.845
μ_p	C	0.200	0.200	0.200	0.200	0.200	0.000	0.200	0.200	0.200
g	C	0.200	0.200	0.200	0.200	0.200	0.000	0.200	0.200	0.200
τ	C	0.200	0.200	0.200	0.200	0.200	0.000	0.200	0.200	0.200
σ_{R^a}	IG1	0.065	0.136	0.498	0.098	0.101	0.005	0.093	0.101	0.109

Table 5: Prior and Posterior parameter distributions.

Intertemporal elasticity parameter, σ, has a Gamma prior with mean of 2. The habits parameter, η, has a Beta distribution with mean 0.7. The parameter controlling the convexity of labor disutility, ν, has Gamma prior with mean 1. The Calvo probability parameter, α_p has a Beta prior with mean 0.7. Price indexation parameter, ι_p has a prior Beta distribution with mean 0.4. Interest rate smoothing parameter, ρ_r has a Beta prior with mean 0.75. Policy rule coefficients on inflation, output gap, and changes in the output gap have Gamma priors with means 1.5, 0.5, and 0.5, respectively. The lag coefficient on all shocks has Beta prior with mean 0.5, except for the inflation target which has prior mean of 0.95. The price markup has ARMA structure with the same AR coefficient as other processes, and MA coefficient with prior mean of 0.5. We also assume that steady state government spending relative to output is 20% and steady state tax rate, τ, is 0.2. All shocks innovations standard deviation have loose Inverse Gamma type 1 priors.

We estimate the model with Bayesian methods, as described by **?**. Bayesian estimation combines prior information on the parameters and the model likelihood to form the posterior distribution. We construct the likelihood using a Kalman filter based on the state-space representation of the rational-expectations solution of the model.

Posterior estimations are somewhat typical, with note for the very high nominal rigidities with a Calvo probability posterior median of 0.97. Policy rule coefficients have relatively standard posteriors.

One important part is the persistency of the shocks. The two shocks that impact the natural rate of interest, the productivity and demand shocks, are each estimated to have very different persistency. The productivity shock as a median AR coefficient of 0.94, while the demand shock is estimated to be very transitory. This will matter for the following parts of the analysis.

4.3 Empirical Estimates of the Natural Rate

With the posterior sample we can extract the states and shocks of the model from the data using the simulation smoother described in Durbin and Koopman (2002). For this part we extend the data through the second quarter of 2025. The exact path is not the main point of this exercise. The model is too simple for this path to be considered the ultimate word on the level of the natural rate or its derivatives. Rather the idea is show in a real example how much the medium-run natural rate can look like and how appealing that may or may not be for policy analysis.

Figure 7 shows the empirical estimate of the natural rate. It also shows the estimate over the sample period for two alternative medium-run estimates, with $d = 8$ and $d = 20$,

