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Real Effects of Nominal Interest Rates*

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Abstract

Nominal interest rates have real effects. Residential mortgages and other real world debt contracts require a sequence of constant nominal payments. Combined with payment-to-income constraints, these nominal payments force borrowers to take on less debt when nominal interest rates rise, regardless of the behavior of the real interest rate. Survey data shows that conditional on the real rate, higher nominal mortgage interest rates reduce home buying sentiment. And increases in nominal mortgage rates reduce mortgage origination more in cities where payment-to-income constraints are more likely to bind. We explore the macroeconomic implications of payment-to-income constraints in a new Keynesian model modified to include a credit good. The payment-to-income constraint amplifies the effect of current short-term nominal interest rates on output and inflation, making the model less forward-looking than the standard new Keynesian model.

*The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

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[§]John Leahy passed away while we were writing this paper. We miss his brilliance, his wisdom, and his perpetual good humor. We could not have written the paper without him; among his many other contributions, he developed the model in section 5.

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1 Introduction

In standard macroeconomic models nominal interest rates are irrelevant; what matters for economic activity are real interest rates, the nominal interest rate net of expected inflation (e.g., Woodford, 2003; Galí, 2015). Nominal rates only matter if, for instance at the zero lower bound, they determine real rates or if agents are subject to money illusion and confuse real and nominal rates. We argue that the real world is different: even absent money illusion or the zero lower bound, the structure of debt contracts means that nominal interest rates have real effects.

The importance of nominal interest rates is easy to overlook in the data since real and nominal rates often move together. It is even easier to overlook the importance of nominal rates in models since the single period real debt in standard models makes only the real rate relevant. But unlike in models, real world debt contracts often require a sequence of constant nominal payments across time. Multi-period nominal debt is characteristic of residential mortgages, car loans, and corporate bonds.

When debt is multi-period and nominal, one-for-one increases in nominal rates and inflation tilt the real payment profile, increasing real payments today and lowering them in the future. If a borrower faces payment-to-income constraints, limits on what share of her income she can pay today, a higher nominal rate conditional on a given real rate is contractionary. It reduces the size of a loan and thus the size of a purchase (e.g., of a house) that she can afford.

That the nominal interest rate has real effects almost directly implies that inflation does as well. In steady state, higher inflation leads to higher nominal interest rates. Outside of steady state, higher inflation often leads central banks to raise nominal interest rates. Our evidence on the real effects of nominal interest rates is thus also an argument that changes in inflation have larger real effects on the economy than standard macro models suggest. In particular, the interaction of higher nominal interest rates and payment-to-income constraints suggests that higher inflation imposes costs on new borrowers (MacGee and Yao, 2025a,b). Indeed, this may be one reason why inflation is so unpopular; the higher nominal interest rates that accompany inflation make it more difficult for households to borrow money to purchase homes, cars, and other durable goods.

In the next section, we expand on the intuition for how multi-period debt and payment-to-income constraints make it likely that the nominal rate will have real effects. We show that when payment-to-income constraints bind, it can be the nominal rather than the real rate that determines housing demand. Equipped with this intuition, in section 3 we turn to empirical evidence on the impact of nominal versus real interest rates in the Michigan Survey of Consumers. The survey evidence shows that it is nominal, not real, interest rates that drive home-buying sentiment. Indeed, conditional on the nominal rate, the real rate has the opposite from predicted effect, with a *lower* real rate depressing home buying sentiment.

In section 4, we use cross-sectional data on U.S. cities (core-based statistical areas, or CBSAs) to see whether the nominal rate effect works through payment-to-income constraints. While the mortgage interest rate does not vary across cities, its effects likely do. Payment-to-income constraints are more likely to bind in cities where mortgage loans are large relative to incomes. In these high leverage cities, increases in nominal interest rates may make a mortgage unaffordable for many households. A proxy for payment-to-income constraints is the average loan-to-income (LTI) ratio in a city, which we calculate from mortgage-level data. We then bin cities into terciles based on their average LTI ratio and regress the growth rate of loans in a city on the nominal mortgage rate and its interaction with LTI terciles. The real mortgage rate and its interaction with LTI terciles is a control variable.

We find that the coefficient on the interaction of the nominal mortgage rate with the highest LTI tercile is consistently negative and statistically significant; in cities where mortgage loans are larger relative to household incomes, an increase in the nominal interest rate leads to a larger reduction in mortgage issuance. The effect is economically significant with a one percentage point increase in the nominal mortgage rate reducing mortgage loan growth by roughly 4-5 percentage points more in cities in the highest LTI tercile relative to cities in the lowest LTI tercile. The size of the estimated nominal rate effect is stable as we add controls for house price growth and the federal funds rate (both interacted with LTI terciles) and city and year fixed effects. To the extent that new mortgage issuance is a good proxy for residential investment, these results suggest that the nominal rate has large real effects on the housing market.

In section 5, we use a new Keynesian model to study the macroeconomic implications of payment-to-income constraints. We modify a standard new Keynesian model to include a credit good subject to a payment-to-income constraint. In the model, as in the data, changes in nominal rates have real effects even when holding real interest rates fixed. The model also has four more surprising implications. First, the credit good amplifies the effect of the current *nominal* interest rate on output holding inflation fixed. The response is larger than in the standard model since the effect of the change in the real rate is the same and there is an additional effect from the change in the nominal rate. Second, the payment-to-income constraint makes the current nominal interest rate relatively more important for output and inflation than future nominal interest rates. The importance of the current relative to future nominal rates helps to address the forward guidance puzzle (Del Negro, Giannoni, and Patterson, 2023). Third, a higher nominal interest rate reduces labor supply through a wealth effect; but despite lower labor supply, consumption demand falls enough for inflation to decline when nominal rates rise. Fourth, the determinacy condition for the model remains the standard Taylor principle; determinacy still requires that the nominal interest rate rise more than one-for-one with inflation.

1.1 Related literature That the structure of mortgage contracts means that nominal rates have real effects was explored by macroeconomists during and in the aftermath of the 1970s Great Inflation. An early paper in this literature is Lessard and Modigliani (1975). They note that a higher nominal interest rate for a given real interest rate tilts the real payment profile, increasing real payments early in the life of a mortgage and decreasing those late in its life. Like us, although without econometric evidence, they argue that this mechanism means that a higher nominal interest rate reduces housing demand. Kearl (1979), Schwab (1982), and Schwab (1983) use models and data to make the same point. More recently, Haight (2008) discusses the logic of how nominal interest rates interact with payment-to-income constraints to affect the housing market. And MacGee and Yao (2025a,b) look at the long-run effects of steady-state inflation on the housing market. They argue that through its effect on nominal interest rates, higher inflation lowers mortgage debt, home ownership, and welfare. Their focus on the long-run impact of nominal interest rates on the housing market complements our focus on the short-run, business cycle, impacts of nominal versus real interest changes.

Most similar in intent though not method to our paper are Wilcox (1989) and Wilcox (1990). Wilcox argues that the nominal rather than the real rate determines aggregate demand. Wilcox (1990) concludes: "The evidence presented here suggests that nominal, as opposed to real interest rates are an important determinant of each category of consumer spending" (p. 36). Unlike our paper, however, Wilcox (1989) and Wilcox (1990) rely on macro time series to infer the importance of nominal versus real rates.

Recent work on housing and the macroeconomy shows the empirical relevance of payment-to-income constraints (e.g., Kuttner and Shim (2016), Bhutta and Ringo (2021), and Bosshardt, Di Maggio, Kakhbod, and Kermani (2024)). And Greenwald (2018) shows how changes in mortgage payment-to-income constraints matter in a macro model. Similarly, Drechsel (2023) looks at a model with income (earnings) based credit constraints on firms. None of these papers, however, link the importance of payment-to-income constraints to the effect of nominal versus real interest rate changes.

The most closely related papers to the theory part of our paper are Garriga, Kydland, and Šustek (2017, 2021) who look at long-term fixed nominal payment mortgages in macro models. Like us, Garriga et al. (2021) examine a new Keynesian model with mortgages. But unlike us, they emphasize the homeowner's loan-to-value rather than payment-to-income constraint. The distinction is crucial because nominal interest rates do not directly affect the loan-to-value ratio, whereas they do directly affect the payment-to-income ratio. Therefore Garriga et al. (2021) do not capture the effect of nominal interest rates on mortgage affordability.

2 Nominal debt contracts

Many real world loans, specifically fixed rate mortgages, have the following three features:

1. Debt is multi-period.

2. One repays debt with a constant nominal payment each period. A homebuyer who obtains a 30-year fixed-rate mortgage in July 2026, for instance, will make 360 identical nominal payments from July 2026 through June 2056.

3. Lenders resist making loans in which the ratio of the mortgage payment to current

income is high. Lenders' resistance may stem from regulatory constraints or from asymmetric information and the risk of default.

These features are familiar to consumers, or at least to homebuyers. But their combination is generally absent from macroeconomic models.

Multi-period nominal debt means that a higher nominal interest rate increases nominal mortgage payments and thus tightens the payment-to-income constraint. To see why, consider the following hypothetical: a household earns $100,000 in year t and wishes to buy as large a house as it can afford. Institutional constraints mean that a bank will not give the household a loan unless the payment in year t (the first year of the loan) plus other household debt payments and property taxes is less than 50 percent of the household's income in year t. Even absent any institutional constraints, many households would be uncomfortable or unable to commit more than 50 percent of their current gross income to paying their mortgage.

Suppose that expected inflation and the real interest rate are zero, so that the nominal interest rate is also zero. Then the annual mortgage payment on a 30-year fixed rate mortgage will be one-thirtieth of the mortgage principal. And the household will be able to take out a $0.5 \cdot \$100,000 \cdot 30 = \$1.5$ million mortgage. Now suppose instead that the real interest rate remains zero, but expected inflation and the nominal rate are 10 percent. In order to keep the mortgage payment in the first year of the mortgage less than $50,000, the household will only be able to borrow roughly $470,000.[1] Rising wages along with inflation will make the mortgage payment a smaller share of income over time, but that does not affect how the payment-to-income constraint binds at the time of mortgage origination.

It will thus be the nominal rate, not the real rate, that determines the consumer's house purchase size. Unless the real rate rises enough to reduce the consumer's optimal home purchase size below $470,000, changes to the real rate will be irrelevant to her housing demand. By contrast, changes to the nominal rate will affect how the payment-to-income constraint binds, and thus how much housing she demands. Figure 1 illustrates. It shows in time $t = 1$ (real) dollars the payment profile for a $1.5 million mortgage when the nominal

[1] The maximum size (principal) of a 30-year mortgage that can be borrowed for a nominal interest rate i and annual payment X is $\frac{X(1-[\frac{1}{1+i}]^{30})}{i}$.

interest rate is zero and inflation is zero, and when the nominal interest rate is 10 percent and inflation is 10 percent. When the nominal rate and inflation are zero, the real payment profile is flat at $50,000, half of the hypothetical household income of $100,000. As discussed above, the household can afford the mortgage. But when the nominal interest rate and inflation are 10 percent, the real payment profile is downward sloping, and the mortgage is unaffordable for the household. Thus even if the real interest rate and the real wage are constant, changes in inflation and the nominal rate will matter for housing affordability.

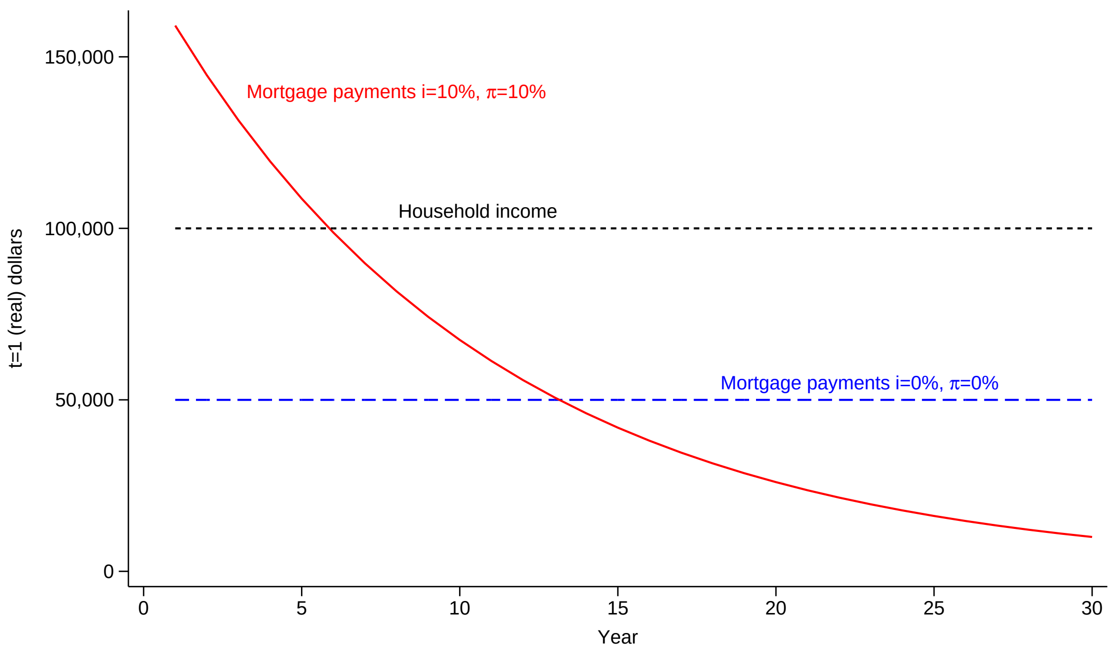

Figure 1 – Annual real mortgage payment on a $1.5 million mortgage compared to real household income

3 Nominal and real rates: Survey evidence

The above intuition suggests that the nominal rate is likely to influence consumer attitudes about homebuying. We test for such a nominal rate effect using data from the Michigan Survey of Consumers. The Michigan survey asks consumers about their inflation expectations and about whether they think it is a good time to buy a home.

Thus we estimate

$$\text{Home buy attitude}_{j,t} = \beta_0 + \beta_1 i_t + \beta_2 r_{j,t} + \varepsilon_{j,t}. \tag{1}$$

Home buy attitude is a dummy variable equal to 1 if the survey respondent says that it is a good time to buy a home.[2] It is measured at the level of a survey respondent j surveyed in month t. i_t is the actual nominal interest rate on a 30-year fixed rate mortgage (from FRED series MORTGAGE30US). $r_{j,t}$ is the respondent-specific real interest rate equal to i_t minus the respondent's expected annual inflation rate over the next five to ten years.[3]

Column (1) of table 1 shows results from estimating equation 1, controlling for a time trend. As expected, the coefficient on the nominal interest rate is negative and statistically significant. The coefficient of -0.11 means that for every percentage point increase in the nominal mortgage rate, a respondent is 11 percentage points less likely to say that it is a good time to buy a home. By contrast, conditional on the nominal interest rate, the coefficient on the real interest rate has the wrong sign: higher real interest rates make it *more* likely that a respondent says it is a good time to buy a home.

In a simple regression of home buying attitudes on the nominal or real interest rate, one would worry about omitted variable bias and reverse causality. Omitted variable bias would result, for instance, from an aggregate demand shock that leads to both lower interest rates and to decreased interest in buying homes. Reverse causality would occur if changing home buying sentiment affected the macroeconomy and thus interest rates. In our setting, omitted variable bias and reverse causality are less of a concern, since we look at the effect of the nominal interest rate conditional on the real interest rate. Thus for omitted variable bias or reverse causality to be a concern, there would need to be a story in which an omitted variable changes both home buying sentiment and the difference between the nominal and real interest rate, i.e. expected inflation. Reverse causality would only result if changes in homebuying sentiment caused changes in expected inflation.

To get some sense of whether omitted variable bias is a concern, in column (2) we add a control for the unemployment rate. The coefficients on the nominal and real interest rate are nearly unchanged, strongly suggesting that any omitted variable bias from macroeconomic shocks affecting both expected inflation and home buying sentiment is minimal.[4] It is less

[2]Respondents are asked: "Generally speaking, do you think now is a good time or a bad time to buy a home?" (Variable HOM.) We make the dummy variable equal to 1 if the response is "good" (HOM=1). See the "Survey Description" at https://data.sca.isr.umich.edu/survey-info.php.

[3]Michigan Survey variable PX5.

[4]Controlling for the unemployment rate in other specifications in table 1 also has almost no effect on the

Table 1 – Michigan survey regressions

	(1)	(2)	(3)	(4)	(5)
	Buy home	Buy home	Buy home	Low int. rate	Buy home
Right hand side variables:					
Mortgage rate (%)	−0.11***	−0.11***	−0.11***	−0.074***	
	(0.0023)	(0.0024)	(0.0059)	(0.0028)	
Real mortgage rate (%)	0.0039***	0.0039***	0.0051***	0.0031***	
	(0.00035)	(0.00035)	(0.00064)	(0.00045)	
Expected house price inflation (%)			0.0081***		0.0081***
			(0.00053)		(0.00054)
Expected 5-10 year inflation (%)					−0.0043***
					(0.00054)
Unemployment rate (%)		−0.0022			
		(0.0022)			
Time trend (monthly)	−0.0021***	−0.0021***	−0.0026***	−0.0015***	
	(0.000051)	(0.000058)	(0.000098)	(0.000074)	
Survey month fixed effects					X
R^2	0.18	0.18	0.25	0.10	0.29
Sample period	2/79-6/25	2/79-6/25	3/07-6/25	2/79-6/25	3/07-6/25
Observations	228,425	228,425	85,356	224,563	85,356

Notes: In columns (1), (2), (3), and (5) the dependent variable is a dummy variable for whether or not the respondent says that it is a good time to buy a home. In column (4), the dependent variable is a dummy variable for whether or not the respondent says that low interest rates are a reason why it is a good time to buy a home. Standard errors are clustered by survey month. $^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$
Since April 1990, the Michigan survey has asked respondents every month about their 5-10 year inflation expectations. Before that, the series has gaps with respondents asked about expected inflation only in some months and not at all in 1988-89.

straightforward to test for reverse causality, for whether changes in home buying sentiment might be causing changes in expected inflation rather than vice-versa. But we can use the fact that in March 2007 the Michigan survey started asking respondents about their expected change in house prices.[5] It is likely that changes in home buying sentiment would affect expected house prices more than they would affect expectations of all consumer prices. So adding expected house price inflation to the regression, as we do in column (3), helps to control for possible reverse causality between home buying sentiment and expected inflation. Adding this control has little effect on the coefficients on the nominal and real rate. And reassuringly the coefficient on house price inflation expectations is positive: higher house

coefficients of interest. Results are available upon request.

[5] Michigan Survey variable HOMPX5.

price inflation expectations are associated with more respondents saying it is a good time to buy a house.

In column (4) we take advantage of another question in the Michigan survey, in which respondents who say it is a good time to buy a house are asked why. The dependent variable in this regression is a dummy variable equal to 1 if the respondent says that it is a good time to buy a home because interest rates are low.[6] Respondents are not asked to specify whether they mean nominal or real interest rates. But the results in column (4) make clear that they mean the nominal rate: the coefficient on the nominal rate is negative while that on the real rate is positive.

In the final column, we look in more detail at what variation is driving our real rate results. We regress the dummy variable for whether or not a respondent says it is a good time to buy a home on expected consumer price inflation and expected house price inflation. We include date (survey month) fixed effects to isolate cross-sectional variation. (The date fixed effects mean we cannot include the nominal mortgage rate, since the nominal mortgage rate varies only across time, not survey respondent.) We obtain results similar to those in column (3); higher expectations of consumer price inflation are associated with worse home-buying sentiment, while higher expectations of house price inflation are associated with better home-buying sentiment.