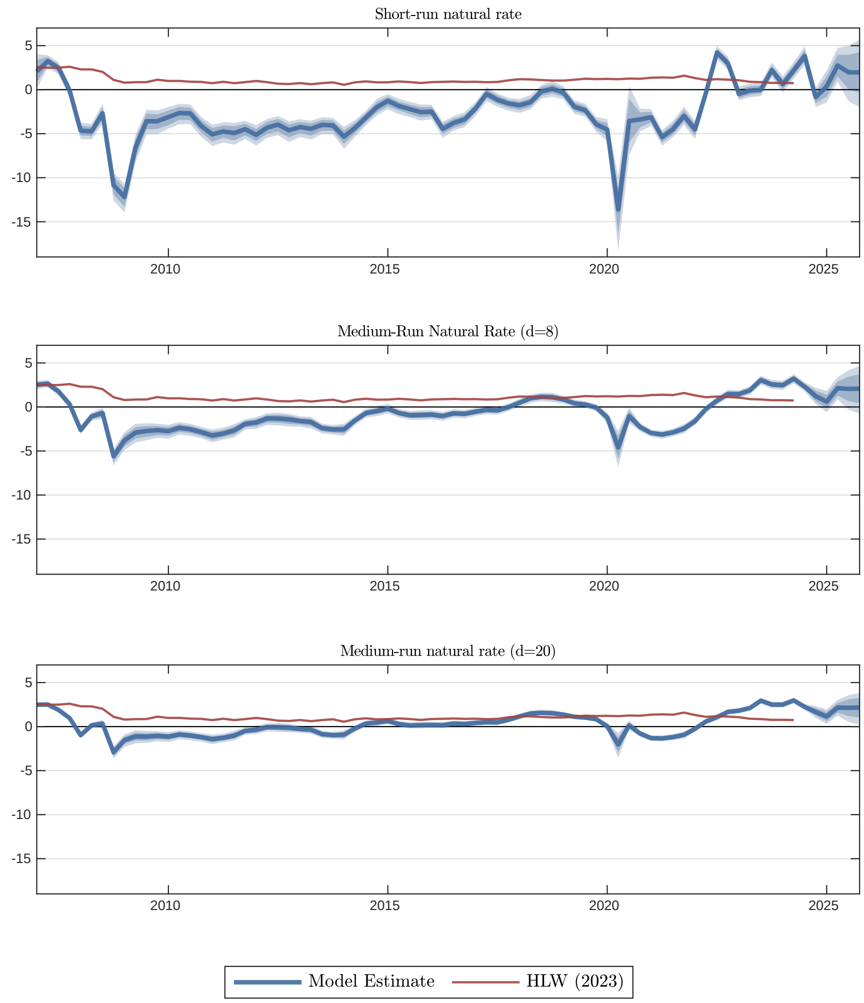

Figure 7: Empirical estimates of the natural rate and two alternative medium-run measures. Compared to the estimates of HLW (2023).

corresponding to durations of 2 years and 5 years respectively. The main point to take out the natural rate in the top panel is that it can fluctuate quite substantially, anywhere between -19 and 6 percentage points. Offering such a measure to a policymaker is not surprisingly a challenge. It's not obvious what they can do or how to communicate to the public that their refererence rate is nearly -15 percentage points, or that it remained at -5 percent for five years.

The medium-run rate with 2-year duration has much less volatility, but still has some considerable spikes. We had discussed previously some calibrated results, suggesting that setting $d = 8$ can be in the range of reasonable options when accounting for different concerns by policymakers. But even this is challenging because it still moves more than what can be effective in communications.

Finally, the 5-year duration medium-run natural rate, with $d = 20$, is more appealing with fluctuations mostly within the range of measures for medium-to-long run neutral rates over recent decades. Easier to talk about -2 percentage points than -15 percentage points. It's good to have a measure that does not fluctuate wildly, beyond what may be considerable reasonable, but is it still useful form a normative point of view?

For the normative analysis, I compute loss function simulations similar to the previous sections, but using the model posterior sample to draw parameters and then get the median results for alternative policy rules. I do this with and without the assumption of measurement error. Table 6 presents the results. Note that in the empirical model, the estimate of the natural rate in the short run has a higher uncertainty than in the calibrated model. We compute the median estimate at 2- to 5-year horizon and get what would be the equivalent standard deviation for an AR(1) process with a coefficient of 0.8, and get values of 2.3 to 2.6, respectively. We set $\sigma_{r^{n,cb}}$ to 2.5 in the simulations with measurement error. I present optimal choice of d for each shock separately in the first two columns. Then I consider all shocks with no measurement error in third column, followed by all shocks and measurement error in the fourth column. Notice that the measurement error only impacts the "all shocks" component.

	ξ_z shocks	ξ_λ shocks	all shocks	all shocks w/ m.e.
$\lambda_{\Delta R} = \lambda_{r^{n,cb}} = 0$	3	4	3	4
$\lambda_{\Delta R} = 1,\ \lambda_{r^{n,cb}} = 0$	3	9	3	4
$\lambda_{\Delta R} = 1,\ \lambda_{r^{n,cb}} = 0.25$	10	100*	10	30

Table 6: Optimal choice of medium-run d, in the empirical model, alternative loss function assumptions. First column assumes only ξ_z shocks; second column only ξ_λ shocks; third column considers all shocks in model but no measurement error, and the fourth column considers all shocks and measurement error. Note: 100* means that the choice of d was truncated at 100.

In considering the empirical results, it is important to notice that the persistency of the productivity (ξ_z) and the demand (ξ_λ) shocks is very different. The productivity shocks have a median AR coefficient estimate of 0.942, compared to a median estimate of 0.035 for the demand shocks. It is not surprising then that the choice of the d is substantially different for each shock separately, consistent with the results in the calibrated model.Across all shocks, it is remarkable that in the simple NK loss function, the optimal is to choose $d = 3$, rather than using the short-run natural rate. That is due to the high degree of interest rate smoothing in the empirical policy rule. Thus, even in the absence of other considerations it is still optimal to average some fluctuations in the natural rate.