The Michigan Survey results suggest that the nominal interest rate has real effects. Conditional on the real rate, higher nominal interest rates have large negative effects on home-buying sentiment. In the next section, we use mortgage origination data to better understand the mechanism and quantitative significance of the nominal rate effect on the housing market.

4 The mortgage market and the nominal rate

We would like to identify the effect of changes in nominal mortgage interest rates on mortgage origination. More precisely, we would like to know β_1 from:

$$\%\Delta \text{Mortgage origination}_t = \beta_0 + \beta_1 \Delta i_t^{\text{exog}} + \beta_2 \Delta r_t + \varepsilon_t, \tag{2}$$

[6]Our dummy variable equals 1 if Michigan Survey variable HOMRN1=16.

where Δi_t^{exog} are exogenous changes in the nominal interest rate, and Δr_t is the change in the real interest rate. Unfortunately, we know of no plausible identification strategy for separately identifying exogenous changes in the nominal and real interest rate. Monetary policy shocks are, for instance, changes to both the nominal and the real rate.

Therefore we turn to cross-sectional evidence. Interest rates do not vary within the U.S. but their effects do. Our hypothesis is that nominal interest rates matter for the volume of mortgage origination because they tighten mortgage payment to income (DTI) constraints.[7] DTI constraints can be either explicitly imposed by lenders, e.g. to conform with Fannie Mae and Freddie Mac guidelines, or implicitly adopted by borrowers, e.g. because borrowers have other consumption commitments or are risk averse. We use the fact that DTI constraints are more likely to bind in cities (core-based statistical areas, CBSAs) where the average mortgage loan size is large relative to average incomes.

DTI ratios are only available at the loan level since 2018, but they are proportional to the loan-to-income (LTI) ratio, the ratio of loan size to the borrower's income.[8] To build intuition for how LTI ratios and nominal interest rates interact and thus for our identification strategy, figures 2 and 3 show the distribution of LTI ratios in Houston and Los Angeles in 2021 and 2023. Over these two years, mortgage rates increased from an average of 3.0 percent in 2021 to 6.8 percent in 2023 (FRED series MORTGAGE30US). And in both cities, the distribution of LTI ratios shifted down.

The shift was not the same in the two cities, however. The distribution of LTI ratios is more right-skewed in Los Angeles than in Houston. We observe relatively few small loans and relatively more large loans in Los Angeles, reflecting high house prices there. And because payment-to-income ratios are higher when loans are larger, the increase in mortgage rates had a larger effect on the affordability of housing in Los Angeles than in Houston. This is reflected in the change in the number of loans issued in the two cities in 2023 compared to 2021. In Los Angeles, the number of loans fell 50 percent from 2021-23, while in Houston the number of loans fell 27 percent.

[7] The industry and literature call the ratio of mortgage payments to income the "debt to income constraint" or DTI. We adopt this convention going forward.

[8] If the DTI ratio includes only mortgage payments, they are proportional by a factor $i\frac{(1+i)^m}{(1+i)^m-1}$, where i is the monthly mortgage rate, and m is the duration of the mortgage in months.

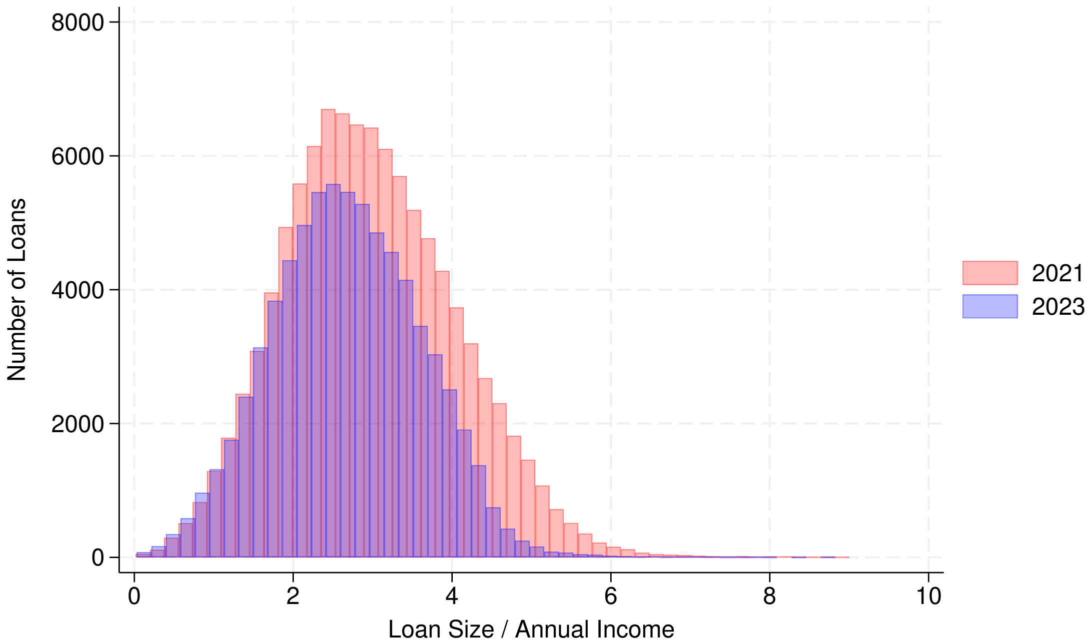

Figure 2 – Loan-to-Income (LTI) Distribution in Houston, 2021 and 2023. Source: HMDA data.

Figure 3 – Loan-to-Income (LTI) Distribution in L.A., 2021 and 2023. Source: HMDA data.

To see the intuition for this mechanism, suppose in Houston households with relatively expensive mortgages have loans of about \$500,000 and incomes of about \$125,000, for an LTI of four. If the nominal mortgage rate is three percent, the annual mortgage payment on a 30-year mortgage will be roughly \$25,000, for a DTI ratio of 20 percent. If the mortgage rate rises to 7 percent, the annual payment will rise to \$40,000, for a DTI ratio of 32 percent. This is well below the cut-off for the DTI ratio above which lenders will not approve mortgages, which is 45 or 50 percent. Even if the borrower has other monthly debt payments (e.g. for a car loan), their DTI may well be below the cut-off.

Now suppose in Los Angeles many borrowers get mortgages of \$1.2 million with incomes of \$200,000 for an LTI of six. The annual payment when the mortgage rate is three percent will be about \$61,000, for a DTI of 31 percent. Any increase in the mortgage rate to above 6.4 percent will push the DTI ratio above 45 percent, and borrowers will be unable to afford the loan. Thus we would expect increases in nominal rates to reduce mortgage issuance more in Los Angeles than in Houston, exactly as we see in figures 2 and 3.

The effect on mortgage issuance in Houston versus Los Angeles is a pure nominal rate effect. It is the nominal, not the real, rate that affects the ratio of payments to income. In the above example, we assume nothing about what happens to expected inflation when the nominal rate increases.

4.1 Specification We estimate

$$\begin{aligned}\%\Delta Loans_{jt} = \beta_0 \Delta i_t + \sum_{k=1}^{3} \gamma_k \text{Tercile } k\text{: LTI}_j + \sum_{k=1}^{3} \theta_k [\Delta i_t \times \text{Tercile } k\text{: LTI}_j] \\ + \delta_0 \Delta r_t + \sum_{k=1}^{3} \zeta_k [\Delta r_t \times \text{Tercile } k\text{: LTI}_j] + \varepsilon_{jt}.\end{aligned} \tag{3}$$

$\%\Delta Loans_{jt}$ is the percent change in the number of mortgages originated in CBSA j and year t; i_t and r_t are the nominal and real mortgage interest rate, which vary across time but not CBSA; and Tercile k: LTI_j is a dummy variable for the loan-to-income bin containing CBSA j. Bin 3 is the tercile of CBSAs with the highest average LTI. Los Angeles, for instance, is in bin 3, and Houston is in bin 1. Our hypothesis is that θ_3 will be negative: conditional on the real interest rate, an increase in the nominal interest rate will reduce mortgage issuance more in places where LTI ratios are highest.

LTI is highly correlated with payment-to-income (DTI) constraints, which our hypothesis suggests are the channel through which nominal rates impact the housing market. Figure 4 is a binscatter showing the correlation between average payment-to-income (DTI) and loan-to-income (LTI) ratios in 2019 across cities (CBSAs). There is a high correlation between DTI ratios and LTI ratios. Figure 5 shows that the average LTI in a city is strongly correlated with the fraction of loans with a high LTI, an LTI above 5.

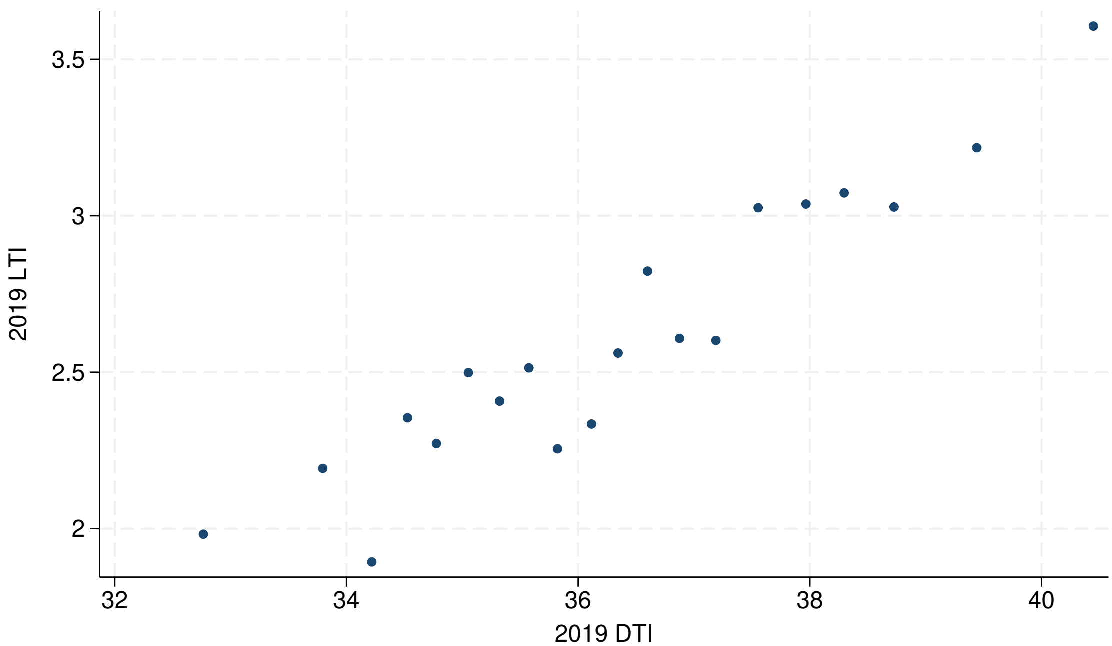

Figure 4 – A binscatter of the distribution of debt-to-income (DTI) and loan-to-income (LTI) ratios in 2019 across CBSAs. Source: HMDA data and authors' calculations.

Note that an advantage of our identification strategy is that it tests both for effects of the nominal rate conditional on the real rate and specifically for our hypothesized channel through which the nominal rate matters, payment-to-income constraints. Whereas the Michigan Survey results are consistent with the importance of payment-to-income constraints or money illusion, our identification strategy in this section rules out money illusion effects: there is no reason why money illusion would have larger effects in cities where payment-to-income constraints bind more.

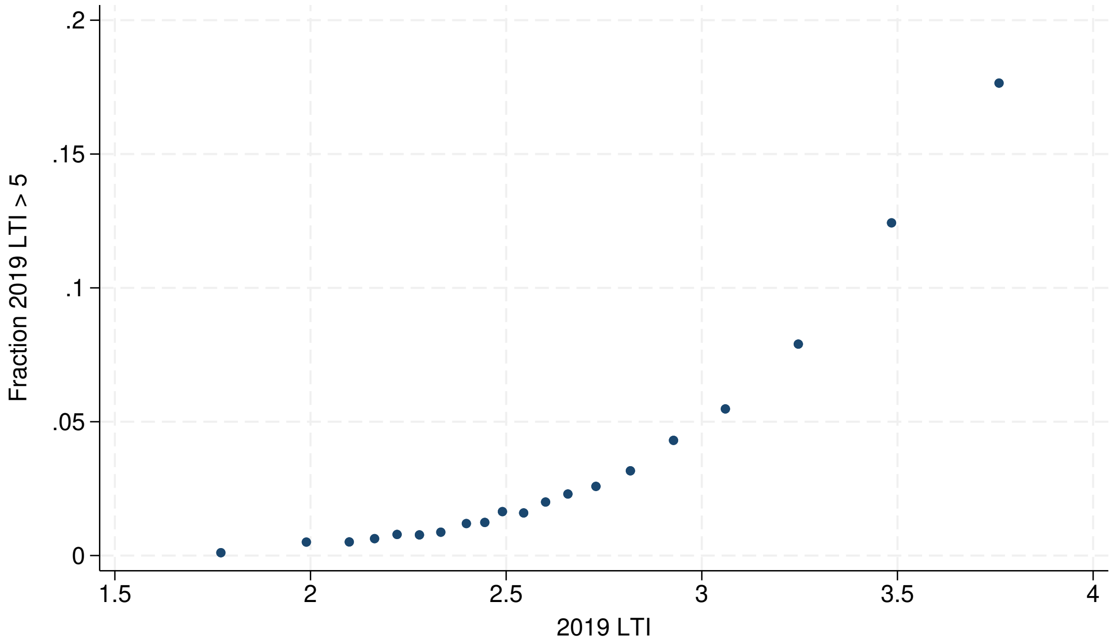

Figure 5 – A binscatter of the average loan-to-income (LTI) ratio and the fraction of mortgage loans with an LTI above 5 in 2019 across CBSAs. Source: HMDA data and authors' calculations.

4.2 Data We measure LTI and mortgage origination across CBSAs from confidential home mortgage disclosure act (cHMDA) data. These data contain application-level characteristics for the majority of originated loans. These are the same data underlying public HMDA data; but while the public data are only annual, we use the confidential data to construct a quarterly measure of the number of originated loans. We start our sample in 1995, when there was sufficiently stable coverage following a significant expansion to the scope and coverage of HMDA reporting (Avery, Brevoort, and Canner, 2007). Since 1995, these data are generally thought to cover at least 80 percent, often more, of the mortgage market. We restrict the sample to purchase loans for owner-occupied / principal occupancy homes, which include both detached and attached single-family homes. Starting in 2004 when the requisite codes become available, we restrict the data to first liens.[9] We then aggregate the data to the CBSA and quarter level to construct our measure of the count of mortgage purchase

[9] We also drop all loans with negative reported application income or those with loan-to-income ratios greater than seven.

originations.[10] To improve precision we also winsorize the top and bottom 2.5 percentiles of the log growth in loan count, and we drop the 25 percent of CBSAs with the smallest number of loans. These are CBSAs that typically have fewer than 30 loans in a quarter.

We use the national 30-year fixed mortgage rate as our nominal interest rate. We construct real mortgage rates using inflation expectations from several sources. From the Michigan Survey we take the 5-10 year mean and the 5-10 year median expected inflation rate.[11] We also use the TIPS 5-year breakeven inflation rate and the Cleveland Fed 5-year expected inflation rate.

Table 2 reports summary statistics for these data. The typical year-over-year growth in purchase loan counts is about one percent but with significant volatility. The statistics on the real rate show important variation across the measures of expected inflation with the survey-based measures generally showing trend declines in the real rate that are less apparent in the other two expected inflation measures.

	Observations	Mean	SD	25th Pct	50th Pct	75th Pct
$100 \times \Delta \ln$ Loan Count (Winsorized)	78,848	0.80	21.65	-12.73	1.97	14.84
Δ Mortgage Rate	78,848	-0.04	0.84	-0.64	-0.17	0.41
$100 \times \Delta \ln$ HPI	78,848	4.74	6.26	2.32	5.15	8.54
Δ Fed Funds	78,848	-0.03	1.47	-0.49	0.01	0.68
Δ Real Rate (Michigan Mean)	78,848	-0.04	0.78	-0.54	-0.11	0.36
Δ Real Rate (Michigan Median)	78,848	-0.03	0.82	-0.51	-0.20	0.36
Δ Real Rate (Cleveland)	78,848	-0.00	0.61	-0.40	-0.13	0.23
Δ Real Rate (Breakeven)	56,320	0.02	1.07	-0.60	-0.14	0.33

Table 2 – This table reports summary statistics for variables in our CBSA-quarter level analysis. All variables are in percentage point terms. We include data from 1995Q1 to 2023Q4 and drop the CBSAs in the bottom 25th percentile of average loan count. All differences are four-quarter differences. We winsorize log loan count at the top and bottom 2.5th percentiles. The loan counts come from HMDA. House prices are from the S&P/Case-Shiller U.S. National Home Price Index (FRED series CSUSHPINSA). The Fed Funds rate is the effective federal funds rate (FRED series DFF). The mortgage rate is the 30-year fixed rate mortgage average from Freddie Mac (FRED series MORTGAGE30US). The Michigan mean and median real rates are calculated with inflation expectations from the Michigan Survey of Consumers. The Cleveland real rate uses the model-implied 5-year expected inflation rate from the Cleveland Fed (FRED series EXPINF5YR). The breakeven real rate uses the 5-year breakeven inflation rate (FRED series T5YIE).

4.3 Results Table 3 shows estimates of equation (3) with inflation expectations measured using mean 5-10 year expectations from the Michigan Survey. Column 1 shows results using

[10] CBSAs are constructed from combinations of counties in order to reflect geographic units that share a high degree of economic and social integration. See here.

[11] Survey respondents are asked about their expectations of inflation "during the next five to ten years" (https://data.sca.isr.umich.edu/fetchdoc.php?docid=75445).

only the real interest rate as a control. Columns 2 and 3 add control variables for house prices (HPI) and the Fed Funds rate. In column 4, we add CBSA and date fixed effects. The coefficient of interest is that on the interaction of the high LTI bin with the nominal interest rate (θ_3 in equation (3)). The coefficient is negative, statistically significant, and stable across specifications. The magnitude of roughly -0.04 means that a one percentage point increase in the nominal interest rate reduces mortgage issuance by 4 percentage points more in the tercile of CBSAs with the highest LTI ratio relative to the tercile of CBSAs with the lowest LTI ratio. This is a large effect. It implies that the four percentage point increase in the mortgage rate from 2020 to 2022 reduced mortgage issuance by 16 percentage points more in high LTI CBSAs than in low LTI CBSAs. We also see evidence that mortgage issuance falls more in the middle LTI tercile than in the lowest LTI tercile, with the coefficient averaging -0.03 across columns.

In contrast, the coefficient on the interaction of the real interest rate with the highest LTI tercile, ζ_3, is positive and statistically significant. This implies that conditional on the nominal interest rate, higher real interest rates lead to *more* mortgage issuance in high LTI CBSAs, the opposite sign from that predicted by standard models. In the middle LTI tercile, the coefficient is again positive, though smaller.

Table 4 repeats the exercise using alternative inflation expectation measures to construct the real mortgage rate. As in column (4) of table 3, we include all controls. While the quantitative magnitude of the coefficient on the interaction of the high LTI bin and the nominal interest rate changes, the qualitative effect remains. There is again no evidence of a negative effect of the real interest rate interacted with the high LTI bin.