As we add a penalty for volatility in the reference natural rate used by thge central bank (last row) then it is optimal to average out fluctuations even further, even for the persistent productivity shock. Accounting for all shocks, it is still best to respond to some of these fluctuations, than to rely only on the steady state level of the natural rate. In the absence of measurement error the optimal d is 10. If in addition we account for central bank's uncertainty about the natural rate then the optimal choice of d is 30.

Where exactly central bank's measurement error lies can be subject to discussion, and possibly depend on specific model features. But it is notable that once it is accounted for in some form the degree of optimal averaging of natural rate fluctuations is higher, and that responding to some of those is better than not responding at all. All in all it seems reasonable to consider that medium-run estimates in the range of 2 to 7 year duration are good candidates to present to policymakers as alternatives for their consideration, with the benefit that they are based on theoretically consistent approaches, compared to more purely empirical or semi-structural measures.

5 Conclusion

This paper considers a simple theoretical model in which we can discuss the concerns of policymakers regarding the DSGE based natural rate of interest. It proposes an alternative medium-run measure that can average over some of the excessive fluctuations and provide more palatable measures for consideration by policymakers. It is up to them to judge how concerned they are about volatility in the measures considered, and uncertainty about them, as well as how easy they find it to communicate decisions that use these as inputs.

Ultimately the analysis supports that responding to some fluctuations in the natural rate is beneficial, despite the averaging out. If we consider all the concerns discussed it is reasonable to use measures like the five year medium-run natural rate as both palatable as well as reasonable from normative perspective.

This analysis is not meant to give the last word on the exact level of the natural rate. Rather it provides some alternatives and discusses different ways in which we can formally discuss the usage of some measures over others. It is left for future research to incorporate this approach and proposed measures into more complex models to apply to more empirically relevant situations.

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A Additional Figures

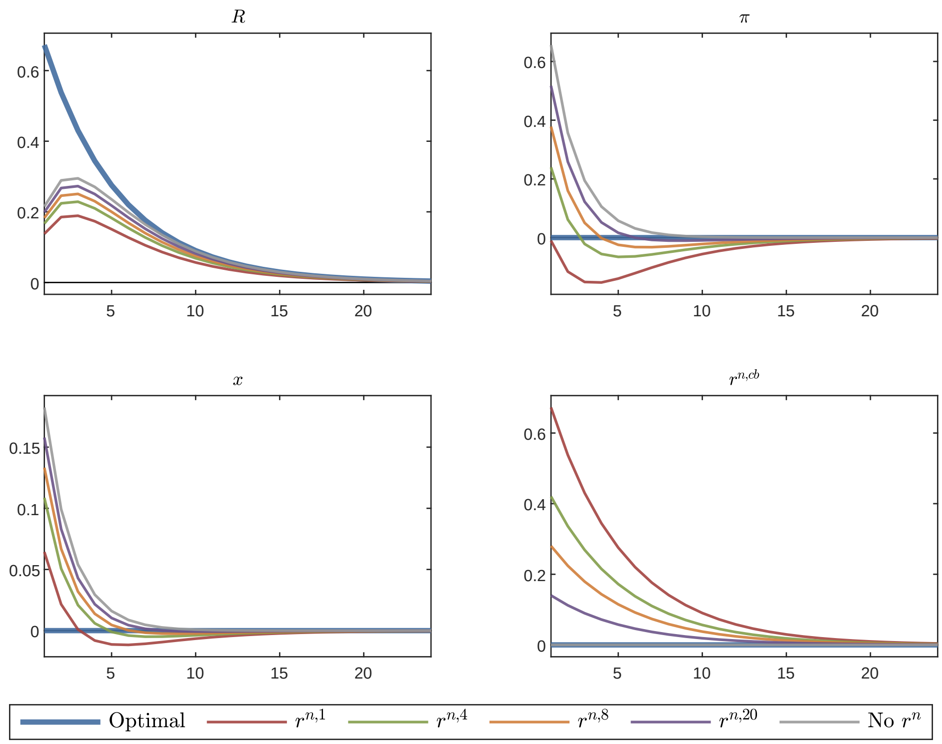

Figure A1: Responses to a productivity shock for alternative policy rules, in the case in which the policy rule has interest rate smoothing.