Table 3 – Δ ln Loan Count (winsorized)

	(1)	(2)	(3)	(4)
Δ Mortgage Rate	-0.044***	-0.139***	-0.157***	
	(0.004)	(0.004)	(0.005)	
Mid LTI x Δ Mortgage Rate	-0.024***	-0.030***	-0.029***	-0.029***
	(0.005)	(0.006)	(0.006)	(0.006)
High LTI x Δ Mortgage Rate	-0.055***	-0.055***	-0.039***	-0.039***
	(0.006)	(0.006)	(0.007)	(0.007)
Δ Real Rate (Michigan Mean)	0.023***	0.102***	0.107***	
	(0.004)	(0.004)	(0.004)	
Mid LTI x Δ Real Rate (Michigan Mean)	0.012**	0.017***	0.017***	0.017***
	(0.006)	(0.006)	(0.006)	(0.006)
High LTI x Δ Real Rate (Michigan Mean)	0.029***	0.029***	0.024***	0.024***
	(0.006)	(0.007)	(0.007)	(0.007)
%Δ HPI		1.250***	1.201***	
		(0.019)	(0.019)	
Mid LTI x %Δ HPI		0.074***	0.077***	0.077***
		(0.028)	(0.027)	(0.027)
High LTI x %Δ HPI		-0.002	0.042	0.042
		(0.032)	(0.031)	(0.031)
Δ Fed Funds			0.014***	
			(0.001)	
Mid LTI x Δ Fed Funds			-0.001	-0.001
			(0.001)	(0.001)
High LTI x Δ Fed Funds			-0.013***	-0.013***
			(0.002)	(0.002)
CBSA FEs				X
Date FEs				X
CBSAs	704	704	704	704
R2	0.03	0.14	0.14	0.55
Number of Observations	78,848	78,848	78,848	78,848

The outcome is the 4-quarter log-change in the number of loans, winsorized at the top and bottom 2.5th percentiles. Column 4 controls for quarterly date and CBSA fixed effects. The sample is 1996q1-2023q4. Standard errors are clustered at the CBSA level.

Table 4 – Δ ln Loan Count (winsorized)

	(1)	(2)	(3)
Mid LTI x Δ Mortgage Rate	-0.056***	-0.011*	-0.010**
	(0.012)	(0.006)	(0.004)
High LTI x Δ Mortgage Rate	-0.095***	-0.018***	-0.019***
	(0.013)	(0.006)	(0.004)
Mid LTI x Δ Real Rate (Michigan Median)	0.044***		
	(0.012)		
High LTI x Δ Real Rate (Michigan Median)	0.081***		
	(0.013)		
Mid LTI x Δ Real Rate (Cleveland)		-0.004	
		(0.007)	
High LTI x Δ Real Rate (Cleveland)		0.002	
		(0.007)	
Mid LTI x Δ Real Rate (Breakeven)			-0.005*
			(0.003)
High LTI x Δ Real Rate (Breakeven)			0.003
			(0.003)
Mid LTI x %Δ HPI	0.096***	0.052*	0.026
	(0.029)	(0.027)	(0.036)
High LTI x %Δ HPI	0.083**	0.016	-0.018
	(0.032)	(0.030)	(0.036)
Mid LTI x Δ Fed Funds	-0.002	-0.001	0.001
	(0.001)	(0.001)	(0.002)
High LTI x Δ Fed Funds	-0.014***	-0.013***	-0.012***
	(0.002)	(0.002)	(0.002)
CBSA FEs	X	X	X
Date FEs	X	X	X
CBSAs	704	704	704
R2	0.55	0.55	0.55
Number of Observations	78,848	78,848	56,320

The outcome is the 4-quarter log-change in the number of loans, winsorized at the top and bottom 2.5th percentiles. All columns control for quarterly date and CBSA fixed effects. The sample is 1996q1-2023q4. Standard errors are clustered at the CBSA level.

5 Macroeconomic Implications of Payment-to-Income Constraints

Our empirical results document the relevance of payment-to-income constraints and thus of nominal interest rates. In this section, we show the macroeconomic implications of such constraints in a simple model. The model is as close as possible to Galí (2015) and is in the spirit of Lucas and Stokey (1987) with a cash good and a credit good. Purchases of the credit good are subject to a payment-to-income constraint like that discussed in section 2.

We keep firm behavior simple by assuming that the two goods are produced using the same production function.

The model shows how a payment-to-income constraint leads to real effects of nominal rates. The model also shows that the payment-to-income constraint amplifies the effect of *real* interest rate shocks resulting from changes in nominal rates. Increases in nominal interest rates shift demand away from the credit good and toward the cash good. Diminishing marginal utility for the cash good then reduces labor supply and output. Perhaps surprisingly, despite the relevance of nominal rates for aggregate outcomes, the conditions for determinacy in the model are unaffected by the presence of the payment-to-income constraint.

5.1 Set-up We consider an economy with one cash good and one credit good. Consumers purchase the credit good today but pay for it tomorrow. We impose a borrowing constraint on the credit good to mimic the way in which nominal interest rates typically affect agents' ability to borrow with real world debt contracts.[12] The production function for both goods is the same, so their prices are the same. This simplification allows us to isolate the role of the payment-to-income constraint.

Time is discrete and indexed by t. The representative household derives utility from the consumption of two non-durable goods, the cash good indexed by n, and the credit good indexed by d. Let P^n and P^d denote the prices of the two goods and C^n and C^d their consumption. The agent chooses consumption of the two goods and labor supply to maximize the present value of utility

$$\sum_t \beta^t U_t \text{ with } U_t = \frac{(C_t^n)^{1-\sigma}}{1-\sigma} + \lambda \frac{(C_t^d)^{1-\sigma}}{1-\sigma} - \frac{1}{1+\gamma} N_t^{1+\gamma}.$$

$1/\sigma$ is the elasticity of intertemporal substitution, γ is the inverse Frisch elasticity of labor supply, and λ is a parameter that determines the relative importance of the cash and credit goods in utility.

[12]Similar constraints appear in the literature on firm borrowing. Drechsel (2023) analyzes the importance of current earnings as a constraint on firm borrowing. Chodorow-Reich and Falato (2022) document the importance of financial covenants in loan contracts, e.g. the interest coverage ratio, in limiting firms' access to credit during the Great Recession.

Purchases are subject to the budget constraint

$$P_t^n C_t^n + (1 + i_{t-1}) P_{t-1}^d C_{t-1}^d + B_t = W_t N_t + \Pi_t + (1 + i_{t-1}) B_{t-1}.$$

B_t is the purchase or issuance of nominal bonds at date t maturing at date $t + 1$.[13] i_t is the nominal rate of return between t and $t + 1$; W_t is the nominal wage, and Π_t is firm profits. What distinguishes the credit good (d) is that the consumer pays the price plus interest in the next period. We assume that these purchases are limited by the payment-to-income constraint:

$$(1 + i_t) P_t^d C_t^d \leq \theta P_t Y_t.$$

Here P is the aggregate price level (to be determined below), so PY is nominal income. θ is the share of income that a consumer can commit to spending on the credit good. It is strictly greater than zero and less than one, ensuring that the consumer can always purchase both the credit and cash good. The payment-to-income constraint depends on current income as in Drechsel (2023), and as is true in the U.S. mortgage market (see e.g., figures 2 and 3).[14] Firms use current income to judge whether or not a borrower is credit worthy; they do not attempt to forecast future income. θ captures the tightness of the payment-to-income constraint.

We assume that debt contracts are one-period nominal bonds. This assumption means that we do not need to keep track of debt as a separate state variable and keeps the model close to the standard new Keynesian framework. Nevertheless, the payment-to-income constraint captures the key feature of real world debt contracts: a higher nominal interest rate increases the payment obligations of borrowers and thus tightens their borrowing constraints.

Labor-leisure choice gives

$$\frac{W_t}{P_t^n} = (C_t^n)^\sigma N_t^\gamma.$$

The two consumption goods are produced from a continuum of intermediate goods by com-

[13] Normally bonds would be in zero net supply. In the model, however, consumers go into debt to purchase the credit (d) good. The debt is issued by producers of the credit good, and these producers are owned by the consumers. In equilibrium $B_t = P_t^d C_t^d$.

[14] Most existing work on mortgages in macro models focuses on other constraints. Iacoviello (2005), for instance, assumes that borrowing is limited by the expected present value of collateral, which in his case is the value of the house. Garriga et al. (2017, 2021) emphasize the importance of revaluation and cash-flow channels of existing long-term debt in response to monetary policy shocks.

petitive firms with access to constant returns to scale technology. Intermediate goods are indexed by $j \in [0,1]$. The production functions in the two sectors are

$$\begin{aligned} Y_t^n &= \left(\int_0^1 \left(y_{t,j}^n \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \text{ and} \\ Y_t^d &= \left(\int_0^1 \left(y_{t,j}^d \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

$y_{t,j}^n$ is the use of input j in the n sector, and $y_{t,j}^d$ is the use of the input in the d sector. Cost minimization yields

$$P_t = P_t^d = P_t^n = \left(\int_0^1 \left(P_{t,j} \right)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.$$

Since the sectors are competitive, symmetric, and constant returns to scale, the price of the cash and credit good are equal and equal to the aggregate price level.

Each of the intermediate goods producers has access to a constant returns to scale production technology:

$$Y_{t,j} = N_{t,j}.$$

The demand for each intermediate good is easily shown to be

$$y_{t,j} = \left(\frac{P_{t,j}}{P_t} \right)^{-\varepsilon} \left(Y_t^n + Y_t^d \right).$$

Intermediate goods producers face a Calvo pricing friction. They can only change their price with probability $(1-\alpha)$. A firm that adjusts its price at date t maximizes

$$\max_{P_{t,j}^*} \sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \left[\left(\frac{P_{t,j}^*}{P_{t+k}} \right) - \frac{W_{t+k}}{P_{t+k}} \right] \left(\frac{P_{t,j}^*}{P_{t+k}} \right)^{-\varepsilon} \left(Y_{t+k}^n + Y_{t+k}^d \right),$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor between t and $t+k$.

We close the model with equilibrium in the goods and labor markets,

$$\begin{aligned} Y_t^n &= C_t^n; \\ Y_t^d &= C_t^d; \\ \int_0^1 N_{t,j} dj &= N_t; \end{aligned}$$

and a monetary policy rule,

$$i_t = \phi \pi_t + \eta_t.$$

η_t is a monetary policy shock.

5.2 Solution

5.2.1 Firm behavior

Firm behavior is entirely conventional. Solving the intermediate firm's optimization problem, linearizing the model, and aggregating yields the standard new Keynesian Phillips curve

$$\pi_t = \kappa(w_t - p_t) + \beta E_t \pi_{t+1},$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$, and $w - p$ is marginal cost.

5.2.2 Consumer demand

Optimal choice of the n good leads to the linearized Euler equation

$$c_t^n = -\frac{1}{\sigma}(i_t - \pi_{t+1}) + c_{t+1}^n.$$

We want to express marginal cost and c^n in terms of the output gap, inflation and the interest rate. Consider first the definition of GDP:

$$Y_t = C_t^n + C_t^d.$$

Linearizing yields

$$\bar{Y} y_t = \bar{C}^n c_t^n + \bar{C}^d c_t^d,$$

where bars indicate steady state values. Linearizing the constraint on the purchase of the d good yields

$$i_t + c_t^d = y_t,$$

and substituting into the aggregate market clearing condition gives

$$\bar{Y} y_t = \bar{C}^n c_t^n + \bar{C}^d \left(y_t - i_t \right),$$

or

$$c_t^n = y_t + \frac{\bar{C}^d}{\bar{C}^n} i_t.$$

The equation says that if the nominal interest rate rises, then C^n must rise or Y must fall. How much they rise or fall depends on the relative size of the d sector.

We can now write the Euler equation in terms of the output gap, inflation, and the nominal interest rate:

$$c_t^n = -\frac{1}{\sigma}(i_t - \pi_{t+1}) + c_{t+1}^n$$

becomes

$$y_t + \frac{\bar{C}^d}{\bar{C}^n} i_t = -\frac{1}{\sigma}(i_t - \pi_{t+1}) + y_{t+1} + \frac{\bar{C}^d}{\bar{C}^n} i_{t+1};$$

or

$$y_t = -\frac{1}{\sigma}(i_t - \pi_{t+1}) + y_{t+1} - \frac{\bar{C}^d}{\bar{C}^n} \left(i_t - i_{t+1}\right). \tag{4}$$

Relative to the standard Euler equation, the d sector adds an additional term, a term that depends on the change in the nominal interest rate. To understand the intuition, we solve the equation forward:

$$y_t = -\frac{1}{\sigma} \lim_{T\to\infty} \sum_{k=0}^{T} \left[(i_{t+k} - \pi_{t+k+1})\right] - \frac{\bar{C}^d}{\bar{C}^n} i_t + \underbrace{\lim_{T\to\infty} y_{t+T}}_{=0}. \tag{5}$$

The first term is the long-term real interest rate, as in the standard Euler equation. The second term shows that higher nominal interest rates reduce current demand, as they tighten the payment-to-income constraint on the credit good. The magnitude of this effect depends on the size of the d sector relative to the n sector.

5.2.3 Forward Guidance

Equation 5 shows that aggregate demand is a function of current not future nominal interest rates. (It is a function of future real rates.) This is important because the degree to which demand is forward-looking in the standard model has been repeatedly criticized for its implications for forward-guidance (e.g., Del Negro et al., 2023; McKay, Nakamura, and Steinsson, 2016; McKay and Wieland, 2022, among others). That only the current nominal interest rate matters is a consequence of the one-period debt contract. With longer-term debt contracts, higher future nominal rates over the duration of the contract also increase current nominal payments, tighten the payment-to-income constraint, and reduce demand for the credit good.

In Appendix A, we derive the Euler equation with M-period debt contracts,

$$y_t = -\frac{1}{\sigma}\sum_{j=0}^{\infty} E_t\big(i_{t+j} - \pi_{t+j+1}\big) - \frac{\bar{C}^d}{\bar{C}^n}\sum_{s=0}^{M-1}\omega_s i_{t+s}, \tag{6}$$

where ω_s are positive weights that are decreasing in s. The payment-to-income constraint depends on the long-term mortgage rate ρ_t, which by the expectations hypothesis is equal to $\rho_t = \sum_{s=0}^{M-1}\omega_s i_{t+s}$. The expected one-year interest rate 29 years in the future affects 30-year mortgage rates less than the expected one-year interest rate one year in the future. Intuitively, future rates matter less for the long-term mortgage rate because of compound interest: the current nominal rate affects all future payments, whereas the $M-1$ period nominal rate only affects the last payment.

Another way to see the intuition is to consider the lender's profit maximization condition. The mortgage principal is paid back over time and lenders can invest these repayments and earn future short rates. In later years of the mortgage term, most of the mortgage principal has been paid back, and so the lender can earn future short rates on most of the mortgage. By contrast, early in the life of the mortgage, only a small amount of principal has been paid back, and thus changes in near term rates (like i_{t+1}) have little effect on what the lender earns from a mortgage. Thus increases in near-term short rates have larger effects on the ρ that a lender will need to charge to earn the same amount on a mortgage as investing in a succession of short-term bonds.[15]

[15] More formally, consider a lender choosing between lending X dollars in a long-term, fixed rate amortizing mortgage at rate ρ or purchasing a succession of one-year bonds paying interest rates $i_1, i_2, \ldots, i_M$. The latter strategy will yield $X(1+i_1)(1+i_2)\cdots(1+i_M)$ dollars. If the lender lends X dollars for a long-term mortgage, it receives a constant payment of interest and principal combined of C every year for M years. C is a function of the size of the mortgage (X) and the mortgage interest rate (ρ): $C(X,\rho) = [X\rho(1+\rho)^M]/[(1+\rho)^M - 1]$. So $\partial C/\partial X > 0$ and $\partial C/\partial \rho > 0$. The lender can reinvest the payments C at the prevailing short-run interest rate, so the payoff to giving the mortgage is $C(X,\rho)\prod_{t=2}^{M}(1+i_t) + C(X,\rho)\prod_{t=3}^{M}(1+i_t) + \cdots + C(X,\rho)(1+i_M) + C(X,\rho)$.

A profit maximizing lender will be indifferent between these two strategies, and will set ρ such that $X(1+i_1)(1+i_2)\cdots(1+i_M) = C(X,\rho)\prod_{t=2}^{M}(1+i_t) + C(X,\rho)\prod_{t=3}^{M}(1+i_t) + \cdots + C(X,\rho)(1+i_M) + C(X,\rho)$. The effect of current and future short-run rates (e.g., i_2 vs i_{M-1}) has a symmetric effect on the left-hand side. But an increase in current interest rates (e.g., i_2) increases the right-hand side less than an increase in future rates (e.g., i_{M-1}).

In short, payment-to-income constraints with long-term debt are a realistic way to make the New Keynesian model less forward-looking, and thus to address the forward guidance puzzle.

5.3 Labor supply We want to express marginal cost in terms of the output gap, inflation, and the interest rate. From labor-leisure choice

$$\frac{W_t}{P_t} = (C_t^n)^{\sigma} N_t^{\gamma}.$$

Linearizing

$$w_t - p_t = \sigma c_t^n + \gamma n_t.$$

Given prices, the share of each intermediate good in output is constant. Hence doubling the labor input will double the output of all intermediate goods. Aggregate output is proportional to labor input

$$y_t = n_t.$$

Using the above derivation of c_t^n, we therefore arrive at the modified new Keynesian Phillips curve:

$$\begin{aligned} \pi_t &= \kappa \left(\sigma c_t^n + \gamma n_t \right) + \beta \pi_{t+1}; \\ &= \kappa \left(\sigma + \gamma \right) y_t + \kappa \sigma \frac{\bar{C}^d}{\bar{C}^n} i_t + \beta \pi_{t+1}. \end{aligned}$$

Nominal interest rates now enter the Phillips curve directly. They do so because a higher nominal interest rate shifts demand from the credit good to the cash good, which reduces the marginal utility of consumption and thus labor supply through a positive wealth effect.

Nevertheless, higher nominal interest rates reduce both output and inflation. To see this, solve the Phillips curve forward:

$$\pi_t = \kappa \lim_{T \to \infty} \sum_{k=0}^{T} \beta^k \left[\left(\sigma + \gamma \right) y_{t+k} + \sigma \frac{\bar{C}^d}{\bar{C}^n} i_{t+k} \right].$$

and then substitute in the expression for y_t derived above. The result is

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \left[(\sigma+\gamma) \left(-\frac{1}{\sigma} \sum_{j=0}^{\infty} [(i_{t+k+j} - \pi_{t+k+j+1})] - \frac{\bar{C}^d}{\bar{C}^n} i_{t+k} \right) + \sigma \frac{\bar{C}^d}{\bar{C}^n} i_{t+k} \right]$$

$$= \kappa \sum_{k=0}^{\infty} \beta^k \left[-\frac{\sigma+\gamma}{\sigma} \sum_{j=0}^{\infty} [(i_{t+k+j} - \pi_{t+k+j+1})] - \gamma \frac{\bar{C}^d}{\bar{C}^n} i_{t+k} \right].$$

Thus, inflation is solely a function of the long-term real interest rate and the discounted path of nominal interest rates. Higher nominal interest rates always reduce inflation holding the real interest rate fixed. This is because lower output from higher nominal interest rates reduces marginal cost by more than the wealth effect increases marginal cost.

We therefore arrive at a modified three equation system

$$\begin{aligned} y_t &= -\frac{1}{\sigma}(i_t - \pi_{t+1}) + y_{t+1} - \frac{\bar{C}^d}{\bar{C}^n}(i_t - i_{t+1}); \\ \pi_t &= \kappa(\sigma+\gamma) y_t + \kappa\sigma \frac{\bar{C}^d}{\bar{C}^n} i_t + \beta E_t \pi_{t+1}; \\ i_t &= \phi \pi_t + \eta_t. \end{aligned}$$

In Appendix B we show that the determinacy condition for this system is identical to that in the standard new Keynesian model, $\phi > 1$. Intuitively, the presence of the payment-to-income constraint only affects the short-run dynamics of the model, not the long-run relationship between inflation, output, and the nominal interest rate.

5.4 Summary of macroeconomic implications of the payment-to-income constraint

Relative to the standard new Keynesian model, the presence of the payment-to-income constraint on the credit good has four novel implications:

1. The constraint on the credit good makes the elasticity of output to current nominal interest rates positive, thereby amplifying the impact of monetary policy on the economy. This is because the credit good does not affect the strength of the standard intertemporal substitution channel but does add a new channel through which nominal interest rates affect demand.

2. Output is less sensitive to future nominal interest rates than to current nominal interest rates. This aspect of the model helps to address the forward guidance puzzle.

3. The constraint on the credit good affects both supply and demand. On the supply side, higher interest rates shift demand to the cash good. Diminishing marginal utility of consumption of the cash good reduces labor supply. Nevertheless, conditional on the real interest rate, higher nominal interest rates still reduce output and inflation.

4. The determinacy condition for the model remains the standard Taylor principle.

6 Conclusion

The multi-period, fixed nominal payment structure of debt contracts means that the nominal interest rate almost certainly has real effects. In particular, higher nominal mortgage rates make it more difficult for households to meet institutional payment-to-income constraints, limiting how much they can spend on a house. We find support for this simple theory from two sources. First, in the Michigan Survey of Consumers, there is strong evidence that higher nominal — but not real — interest rates reduce housing demand. Second, and consistent with the importance of payment-to-income constraints, the nominal rate has larger impacts on mortgage originations in cities where loan-to-income ratios are higher.

In the standard new Keynesian model, there is no such nominal rate effect, since households do not face a payment-to-income constraint. We show that when the new Keynesian model is modified to include a (realistic) payment-to-income constraint, the nominal rate has real effects. As in the real world, higher nominal rates tighten the payment-to-income constraint, limiting how much consumers can spend on a credit good.

The effect of nominal interest rates on consumers' ability to purchase goods on credit may be a partial explanation for why consumers dislike inflation. Of course, the argument that high interest rates are unpopular is hardly novel. But standard models miss that nominal rates themselves have real effects, so consumers may be unhappy when nominal rates are high even if real rates are low. And inflation is almost inevitably accompanied by higher nominal interest rates. These higher nominal interest rates, while unmeasured in the CPI, make it more difficult for consumers to purchase houses, cars, and other durable goods. Consumers are likely to be unhappy about the decline in their purchasing power, and they may correctly blame inflation.

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A Extension With Long-Term Debt

This appendix derives the household Euler equation and the New Keynesian Phillips curve when the payment-to-income (PTI) constraint depends on a long-term mortgage rate ρ_t rather than the one-period nominal rate i_t.

A.1 Long-term mortgage rate as a function of short rates Assume a fixed-rate mortgage of maturity M periods. The mortgage rate ρ_t is set at time t such that the present value of a unit stream discounted at the mortgage rate equals the present value discounted at the sequence of one-period nominal rates:

$$\sum_{s=0}^{M-1} \frac{1}{\prod_{k=0}^{s}(1+\rho_t)} = \sum_{s=0}^{M-1} \frac{1}{\prod_{k=0}^{s}(1+i_{t+k})}. \tag{7}$$

Define

$$F(\rho) \equiv \sum_{s=0}^{M-1}(1+\rho)^{-(s+1)}, \tag{8}$$

$$G_t \equiv \sum_{s=0}^{M-1}\prod_{k=0}^{s}(1+i_{t+k})^{-1}. \tag{9}$$

Then (7) is

$$F(\rho_t) = G_t. \tag{10}$$

$F(\rho)$ is a finite geometric series:

$$\begin{aligned} F(\rho) &= (1+\rho)^{-1}\sum_{s=0}^{M-1}(1+\rho)^{-s} = (1+\rho)^{-1}\cdot\frac{1-(1+\rho)^{-M}}{1-(1+\rho)^{-1}} \\ &= (1+\rho)^{-1}\cdot\frac{1-(1+\rho)^{-M}}{\rho/(1+\rho)} = \frac{1-(1+\rho)^{-M}}{\rho}. \end{aligned} \tag{11}$$

A.2 First-order approximation of ρ_t in terms of $\{i_{t+s}\}$ Let the steady state be constant: $i_{t+s} = i$ for all s and $\rho = i$. We log-linearize/linearize around this steady state.

A first-order Taylor expansion of (10) around ($\rho = i$, $i_{t+s} = i$) gives

$$F'(i)\,(\rho_t - i) = \sum_{s=0}^{M-1} \frac{\partial G_t}{\partial i_{t+s}}\bigg|_{ss} (i_{t+s} - i). \tag{12}$$

Step 1: compute $F'(i)$. From (11),

$$F(\rho) = \frac{1 - (1+\rho)^{-M}}{\rho}.$$

Let $A(\rho) = 1 - (1+\rho)^{-M}$ so $A'(\rho) = M(1+\rho)^{-M-1}$. By the quotient rule,

$$F'(\rho) = \frac{A'(\rho)\rho - A(\rho)}{\rho^2} = \frac{M\rho(1+\rho)^{-M-1} - \left[1 - (1+\rho)^{-M}\right]}{\rho^2}. \tag{13}$$

Evaluate at $\rho = i$:

$$F'(i) = \frac{Mi(1+i)^{-M-1} - \left[1 - (1+i)^{-M}\right]}{i^2}. \tag{14}$$

Define the positive constant

$$\Gamma \;\equiv\; -F'(i) = \frac{\left[1 - (1+i)^{-M}\right] - Mi(1+i)^{-M-1}}{i^2} \;>\; 0. \tag{15}$$

Step 2: compute $\partial G_t/\partial i_{t+s}|_{ss}$. Write the m-th discount factor term as

$$Q_m \equiv \prod_{k=0}^{m} (1 + i_{t+k})^{-1}.$$

Then $G_t = \sum_{m=0}^{M-1} Q_m$. For a given s, i_{t+s} affects Q_m only for $m \geq s$. For $m \geq s$,

$$\frac{\partial Q_m}{\partial i_{t+s}} = Q_m \cdot \frac{\partial}{\partial i_{t+s}}\big(-\log(1 + i_{t+s})\big) = -\frac{Q_m}{1 + i_{t+s}}.$$

Hence

$$\frac{\partial G_t}{\partial i_{t+s}} = -\sum_{m=s}^{M-1} \frac{Q_m}{1 + i_{t+s}}. \tag{16}$$

At the steady state, $Q_m = (1+i)^{-(m+1)}$ and $1 + i_{t+s} = 1 + i$, so

$$\frac{\partial G_t}{\partial i_{t+s}}\bigg|_{ss} = -\sum_{m=s}^{M-1} \frac{(1+i)^{-(m+1)}}{1+i} = -\sum_{m=s}^{M-1} (1+i)^{-(m+2)}. \tag{17}$$

Rewrite the sum (geometric series):

$$\sum_{m=s}^{M-1}(1+i)^{-(m+2)} = (1+i)^{-(s+2)} \sum_{u=0}^{M-1-s} (1+i)^{-u}$$
$$= (1+i)^{-(s+2)} \cdot \frac{1-(1+i)^{-(M-s)}}{1-(1+i)^{-1}}$$
$$= (1+i)^{-(s+2)} \cdot \frac{1-(1+i)^{-(M-s)}}{i/(1+i)}$$
$$= \frac{(1+i)^{-(s+1)}\left[1-(1+i)^{-(M-s)}\right]}{i}. \tag{18}$$

Therefore

$$\left.\frac{\partial G_t}{\partial i_{t+s}}\right|_{ss} = -\frac{(1+i)^{-(s+1)}\left[1-(1+i)^{-(M-s)}\right]}{i}. \tag{19}$$

Step 3: solve for ρ_t as a weighted sum of (expected) short rates. Plug (19) and $F'(i) = -\Gamma$ into (12):

$$(-\Gamma)(\rho_t - i) = \sum_{s=0}^{M-1}\left(-\frac{(1+i)^{-(s+1)}\left[1-(1+i)^{-(M-s)}\right]}{i}\right)(i_{t+s} - i). \tag{20}$$

Multiply by -1:

$$\Gamma(\rho_t - i) = \sum_{s=0}^{M-1} a_s\,(i_{t+s} - i), \qquad a_s \equiv \frac{(1+i)^{-(s+1)}\left[1-(1+i)^{-(M-s)}\right]}{i}. \tag{21}$$

Divide by Γ and define weights

$$\omega_s \equiv \frac{a_s}{\Gamma}. \tag{22}$$

Then

$$\rho_t - i = \sum_{s=0}^{M-1} \omega_s\,(i_{t+s} - i). \tag{23}$$

The weights satisfy $\omega_s > 0$ and decline with s,

$$\frac{\partial \omega_s}{\partial s} = \frac{1}{\Gamma}\frac{\partial a_s}{\partial s} = \frac{1}{i\Gamma}\frac{\partial(1+i)^{-(s+1)}}{\partial s} = -\frac{\log(1+i)}{i\Gamma}(1+i)^{-(s+1)} < 0. \tag{24}$$

In what follows we treat ρ_t and i_t as their log-linearized values, so (23) becomes

$$\rho_t = \sum_{s=0}^{M-1} \omega_s i_{t+s}. \tag{25}$$

This is the standard expectations hypothesis formula for the long-term interest rate as a weighted average of current and expected future short-term interest rates.

A.3 PTI constraint with long-term rate Replace the one-period PTI constraint with a constraint that depends on the long mortgage rate:

$$(1+\rho_t)P_t C_t^d \leq \theta P_t Y_t. \tag{26}$$

Assume it binds. Cancel P_t:

$$(1+\rho_t)C_t^d = \theta Y_t. \tag{27}$$

Linearize the binding condition around the steady state ($\rho = i$). As in the main text, we use the normalization that yields the compact form

$$\rho_t + c_t^d = y_t. \tag{28}$$

(If one keeps exact scaling, the term ρ_t becomes $\frac{1}{1+i}(\rho_t - i)$; this only rescales constants below.)

A.4 Goods-market clearing

$$Y_t = C_t^n + C_t^d. \tag{29}$$

Linearize:

$$\bar{Y}\, y_t = \bar{C}^n\, c_t^n + \bar{C}^d\, c_t^d. \tag{30}$$

Use (28) to eliminate c_t^d:

$$c_t^d = y_t - \rho_t. \tag{31}$$

Substitute (31) into (30):

$$\bar{Y}\, y_t = \bar{C}^n\, c_t^n + \bar{C}^d\,(y_t - \rho_t). \tag{32}$$

Use $\bar{Y} = \bar{C}^n + \bar{C}^d$ and rearrange:

$$\begin{aligned}(\bar{C}^n + \bar{C}^d)y_t &= \bar{C}^n c_t^n + \bar{C}^d y_t - \bar{C}^d \rho_t \\ \bar{C}^n y_t &= \bar{C}^n c_t^n - \bar{C}^d \rho_t \\ c_t^n &= y_t + \frac{\bar{C}^d}{\bar{C}^n}\rho_t = y_t + \chi\rho_t. \end{aligned} \tag{33}$$

A.5 Euler equation The optimality condition for C_t^n delivers the same linearized Euler equation as in the baseline:

$$c_t^n = -\frac{1}{\sigma}\big(i_t - \pi_{t+1}\big) + c_{t+1}^n. \tag{34}$$

Using (33) at t and $t+1$,

$$c_t^n = y_t + \chi\rho_t, \qquad c_{t+1}^n = y_{t+1} + \chi\rho_{t+1}.$$

Plug into (34):

$$y_t + \chi\rho_t = -\frac{1}{\sigma}\big(i_t - \pi_{t+1}\big) + \big(y_{t+1} + \chi\rho_{t+1}\big). \tag{35}$$

Rearrange:

$$y_t = -\frac{1}{\sigma}\big(i_t - \pi_{t+1}\big) + y_{t+1} - \chi\big(\rho_t - \rho_{t+1}\big). \tag{36}$$

Solving (36) forward gives

$$y_t = -\frac{1}{\sigma}\sum_{j=0}^{\infty} E_t\big(i_{t+j} - \pi_{t+j+1}\big) - \chi\sum_{j=0}^{\infty} E_t\big(\rho_{t+j} - \rho_{t+j+1}\big) \tag{37}$$

$$= -\frac{1}{\sigma}\sum_{j=0}^{\infty} E_t\big(i_{t+j} - \pi_{t+j+1}\big) - \chi\rho_t. \tag{38}$$

This is the Euler/IS equation with long-term PTI: relative to the standard IS curve, there is an additional term involving the change in the long mortgage rate. Relative to the one-period PTI Euler equation (4), the long-term mortgage rate replaces the one-period nominal rate.

We can express the Euler equation purely in terms of short rates by substituting (25),

$$y_t = -\frac{1}{\sigma}\sum_{j=0}^{\infty} E_t\big(i_{t+j} - \pi_{t+j+1}\big) - \chi\sum_{s=0}^{M-1} \omega_s i_{t+s}. \tag{39}$$

This makes explicit that current and expected future nominal short rates enter aggregate demand with declining weights.

A.6 Phillips curve with long-term PTI Labor supply (from the main text):

$$\frac{W_t}{P_t} = (C_t^n)^\sigma N_t^\gamma. \tag{40}$$

Linearize:

$$w_t - p_t = \sigma c_t^n + \gamma n_t. \tag{41}$$

With linear technology $Y_t = N_t$, aggregate output is proportional to labor input:

$$y_t = n_t. \tag{42}$$

Combine (41)–(42):

$$w_t - p_t = \sigma c_t^n + \gamma y_t. \tag{43}$$

Use (33) to eliminate c_t^n:

$$w_t - p_t = \sigma(y_t + \chi\rho_t) + \gamma y_t = (\sigma + \gamma)y_t + \sigma\chi\rho_t. \tag{44}$$

The Calvo price-setting block is unchanged, so the NKPC is

$$\pi_t = \kappa(w_t - p_t) + \beta\pi_{t+1}, \qquad \kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}. \tag{45}$$

Substitute (44) into (45):

$$\pi_t = \kappa(\sigma+\gamma)y_t + \kappa\sigma\chi\rho_t + \beta\pi_{t+1}. \tag{46}$$

Thus, the long-term mortgage rate enters inflation dynamics directly via its effect on the composition of demand and hence marginal utility and labor supply.

Using (25) to express ρ_t in terms of short rates, the NKPC can be written purely in terms of expected short rates:

$$\pi_t = \kappa(\sigma+\gamma)y_t + \kappa\sigma\chi\sum_{s=0}^{M-1}\omega_s i_{t+s} + \beta\pi_{t+1}. \tag{47}$$

B Determinacy Conditions

The system of equations is

$$\begin{aligned} y_t &= y_{t+1} - \frac{1}{\sigma}(i_t - \pi_{t+1}) - \chi(i_t - i_{t+1}),\\ \pi_t &= \kappa(\sigma+\gamma)y_t + \kappa\sigma\chi i_t + \beta\pi_{t+1},\\ i_t &= \phi\pi_t + \eta_t, \quad \text{where } \chi = \frac{\bar{C}^d}{\bar{C}^n}. \end{aligned}$$

Substituting the Taylor rule and using $i_{t+1} = \phi\pi_{t+1}$ (setting $\eta_t = 0$), the IS curve becomes

$$y_t = y_{t+1} - \Big(\frac{\phi}{\sigma} + \chi\phi\Big)\pi_t + \Big(\frac{1}{\sigma} + \chi\phi\Big)\pi_{t+1}. \tag{IS}$$

The Phillips curve is

$$\pi_t = \kappa(\sigma+\gamma)y_t + \kappa\sigma\chi\phi\pi_t + \beta\pi_{t+1}. \tag{PC}$$

Solving the Phillips curve for y_t and one step ahead yields

$$\begin{aligned} y_t &= \frac{\pi_t - \kappa\sigma\chi\phi\pi_t - \beta\pi_{t+1}}{\kappa(\sigma+\gamma)},\\ y_{t+1} &= \frac{\pi_{t+1} - \kappa\sigma\chi\phi\pi_{t+1} - \beta\pi_{t+2}}{\kappa(\sigma+\gamma)}. \end{aligned}$$

Substituting into the IS curve and multiplying by $\kappa(\sigma+\gamma)$ gives:

$$(1-\kappa\sigma\chi\phi)\pi_t - (\beta+1-\kappa\sigma\chi\phi)\pi_{t+1} + \beta\pi_{t+2} = -\kappa(\sigma+\gamma)\phi\Big(\frac{1}{\sigma}+\chi\Big)\pi_t + \kappa(\sigma+\gamma)\Big(\frac{1}{\sigma}+\chi\phi\Big)\pi_{t+1}.$$

Collecting terms yields

$$A_0\pi_t + A_1\pi_{t+1} + A_2\pi_{t+2} = 0, \tag{48}$$

where

$$
\begin{aligned}
A_0 &= 1 + \kappa\phi\Big(\frac{\sigma+\gamma}{\sigma} + \chi\gamma\Big), \\
A_1 &= -\beta - 1 - \kappa\frac{\sigma+\gamma}{\sigma} - \kappa\chi\gamma\phi, \\
A_2 &= \beta.
\end{aligned}
$$

The characteristic equation is

$$
A_2 r^2 + A_1 r + A_0 = 0. \tag{49}
$$

For determinacy, the two roots must lie outside the unit circle. The solution for the two roots is

$$
\begin{aligned}
r_{1,2} &= \frac{-A_1 \pm \sqrt{A_1^2 - 4A_0A_2}}{2A_2} \\
&= \frac{1}{2\beta}\Bigg[\beta + 1 + \kappa\frac{\sigma+\gamma}{\sigma} + \kappa\chi\gamma\phi \pm \sqrt{\Big(-\beta - 1 - \kappa\frac{\sigma+\gamma}{\sigma} - \kappa\chi\gamma\phi\Big)^2 - 4\beta\Big(1 + \kappa\phi\big(\frac{\sigma+\gamma}{\sigma} + \chi\gamma\big)\Big)}\Bigg].
\end{aligned}
$$

For the second root to lie outside the unit circle, we need $|r_2| > 1$. This requires

$$
\begin{aligned}
-\beta + 1 + \kappa\frac{\sigma+\gamma}{\sigma} + \kappa\chi\gamma\phi &> \sqrt{\Big(-\beta - 1 - \kappa\frac{\sigma+\gamma}{\sigma} - \kappa\chi\gamma\phi\Big)^2 - 4\beta\Big(1 + \kappa\phi\big(\frac{\sigma+\gamma}{\sigma} + \chi\gamma\big)\Big)} \\
(-\beta + 1 + \kappa\frac{\sigma+\gamma}{\sigma} + \kappa\chi\gamma\phi)^2 &> \Big(-\beta - 1 - \kappa\frac{\sigma+\gamma}{\sigma} - \kappa\chi\gamma\phi\Big)^2 - 4\beta\Big(1 + \kappa\phi\big(\frac{\sigma+\gamma}{\sigma} + \chi\gamma\big)\Big) \\
-4\beta(1 + \kappa\frac{\sigma+\gamma}{\sigma} + \kappa\chi\gamma\phi) &> -4\beta\Big(1 + \kappa\phi\big(\frac{\sigma+\gamma}{\sigma} + \chi\gamma\big)\Big) \\
\phi &> 1,
\end{aligned}
$$

which is the standard Taylor principle.